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Effective Magnitude and Effective Threshold

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Effective magnitude (M') and effective threshold (T') are important because they try to express quantitatively a major aspect of electoral systems, namely the degree of squeeze they put on representation of small parties. Three relationships have previously been proposed between M' and T'. Of these, $T' = 75\%/(M' + 1)$ is found here to have the most desirable characteristics. However, regardless of the precise equation used, a disturbing discrepancy is observed in the case of single-member districts: the effective threshold predicted is much too high, if applied nationwide. This points out a more general need to keep district-level and nationwide indicators carefully separate. An appendix proposes a new formula to find effective magnitude when district magnitudes within a country vary.

Keywords: effective magnitude, effective threshold, small party representation, district and national levels

Arguably the most important aspect of an electoral system is the degree of squeeze it puts on representation of small parties, which influences the number of parties and is reflected in deviation from proportional representation (PR). Small-party representation is affected by district magnitude (the number of seats allocated, M), legal thresholds (T), adjustment seats, other features of electoral rules, and their combinations.

Attempts have been made to express the totality of these features with a single number, based on either of the two questions: ‘Which simple electoral system with a uniform district magnitude would lead to *approximately* the same results as the actual, more complex system?’ or ‘Which simple system with a legal threshold would lead to *approximately* the same results as the actual, more complex system?’ These measures are called, respectively, effective magnitude (M') and effective threshold (T') of the system.

To the extent that such measures can be operationally defined, the task of comparing the impact of various electoral systems on small party fortunes is greatly simplified. The two are interrelated. Reducing a legal threshold or increasing the district magnitude ‘can be seen as two sides of the same coin’ (Lijphart, 1994, p. 12) because they have a somewhat similar favorable effect on the smaller parties.

The purpose of the present study is to put this relationship on firmer ground and to ponder under which circumstances it is more advantageous to think in terms of M' or of T'. While so doing, an apparent discrepancy arises in the case of single-member districts ($M = 1$): the corresponding effective threshold T' makes sense in some ways but is much too high when looked at from a different angle. The explanation is that one must be very careful not to confuse district-level and nationwide parameters — as everyone has done up to now (including the present author).

The difficulty of determining the effective magnitude is illustrated in the Appendix. When all seats are allocated in districts according to a simple formula, but these districts have different magnitudes, the answer seems simple: effective magnitude is the arithmetic mean of the district magnitudes. This is the way it has been done by all researchers up to now (including myself). However, the Appendix shows that even in such an apparently straightforward case a more complex approach is needed.

Given such difficulties, would it be better to give up on the notions of effective magnitude and threshold? Far from it. In order to compare systematically the outputs of various electoral systems (such as the number of parties and deviation from PR), there is considerable advantage in dealing with a single input variable. In the case of stable electoral systems even the present imperfectly defined M' and T' offer definite correlations with the output variables. It's a question of fine tuning — and this is what the present study is about.

Background

'Effective magnitude' and 'average threshold' were introduced by Taagepera and Shugart (1989, pp. 117, 126–141, 266–269 and 274–277) as rough measures of the effect of a given electoral system on small-party representation. They proposed the approximations

$$T' = 50\%/M \text{ and } M' = 50\%/T \qquad (1)$$

respectively, to express the district magnitude as a roughly equivalent threshold or the legal threshold as a roughly equivalent magnitude. In subsequent analysis they focused on the effective magnitude aspect.

In contrast, Lijphart (1994, p. 25–30) preferred an emphasis on 'effective threshold.' It was analogous to the earlier average threshold but used a different averaging formula:

$$T' = 50\%/(M + 1) + 50\%/(2M), \text{ for } M > 1 \qquad (2)$$

At a given M, Equation (2) yields a higher value of T' than Equation (1), except for $M = 1$. For $M = 1$ both equations yield $T' = 50\%$, which is too high, given that the single seat is most often won with fewer votes (be it plurality or first round of majority systems). Therefore, while using Equation (2) for $M > 1$, Lijphart (1994, p. 28) set the threshold at 35% for $M = 1$. In order to have a single equation apply at all M (including $M = 1$) Taagepera suggested (cf. Lijphart, 1994, p. 183)

$$T' = 75\%/(M + 1) \qquad (3)$$

which yields practically the same values as Equation (2) for $M \gg 1$ but for $M = 1$ results in $T' = 37.5\%$, very much in line with the value chosen by Lijphart. For the sake of comparability of results, it would be desirable for the profession to settle on one of these equations.

