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# Information Aggregation, Currency Swaps, and the Design of Derivative Securities

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# Information Aggregation, Currency Swaps, and the Design of Derivative Securities

## Abstract

A model of security design based on the principle of information aggregation and alignment is used to show that (i) firms needing to finance their operations should issue different securities to different groups of investors in order to aggregate their disparate information and (ii) each security should be highly correlated (closely aligned) with the private information signal of the investor to whom it is marketed. This alignment reduces the adverse selection penalty paid by a firm with superior information. Adverse selection costs are often contingent on ex post publicly observable and contractible state variables such as exchange rates. In such cases, debt contracts are dominated by currency swaps and optimal securities, in general, are derivative contracts that are contingent on state variables that influence adverse selection costs. This is because the netting of cash flows in these derivative contracts, in effect, alters the state-by-state seniority of different claims in a desirable way.

JEL classification: G10

One of the long held tenets of financial economics is that in a perfect capital market, the precise packaging of securities is irrelevant. However, the practice of finance in the 1980's and 1990's is largely noted for the proliferation of new contractual arrangements that package securities payoffs in different ways. As Ross (1989) suggested in his presidential address to the American Finance Association, we still do not understand why firms go through the trouble of creating such redundant assets and liabilities. Surely, such financial engineering is costly.

The role of standard securities like debt and equity in the financing of real investments has been explored in a rapidly burgeoning literature on security design.<sup>1</sup> However, only recently has research in financial economics begun to address why seemingly trivial packagings of securities are so popular. Allen and Gale (1988) show that when it is costly to issue securities and when different groups of investors place different values on the same security, optimal securities split up the firm's state-contingent cash flows, allocating all cash flow in a given state to the investor who values it the most. Madan and Soubra (1991) introduce marketing costs into the Allen and Gale model and show that the sharing of cash flow in several states may be optimal in the presence of marketing costs. Ross (1989) also explores the implications of marketing costs and shows that financial innovation can reduce the costs of marketing securities; Pesendorfer (1991) generalizes this result to a general equilibrium framework. Boot and Thakor (1993) argue that selling multiple financial claims partitions a firm's total cash flow into "informationally sensitive" and "informationally insensitive" components. This encourages the acquisition of information by investors, which enhances firm revenue. DeMarzo and Duffie (1995) analyze the effect of adverse selection when issuing firms possess superior information. In their model, the signal from the quantity issued generates a downward sloping convex demand curve for the security. They then show that the design of securities like Collateralized Mortgage Obligations allows intermediaries to retain the portion of the security's return for which adverse selection, due to private information, is greatest, thereby reducing the demand curve effect on revenues collected.

We model a situation in which different agents possess signals about different components of a firm's aggregate cash flow and show that the joint pricing of the firm's securities may reveal these signals to all of the agents.<sup>2</sup> For example, suppose domestic banks receive

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<sup>1</sup>See Allen and Winton (1994) for a recent survey of literature on security design.

<sup>2</sup>An example of this is also found in Kraus and Smith (1995).

private signals about a firm's domestic cash flows and foreign banks receive private signals about its foreign cash flows. Then the joint pricing of domestic and foreign debt instruments may reveal information about foreign cash flows to domestic banks and vice versa. Indeed, in the models developed here, full revelation of investor information occurs in equilibrium. This may explain why firms raise capital from several investors simultaneously.

The more elaborate versions of our models show that all investor information, but not all issuer information, is revealed in equilibrium. In this case, securities with payoffs that are highly correlated or "aligned" with the private information signals of the investors to whom the securities are sold are superior to less aligned securities. Such information alignment reduces the adverse consequences of a high quality issuer being pooled with low quality issuers. This not only reduces financing costs for high quality issuers, but also for low quality issuers, who would otherwise be revealed as low quality types if they do not follow the optimal security design of the high quality issuers.

Such a model argues that in each bankrupt state, claims should be designed so that only one cash flow claimant { the one facing the least amount of adverse selection in the realized state { owns all of the firm's assets. Such claims are derivatives, in the sense that their payoffs in bankruptcy depend on economic variables that determine the state-contingent adverse selection of each investor. Moreover, the netting of cash flows in many derivative contracts alters the state-by-state seniority of many issues in a desirable way.

A point that is often missed in thinking about derivatives is that while the out-of-bankruptcy cash flows of derivatives can be replicated by a portfolio of investments in more fundamental securities, it may be difficult to replicate the cash flows of the derivatives in bankruptcy with the same portfolio. For example, if the swap rates, swap notional amounts, and face values are set correctly, two currency swaps with opposite exchanges of domestic for foreign currency can have their out-of-bankruptcy cash flows replicated by a combination of domestic and foreign debt. However, in bankruptcy, there will be states of nature where one swap owes money to the firm and the other swap is owed money. This state-contingent seniority pattern, which arises from the cash flow netting in a swap, can never be replicated by two debt contracts. Hence, if the recipient of a cash flow in a particular bankrupt state is as important for valuation as the asset's distribution of promised payouts, the firm may perceive a pair of swaps to be very different from a pair of seemingly equivalent debt issues.

The model of financing that we develop motivates the existence of currency swaps even in the absence of hedging needs. For example, if the exchange rate determines the degree of adverse selection faced by domestic and foreign banks who will share the firm's assets in bankruptcy, contracts with exchange rate contingent payoffs would be appropriate. Properly designed currency swaps can ensure that in bankruptcy, only one counterparty owes money when the other is owed money. This is never true for pari passu domestic and foreign debt, which is always suboptimal. Nor is it true for a senior-subordinated debt structure which, in effect, cannot switch the ordering of priority, depending on the realized exchange rate in bankruptcy. The firm can raise the same amount of money and promise less to investors by entering into "off-market" currency swap agreements.<sup>3</sup>

The outline of the paper is as follows. Section I develops a model of security design for securities with linear payoffs. It demonstrates that investor information revelation generally occurs and computes the difference between revenue and cost to the firm for each linear security design. Section II applies the model to analyze optimal security design when we add stochastic elements to the payoff and design space and shows that currency swaps dominate debt contracts. Section III discusses the principles of optimal security design in more general models. Section IV briefly concludes the paper.

## I. A Model with Linear Payoffs

In this section we present two simple models of security design. These are used to develop some preliminary results that apply more generally. They also help us develop intuition for the model that analyzes currency swaps, which is developed in the next section of the paper. Specifically, we analyze a particularly strong initial information asymmetry between the firm and its investors.

In the first subsection, we develop a model where all information about a firm is aggregated as a consequence of investors bidding for its securities. The distinguishing feature of the model is that these investors have different information about various components of the firm's cash flows. We draw conclusions about the number of securities that the firm should issue. Beyond this, however, the model has little to say about security design.

In the second subsection, we introduce an additional source of uncertainty that prevents

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<sup>3</sup>In an off-market swap, the firm may receive or pay cash payments up-front.

the full aggregation of information in equilibrium. Security design plays a role in this slightly more complex model. In particular, security design can mitigate adverse selection arising from residual information asymmetry between the firm and its investors.

#### A. Effectively Complete Markets

We begin by analyzing a simple two date world (dates 0 and 1) in which securities issued to investors at date 0 are assumed to have a date 1 linear payout of the form

$$\alpha x + \beta y:$$

Here,  $x$  and  $y$  are nonnegative random variables, which can be thought of as cash flow components, and  $\alpha$  and  $\beta$  denote security design parameters, which are assumed to be common knowledge. Later in the paper, we will analyze the security design problem in the presence of some interesting contracting constraints that can be modeled as constraints on the security design parameters. For instance, we will consider the case in which contracts cannot be written on the individual cash flow components  $x$  and  $y$  but only on the aggregate cash flow  $x + y$ . In this case, we will impose the constraint that  $\alpha = \beta$ . Another interesting case arises when  $x$  represents cash flow in domestic currency and  $y$  represents cash flow in foreign currency. The aggregate cash flow in this case is  $x + sy$ , where  $s$  denotes the exchange rate. If contracts can only be written on aggregate cash flow, this is modeled as the constraint  $\beta(s) = s\alpha(s)$ .

The realized values of  $x$  and  $y$  are known to the issuing firm, but not to all investors. Specifically, we assume that there are two investor-types  $X$  and  $Y$  with respective information sets generated by private signals and the observation of securities prices.<sup>4</sup> The realization of  $x$  is known to  $X$ -type investors. That of  $y$  is known to a distinct set of  $Y$ -type investors. Neither investor-type can credibly announce their private signals to the other investor-type. Moreover, there is no common knowledge about the prior distributions of  $x$  and  $y$  held by  $Y$ -types and  $X$ -types, respectively.

For expositional clarity, we will often simplify the discussion of "the market" by treating it as though there is a single  $X$ -type investor, denoted  $X$ , and a single  $Y$ -type investor, denoted  $Y$ . We will analyze a pair of securities: one issued to  $X$  at an agreed to price of  $P_X$ , with coefficients  $\alpha_X$  and  $\beta_X$ , and the other issued to  $Y$ , at an agreed to price of  $P_Y$ ,

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<sup>4</sup>The model can easily be generalized to multiple investor-types.

with coefficients  $\alpha_Y$  and  $\beta_Y$ . We will later see that the issuance of more than two securities is superfluous.

Investors are assumed to be risk-neutral, which allows us to demonstrate (in the next section) that the use of derivatives need not be driven by any hedging motivation due to risk aversion.<sup>5</sup> For expositional simplicity, we set the risk-free rate in this one-period model to zero. From a valuation perspective, the information of investor-types X and Y is thus summarized by the expectation operators  $E^X$  and  $E^Y$  respectively.

