Unit 054 - Representing Fields

by Michael F. Goodchild
Department of Geography, University of California, Santa Barbara

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Advanced Organizer

Topics covered in this unit

- What is a field?
- What types of fields are there?
- What is the difference between fields and discrete entities?
- How are fields represented?

Intended learning outcomes

After reading this unit, you should be able to:

- define a field and cite examples
- determine whether a conceptualization satisfies the requirements of a field, a collection of discrete entities, or neither
- argue for and against field conceptualizations
- describe the common methods for representing fields in GIS
- explain the relationships between terminologies in GIS and other areas

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Metadata and Revision History

Unit 054 - Representing Fields

1. What is a field?

- a conceptual model of geographic variation
    - one of several such models
    - the differences between this and other models are conceptual, that is, they exist in the human mind
- a model of variation within a spatio-temporal frame
    - at every point in the frame there exists a single value of a variable
        - e.g. a field of temperature
        - e.g. a field of land surface elevation
        - e.g. a field of land ownership
    - the variable may be measured on any scale
        - temperature - degrees Celsius
        - elevation - meters above sea level
        - land ownership - name of the owner, parcel ID
    - for geographic information, the frame may be defined by:
        - two spatial dimensions (x,y)
        - three spatial dimensions (x,y,z)
        - spatial dimensions and time (e.g. x,y,t)
    - the field variable can be thought of as a function of these dimensions
        - e.g. z(x,y) might denote elevation as a function of two spatial dimensions
        - generally, z(**x**) is the value at the location defined by the vector **x**, which will have as many components as there are dimensions of the spatio-temporal frame
    - in GIS applications, x and y are usually interchangeable
        - they have similar characteristics
        - rotation is possible to new axes in the plane
    - however, z and t are not similar
        - it would make little sense to rotate to new axes that are oblique to t
        - z is often sampled at different density to x and y
        - t is expressed in different units of measurement
- a *vector field* has a vector (direction and magnitude, or possibly just direction) rather than a single value, at every point
    - a field having a single value is also known as a *scalar field*
    - examples of vector fields with geographic relevance include
        - gradient of the land surface (aspect and slope)
        - wind (direction and magnitude)
        - flow direction, perhaps also speed
    - a vector field could be represented by storing two linked fields
        - either one for direction, the other for magnitude
        - or one for the x component of the vector, the other for the y component
    - vector fields can be displayed using arrays of arrows

it is possible to think of geographic variation entirely in terms of fields
    - geography consists of a number of variables with single values everywhere on the Earth's surface
        - these variables keep recurring in GIS applications:
        - land elevation
        - population density
        - vegetation cover type
        - soil type
        - surficial geology
        - depth to water table
        - soil pH
        - mean annual precipitation
        - January mean rainfall
        - etc etc.
    - the plates in the front section of any atlas
- in general, it is impossible to achieve a perfect representation of a field
    - fields can be infinitely complex
        - it could take an infinite amount of information to represent a field perfectly
    - representations of fields must be approximate
        - the space available in a digital computer is always limited
    - often, representations capture only the coarser aspects of variation
        - the details or high-resolution elements are not captured
        - they constitute part of the uncertainty of the representation

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2. What types of fields are there?

- scalar and vector fields already covered

- types of fields based on scales of measurement
    - nominal
        - a measurement is nominal if the measured value has no meaning other than identification
        - a social security number serves only as identification
        - it has no significance as a number, does not establish quantity
        - adding, multiplying, dividing make no sense
        - a larger number indicates nothing, so social security numbers do not establish order
        - examples: name of owner, class of vegetation cover, class of land use, name of street
    - ordinal
        - a measurement is ordinal if its value establishes order
        - e.g. 2 might be between 1 and 3
        - e.g. yellow might be between red and green
        - ordinal information is common in studies of preferences, market research
    - interval
        - a measurement is interval if differences on the scale make sense

        - e.g. it makes sense to say that 80 degrees is 10 degrees hotter than 70 degrees
        - the difference between 80 and 70 is equal to the difference between 70 and 60
    - ratio
        - a measurement is ratio if division makes sense
        - a 200 kg person is twice as heavy as a 100 kg person
        - 1 km is ten times as far as 100m
        - but 80 degrees Celsius is not twice as hot as 40 degrees Celsius
        - to be ratio, a scale must have an absolute zero point
        - negative values are often not meaningful for ratio data
        - Kelvin temperature is ratio, but Celsius and Fahrenheit are only interval
- implications for GIS
    - it makes no sense to perform certain operations on certain kinds of variables
        - arithmetic on nominal data is meaningless
        - a nominal variable doesn't have a mean
        - instead, use the commonest or modal value
        - for ordinal data, use the median, not the mean
        - it makes no sense to divide a ratio variable by an ordinal one
    - GIS software won't protect the user from meaningless operations like these
        - software doesn't keep track of the type of a variable
- the strange case of cyclic data
    - e.g. aspect, measured as a direction from 0 to 360
        - consider averaging aspects of 5, 10, 355, 350
        - all are close to North
        - total and divide by 4, the result is 180
    - cyclic data is a special type, encountered in GIS:
        - aspect
        - flow direction
        - wind direction
    - best to use a vector field, even though there is no variation in magnitude
        - but many GIS do not support vector fields
    - be careful with cyclic variables in GIS
- the term *continuous*
    - a field is spatially continuous by definition
        - values exist everywhere
    - the term continuous' can also be applied to the variable
        - implying that it is measured on a continuous scale
        - all values of the variable are possible
        - between limits if any exist
        - in this sense, continuous' implies ordinal, interval, or ratio
        - a nominal variable can't have a continuous scale
        - a nominal variable must be measured on a discrete scale where only certain values are possible
- types of fields based on continuity properties
    - defined only for fields of continuous variables (ordinal, interval, ratio)
    - a field can be smooth or rugged
        - locally smooth or locally rugged

