Adaptive Robust Control for Pointing Tracking of Marching Turret-Barrel Systems: Coupling, Nonlinearity and Uncertainty

Qinqin Sun, Xiuye Wang, Guolai Yang, Ye-Hwa Chen, and Fai Ma

***Abstract*— Pointing tracking control of marching turret-barrel system is one of the important topics in exploration of intelligent ground combat platform. This paper focuses on an adaptive robust control scheme for pointing tracking of marching turret-barrel system driven by a motor and an electric cylinder. Three types of possibly fast time-varying but bounded uncertainty are considered: system modeling error, external disturbance and road excitation. The uncertainty bounds are not necessary to be known. First, the pointing tracking system is constructed as a coupled, nonlinear and uncertain dynamical system with two interconnected (horizontal and vertical) subsystems. Second, a tracking error e is defined as a gauge of control objective, and then the dynamical equation of the pointing tracking system is built in state-space form. Third, for uncertainty control, a comprehensive uncertainty bound α is derived to measure the most conservative influence of the uncertainty, and then an adaptive law is proposed to evaluate it in real time. Finally, for pointing tracking control, an adaptive robust control is proposed to render the pointing tracking system to be practically stable; thereout, the objective of pointing tracking is achieved. This work should be among the first ever endeavours to cast all the *coupling*, *nonlinearity* and *bound-unknown uncertainty* into the pointing tracking framework of marching turret-barrel system.**

***Index Terms*— Intelligent ground combat platform, turret-barrel system, intelligent control, pointing tracking, uncertainty, adaptive robust control.**

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Qinqin Sun is with the School of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics, Jiangsu, Nanjing 210016, China (e-mail: sun421529@126.com).

Xiuye Wang and Guolai Yang are with the School of Mechanical Engineering, Nanjing University of Science and Technology, Jiangsu, Nanjing 210094, China (e-mail: xiuyewang@126.com; yyanggl@njust.edu.cn).

Ye-Hwa Chen is with the George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA, and also with the Key Laboratory of Road Construction Technology and Equipment of MOE, Chang'an University, Shanxi, Xi'an 710064, China (e-mail: yehwa.chen@me.gatech.edu).

Fai Ma is with the Department of Mechanical Engineering, University of California at Berkeley, Berkeley, CA 94720 USA (e-mail: fma@berkeley.edu).

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I. Introduction

Driven by intelligent technology, the intelligent development of ground combat platform has attracted much attention, while intelligent control for pointing tracking of marching turret-barrel system is one of the important topics. Pointing tracking system of turret-barrel system involves a linkage control of the concatenated turret and barrel on a vehicle platform. In the past, the turret and the barrel are driven respectively by a set of motor servo system and a set of electrohydraulic servo system to rotate around the rotating shafts of the turret and the trunnion [1], [2]. However, with the development of intelligent and pure electric weapons, in recent years, the barrel tends to be driven by electric cylinder instead of electrohydraulic servo system. It is frankly to say that, future pointing tracking system may be a coupled system of mechanical, motor, electric cylinder and control system. Meanwhile, nonlinearity (such as change of fluid direction, friction, *et al.*) and uncertainty (such as modeling error, disturbance, *et al.*) are inevitable in the pointing tracking system [3]. By this, for pointing tracking control, the pointing tracking system should be taken as a *coupled*, *nonlinear* and *uncertain* dynamic system.

The pointing tracking problem usually can be cast into a position adjustment control problem of keeping the barrel at a desired position, or a trajectory tracking control problem of driving the barrel to follow a desired fire control command. At present, the existing pointing tracking strategies are mainly designed based on a simple linear time-invariant system model and relatively classical control theory (such as the well-known PID control) [4]–[6], such that cannot effectively handle the coupling, non-linearity and uncertainty of the pointing tracking system. Although some efforts [1], [2], [7]–[10] on uncertainty management in pointing tracking control design have been done, they mainly refer to simple constant uncertainty. By this, more explorations on coupling dynamics modeling and control as well as uncertainty control in pointing tracking problem are expected. In this sense, exploring an effective way to handle all the coupling, nonlinearity and uncertainty in the process of pointing tracking control for marching turret-barrel system is the critical motivation of this work.

In recent years, some modern control theories such as adaptive control, sliding mode control, and fuzzy control have been applied on pointing tracking of turret-barrel system [7]–[10]. However, they usually simplify the dynamics model of the pointing tracking system as a linear transfer function, and ignore the coupling relationships between the mechanical structure, the power-driven system and the control system; thereout, the resulted control strategies are usually not optimal. Although some current works gradually concern on the coupling characteristic of the pointing tracking system [11]–[14], they are limited to dynamical analysis and modeling, and have not been developed in control design. As a pioneer work, this paper formulates the pointing tracking system as a coupled nonlinear dynamic system with possibly fast time-varying but bounded uncertainty. By this, constructing the dynamic model of the pointing tracking system under the consideration of all the coupling, nonlinearity and uncertainty is the first branch of the motivation of this work.

Many approaches [15]–[24] have been proposed in the field of uncertainty control. Li *et al.* [25] proposed a finite-time controller for a class of uncertain nonlinear systems by applying the method of adaptive neural networks control, meanwhile, they [26] also proposed another type of control method of observer-based fuzzy adaptive inverse optimal output feedback control for uncertain nonlinear systems. Liu *et al.* [27] developed an adaptive control method for a class of uncertain strict-feedback switched nonlinear systems. All the above studies have given effective methods for uncertainty control, and each method has its own characteristics. However, in the field of pointing tracking, the existing control approaches usually can only deal with uncertainty with known bound. However, in practical problem such as pointing tracking, the uncertainty bound may be difficult to be determined. In recent years, Chen *et al.* [28]–[31] have done some innovative works on control of uncertainty with *unknown bound*. It is expected to handle such kind of bound-unknown uncertainty in pointing tracking. Along this way, this paper constructs a comprehensive uncertainty bound α to measure the most conservative influence of the uncertainty, and then proposes an adaptive law to evaluate it in real time, based on which an adaptive robust control is then proposed to render the pointing tracking system to be practically stable. By this, proposing a novel control scheme to handle uncertainty with unknown bound in pointing tracking control for marching turret-barrel system is the second branch of the motivation of this work.

The main contributions of this paper are threefold. First, a coupled nonlinear dynamic model for the pointing tracking system of marching turret-barrel system with bound-unknown uncertainty is constructed. Second, an adaptive law is proposed for uncertainty control to evaluate a comprehensive uncertainty bound α, such that the most conservative influence of the uncertainty is overcame. It is worth emphasizing that the uncertainty may be nonlinear (possibly fast) time varying but bounded and the bound is unknown. Third, an adaptive robust control is proposed to render the pointing tracking system to be practically stable; thereout, the pointing tracking is achieved under the consideration of all the *coupling*, *nonlinearity* and *uncertainty*.

Meanwhile, the originality of this paper can be summarized as the follows. First, it does the first effort that gives out a coupled dynamic model of the controlled object (i.e., the marching turret-barrel system) and the control actuator (i.e., the motor servo system and the electric cylinder system) in pointing tracking. Second, it can specially handle rather complex uncertainty in pointing tracking control, which may be nonlinear (possibly fast) time varying but bounded and the bound is unknown. Third, it gives out a comprehensive way to handle the coupling, nonlinearity and uncertainty in pointing tracking of marching turret-barrel system.

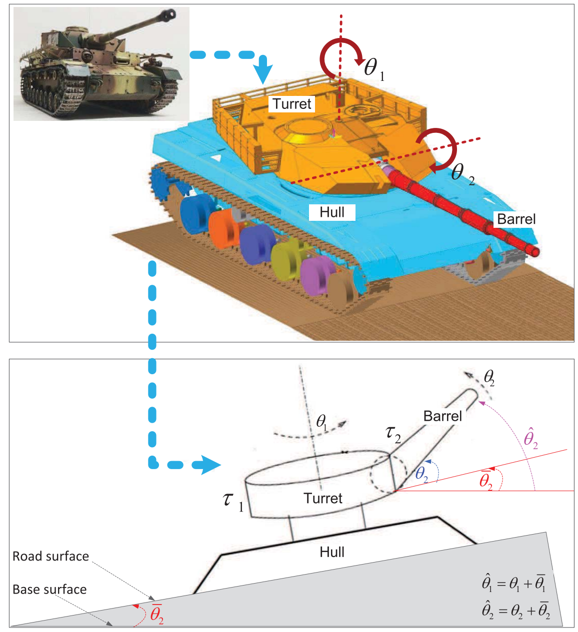

Fig. 1. A marking turret-barrel system.

II. Dynamical Model of Pointing Tracking System

We now formulate the dynamical model of pointing tracking system in turret-barrel system. It is composed of a horizontal mechanical system driven by motor (i.e., the turret-motor subsystem) and a vertical mechanical system driven by electric cylinder (i.e., the barrel-electric-cylinder subsystem).

A. Marching Turret-Barrel System

First, we focus on a marching turret-barrel system that is abstracted from the marching tank gun. Shown as Fig. 1, it consists of a rotating base (turret) with a tilting arm (barrel) attached to the base, where $\hat{\theta}_{1,2} = \theta_{1,2} + \bar{\theta}_{1,2}$ are the angular positions of the turret load and the barrel load respect to the base space, $\theta_{1,2}$ are the angular positions of the turret load and the barrel load (i.e., the horizontal and vertical pointing angles) respect to the joint space, and $\bar{\theta}_{1,2}$ are the horizontal and vertical angular position fluctuation resulted from the road excitation. The dynamical model of the turret-barrel system can be described as [32]

$$\begin{aligned}(&\frac{1}{2}m_1 R_1^2 + m_2 R_1^2 + m_2 R_1 R_2 \cos\theta_2(t) \\ &+ \frac{1}{3}m_2 R_2^2 \cos^2\theta_2(t))\ddot{\theta}_1(t) \text{-} (m_2 R_1 R_2 \sin\theta_2(t) \\ &+ \frac{1}{3}m_2 R_2^2 \sin(2\theta_2(t)))\dot{\theta}_1(t)\dot{\theta}_2(t) + d_1(t) = \tau_1(t), \quad (1a)\end{aligned}$$

$$\frac{1}{3}m_2R_2^2\ddot{\theta}_2(t)+\frac{1}{2}m_2R_1R_2\sin\theta_2(t)\dot{\theta}_1^2(t)+\frac{1}{6}m_2R_2^2\sin(2\theta_2(t))\,\dot{\theta}_1^2(t)+\frac{1}{2}m_2\tilde{g}R_2\cos\theta_2(t)+d_2(t)=\tau_2(t), \tag{1b}$$

where $t\in\mathbf{R}$ is the time, $\tau_{1,2}(t)\in\mathbf{R}$ are the torque inputs, $d_{1,2}(t)\in\mathbf{R}$ are the external disturbances, m_1 is the mass of turret, m_2 is the mass of barrel, R_1 is the radius of turret, R_2 is the length of barrel, and $\tilde{g}$ is the gravitational constant.

Remark: The state of the turret-barrel system is coupled with a horizontal state θ_1 and a vertical state θ_2.