Investors are assumed to behave rationally and competitively. Let  $V_i^j$  denote the valuation of security  $i$  by investor  $j$ .

**Definition 1**  $P_X$  and  $P_Y$  are rational and competitive bids for their respective securities by investor types X and Y if and only if,

$$P_X = V_X^X = \alpha_X x + \beta_X E^X y; \quad (1)$$

$$P_Y = V_Y^Y = \alpha_Y E^Y x + \beta_Y y; \quad (2)$$

**Definition 2** Given  $\alpha_X, \beta_X, \alpha_Y, \beta_Y$ , an equilibrium is a pair of bids  $P_X^*$  and  $P_Y^*$  such that

1. the bids are rational and competitive and
2. the firm accepts the bids with the knowledge that no rational competitive bid pair,  $P_X$  and  $P_Y$ , with  $P_X + P_Y > P_X^* + P_Y^*$  is possible.

What is missing at this point is the rules, which we call conjecture functions, that X- and Y-type investors use to map observed prices and private signals into posterior distributions that generate their expectation operators and ultimately, their bids. A pair of rules that investors use to generate  $V_X$  and  $V_Y$  are rational if they are consistent with Bayes's rule and generate the observed  $P_X$  and  $P_Y$  as fixed points.

We now discuss the bidding process for these securities and the information it generates. Consider investor X, who knows little about  $y$ , bidding for a security with a positive  $\beta_X$ :<sup>6</sup>

<sup>5</sup>Alternatively, we can view expectations as being generated by the equivalent martingale measure that follows from no arbitrage provided that this measure is invariant to the design of securities by the firms we study.

<sup>6</sup>We will later explore reasons that might explain why  $\beta_X > 0$ .

In the absence of further information, he is at a disadvantage when trading with the issuing firm, which knows  $y$  (and  $x$ ) perfectly. Of course, investor  $Y$  knows  $y$  perfectly, but it is not obvious how he can credibly communicate this information to  $X$ . If investors  $X$  and  $Y$  bid for their securities using their prior distributions about  $y$  and  $x$ , respectively, and, importantly, if these priors are common knowledge, then the bids reveal each investor's private information to all investors. In this case, a bidding process in which investors are allowed to revise their bids after observing the bids of other investors will surely end on the second bid since all information asymmetry will be resolved by then. However, common knowledge, particularly with an infinite state space, is a very strong assumption despite its prevalent use in models with asymmetric information.<sup>7</sup> In the absence of common knowledge about each others' priors, it is not obvious that full revelation will occur even with a bidding process that permits revision. Indeed, unless the securities issued to  $X$  and  $Y$  are distinct, full revelation will not generally occur and market failure may result.

It may be useful to picture a real-world analogy to the game modeled here. Consider an investment bank using its sales force to shop among partially informed investors for the best deal for a corporate client. It offers a security to  $X$ , informs him that it will also be offering another security to  $Y$ , and asks  $X$  what he will price his security for.  $X$  returns a price,  $P_X$ . This price quote is not firm until  $X$  sees  $Y$ 's price for his security,  $P_Y$ , and it is common knowledge that 1) both  $X$  and  $Y$  are jointly satisfied with their quotes knowing each other's quotes and 2) the issuing firm is also satisfied with the quotes for the securities. The bank then approaches  $Y$ , informs him of  $X$ 's  $P_X$  quote and asks for his  $P_Y$  quote. Once again, the quote is not firm unless it is common knowledge that neither investor has regrets about his price quotes. This process may iterate as the investment bank shuttles pricing information between the two investor-types until an equilibrium pair of prices is reached.<sup>8</sup> Thus, the information sets of the two investors not only contain the private signals,  $x$  or  $y$ , and securities prices,  $P_X$  and  $P_Y$ , but also knowledge that all investors are happy with their price quotes given this information and that the firm does not think it can do better. The joint "no regrets" condition limits the rational competitive bid that each investor can make, based on their expectation about the "unknown variables,"  $y$  for  $X$  and  $x$  for  $Y$ .

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<sup>7</sup>See Geanakoplos (1992).

<sup>8</sup>The investment bank also could, but need not, suggest prices that the firm finds acceptable, in the interest of speeding the interactive process along. Our description of the negotiations between different parties is not far from reality; see Allen (1987).

The following lemma illustrates that rational competitive bids can be generated.

Lemma 1 The following pair of conjecture functions

$$E^Y x = \frac{P_X i^{-X} y}{\theta_X};$$

$$E^X y = \frac{P_Y i^{\theta_Y} x}{-Y};$$

lead to rational competitive bids and full revelation of investor information if  $\frac{\theta_X}{X} \in \frac{\theta_Y}{Y}$ .<sup>9</sup>

Proof: See the Appendix. ■

Lemma 1 implies that if the security design coefficients satisfy  $\frac{\theta_X}{X} \in \frac{\theta_Y}{Y}$ , the firm need never settle for a pair of bids that undervalue the securities. This immediately implies Lemma 2.

Lemma 2 If  $\frac{\theta_X}{X} \in \frac{\theta_Y}{Y}$  then the conjecture made by X about y,  $E^X y$ , and the conjecture made by Y about x,  $E^Y x$ , imply rational and competitive bids if and only if X and Y agree about the valuation of the securities issued to X and Y. That is,

$$V_X^Y = P_X; \quad (3)$$

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<sup>9</sup>Lemma 1, in addition to providing an example of a situation in which prices aggregate all information, also suggests an underlying iterative inference process by which such an outcome may be realized in equilibrium. Let  $\{x_t^a\}$ ,  $\{y_t^a\}$  denote the sequence of conjectures and let  $\{P_{X;t}\}$ ,  $\{P_{Y;t}\}$  denote the sequence of price quotes. Let us assume that X moves first with the following initial conjecture:

$$y_1^a = E(y | \hat{A}_X^0);$$

where  $\hat{A}_X^0$  denotes the initial information set of investor X which is his private information and is not observed by any other investor. Using this conjecture, X quotes the first price as

$$P_{X;1} = \theta_X x + \bar{x} y_1^a;$$

Observing  $P_{X;t}$ , investor Y makes the following conjecture:

$$x_t^a = \frac{P_{X;t} i^{-X} y}{\theta_X} \quad \text{for } t \geq 1$$

to determine the price quote for his security

$$P_{Y;t} = \theta_Y x_t^a + \bar{y} y \quad \text{for } t \geq 1;$$

Observing  $P_{Y;t}$ , investor X makes the following conjecture:

$$y_{t+1}^a = \frac{P_{Y;t} i^{\theta_Y} x}{-Y} \quad \text{for } t \geq 1$$

to determine the next price quote for his security

$$P_{X;t+1} = \theta_X x + \bar{x} y_{t+1}^a \quad \text{for } t \geq 1;$$

This iterative process converges to the equilibrium specified in Lemma 1 if  $\frac{\theta_X}{X} > \frac{\theta_Y}{Y}$ .

$$V_Y^X = P_Y : \tag{4}$$

Proof: See the Appendix.■

Both investor-types realize that the structure of the game means that they cannot rationally disagree about the value of the two securities if the firm accepts their bids. This is because the firm will accept the bids only when it generates revenues that are at least as high as those that are generated using the conjecture functions specified in Lemma 1. Disagreement, therefore, implies that the two investors must be overpaying for the two securities in the aggregate. Clearly, rational conjectures cannot be such that any investor thinks that he himself is overpaying. This implies that each investor thinks that the other investor is overpaying when they disagree about the valuation.

However, believing that only the other investor is overpaying cannot be rational either. For example, suppose that X, knowing  $x$ , thinks that Y's conjecture about  $x$  is 10 when he knows that  $x$  is 8. Then X must believe, not only that Y is overpaying for Y's security, but also that Y believes that X is underpaying for X's security.<sup>10</sup> The latter belief is not consistent with X and Y having common knowledge about the structure of the game; a structure which implies that X has to think that Y thinks X cannot be underpaying. Thus, both investors realize that beliefs that lead to disagreement about securities values cannot be rational in equilibrium.

The following lemma shows that rational competitive bids that are consistent with agreement about the values of the two securities lead to full revelation.

Lemma 3 If  $\frac{\partial x}{\partial x} \in \frac{\partial y}{\partial y}$  then the only conjectures associated with rational competitive bids are those for which  $E^Y x = x$  and  $E^X y = y$ .

Proof: See the Appendix.■

Since Lemma 1 proves the existence of rational competitive bids, we obtain the following proposition.

Proposition 1 All investor information is revealed in equilibrium when firms issue two

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<sup>10</sup>Since Y has superior information about  $y$ , X has to believe that it is only Y's conjecture about  $x$  that could possibly lead to Y making a valuation error. Similarly, only Y's view of X's conjecture about  $y$  enters into Y's opinion about X's valuation error.

distinct securities to investors with distinct information sets.

Proof: Follows from Lemmas 1-3. ■

Proposition 1 suggests that when the information set of investor X and the information set of investor Y can be spanned by the cash flows of some portfolio of the two securities, the firm gets full value for its securities. In this case, it is because the joint pricing of the securities reveals both  $x$  and  $y$  to investors Y and X, respectively. Note that it is not the securities issuance per se that reveals firm type, as in typical financial signaling models. Rather it is the information aggregating property of the joint pricing of the securities that reveals the disparate information. In this sense, the model developed here is more closely related to the rational expectations equilibrium work of Grossman (1976) and Admati (1985) than it is to the models in the signaling literature like Ross (1977) or Leland and Pyle (1977).

