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REVIEW ARTICLE

On the topology of topography: a review

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ABSTRACT

In the field of terrain analysis, a primary goal is to effectively identify topographic features for a better understanding of their associated processes. The relationships among features are, therefore, of particular importance. The concept of the surface network, involving and defined by such features as peaks, pits, various saddles, ridge lines, and the opposite course lines, can be a beneficial construct for describing and modeling any mathematical surface and, perhaps, topographic surfaces, as well. However, limitations of terrain data collection, storage, and computational processing have presented difficulties when attempting to make the jump from such logical constructs and their supporting mathematical theories to the development of tools and mapped products representing the measured topography of a landscape. Compared to feature extraction, less attention has been given to the topological relationships among topographic features. This article provides a chronological review of the development of surface network and critical point theory, the study of topography, and the progression of terrain analysis with particular consideration given to the application of surface network theory to represent the topology of topography. Any possible true computed surface network is concluded to be scale-dependent, fuzzy, and vague and its undisputed calculation elusive.

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Surface network; terrain analysis; topology; Warntz network

Introduction

Much of cartography and geography and all of terrain analysis involve inductive reasoning from specific land surface features to more general rules or principles. In map interpretation, we are taught to recognize patterns and forms on the land- and seascape that reveal process from form, be it human, glacial, fluvial or coastal. As cartography has yielded to automation, so too are these human interpretive tasks likewise yielding to the computer. As in other tasks, automation holds the promise of removing repetitive and error-prone human efforts, of forcing us to create precise definitions and ontologies for the key constructs, and of freeing the user for more demanding mental questions. Yet as before, the focus has moved to data acquisition and analysis and to the creation and application of computer algorithms to terrain data that reliably and accurately yield the features and forms we seek. Over the history of research on the topology of topography–the terrain surface network and its formative points and lines–early abstract theory has yielded to computational algorithms and heuristics that are subject to discretization and numerical errors in their computation. Perhaps the time has now come to revisit the basics of the theory.

For much of the recent history of cartography, data collection has been slow, coarse and inaccurate. Nowhere is this more true than with land surface topography. Initial experiments with digital models of terrain date from the 1960s, characterized by the emergence of cartographic data structures for terrain (Peucker and Chrisman 1975) and eventual dominance by the digital elevation model (DEM) and the triangular irregular network or TIN (Wilson and Gallant 2000). Both of these data structures are suitable for point sampling of terrain, and fit well with the photogrammetric methods used for early terrain data collection. While the TIN is more efficient for data storage, the more redundant regular grid of the DEM has been favored for the analysis of terrain using moving window methods. Later, DEMs became more available, as remote sensing took over as the primary source of born-digital terrain data. Resolution increased, from the coarse ETOPO5 DEM assembled in 1988 on a 5-minute grid, to USGS DEMs at 30 m, to GTOPO30 at 30-arcseconds in 1996, and eventually to global 30 m resolution in the ASTER GDEM and SRTM efforts (Maune 2007). Similarly, accuracy has increased, although major errors were induced by radar backscatter and the uneven nature of geodetic frameworks. More recently, lidar has emerged as a primary source of terrain data. Lidar offers both high resolution and high accuracy, due to its direct link to the Global

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Positioning System and high data densities. DEMs are now routinely available at better than 1 m spatial resolution and as both point clouds and grids. Lidar can separate the tops of the landforms and features and the bare earth below vegetation and buildings (Pingel, Clarke, and McBride 2013).

Fractal theory tells us that as measurements become increasingly detailed at higher spatial granularity, we simply measure more and more length and area (Dauphiné 2012). This is the case with terrain, as lidar and low altitude photogrammetry can reveal details of even minor forms on the land surface. Of course, this has led to a massive increase in the amount of terrain data to be processed, with point densities in the dozens per square meter or better. Consequently, higher resolution data has also introduced new sources of error, from reflections from birds and moving vehicles to algorithmic error around buildings and through vegetation. Broadly speaking, terrain analysis has moved from sparse and incomplete data to rich and ubiquitous data. While this is true of map detail, what is the case for the broader scale landscape features with which we introduced this discussion? By far the majority of today's algorithms designed to process topographic data into landscape features – primarily stream course lines, ridges, peaks, pits and saddle points – were designed for DEM data at spatial resolutions of about 30 m and with integer elevations in feet or meters. Obviously, increased data leads to new numerical and computational issues, including dealing with the fractal nature of terrain (Chase 1992). But what has been the impact of such high resolution data on surface models, especially the terrain surface network? In other work, we have used an empirical approach (Romero and Clarke 2013). In this paper, we seek to examine this question from the theoretical point of view.