B. Horizontal Pointing Tracking System

The horizontal pointing tracking system can be seen as a mechanical system driven by motor that includes the turret and the motor. Considering the fact that the armature current dynamics can be neglected resulting from the small value of armature inductance, the dynamical model of the motor can be expressed as [33]:

$$J\dot{\omega}_m(t)=T_a(t)-B_m\omega_m(t)-T_m(t), \tag{2}$$
$$T_a(t)=k_tu_m(t)-k_e\omega_m(t), \tag{3}$$

where $\omega_m(t)\in\mathbf{R}$ is the angular velocity of the motor, $T_a(t)\in\mathbf{R}$ is the motor torque, $T_m(t)\in\mathbf{R}$ is the gear input torque, $u_m(t)\in\mathbf{R}$ is the control input voltage, J is the moment of inertia of the motor, B_m is viscous damping coefficient of the motor, k_t and k_e are the constants of the motor torque and the electromotive force.

Due to the fact that gears exist to transfer the torque from the motor to the turret, the backlash nonlinearity between gears is generally given as

$$\tau_1(t)=NT_m(t)+d_t(t), \tag{4}$$

where τ_1 is the gear output torque (i.e., actually the torque input on the turret described as (1a)), N is the gear ratio, and $d_t(t)\in\mathbf{R}$ is the transmission error. Taking T_a as (3), T_m as (4), $\omega_m=N\dot{\theta}_1$ and $\dot{\omega}_m=N\ddot{\theta}_1$ into (2), yields

$$JN^2\ddot{\theta}_1=k_tNu_m-k_eN^2\dot{\theta}_1-B_mN^2\dot{\theta}_1-\tau_1+d_t. \tag{5}$$

Focusing on τ_1, and taking it into (1a), a lengthy but straightforward algebra shows that the dynamic equation of the horizontal pointing tracking system can be expressed as

$$\begin{aligned}\ddot{\theta}_1(t)&=(\frac{1}{2}m_1R_1^2+m_2R_1^2+m_2R_1R_2\cos\theta_2(t)\\&\quad+\frac{1}{3}m_2R_2^2\cos^2\theta_2(t)+JN^2)^{-1}(m_2R_1R_2\sin\theta_2(t)\\&\quad+\frac{1}{3}m_2R_2^2\sin(2\theta_2(t)))\dot{\theta}_1(t)\dot{\theta}_2(t)\text{-}(\frac{1}{2}m_1R_1^2+m_2R_1^2\\&\quad+m_2R_1R_2\cos\theta_2(t)+\frac{1}{3}m_2R_2^2\cos^2\theta_2(t)+JN^2)^{-1}\\&\quad\times(k_eN^2+B_mN^2)\dot{\theta}_1(t)+k_tN(\frac{1}{2}m_1R_1^2+m_2R_1^2\\&\quad+m_2R_1R_2\cos\theta_2(t)+\frac{1}{3}m_2R_2^2\cos^2\theta_2(t)+JN^2)^{-1}u_m(t)\\&\quad-(\frac{1}{2}m_1R_1^2+m_2R_1^2+m_2R_1R_2\cos\theta_2(t)\\&\quad+\frac{1}{3}m_2R_2^2\cos^2\theta_2(t)+JN^2)^{-1}(d_1(t)-d_t(t)).\end{aligned} \tag{6}$$

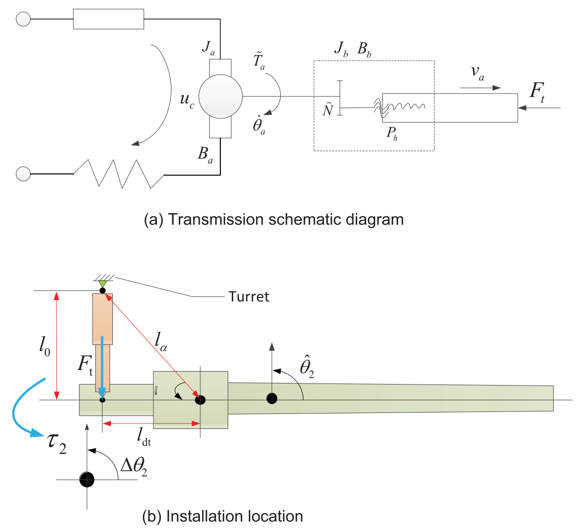

Fig. 2. Transmission schematic diagram and installation location of the electric cylinder.

Remark: The dynamic equation of the horizontal pointing tracking system is derived from the horizontal dynamics (1a) of the turret-barrel system and the dynamics of motor (2). It is nonlinear and contains the information of both the turret-barrel system and the motor; hence, a nonlinear coupled system.

C. Vertical Pointing Tracking System

The vertical pointing Tracking system can be seen as a mechanical system driven by electric cylinder that includes the barrel and the electric cylinder system. The transmission schematic diagram and installation location of the electric cylinder are shown as Fig. 2 The dynamics of the electric cylinder can be described as [34]:

$$\tilde{T}_a(t)-T_b(t)=J_a\ddot{\theta}_a(t)+B_a\dot{\theta}_a(t) \tag{7}$$
$$\tilde{T}_a(t)=K_mu_c(t) \tag{8}$$
$$T_b(t)-\tilde{T}_m(t)=J_b\ddot{\theta}_a(t)+B_b\dot{\theta}_a(t) \tag{9}$$
$$\tilde{T}_m(t)=\frac{P_hF_t(t)}{2\pi\eta\tilde{N}} \tag{10}$$
$$y(t)=\frac{P_h\theta_a(t)}{2\pi\tilde{N}}, \tag{11}$$

where $\tilde{T}_a(t)\in\mathbf{R}$ is the motor torque, $T_b(t)\in\mathbf{R}$ is the output moment of motor, J_a is the moment of inertia of motor, B_a is the viscous friction coefficient of motor, $\theta_a(t)\in\mathbf{R}$ is the output rotation angle of motor, K_m is the electromagnetic torque coefficient, $u_c(t)\in\mathbf{R}$ is the input voltage, $\tilde{T}_m(t)\in\mathbf{R}$ is the output moment of electric cylinder, J_b is the moment of inertia of drive system, B_b is the viscous friction coefficient of drive system, $F_t(t)\in\mathbf{R}$ is the load force, P_h is screw lead, η is the mechanical transmission efficiency, $\tilde{N}$ is the transmission ratio, and $y(t)\in\mathbf{R}$ is the stretching length of piston rod.

Shown as Fig. 2, a is the corresponding vertex angle of the electric cylinder, a_0 is the initial vertex angle of the electric cylinder, l_0 is the initial length of electric cylinder, l_a is the distance between the center of the trunnion and the installation position of the electric cylinder on the turret, l_{dt} is the distance between the center of the trunnion and the driving point location of electric cylinder on the cradle. It shows that $a = \theta_2 + a_0$, and then the displacement of piston rod can be expressed by

$$\begin{aligned} y &= (l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(a))^{\frac{1}{2}} - l_0 \\ &= (l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{\frac{1}{2}} - l_0. \end{aligned} \tag{12}$$

Taking it into (11), we have

$$\theta_a = \frac{2\pi\tilde{N}}{P_h}\Big[(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{\frac{1}{2}} - l_0\Big]. \tag{13}$$

By differentiation, we have

$$\begin{aligned} \dot{\theta}_a = \frac{2\pi\tilde{N} l_a l_{dt}}{P_h}\Big[&(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{1}{2}} \\ &\times \sin(\theta_2 + a_0)\dot{\theta}_2\Big], \end{aligned} \tag{14}$$

and

$$\begin{aligned} \ddot{\theta}_a = \frac{2\pi\tilde{N} l_a l_{dt}}{P_h}\Big[&-l_a l_{dt}(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{3}{2}} \\ &\times \sin^2(\theta_2 + a_0)\dot{\theta}_2^2 + (l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{1}{2}} \\ &\times \cos(\theta_2 + a_0)\dot{\theta}_2^2 + (l_a^2 + l_{dt}^2 - 2l_a l_{dt} \\ &\times \cos(\theta_2 + a_0))^{-\frac{1}{2}}\sin(\theta_2 + a_0)\ddot{\theta}_2\Big]. \end{aligned} \tag{15}$$

With (7) to (9), we have

$$\tilde{T}_m = K_m u_c - (J_a + J_b)\ddot{\theta}_a - (B_a + B_b)\dot{\theta}_a. \tag{16}$$

Fig. 2 further shows that the torque input on the barrel described as (1b) is $\tau_2 = F_t l_{dt}$. With (10) and (16), we have

$$\begin{aligned} \tau_2 = &\frac{2\pi\eta\tilde{N} l_{dt} K_m}{P_h}u_c - \frac{2\pi\eta\tilde{N} l_{dt}(J_a + J_b)}{P_h}\ddot{\theta}_a \\ &- \frac{2\pi\eta\tilde{N} l_{dt}(B_a + B_b)}{P_h}\dot{\theta}_a. \end{aligned} \tag{17}$$

Using (14) and (15) in it, we have

$$\begin{aligned} \tau_2 = &\frac{2\pi\eta\tilde{N} l_{dt} K_m}{P_h}u_c - \frac{4\pi^2\eta\tilde{N}^2 l_a l_{dt}^2(J_a + J_b)}{P_h^2} \\ &\times \Big[-l_a l_{dt}(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{3}{2}} \\ &\times \sin^2(\theta_2 + a_0)\dot{\theta}_2^2 + (l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{1}{2}} \\ &\times \cos(\theta_2 + a_0)\dot{\theta}_2^2 + (l_a^2 + l_{dt}^2 - 2l_a l_{dt} \\ &\times \cos(\theta_2 + a_0))^{-\frac{1}{2}}\sin(\theta_2 + a_0)\ddot{\theta}_2\Big] \\ &- \frac{4\pi^2\eta\tilde{N}^2 l_a l_{dt}^2(B_a + B_b)}{P_h^2} \\ &\times \Big[(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{1}{2}}\sin(\theta_2 + a_0)\dot{\theta}_2\Big]. \end{aligned} \tag{18}$$

Let

$$\begin{aligned} &s_1 := \frac{2\pi\eta\tilde{N} l_{dt} K_m}{P_h}, \quad s_2 := \frac{4\pi^2\eta\tilde{N}^2 l_a l_{dt}^2(J_a + J_b)}{P_h^2}, \\ &s_3 := \frac{4\pi^2\eta\tilde{N}^2 l_a l_{dt}^2}{P_h^2}, \end{aligned} \tag{19}$$

we then have

$$\begin{aligned} \tau_2 = &s_1 u_c + s_2 l_a l_{dt}(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{3}{2}} \\ &\times \sin^2(\theta_2 + a_0)\dot{\theta}_2^2 - s_2(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{1}{2}} \\ &\times \cos(\theta_2 + a_0)\dot{\theta}_2^2 - s_2(l_a^2 + l_{dt}^2 - 2l_a l_{dt} \\ &\times \cos(\theta_2 + a_0))^{-\frac{1}{2}}\sin(\theta_2 + a_0)\ddot{\theta}_2 - s_3(B_a + B_b) \\ &\times (l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2 + a_0))^{-\frac{1}{2}}\sin(\theta_2 + a_0)\dot{\theta}_2. \end{aligned} \tag{20}$$

