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FLAVOR CHANGING NEUTRAL PROCESSES AND B_d^0–$\bar{B}_d^0$ MIXING

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We propose that the observed B_d^0–$\bar{B}_d^0$ mixing reported by ARGUS and CLEO is due to the tree-level flavor changing neutral coupling of the standard model Higgs scalar, H^0, or the Z^0, induced by new physics with a mass scale beyond the standard model. The strengths of the flavor changing couplings of H^0 and Z^0 are shown to be increasing with the masses of the fermion flavors involved. If the observed B_d^0–$\bar{B}_d^0$ mixing is due to the flavor changing coupling of H^0, the key predictions are D^0–$\bar{D}^0$ mixing of O(10%) of the present experimental upper limit and $BR(\mu^-\to e^-\gamma)\simeq(1.1\pm0.6)\times10^{-12}$, and the mass of the Higgs scalar $M_H\simeq(200\text{–}300)$ GeV. In case the observed B_d^0–$\bar{B}_d^0$ mixing is due to the flavor changing coupling of Z^0, the rare decay mode $\mu^-\to e^-e^+e^-$ is predicted to be observable at any time in the near future with the branching ratio in the neighborhood of the present experimental upper limit, while other predictions include: D^0–$\bar{D}^0$ mixing of O(1–10)% of the present upper limit, $BR(B_s^0\to\mu^+\mu^-X)\simeq(8.5\pm4.2)\times10^{-5}$, $BR(\tau^-\to\mu^-\mu^+\mu^-)\simeq(8.8\pm4.8)\times10^{-8}$, and the branching ratios for the flavor changing decay modes of Z^0, $BR(Z^0\to b\bar{s}+s\bar{b})\times10^7\simeq(14\pm7)$, $BR(Z^0\to t\bar{c}+c\bar{t})\times10^7\simeq(1500\pm700)(m_t/60\ \text{GeV})$, $BR(Z^0\to b'\bar{b}+b\bar{b}')\times10^7\simeq(4800\pm2300)(m_{b'}/50\ \text{GeV})$, $BR(Z^0\to\mu^-\tau^++\mu^+\tau^-)\times10^7\simeq3.6\pm1.8$, and $BR(Z^0\to\tau'\bar{\tau}+\bar{\tau}'\tau)\times10^7\simeq(1300\pm600)\ (m_{\tau'}/40\ \text{GeV})$. These flavor changing branching ratios of Z^0 can be tested at LEP with 10^7 Z^0's. From the observed strength of B_d^0–$\bar{B}_d^0$ mixing the scale of new physics can be inferred to be $M\simeq250$ GeV.

1. Introduction

In our present understanding of particle physics, there is one potentially important piece of experimental data whose explanation may require new physics beyond the standard model. It is the surprisingly large B_d^0–$\bar{B}_d^0$ mixing reported by the ARGUS Collaboration [1], which has been confirmed by the CLEO Collaboration [2].

Although such a large mixing can be understood within the standard KM model by assuming [3] a large t-quark mass ($m_t\gtrsim100$ GeV) or certain KM matrix elements in the neighborhood of their present upper limits, this will fail to provide the solution if the t-quark is discovered below 100 GeV or so in the near future and the $|V_{ub}|$ element is measured to be well below (e.g. $|V_{ub}|/|V_{cb}|\simeq0.06\text{–}0.08$ [4]) the present upper limit, $|V_{ub}|/|V_{cb}|\leqslant0.21$ [5]. Another resolution of the dilemma could come from the existence of the fourth family of quarks (and leptons); the t′-quark contribution to the usual box diagram could account for the magnitude of B_d^0–$\bar{B}_d^0$ mixing. This would require special values for $m_{t'}$ and for the four family KM element $V_{t'd}V_{t'b}^*$. While there exist practically no experimental constraints on these parameters, a recent investigation [6] suggests that this may not be the case. This is because the t′-quark exchange diagrams, which are required to bring the B_d^0–$\bar{B}_d^0$ mixing to the observed level, inevitably enhance the *CP*-violating amplitude in K^0–$\bar{K}^0$ mixing, giving too large a contribution [6] to $\mathrm{Re}\,\epsilon_K$ ($=1.62\times10^{-3}$).