The theory behind the terrain surface has been much slower to develop than the technical ability to gather data on heights. Three kinds of surface theory have developed over time: (1) mathematical surface abstraction based on Morse theory (Gyulassy et al. 2007, 2008); (2) identification of Very Important Points (VIPs), those inflection and other points where breaks in slope continuity imply form, or create new lines such as vertical cliffs, breaks of slope and ridge lines; and (3) surface abstraction into surface networks that include points, lines and areas. Reviews and syntheses of this theory can be found in Rana and Morley (2002) and Rana (2004). Software implementations of surface theory vary remarkably in terms of assumptions, algorithms, decision points, and thresholds, especially with respect to the definition of the surface at a point, its neighborhood, and the assumed direction of movement of surface water at that point. These thresholds are related to downslope flow, flow partitioning, and flow accumulation and include both methodological details and spatial resolution. While the surface network has been proposed as a robust topological structure ruled by mathematical laws, the rigor and laws have now been challenged by new data collection methods, algorithms, and definitions that suggest far less law and more heuristics and thresholds.

The purpose of this paper is to revisit the assumptions, algorithms, decision points, and thresholds that surround contemporary methods for terrain analysis. For example, on a 30 m DEM, using D8 flow, a level of downslope flow accumulation must be chosen to represent a drainage channel. Too low and every tiny stream branch is included, but coarsely. Too high and only the main channel is extracted, leaving lower levels of the network undefined. A possible approach would be to regard the terrain surface as a region to be partitioned into nonoverlapping features with crisp boundaries. Instead, we take the approach that surface features are multiscale clues to the processes that create surface forms and that such forms are dynamic and occasionally transient, as have other researchers (e.g. Gerçek 2010; Wood 1996). Most land forming processes are dominated by the force of gravity and by the carrying and erosional capacity of moving wind and water. Since various constituents of the landscape have different degrees of endurance, there should be no statistical or mathematical assumption of terrain continuity, smoothness, or invariance as is necessary for Morse theory. Theory also should apply across scales, across levels of measurement and include both surface details and the large size features common in geography, from mountain ranges to boulders. In mathematics, the study of relations that hold true regardless of specific geometry is termed topology. Topographic topology is only a subset of that dealt with in mathematics, but it is an important subset and surface theory has practical value in an applied mathematical sense. These mathematics are built upon prior work by Cayley, Maxwell, Morse, Warntz, and others. We will first examine, therefore, the theory behind the topology of topography. Second, we will investigate the consequences of that theory for both historical and contemporary terrain mapping and analysis. We will conclude by posing the question: does surface theory help or hinder the dense and accurate terrain mapping and processing now possible?

Surface theory

The theory behind surface networks has origins in the writings of British pure mathematician Arthur Cayley in 1859. In his paper "On Contour Lines and Slope

Lines", Cayley introduced a discussion based on contour lines and submerged islands, noting that features fall onto critical points at the limits. The paper's comparisons to terrain terms in French may be references to an earlier paper by Reech (1858). Cayley's observations concerned contour lines and slope lines, where contour lines are traced horizontally as loops by specific elevations and slope lines run vertically at right angles to contours both up and down the terrain and showed that there were both outloops around summits and inloops around pits. Cayley illustrated the extremes of summits, "immits" (pits, at the bottom of depressions), included "knots" (saddles) and noted that special slope lines, ridge lines and course lines, connect the critical points. He noted that at each saddle point, the slope lines of steepest ascent follow ridge lines, and climb to summits and that the slope lines of steepest descent from a saddle point are course lines and trace down to pits. Connecting the lines together reveals watersheds. The naming of these key points and the statement of their relations was the foundation of surface network theory.