The starting point for all three equations is the same. For a district of given M and allocation rule, establish the theoretical inclusion and exclusion thresholds. These are, respectively, the minimum vote share a party needs to win a seat under the most favorable conditions, and the maximum vote share at which it still could fail to do so under the worst conditions. These extremes depend on the distribution of votes among the competitors and, for inclusion threshold, also on the number of parties competing (p). Initiated by Rokkan (1968), the study of inclusion and exclusion thresholds was continued by Rae *et al.* (1971), Lijphart and Gibberd (1977), Laakso (1979) and Gallagher (1992). Acquisition of the first seat must occur in the zone between the inclusion and exclusion thresholds, and it most often can be expected to happen somewhere in the middle between the two boundaries.

To have a broadly applicable effective threshold, we need to establish a judicious average level of votes at which the first seat is expected to be won in the presence of a reasonable number of competitors and applying a variety of allocation rules. Considering the usual PR formulas, Taagepera and Shugart (1989, pp. 274–277) took the arithmetic mean of the highest exclusion threshold exhibited by any of the usual allocation rules — $1/(M+1)$ (for d'Hondt) — and the lowest inclusion threshold, $1/Mp$ (Largest Remainders with Simple Hare Quota). Recall that p stands for the number of parties competing. Simplifying assumptions led to Equation (1).

Lijphart (1994, p. 26–27) pointed out that while $1/Mp$ was theoretically possible, it was much below the actual lowest values observed, which were close to one-half of simple (Hare) quota ($1/M$). With the same higher limit of $1/(M + 1)$, it led to Equation (2). This equation can be recast as $T' = 25\%\ (3M + 1)/[M(M + 1)]$ which, for large M, can be simplified to yield Equation (3). Extending the use of Equation (3) to small M produces a realistically low value of $T' = 37.5\%$ for $M = 1$.

I now have a more direct way to introduce Equation (3). While $1/Mp$ (the basis for Equation (1)) seriously understates the realistic lower limit of inclusion, $1/2M$ overstates it. With LR-Hare, quite a few parties do obtain a seat with somewhat less than $1/2M$, and one simple way to reflect it is to assume a practical lower limit close to $1/[2(M + 1)]$. Equation (3) directly follows.

Choosing between Three Equations

Three criteria are considered here in evaluating the three equations proposed for connecting M' and T': a realistic value of T' at $M = 1$; ease of convertibility from T' to M'; and correlation with output variables.

Effective Threshold for Single-Member Districts

Figure 1 shows T' graphed against M' (both on logarithmic scales), using Equations (1)–(3). For $M' > 4$, Equation (2) and Equation (3) yield practically the same values of T', which tend to be 1.5 times higher than those given by Equation (1). At $M = 1$ only Equation (3) yields a value ($T' = 37.5\%$) close to the 35% deemed reasonable by Lijphart.[1] Hence this criterion tilts the choice in favor of Equation (3).

Reversibility of Relationship

Based on Equation (1), one can easily travel in either direction:

$$T' = 50\%/M' < - - - - - > M' = 50\%/T'$$

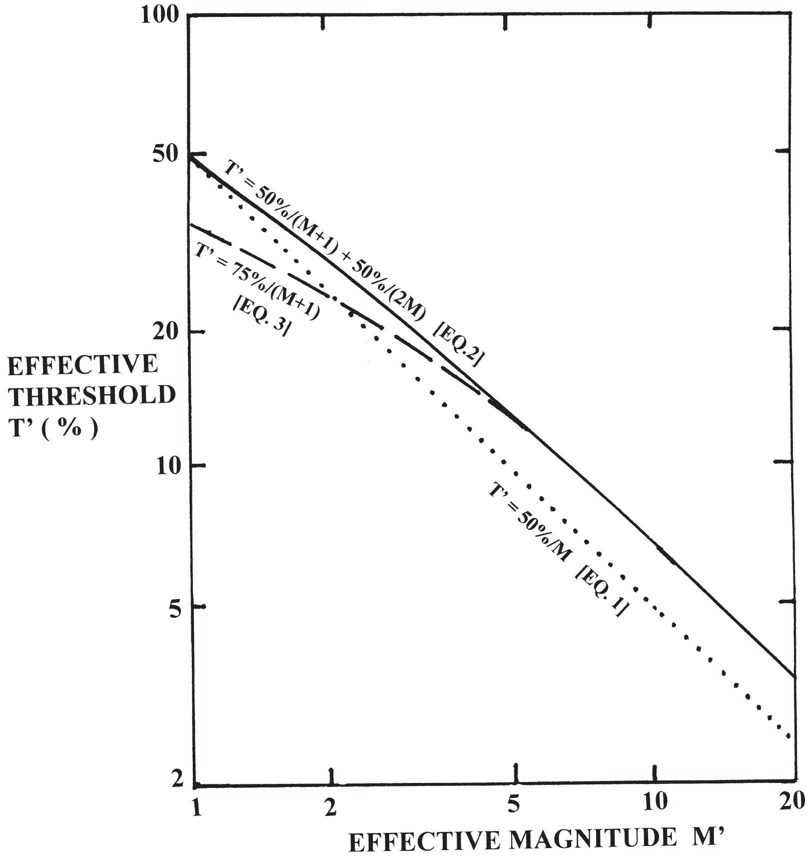

Fig. 1. Three equations to connect effective threshold and effective magnitude

The roles of M' and T' are only slightly asymmetrical for Equation (3):

$$T' = 75\%/(M' + 1) < - - - - - - - - > M' = (75\%/T') - 1.$$

For Equation (2), however, the reverse direction requires the solution of a second-degree equation:

$$T' = 50\%/(M' + 1) + 50\%/(2M')$$
$$< - - - - - > M' = \{75 - T' + [(75 - T')^2 + 100T']^5\}/2T'$$

By this criterion, Equation (1) is the most elegant in its simple symmetry. Equation (3) comes second. Equation (2) allows easy calculation of T' from M' but not of M' from T'.

Correlation with Output Variables

A number of correlations have been observed between M' and various output variables. The foremost one is the continuous form of Duverger's rule ('law' + 'hypothesis'), relating the effective number of parties to M' (Taagepera and Shugart, 1989, p. 144 and Taagepera and Shugart, 1993).[2] But also, deviation from PR tends to vary as inverse square root of M', and so does the break-even point (in terms of votes) between small-party penalty and large-party bonus (Taagepera and Shugart, 1989, pp. 118, 140–141). All these correlations could as well be expressed in terms of T' instead of M'. The question is whether correlation would improve, if Equation (2) or Equation (3) were applied instead of Equation (1) used by Taagepera and Shugart (1989). This criterion would carry considerable weight when deciding between the three forms, because ability to predict various output properties of the electoral system is the main reason why M' and T' are of interest. Which way to calculate T' offers more explanatory power?

Lijphart (1994, pp. 27–28) reports regressing deviation from PR (in its least-square form) linearly on actual T in 20 systems which have clear legal thresholds; he finds a slope of 0.42. He also calculated effective threshold, using both Equation (1) and Equation (2), for 37 systems in which district magnitude predominates (because legal threshold does not exist or is low). Presumably only multiseat districts were included. The slopes were 0.50 for T' from Equation (1) 0.40 for T' from Equation (2). Lijphart concludes that the latter is more in line with the slope arising from actual legal thresholds. Taking into account the specific allocation formula made it even more explicit (0.54 vs 0.42). This finding favors Equation (2) over Equation (1) but not over Equation (3), because Equation (2) and Equation (3) yield practically the same T' for $M > 4$. Intercepts of the regression equations would also be of interest but were not reported.