In this article, we propose that this mixing is due to tree-level flavor changing couplings of the standard model Higgs scalar, H^0, or the Z^0, induced by new physics beyond the standard model. In section 2, we present an illustrative example, where physics beyond the standard model could be responsible for such flavor changing neutral current processes (FCNP) of the known quarks and leptons. In section 3, we set up our conventions for such couplings and look at the present experimental bounds. As mentioned, we assume that the large B_d^0–$\bar{B}_d^0$ mixing is due to such FCNP and this anchors one of these couplings. In section 4, we look at various theoretical ex-

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pectations for the flavor dependence of these couplings and in section 5 we show how these expectations hold up in view of the previously studied bounds. The results are pleasing in that not only are the bounds not seriously violated, but several unseen reactions are on the verge of observability. These discussions, predictions for flavor changing decays of the Z^0, and speculations on the scale of this new physics responsible for the FCNP are presented in section 6.

2. An illustrative example: vector singlet model

As a simple, illustrative example of the general class of models, in which tree-level neutral flavor changing couplings of H^0 and Z^0 between ordinary quarks and leptons are generated through the effect of mixings with heavy exotic fermions, we consider a model [#1] with an $SU(2)_L$ vector singlet of charge $-1/3$ quarks, D_L and D_R, plus the three standard families of quarks and leptons.

In the basis of weak-eigenstates d^0_{iL} and d^0_{jR}, the mass and the Yukawa couplings of the charge $-1/3$ quarks are given by

$$-L_Y = [\overline{d^0_{iL}}(M^d)_{ij} d^0_{jR} + \overline{d^0_{iL}}(y^d)_{ij} d^0_{jR} H^0/\sqrt{2}] + \text{h.c.}, \tag{1}$$

where

$$(y^d) = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

$$(M^d) = (y^d) v/\sqrt{2} + (M'),$$
$$v \equiv (\sqrt{2} G_F)^{-1/2} = 246\ \text{GeV}, \tag{3}$$

with

$$(M') = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M \end{bmatrix}. \tag{4}$$

[#1] Generalization to the case with a leptonic vector singlet with charge -1 (E^-_L and E^-_R) and/or charge 2/3 quarks (U_L and U_R) is straightforward. For earlier works on models with exotic heavy fermions, see ref. [7] and references therein.

In eq. (1), $i, j = 1, 2, 3$ correspond to the three families of ordinary d-type quarks (d, s, b), while $i, j = 4$ corresponds to the heavy exotic ones, D_L and D_R. (M') of eq. (4) is due to the bare mass term M and takes the given form without any loss of generality [#2]. The mass matrix (M^d) can be diagonalized by the unitary matrices V_L and V_R,

$$(V_L)^\dagger (M^d)(V_R) \equiv M^d_{\text{diag.}}. \tag{5}$$

Defining the mass-eigenstates d_{iL} and d_{iR}, for $i = 1, 2, 3, 4$, by

$$d^0_{iL} \equiv (V_L)_{ij} d_{jL}, \quad d_{iL} \equiv (V^\dagger_L)_{ij} d^0_{jL},$$
$$d^0_{iR} \equiv (V_R)_{ij} d_{jR}, \quad d_{iR} \equiv (V^\dagger_R)_{ij} d^0_{jR}, \tag{6}$$

the Yukawa couplings of the (standard model) Higgs scalar H^0 are given by

$$-L^{H^0}_Y = \overline{d_L}[(V^\dagger_L)(y^d)(V_R)] d_R H^0/\sqrt{2} + \text{h.c.}$$
$$= \overline{d_{iL}}(M^d_{\text{diag.}})_{ii} d_{iR} H^0/v$$
$$- (M/v) \overline{d_{iL}} (V_L)^*_{4i} (V_R)_{4j} d_{jR} H^0 + \text{h.c.} \tag{7}$$

where we have used $(y^d) = [(M^d) - (M')](\sqrt{2}/v)$, from eq. (3). Thus the flavor changing coupling of H^0 is given by

$$L^{H^0}_{\text{F.C.}} = y'_{ij} \overline{d_{iL}} d_{jR} H^0 + \text{h.c.}, \quad \text{for } i \neq j, \tag{8}$$

where

$$y'_{ij} = (M/v)(V_L)^*_{4i}(V_R)_{4j}, \quad \text{for } i \neq j. \tag{9}$$

In the basis of weak-eigenstates, the neutral current coupling of Z^0 is

$$L^{Z^0}_{\text{N.C.}} = (e/\sin\theta_W \cos\theta_W) J^0_\mu Z^\mu, \tag{10}$$

where

$$J^0_\mu = J^3_\mu - \sin^2\theta_W J^{\text{em}}_\mu \tag{11}$$
$$= \overline{d^0_{iL}} (t^L_3)_{ii} \gamma_\mu d^0_{iL}$$
$$- (-\tfrac{1}{3}) \sin^2\theta_W (\overline{d^0_{iL}} \gamma_\mu d^0_{iL} + \overline{d^0_{iR}} \gamma_\mu d^0_{iR}), \tag{12}$$