Maxwell (1870) acknowledged Cayley when he reintroduced the concept in his 1870 paper "On Hills and Dales", which renamed many of Cayley's terms but drew similar conclusions. However, Maxwell added two types of "districts" on a surface, those bounded by ridge and course lines. A district surrounding a peak and bounded by course lines was termed a hill, and a district surrounding a pit and bounded by ridge lines was termed a dale. Hills and dales overlap as any point in a dale is also on a hillslope. Maxwell noted that slope lines within a hill climb to a single peak, and that similarly slope lines in a dale descend to a single pit. Maxwell provided new terminology and added new features: top (summit), bottom (pit), pass (knot, or saddle, at the intersection of an inloop contour), and bar (knot, or saddle, at the intersection of an outloop contour), noting that at a saddle point there can be multiple meeting points between hills and depressions. He also introduced a set of numerical relations that characterized a complete surface network, including that the number of summits is the number of passes plus one. After defining regions of hills delimited by watercourses (course lines) and dales delimited by watersheds (ridge lines), he deduced a number of regions, two, for use in re-forming Listing's rule to find that the number of faces is the number of lines minus the number of points plus two; that the number of dales was the number of watersheds less the sum of the summits, passes and bars plus two; and that the number of hills was the number of watercourses less the sum of the bottoms, passes and bars, plus two. It has been shown that Maxwell's relations correspond to the Euler-Poincaré formula which describes the relationship among the number of vertices, the number of edges and the number of faces for a manifold.

Marston Morse in 1925 advanced the mathematical understanding of Euclidean surfaces by examining the nature of critical points on the surface. Assuming a homomorphic surface (no holes, cliffs or overhangs), Morse (1925) noted that the critical points corresponded to the surface derivatives in each dimension, and that at extrema, these formed elliptic functions and at saddles, they formed hyperbolics. His nine theorems determined numerical relations among critical points in n dimensions and added inflection points to the set of critical surface features. While Morse's surface network was largely abstract, leaving the critical points in place, one can trace out a set of critical slope lines that define the surface and complement a map of the perpendicular contour lines. Though there is a large set of "space-filling" slope lines that traverse each slope, their critical limits, with maximum gradient, define the ridge and course lines of Cayley and Maxwell's formulations and converge at critical points. Morse's theories have proven invaluable in solid modeling in three dimensions and in multi-dimensional analysis.

William Warntz's work in the 1960s extended Maxwell's idea of regions and further developed the theory (Warntz 1966; Warntz and Woldenberg 1967; Hessler 2009). Warntz defined operational contours as those associated with critical points and reinforced associations of ridges with outloops and courses with inloops. Along with referring to bars as pales, new concepts were territories, the overlaps between hills and dales (i.e. regions bounded by two ridge lines and two course lines), and the listing of the vergency of forces that flow upon a surface (peaks, ridges, and hills are divergent; pits, courses, and dales are convergent; passes and pales have mixed vergency). While Maxwell enlarged the theory to include areas (in addition to points and lines), Warntz suggested that the surface features were part of a complete surface network and that this network was common to topography and other geographical surfaces, such as socioeconomic variables. Warntz and Woldenberg (1967) also expanded Cayley's original idea of a submerged mountainous island using a series of figures to show that the points and lines emerged as the water level fell (Figure 1). A peak is a point where the tip of a hill or mountain emerges from the water, a pass where separate contours expanding from emergent peaks touch to become a single outloop contour (like a "figure 8"), a pale where a contour wraps around to touch and enclose another region as an inloop contour, and a

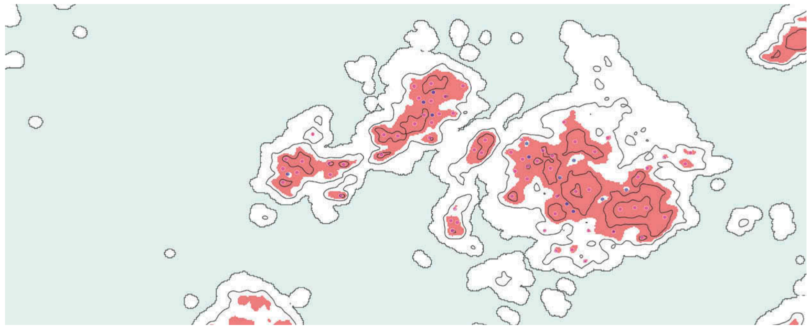

Figure 1. The surface network metaphor of submerged mountains (Swans Island, ME).

pit where enclosed holes converge to a point. In Warntz's honor, the surface network has since been called the *Warntz network* in much of the cartography and GIScience literature, although it also became called the *surface network* for connected graph applications by Pfaltz (1976). Earlier work in graph theory produced similar networks (Reeb 1946) that provided a link between contours, critical points, a surface hierarchy, contour trees and surface networks. Eventually, Mark (1978) reintroduced the theory into cartography and GIS, using this contour tree model.

