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Solid Angle Subtended by a Finite Rectangular Counter

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RADIATION LABORATORY

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Radiation Laboratory

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SOLID ANGLE SUBTENDED BY A FINITE RECTANGULAR COUNTER

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January 27, 1953

Berkeley, California

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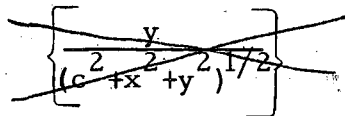
A geometry problem that sometimes arises in particle detection is the calculation of the solid angle subtended by a "finite" detector at a source of particles. This will often involve numerical integration. However for the case of a rectangular detector and a point source, a particularly simple formula can be obtained for the integrated solid angle.

We first consider the special case where the point source P is located a distance c perpendicularly above a corner of a rectangle of length a and width b. See Fig. 1. Then

$$d\Omega = \frac{dx dy \cos \theta}{r^2} = \frac{c dx dy}{(c^2 + x^2 + y^2)^{3/2}}, \text{ where}$$

$$r = (c^2 + x^2 + y^2)^{1/2} \text{ and } \cos \theta = c/r.$$

$$\text{Then } \Omega = c \int_0^a dx \int_0^b \frac{dy}{(c^2 + x^2 + y^2)^{3/2}} = c \int_0^a \frac{dx}{(c^2 + x^2)} \int_0^b d \left\{ \frac{y}{(c^2 + x^2 + y^2)^{1/2}} \right\}$$



$$= cb \int_0^a \frac{dx}{(c^2 + x^2)(c^2 + x^2 + b^2)^{1/2}} = \tan^{-1} \frac{ab}{cd}$$

where $d = (c^2 + a^2 + b^2)^{1/2}$ *

We may write this as $\tan \Omega = \frac{ab}{r^2_{\text{eff}}}$,

where $ab = \text{area of rectangle}$

and $r_{\text{eff}} = (cd)^{1/2} = \text{geometric mean of smallest and largest distances from P to the rectangle.}$

* Pierce: A Short Table of Integrals, 3rd Rev. Ed., 229.

Thus the finite solid angle formula is obtained from that of an infinitesimal detector by replacing r^2 by r^2_{eff} , and Ω by $\tan \Omega$.

The above holds only for the special case that the perpendicular from P to the plane of the detector intersects one corner of the detector. We can now use this result to obtain the solid angle subtended by a rectangle oriented arbitrarily with respect to P. Let σ be the intersection with the plane of the rectangle of the perpendicular from P to the plane of the rectangle. If σ lies inside the rectangle, (Fig. 2), then we simply apply the formula to the four sub rectangles A, B, C, and D and add the results. If σ lies outside the rectangle, (Fig. 3), then we apply the formula to the four rectangles A+B+C+D, B+C, C+D, and C and combine the results using the fact that $A=(A+B+C+D) - (C+D) - (B+C) + C$.

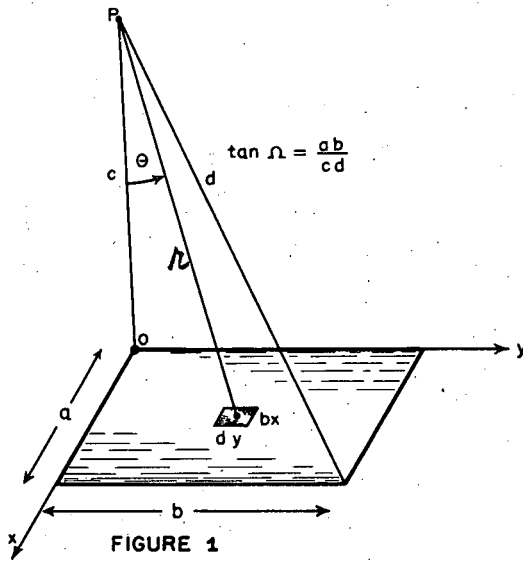
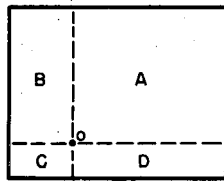


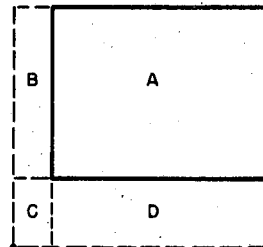
FIGURE 1



$$\Omega = \Omega A + \Omega B + \Omega c + \Omega D$$

FIGURE 2

FIGURE 3



$$\Omega = \Omega A + B + C + D$$

$$- \Omega B + c$$

$$- \Omega c + D$$

$$+ \Omega c$$