[#2] One may think that the most general form of the matrix (M') has four nonvanishing elements in the last row, i.e. $(M')_{4j} \equiv M'_j \neq 0$. However, a unitary rotation in the space of the four d^0_{jR}'s ($j = 1, 2, 3, 4$), can always bring the matrix (M') to the form given in eq. (4) with $M \equiv (|M'_1|^2 + |M'_2|^2 + |M'_3|^2 + |M'_4|^2)^{1/2}$.

$$(t_3^{\mathrm{L}}) = \mathrm{diag.}(-\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, 0)$$
$$= -\tfrac{1}{2}[\mathrm{diag.}(1,1,1,1) - \mathrm{diag.}(0,0,0,1)]. \quad (13)$$

In terms of the mass-eigenstates of eq. (6), eq. (11) becomes

$$J_\mu^0 = -\tfrac{1}{2}\overline{\mathrm{d}_{i\mathrm{L}}}\gamma_\mu \mathrm{d}_{i\mathrm{L}} + \tfrac{1}{2}\overline{\mathrm{d}_{i\mathrm{L}}}(V_\mathrm{L})^*_{4i}(V_\mathrm{L})_{4j}\mathrm{d}_{j\mathrm{L}}$$
$$-(-\tfrac{1}{3})\sin^2\theta_\mathrm{W}(\overline{\mathrm{d}_{i\mathrm{L}}}\gamma_\mu \mathrm{d}_{i\mathrm{L}} + \overline{\mathrm{d}_{i\mathrm{R}}}\gamma_\mu \mathrm{d}_{i\mathrm{R}}). \quad (14)$$

Thus, the flavor changing coupling of Z^0 is

$$L_{\mathrm{F.C.}}^{\mathrm{Z}^0} = (e/2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W})\hat{g}^\mathrm{L}_{ij}\overline{\mathrm{d}_{i\mathrm{L}}}\gamma^\mu \mathrm{d}_{j\mathrm{L}}\mathrm{Z}^0_\mu,$$
$$\text{for } i \neq j, \quad (15)$$

with

$$\hat{g}^\mathrm{L}_{ij} = (V_\mathrm{L})^*_{4i}(V_\mathrm{L})_{4j}, \quad \text{for } i\neq j. \quad (16)$$

The main results [#3] of this section show that the flavor changing couplings of H^0 and Z^0 are given by eqs. (8), (9), (15), and (16), with strengths proportional to the product of the mixing angles $(V_\mathrm{L})^*_{4i}$ and $(V_{\mathrm{L,R}})_{4i}$ of the unitary matrices V_L and V_R.

3. Experimental constraints on the flavor changing couplings of H^0 and Z^0

In order to discuss the physics of flavor changing couplings of H^0 and Z^0 we need to know their present experimental constraints. We shall use the notation and conventions described below. The most general form of couplings of the Z^0 to ordinary quarks and leptons is

$$L_{\mathrm{Z}^0} = \sum_{i,j}(g^\mathrm{L}_{ij}\overline{f_{i\mathrm{L}}}\gamma^\mu f_{j\mathrm{L}} + g^\mathrm{R}_{ij}\overline{f_{i\mathrm{R}}}\gamma^\mu f_{j\mathrm{R}})\mathrm{Z}^0_\mu + \mathrm{h.c.}$$
$$\equiv \frac{e}{2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W}}\left(\sum_{i,j}(\hat{g}^\mathrm{L}_{ij}\overline{f_{i\mathrm{L}}}\gamma^\mu f_{j\mathrm{L}}\right.$$
$$\left.+\hat{g}^\mathrm{R}_{ij}\overline{f_{i\mathrm{R}}}\gamma^\mu f_{j\mathrm{R}})\mathrm{Z}^0_\mu + \mathrm{h.c.}\right). \quad (17)$$