The use of the theory in digital cartography dates from Steven Morse's work in electrical engineering on the abstraction of adjacent contour lines (Morse 1969). His initial problem was mapping ground tracks of aircraft when only the elevation and flight bearing were known. Morse proposed a formalization of contours as continuous loops and then derived a network model that abstracted the adjacencies among loops. This model was attached to a Freeman code abstraction of the line itself and required branching when two peaks or pits were contained within a single contour loop (Figure 2). This enabled a flow network, in which the rise in elevation from point to point could be tracked to see which contours were crossed. Morse (1969, 147) noted that "the model provides the necessary structure for developing formal algorithms, based on topological and geometrical properties, that can be used in the solution of contour map problems." In the original paper, adjacent loops were connected with straight lines at the edges, and the interior space or "maze" was assumed to be the possible path of an aircraft ground track. Surface network theory, consequently, has been based on the contours and not the slope lines discussed by Cayley, Maxwell, and Warntz. This early work was also demonstrating the development of a data structure to support rapid query of both contour lines and their elevations, rather than a model of the form of the landscape.

Implementations of surface network theory had remained a set of abstractions surrounding contour map interpretation. However, in the decade following the work by Warntz, the emergence of the digital computer and stored programs led to the intense pursuit of automated methods for extracting and labeling parts of the terrain surface network from digital terrain models. As the accuracy and resolution of the input data improved, so also did the processing power of faster CPUs and the capability of computer code. Terrain surface and network theory chased–perhaps continues to chase–the somewhat elusive goal of "solving" the surface network for any particular data set.

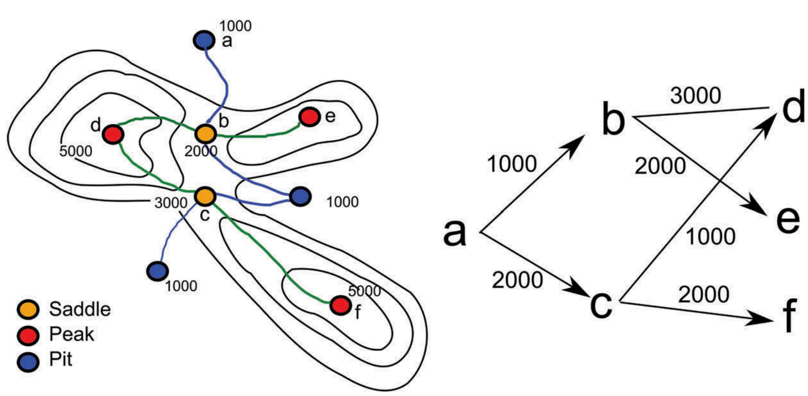

Figure 2. Surface networks from contours (adapted from Rana 2007).

Algorithms

Surface flow and network algorithms were driven by the application of surface networks in hydrology, flood modeling, and to a host of other geographical problems primarily using rasters and DEMs. A recent review of surface network extraction algorithms using Morse-Smale theory is that by Čomić et al. (2014). The surface networks were also seen as models for terrain that conveniently fit the point/line/area geometry of computer cartography in vector mode and as a possible means for the reduction of large digital elevation models as tiled grids in raster mode into smaller partitions for processing. Pfaltz (1976, 92) noted of surface networks: "They are of practical use as one means of condensed surface description; particularly since they may be used as a directory in conjunction with a computer representation, and also they support an automatic abstraction process." This fits the Triangulated Irregular Network model well, in which a terrain surface is abstracted as a set of triangles that are allocated by Delaunay triangulation and processed within these facets. Peucker and Douglas (1975) were among the first to suggest how critical or very important points could be detected in an array, providing a comparison of several methods. Important to this work was the computation of the number of surface zero crossings (Figure 3). If the neighboring points in sequence rise then fall, the point sits on a slope, but if there are two crossings from high to low, the point is a saddle. There can also be multiple crossings, like the multiple rise and dip saddles noted by Maxwell, occasionally called monkey saddles.