It may not be immediately obvious how each investor-type is able to extract relevant information by observing each other's price quotes. Each investor-type lacks one piece of information ( $y$  for X and  $x$  for Y) that will be relevant in pricing his security and knows neither the conjecture nor the private information signal of the other investor-type which is to generate the observed price of the other security. We also know from Grossman (1977) that, generally, equilibrium prices do not perfectly aggregate information. At first glance, it thus seems possible that both investor-types could be simultaneously overoptimistic or underoptimistic in their conjectures. However, this is where the issuance of two distinct securities, i.e.,  $\frac{\partial x}{\partial X} \neq \frac{\partial y}{\partial Y}$ , becomes important. The equilibrium is attained when each investor-type is convinced that not only is his valuation of his own security the same as the other investor-type's valuation, but also that his valuation of the other security (which he may not necessarily purchase himself) also agrees with the other investor-type's valuation. If, for instance, investor X was overly optimistic about  $y$  and investor Y was overly optimistic about  $x$ , their valuations could agree for one security, but could not agree for the other since the two securities have different relative sensitivities to the  $x$  and  $y$  components.

Full revelation cannot occur if the same security is issued to X and Y, i.e.,  $\frac{\partial x}{\partial X} = \frac{\partial y}{\partial Y}$ . Consider the scenario where a security paying  $x + y$  is issued to both X and Y. Since X prices the security at  $x + E^X y$  and Y prices the security at  $E^Y x + y$ , both  $E^Y x$  and  $E^X y$  could overestimate  $x$  and  $y$  by the same amount and the investors would be agreeing on the values of both securities. There is no way to distinguish these conjectures from states where

the conjectures underestimate  $x$  and  $y$ . Hence, in contrast to the pricing of two distinct securities, the pricing of two identical securities cannot fully reveal information.<sup>11</sup>

In general, full revelation of investor information is obtained if there exists a portfolio of the linear securities with payoffs that span each investor-type's private information set. This conclusion is robust to the number of pieces of private information and the distribution of private information across investor-types. By simply issuing as many securities as there are pieces of information among this set of investors, and selecting any securities coefficient matrix that is of full rank, one can achieve full revelation of the private information held by this subgroup. This result thus provides an explanation for why firms raise financing from several different sources even when a single financial intermediary may be capable of satisfying the financing requirements of any given firm.

Finally, the results in this subsection suggest that it is easy to circumvent many security design constraints, such as not being able to spin off a part of the firm that just pays  $x$  to  $X$ , or just pays  $y$  to  $Y$ . Indeed, for the model developed in this subsection, no aspect of security design, beyond sheer numbers of securities, is relevant. However, the effectively complete market that drives this result is not apparent when one examines the real world. In the remainder of this paper, we will explore barriers to the issuance of the types of securities that eliminate the asymmetric information problem described above. If complete contracting is not possible, we have more to say about security design.

## B. Incomplete Markets

In our model, an incomplete market is generated by introducing a set of state variables  $\{z\}$  that affect the pricing of securities, but are not publicly observable, so that all investors cannot easily write contracts on the variables  $x$  and  $y$ . The  $z$ 's exist merely to provide some uncertainty about the source of the variation in equilibrium prices. In this sense, they serve the same function as noise trading in rational expectations equilibrium models. Here, however, there is a more appealing interpretation of the  $z$ 's (which we will discuss further in the next section). In this model, the  $z$ 's represent the difference in the

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<sup>11</sup>If there is common knowledge about the supports of investors' priors, then in some cases full revelation may occur. For instance, this may happen when the state space is finite, as we learned from an interesting example provided to us by Peter DeMarzo. However, in a continuous state space it is straightforward to prove that revelation cannot occur unless both  $x$  and  $y$  lie on the same boundary (lower or upper). The proof is available from the authors upon request.

payoffs of each of the two state variables to the two investor-types. For example, the first state variable pays  $x$  to Y-type investors and  $x + z_x$  to the X-type investors. Private signals communicate  $x + z_x$  (but not  $x$ ) to X-type investors and  $y + z_y$  (but not  $y$ ) to Y-type investors. As before, X-type investors do not know  $y$  and Y-type investors do not know  $x$ . The firm knows everything,  $x; y; z_x$ , and  $z_y$ , as before.

In algebraic terms, we assume that the security issued to X has (positive) payoff

$$p_X(x + z_x) + p_{XY}y$$

and the security issued to Y has (positive) payoff

$$p_{YX}x + p_Y(y + z_y):$$

The conditional expectation functions

$$x(x + z_x) \uparrow E(x|x + z_x);$$

$$y(y + z_y) \uparrow E(y|y + z_y);$$

are assumed to be common knowledge and monotonically increasing in their arguments.<sup>12</sup>

In spite of this common knowledge, the payoff structure implies that an asymmetry between the information of the issuing firm and the information of its investors cannot be eliminated. A Y investor observing a high price for a security issued to X cannot distinguish between a high value of  $x$  and a high  $z_x$ . Such an investor must pool firms with high  $x$ 's and low  $z_x$ 's together with firms that have low  $x$ 's and high  $z_x$ 's. The latter firm types benefit by being in the pool and get favorable pricing on the securities they issue to Y. The former firms receive unfair pricing on securities issued to Y given their information set. They will do anything they can to either break out of the pool or to alter the composition of the intrinsic security values in the pool so that their securities pricing is not so disadvantageous. This, as we will show, is accomplished via security design. The low  $x$  high  $z_x$  firms will remain in the pool by mimicking the high  $x$  low  $z_x$  firms' security design. In this pooling equilibrium, all firms issue the same securities. Hence, pools distinguish themselves by the prices they receive for securities from X and Y.

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<sup>12</sup>This condition obtains when  $x$  and  $x + z_x$  are affiliated (i.e., loosely speaking, positively correlated; see Milgrom and Weber (1982), Theorem 5), and similarly  $y$  and  $y + z_y$  are affiliated.

More formally, let  $V_i^j$  denote the valuation of security  $i$  by investor  $j$ . Specifically,

$$V_X^X = \alpha_X(x + z_x) + \beta_X E^X y; \quad (5)$$

$$V_X^Y = \alpha_X E^Y(x + z_x) + \beta_X E^Y y; \quad (6)$$

$$V_Y^Y = \alpha_Y E^Y x + \beta_Y(y + z_y); \quad (7)$$

$$V_Y^X = \alpha_Y E^X x + \beta_Y E^X(y + z_y); \quad (8)$$

Then, rational competitive bids  $P_X$  and  $P_Y$  satisfy

$$P_X = V_X^X = \alpha_X(x + z_x) + \beta_X E^X y; \quad (9)$$

$$P_Y = V_Y^Y = \alpha_Y E^Y x + \beta_Y(y + z_y); \quad (10)$$

As in Section A, an equilibrium is defined by Definition 2.

Once again, we first establish a pair of rational competitive bids. These make it possible to rule out candidate equilibria where the firm undercharges relative to the aggregated private information sets of all investors.

Lemma 4 The following pair of conjecture functions

$$E^Y x = x \frac{\alpha_X P_X + \beta_X y}{\alpha_X};$$

$$E^X y = y \frac{\alpha_Y P_Y + \beta_Y x}{\alpha_Y};$$

lead to rational competitive bids and full revelation of investor information.<sup>13</sup>

Proof: See the Appendix. ■

Lemma 5 The conjecture made by  $X$  about  $y$ ,  $E^X y$ , and the conjecture made by  $Y$  about  $x$ ,  $E^Y x$ , are rational and competitive if and only if  $X$  and  $Y$  agree about the valuation of the securities issued to  $X$  and  $Y$ . That is,

$$V_X^Y = P_Y; \quad (11)$$

$$V_Y^X = P_X; \quad (12)$$

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<sup>13</sup>There could be degenerate cases in this model { as found, for example, in Section A { where full revelation cannot occur. In contrast to Section A, where such degenerate cases arose only when the firm issued two identical securities, the degenerate cases here may arise with distinct securities with specific security design parameters. Using analogous arguments to those found in Section A, one can show that it is not in the interests of the firm to issue a degenerate pair of securities that cannot lead to full revelation. For Lemma 4 and the remainder of this paper, we simply rule out such degenerate cases by assumption.

Proof: See the Appendix. ■

Lemma 6 The only conjectures associated with rational competitive bids are those for which  $E^Y x = E^X x$  and  $E^X y = E^Y y$ .

Proof: See the Appendix. ■

Proposition 2 All investor information is revealed in equilibrium when firms issue two nondegenerate securities to investors with distinct information sets.

Proof: Follows from Lemmas 4-6. ■

The equilibrium specified in Proposition 2 implies that low  $z_x$  high  $x$  issuers are pooled with high  $z_x$  low  $x$  issuers of the same pair of securities when  $Y$  makes the conjecture  $E^Y x$ . Similarly, high  $z_y$  low  $y$  issuers are pooled with low  $z_y$  high  $y$  issuers when  $X$  makes the conjecture  $E^X y$ . The implications of this pooling for security design are profound. For example, in the pool of firms that has the same sum,  $x + z_x$ , and issues the same security at the same price to  $X$ , the high  $x$  firms in the pool are receiving financing from  $Y$ , on a separate security, on relatively unfavorable terms. These good firms would like to break out of the pool if possible. However, any attempt to do this via security issuance is doomed to failure. Suppose the high  $x$  firms attempt to issue a security to  $X$  with a different value of  $\frac{x}{x+z_x}$ . Low  $x$  firms in the pool will then mimic the high  $x$  firms' security issuance so that they can continue to pool with high  $x$  firms and thus receive financing on favorable terms from  $Y$ . This suggests that security design is dictated by the preferences of types with  $x > \bar{x}(x + z_x)$  and  $y > \bar{y}(y + z_y)$ , who, in effect, finance their projects at unfavorable rates. With the next proposition, we show that the design of the securities issued affects the financing revenue for these high quality issuers who are pooled (by pricing) with lower quality (higher  $z$ ) issuers. This proposition describes the optimal way to divide up the cash flow components to the two investor-types.