From hermiticity

$$\hat{g}^\mathrm{L}_{ji} = \hat{g}^{\mathrm{L}*}_{ij} \quad \text{and} \quad \hat{g}^\mathrm{R}_{ji} = \hat{g}^{\mathrm{R}*}_{ij}. \quad (18)$$

[#3] Similar tree-level flavor changing couplings of H^0 and Z^0 exist for vector doublet models ($\hat{g}^\mathrm{R}_{ij}$, y_{ij}) and mirror fermion models ($\hat{g}^\mathrm{L}_{ij}$, $\hat{g}^\mathrm{R}_{ij}$, y_{ij}); generalizations of our discussions to these models are straightforward.

The indices i and j stand for flavors of the ordinary quarks and leptons. Similarly, the most general form of couplings of the Higgs scalar, H^0, to the ordinary quarks and leptons is given by

$$L_{\mathrm{H}^0} = \sum_{i,j}(\overline{f_{i\mathrm{L}}}y^\mathrm{R}_{ij}f_{j\mathrm{R}} + \overline{f_{i\mathrm{R}}}y^\mathrm{L}_{ij}f_{j\mathrm{L}})\mathrm{H}^0 + \mathrm{h.c.}$$
$$\equiv \frac{e}{2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W}}\left(\sum_{i,j}(\overline{f_{i\mathrm{L}}}\hat{y}^\mathrm{R}_{ij}f_{j\mathrm{R}}\right.$$
$$\left.+\overline{f_{i\mathrm{R}}}\hat{y}^\mathrm{L}_{ij}f_{j\mathrm{L}})\mathrm{H}^0 + \mathrm{h.c.}\right), \quad (19)$$

with

$$\hat{y}^\mathrm{R}_{ji} = \hat{y}^{\mathrm{L}*}_{ij} \quad \text{and} \quad \hat{y}^\mathrm{L}_{ji} = \hat{y}^{\mathrm{R}*}_{ij}. \quad (20)$$

The unknown mass of the Higgs scalar, M_H, will be expressed in terms of $\hat{M}_\mathrm{H}$, where

$$\hat{M}_\mathrm{H} \equiv M_\mathrm{H}/M_\mathrm{Z} = M_\mathrm{H}/(92\ \mathrm{GeV}). \quad (21)$$

Note that for $\sin^2\theta_\mathrm{W} = 0.23$, $e/(2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W}) = 0.36$. The advantage of expressing all couplings in these units is that the relations

$$(e/2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W})^2/M_\mathrm{Z}^2 = \sqrt{2}G_\mathrm{F},$$
$$y^{\mathrm{L,R}}_{ij}y^{\mathrm{L,R}}_{kl}/M_\mathrm{H}^2$$
$$= \hat{y}^{\mathrm{L,R}}_{ij}\hat{y}^{\mathrm{L,R}}_{kl}(e/2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W})^2/M_\mathrm{H}^2$$
$$= \sqrt{2}G_\mathrm{F}\hat{M}_\mathrm{H}^{-2}\hat{y}^{\mathrm{L,R}}_{ij}\hat{y}^{\mathrm{L,R}}_{kl} \quad (22)$$

simplify the expressions appearing in the effective lagrangians. We have investigated a variety of FCNP which are likely to provide the most stringent constraints on the flavor changing couplings of H^0 and Z^0; details will be published elsewhere [8]. Our results are summarized in the first three columns of tables 1 and 2 [#4]. Note that the results for $\mathrm{B^0_d}$–$\overline{\mathrm{B}^0_\mathrm{d}}$ mixing are not to be taken as a bound, but, in the spirit of this work, as a positive result fixing the parameters coupling the b-quark to the d-quark.

[#4] The full expressions, of course, involve the coupling constants in a complicated manner. For example, in the case of H^0 exchange, the constraint from ΔM_K is $(1/\hat{M}_\mathrm{H})|(\hat{y}^\mathrm{L}_{\mathrm{sd}})^2 + (\hat{y}^\mathrm{R}_{\mathrm{sd}})^2 - \frac{2}{5}(6+\beta_\mathrm{K}^{-1})(\hat{y}^\mathrm{L}_{\mathrm{sd}})(\hat{y}^\mathrm{R}_{\mathrm{sd}})|^{1/2} \leqslant 9.5\times10^{-5}(10/\beta_\mathrm{K})^{1/2}$, with $\beta_\mathrm{K} \equiv [m_\mathrm{K}/(m_\mathrm{s}+m_\mathrm{d})]^2 \simeq (10\pm5)$. Since we cannot disentangle the various parts of this expression, we assume that there are no fortuitous cancellations and present results for a generic coupling $|\hat{y}^{\mathrm{L,R}}_{\mathrm{sd}}|$.