Fowler and Little (1979) presented a set of simple point labeling methods that allowed critical points to be extracted from arrays to build a TIN or other surface model. For any grid cell, expedient directional evaluation occurs for only four cells, including the "central cell" and three neighbors to the right and below the central cell. The lowest of the four cells is not a ridge candidate (the highest is not a course candidate). Using the complementary sets of ridge (or course) candidates, passes are found by comparing all neighboring ridge (or course) candidates. If a cell is the lowest (highest) of the ridge (course) candidates, then it is a pass. Although complication lies in that neighboring points can be equal, especially with integer elevations, Fowler and Little's algorithm remains robust for finding VIPs. Using the saddles as start points, ridges can then be followed to peaks (and course lines to pits). Difficulties with the network occur when nearby ridge cells, for example, have equal values, creating regions, or the lines meet rather quickly, perhaps creating a pit within, having no course lines at that scale of analysis.

The fact that digital images can be treated as terrain surfaces led the field of image processing to adopt surface network theory. Haralick (1983) used ridge and valley detection with zero crossings as a way of partitioning images to analyze content, while Toriwaki and Fukumura (1978) used surface networks as one method for determining image structure. It is clear from this early work that the networks were not fully extracted, nor were the lines they created continuous. This problem carried over into further work on stream

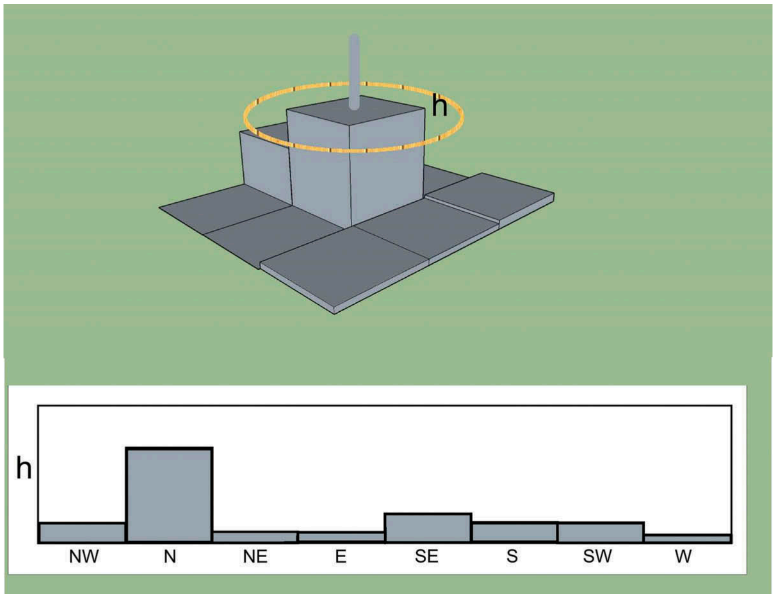

Figure 3. Eight cell neighbors around a peak, with the zero crossing graph.

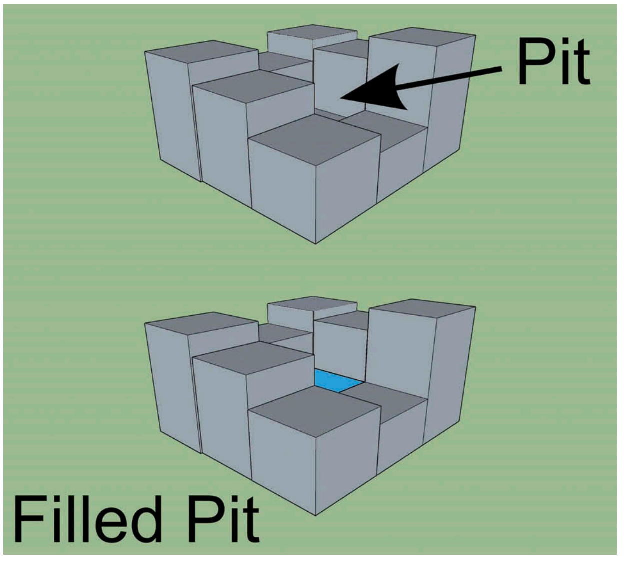

Figure 4. Pit filling up to the lowest pour point.

networks by O'Callaghan and Mark (1984), Jenson and Domingue (1988) and Band (1986). This and subsequent work around the 1990s focused almost exclusively on the digital elevation model or grid, and on finding means to circumvent two problems: (1) dealing with pits, usually by filling them until they reached their tip-over point into adjacent features (Figure 4) (Hutchinson 1989); and (2) the discontinuity of the extracted features (Kweon and Kanade 1994; Takahashi et al. 1995; Wilcox and Moellering 1995). One method of connecting the features was by "burning-in" or deliberately deepening, and thereby removing, the barriers between stream cells (Garbrecht and Martz 1997). A limited number of papers during this period dealt with the abstraction of continuous surfaces for data structures and generalization, for example with Morse functions, trend surface analysis and Fourier series, without considering the surface network (Clarke 1988; Wolf 1991; Li, Zhu, and Gold 2004).