Taking (20) into (1b), a lengthy but straightforward algebra shows that the dynamic equation of the vertical pointing tracking system can be expressed as

$$\begin{aligned} &\ddot{\theta}_2(t) \\ &= \Big[\frac{1}{3}m_2 R_2^2 + s_2(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2(t) + a_0))^{-\frac{1}{2}} \\ &\quad \times \sin(\theta_2(t) + a_0)\Big]^{-1}\Big\{s_2 l_a l_{dt}(l_a^2 + l_{dt}^2 \\ &\quad - 2l_a l_{dt}\cos(\theta_2(t) + a_0))^{-\frac{3}{2}}\sin^2(\theta_2(t) + a_0)\dot{\theta}_2^2(t) \\ &\quad - s_2(l_a^2 + l_{dt}^2 - 2l_a l_{dt}\cos(\theta_2(t) + a_0))^{-\frac{1}{2}} \\ &\quad \times \cos(\theta_2(t) + a_0)\dot{\theta}_2^2(t) - s_3(B_a + B_b)(l_a^2 + l_{dt}^2 \\ &\quad - 2l_a l_{dt}\cos(\theta_2(t) + a_0))^{-\frac{1}{2}}\sin(\theta_2(t) + a_0)\dot{\theta}_2(t) \\ &\quad - \frac{1}{2}m_2 R_1 R_2\sin\theta_2(t)\dot{\theta}_1^2(t) - \frac{1}{6}m_2 R_2^2\sin(2\theta_2(t))\dot{\theta}_1^2(t) \\ &\quad - \frac{1}{2}m_2\tilde{g}R_2\cos\theta_2(t) - d_2(t) + s_1 u_c(t)\Big\}. \end{aligned} \tag{21}$$

Remark: The dynamic equation of the vertical pointing tracking system is derived from the vertical dynamics (1b) of the turret-barrel system and the dynamics of electric cylinder (7) to (11). It is nonlinear and contains the information of both the turret-barrel system and the electric cylinder; hence, also a nonlinear coupled system.

III. Problem Statement: Pointing Tracking of Marching Turret-Barrel Systems

The problem of pointing tracking for marching turret-barrel systems can be formulated as a control problem of driving the turret angular position $\hat{\theta}_1(t)$ and the barrel angular position $\hat{\theta}_2(t)$ to track a desired reference command signal $\theta_1^d(t)$ and $\theta_2^d(t)$ with a satisfactory performance, regardless of the uncertainty. Assume that $\theta_1^d(\cdot) : [t_0, \infty] \to \mathbf{R}$ and $\theta_2^d(\cdot) : [t_0, \infty] \to \mathbf{R}$ are of class C^2. Denote the tracking errors as

$$e_1(t) := \hat{\theta}_1(t) - \theta_1^d(t) = \theta_1(t) + \bar{\theta}_1(t) - \theta_1^d(t), \tag{22}$$

$$e_2(t) := \hat{\theta}_2(t) - \theta_2^d(t) = \theta_2(t) + \bar{\theta}_2(t) - \theta_2^d(t), \tag{23}$$

and then we have,

$$\theta_1(t) = e_1(t) - \bar{\theta}_1(t) + \theta_1^d(t),$$

$$\begin{aligned}\dot{\theta}_1(t) &= \dot{e}_1(t) - \dot{\bar{\theta}}_1(t) + \dot{\theta}_1^d(t),\\ \ddot{\theta}_1(t) &= \ddot{e}_1(t) - \ddot{\bar{\theta}}_1(t) + \ddot{\theta}_1^d(t),\\ \theta_2(t) &= e_2(t) - \bar{\theta}_2(t) + \theta_2^d(t),\\ \dot{\theta}_2(t) &= \dot{e}_2(t) - \dot{\bar{\theta}}_2(t) + \dot{\theta}_2^d(t),\\ \ddot{\theta}_2(t) &= \ddot{e}_2(t) - \ddot{\bar{\theta}}_2(t) + \ddot{\theta}_2^d(t).\end{aligned} \tag{24}$$

Let $x_1 := [x_{11}\ x_{12}]^T = [e_1\ \dot{e}_1]^T$ and $x_2 := [x_{21}\ x_{22}]^T = [e_2\ \dot{e}_2]^T$. Using them in (24), yields

$$\begin{aligned}\theta_1(t) &= x_{11}(t) - \bar{\theta}_1(t) + \theta_1^d(t),\\ \dot{\theta}_1(t) &= x_{12}(t) - \dot{\bar{\theta}}_1(t) + \dot{\theta}_1^d(t),\\ \ddot{\theta}_1(t) &= \dot{x}_{12}(t) - \ddot{\bar{\theta}}_1(t) + \ddot{\theta}_1^d(t),\\ \theta_2(t) &= x_{21}(t) - \bar{\theta}_2(t) + \theta_2^d(t),\\ \dot{\theta}_2(t) &= x_{22}(t) - \dot{\bar{\theta}}_2(t) + \dot{\theta}_2^d(t),\\ \ddot{\theta}_2(t) &= \dot{x}_{21}(t) - \ddot{\bar{\theta}}_2(t) + \ddot{\theta}_2^d(t).\end{aligned} \tag{25}$$

Introducing the resulted $\theta_2,\ \dot{\theta}_{1,2},\ \ddot{\theta}_1$ in (25) into (6) and focusing on $\dot{x}_{12}$, we have

$$\begin{aligned}\dot{x}_{12} =& \left(M_1 + JN^2\right)^{-1} G_1 H_1 (x_{12} - \dot{\bar{\theta}}_1 + \dot{\theta}_1^d)\\ &- \left(M_1 + JN^2\right)^{-1} (k_e N^2 + B_m N^2)(x_{12} - \dot{\bar{\theta}}_1 + \dot{\theta}_1^d)\\ &+ k_t N \left(M_1 + JN^2\right)^{-1} u_m\\ &+ \ddot{\bar{\theta}}_1 - \ddot{\theta}_1^d - \left(M_1 + JN^2\right)^{-1} (d_1 - d_t),\end{aligned} \tag{26}$$

with the definitions

$$\begin{aligned}M_1(x_{21}) :=& \frac{1}{2} m_1 R_1^2 + m_2 R_1^2 + m_2 R_1 R_2 \cos(x_{21} - \bar{\theta}_2 + \theta_2^d)\\ &+ \frac{1}{3} m_2 R_2^2 \cos^2(x_{21} - \bar{\theta}_2 + \theta_2^d),\end{aligned} \tag{27}$$

$$\begin{aligned}G_1(x_{21}) :=& m_2 R_1 R_2 \sin(x_{21} - \bar{\theta}_2 + \theta_2^d)\\ &+ \frac{1}{3} m_2 R_2^2 \sin(2x_{21} - 2\bar{\theta}_2 + 2\theta_2^d),\end{aligned} \tag{28}$$

$$H_1(x_{22}) := x_{22} - \dot{\bar{\theta}}_2 + \dot{\theta}_2^d, \tag{29}$$

Furthermore, introducing the resulted $\theta_2,\ \dot{\theta}_{1,2},\ \ddot{\theta}_2$ in (25) into (21) and focusing on $\dot{x}_{22}$, we have

$$\begin{aligned}\dot{x}_{22} =& M_2 G_2 \left(x_{22} - \dot{\bar{\theta}}_2 + \dot{\theta}_2^d\right)^2 + M_2 H_2 (B_a + B_b)\\ &\times \left(x_{22} - \dot{\bar{\theta}}_2 + \dot{\theta}_2^d\right) + M_2 Y_2 \left(x_{12} - \dot{\bar{\theta}}_1 + \dot{\theta}_1^d\right)^2\\ &- \frac{1}{2} M_2 m_2 \tilde{g} R_2 \cos(x_{21} - \bar{\theta}_2 + \theta_2^d)\\ &+ M_2 s_1 u_c + \ddot{\bar{\theta}}_2 - \ddot{\theta}_2^d - M_2 d_2.\end{aligned} \tag{30}$$

with the definitions

$$\begin{aligned}M_2(x_{21}) :=& \Big[\frac{1}{3} m_2 R_2^2 + s_2 (l_a^2 + l_{dt}^2\\ &- 2 l_a l_{dt} \cos(x_{21} - \bar{\theta}_2 + \theta_2^d + a_0))^{-\frac{1}{2}}\\ &\times \sin(x_{21} - \bar{\theta}_2 + \theta_2^d + a_0)\Big]^{-1},\end{aligned} \tag{31}$$

$$\begin{aligned}G_2(x_{21}) :=& s_2 l_a l_{dt} (l_a^2 + l_{dt}^2\\ &- 2 l_a l_{dt} \cos(x_{21} - \bar{\theta}_2 + \theta_2^d + a_0))^{-\frac{3}{2}}\\ &\times \sin^2(x_{21} - \bar{\theta}_2 + \theta_2^d + a_0) - s_2 (l_a^2 + l_{dt}^2\\ &- 2 l_a l_{dt} \cos(x_{21} - \bar{\theta}_2 + \theta_2^d + a_0))^{-\frac{1}{2}}\\ &\times \cos(x_{21} - \bar{\theta}_2 + \theta_2^d + a_0),\end{aligned} \tag{32}$$

$$\begin{aligned}H_2(x_{21}) :=& -s_3 (l_a^2 + l_{dt}^2\\ &- 2 l_a l_{dt} \cos(x_{21} - \bar{\theta}_2 + \theta_2^d + a_0))^{-\frac{1}{2}}\\ &\times \sin(x_{21} - \bar{\theta}_2 + \theta_2^d + a_0),\end{aligned} \tag{33}$$

$$\begin{aligned}Y_2(x_{21}) :=& -\frac{1}{2} m_2 R_1 R_2 \sin(x_{21} - \bar{\theta}_2 + \theta_2^d)\\ &- \frac{1}{6} m_2 R_2^2 \sin(2(x_{21} - \bar{\theta}_2 + \theta_2^d)).\end{aligned} \tag{34}$$

For the horizontal pointing tracking system (26), consider the moment of inertia of the motor J, the viscous damping coefficient of the motor B_m, the external disturbance d_1, and the transmission error d_t as the uncertainty, and decompose them into nominal and uncertain portions as: $J = \bar{J} + \Delta J,\ B_m = \bar{B}_m + \Delta B_m,\ d_1 = \bar{d}_1 + \Delta d_1,\ d_t = \bar{d}_t + \Delta d_t$, where $\bar{J},\ \bar{B}_m,\ \bar{d}_1,\ \bar{d}_t$ are the nominal portions, and $\Delta J,\ \Delta B_m,\ \Delta d_1,\ \Delta d_t$ are the uncertain portions, which are possibly fast time-varying but bounded, and the bounds are $|\Delta J| \le \hat{J}$, $|\Delta B_m| \le \hat{B}_m$, $|\Delta d_1| \le \hat{d}_1$, and $|\Delta d_t| \le \hat{d}_t$. Let $D := (M_1 + JN^2)^{-1}$. With the decomposition of J, we then have

$$\begin{aligned}&D(x_{21}, \Delta J)\\ &= \frac{1}{M_1(x_{21}) + N^2 \bar{J}}\\ &\quad - \frac{N^2 \Delta J}{(M_1(x_{21}) + N^2 \bar{J})(M_1(x_{21}) + N^2 \bar{J} + N^2 \Delta J)}\\ &=: \bar{D}(x_{21}) + \Delta D(x_{21}, \Delta J).\end{aligned} \tag{35}$$