- cliffs may or may not be allowed
  - cliffs are zero-order' discontinuities
  - a field with no cliffs is zero-order continuous
  - mathematically, z(**x**+**dx**) - z(**x**) tends everywhere to zero as **dx** tends to zero if there are no cliffs
- sharp ridges and valleys and breaks of slope may or may not be allowed
  - breaks of slope are second-order discontinuities

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3. Fields and discrete entities

- the main competitor to the field conceptualization
  - geography consists of an otherwise empty space littered with discrete entities
  - as with fields, this is a question of conceptualization, not digital representation
  - a point can lie in any number of entities, including zero
  - entities can be points, lines, areas, or volumes in three or more dimensions
  - entities can have any number of characteristics (attributes) associated with them
    - the attributes apply to the entire entity
  - the discrete entity conceptualization is the subject of the unit on Representing Discrete Objects

3.1. Examples to clarify the dichotomy

3.1.1. Weather forecasting

- an example of the use of fields
- the processes operating in the atmosphere can be described by physical laws
- the include the Gas Law, the Navier Stokes equation governing the behavior of a viscous fluid, and others
- the laws are valid at the levels of resolution appropriate to the atmosphere, but break down at very detailed molecular scales
- these laws are written in terms of fields
  - many of them are partial differential equations governing the rates of change of field variables in time and space
  - the variables include pressure, temperature, wind
  - such variables can be defined at any point in the spatio-temporal frame
  - that is, they are fields
- these days, computer models are used to predict the behavior of the atmosphere
  - they must work with digital representations
  - important decisions must be made about the level of detail of the representation
  - is it sufficient to represent the atmosphere by sampling every 100km horizontally, every 100m vertically, every 1 hour in time, or is greater detail needed for accurate forecasting?
  - how will errors accumulate and limit the time horizon of the forecast?
  - today, forecasting is limited to about 5 days for reliable estimates
- the inputs to these models are representations of fields

    - as are the outputs
- however, weather forecasts for consumption by the general public translate these fields into more understandable terms
    - e.g. for pressure fields, highs and lows and fronts
    - forecasts may be described as apparent behaviors of these discrete entities
    - e.g., this front will stall
    - e.g., this high will weaken
    - scientific models may work with fields, but people may find discrete entities more acceptable, more easily understood
    - natural language provides much better ways of talking about discrete entities
    - it is comparatively difficult to describe a field
    - as Helen Couclelis writes, "People manipulate objects, but cultivate fields" (Couclelis, 1992)

3.1.2. Lakes in Minnesota

- the Minnesota license plate refers to 10,000 lakes
- who counted them? a student hired for a summer?
- what scale of map was used?
    - the result will depend on the scale
    - with more detailed maps, the total would surely be higher
- who defined a lake?
    - how to tell when one lake with a narrow part is really two?
    - how to tell when a swamp is a lake?
- the result will vary from one person to another
    - it is impossible to define lake' with sufficient accuracy to have one right answer
    - the task boils down to trying to count discrete geographic entities conceived as littering an otherwise empty space
- what is the equivalent field conceptualization?
- define a variable lakeness' with a single value everywhere in the state
    - L=0 for well-drained sandy soils that are never waterlogged
    - L=2 for areas that are swampy in Spring
    - L=5 for permanent swamp
    - L=8 for areas inundated except in very dry summers
    - L=10 for deep, permanent water
- this is a field, rigorously defined at every point
    - two people could agree on its value
    - maps of the field would be useful to others
- its general properties could be computed
    - its mean value over the state
    - the license plate might refer to mean lakeness
    - welcome to Minnesota, mean lakeness 2.8
    - 13% of Minnesota has lakeness 9 or higher

3.1.3. Benefits and disbenefits of fields

- what can we learn from these examples about the benefits, disbenefits of fields?