By the late 1990s, variations on the most commonly used algorithms were being adopted into GIS and general purpose terrain analysis packages, such as TAPES-G (Gallant and Wilson 1996) and later Landserf (Wood 2008). Many packages standardized their approaches to stream network extraction, especially for hydrological modeling (Moore, Grayson, and Ladson 1991). The most common approach was to fill the pits, find the local maxima and minima, then process the DEM to determine the flow direction and then the flow accumulation (Clarke and Lee 2007). Flow accumulation is then thresholded and large values thinned to reveal stream lines (Figures 5 and 6). Flow direction is the direction water would flow out of a pixel, usually the adjacent pixel with the greatest downward elevation gradient. Its calculation can use the four-cell neighbors (termed D4), eight cell neighbors (D8) or hybrid adjacency methods such as multiple-, divergent-, or divided-flow (D-infinity, or D∞) (Freeman 1991; Holmgren 1994; Tarboton 1997). Choosing a neighborhood can bias the calculation

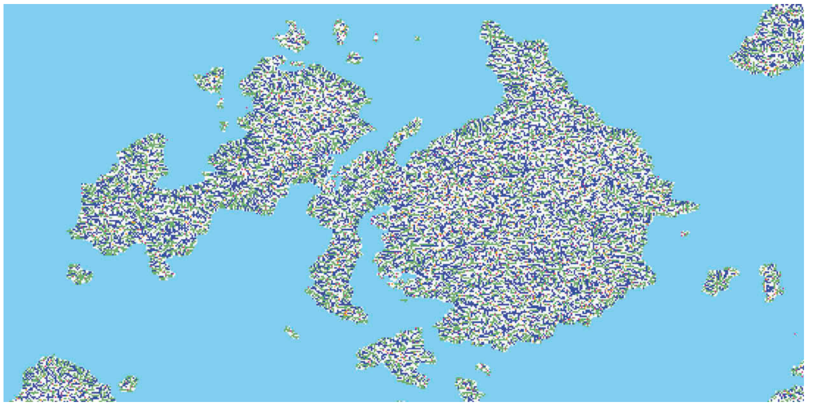

Figure 5. Surface network from 30 m DEM for Swans Island ME.

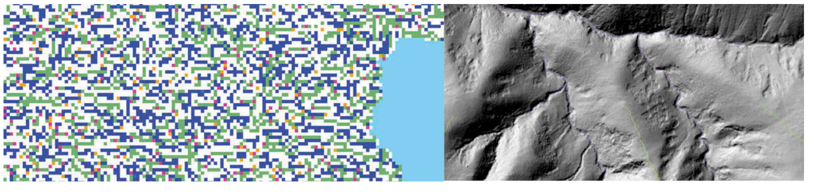

Figure 6. Computed surface network with critical points at two resolutions. Left: Swans Island Maine at 30 m (detail). Right: Part of Santa Cruz Island, California at 1 m (detail). Stream blue, ridge green, saddle orange, pit dark blue, summit pink.

of gradients because the linear distance is either one or root 2. In a thorough review of the various algorithms, Florinsky (1998) looked at the comparative accuracy of the various methods. Several scholars noted the effects of scale and elevation errors in many of the methods (Wilson, Repetto, and Snyder 2000; Wechsler 2007; Thompson, Bell, and Butler 2001; Lee, Snyder, and Fisher 1992). Rarely were these algorithms applied at high resolution, where TINs provided an easier solution of the surface network.