In view of the closeness of the curves in Fig. 1, the differences found by Lijphart are remarkable. Though problems remain,[3] it is likely that in their effect on outputs such as deviation from PR, the values of T' calculated from Equation (2) are more in line with legal thresholds, compared to the ones based on Equation (1). Thus, the correlation criterion puts Equation (1) last, but cannot discriminate between Equation (2) and Equation (3).

In conclusion, taking into account all three criteria, the following emerges. Equation (1) is simple and easy to reverse, but it yields an excessively high T' at $M' = 1$ and possibly too low T' at large M'. Equation (2) is anything but simple; it is messy to reverse and yields excessively high T' at $M = 1$. However, it is more powerful than Equation (1) in explaining deviation from PR. Equation (3) is fairly simple and easily reversible. At large M it yields the same T' as Equation (2) and hence shares in its ability to explain deviation from PR. For $M = 1$ plurality rule Equation (3) is the only one to give a value of T' that agrees with some observed district-level values. Thus Equation (3), $T' = 75\%/(M + 1)$, is to be preferred to the two earlier variants.

Should We Prefer Effective Magnitude or Threshold — or Neither?

In some electoral systems magnitude is unambiguously given, and threshold is a derivative, an 'effective' measure. This is so in Malta, where all districts have $M = 5$. In some other systems threshold is legally stipulated, and the corresponding effective magnitude is a derivative. In many more cases both are derivative. Finland has no legal threshold, but district magnitudes vary from 1 to 27, so that an effective magnitude must be chosen (and the Appendix shows that arithmetic mean is a poor choice) — thus both magnitude and threshold can be expressed only as ‘effective’ ones. In a broad generalization all the T and M in Equations (1)–(3) should really be shown as T' and M'. Which of the two is more basic?

First of all, it should be pointed out that no one has ever pretended that a legal threshold of, say, 5% for nationwide allocation of seats would have exactly the same consequences as allocation in 14-seat districts (as suggested by Equation (3)). With a legal T = 5%, a party with 4.5% votes would receive no seats, while a party with 5.5% votes would receive about 5.5% of seats. In contrast, with 14-seat districts both parties could expect about 2.5% of seats, on average. Legal threshold means a step function, a sudden cutoff, while allocation in districts produces gradual improvement in the advantage ratio (A = % seats/% votes) as a party’s vote share increases. This is a distinct difference.

Different allocation formulas, unequal district magnitudes, adjustment seats, multiple tier allocations, etc. all have their specific effects. Yet some of their effects are similar in their impact on the fortunes of smaller parties. This general impact is what M' and T' try to capture, so that we are able to compare widely divergent systems. Of course, whenever we generalize we lose detail as a price for increased ability to draw comparisons.

T' and M' are, broadly speaking, the reciprocals of one another (Lijphart’s ‘two sides of the same coin’). This is most manifest in Equation (1), which can be recast in the symmetric form $M'T' = 50\%$.[4] Equation (3) does the same with $M' + 1$: $T'(M' + 1) = 75\%$.

Effective magnitude is a more convenient building block for theoretical models, because a district with a certain number of seats is easy to understand. In a district with M seats, the number of seat-winning parties can range from 1 to M. This is the starting point for a quantitative model of Duverger’s ‘mechanical effect’ on the number of parties (Taagepera and Shugart, 1993). The aforementioned extensive work on inclusion and exclusion thresholds also represents modeling in terms of M as independent variable. I am not aware of any quantitative models starting out from threshold as independent variable.

When it comes to practical conclusions and recommendations to decisionmakers, threshold is more suitable. Mention a 5% effective threshold to the leader of a party that usually obtains 3% of the votes and he or she knows very directly what it would mean for the party. Tell about an effective magnitude of 14, and you get a blank, although the impact is roughly the same according to Equation (2) and Equation (3).

In this specific sense, magnitude is more of an input variable, while threshold is more of an output variable, and their usefulness varies accordingly. There is no reason to prefer one to the other, as long as one can quickly shift between them.

The Paradox of Effective Threshold for Single-Member Districts

For M much larger than 1 it is fairly easy to visualize an effective threshold for a single district of a given magnitude. However, an apparent contradiction crops up when trying to do the same at $M = 1$. Equation (3) yields $T' = 37.5\%$ for $M = 1$. Two questions arise.