- fields can be well-defined
  - if the variable is well-defined
- they are the basis of much physical modeling
  - models of social systems using fields are rare
  - population density is a notable exception
  - a good reference is Angel and Hyman (1976)
  - social systems are mostly modeled as collections of discrete entities
- fields are hard to describe in natural language
  - as humans we prefer to reason, remember, describe our surroundings in terms of discrete entities, not continuously varying fields
  - a tourist headed for Minnesota is more likely to be attracted by the idea that Minnesota contains a large number of discrete lakes than to know what percentage of the state is covered by them, even though scientific rigor and rationality appear to favor the other side

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4. How are fields represented?

- there are many ways of representing fields
  - not all are implemented in GIS
  - different terminologies exist in different disciplines
  - this discussion begins with what is normal in GIS, discusses other disciplines at the end
- six major representations, with example uses in each case
  - this discussion deals mostly with two-dimensional frames
    - see later discussion for higher dimensionality
  - some of these six representations give values for the field at all points (they are *complete*)
    - some define the field only at certain points, require additional methods to make estimates elsewhere (they are *incomplete*)

4.1. Rectangular cells

- see Figure 1(a)
- value in each cell is an average, total, or some other aggregate property of the field within the cell
  - the representation defines a value everywhere, so is complete
  - however, all within-cell variation is lost
  - if necessary, it must be reconstructed by some method of intelligent guesswork
- e.g. remote sensing data and other kinds of digital imagery
- see the unit on Rasters for more on grids and cells

4.2. Rectangular grid of points

- see Figure 1(b)
- e.g. measurements of land surface elevation in a digital elevation model (DEM)

spacing of measurements is critical to accuracy of representation
    - all variation between sample points is lost
    - elevations at other points must be estimated by some method of intelligent guesswork (the representation is incomplete)

4.3. Irregularly spaced points

- see Figure 1(c)
- the field's value is defined at a set of sample points scattered in the frame
    - values of the field at other points must be interpolated
    - representation is incomplete
- e.g. weather data, available at scattered weather stations
- accuracy depends on the density of points
    - it is not clear what measure best defines accuracy - density per unit area, minimum distance between sample points, maximum distance

4.4. Digitized contours

- see Figure 1(d)
- the field is represented as a set of isolines, each connecting points of constant value
    - representation is incomplete
- the scale of measurement of the variable must be at least ordinal
    - isolines cannot be defined for nominal data
- each isoline is represented as a polyline
- e.g. data obtained from topographic maps
- accuracy depends on:
    - the number of contoured values, or the contour interval
    - the density of polyline points

4.5. Polygons

- see Figure 1(e)
- the frame is partitioned into irregular areas (volumes for 3 or more dimensions)
- value in each area is an average, total, or some other aggregate property of the field within the area
    - the representation is complete
    - all variation within areas is lost
- e.g. data obtained from maps of vegetation cover class, soil type
- the boundaries of areas are continuously curved lines
    - represented digitally as *polylines* - an ordered sequence of points connected by straight lines
    - the denser the points, the more accurate the polyline as a representation of a continuous curve
    - accuracy depends both on the size of polygons and on the density of polyline points
    - it is not clear what measure of polygon size - average, minimum - best defines accuracy

- every point in the frame lies in exactly one polygon
  - except for points on the boundaries
  - the polygons cannot overlap, must exhaust the frame
  - they are said to *tesselate* the space, they form an irregular *tesselation*
- see the unit on Polygon Coverages

4.6. Triangulated irregular networks (TINs)

- see Figure 1(f)
- the frame is covered with a mesh of irregular triangles
  - every point lies in exactly one triangle, or on a triangle edge
- the value of the field is known at every triangle vertex
  - within triangles and along edges it is assumed to vary linearly
  - the representation is complete
  - contours drawn across triangles will therefore always be straight and parallel
  - across triangle edges there will be breaks of slope, but not cliffs
  - contours will kink at edges
- the scale of measurement of the variable must be at least interval
  - variation within triangles cannot be defined for nominal or ordinal variables
- accuracy depends on:
  - how carefully the vertices were located on the surface
  - how well the planes defined within each triangle fit the actual surface
  - the sizes of triangles
  - but it is not clear what property of triangle size best defines accuracy - average, smallest, largest

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5. Other representations

- in modeling outside the context of GIS
  - grids and cells are often known as *finite difference methods*
  - *finite element methods* use irregular primitive elements
    - TINs are a form of finite element method
  - both are loosely described as grids
  - *adaptive grids* redefine the grid during execution of the model
    - e.g. after every step or iteration of a dynamic model
    - in GIS the representation normally remains constant
- TINs are known as *triangular meshes* in many areas of computer graphics
  - used for visualizing solid objects
  - the sharp breaks of slope in a TIN make the triangles visible to the eye, which may be undesirable
    - instead, the field within each triangle is sometimes described by a higher-order mathematical function that achieves first-order (no break of slope) continuity across triangle edges
- many of the concepts discussed above are valid for fields in frames with three or more dimensions
  - note, however, the earlier comments about the lack of equivalence of z and t with

x and y
  - grids and cells extend naturally to three and four dimensions, with appropriately defined sampling intervals
  - with a third spatial dimension, polygons become polyhedra with polygonal faces
    - e.g. in subsurface geology
    - see the unit on Representing time and storing temporal data for discussion of adding the time dimension
  - with a third spatial dimension, isolines become isosurfaces and are widely used in 3D GIS
    - it's more difficult to imagine the equivalent of isolines when time is added
  - irregular point samples can be taken in three or four dimensions
  - TINs in three spatial dimensions are widely used in computer graphics for visualizing solid objects as objects covered with triangular meshes
    - TINs don't extend as easily to the temporal dimension