Table 1
Present experimental constraints on the flavor changing couplings (times Higgs mass factor) of the Higgs scalar, H^0, and their predicted values for $p=1/2$ and $p=1$ by eq. (34). In the first row, $\beta_K \equiv [m_K/(m_s+m_d)]^2 \simeq (10\pm5)$. For p between 1/2 and 1, the predicted values are between the values for $p=1/2$ and $p=1$.

Source	Coupling	Experimental upper bound	Predictions: $p=1/2$	Predictions: $p=1$
$\Delta M_K = 3.52\times10^{-15}$ GeV [5]	$\hat{M}_H^{-1}\lvert\hat{y}_{sd}^{L,R}\rvert$	$9.0\times10^{-5}\sqrt{10/\beta_K}$	$(1.6\pm0.4)\times10^{-4}$	$(2.6\pm0.7)\times10^{-5}$
D^0-$\bar{D}^0$ mixing [9]	$\hat{M}_H^{-1}\lvert\hat{y}_{cu}^{L,R}\rvert$	$7.2\times10^{-4}(0.16\text{ GeV}/f_D)$	$(3.9\pm0.9)\times10^{-4}$	$(1.6\pm0.4)\times10^{-4}$
B_d^0-$\bar{B}_d^0$ mixing: ARGUS, CLEO [1,2]	$\hat{M}_H^{-1}\lvert\hat{y}_{bd}^{L,R}\rvert$	$(9.5\pm2.3)\times10^{-4}(0.15\text{ GeV}/f_B)$	input	input
$BR(B_s^0\to\mu^+\mu^-X)\leqslant1.2\times10^{-3}$ [10]	$\hat{M}_H^{-2}\lvert\hat{y}_{bs}^{L,R}\rvert$	$10.3(\lvert V_{cb}\rvert/0.043)$	$\hat{M}_H^{-1}(4.1\pm1.0)\times10^{-3}$	$\hat{M}_H^{-1}(1.8\pm0.4)\times10^{-2}$
-	$\hat{M}_H^{-1}\lvert\hat{y}_{tc}^{L,R}\rvert$	-	$(4.2\pm1.0)\times10^{-2}\sqrt{m_t/60\text{ GeV}}$	$(1.9\pm0.5)(m_t/60\text{ GeV})$
-	$\hat{M}_H^{-1}\lvert\hat{y}_{b'b}^{L,R}\rvert$	-	$(7.5\pm1.8)\times10^{-2}\sqrt{m_{b'}/50\text{ GeV}}$	$(5.9\pm1.4)(m_{b'}/50\text{ GeV})$
$BR(\mu^-\to e^-\gamma)\leqslant4.9\times10^{-11}$ [11]	$\hat{M}_H^{-1}\lvert\hat{y}_{\mu e}^{L,R}\rvert$	1.6×10^{-2}	$(1.1\pm0.3)\times10^{-5}$	$(1.2\pm0.3)\times10^{-7}$
$\delta a_e<2\times10^{-10}$ [12]	$\hat{M}_H^{-1}\lvert\hat{y}_{\tau e}^{L}\hat{y}_{\tau e}^{R}\rvert^{1/2}$	0.38	$(1.4\pm0.3)\times10^{-4}$	$(2.0\pm0.5)\times10^{-5}$
$BR(\mu^-\to e^-\gamma)\leqslant4.9\times10^{-11}$ [11]	$\hat{M}_H^{-2}\lvert\hat{y}_{\tau e}^{R}\hat{y}_{\tau\mu}^{L}\rvert$	1.9×10^{-6}	$(2.8\pm0.7)\times10^{-7}$	$(8.4\pm4.0)\times10^{-8}$
$\delta a_\mu<3\times10^{-8}$ [12]	$\hat{M}_H^{-1}\lvert\hat{y}_{\tau\mu}^{R}\hat{y}_{\tau\mu}^{L}\rvert^{1/2}$	23.8	$(2.0\pm0.5)\times10^{-3}$	$(4.2\pm1.0)\times10^{-3}$
$BR(\tau^-\to\mu^-\mu^+\mu^-)\leqslant2.9\times10^{-5}$ [11]	$\hat{M}_H^{-2}\lvert\hat{y}_{\tau\mu}^{L,R}\rvert$	35.3	$\hat{M}_H^{-1}(2.0\pm0.5)\times10^{-3}$	$\hat{M}_H^{-1}(4.2\pm1.0)\times10^{-3}$
-	$\hat{M}_H^{-1}\lvert\hat{y}_{\tau'\tau}^{L,R}\rvert$	-	$(3.9\pm0.9)\times10^{-2}\sqrt{m_{\tau'}/40\text{ GeV}}$	$(1.6\pm0.4)(m_{\tau'}/40\text{ GeV})$