(1) Does it mean that a party typically does obtain a seat with more than 37.5% votes in the given district, and does not get it with fewer votes? It will be shortly shown that the answer may well be yes.

(2) Does it mean that single-member districts could be replaced by nationwide allocation of seats subject to a legal threshold of 37.5%, and the outcome would be broadly the same? The answer is an emphatic no. Such a threshold would in all too many cases produce a

single-party parliament.[5] Indeed, in one New Zealand election (1928) any threshold of 33% or higher would have eliminated ALL parties. This is far from 'effectively' the same outcome as with plurality in single-member districts.

Note that the first question deals with the district and the second with the nationwide. Let us keep these levels apart and start with the district level.

District Level

Consider the British 1983 election data, as reported by McAllister and Rose, 1984 (pp. 222–242). Figure 2 shows the observed probability of a party winning with a given percentage of votes in that particular one-seat district. This probability is obtained by the following procedure.

In every 2% votes bracket I counted how many parties won and how many lost; e.g. in the 36.0–37.9% bracket England had 10 cases where a party won (in face of splintered competition) and 49 cases where it lost to a competitor with even more votes; thus the observed probability of winning a seat with 37% votes comes out as 16.9%. I define the observed effective threshold T' at district level as the votes share at which the number of candidates winning with less than T' equals the number of candidates losing with more than T'.

Two separate curves are shown in Fig. 2. England had an average of 2.5 effective electoral parties at district level, and it may be representative of moderately fractionalized district-level

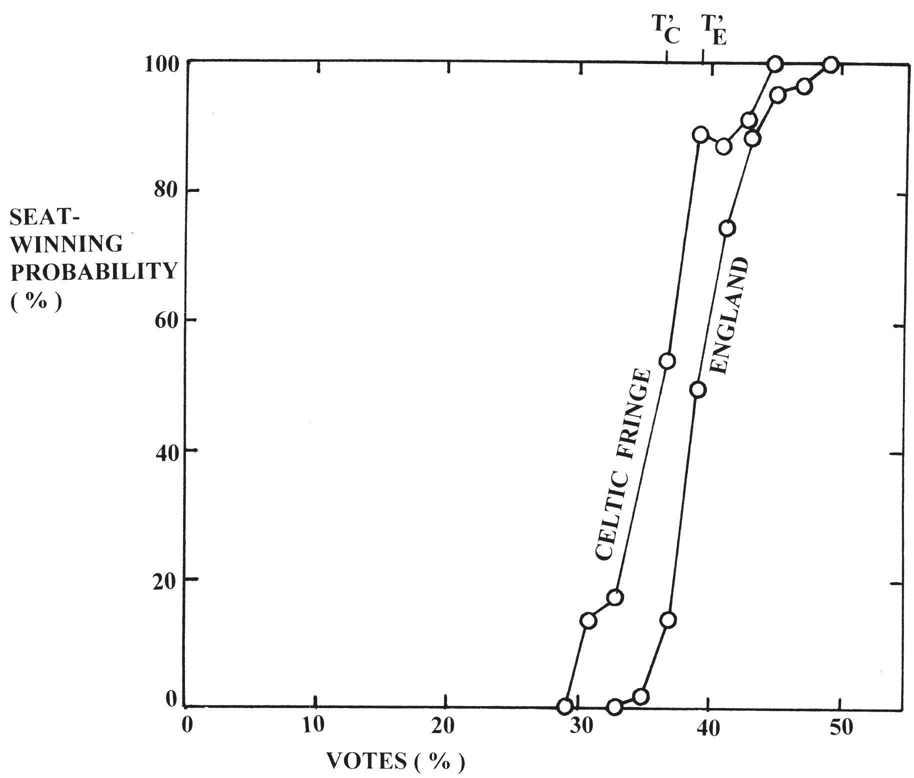

Fig. 2. Observed probability of winning a seat with given per cent votes, UK 1983

competition, such as occurs in India (Chhibber and Kollman, 1996) and pre-1995 New Zealand.[6] The Celtic Fringe (Wales, Scotland, Northern Ireland) had 3.2 effective electoral parties and is representative of a more fractionalized within-district competition that might apply in Canada. For England 1983 the observed T' is 39.45%. For the more fractionalized Celtic Fringe it is 36.65. These values straddle the 37.5% point proposed in Equation (3). The overall figure for the UK is around 39% because of England's preponderance.