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6. References

- Angel, S. and G.M. Hyman (1976) *Urban Fields: A Geometry of Movement for Regional Science*. London: Pion.
- Couclelis, H. (1992) People manipulate objects (but cultivate fields): beyond the raster-vector debate in GIS. In A.U. Frank, I. Campari, and U. Formentini (editors) *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*. Lecture Notes in Computer Science 639. Berlin: Springer-Verlag, pp. 65-77.

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7. Exam and discussion questions

1. Give other examples to illustrate the use of fields in scientific research, and discrete entities in human cognition and reasoning.

2. After studying this unit and unit 065, make and illustrate a list of the most viable methods for representing fields in two spatial dimensions and time.

3. "There appear to be no viable uses of digitized isolines" - discuss.

4. If you were asked to design a GIS to handle representations of vector fields, what functions would you want it to perform, and what applications could you find for it?

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8. Exercise

Exercise in Digital Elevation/Terrain Models: From Point to Mathematics

by Ahmad S. Massasati, United Arab Emirates University

This paper presents a practical way to teach about elevation models. These currently include solutions geographers refer to as digital elevation/terrain models such as point data, contour lines, triangular irregular networks, and mathematical models. The apparent complexity of data transfer in these methods, however, seems difficult to students and other first time users. In the author's classroom experience, the pyramids of Egypt have proved to be an excellent and efficient example for teaching digital elevation/terrain models.

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Unit 054 - Representing Fields

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Unit 054 - Representing fields

Metadata and Revision History

1. About the main contributors

- Michael F. Goodchild

2. Details about the file

- unit title
  - Representing fields
- unit key number
  - 054

3. Key words

4. Index words

5. Prerequisite units

6. Subsequent units

7. Other contributors to this unit

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- Created July 24, 1997
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Back to the Unit.