Proposition 3 Consider two securities, one issued to  $X$  and the other to  $Y$ . Let

$$P_X \text{ ; } C_X = [\theta_X(x + z_x) + \bar{x} E^X y] \text{ ; } [\theta_X(x + z_x) + \bar{x} y]; \quad (13)$$

$$P_Y \text{ ; } C_Y = [\theta_Y E^Y x + \bar{y}(y + z_y)] \text{ ; } [\theta_Y x + \bar{y}(y + z_y)]; \quad (14)$$

denote the difference between revenue and cost to the firm from security issuance to X and Y, respectively. If the adding up constraints:

$$\alpha_X + \alpha_Y = 1; \tag{15}$$

$$\beta_X + \beta_Y = 1; \tag{16}$$

hold then the aggregate profit,  $P - C$ , is increasing in  $\alpha_X$  and  $\beta_Y$ .

Proof: Substituting from (5) and (6), into (3) and (4), adding, and simplifying, we get:

$$P - C = [P_X + P_Y] - [C_X + C_Y] = \beta_X [(x - E^Y x) + (y - E^X y)] + \alpha_X (x - E^Y x) + \beta_Y (y - E^X y); \tag{17}$$

Since  $x > E^Y x$  and  $y > E^X y$  for pool leaders (as described in the paragraph preceding the proposition), the result immediately follows. ■

Investor X has precise information about  $(x + z_x)$  but knows  $y$  only imperfectly. A high value of  $\alpha_X$  and a low value of  $\beta_X$  thus more closely aligns the first security with his private information. Similarly, Y has precise information about  $(y + z_y)$  but knows  $x$  only imperfectly. A low value of  $\alpha_Y$  and high value of  $\beta_Y$  aligns this security more closely with his private information. Such alignment reduces the impact of the adverse selection:  $y - E^X y$  for X and  $x - E^Y x$  for Y, thus increasing profit.

The proposition is based on an adding up constraint for the nonnegative coefficients, which can be loosely interpreted as suggesting that the firm's assets must be divided up between the two cash flow claimants, X and Y. If  $\alpha_X$  and  $\beta_Y$  are individually unconstrained, Proposition 3 suggests that the optimal issuance strategy is "the spin-off solution": issue a security to X that pays  $x + z_x$  only, and a security to Y that pays  $y + z_y$  only. This design eliminates all adverse selection. In the real world, however, security design may be constrained. For example, it may be impossible, because of technological constraints, regulation, accounting standards, transaction costs, or agency issues to completely unbundle  $y$  from  $x$ . If there exist independent constraints on the maximum values for  $\alpha_X$  and  $\beta_Y$ , Proposition 3 suggests that the issuer should design securities to get as close as possible to the spin-off solution as the constraints allow.

There are other interesting constraints that involve dependence between the security design coefficients. Consider cash flow components that cannot be unbundled. In this case,

all financial claims must be written on the aggregate cash flow rather than the cash flow components. In terms of our model, this amounts to imposing the constraints that  $\theta_X = \bar{x}$  and  $\theta_Y = \bar{y}$ . Substituting these constraints into Equation 17, we get

$$P \downarrow C = \downarrow (x \downarrow E^Y x) + \theta_X [(x \downarrow E^Y x) \downarrow (y \downarrow E^X y)];$$

If

$$x \downarrow E^Y x > y \downarrow E^X y;$$

(i.e., pool leading X-type investors experience less adverse selection), the profit  $P \downarrow C$  is increasing in  $\theta_X$  and vice versa.<sup>14</sup>

Here, the determinant of optimal security design, the relative magnitudes of  $x \downarrow E^Y x$  and  $y \downarrow E^X y$ , are constant, as is the relevant security design parameter  $\theta_X$ . In the next section, we consider one of the cash flow components,  $y$ , to be denominated in a foreign currency. This makes the domestic currency value of the foreign currency component of the cash flow contingent on the exchange rate. With adverse selection costs contingent on the exchange rate, it becomes optimal to have security design parameters that are also contingent on the exchange rate, as we show in the next section.

## II. Optimal Security Design and Currency Swaps

In this section, we present a modest elaboration of the abstract model at the end of the last section and provide concrete interpretations of the variables used there.

### A. The Model

Consider a multinational operating in two countries, which is attempting to secure financing of  $I$  to fund projects with domestic and foreign cash flows. The values of these projects depend on whether the firm is in bankruptcy or not, and, if in bankruptcy, on the structure of the firm's financial claims. We also assume asymmetric information about the value of firm's assets in bankruptcy. We will contrast the efficiency of different combinations of financial instruments for achieving this financing. Following Townsend (1979) and Diamond (1984), the non-verifiability of some of the payoffs make it impossible to issue equity to obtain this financing. However, debt need not be the optimal issuance here because there

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<sup>14</sup>We still need to issue two securities to obtain full revelation.

are some publicly observable and verifiable state variables, such as the exchange rate,  $s$ , on which contractual payoffs can be made contingent.<sup>15</sup>

After investing the financing, the out-of-bankruptcy cash flows from the assets at date 1 are  $A(s; \mu)$ .<sup>16</sup> In contrast to  $s$ , contracts cannot be written that are contingent on  $\mu$  (perhaps because realized  $\mu$  at date 1 is unobservable). With  $\mu$  noncontractible, bankruptcy cannot be eliminated with a security design that hedges away all risk. In bankruptcy, if the firm is run efficiently, its domestic assets have cash flows of  $x + z_x$  and the foreign assets have cash flows (denominated in foreign currency) of  $y + z_y$ .

The joint distribution of  $[s; A(s; \mu)]$  is assumed to be common knowledge. The information structure for  $x$ ,  $z_x$ ,  $y$ , and  $z_y$  is identical to that described in the model of the previous section, except that the common knowledge expectations,  $\bar{x}(x + z_x)$  and  $\bar{y}(y + z_y)$ , are expectations conditional on bankruptcy.<sup>17</sup> Consistent with a long literature on bankruptcy costs, we assume that for all  $s$ ,  $A(s; \mu) \geq (x + z_x) + s(y + z_y)$ .

$x$  and  $sy$ , which are assumed to be nonnegative, can be thought of as the values of the tangible component of the domestic and foreign assets of the firm in bankruptcy. By tangible component of the assets, we mean that component of asset value that is unaffected by who runs the firm in bankruptcy. The intangible component of asset values varies depending on whether the owner/manager of the assets in bankruptcy is efficient at employing them. For example, it is not unreasonable to presume that a key employee will work for the firm if  $X$  is the owner, but quit if  $Y$  is the owner. In the finance literature, Titman (1984) discusses how difficult it is for firms in financial distress to efficiently employ their assets. He notes that different financial claimants have different incentives about the efficient liquidation policy for assets.<sup>18</sup>

Along these lines, we assume that, in bankruptcy, the cash flows of the domestic and foreign assets depend on who controls the assets. If either equity holders or a home country bank controls the assets, the domestic asset cash flows are  $x + z_x$  and the foreign asset

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<sup>15</sup>Debt contracts are optimal when state verification is costly and done nonstochastically. Border and Sobel (1987), Townsend (1988), and Mookherjee and Png (1989) show that with costly state verification stochastic verification is optimal. Boyd and Smith (1994) argue that the welfare loss, calibrated for realistic parameter values, from exogenously restricting state verification to be nonstochastic, is small. We therefore restrict our attention to contracts with nonstochastic verification of states that are not publicly observable.

<sup>16</sup>Unless specified otherwise, all cash flows are denominated in the domestic currency.

<sup>17</sup>For expositional simplicity, we do not add uncertainty to the cash flows in bankruptcy. However, it is possible to view  $x + z_x$  and  $y + z_y$  as mean cash flows without altering our results.

<sup>18</sup>Market imperfections may prohibit perfectly efficient contracting solutions to this problem.

cash flows (denominated in foreign currency) are  $y + z_y$ . If an inferior investor, such as a non-home bank ends up with the assets in bankruptcy, only the tangible component of value is captured. Hence, for a foreign bank controlling domestic assets, the domestic cash flow is  $x$ . For a domestic bank controlling foreign assets, the foreign cash flow (in foreign currency) is  $y$ .

Fractional ownership of the assets produces proportional cash flows. Hence, if the domestic bank owns the fraction  $f_X$  of the domestic and foreign assets, its cash flow would be  $f_X[x + z_x + sy]$ , while the foreign bank's cash flow would be  $[1 - f_X][x + s(y + z_y)]$ . Clearly, there are rents attached to assigning control of the cash flows to existing management team rather than have inferior investors control the assets of the firm. The dead weight loss associated with the transfer of operating control of the firm to inferior investors suggests a rationale for why absolute priority is often violated in practice. Reorganizations and debt restructurings occur in bankruptcy, which allow equityholders to maintain a measure of control of the firm. In the United States, for example, Chapter 11 bankruptcy involves a reorganization of the firm as an operating entity.