Results are different for the United States with its unusually pure two-party competition. For the US House elections of 1970, the observed district-level T' was as high as 49.55%, reflecting an almost total absence of third parties.

In this way, the abstract formulations for effective threshold gain substance for single-member districts. Equation (1) and Equation (2) (which imply a T' of 50% for $M = 1$) fit in cases of extremely clear-cut two-party constellation. For the more usual cases with some third-party votes Equation (3) is rather on the mark with its estimate of $T' = 37.5\%$ for $M = 1$. Lijphart's special estimate of $T' = 35\%$ for $M = 1$ is slightly on the low side.

Nationwide Level

It should not surprise us that Equation (3) works and makes sense at district level — it was designed on the basis of district-level inclusion and exclusion considerations. But what does it mean nationwide? We have seen that, at least for $M = 1$, T' cannot be interpreted as a nationwide threshold to exclude *parties*. It will now be shown that it could be applied nationwide to individual *candidates*, using the following odd-looking rule (which I do not advocate for actual adoption!).

To obtain an assembly of S members, divide the country into S districts, but instead of plurality within district stipulate genuine *nationwide plurality for candidates*, meaning that seats go to the S candidates, nationwide, who receive the highest percent votes in their particular constituencies. In some districts two candidates may receive seats (e.g. if votes go 45–45–10), while in some others no one does (e.g. for 30–30–20–20). (As long as $T' > 33.4$ three candidates cannot win in the same district.) This is equivalent to specifying a legal T' retroactively, as done in Fig. 2 with British 1983 data, balancing the number of losers and winners.[7]

Of course, seat distribution among parties can shift somewhat.[8] Still, such a nationwide threshold for candidates would produce approximately the same outcome as the usual rule of local plurality in single-member districts. Complete equivalence of different rules is not to be expected.

The foregoing explains what the effective threshold, as presently calculated, means at the district level and what it could mean nationwide. But it leaves begging the basic question: what about an effective nationwide threshold that corresponds to the actual plurality rule in single-member districts?

To be more specific, which legal threshold would yield about the same deviation from PR and the same effective number of assembly parties, if there were nationwide PR allocation of seats subject to a legal threshold? It should be clear by now that such a nationwide effective threshold is quite different from the district-level one.

For the UK and New Zealand 1959–87 (two-party dominance) and also New Zealand 1919–28 (even struggle among three parties) I found that a nationwide threshold of about 24% could reproduce the actual effective number of assembly parties and deviation from PR to a fair degree. On the other hand, in India 1952–84 even a 11% threshold would have eliminated all parties except one in four elections out of six. The problem remains to be resolved.

The discrepancy between the district-level and nationwide effective thresholds is most manifest for $M = 1$. But the difference is likely to exist in low-magnitude multiseat districts too, though to a milder (and hence less detectable) degree.

Conclusion

The insights drawn from this study involve connection between effective magnitude and threshold, distinction between the district and national levels, and on a new note, seat allocation ability of various electoral rules.

Connection between Effective Magnitude and Effective Threshold

To the imperfect extent that the effects of the various ingredients of electoral rules can be expressed by a single variable, both effective magnitude and effective threshold do the job. They can be best converted into each other by using Equation (3): $T' = 75\%/(M' + 1)$.

District and National Levels

The discrepancy observed in the case of $M = 1$ has serious broader implications that apply to multiseat districts, too. All existing work on exclusion and inclusion thresholds has concerned a single district. Therefore, the generalized equation based thereon (Equation (3)) also applies at district level only. The assumption that the nationwide effective threshold is just an average of district thresholds has been blithely made in practically all existing work (including mine) — but this is patently false in the case of single-member districts.

