Title
Solid Angle Subtended by a Finite Rectangular Counter

Author
Crawford, Frank S, Jr.

Publication Date
1953

 UCRL- 1753

UNIVERSITY OF CALIFORNIA — BERKELEY

RADIATION LABORATORY

DISCLAIMER

UCRL-1753
Unclassified-Instrumentation Distribution

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

Contract No. W-7405-eng-48

SOLID ANGLE SUBTENDED BY A FINITE RECTANGULAR COUNTER

Frank S. Crawford Jr.

January 27, 1953

Berkeley, California

SOLID ANGLE SUBTENDED BY A FINITE RECTANGULAR COUNTER

Frank S. Crawford Jr.

Radiation Laboratory, Department of Physics,
University of California, Berkeley, California

January 27, 1953

A geometry problem that sometimes arises in particle detection is the calculation of the solid angle subtended by a "finite" detector at a source of particles. This will often involve numerical integration. However for the case of a rectangular detector and a point source, a particularly simple formula can be obtained for the integrated solid angle.

We first consider the special case where the point source P is located a distance c perpendicularly above a corner of a rectangle of length a and width b. See Fig. 1. Then

$$d\Omega = \frac{dx\,dy\cos\theta}{r^2} = \frac{c\,dx\,dy}{(c^2+x^2+y^2)^{3/2}}, \text{ where}$$

$$r = (c^2+x^2+y^2)^{1/2} \quad \text{and} \quad \cos\theta = c/r.$$

$$\text{Then} \quad \Omega = c\int_0^a dx\int_0^b \frac{dy}{(c^2+x^2+y^2)^{3/2}} = c\int_0^a \frac{dx}{(c^2+x^2)}\int_0^b d\left\{\frac{y}{(c^2+x^2+y^2)^{1/2}}\right\}$$

$$\left[\frac{y}{(c^2+x^2+y^2)^{1/2}}\right]$$

$$= cb\int_0^a \frac{dx}{(c^2+x^2)(c^2+x^2+b^2)^{1/2}} = \tan^{-1}\frac{ab}{cd},$$

where $d = (c^2+a^2+b^2)^{1/2}$ *

We may write this as $\tan\Omega = \dfrac{ab}{r^2\,\text{eff}}$,

where ab = area of rectangle

and $r\text{ eff} = (cd)^{1/2}$ = geometric mean of smallest and largest distances from P to the rectangle.

---

* Pierce: A Short Table of Integrals, 3rd Rev. Ed., 229.

Thus the finite solid angle formula is obtained from that of an infinitesimal detector by replacing r^2 by r^2eff, and Ω by tan Ω.

The above holds only for the special case that the perpendicular from P to the plane of the detector intersects one corner of the detector. We can now use this result to obtain the solid angle subtended by a rectangle oriented arbitrarily with respect to P. Let σ be the intersection with the plane of the rectangle of the perpendicular from P to the plane of the rectangle. If σ lies inside the rectangle, (Fig. 2), then we simply apply the formula to the four sub rectangles A, B, C, and D and add the results. If σ lies outside the rectangle, (Fig. 3), then we apply the formula to the four rectangles A+B+C+D, B+C, C+D, and C and combine the results using the fact that A=(A+B+C+D) - (C+D) - (B+C) +C.

Information Division
1/27/53 dm

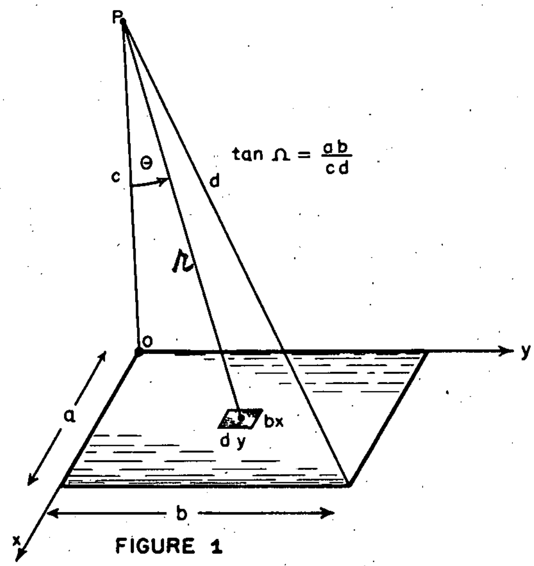
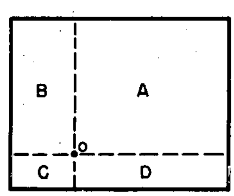


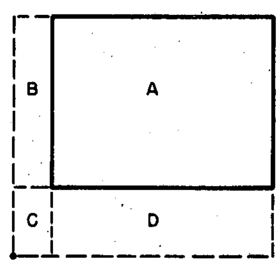
FIGURE 1



$\Omega = \Omega A + \Omega_B + \Omega c + \Omega D$

FIGURE 2

FIGURE 3



$\Omega = \Omega A + B + C + D$
$- \Omega B + c$
$- \Omega c + D$
$+ \Omega c$

MU-4773