As $|\Delta J| \le \hat{J}$, ΔD has a bound $|\Delta D| \le \hat{D}$. Let $\sigma_1 := [\Delta J\ \ \Delta B_m\ \ \Delta d_1\ \ \Delta d_t]^T$. By introducing the decompositions of $J,\ B_m,\ d_1,\ d_t,\ D$ into (26), and classifying the "nominal" and "uncertain" portions, it can be rewritten as

$$\dot{x}_{12} = \bar{f}_{12} + \Delta f_{12} + (B_{12} + \Delta B_{12})\,(u_m - \varpi_m)\,, \tag{36}$$

with the definition of $\bar{f}_{12}(\cdot) \in \mathbf{R}$ as a function of $x_{11}, x_{12}, x_{21}, x_{22}$ and t, and

$$\begin{aligned}&\Delta f_{12}(x_{11}, x_{12}, x_{21}, x_{22}, \sigma_1, t)\\ &:= \Big[\left(G_1(x_{21}) H_1(x_{22}) - k_e N^2 - N^2 \bar{B}_m\right) \Delta D(x_{21}, \Delta J)\\ &\quad - N^2 \bar{D}(x_{21}) \Delta B_m - N^2 \Delta D(x_{21}, \Delta J) \Delta B_m\Big]\\ &\quad \times (x_{12} - \dot{\bar{\theta}}_1 + \dot{\theta}_1^d)\\ &\quad - (\bar{d}_1 - \bar{d}_t) \Delta D(x_{21}, \Delta J) - \bar{D}(x_{21})(\Delta d_1 - \Delta d_t)\\ &\quad - \Delta D(x_{21}, \Delta J)(\Delta d_1 - \Delta d_t) + \omega(x_{21}, x_{22})\\ &\quad + f_{12}(x_{12}, x_{21}, x_{22})\\ &\quad + \underbrace{\bar{D}^{-1}(x_{21}) \Delta D(x_{21}, \Delta J) \bar{f}_{12}(x_{11}, x_{12}, t)}_{=\Delta B_{12} B_{12}^{-1} \bar{f}_{12}},\end{aligned} \tag{37}$$

$$B_{12}(x_{21}) := k_t N \bar{D}(x_{21}), \tag{38}$$

$$\Delta B_{12}(x_{21}, \sigma_1) := k_t N \Delta D(x_{21}, \Delta J), \tag{39}$$

$$\begin{aligned}&\varpi_m(x_{11}, x_{12}, x_{21}, x_{22}, t)\\ &:= B_{12}^{-1}(x_{21}) \bar{f}_{12}(x_{11}, x_{12}, x_{21}, x_{22}, t),\end{aligned} \tag{40}$$

where

$$\begin{aligned} & f_{12}(x_{12}, x_{21}, x_{22}) \\ & \quad := \Big(G_1(x_{21})H_1(x_{22})\bar{D}(x_{21}) - k_e N^2 \bar{D}(x_{21}) \\ & \qquad - N^2 \bar{D}(x_{21})\bar{B}_m\Big) x_{12}, \quad (41) \\ & \omega(x_{21}, x_{22}) \\ & \quad = \Big(G_1(x_{21})H_1(x_{22})\bar{D}(x_{21}) \\ & \qquad - k_e N^2 \bar{D}(x_{21}) - N^2 \bar{D}(x_{21})\bar{B}_m\Big)(-\dot{\bar{\theta}}_1 + \dot{\theta}_1^d) \\ & \qquad + \ddot{\bar{\theta}}_1 - \ddot{\theta}_1^d - \bar{D}(x_{21})(\bar{d}_1 - \bar{d}_t). \quad (42) \end{aligned}$$

As a result, the horizontal pointing tracking system (6) can be written in state-space form as

$$\begin{aligned} \dot{x}_1(t) = & \begin{bmatrix} x_{12}(t) \\ \bar{f}_{12}(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), t) \end{bmatrix} \\ & + \begin{bmatrix} 0 \\ \Delta f_{12}(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), \sigma_1(t), t) \end{bmatrix} \\ & + \left(\begin{bmatrix} 0 \\ B_{12}(x_{21}) \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta B_{12}(x_{21}(t), \sigma_1(t)) \end{bmatrix} \right) \\ & \times \left(u_m(t) - \varpi_m(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), t)\right). \quad (43) \end{aligned}$$

For the vertical pointing tracking system (30), consider the viscous friction coefficient of the motor B_a, the viscous friction coefficient of the drive system B_b, and the external disturbances d_2 as the uncertainty, and decompose them into nominal and uncertain portions as: $B_a = \bar{B}_a + \Delta B_a$, $B_b = \bar{B}_b + \Delta B_b$, and $d_2 = \bar{d}_2 + \Delta d_2$, where $\bar{B}_a$, $\bar{B}_b$, $\bar{d}_2$ are the nominal portions, and ΔB_a, ΔB_b, Δd_2 are the uncertain portions, which are possibly fast time-varying but bounded, and the bounds are $|\Delta B_a| \leq \Delta \hat{B}_a$, $|\Delta B_b| \leq \Delta \hat{B}_b$, and $|\Delta d_2| \leq \Delta \hat{d}_2$. Let $\sigma_2 := [\Delta B_a \ \Delta B_b \ \Delta d_2]^T$. By introducing the decompositions of B_a, B_b, d_2 into (30), and classifying the "nominal" and "uncertain" portions, it can be rewritten as

$$\dot{x}_{22} = \bar{f}_{22} + \Delta f_{22} + B_{22}(u_c - \varpi_c), \quad (44a)$$

with the definition of $\bar{f}_{22}(\cdot) \in \mathbf{R}$ as a function of x_{11}, x_{12}, x_{21}, x_{22} and t, and

$$\begin{aligned} & \Delta f_{22}(x_{12}, x_{21}, x_{22}, \sigma_2, t) \\ & \quad := M_2(x_{21})H_2(x_{21})(\Delta B_a + \Delta B_b)\Big(x_{22} - \dot{\bar{\theta}}_2 + \dot{\theta}_2^d\Big) \\ & \qquad - M_2(x_{21})\Delta d_2 + f_{22}(x_{12}, x_{21}, x_{22}), \quad (45) \\ & B_{22}(x_{21}) := M_2(x_{21})s_1, \quad (46) \\ & \varpi_c(x_{11}, x_{12}, x_{21}, x_{22}, t) := B_{22}^{-1}(x_{21})\bar{f}_{22}(x_{11}, x_{12}, x_{21}, x_{22}, t), \quad (47) \end{aligned}$$

where

$$\begin{aligned} & f_{22}(x_{12}, x_{21}, x_{22}) \\ & \quad := M_2(x_{21})G_2(x_{21})\Big(x_{22} - \dot{\bar{\theta}}_2 + \dot{\theta}_2^d\Big)^2 \\ & \qquad + M_2(x_{21})Y_2(x_{21})\Big(x_{12} - \dot{\bar{\theta}}_1 + \dot{\theta}_1^d\Big)^2 \\ & \qquad - \frac{1}{2}M_2(x_{21})m_2\tilde{g}R_2\cos(x_{21} - \bar{\theta}_2 + \theta_2^d) \\ & \qquad + M_2(x_{21})H_2(x_{21})\Big(\bar{B}_a + \bar{B}_b\Big)\Big(x_{22} - \dot{\bar{\theta}}_2 + \dot{\theta}_2^d\Big) \\ & \qquad + \ddot{\bar{\theta}}_2 - \ddot{\theta}_2^d - M_2(x_{21})\bar{d}_2. \quad (48) \end{aligned}$$

As a result, the vertical pointing tracking system can be written in state-space form as

$$\begin{aligned} \dot{x}_2(t) = & \begin{bmatrix} x_{22}(t) \\ \bar{f}_{22}(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), t) \end{bmatrix} \\ & + \begin{bmatrix} 0 \\ \Delta f_{22}(x_{12}(t), x_{21}(t), x_{22}(t), \sigma_2(t), t) \end{bmatrix} \\ & + \begin{bmatrix} 0 \\ B_{22}(x_{21}(t)) \end{bmatrix} \\ & \times \left(u_c(t) - \varpi_c(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), t)\right). \quad (49) \end{aligned}$$

Let $x := \left[x_1^T \ x_2^T\right]^T$, $u_1 := [u_m \ u_c]^T$, $u_2 := [\varpi_m \ \varpi_c]^T$, $u := u_1 - u_2$, and $\sigma := \left[\sigma_1^T \ \sigma_2^T\right]^T$. With (43) and (49), the whole pointing tracking system can be written in state-space form as

$$\begin{aligned} \dot{x}(t) = & \begin{bmatrix} x_{12}(t) \\ \bar{f}_{12}(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), t) \\ x_{22}(t) \\ \bar{f}_{22}(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), t) \end{bmatrix} \\ & + \begin{bmatrix} 0 \\ \Delta f_{12}(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), \sigma_1(t), t) \\ 0 \\ \Delta f_{22}(x_{12}(t), x_{21}(t), x_{22}(t), \sigma_2(t), t) \end{bmatrix} \\ & + \left(\begin{bmatrix} 0 & 0 \\ B_{12}(x_{21}(t)) & 0 \\ 0 & 0 \\ 0 & B_{22}(x_{21}(t)) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \Delta B_{12}(x_{21}(t), \sigma_1(t)) & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \right) \\ & \times \begin{bmatrix} u_m(t) - \varpi_m(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), t) \\ u_c(t) - \varpi_c(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t), t) \end{bmatrix} \\ =: & \ f(x(t), t) + \Delta f(x(t), \sigma(t), t) \\ & + \left(B(x(t)) + \Delta B(x(t), \sigma(t))\right) u(t), \quad (50) \end{aligned}$$

where $t \in \mathbf{R}$ is the time, $x(t) \in \mathbf{R}^4$ is the state, and $u(t) \in \mathbf{R}^2$ is the control, and $\sigma(t) \in \Sigma \subset \mathbf{R}^7$ is the uncertain parameter. $\Sigma \subset \mathbf{R}^7$ is compact and unknown, and stands for the possible bounding of σ. what is more, $f(x, t)$, $\Delta f(x, \sigma, t)$, $B(x)$, $\Delta B(x, \sigma)$ and $\varpi(x, t)$ are matrices of appropriate dimensions, the functions $f(\cdot)$, $\Delta f(\cdot)$, $B(\cdot)$, $\Delta B(\cdot)$ and $\varpi(\cdot)$ are continuous, and can be generalized to be Lebesgue measurable in t. Note that, $u_1(t) = u(t) + u_2(t)$ is the actual control input of the control actuator (i.e., the motor servo system and the electric cylinder system).

Remark: The later proposed control is designed under the guidance of the boundedness control theory that was proposed by Corless and Leitmann in 1981 [35]. According to such theory, the functions $\bar{f}_{12}(\cdot)$ and $\bar{f}_{22}(\cdot)$ should be chosen to let the uncontrolled nominal systems $\dot{x}(t) = f(x(t), t)$ to be uniformly asymptotically stable at the origins $x = 0$.

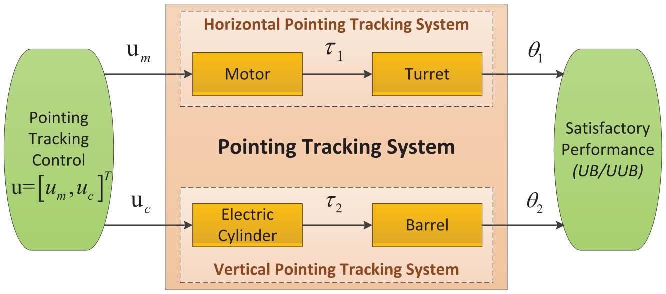

Fig. 3. Structure of pointing tracking system.