The discrepancy may become smaller as district magnitude increases, but it is likely still to exist. In this light, one should be very cautious when comparing the impact of nationwide legal thresholds to that of district-level effective thresholds calculated from Equation (3). Further work needs to be done.

Seat Allocation Ability of Threshold Rules

The study of thresholds for $M = 1$ incidentally highlights one marked difference between allocating seats in districts (of any M) and by nationwide PR subject to threshold. Any allocation rule should be able to allocate all the seats at stake, regardless of turnout and votes distribution among candidates or parties. In this respect the usual PR rules are quite robust. The Largest Remainder rule can allocate all the M seats in the district, as long as at least M lists receive at least one vote each. With d'Hondt, a single vote for a single list will do.

In contrast, with a nationwide threshold, there is always the theoretical risk that all parties are eliminated, so that no seats can be allocated. For moderate thresholds like 5% this danger seemed remote, until the utterly fractionalized Polish and Russian elections of the 1990s made it look quite real. The probability is much higher for ending up with a single-party assembly — or an assembly of a few parties, all based on a small minority of votes. For higher thresholds the risk increases.

To those who consider every electoral system *sui generis* and deny the possibility of comparative analysis based on universal indicators, the complications discussed in this study may look like vindication of their views. To me it looks like another step in clarifying our analytical concepts, pointing out directions for further work.

Notes

1. For $M = 1$ and plurality, the inclusion and exclusion thresholds are 100%/p and 50%, respectively, where p is the number of parties competing. Their average is 50% for p = 2, 41.8% for p = 3, 37.5% for p = 4, and 35.0% for p = 5. Thus Equation (2) implies two competing candidates, while Equation (3) implies four candidates (independents included).
2. The effective number of parties is given by $N = P^2/\Sigma P_i^2$, where P_i is the number of votes or seats for the ith party and P is their sum (cf. Taagepera and Shugart, 1989, p. 81 and Lijphart, 1994, p. 68).
3. One difficulty is that systems with clear-cut nationwide legal thresholds and clear-cut uniform district magnitudes are relatively rare (and Lijphart's second group included complex cases like Austria and Belgium). The historical particularities of individual countries and the noise from converting complex systems to the simple $M' - T'$ format may distort the outcomes.
4. In this form the relationship could be labeled 'the law of conservation of magnitude–threshold product.' Physics offers examples of reciprocal quantities where neither has precedence. In electricity one may think in terms of resistance or its inverse, conductance, depending on the problem at hand. In contrast, length (in cm) is used much more often than 'shortness' (in inverse cm), although the unit cm^{-1} also does occur (for the so-called wave number). In our case, it is more like resistance and conductance: Both M' and T' have advantages.
5. In the 28 UK elections from 1885 to 1987 a nationwide 37.5% legal threshold would have resulted in 11 cases of single-party parliament, because all other parties fell below 37.5% of votes. This would also have been the case for 14 elections out of 32 in New Zealand, 1890–1987, and 8 elections out of 8 in India, 1952–84. A 50% legal threshold (resulting from Equation (1) or Equation (2)) would make it worse, but even the lower 35% effective threshold proposed by Lijphart for $M = 1$ would not eschew the problem. Note that it is not a question of shift in allocation rule (PR to plurality). All usual list PR rules can be applied at $M = 1$, and outcome is equivalent to plurality.
6. Nationwide fractionalization is irrelevant. India, for instance, has many effective electoral parties countrywide but only about 2.5 in an average district (Chhibber and Kollman, 1996).
7. If the nationwide threshold is specified in advance, say at 37.5%, then the total number of seats won may change from election to election. For England 1983 the balanced $T' = 39.45\%$ exceeds 37.5%; as a result the actual number of seats would exceed the present number by about 30, while the Celtic Fringe (balanced $T' = 36.65\%$) would have a shortfall of about 6.
8. In the British 1983 case, with the total number of seats fixed, Alliance would add 10 seats in England at the expense of mainly Labour (8 lost), reflecting the many instances where Alliance narrowly lost the plurality contest despite high per cent votes. In the Celtic Fringe 5 seats would change hands, the main loser being again Labour (3 seats), and the main beneficiary the North Ireland SDL (2 seats gained). These shifts are marginal for the major parties, though nearly doubling the Alliance seats. Of course, a change in rules would also produce alterations in tactics by parties and voters. With nationwide T', the goal would change from beating the closest competitor to (1) beating the threshold and, if possible, (2) reaching $100\% - T'$ votes so as to preclude another party from also winning a seat. Taken together, these goals may impose on the parties about the same tactics as $M = 1$ plurality. The voters, on the other hand, would be strongly motivated to vote strategically for the two major parties only, because otherwise the district might not get a representative at all.