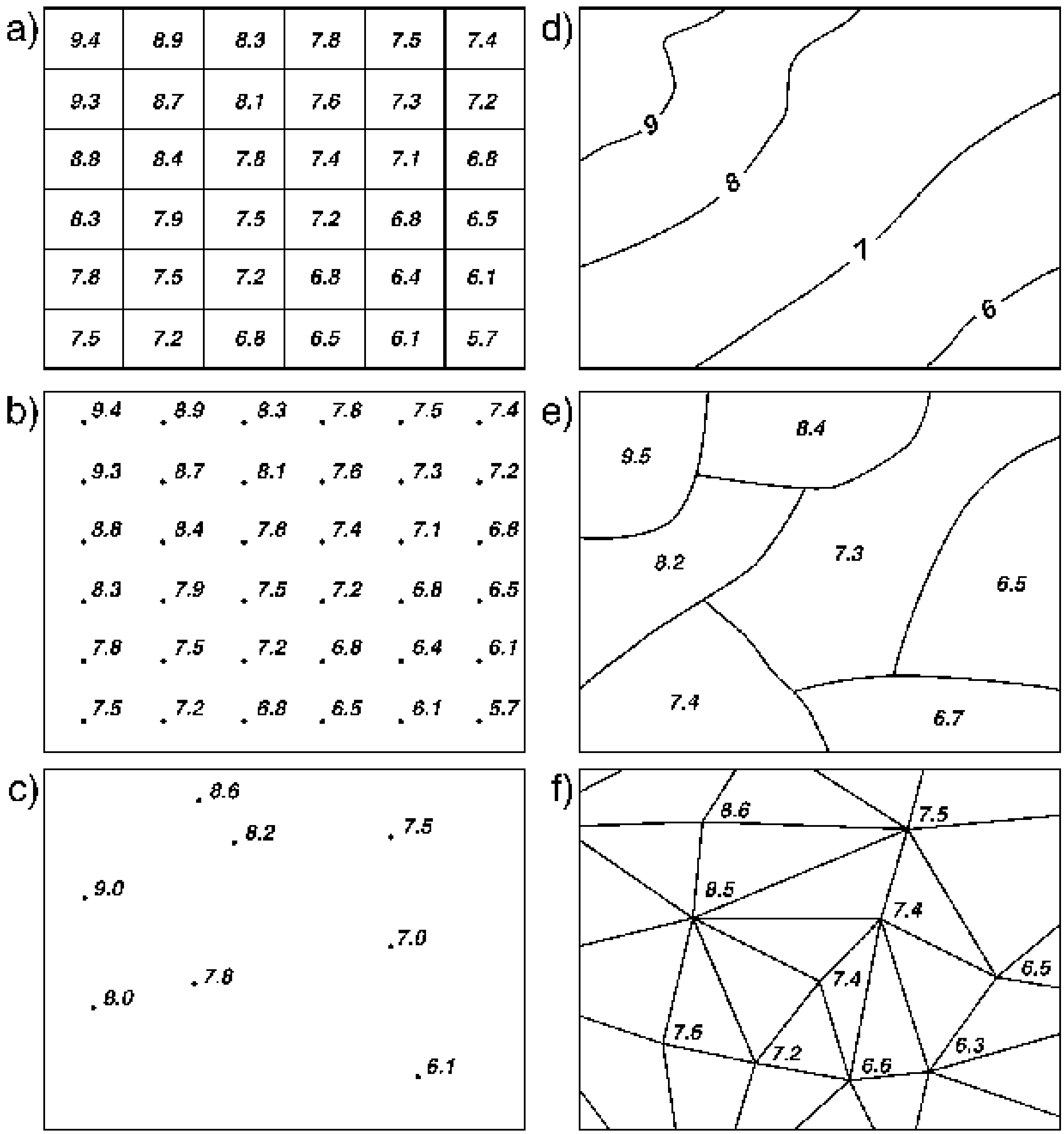

Figure 1 (a) rectangular cells; (b) rectangular grid of points; (c) irregularly spaced points; (d) digitized contours; (e) polygons; (f) triangulated irregular network (TIN).

An Exercise in Digital Elevation/Terrain Models: From Point to Mathematics

by Ahmad S. Massasati
United Arab Emirates University

Advanced Organizer

Abstract

This paper presents a practical way to teach about elevation models. These currently include solutions geographers refer to as digital elevation/terrain models such as point data, contour lines, triangular irregular networks, and mathematical models. The apparent complexity of data transfer in these methods, however, seems difficult to students and other first time users. In the author's classroom experience, the pyramids of Egypt have proved to be an excellent and efficient example for teaching digital elevation/terrain models.

Key words: geographic education, digital elevation models (DEM), digital terrain models (DTM), contour lines, triangular irregular network (TIN), geographic information systems (GIS).

Metadata and Revision History

An Exercise in Digital Elevation/Terrain Models: From Point to Mathematics

Introduction

Teaching elevation models in the classroom has always been a challenge. The difficulties come from the fact that data development is done with highly technical specialists and advanced computer technology while the use of the models is spread wide among different fields of sciences such as geology and geography.

Useful models for representing elevation data are point data, contour lines, triangular irregular network (TIN), digital elevation models (DEM) (Burrough, 1998) or digital terrain models (DTM) (Maguire, 1991), and mathematical models. A derivative of these models comes from traditional surveying practice and techniques in which a surveyor would collect the elevation of point data in the field, then transfer it to various methods of presentation on the map (Moffitt, 1982). Though these techniques are very precise, they are hard, require technical specialty, and are time consuming. The introduction of aerial photography made it possible to transfer the bulk of fieldwork to the office and derive elevation data using three-dimensional aerial photography. Recently the introduction of softcopy photogrammetry make it possible to derive elevation data with even more automation and less human involvement by using computers (Greve, 1996). For that, without hands on field experience, students of GIS might find it difficult to understand the variation between elevation models and how they compare to each other. The following exercise is designed to help understand these models, simplify the transformation from one to another, and clarify the differences among them.

The great pyramid of Khufu (Gizah, Egypt) (Fowler, 1999) is simple to imagine and calculate. The real dimensions of the pyramid are 755 feet. at the base with 481.4 feet height and a 52 degrees slope (Figure 1) (Edwards, 1993). Round these to 750 feet at the base and 450 feet height. The assumed orientation of the pyramid will be north-south and east-west. The coordinate of the southwest corner is zero east, zero north, and zero elevation.