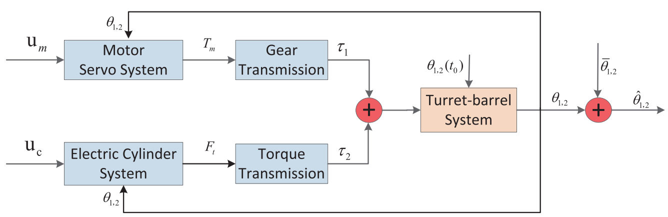

Fig. 4. Block diagram of the control procedure and signals.

Remark: Shown as Fig. 3, the problem of pointing tracking for turret-barrel system can be solved by designing appropriate control u to drive the pointing tracking system (50) to render satisfactory performance (i.e., uniform boundedness and uniform ultimate boundedness). Meanwhile, the corresponding control procedure and signals are shown as Fig. 4.

IV. Adaptive Robust Control for Pointing Tracking

An adaptive robust control scheme is proposed for pointing tracking of turret-barrel system in this Section. First, the bounding and structural conditions of the concerned system (50) is discussed. Choose functions $\bar{f}_{12}(\cdot)$ and $\bar{f}_{22}(\cdot)$ to let $f(0,t) = 0$ for all $t \in \mathbf{R}$ and the uncontrolled nominal system $\dot{x}(t) = f(x(t),t)$ to be uniformly asymptotically stable at the origin $x = 0$, and then there are a C^1 function $V_1(\cdot) : \mathbf{R}^4 \times \mathbf{R} \to \mathbf{R}_+$ and continuous, strictly increasing functions $\gamma_i(\cdot) : \mathbf{R}_+ \to \mathbf{R}_+$, $i = 1, 2, 3$, which satisfy (see [36], [37])

$$\gamma_i(0) = 0, \quad i = 1,2,3 \tag{51}$$

$$\lim_{r\to\infty} \gamma_i(r) = \infty, \quad i = 1,2 \tag{52}$$

such that for all $(x_1, t) \in \mathbf{R}^2 \times \mathbf{R}$

$$\gamma_1(\|x\|) \leqslant V_1(x,t) \leqslant \gamma_2(\|x\|), \tag{53}$$

$$\mathscr{L}_0(x,t) := \frac{\partial V_1(x,t)}{\partial t} + \nabla_x^T V_1(x,t) f(x,t) \leqslant -\gamma_3(\|x\|). \tag{54}$$

We then discuss the bounding condition for the uncertain portions Δf and ΔB of system (50). First, focus on Δf. It can be decomposed as

$$\Delta f(x,\sigma,t) = B(x)h(x,\sigma,t), \tag{55}$$

$$\Delta B(x,\sigma) = B(x)E(x,\sigma). \tag{56}$$

with

$$h(x,\sigma,t) = \begin{bmatrix} B_{12}^{-1}(x_{21})\Delta f_{12}(x_{11},x_{12},x_{21},x_{22},\sigma_1,t) \\ B_{22}^{-1}(x_{21})\Delta f_{22}(x_{12},x_{21},x_{22},\sigma_2,t) \end{bmatrix} =: \begin{bmatrix} h_{12}(x_{11},x_{12},x_{21},x_{22},\sigma_1,t) \\ h_{22}(x_{12},x_{21},x_{22},\sigma_2,t) \end{bmatrix}, \tag{57}$$

$$E(x,\sigma) = \begin{bmatrix} B_{12}^{-1}(x_{21})\Delta B_{12}(x_{21},\sigma_1) & 0 \\ 0 & 0 \end{bmatrix} =: \begin{bmatrix} E_{12}(x_{21},\sigma_1) & 0 \\ 0 & 0 \end{bmatrix}. \tag{58}$$

Focus on E_{12}, with B_{12} as (38), ΔB_{12} as (39), and $\bar{D}$, ΔD as in (35), we have

$$E_{12} = B_{12}^{-1}\Delta B_{12} = (k_t N\bar{D})^{-1} k_t N \Delta D = \frac{-N^2\Delta J}{M_1 + N^2\bar{J} + N^2\Delta J}. \tag{59}$$

Recalling M_1 as in (27), and then we have

$$\underline{M}_1 \le M_1 \le \overline{M}_1, \tag{60}$$

with

$$\underline{M}_1 = \frac{1}{2}m_1R_1^2 + m_2R_1^2 - m_2R_1R_2,$$
$$\overline{M}_1 = \frac{1}{2}m_1R_1^2 + m_2R_1^2 + m_2R_1R_2 + \frac{1}{3}m_2R_2^2. \tag{61}$$

Define a function

$$\eta(\Delta J) := \frac{-N^2\Delta J}{M_1 + N^2\bar{J} + N^2\Delta J}, \tag{62}$$

and then take its first order derivative respect to ΔJ to yield

$$\frac{\partial\eta}{\partial\Delta J} = -\frac{N^2M_1 + N^4\bar{J}}{\left(M_1 + N^2\bar{J} + N^2\Delta J\right)^2} < 0, \tag{63}$$

with M_1, N, $\bar{J} > 0$; hence, $\eta(\cdot)$ is strictly decreasing in ΔJ, and we have

$$\eta \ge \frac{-N^2\Delta\overline{J}}{M_1 + N^2\bar{J} + N^2\Delta\overline{J}}. \tag{64}$$

By this, as usually $\Delta\overline{J} \ge 0$, we have

$$\frac{-N^2\Delta\overline{J}}{M_1 + N^2\bar{J} + N^2\Delta\overline{J}} > \frac{-N^2\Delta\overline{J}}{\overline{M}_1 + N^2\bar{J} + N^2\Delta\overline{J}} > -1; \tag{65}$$

hence, $E_{12} > -1$. It can be seen that there exists a constant $\rho_E > -1$, such that $\frac{1}{2}\lambda_{min}(E + E^T) \ge \rho_E$, λ_{min} denotes the minimum eigenvalue.

As a preliminary for control design, we now do some analysis specially on the bound of h. Recalling Δf_{12} as (37), B_{12} as (38), Δf_{22} as (45), and B_{22} as (46), we have

$$\begin{aligned} h_{12} &= B_{12}^{-1}\Delta f_{12} \\ &= B_{12}^{-1}\Big\{\Big[\Big(G_1H_1 - k_eN^2 - N^2\bar{B}_m\Big)\Delta D - N^2\bar{D}\Delta B_m \\ &\quad - N^2\Delta D\Delta B_m\Big](x_{12} - \dot{\bar{\theta}}_1 + \dot{\theta}_1^d)\text{-}(\bar{d}_1 - \bar{d}_t)\Delta D \\ &\quad - \bar{D}(\Delta d_1 - \Delta d_t) - \Delta D(\Delta d_1 - \Delta d_t) + \omega + f_{12} \\ &\quad + \bar{D}^{-1}\bar{f}_{12}\Delta D\Big\}, \end{aligned} \tag{66}$$

$$\begin{aligned} h_{22} &= B_{22}^{-1}\Delta f_{22} \\ &= B_{22}^{-1}\left\{M_2H_2\left(\Delta B_a+\Delta B_b\right)\left(x_{22}-\dot{\bar{\theta}}_2+\dot{\theta}_2^d\right)\right. \\ &\quad \left. -M_2\Delta d_2+f_{22}\right\}. \end{aligned} \tag{67}$$

Aiming at $\|h\|$, we have

$$\begin{aligned} \|h\| &\le \|h_{12}\|+\|h_{22}\| \\ &\le B_{12}^{-1}\left(\left\|\left(G_1H_1-k_eN^2-N^2\bar{B}_m\right)\right.\right. \\ &\qquad \left.\left.\times\left(x_{12}-\dot{\bar{\theta}}_1+\dot{\theta}_1^d\right)\right\|+\left\|\bar{d}_1-\bar{d}_t-\bar{D}^{-1}\bar{f}_{12}\right\|\right)\hat{D} \\ &\quad +B_{12}^{-1}N^2\bar{D}\left\|x_{12}-\dot{\bar{\theta}}_1+\dot{\theta}_1^d\right\|\hat{B}_m \\ &\quad +B_{12}^{-1}N^2\left\|x_{12}-\dot{\bar{\theta}}_1+\dot{\theta}_1^d\right\|\hat{D}\hat{B}_m \\ &\quad +B_{12}^{-1}\bar{D}\left(\hat{d}_1+\hat{d}_t\right)+B_{12}^{-1}\hat{D}\left(\hat{d}_1+\hat{d}_t\right) \\ &\quad +B_{12}^{-1}\|\omega+f_{12}\| \\ &\quad +\left\|B_{22}^{-1}M_2H_2\left(x_{22}-\dot{\bar{\theta}}_2+\dot{\theta}_2^d\right)\right\|\left(\hat{B}_a+\hat{B}_b\right) \\ &\quad +\left\|B_{22}^{-1}M_2\right\|\hat{d}_2+\left\|B_{22}^{-1}f_{22}\right\|. \end{aligned} \tag{68}$$

Let

$$\begin{aligned} \rho_1 &:= B_{12}^{-1}\left(\left\|\left(G_1H_1-k_eN^2-N^2\bar{B}_m\right)\right.\right. \\ &\qquad \left.\left.\times\left(x_{12}-\dot{\bar{\theta}}_1+\dot{\theta}_1^d\right)\right\|+\left\|\bar{d}_1-\bar{d}_t-\bar{D}^{-1}\bar{f}_{12}\right\|\right), \end{aligned} \tag{69}$$

$$\rho_2 := B_{12}^{-1}N^2\bar{D}\left\|x_{12}-\dot{\bar{\theta}}_1+\dot{\theta}_1^d\right\|, \tag{70}$$

$$\rho_3 := B_{12}^{-1}N^2\left\|x_{12}-\dot{\bar{\theta}}_1+\dot{\theta}_1^d\right\|, \tag{71}$$

$$\rho_4 := B_{12}^{-1}\bar{D}, \tag{72}$$

$$\rho_5 := B_{12}^{-1}, \tag{73}$$

$$\rho_6 := \left\|B_{22}^{-1}M_2H_2\left(x_{22}-\dot{\bar{\theta}}_2+\dot{\theta}_2^d\right)\right\|, \tag{74}$$

$$\rho_7 := \left\|B_{22}^{-1}M_2\right\|, \tag{75}$$

$$\rho_8 := B_{12}^{-1}\|\omega+f_{12}\|+\left\|B_{22}^{-1}f_{22}\right\|. \tag{76}$$