After 2000, the variety of approaches of the pioneer era returned. Morse theory was expanded considerably for mapping and modeling applications (Robins, Wood, and Sheppard 2011; Bremer et al. 2003; Čomić, De Floriani, and Papaleo 2005), multiscale terrain modeling began (Wood 1996; Danovaro et al. 2003; Danovaro, De Floriani, and Vitali 2007; Schmidt and Andrew 2005), and grid-based surface patch methods emerged to compete with the TIN (Schneider 2005). Danovaro, De Floriani, and Vitali (2007) combined Morse methods and the multiscale approach, while Florinsky (2009) moved beyond the second to the third terrain derivatives. Rana (2004, 2010) provided comprehensive reviews of methods and issues in topological surface networks, while others considered the uncertainties in the data and processing (Lindsay 2006; Pathmanabhan and Dinesh 2007; O'Neil and Shortridge 2013). Importantly, Fisher, Wood, and Cheng (2004) linked the multiscale uncertainties of the surface network with toponymy and place names, introducing fuzzy set theory into the issue of surface network characterization. The implication is that not only are data subject to error and uncertainty, surface abstraction methods are also subject to variation due to their parameters and assumptions, and worst of all, humans are vague about the definitions of the features they seek to extract. The paper by Fisher et al. used mountains in its argument, but the reasoning applies to all topographical features. Using this reasoning, the surface network may be a mathematical simplification that is just incapable of representing the complexity of real terrain. Ironically, there might actually be a true surface network measureable in reality, but not one that we can represent accurately with computers or terrain surface data structures.

Contemporary research on the surface network

The last decade has seen continued interest in the concept of the terrain surface network, its computation and extraction and its broader meaning. New algorithms continue to appear, for example by Magillo et al. (2007, 2008), Hashemi (2008) for peaks, Rana (2010), Wang (2014) for drainage networks, and Chen and Zhou (2013) for multiscale VIPs. Mitasova et al. (2012) have rigorously explored the role of the surface network in terrain visualization and made use of open source tools in raising terrain analysis to new levels of sophistication, including adding back flow lines. Guilbert (2013) continues to explore the multiscale effects on terrain and its network, especially with new higher resolution data. Orlandini, Moretti, and Gavioli (2014) for analytical representation and Mower (2009) for visualization have re-examined the role of slope lines as used in the original work. Hu, Miller, and Li (2014) have shown that surface network theory has use in the hotspot analysis of mobile objects, such as vehicles in traffic or potentially diffusion of disease in a populace. Wolf (2014) has examined the use of the surface network in nanotechnology. Research has also moved toward more theoretical and analytical approaches to slope line and surface network extraction (Orlandini, Moretti, and Gavioli 2014; Le and Kumar 2014; Jeong et al. 2014). Finally, Rana (2007) has suggested that once the surface network is computed, network metrics can be used to empirically search for spatial structure in terrain. He proposes measures of network length, depth, diameter, mean depth, and degree that have parallels in the application of network theory to other sorts of networks, such as street patterns and social networks.

A new dimension to surface network theory has derived from the addition of both uncertainty and language into the mix (Fisher, Wood, and Cheng 2004). The rigor of ontologies in data and computer science is not matched by that of more casual definitions of surface feature terms in geomorphology. Mapping has traditionally been where the two have coincided, because it has always necessitated precise definitions of features. Nevertheless, many terms common in land use mapping, geology, and human geography are inherently vague. Mark and Sinha (2006) have sought to define "topographic eminences," while lidar mapping has needed definitions of terms such as bare earth, digital surface model, digital feature model, and digital terrain model (Pingel, Clarke, and Ford 2015). One impact of the new focus is a broadening of the feature set considered part of the surface network (Gerçek 2010), while others have investigated the degree of membership in the fuzzy set context, particularly for the significance of drainage divides and "valleyness" (Straumann and Purves 2011; Lindsay and Seibert 2013). Some research has sought to link ontology, landforms, and the surface network, with the aim of automated identification and labeling of features by type (Chaudhry and Mackaness 2008; Blanc,

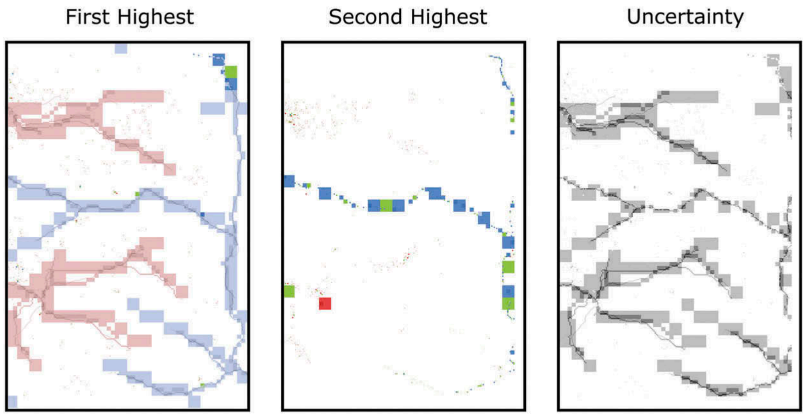

Figure 7. Fuzzy representation of surface network features across methods and scales for Santa Cruz Island (detail). Left: Membership in features class set {peak, pit, saddle, ridge, stream} by most common class; Center: Same by second most common class; Right overall feature uncertainty, black is high, white low) (based on Romero and Clarke 2013).