Table 2
Present experimental constraints on the flavor changing couplings of the Z^0, and their predicted values for $p=1/2$ and $p=1$ by eq. (34). For p between 1/2 and 1, the predicted values are between the values for $p=1/2$ and $p=1$.

Source	Coupling	Experimental upper bound	Predictions: $p=1/2$	Predictions: $p=1$
$\Delta M_K=3.52\times10^{-15}$ GeV [5]	$\lvert\hat{g}_{sd}^{L,R}\rvert$	2.2×10^{-4}	$(1.3\pm0.3)\times10^{-4}$	$(2.1\pm0.5)\times10^{-5}$
$BR(K_L\to\mu^+\mu^-)=9.1\times10^{-9}$ [5]	$\lvert\mathrm{Re}(\hat{g}_{sd}^{R}-\hat{g}_{sd}^{L})\rvert$	1.34×10^{-5}	$(1.3\pm0.3)\times10^{-4}$	$(2.1\pm0.5)\times10^{-5}$
D^0-$\bar{D}^0$ mixing [11]	$\lvert\hat{g}_{cu}^{L,R}\rvert$	$7.9\times10^{-4}(0.16\text{ GeV}/f_D)$	$(3.0\pm0.7)\times10^{-4}$	$(1.2\pm0.3)\times10^{-4}$
B_d^0-$\bar{B}_d^0$ mixing: ARGUS, CLEO [1,2]	$\lvert\hat{g}_{bd}^{L,R}\rvert$	$(7.5\pm1.8)\times10^{-4}(0.15\text{ GeV}/f_B)$	input	input
$BR(B_s^0\to\mu^+\mu^-X)\leqslant1.2\times10^{-3}$ [10]	$\lvert\hat{g}_{bs}^{L,R}\rvert$	$1.2\times10^{-2}(\lvert V_{cb}\rvert/0.043)$	$(3.2\pm0.8)\times10^{-3}$	$(1.4\pm0.3)\times10^{-2}$
-	$\lvert\hat{g}_{tc}^{L,R}\rvert$	-	$(3.3\pm0.8)\times10^{-2}\sqrt{m_t/60\text{ GeV}}$	$(1.5\pm0.4)(m_t/60\text{ GeV})$
-	$\lvert\hat{g}_{b'b}^{L,R}\rvert$	-	$(5.9\pm1.4)\times10^{-2}\sqrt{m_{b'}/50\text{ GeV}}$	$(4.7\pm1.1)(m_{b'}/50\text{ GeV})$
$BR(\mu^-\to e^-e^+e^-)\leqslant1.0\times10^{-12}$ [11]	$\lvert\hat{g}_{\mu e}^{L,R}\rvert$	2.3×10^{-6}	$(8.4\pm2.0)\times10^{-6}$	$(9.5\pm2.3)\times10^{-8}$
$BR(\tau^-\to e^-\mu^+\mu^-)\leqslant3.3\times10^{-5}$ [11]	$\lvert\hat{g}_{\tau e}^{L,R}\rvert$	3.9×10^{-2}	$(1.1\pm0.3)\times10^{-4}$	$(1.6\pm0.4)\times10^{-5}$
$BR(\mu^-\to e^-\gamma)\leqslant4.9\times10^{-11}$ [11]	$\lvert\hat{g}_{\tau e}^{L,R}\hat{g}_{\tau\mu}^{R,L}\rvert$	7.5×10^{-6}	$(1.7\pm0.7)\times10^{-7}$	$(5.3\pm2.4)\times10^{-8}$
$BR(\tau^-\to\mu^-\mu^+\mu^-)\leqslant2.9\times10^{-5}$ [11]	$\lvert\hat{g}_{\tau\mu}^{L,R}\rvert$	2.9×10^{-2}	$(1.6\pm0.4)\times10^{-3}$	$(3.3\pm0.8)\times10^{-3}$
-	$\lvert\hat{g}_{\tau'\tau}^{L,R}\rvert$	-	$(3.1\pm0.7)\times10^{-2}\sqrt{m_{\tau'}/40\text{ GeV}}$	$(1.3\pm0.3)(m_{\tau'}/40\text{ GeV})$