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APPENDIX
Effective Magnitude for Districts of Unequal Magnitude

This is an example of problems that arise in determining M' even under apparently simple conditions, namely when all seats are allocated within districts, using a simple allocation formula. The problem is that district magnitudes often vary within a country, and the arithmetic mean usually reported in such cases fails to reflect the political implications of uneven district magnitudes.

Take Finland. Its 200 parliament seats are distributed among 15 districts, so that the mean M is 13.3, corresponding to $T' = 5.2\%$ according to Equation (3). However, the individual districts in 1983 ranged from 1, 7 and 8 in the periphery to 20 in Helsinki and 27 in Uusimaa province around it. In Uusimaa, the effective threshold is not 5.2 but 2.7%, and this is the threshold that really determines whether a small party can survive in Finland. Parties with only one or a few parliament seats typically obtain them in the two largest districts, conceding not only North Karelia ($M = 7$, hence $T' = 9.4\%$) but also the medium-magnitude districts. The result is that party fractionalization in Finland is greater than it would be if all districts were of average size.

Spain, Austria and France (during its PR period) offer similar examples of some extra-large districts (typically around the capital) supplying refuge to small parties that otherwise would vanish from the political scene. But what is one to make of Russia? Its Duma is elected in 225 single-member districts plus one nationwide 225-seat district with 5% threshold (but no German-type compensatory PR).

The arithmetic mean magnitude ($M = 2.0$, hence $T' = 25\%$) vastly understates the opportunities available for minor Russian parties, even if and when the present instability subsides and small parties and independents no longer can carry most one-seat districts. The real effective threshold for small parties will be the legal 5 per cent threshold.

If one is interested in the number of parties that make it into the assembly with at least one member, then effective magnitude would be that of the largest district, M_{max}. However, using M_{max} would overestimate the effective number of parties (N), which largely depends on the largest parties. While N is larger than one would expect on the basis of arithmetic mean magnitude, it is still smaller than predicted by M_{max}. To reflect this, the following formula might be used:

$$M' = \Sigma M_i^2/S$$

where M_i is the magnitude of the ith district and $S = \Sigma M_i$ is the total number of seats in the assembly. This 'self-weighted average' or 'effective size' was proposed by Feld and Grofman (1977) to explain a paradox in different perceptions of class size by administrators and by students. It is tied to the notion of effective number of parties in the following way (Taagepera and Grofman, 1981). Apply the formula for N (note 2) to calculate the effective number of *districts* (rather than parties). Dividing the total number of assembly seats by this effective number of components yields the 'effective size' of districts; this is the equation above.

For Finland this formula yields $M' = 15.9$ (compared to arithmetic mean 13.3), lowering T' to 4.4% (instead of 5.2). This is still appreciably higher than the threshold in the largest district ($T' = 2.7\%$). Hence the correction may seem a minor one. However, the difference becomes huge in the case of Russia.

For the Russian Duma $M' = 113$, as compared to the arithmetic mean M of 2.0. Accordingly, T' is lowered to 0.66%, instead of 25%. The legal threshold (5%) is in between. If one took the arithmetic mean magnitude at face value, then the 5% legal threshold would look pointless, because $M = 2.0$ would override it with a much higher effective threshold of 25%. In reality the legal threshold eliminated a large number of parties, as one would expect, if the effective barrier at district level is only 0.66%.

Detailed testing of this correction (in terms of improving correlation with various output variables) is complex and will not be undertaken here. In face of the obvious understatement of effective magnitude by the arithmetic mean the equation above is at least a move in the right direction.