In this spirit, when bankruptcy occurs in our model, we assume that  $s$ ,  $x$ , and  $y$  are revealed, and negotiations go on between these managers and the senior claimants over the payment owed to the claimants. In this case,  $X$  owns the fraction  $f_X(s)$  of the firm, which produces a cash flow of  $f_X(s)[(x + z_x) + sy]$  if run by  $X$  and  $f_X(s)[(x + z_x) + s(y + z_y)]$  if run by the existing management. For expositional purposes, we assume that all of the bargaining power belongs to the equity-maximizing managers. Hence, equityholders can reorganize the firm by paying  $f_X(s)[(x + z_x) + sy]$  to  $X$  and keeping the rent  $f_X(s)sz_y$  for themselves. Similarly, equityholders can pay  $[1 - f_X(s)][x + s(y + z_y)]$  to  $Y$  and keep  $[1 - f_X(s)]z_x$  for themselves.<sup>19</sup>

The model allows firms to write contracts that make promised payments to  $X$ -type and  $Y$ -type investors contingent on the exchange rate. The design of these contracts is common knowledge and they are priced simultaneously using the "joint no-regrets criterion" described in the previous section.  $X$ -type investors (domestic banks) receive the state-contingent promise of  $F_X(s)$  and  $Y$ -type investors (foreign banks) receive the state-

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<sup>19</sup>Rather than negotiating this arrangement, this solution may also be mandated by a bankruptcy court judge.

If the rents are shared, simply redefine  $y$  and  $x$  to be the sum of the tangible component plus the rent captured in negotiation by the domestic and foreign banks, respectively.

contingent promise of  $F_Y(s)$ . We allow  $F_X(s)$  and  $F_Y(s)$  to be positive or negative at different values of  $s$ .

Bankruptcy occurs whenever the promised claims exceed the cash flow the firm can provide when run by management, i.e.,

$$F_X(s) + F_Y(s) > A(s; \mu):$$

Note that the probability of bankruptcy is common knowledge.

A key assumption of the model is that assets cannot easily be spun off. For example, moral hazard may make it impossible to separate the foreign and domestic assets in bankruptcy. There may be too much fungibility of domestic and foreign assets to make financing secured by domestic or foreign assets alone feasible. In this case, while financial claims have rights, in bankruptcy, to some fraction of the firm's aggregate assets, they cannot own different fractions of the domestic and foreign assets. In algebraic terms, if we think of the payoffs to  $X$  and  $Y$  in bankruptcy as  $s$ -contingent linear functions of the domestic and foreign cash flows, i.e., having payoffs to  $X$  of

$$\alpha_X(s)(x + z_x) + \beta_X(s)y;$$

and payoffs to  $Y$  of

$$[1 - \alpha_X(s)]x + [1 - \beta_X(s)]s(y + z_y);$$

then  $\alpha_X(s) = \beta_X(s)$ . A key point that we will return to in the next subsection is that even if  $\alpha_X(s) = \beta_X(s)$ , the effective coefficient on  $y$  for  $X$ , which is the expectation  $E[\alpha_X(s)]$  differs from  $E[\beta_X(s)]$  which is the effective coefficient on  $x$  for  $X$ . Because of the "adding-up constraint," this implies that the relative coefficients on  $x$  and  $y$  differ for  $X$  and  $Y$ . Hence, based on our analysis of the previous section's models, information will be revealed by issuing two securities. We will shortly prove this formally.

Clearly, in bankruptcy, the firm cannot meet its aggregate contractually promised payment,

$$F(s) = F_X(s) + F_Y(s):$$

Hence, there must be some rule that divides up what the firm can pay in bankruptcy to its cash flow claimants. In principle, the rule for determining the realized payoffs in bankrupt states need not be tied to the promised payoffs,  $F_X(s)$  and  $F_Y(s)$ . As a practical

matter, however, what a cash flow claimant gets in bankruptcy is related to what he is promised. This perhaps has to do with the nebulous nature of what constitutes bankruptcy and the moral hazard attached to legally pushing a firm into bankruptcy if the relative state-contingent payoffs can be distorted by this legal event. In this vein, we assume that the asset fraction received by X in bankruptcy is

$$f_X(s) = \begin{cases} 1 & \text{if } F_X(s) \geq 0; F_Y(s) < 0 \\ \frac{F_X(s)}{F_X(s) + F_Y(s)} & \text{if } F_X(s) \geq 0; F_Y(s) \geq 0 \\ 0 & \text{if } F_X(s) < 0; F_Y(s) \geq 0 \end{cases}$$

and  $1 - f_X(s)$  is the fraction received by Y in default. Given this constraint, a security design is defined by  $F_X(s)$  and  $F_Y(s)$ .

The aggregate proceeds,  $P$ , of the firm's securities issuance is the probability-weighted average of conditional expectations, which are denoted by subscripts next to the expectation operator. Specifically,

$$P = \Pr[A(s; \mu) \geq F(s)]E_{A(s; \mu) \geq F(s)}[F(s)] \\ + \Pr[A(s; \mu) < F(s)]E_{A(s; \mu) < F(s)}[f_X(s)f(x + z_x) + sy^a g + [1 - f_X(s)]fx^a + s(y + z_y)g];$$

Note that if  $F_X(s) < 0$  or  $F_Y(s) < 0$  in bankruptcy, there will be a transfer payment between X and Y. However, such a transfer payment does not affect  $P$ . Let  $T$  denote the set of all bankrupt states where such transfers occur.

As in the previous model, the pricing of the securities issued to X and Y reveal the private information signals  $x + z_x$  and  $y + z_y$ , as proved in the following proposition. To simplify exposition, we denote  $x^a \equiv \hat{x}(x + z_x)$  and  $y^a \equiv \hat{y}(y + z_y)$ .

**Proposition 4** All investor information is revealed in equilibrium when firms issue two nondegenerate securities to investors with distinct information sets. The price of the security issued to X is

$$P_X = \Pr[A(s; \mu) \geq F(s)]E_{A(s; \mu) \geq F(s)}[F_X(s)] \\ + \Pr[A(s; \mu) < F(s)]E_{A(s; \mu) < F(s)}[f_X(s)f(x + z_x) + sy^a g] \\ + \Pr[T]E_T[\min(0; F_X(s)) + \max(0; -F_Y(s))]$$

and the price of the security issued to Y is

$$\begin{aligned}
P_Y &= \Pr[A(s; \mu) \geq F(s)]E_{A(s; \mu) \geq F(s)}[F_Y(s)] \\
&\quad + \Pr[A(s; \mu) < F(s)]E_{A(s; \mu) < F(s)}[1 - f_X(s)]fx^a + s(y + z_y)g \\
&\quad + \Pr[T]E_T[\max(0; \int F_X(s)) + \min(0; F_Y(s))]:
\end{aligned}$$

Proof: See the Appendix. ■

## B. The Optimal Security Design for a Given Bankruptcy Boundary

Note that, in bankruptcy, the firm is giving up its assets to X and Y and then, in effect, buying them back at the reservation prices of X and Y, leaving a surplus for equity holders. Thus, the equityholders of the firm have cash not only in non-bankrupt states, but in bankrupt states as well because absolute priority is generally violated in reorganizational-type bankruptcies. In particular, equity value,  $V$ , is cash raised, less cash invested, plus cash flows to equity holders after payments to senior cash flow claimants, i.e.,

$$\begin{aligned}
V &= \int I + \Pr[A(s; \mu) \geq F(s)]E_{A(s; \mu) \geq F(s)}[F(s)] \\
&\quad + \Pr[A(s; \mu) < F(s)]E_{A(s; \mu) < F(s)}[f_X(s)f(x + z_x) + sy^a g + [1 - f_X(s)]fx^a + s(y + z_y)g] \\
&\quad + \Pr[A(s; \mu) \geq F(s)]E_{A(s; \mu) \geq F(s)}[A(s; \mu) - F(s)] \\
&\quad + \Pr[A(s; \mu) < F(s)]E_{A(s; \mu) < F(s)}[f(x + z_x) + s(y + z_y)g] \\
&\quad + \Pr[A(s; \mu) < F(s)]E_{A(s; \mu) < F(s)}[f_X(s)f(x + z_x) + syg + [1 - f_X(s)]fx + s(y + z_y)g] \\
&= \int I + \Pr[A(s; \mu) \geq F(s)]E_{A(s; \mu) \geq F(s)}F(s) \\
&\quad + \Pr[A(s; \mu) \geq F(s)]E_{A(s; \mu) \geq F(s)}[A(s; \mu) - F(s)] \\
&\quad + \Pr[A(s; \mu) < F(s)]E_{A(s; \mu) < F(s)}[(x + z_x) + s(y + z_y)] \\
&\quad + \Pr[A(s; \mu) < F(s)]E_{A(s; \mu) < F(s)}[f_X(s)s(y^a - y) + [1 - f_X(s)](x^a - x)] \tag{18}
\end{aligned}$$

For a pool leader, equity value is increased whenever a security design,  $[F_X(s); F_Y(s)]$  can reduce the claims in non-bankrupt states,  $E_{A(s; \mu) > F(s)}[F(s)]$  or reduce the number of bankrupt states, ceteris paribus, or both and still raise at least  $I$ . Holding the aggregate promised claim,  $F(s) = F_X(s) + F_Y(s)$ , fixed, it follows that the maximum equity value is attained with the security design described below:

Proposition 5 Holding  $F(s)$  fixed, the maximum equity value of a firm is achieved for the (bankrupt state) security design when  $f_X(s) = 1$  when  $x - x^a > s(y - y^a)$  and 0 otherwise.