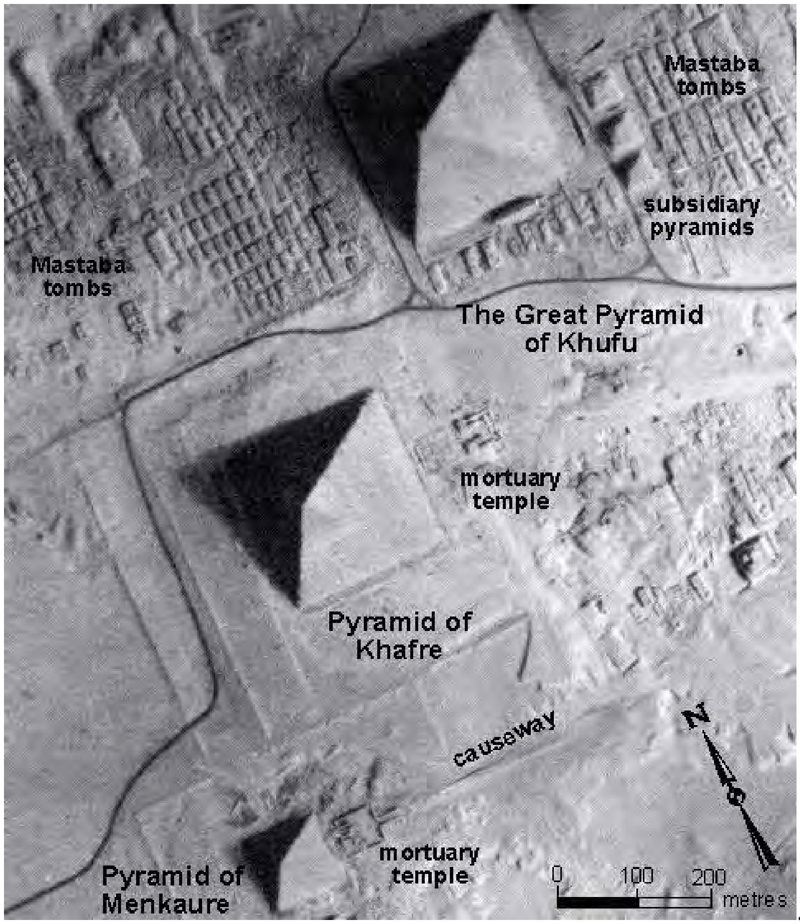

Figure 1. SPIN-2 Panchromatic digital image of Gizah 1997
(from Fowler, 1999. Image provided courtesy of SPIN-2 at Aerial Images Inc.)

The task will be to represent the pyramid in each of the five elevation models. These models are:

1. Point model.
2. Contour lines model.
3. TIN.
4. DEM/DTM.
5. Mathematical model.

1. The Point Model

The pyramid can be presented simply in five points, four at the base and one at the top (Figure 2). The x, y, and z coordinate of these points are;

```
A (0,0,0), B (0,750,0), C (750,750,0), D (750,0,0), and E (375,375,450).
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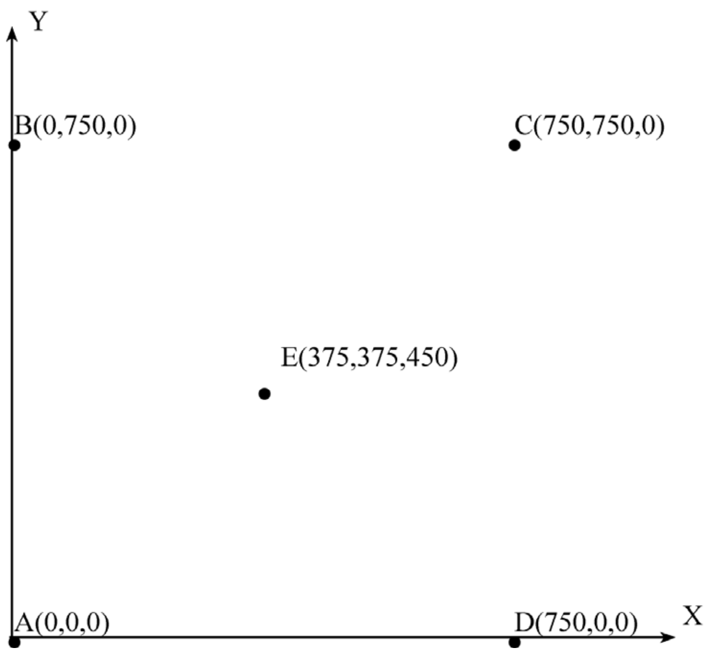

Figure 2. The point model for the pyramid of Khufu.

2. The Contour Line Model

Assuming a 50 meter interval, the four edges of the pyramid would be divided into 50 (450/50 = 9) resulting in ten contour line considering the top one as a point (Figure 3).

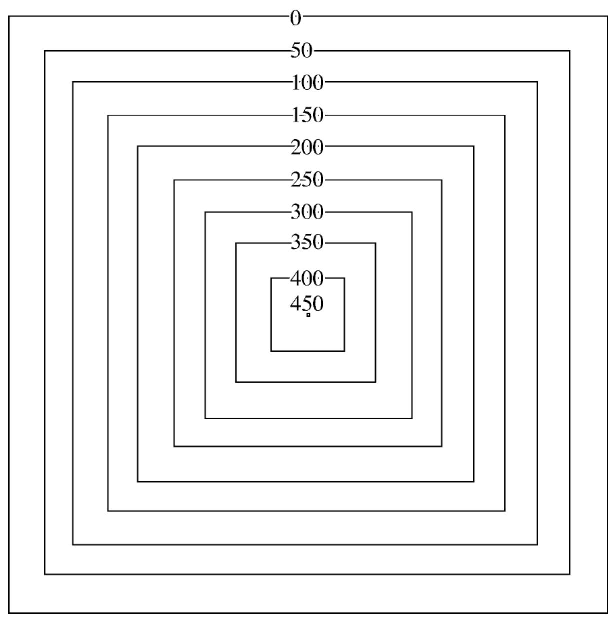

Figure 3. The contour lines for the pyramid of Khufu.

3. TIN

Standing on the top of the pyramid, the surface can be generalized and divided into four triangles (Figure 4). Each of the four triangles is identified with the coordinates of the three points forming it. It is important to note that any extra point on each of the four triangles is unnecessary to provide additional details of that surface.