Grime, and Blateyron 2011; Blazquez 2011). This approach has even been promoted as a move toward unifying theory in landscape and ecological modeling (Gaucherel et al. 2014). In addition, the combination of uncertainties inherent in both resolution and algorithm has been shown to classify pixels in a DEM not only as partially belonging to each of the feature types, such as a ridge, course, peak, pit, and saddle, but also to classify a pixel as a combination, *across* these classes, depending on terrain type, resolution, and algorithm (Romero and Clarke 2013) (Figure 7).

On the topology of topography

The surface network has survived as a geographical concept. Early work in applied mathematics was largely based on the rising popularity of contour lines from the hand-drawn maps of the various national topographic mapping programs. As we have seen, the advent of the computer led to application of the early theory, with great value to fields such as terrain analysis and hydrology. The vast number of algorithms for extracting the surface network and its various features led to competing implementations that did not produce consistent or even complete results and that were also scale-sensitive. Research designed to overcome these as practical limitations instead has determined that they are also problems of theory: of uncertainty, scale, semantics and implementation. Today there is a broad choice of methods to compute and extract parts of the surface network, and the methods are available in many GIS packages. There can be little doubt that this availability has promoted the use of the surface network as a tool for data modeling, analysis and for applications that now extend far beyond the realm of contour maps and digital terrain models. Nevertheless, the "solution" approach that produces a complete network automatically for any terrain remains elusive. At the same time, the need for the surface network as a data structure for dealing with massive data sets has lessened, as systems for dealing with "big data" quantities of accurate and precise terrain data are becoming more available.

As we have seen, the move toward using the terrain surface network as a descriptive tool has value in landscape classification, feature identification, and labeling. What is impressive about this is that most feature extraction, identification, and labeling has taken place without using the geographic or spatial context of the topographic data. Yet Tobler's law informs us of the power of locality in geographic space. This locality is wholly contained within a three dimensional space bounded by the surface extremes, what might be called the digital feature model, but more data reveals more surface and hence more features. Within this space, the forces at work on modifying real topographic surfaces are not symmetrical: they are dominated by gravity and the movement rock by wind, water, and ice. Over time, this tends to level slopes, reduce extremities, and fill pits. In other ways, processes are also disruptive, with uplift, landslides, folding, and faulting. Topographic surfaces are, therefore, quite different than other types of surfaces or mathematical models that are not subject to such process-oriented "potentials," directed forces, or discontinuities.

An appealing aspect of the surface network is that it has spatial relations, point to line, and line to area. Warntz and Maxwell's regions have more than just number, they also have structure. For example, radiating outward from a single high peak should be ridge lines and stream lines that alternate, and there should be equal numbers of ridge lines and stream lines (Figure 8). While it is tempting to mathematically outline all possible combinations of region adjacency, it

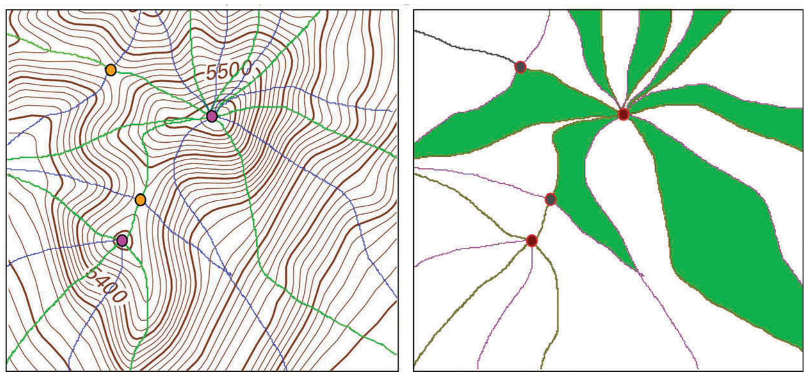

Figure 8. Adjacency of surface network features and facets in alternating configuration around a peak.

might be more informative to extract surface networks in bulk consistently from large quantities of terrain and to data mine the result for common structures. When combined with the place names, for GNIS or GeoNames, it should be possible to match toponyms for features to their geometric forms. This would truly be a topology of topography.

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