Proof: The set of bankrupt states depends only on  $F_X(s) + F_Y(s)$ , which is fixed. Hence, every term but the last expectation in Equation 18 is fixed. This expectation is

$$\begin{aligned} & E_{A(s;\mu) < F(s)} [f_X(s)s(y^a \downarrow y) + [1 \downarrow f_X(s)](x^a \downarrow x)] \\ &= E_{A(s;\mu) < F(s)} [x^a \downarrow x + f_X(s)f(r \downarrow s)(y \downarrow y^a)g]; \end{aligned}$$

where  $r = \frac{x \downarrow x^a}{y \downarrow y^a}$ . This expression is maximized on a state by state basis by the security design with

$$f_X(s) = \begin{cases} 1 & \text{when } r > s, \text{ i.e., } x \downarrow x^a > s(y \downarrow y^a) \\ 0 & \text{otherwise:} \end{cases} \blacksquare$$

Proposition 5 shows that the proceeds from bankrupt states are maximized by minimizing adverse selection on a state by state basis, where states are defined by the realized exchange rate. Alternatively, this can be viewed as writing a state-contingent contract that allocates each state contingent cash flow to the claimant who values it the most.<sup>20</sup> Obviously, two pari passu debt contracts, e.g., foreign and domestic debt, cannot be optimal because bankruptcy proceeds are shared. Also, senior and junior debt are suboptimal because they cannot reverse priority contingent on  $s$ . However, two currency swaps, one issued to the domestic bank and one to the foreign bank, can implement the design suggested in Proposition 4, as we show later.

The  $f_X(s)$  pattern suggested in Proposition 5 is also one that maximizes  $\frac{E_{A(s;\mu) < F(s)} [f_X(s)]}{E_{A(s;\mu) < F(s)} [sf_X(s)]}$  and minimizes  $\frac{E_{A(s;\mu) < F(s)} [1 \downarrow f_X(s)]}{E_{A(s;\mu) < F(s)} [s(1 \downarrow f_X(s))]}$  given the constraint that the domestic and foreign cash flow cannot be unbundled. Hence, consistent with Proposition 2, Proposition 3 suggests that one should design securities as close to the spin-off<sup>®</sup> solution as possible, given constraints.

### C. Characterizing the Optimal Security Design

Proposition 4 implies that there is only one cash flow claimant to the firm's asset in bankruptcy with the optimal security design. To see this, note that the sum of the aggregate promised claims in a security design,  $F(s) = F_X(s) + F_Y(s)$  can always be mimicked by an alternative security design where

$$F_X(s) = \begin{cases} \text{positive number} & \text{when } r > s, \text{ i.e., } x \downarrow x^a > s(y \downarrow y^a) \\ \text{nonpositive number} & \text{otherwise} \end{cases}$$

<sup>20</sup>Thus, our result is consistent with the results of Allen and Gale (1989).

and

$$F_Y(s) = \begin{cases} \text{nonpositive number} & \text{when } r > s, \text{ i.e., } x_j - x^a > s(y_j - y^a) \\ \text{positive number} & \text{otherwise:} \end{cases}$$

For this alternative security design the bankruptcy boundary is kept the same, and the aggregate claims out of bankruptcy are kept the same, but, by Proposition 3, it dominates the first security design by minimizing adverse selection within bankruptcy.

#### D. Security Design and Currency Swaps

A plain vanilla currency swap involves the exchange of the cash flows of a domestic bond for the cash flows of a foreign bond. The bonds may be fixed or floating. Both interest and principal are typically exchanged. There may even be an exchange of cash for foreign currency at the initiation of the swap. Traditional analyses have ignored the default risk inherent in these contracts.<sup>21</sup>

It is noteworthy that many currency swap participants have credit risk. Indeed, some swap investors (e.g., Disney in the mid-80's) explain their use of swaps as being driven by credit risk.<sup>22</sup> Solnik (1994) notes that firms with default risk are charged a markup over the market swap prices that are determined (and quoted) using traditional methods that ignore default risk considerations. Moreover, Głeczy, Minton, and Schrand (1995) have shown that the use of currency swaps is positively correlated with the debt to equity ratios of firms. Because default risk is critical to our motivation for swaps, the models in this paper are designed to explain the currency swap market between corporations and banks, as opposed to the marked-to-market swap market that almost exclusively involves money center banks and investment banks. The latter swap market operates more like a futures market and exhibits negligible default risk.

In this subsection, we analyze simplified representations of currency swaps and show that they can be used to implement the optimal security design. A currency swap to bank X in the home country, (which uses dollars as currency units), contractually obligates the firm to pay  $n_X$  dollars to bank X and in return the bank is obligated to pay  $r_X n_X$  in foreign currency to the firm. Clearly, if  $r_X = 0$ , the instrument is domestic debt with face value  $n_X$  dollars. Similarly, a currency swap issued to bank Y in the foreign country contractually

<sup>21</sup>However, see Cooper and Mello (1991) and Litzenberger (1992).

<sup>22</sup>See Allen (1987).

obligates the firm to pay  $n_Y$  units of foreign currency to bank Y and in return the bank is obligated to pay  $r_Y n_Y$  units of dollars to the firm. Here,  $r_Y = 0$  represents foreign debt. In general, for positive values of  $r_X$  and  $r_Y$  these instruments represent currency swap contracts. We assume that in bankruptcy, each swap has a pari passu claim in proportion to its contractually promised payment. Using the notation of the previous subsection, this means

$$F_X(s) = n_X(1 - sr_X);$$

$$F_Y(s) = n_Y(s - r_Y);$$

Proposition 6 Two currency swaps, one issued to X and the other to Y, with

$$\frac{1}{r_X} = r_Y = \frac{x - x^n}{y - y^n}$$

implement the maximal equity value bankruptcy design described in Proposition 3.

Proof: If  $s < r_Y = \frac{x - x^n}{y - y^n}$ ,  $F_X(s) > 0$  and  $F_Y(s) < 0$  and in bankruptcy  $f_X(s) = 1$  and  $x - x^n > y - y^n$ . The complementary case is symmetric. ■

In our model, the exchange rate is intimately tied to both probability of bankruptcy and adverse selection in bankruptcy. Propositions 5 and 6 describe securities that minimize adverse selection in bankruptcy. Optimally designed securities, however, must determine the bankruptcy boundary as well as the cash flow allocation rules in bankruptcy simultaneously. This complicates the description of optimal securities. However, it is possible to show that currency swaps dominate debt.

Proposition 7 Any pair of debt contracts as well as any pairing of a currency swap with debt is dominated by a pair of currency swaps.

Proof: We show this by proving that a pair of currency swaps alone (not necessarily the pair described in Proposition 5) can achieve the same bankruptcy boundary as the design involving debt and yet realize lower adverse selection for each realization of  $s$ .

We first note that a pair of swaps that promise  $n_X(1 - sr_X)$  to X and  $n_Y(s - r_Y)$  to Y with the swap rates  $r_X$  or  $r_Y$  or both set to zero (i.e., debt) has the same  $s$ -contingent bankruptcy boundary for any realization of  $\mu$  as a pair of swaps with swap rates  $R_X$  and

$R_Y$  that are closer to  $1=r$  and  $r$ , respectively. The respective notional amounts for the alternative design that achieves this are

$$N_X = \frac{n_X(1 - r_X R_Y) + n_Y(R_Y - r_Y)}{1 - R_X R_Y}$$

and

$$N_Y = \frac{n_Y(1 - R_X r_Y) + n_X(R_X - r_X)}{1 - R_X R_Y}.$$

There is now complete freedom to select  $R_X$  and  $R_Y$  to reduce adverse selection. It follows that the equity value from bankrupt states is higher and the equity value from non-bankrupt states is the same. Hence, equity value is larger with the alternative design than with the proposed debt-based design.<sup>23</sup> ■

### III. Generalizing the Adverse Selection Model

The need for contractual derivatives in the optimal security design strikes us as a fairly generic conclusion that will hold in more complex models. The fact that these are implemented in a risk-neutral world makes this a conservative solution and one that would only be made stronger by the formal introduction of risk aversion into the model.

There are a few general conclusions that one can draw from the model in the last section. One is that securities should be designed so as to minimize the penalty from adverse selection. In the model above, with one state variable, the cost of adverse selection is either a constant or a linear function of  $s$  depending on whether  $X$  or  $Y$  owns the firm (see Figure 1). In general, securities generate cost of adverse selection that is a weighted average of these lines. It is obvious, as seen in Figure 1, that two debt contracts cannot minimize

<sup>23</sup>For example, if  $r_X = r_Y = 0$ , (two debt contracts) then, letting all expectations be conditional on  $A(s; \mu) < F(s)$ , if

$$E \frac{h}{n_X + n_Y s} \frac{r - s}{s} > 0;$$

set  $R_X = 0$  and  $R_Y > 0$  but small. In this case, the portion of equity value that is affected by security design (see Proposition 3) is

$$E[f_X(s)(r - s)] = \Pr(s < R_Y) E_{s < R_Y} \left[ \frac{h}{n_X + n_Y s} \frac{r - s}{s} \right] + \Pr(s > R_Y) E_{s > R_Y} \left[ \frac{h}{n_X + n_Y s} \frac{r - s}{s} \right] [n_X + n_Y R_Y];$$

which is increasing in  $R_Y$  for small  $R_Y$ . For the complementary case, set  $R_Y = 0$  and  $R_X > 0$  but small. In this case,

$$E[f_X(s)(r - s)] = \Pr(s < 1 - R_X) E_{s < 1 - R_X} \left[ \frac{h}{n_X + n_Y s} \frac{r - s}{s} \right] [n_X(1 - s R_X)];$$

which is increasing in  $R_X$  for small  $R_X$ .

the cost of adverse selection, but two currency swaps can.<sup>24</sup> Even in a world with multiple state variables, with a different (nonlinear) model of asymmetric information, there would always be an investor for each  $s$  such that the cost of adverse selection will be minimized if he owned the firm. In an optimal security design, that investor should own the firm for that realization of  $s$ . In contrast to our model, where the cost of adverse selection is linear in  $s$  if either investor owned the firm, in more general models, the realizations of  $s$  for which a particular investor should own the firm in bankruptcy may be non-contiguous.