We then can rewrite (68) as

$$\begin{aligned} \|h\| &\le \rho_1\hat{D}+\rho_2\hat{B}_m+\rho_3\hat{D}\hat{B}_m \\ &\quad +\rho_4\left(\hat{d}_1+\hat{d}_t\right)+\rho_5\hat{D}\left(\hat{d}_1+\hat{d}_t\right) \\ &\quad +\rho_6\left(\hat{B}_a+\hat{B}_b\right)+\rho_7\hat{d}_2+\rho_8 \\ &\le \left[\hat{D}^2+\hat{B}_m^2+\hat{D}^2\hat{B}_m^2+(\hat{d}_1+\hat{d}_t)^2\right. \\ &\quad +\hat{D}^2\left(\hat{d}_1+\hat{d}_t\right)^2+(\hat{B}_a+\hat{B}_b)^2+\hat{d}_2^2 \\ &\quad \left.+1\right]^{\frac{1}{2}}\left(\rho_1^2+\rho_2^2+\rho_3^2+\rho_4^2+\rho_5^2+\rho_6^2+\rho_7^2+\rho_8^2\right)^{\frac{1}{2}} \\ &=: \alpha\hat{\Pi}(x,t) \\ &=: \Pi(\alpha,x,t), \end{aligned} \tag{77}$$

where

$$\begin{aligned} \alpha &:= \left[\hat{D}^2+\hat{B}_m^2+\hat{D}^2\hat{B}_m^2+(\hat{d}_1+\hat{d}_t)^2\right. \\ &\quad \left.+\hat{D}^2\left(\hat{d}_1+\hat{d}_t\right)^2+(\hat{B}_a+\hat{B}_b)^2+\hat{d}_2^2+1\right]^{\frac{1}{2}}, \end{aligned} \tag{78}$$

$$\hat{\Pi} := \left(\rho_1^2+\rho_2^2+\rho_3^2+\rho_4^2+\rho_5^2+\rho_6^2+\rho_7^2+\rho_8^2\right)^{\frac{1}{2}}. \tag{79}$$

Note that, $\alpha \in \mathbf{R}^+$ reflects a combined effect of uncertainty bound. When we take α as a comprehensive influence measure, we actually take the bounds of the uncertainty to measure their influences. However, in fact, the actual influences of the uncertainty should not exceed their bounds, so when we take the bounds as the influence measure, we actually consider the maximum (most conservative) influence of the uncertainty. By this, this paper considers a rather conservative condition for the uncertainty that it is nonlinear and possibly fast time varying but bounded, and the bound is unknown; hence, α is unknown here. We design the following leakage type of adaptive law to evaluate α:

$$\dot{\hat{\alpha}}(t) = \frac{1}{2}k_1\hat{\Pi}(x(t),t)\left\|\nabla_x^T V_1(x(t),t)B(x(t),t)\right\| - k_2\hat{\alpha}(t), \tag{80}$$

where $k_{1,2} > 0$ are scalar constants. Based on this adaptive law, we then propose the following adaptive robust control for the system (50)

$$u(t) = -\gamma B^T(x(t),t)\nabla_x V_1(x(t),t)\Pi^2(\hat{\alpha}(t),x(t),t), \tag{81}$$

with a constant $\gamma > 0$.

Remark: Three kinds of uncertainty, including system modeling error, external disturbance and road excitation, are considered in the control design. For uncertainty handling, we first find a parameter α to measure the combined effect of the uncertainty in a rather conservative condition by taking the uncertainty bounds as influence measure. As α is unknown, an adaptive law described as (80) is designed to evaluate the unknown parameter online. By this, for control design, except that the uncertainty is bounded, no more information about the uncertainty is needed.

Theorem 1: Let $\varsigma(t) := \left[x^T(t)\ \left(\hat{\alpha}(t)-\alpha\right)^T\right]^T \in \mathbf{R}^5$. Suppose that the control (81) is applied to the system (50). The solution of the controlled system renders the following performance of practically stable:

(i) Uniform boundedness: For any $r > 0$, there is a $d(r) < \infty$ such that if $\|\varsigma(t_0)\| \le r$, then $\|\varsigma(t)\| \le d(r)$ for all $t \ge t_0$;

(ii) Uniform ultimate boundedness: For any $r > 0$ with $\|\varsigma(t_0)\| \le r$, there exists a $\underline{d} > 0$ such that $\|\varsigma(t)\| \le \overline{d}$ for any $\overline{d} > \underline{d}$ as $t \ge t_0 + T\left(\overline{d}, r\right)$, where $T\left(\overline{d}, r\right) < \infty$.

(iii) Uniform Stability: For any $\overline{d} > \underline{d}$, there is a constant $\psi(\bar{d}) > 0$ such that $\|\varsigma(t_0)\| \le \psi(\bar{d})$ implies that $\|\varsigma(t)\| \le \bar{d}$ for all $t \ge t_0$.

Proof: Choose $V\left(x, \hat{\alpha}-\alpha, t\right) = V_1(x,t) + V_2(\hat{\alpha}-\alpha)$ as a legitimate Lyapunov function candidate, where $V_1(x,t)$ subjects to (53) -(54) and $V_2(\hat{\alpha}-\alpha) = k_1^{-1}(\hat{\alpha}-\alpha)^T(\hat{\alpha}-\alpha)$. The Lyapunov derivative for the system (50) is given by

$$\begin{aligned} \mathcal{L} := \mathscr{L}_0 + \nabla_x^T V_1\left[\Delta f(x,\sigma,t) + (B+\Delta B(x,\sigma))u\right] \\ + 2k_1^{-1}\left(\hat{\alpha}-\alpha\right)^T\dot{\hat{\alpha}}. \end{aligned} \tag{82}$$

Recalling $\Delta f = Bh,\ \Delta B = BE$ and with (54), we have

$$\begin{aligned} \mathcal{L} \le -\gamma_3(\|x\|) + \nabla_x^T V_1\left[Bh(x,\sigma,t) + Bu\right. \\ \left. + BEu\right] + 2k_1^{-1}\left(\hat{\alpha}-\alpha\right)^T\dot{\hat{\alpha}} \end{aligned}$$

$$= -\gamma_3(\|x\|) + \nabla_x^T V_1 B h(x,\sigma,t) + \nabla_x^T V_1 B u + \nabla_x^T V_1 B E u + 2k_1^{-1}\left(\hat{\alpha}-\alpha\right)^T \dot{\hat{\alpha}}. \quad (83)$$

With (80) and (81), we have

$$\begin{aligned}\mathcal{L} \leq & -\gamma_3(\|x\|) + \nabla_x^T V_1 B h(x,\sigma,t) \\ & - \gamma\, \nabla_x^T V_1 B B^T \nabla_x V_1 \Pi^2(\hat{\alpha},x,t) \\ & - \gamma\, \nabla_x^T V_1 B E B^T \nabla_x V_1 \Pi^2(\hat{\alpha},x,t) \\ & + \left(\hat{\alpha}-\alpha\right)^T \hat{\Pi}(x,t) \left\|\nabla_x^T V_1 B\right\| - 2k_1^{-1}k_2\left(\hat{\alpha}-\alpha\right)^T \hat{\alpha},\end{aligned} \quad (84)$$

Let $\phi := \nabla_x^T V_1 B$, and then (84) can be rewritten as

$$\begin{aligned}\mathcal{L} \leq & -\gamma_3(\|x\|) + \|\phi\| \, \|h(x,\sigma,t)\| \\ & - \gamma\, \|\phi\|^2 \Pi^2(\hat{\alpha},x,t) - \gamma\, \phi E(x,\sigma,t)\phi^T \Pi^2(\hat{\alpha},x,t) \\ & + \left(\hat{\alpha}-\alpha\right)^T \hat{\Pi}(x,t)\, \|\phi\| - 2k_1^{-1}k_2\left(\hat{\alpha}-\alpha\right)^T \hat{\alpha}, \end{aligned} \quad (85)$$

Recalling $\|h\| \leq \Pi(\alpha,x,t)$ and (77), we have

$$\begin{aligned}\mathcal{L} \leq & -\gamma_3(\|x\|) - 2k_1^{-1}k_2\left(\hat{\alpha}-\alpha\right)^T \hat{\alpha} \\ & + \|\phi\|\, \Pi(\hat{\alpha},x,t) - \gamma\, \|\phi\|^2 \Pi^2(\hat{\alpha},x,t) \\ & - \gamma\, \phi E \phi^T \Pi^2(\hat{\alpha},x,t). \end{aligned} \quad (86)$$

Recalling $\frac{1}{2}\lambda_{min}(E+E^T) \geq \rho_E > -1$ and

$$\lambda_{\min}(E+E^T)\, \|\phi\|^2 \leq \phi\left(E+E^T\right)\phi^T, \quad (87)$$

we have

$$\begin{aligned}-\gamma\, \phi E \phi^T \Pi^2(\hat{\alpha},x,t) &= -\frac{1}{2}\gamma\, \phi\left(E+E^T\right)\phi^T \Pi^2(\hat{\alpha},x,t) \\ &\leq -\frac{1}{2}\gamma\, \lambda_{\min}(E+E^T)\, \|\phi\|^2 \Pi^2(\hat{\alpha},x,t) \\ &= -\gamma \rho_E\, \|\phi\|^2 \Pi^2(\hat{\alpha},x,t). \end{aligned} \quad (88)$$

We then have

$$\begin{aligned}\mathcal{L} \leq & -\gamma_3(\|x\|) - 2k_1^{-1}k_2\left(\hat{\alpha}-\alpha\right)^T \hat{\alpha} \\ & + \|\phi\|\, \Pi(\hat{\alpha},x,t) - \gamma\,(1+\rho_E)\, \|\phi\|^2 \Pi^2(\hat{\alpha},x,t) \\ \leq & -\gamma_3(\|x\|) - k_1^{-1}k_2 \left\|\hat{\alpha}-\alpha\right\|^2 \\ & + k_1^{-1}k_2\, \|\alpha\|^2 + \frac{1}{4\gamma\,(1+\rho_E)}. \end{aligned} \quad (89)$$

Recalling $\varsigma = \left[x^T, (\hat{\alpha}-\alpha)^T\right]^T$, there exits a function

$$\hat{\gamma}_3(\|\varsigma\|) \leq \gamma_3(\|x\|) + k_1^{-1}k_2 \left\|\hat{\alpha}-\alpha\right\|^2, \quad (90)$$

with which we have

$$\begin{aligned}\mathcal{L} \ \leq & \ -\hat{\gamma}_3(\|\varsigma\|) + k_1^{-1}k_2\, \|\alpha\|^2 + \frac{1}{4\gamma\,(1+\rho_E)} \\ =: & -\hat{\gamma}_3(\|\varsigma\|) + \delta, \end{aligned} \quad (91)$$

where

$$\delta = k_1^{-1}k_2\, \|\alpha\|^2 + \frac{1}{4\gamma\,(1+\rho_E)}. \quad (92)$$

Upon invoking the standard arguments as in ([35], [38]), the performance described as Theorem 1 is guaranteed. First, the uniform boundedness follows. That is, given any $r > 0$ with $\|\varsigma(t_0)\| \leq r$, where t_0 is the initial time, there is a $d(r)$ given by

$$d(r) = \begin{cases} (\gamma_1^{-1} \circ \gamma_2)(r), & if\ r > R, \\ (\gamma_1^{-1} \circ \gamma_2)(R), & if\ r \leq R, \end{cases} \quad (93)$$

with

$$R = \gamma_3^{-1}(\delta), \quad (94)$$

such that $\|\varsigma(t)\| \leq d(r)$ for all $t \geq t_0$. Second, the uniform ultimate boundedness follows. That is, given any $\overline{d}$ with

$$\overline{d} > (\gamma_1^{-1} \circ \gamma_2)(R), \quad (95)$$

we have $\|\varsigma(t)\| \leq \overline{d},\ \forall t \geq t_0 + T(\overline{d}, r)$, with

$$T(\overline{d}, r) = \begin{cases} 0, & if\ r \leq \overline{R}, \\ \dfrac{\gamma_2(r) - \gamma_1(\overline{R})}{\hat{\gamma}_3(\overline{R}) - \delta}, & otherwise, \end{cases} \quad (96)$$

$$\overline{R} = (\gamma_2^{-1} \circ \gamma_1)(\overline{d}). \quad (97)$$

Finally, the uniform stability follows by letting $\psi(\bar{d}) = R$. □

Remark: With the proposed adaptive robust control u as (81), the turret angular position $\hat{\theta}_1$ and the barrel angular position $\hat{\theta}_2$ can be driven to respectively track a desired reference command signal θ_1^d and θ_2^d with the performance of practically stable described as Theorem 1, regardless of the uncertainty σ; thereout, the control objective of pointing tracking is achieved.