4. Theoretical expectations on the flavor dependence of the flavor changing couplings

In section 2, eq. (9) and eq. (16), we have seen that, in the context of a simple vector singlet model, the flavor changing couplings are proportional to the product of mixing angles $(V_{L,R})^*_{4i}(V_{L,R})_{4j}$. We expect similar results to hold in other models involving heavy exotic fermions. In this section, we consider how such mixing angles ($(V_{L,R})_{4j}$'s) should depend on the generation (family) index j. From experience with KM angles, we expect

$$|(V_{L,R})_{41}| \ll |(V_{L,R})_{42}| \ll |(V_{L,R})_{43}| \ll 1. \tag{23}$$

Lighter fermions are expected to have smaller mixing with the heavy exotic ones in order to keep their masses small; too much mixing would spoil this smallness. This may be the reason why the flavor changing neutral processes between the first two lightest families (i.e., $d \leftrightarrow s$, $e \leftrightarrow \mu$) have not been observed thus far, and the GIM [13] mechanism has been so successful, since these are the ones that are likely to be the most suppressed in terms of the mixing angles.

To make a reasonable estimate on these mixing angles, we consider the following simple case of 2×2 mixing as a guide, which should shed some light on the general relation [#5] between the mixing angles and the mass-eigenvalues. Consider a 2×2 real symmetric matrix,

$$(m) = \begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix} \tag{24}$$

which is diagonalized by an orthogonal matrix $R(\theta)$,

$$R(\theta)^{\mathrm{T}}(m)R(\theta) \equiv m_{\text{diag.}} \equiv \begin{bmatrix} -m_1 & 0 \\ 0 & m_2 \end{bmatrix},$$

$$R(\theta) \equiv \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \tag{25}$$

Note that the matrix (m) contains two free parameters excluding the overall mass scale, while $R(\theta)$ has only one. Therefore, given the mass-eigenvalue hierarchy,

$$r \equiv m_1/m_2 \ll 1, \tag{26}$$

the mixing angle θ will be a function of r and one additional parameter (say α). Choosing the unit of the mass scale such that $m_2 = 1$, eqs. (24) and (25) imply

$$\begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix} = R(\theta) \begin{bmatrix} -r & 0 \\ 0 & 1 \end{bmatrix} R(\theta)^{\mathrm{T}}$$
$$= \begin{bmatrix} -r + (1+r)\sin^2\theta & -(1+r)\sin\theta\cos\theta \\ -(1+r)\sin\theta\cos\theta & 1-(1+r)\sin^2\theta \end{bmatrix}. \tag{27}$$

Thus we have

$$\alpha = -r + (1+r)\sin^2\theta,$$

which is equivalent to

$$\sin^2\theta = \frac{r+\alpha}{1+r}. \tag{28}$$

Now, the eigenvalue equation (trace and determinant of eq. (27)) implies

$$\alpha - \beta^2 \simeq -r. \tag{29}$$

Assuming the mass hierarchy of eq. (26) is not due to a fine-tuned cancellation between α and β^2, one expects α to be at most $O(r)$ from eq. (29). Therefore, in the absence of a fine-tuned cancellation in $r+\alpha$ in eq. (28), $\sin\theta$ is expected to be

$$\sin\theta \simeq O(\sqrt{r}) = O(\sqrt{m_1/m_2}). \tag{30}$$

If $\alpha = 0$ for some symmetry reason, then

$$\sin\theta = \sqrt{r/(1+r)} \simeq \sqrt{m_1/m_2}. \tag{31}$$

If a fine-tuned cancellation exists in $r+\alpha$, so that their sum is of higher order, i.e., $r+\alpha = O(r^2)$, then

$$\sin\theta \simeq O(m_1/m_2). \tag{32}$$

Thus the most natural relation between the mixing angle and the mass ratio is $\sin\theta \simeq O(\sqrt{m_1/m_2})$, and a particularly interesting one is that of eq. (31), where the mixing angle is exactly the square root of the mass ratio. Several examples of this relation of mixing an-

[#5] For a recent review on this subject, see ref. [14].

gles as square roots of mass ratios already exist in the literature [#6].