It is tempting to think that having a single cash flow claimant for each state of nature in bankruptcy is one of the more robust empirical conclusions to come out of the model

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<sup>24</sup>A call and put option on foreign currency could also implement this.

developed in the last section. However, this conclusion is driven by the production and bankruptcy technology of our model. Clearly, if there were advantages to teaming up with other cash flow claimants in running the firm, this conclusion need no longer hold.

For example, if the firm's assets consist of a Japanese and an American factory, it may be useful to have Japanese and American banks jointly controlling both factories with the Japanese having more control over the operations of the Japanese factory and the Americans having control over more control over the American factory. This generalization divorces management of bankrupt assets from fractional ownership of promised claims and may introduce a whole host of agency problems that complicate the analysis. However, these agency problems may be small in relation to the efficiency drain arising from attempts to trade assets or liquidate them. Within this agency framework, there may be a precise mix of ownership that minimizes the inefficiencies of the agency relationship. Such a mix would argue for a portfolio of cash flow claimants rather than a single one.

We have also suggested that moral hazard may prevent the spinning off of assets. However, in weighing the loss from moral hazard, such as the costs of monitoring, against the loss from adverse selection, it may be that partial spinoffs of the assets mitigate the adverse selection cost more than they increase the moral hazard problem. This could be beneficial to the firm. For example, bankruptcy could imply that the Japanese bank's fractional claim of the Japanese factory could exceed his fractional claim of the firm's promised aggregate cash flow, and his fractional claim to the American factory in bankruptcy could be less than his fractional claim to the firm's promised aggregate cash flow. i.e.,  $1_j f_X(s) \notin \frac{F_Y(s)}{F_X(s)+F_Y(s)}$ . However, the benefit of this must be weighed against the possibility that management may cut a deal with one of the cash flow claimants and shift assets between factories (through such mechanisms as favorable transfer pricing) for the benefit of management and one of the cash flow claimants. If this latter problem is sufficiently small, then even if asset management is tied to asset ownership in bankruptcy, a nonlinear asset sharing rule may be optimal. Such a nonlinear rule may generate an adverse selection function that is minimized with a portfolio of claimants in a particular state.

#### IV. Conclusion

This paper has argued that different groups of investors may be asymmetrically informed

about different components of the cash flows generated by firms. For instance, banks in a given country may be better informed about a multinational firm's costs and revenues in that country than about the firm's costs and revenues from its operations in other countries. In this case, firms should allocate cash flows from different countries to different securities. It should then market each of these securities to the group of investors that possesses the most precise and, because of the possibility of adverse selection most favorable information about that particular country's cash flow. This intuition is consistent with the results obtained in Allen and Gale (1988). However, such partitioning of cash flows { which is essentially equivalent to a firm spinning off its different operations { may be infeasible because of contractual, regulatory, or operational constraints. If each feasible security contains several different cash flow components, any group of investors buying any given security will have superior information only about some components of the relevant cash flows.

This is not problematic for superior firms in an effectively complete market. In such a market, the joint pricing of securities can reveal all information relevant for pricing securities to all investors. However, if the cash flows of the assets differ, depending on the agent who owns and operates the asset, then the observable pricing of a security may not perfectly reveal information possessed by the firm.

In order to minimize an adverse selection distortion caused by such imperfect revelation, firms should design securities that are aligned with the private information of investors. The non-stochastic model of Section I can be linked to the stochastic currency swap model of Section II by taking expectations of the latter's stochastic coefficients. In this case, we can show that currency swaps are more aligned than any other debt-oriented security. The use of securities such as a currency swap may be explained by their superior alignment. A multinational firm will typically have cash flows denominated in different currencies. The domestic currency value of cash flows denominated in foreign currencies is highly sensitive to the relevant exchange rate whereas the domestic currency value of cash flows denominated in the domestic currency is relatively insensitive to exchange rate movements. Since one could write financial contracts contingent on the ex post observable value of exchange rates, an appropriate contingent contract may allow one to design securities that have different sensitivities to cash flows denominated in different currencies. Domestic debt and foreign debt have relatively similar sensitivities to domestic and the foreign cash flows. On the other hand, it is possible to show that an appropriately designed currency swap has very different

sensitivities to the two cash flow components. This may explain, not only why a redundant security, such as a currency swap, exists, but also why firms simultaneously engage in financial contracts with different intermediaries when a single financial intermediary could carry out all of the firm's contracts.

The model used to show this did not make use of risk aversion and hedging needs to motivate the purchase of derivative securities. This does not fundamentally alter our results and indeed strengthens many of them since we are able to show that firms may issue securities such as a currency swap based purely on issue-cost minimization considerations. We believe that this also sheds light on the behavior of many corporations that issue derivative securities of various types when a motive based purely on risk-management considerations seems implausible.

## Appendix

### Proof of Lemma 1

Substituting the given conjecture functions in the pricing relations (1) and (2), we get

$$P_X = \theta_X X + \frac{P_Y - \theta_Y X}{\theta_Y}; \quad (19)$$

$$P_Y = \theta_Y \frac{P_X - \theta_X Y}{\theta_X} + \theta_Y Y; \quad (20)$$

Substituting (20) into (19) and rearranging and simplifying, we get,

$$(1 - \hat{A})P_X = (1 - \hat{A})\theta_X X + (1 - \hat{A})\theta_Y Y;$$

where

$$\hat{A} = \frac{\theta_Y \theta_X}{\theta_X \theta_Y};$$

$\hat{A} = 1$  represents the degenerate case for which the two securities are identical. For the nondegenerate cases assumed here,  $\hat{A} \neq 1$ , implying

$$P_X = \theta_X X + \theta_Y Y;$$

which upon substitution into (20) yields

$$P_Y = \theta_Y X + \theta_Y Y;$$

The results then follow immediately. ■

### Proof of Lemma 2

Let  $V_X$  and  $V_Y$  denote the full information values of the two securities. From Lemma 1, we know that there exists an equilibrium in which the bids for the two securities equal their full information value. This implies that for any pair of bids  $P_X$  and  $P_Y$  to be a candidate for equilibrium, it must be the case that

$$P_X + P_Y \leq V_X + V_Y;$$

which implies that

$$(P_X - V_X) + (P_Y - V_Y) \leq 0; \quad (21)$$

Taking expectations of (21) with respect to  $X$ 's and  $Y$ 's information sets respectively we get,

$$c_X^X + c_Y^X \leq 0; \quad (22)$$

$$c_X^Y + c_Y^Y \leq 0; \quad (23)$$

where  $c_i^j$  represents the amount by which the price of security  $i$  exceeds  $j$ 's valuation of the security. Clearly, for the bids to be rational

$$c_X^X \leq 0;$$

$$c_Y^Y \leq 0;$$

which from (22) and (23) implies that

$$c_Y^X \leq 0; \quad (24)$$

$$c_X^Y \leq 0; \quad (25)$$

Taking expectations of (24) with respect to  $Y$ 's information set, and of (25) with respect to  $X$ 's information set, we get

$$E^Y [c_Y^X] \leq 0; \quad (26)$$

$$E^X [c_X^Y] \leq 0; \quad (27)$$

Now,

$$c_Y^X \leq P_Y - V_Y^X = \alpha_Y [E^Y x - x] + \beta_Y [y - E^X y]; \quad (28)$$

$$c_X^Y \leq P_X - V_X^Y = \alpha_X [x - E^Y x] + \beta_X [E^X y - y]; \quad (29)$$

Taking expectations of (28) and (29) with respect to the information sets of  $X$  and  $Y$ , we get:

$$E^X [c_Y^X] = c_Y^X = \alpha_Y [E^X E^Y x - x]; \quad (30)$$

$$E^X [c_X^Y] = \alpha_X [x - E^X E^Y x]; \quad (31)$$

$$E^Y [c_Y^X] = \beta_Y [y - E^Y E^X y]; \quad (32)$$

$$E^Y [c_X^Y] = c_X^Y = \beta_X [E^Y E^X y - y]; \quad (33)$$

From (31) and (30), we get

$$E^X [c_X^Y] = \frac{\alpha_X}{\alpha_Y} c_Y^X; \quad (34)$$

Since  $E^X [c_X^Y] \leq 0$  from (27), combining it with (34), we get

$$c_Y^X \leq 0; \quad (35)$$

From (35) and (24), we get  $c_Y^X = 0$ :

Similarly, from (32) and (33), we get

$$E^Y [c_Y^X] = \frac{\bar{Y}}{\bar{X}} c_X^Y: \quad (36)$$

Since  $E^Y [c_Y^X] \geq 0$  from (26), combining it with (36), we get

$$c_X^Y \leq 0: \quad (37)$$

From (37) and (25), we get  $c_X^Y = 0$ .<sup>25</sup>

Proof of Lemma 3

From Lemma 2,  $c_X^Y = c_Y^X = 0$  which implies from (28) and (29) that

$$\frac{\otimes_X}{\bar{X}} [x \mid E^Y x] = [y \mid E^X y]: \quad (40)$$

$$[x \mid E^Y x] = \frac{\bar{Y}}{\otimes_Y} [y \mid E^X y]: \quad (41)$$

Substituting for  $[x \mid E^Y x]$  from (40) into (41), rearranging and simplifying, we get:

$$[y \mid E^X y] = \hat{A} [y \mid E^X y]:$$

Since, for the nondegenerate cases assumed,  $\hat{A} \notin 1$ ,

$$y \mid E^X y = 0:$$

This implies that  $E^X y = y$  which from (41), in turn implies that  $E^Y x = x$ . $\blacksquare$

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<sup>25</sup>If we assume that X and Y have common knowledge that x and y are bounded, then it is possible to prove this lemma without the assumption that firms only accept the best rational competitive bids. We hinted at this earlier. Taking expectation  $E^X$  of (36), substituting from (34) and rearranging, we get

$$c_Y^X = \hat{A} E^X E^Y [c_Y^X]; \quad (38)$$

where

$$\hat{A} = \frac{\otimes_Y = \bar{Y}}{\otimes_X = \bar{X}}:$$

Repeated substitution for  $c_Y^X$  from (38) into the R.H.S. of (38) yields:

$$c_Y^X = \hat{A}^n (E^X E^Y)^n [c_Y^X] \quad \text{for } n = 1; 2; 3; \dots: \quad (39)$$

where  $(E^X E^Y)^n$  represents the  $E^X E^Y$  operator applied n times.