Remark: It is worth emphasizing that, this paper addresses the pointing tracking system as a horizontal state x_1 and vertical state x_2 coupled dynamical system as (50). This is very different from the past works [7]–[10] that deal with pointing tracking in (horizontal and vertical) separate channel.

V. Design Procedure

This paper proposes an adaptive robust control for pointing tracking of marking turret-barrel system. The design procedure (shown as Fig. 5) can be summarized as:

(i) Obtain the dynamic equations of the horizontal and vertical pointing tracking systems as (6) and (21).

(ii) Recalling the definition of pointing tracking, define the tracking errors $e_{1,2}$ as (22), and rewrite the dynamic equations of the pointing tracking system in state-space form as (50).

(iii) Determine $\bar{f}_{12}(\cdot)$, $\bar{f}_{22}(\cdot)$ to let the uncontrolled nominal systems $\dot{x} = f(x,t)$ to be uniformly asymptotically stable at the origins $x = 0$, and choose $\gamma_{1,2,3}(\cdot)$, $V_1(\cdot)$ according to (51)-(54);

(iv) Discuss the bounding condition of the uncertain portions Δf in system (50) to yield $\Pi(\cdot)$ as (77) and $\hat{\Pi}(\cdot)$ as (79). Choosing $k_{1,2} > 0$ and with V_1 and $\hat{\Pi}$, design the adaptive law $\hat{\alpha}(\cdot)$ as (80).

(v) Choosing $\gamma > 0$ and with V_1, Π and the adaptive law $\hat{\alpha}$, design the adaptive robust control $u(\cdot)$ as (81).

Remark: The main rule for determining the functions $\bar{f}_{12}(\cdot)$ and $\bar{f}_{22}(\cdot)$ is to let the uncontrolled nominal systems $\dot{x}(t) = f(x(t),t)$ to be uniformly asymptotically stable at the origin $x = 0$. Specifically, for the sake of simplicity, we can choose linear functions $\bar{f}_{12}(\cdot)$ and $\bar{f}_{22}(\cdot)$ to let the uncontrolled

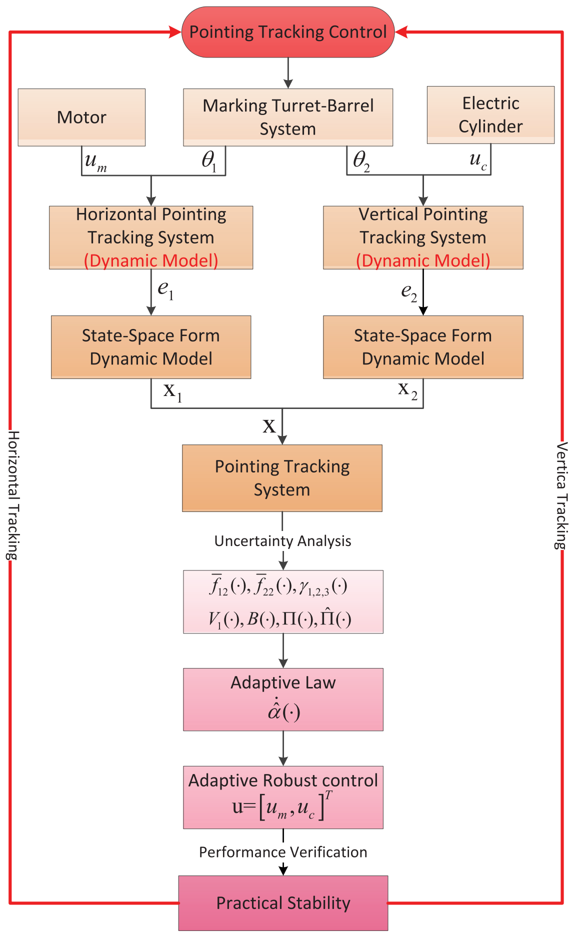

Fig. 5. Control design procedure.

nominal system to be a linear system $\dot{x}(t) = Ax(t)$ with A as a Hurwitz matrix.

Remark: There are three major advantages of the proposed control method. First, it has considered all the coupling, nonlinearity and uncertainty in tracking control. Second, it can handle rather complex time-varying uncertainty with unknown bound. Third, it can realize the pointing tracking of marching turret-barrel system in one horizontal-vertical coupled channel.

VI. Simulations

A. Parameter Selections

In the simulation, we desire the marking turret-barrel system to adjust its horizontal and vertical angular positions respectively from $\hat{\theta}_{1,2}(0) = 10rad$ to $\hat{\theta}_{1,2} = 5rad$. For this, we choose the desired signals $\theta_{1,2}^d = 5rad$, the initial tracking errors $e_{1,2}(0) = 5rad$ (i.e., $x_{11}(0) = x_{21}(0) = 5$), and other initial states $x_{12}(0) = x_{22}(0) = \hat{\alpha}(0) = 0$.

Three types of uncertainty of *system modeling error*, *external disturbance* and *road excitation* are considered. For system modeling error, choose $J = 0.021 + 0.0021\sin(10t)kg{\cdot}m^2$, $B_m = 0.0088 + 0.00088\sin(10t)N{\cdot}m{\cdot}s/rad$, $d_t = 10\sin(10t)N{\cdot}m$, $B_a = 0.0153 + 0.00153\sin(10t)N{\cdot}m{\cdot}s/rad$, $B_b = 0.0063 + 0.00063\sin(10t)N{\cdot}m{\cdot}s/rad$. For external disturbance, choose $d_1 = 10\sin(10t)N{\cdot}m$, $d_2 = 10\sin(10t)N{\cdot}m$ and $f_t = 10\sin(10t)N{\cdot}m$. For road excitation, choose

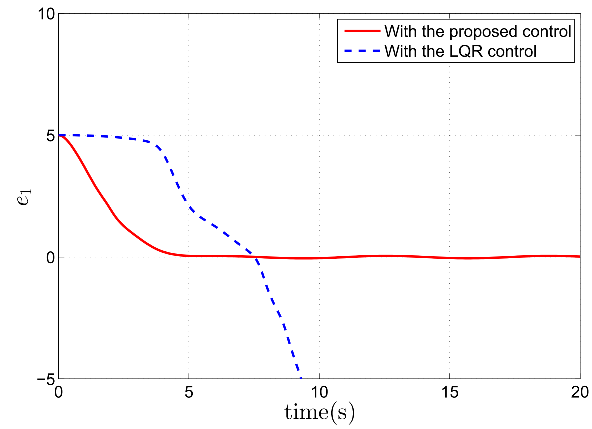

Fig. 6. Comparison of the tracking error e_1.

$\bar{\theta}_{1,2} = 0.1\sin(t)rad$, $\dot{\bar{\theta}}_{1,2} = 0.1\cos(t)rad/s$ and $\ddot{\bar{\theta}}_{1,2} = -0.1\sin(t)rad/s^2$.

To let the uncontrolled nominal system $\dot{x} = f(x,t)$ to be uniformly asymptotically stable at the origin $x = 0$, we choose the functions

$$\bar{f}_{12} = -a_1x_{11} - b_1x_{12}, \tag{98}$$

$$\bar{f}_{22} = -a_2x_{21} - b_2x_{22}. \tag{99}$$

According to (51)-(54), we choose $V_1 = x_1^T P_1 x_1 + x_2^T P_2 x_2$. By choosing $a_1 = 0,\ a_2 = 1,\ b_1 = -1,\ b_2 = 2$, the matrix $P_{1,2}$ can be determined as $P_{1,2} = [3\ 1;\ 1\ 1]$ with the equation $A_{1,2}^T P_{1,2} + P_{1,2}A_{1,2} + Q_{1,2} = 0$ and $Q_{1,2} = [2\ 0;\ 0\ 2]$.

As LQR control is a well studied robust control scheme in both theory and practice. Simulations are carried out along with a comparison between the proposed control $u = [u_m, u_c]^T$ and a standard LQR control. For the LQR control, the dynamic equation of the pointing tracking system (50) can be linearized as

$$\dot{x} = \tilde{A}x + \tilde{B}u, \tag{100}$$

for which we consider the following Riccati equation

$$\tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} - 2\tilde{P}\tilde{B}\tilde{R}^{-1}\tilde{B}^T\tilde{P} + \tilde{Q} = 0, \tag{101}$$

where $\tilde{Q}, \tilde{R} > 0$. The LQR control is

$$u = -\tilde{R}^{-1}\tilde{B}^T\tilde{P}x. \tag{102}$$

For simulation, we choose $\tilde{Q} = I,\ \tilde{R} = I$.

Finally, for simulation, we choose the parameters of the turret-barrel system as $m_1 = 5200kg$, $m_2 = 2088.15kg$, $R_1 = 4.12m$, $R_2 = 8.5m$, $G = 9.8m/s^2$, the parameters of the motor as $N = 20$, $k_t = 1.85N{\cdot}m/A$, $k_e = 23.7725V{\cdot}m/A$, the parameters of the electric cylinder system as $J_a = 0.0495kg{\cdot}m^2$, $J_b = 0.012kg{\cdot}m^2$, $\tilde{N} = 12$, $P_h = 0.2m$, $K_m = 2.7 \times 10^{-3}N{\cdot}m/A$, $\eta = 0.8$, the parameters of the installation location of electric cylinder as $l_0 = 0.3m$, $l_{dt} = 0.4m$, $l_a = 0.5m$, $a_0 = \arcsin(0.6)$, and the control parameters as $\gamma = 100$, $k_1 = 0.01$, $k_2 = 1$.

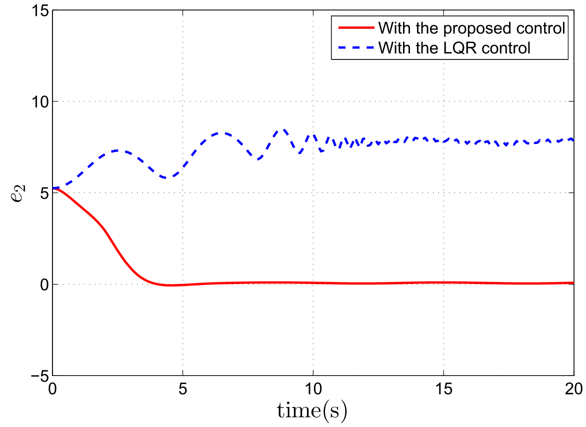

Fig. 7. Comparison of the tracking error e_2.