```
Triangle one is A (0,0,0), B (0,750,0), and E (375,375,450).
Triangle two is B (0,750,0), C (750,750,0), and E (375,375,450).
Triangle three is C (750,750,0), D (750,0,0), and E (375,375,450).
Triangle four is D (750,0,0), A (0,0,0), and E (375,375,450).
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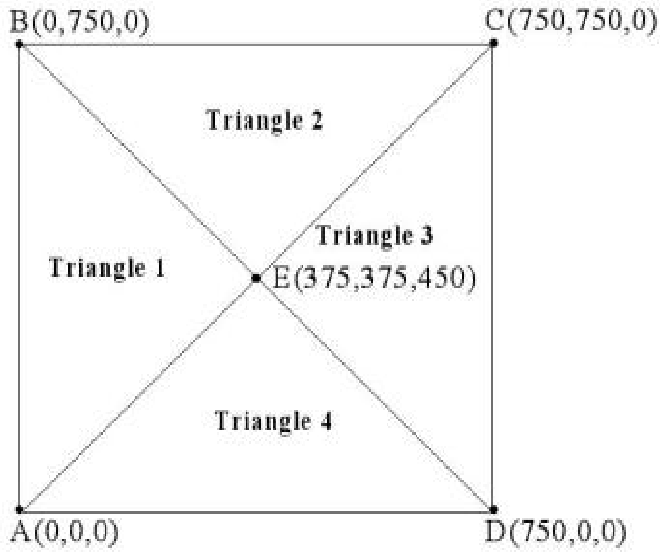

Figure 4. The TIN model for the pyramid of Khufu.

4. DEM/DTM

Assuming a 50x50 foot resolution, the elevation of the pixels at the bottom will have elevation ranges 0-50 feet and the single pixel at the top will have an elevation range from 400-450 feet (Figure 5). Stored in a computer file, the number of rows and columns are 17x17. If each pixel's elevation is set at the minimum elevation in each, the cell values of the file will vary between 0 and 400. Thus the minimum computer storage per file is 17x17x(9 bits per pixel) = 2601 bits of data. (Note: usually using an eight bit binary notion, you can express a number that varies between 0 and 255.) From a computer programming point of view, it is possible to reduce the file size using index value such as using 1 instead of 50, 2 instead of 100, and so on. This will allow the use of fewer bits per pixel therefore reducing the size of the computer file as well as processing time (Figure 6). For a small file, this saving might be minute, but DEM data is large and saving in file size is eventually significant (Burrough, 1998).

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	0
0	50	100	100	100	100	100	100	100	100	100	100	100	100	100	50	0
0	50	100	150	150	150	150	150	150	150	150	150	150	150	100	50	0
0	50	100	150	200	200	200	200	200	200	200	200	200	150	100	50	0
0	50	100	150	200	250	250	250	250	250	250	250	200	150	100	50	0
0	50	100	150	200	250	300	300	300	300	300	250	200	150	100	50	0
0	50	100	150	200	250	300	350	350	350	300	250	200	150	100	50	0
0	50	100	150	200	250	300	350	400	350	300	250	200	150	100	50	0
0	50	100	150	200	250	300	350	350	350	300	250	200	150	100	50	0
0	50	100	150	200	250	300	300	300	300	300	250	200	150	100	50	0
0	50	100	150	200	250	250	250	250	250	250	250	200	150	100	50	0
0	50	100	150	200	200	200	200	200	200	200	200	200	150	100	50	0
0	50	100	150	150	150	150	150	150	150	150	150	150	150	100	50	0
0	50	100	100	100	100	100	100	100	100	100	100	100	100	100	50	0
0	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5. The DEM for the pyramid of Khufu with actual values. (click for a large image)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	1	2	2	2	2	2	2	2	2	2	2	2	2	2	1	0
0	1	2	3	3	3	3	3	3	3	3	3	3	3	2	1	0
0	1	2	3	4	4	4	4	4	4	4	4	4	3	2	1	0
0	1	2	3	4	5	5	5	5	5	5	5	4	3	2	1	0
0	1	2	3	4	5	6	6	6	6	6	5	4	3	2	1	0
0	1	2	3	4	5	6	7	7	7	6	5	4	3	2	1	0
0	1	2	3	4	5	6	7	8	7	6	5	4	3	2	1	0
0	1	2	3	4	5	6	7	7	7	6	5	4	3	2	1	0
0	1	2	3	4	5	6	6	6	6	6	5	4	3	2	1	0
0	1	2	3	4	5	5	5	5	5	5	5	4	3	2	1	0
0	1	2	3	4	4	4	4	4	4	4	4	4	3	2	1	0
0	1	2	3	3	3	3	3	3	3	3	3	3	3	2	1	0
0	1	2	2	2	2	2	2	2	2	2	2	2	2	2	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Index Value

0 = 0 ft
1 = 50 ft
2 = 100 ft
3 = 150 ft
4 = 200 ft
5 = 250 ft
6 = 300 ft
7 = 350 ft
8 = 400 ft

Figure 6. The DEM for the pyramid of Khufu with index values.