Since it is common knowledge that x and y are bounded it implies that it must be common knowledge that  $c_Y^X$  is also bounded. If  $\hat{A} < 1$ , then from (39) it implies that  $c_Y^X = 0$ . If  $\hat{A} > 1$  then  $\hat{A}^n$  becomes unbounded as n becomes large. The only way the R.H.S. of (39) will be bounded in that case is if  $(E^X E^Y)^n [c_Y^X] = 0$  which from (39) implies that  $c_Y^X = 0$ . The case when  $\hat{A} = 1$  is ruled out by assumption since that implies that both securities are identical.

Analogous arguments can be used to show that  $c_X^Y = 0$ .

Proof of Lemma 4

Substituting the given conjecture functions in the pricing relations (9) and (10), we get

$$P_X = \theta_X(x + z_x) + \frac{\mu}{\theta_X} \frac{P_Y + \theta_Y(x + z_x)}{\theta_Y}; \quad (42)$$

$$P_Y = \theta_Y(x + z_x) + \frac{\mu}{\theta_X} \frac{P_X + \theta_X(y + z_y)}{\theta_X}; \quad (43)$$

Substituting (43) into (42), we get,

$$P_X = \theta_X(x + z_x) + \frac{\mu}{\theta_X} \frac{P_X + \theta_X(y + z_y)}{\theta_X} + \frac{\theta_Y}{\theta_X} \frac{\mu}{\theta_X} \frac{P_X + \theta_X(y + z_y)}{\theta_X};$$

Subtracting  $\theta_X(x + z_x) + \frac{\mu}{\theta_X} \frac{P_X + \theta_X(y + z_y)}{\theta_X}$  from both sides, we get

$$\begin{aligned} P_X & - \left[ \theta_X(x + z_x) + \frac{\mu}{\theta_X} \frac{P_X + \theta_X(y + z_y)}{\theta_X} \right] \\ & = \frac{\theta_Y}{\theta_X} \frac{\mu}{\theta_X} \frac{P_X + \theta_X(y + z_y)}{\theta_X} \\ & = \frac{\theta_Y}{\theta_X} m_Y \frac{\mu}{\theta_X} \frac{P_X + \theta_X(y + z_y)}{\theta_X} \\ & = \frac{\theta_Y}{\theta_X} m_X m_Y [P_X + \theta_X(x + z_x) + \theta_X(y + z_y)] \end{aligned}$$

where  $m_X$  and  $m_Y$  are given by the mean value theorem.

Rearranging, we get

$$(1 - \hat{A}^0) [P_X + \theta_X(x + z_x) + \theta_X(y + z_y)] = 0;$$

where

$$\hat{A}^0 = \frac{\theta_Y}{\theta_X} m_X m_Y;$$

For nondegenerate cases (see footnote 13),  $\hat{A}^0 < 1$ . Therefore,

$$P_X = \theta_X(x + z_x) + \theta_X(y + z_y);$$

which, upon substitution into (43) implies that

$$P_Y = \theta_Y(x + z_x) + \theta_Y(y + z_y); \blacksquare$$

Proof of Lemma 5

Let,

$$c_Y^X \sim P_Y \mid V_Y^X = \otimes_Y [E^Y x \mid E^X x] + \bar{\ }_Y [(y + z_y) \mid E^X (y + z_y)]; \quad (44)$$

$$c_X^Y \sim P_X \mid V_X^Y = \otimes_X [(x + z_x) \mid E^Y (x + z_x)] + \bar{\ }_X [E^X y \mid E^Y y]; \quad (45)$$

Taking expectations of (44) and (45) with respect to the information sets of X and Y yields:

$$E^X [c_Y^X] = c_Y^X = \otimes_Y [E^X E^Y x \mid E^X x]; \quad (46)$$

$$E^X [c_X^Y] = \otimes_X [(x + z_x) \mid E^X E^Y (x + z_x)]; \quad (47)$$

$$E^Y [c_Y^X] = \bar{\ }_Y [(y + z_y) \mid E^Y E^X (y + z_y)]; \quad (48)$$

$$E^Y [c_X^Y] = c_X^Y = \bar{\ }_X [E^Y E^X y \mid E^Y y]; \quad (49)$$

Since  $\mathfrak{x}$  and  $\mathfrak{y}$  are increasing in their arguments, we obtain

$$E^X x \mid E^Y x = \mathfrak{x}(x + z_x) \mid E^Y \mathfrak{x}(x + z_x) = m_X^0 [(x + z_x) \mid E^Y (x + z_x)]; \quad (50)$$

$$E^Y y \mid E^X y = \mathfrak{y}(y + z_y) \mid E^X \mathfrak{y}(y + z_y) = m_Y^0 [(y + z_y) \mid E^X (y + z_y)]; \quad (51)$$

where  $m_X^0 > 0$  and  $m_Y^0 > 0$  by the mean value theorem.

From (47), (46) and (50), we get

$$E^X [c_X^Y] = \mid \frac{\otimes_X}{\otimes_Y} \frac{1}{m_X^0} c_Y^X:$$

From (48), (49) and (51), we get

$$E^Y [c_Y^X] = \mid \frac{\bar{\ }_Y}{\bar{\ }_X} \frac{1}{m_Y^0} c_X^Y:$$

The arguments used to prove Lemma 2 immediately apply.■

Proof of Lemma 6

From Lemma 5,  $c_X^Y = c_Y^X = 0$  which from (45) and (44) implies that

$$\frac{\otimes_X}{\bar{\ }_X} [(x + z_x) \mid E^Y (x + z_x)] = [E^Y y \mid E^X y]; \quad (52)$$

$$[E^X x \mid E^Y x] = \frac{\bar{\ }_Y}{\otimes_Y} [(y + z_y) \mid E^X (y + z_y)]; \quad (53)$$

Substituting for  $[E^X x \ ; \ E^Y x]$  from (53) into the L.H.S. of (50), we get

$$m_X^0 [(x + z_x) \ ; \ E^Y (x + z_x)] = \frac{\bar{Y}}{\otimes_Y} [(y + z_y) \ ; \ E^X (y + z_y)]: \quad (54)$$

Substituting from (52) into the L.H.S. of (54), we get:

$$m_X^0 \frac{\bar{X}}{\otimes_X} [E^Y y \ ; \ E^X y] = \frac{\bar{Y}}{\otimes_Y} [(y + z_y) \ ; \ E^X (y + z_y)]: \quad (55)$$

Substituting from (53) into the R.H.S. of (55) and simplifying, we get

$$\hat{A}^0 [E^Y y \ ; \ E^X y] = [E^Y y \ ; \ E^X y]:$$

For nondegenerate cases,  $\hat{A}^0 \Subset 1$ , implying

$$E^Y y \ ; \ E^X y = 0:$$

This implies that  $E^X y = E^Y y$  which from (52) and (50), in turn implies that  $E^Y x = E^X x$ .■

Proof of Proposition 4

The following pair of conjecture functions lead to rational competitive bids and  $y + z_y$  being revealed to X and  $x + z_x$  being revealed to Y:

$$E^Y x = \frac{P_X \ ; \ \Pr[A(s; \mu) \ ; \ F(s)] E_{A(s; \mu) \ ; \ F(s)} [F_X(s)] \ ; \ \Pr[T] E_T [\min(0; F_X(s)) + \max(0; \ ; \ F_Y(s))]}{\Pr[A(s; \mu) < F(s)] E_{A(s; \mu) < F(s)} [f_X(s)]} \ ; \ \frac{E_{A(s; \mu) < F(s)} [s f_X(s)] y (y + z_y)}{E_{A(s; \mu) < F(s)} [f_X(s)]};$$

$$E^X y = \frac{P_Y \ ; \ \Pr[A(s; \mu) \ ; \ F(s)] E_{A(s; \mu) \ ; \ F(s)} [F_Y(s)] \ ; \ \Pr[T] E_T [\max(0; \ ; \ F_X(s)) + \min(0; F_Y(s))]}{\Pr[A(s; \mu) < F(s)] E_{A(s; \mu) < F(s)} [1 \ ; \ f_X(s)]} \ ; \ \frac{E_{A(s; \mu) < F(s)} [1 \ ; \ f_X(s)] x (x + z_x)}{E_{A(s; \mu) < F(s)} [s (1 \ ; \ f_X(s))]};$$

The proof of this is a trivial extension of Lemma 4. The remainder of the proof follows directly from Lemmas 5 and 6.■

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