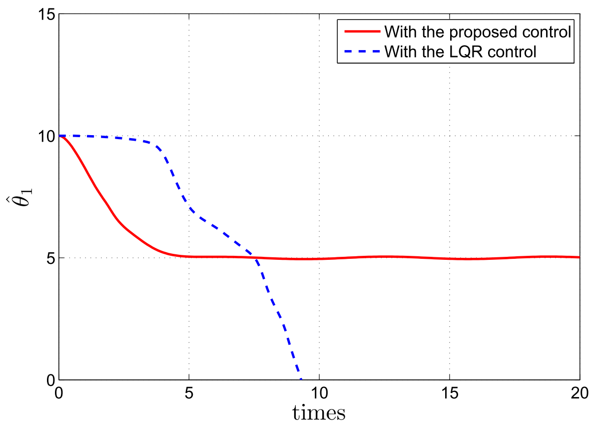

Fig. 8. Comparison of the angular position $\hat{\theta}_1$.

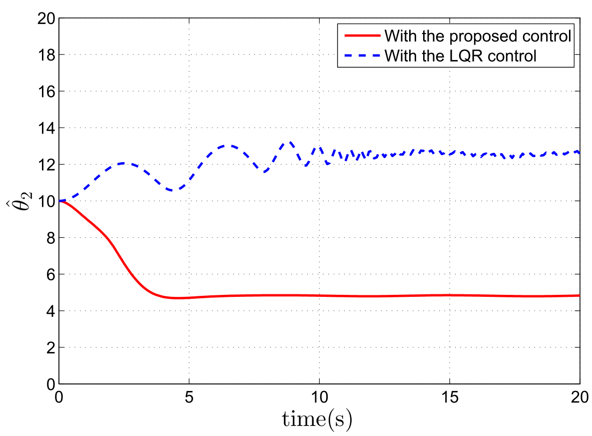

Fig. 9. Comparison of the angular position $\hat{\theta}_2$.

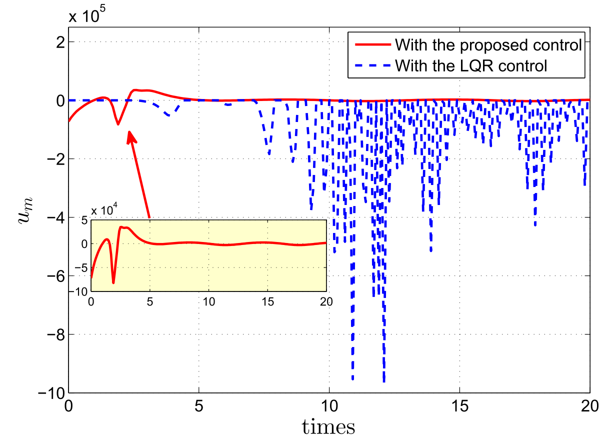

Fig. 10. Comparison of the control input u_m of the motor.

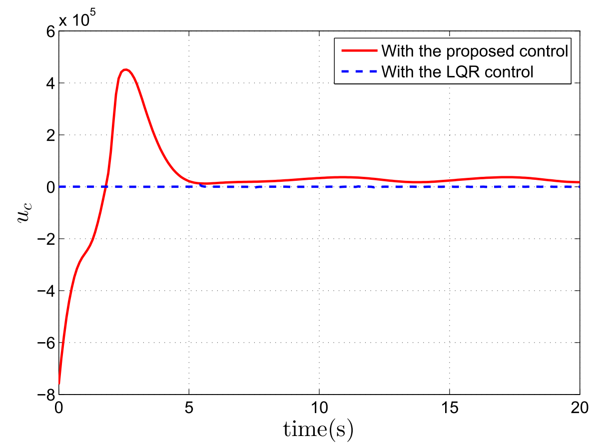

Fig. 11. Comparison of the control input u_c of the electric cylinder.

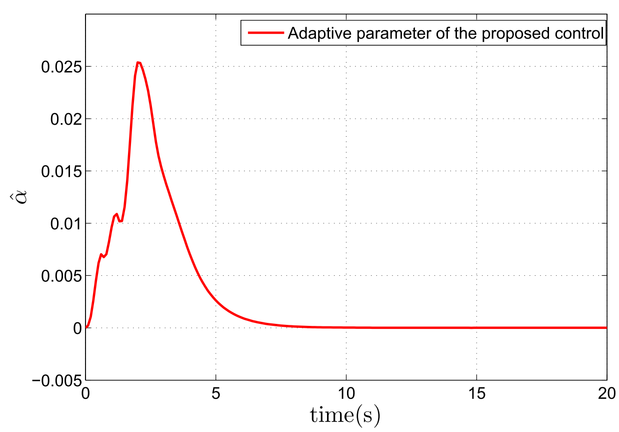

Fig. 12. History of the adaptive parameter $\hat{\alpha}$.

B. Simulation Results

The simulation results are shown as Figs. 6-13. Figs. 6 and 7 present the comparison of the history of the tracking errors $e_{1,2}$. It can be seen that, with the proposed control, the horizontal tracking error e_1 approaches to a neighborhood close to 0 before $t = 4s$, meanwhile, the vertical tracking error e_2 approaches to a neighborhood close to 0 before $t = 3.8s$. By contrast, with the LQR control, the horizontal tracking error e_1 becomes divergent, meanwhile, the vertical tracking error e_2 approaches to a neighborhood far away from the equilibrium point (actually close to 7.5) after $t = 10s$. By this, we can see that the LQR control does not result in any finite time settling, while the proposed control can drive the controlled system to be practically stable; thereout, Theorem 1 is further verified by the way of simulation.

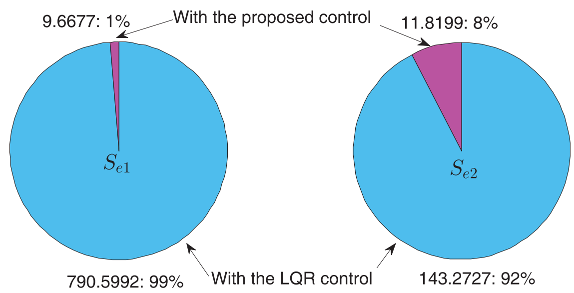

Fig. 13. Comparison of the accumulative tracking error $S_e = [S_{e1}\ S_{e2}]^T$.

Figs. 8 and 9 present the comparison of the history of the horizontal angular position $\hat{\theta}_1$ and the vertical angular position $\hat{\theta}_2$. It can be seen that, with the proposed control, the horizontal angular position $\hat{\theta}_1$ reaches to a neighborhood close to the desired angle $\theta_1^d = 5$ before $t = 4s$, while, the vertical angular position $\hat{\theta}_2$ reaches to a neighborhood close to the desired angle $\theta_2^d = 5$ before $t = 3.8s$. By contrast, with the LQR control, the horizontal angular position $\hat{\theta}_1$ becomes divergent, meanwhile, the vertical angular position $\hat{\theta}_2$ approaches to a neighborhood far away from the desired angle $\theta_2^d = 5$ (actually close to 14) after $t = 10s$. By this, we can see that the LQR control is ineffective in pointing tracking, while the proposed control shows an almost perfect control effect and realizes the pointing tracking well.

Figs. 10-12 present the comparison of the corresponding control input and the history of adaptive parameter $\hat{\alpha}$. It can be seen that, with the proposed control, the control input u_m, u_c and the adaptive parameter $\hat{\alpha}$ stabilize around 0 after the tracking error $e_{1,2}$ being around 0. By contrast, with the LQR control, the fluctuation of the control input u_m of the motor is frequent and the peak value is very large, while the control input u_c of the electric cylinder stays around 0. We can see that the motor and the electric cylinder can not coordinate well under the LQR control drive. Fig. 13 shows that the proposed control renders much smaller accumulative tracking error S_e (i.e., the area between the tracking errors $e_{1,2}$ and 0) than the LQR control.

VII. Conclusion

To improve the performance of pointing tracking for marking turret-barrel system, this paper proposes a novel adaptive robust control scheme with deep explorations on dynamics modeling, control design and uncertainty handling. The uncertainty may be (fast) time varying but bounded, and the bound is unknown. The pointing tracking system is firstly constructed as a coupled, nonlinear and uncertain dynamical system with two interconnected subsystems of a horizontal turret-motor system and a vertical barrel-electric-cylinder system. For uncertainty handling, an adaptive law is proposed to evaluate a comprehensive uncertainty bound α. For pointing tracking, an adaptive robust control is proposed to render the pointing tracking system to be practically stable, such that the objective of pointing tracking is achieved. This paper realizes the pointing tracking for marching turret-barrel system in one horizontal and vertical coupled channel rather than two separate ones; thereout, brings to a new technical support for intelligent development of ground combat platform.

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Qinqin Sun received the Ph.D. degree in mechanical engineering from the Nanjing University of Science and Technology, Nanjing, China, in 2021.

She worked with the Department of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, USA, as a Research Assistant, from 2014 to 2016, and visited the Department of Mechanical Engineering, University of California at Berkeley, Berkeley, CA, USA, as a Visiting Ph.D. student, from 2020 to 2021. She is currently an Associate Research Fellow of Aerospace Propulsion Theory and Engineering with the School of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing. Her research interests include complex system dynamics and control, fuzzy optimal design, and game theory.

Xiuye Wang received the Ph.D. degree in mechanical design and theory from the Hefei University of Technology, Hefei, China, in 2016.

He visited the Department of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, USA, as a Visiting Ph.D. student, from 2014 to 2016. He is currently an Assistant Professor in mechanical engineering with the School of Mechanical Engineering, Nanjing University of Science and Technology, Nanjing, China. His research interests include multibody system dynamics, robust control of mechanical system, fuzzy dynamical systems, and optimal design.

Guolai Yang received the Ph.D. degree in automatic weapons and ammunition engineering from the Nanjing University of Science and Technology, Nanjing, China, in 1999.

He visited the Department of Mechanical Engineering, Texas Tech University, Lubbock, TX, USA, as a Visiting Professor, in 2010. He is currently a Professor in mechanical engineering with the School of Mechanical Engineering, and works as the Dean of the School of International Education, Nanjing University of Science and Technology. His research interests include theory and experimental method of time-varying mechanics, mechanical system dynamics, virtual design, and simulation technique.

Ye-Hwa Chen received the Ph.D. degree in control science and engineering from the University of California at Berkeley, Berkeley, CA, USA, in 1985.

He is currently a Professor in mechanical engineering with the George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, USA. His research interests include complex system dynamics, fuzzy dynamical systems, fuzzy reasoning, and modeling and control of mechanical systems. He received the IEEE TRANSACTIONS ON FUZZY SYSTEMS Outstanding Paper Award from the IEEE Neural Networks Council in 2001. He has been serving as a regional editor and/or an associate editor for six journals.

Fai Ma received the Ph.D. degree in applied mathematics from the California Institute of Technology, Pasadena, CA, USA, in 1981.

He was formerly a Research Engineer with the IBM Thomas J. Watson Research Center and Standard Oil Company. He is currently a Professor in applied mechanics with the Department of Mechanical Engineering, University of California at Berkeley. He is the author or coauthor of more than 190 technical publications. His research interests include vibration, nonlinear damping, and system uncertainties. He is a fellow of the American Society of Mechanical Engineers.