5. The mathematical model

Each surface (triangle) can be presented with the following mathematical model

$Z = a X + b Y + c$

Where:
X, Y and Z are surface point coordinates and
a, b, and c are constants that can be determined for each surface by solving the equation using the three corner points (figure 7).

Calculation for surface 1;

$0 = a * 0 + b * 0 + c$ (1)
$0 = a * 750 + b * 0 + c$ (2)
$450 = a * 375 + b * 375 + c$ (3)

By solving the three equations, b = 1.2 and the equation is

$Z = 1.2 Y$
Where: $0 \le Y \le 375$, $0 \le Z \le 450$

Likewise, the equations for surfaces 2, 3, and 4 are:

$Z = 1.2 X$ Where: $0 \le X \le 375$, and $0 \le Z \le 450$
$Z = 1.2 (750-Y)$ Where: $375 \le Y \le 750$, and $0 \le Z \le 450$
$Z= 1.2 (750-X)$ Where: $375 \le X \le 750$, and $0 \le Z \le 450$

The equations for the line that borders the triangles can be obtained by simply solving for any two surfaces.

Line AE:
$Z = 1.2X=1.2Y$ or $X=Y$,
where: $0 \le X \le 375$, and $0 \le Y \le 375$

Line BE:
$Z= 1.2X=1.2(750-Y)$ or $X+Y=750$,
where: $0 \le X \le 375$, and $375 \le Y \le 750$

Line CE:
$Z= 1.2 (750-X)=1.2(750-Y)$ or $X=Y$,
where: $0 \le X \le 375$, and $375 \le Y \le 750$

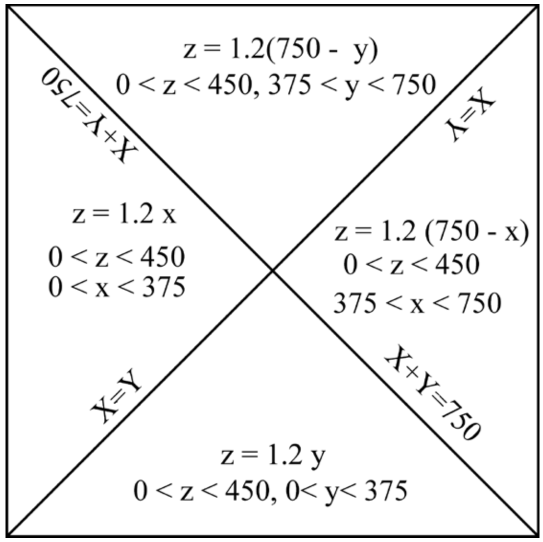

Figure 7. The mathematical model for the pyramid of Khufu.

Conclusion

The mysteries behind elevation models seem to disappear when pyramids are used in examples at the classroom level. To derive elevation data is no longer limited to engineers or surveyors. Computers made it possible to use GIS and Softcopy Photogrammetry technology to bring elevation data and modeling development to interested students in all fields of science. Terms such as DEM or DTM were debated (Maguire, 1991). Advancement in computer technology will add more elements to such debate. Raster and vector data that are GIS terms need to be considered. Using the term Raster Elevation Data (RED) is an appropriate alternative to DTM or DEM in raster contents.

New terms and models for elevation data are emerging. Only recently, a term such as Voxel has been introduced where the principle unit is a volume cell rather than a grid cell (Burrough, 1998). The pyramid can be presented in a voxel model where each building block is accounted for. Errors and error analysis need to be addressed. When transformation is made between the various types of data models, generalization process takes place and some accuracy could be compromised (João, 1998). Understanding the nature of errors is a key element in data transfer between the different models of elevation data. It is also important also to realize that these models are interchangeable but not without compromising accuracy (Maguire, 1991). This paper demonstrated the possibility of simplifying elevation models. It further suggest that using the pyramid as an education tool will make it possible to solve a more difficult problems such as voxel analysis and error analysis.

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Extra 001 - An Exercise in Digital Elevation/Terrain Models: From Point to Mathematics

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1. About the main contributor

- author:
  Ahmad S. Massasati
  Department of Geography
  Faculty of Humanities and Social Sciences
  United Arab Emirates University
  P.O. Box 17771
  Al-Ain, United Arab Emirates

- email: A.Massasati@uaeu.ac.ae

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