From the above discussion on the mixing angles and the mass ratios, we see that the most reasonable estimate on $(V_{\rm L,R})_{4j}$ is

$$(V_{\rm L,R})_{4j} \simeq (m_j/M)^p, \quad \text{with } \tfrac{1}{2} \lesssim p \lesssim 1 \tag{33}$$

and the generation dependence of the flavor changing couplings is expected to be

$$\frac{\hat{y}_{ij}}{\hat{y}_{kl}} \simeq \left(\frac{m_i m_j}{m_k m_l}\right)^p \simeq \frac{\hat{g}_{ij}}{\hat{g}_{kl}}, \quad \text{with } \tfrac{1}{2} \lesssim p \lesssim 1. \tag{34}$$

Moreover, $p \simeq 1/2$ is expected to be more realistic than $p \simeq 1$.

5. Comparison with existing data and predictions for future experiments

Considering B^0_d–$\bar{B}^0_d$ mixing as an anchor for the FCNP, we use the results of the last section as summarized in eq. (34) to predict the expected values for other coupling constants and compare these with experimental data on positive results or on bounds. The results for $p = \frac{1}{2}$ and for $p = 1$ are shown in the last two columns of tables 1 and 2. Before looking at the details, we should discuss two caveats. First, we use eq. (34) to extend the coupling constants, not only to systems made out of charge $-1/3$ quarks, but also to those of charge 2/3 quarks, and to leptons. This would be valid if the mass scale responsible for the breaking of the GIM mechanism would be the same for all three of the above systems. Even though this may be unlikely, we do not expect these masses to be orders apart; thus there may be a rescaling by a small factor as we go from group to group. Second, as discussed in footnote 4, we are presenting results for a common coupling constant for each process, while the detailed expressions may involve complicated sums of products of left and right handed couplings. Thus we are ignoring possible detailed cancellations or enhancements. Due to these two caveats, all of our results should be viewed as valid only up to a factor not too different from one. With these remarks in mind we see that we have no gross violations of any present experimental bounds. We also note that predictions for several, as yet unobserved processes are close to their present bounds. We shall discuss these in some detail.

In case the observed B^0_d–$\bar{B}^0_d$ mixing is due to flavor changing couplings of the Higgs scalar H^0, table 1 predicts two flavor changing couplings which are slightly below the present experimental upper limit. These are the ones for D^0–$\bar{D}^0$ mixing and $BR(\mu^- \to e^-\gamma)$. Predictions on these quantities are given in table 3. If we also take eq. (33) literally, the required strength of $|\hat{y}^{\rm L,R}_{\rm bd}|$ (of table 1) for B^0_d–$\bar{B}^0_d$ mixing predicts

$$M_{\rm H} \simeq (2.4 \pm 0.6)\,[f_{\rm B}/(0.15\ {\rm GeV})]\,M_Z$$
$$\simeq (200\text{–}300)\ {\rm GeV}, \tag{35}$$

for $p = 1/2$, and $\hat{M}_{\rm H} M \simeq (0.5 \pm 0.1)(f_{\rm B}/0.15\ {\rm GeV})$ GeV for $p = 1$. Thus the latter case predicts a rather unrealistic, low value of $M \lesssim 5$ GeV (since we know $\hat{M}_{\rm H} \gtrsim 0.1$) and thus we conclude that $p = 1/2$ would be much closer to reality than $p = 1$.

In case the observed B^0_d–$\bar{B}^0_d$ mixing is due to the flavor changing coupling of Z^0, table 2 predicts several flavor changing couplings of Z^0 which may, in the near future, have observable consequences, namely, $BR(\mu^- \to e^- e^+ e^-)$, D^0–$\bar{D}^0$ mixing, $BR(B^0_s \to \mu^+\mu^- X)$, $BR(\tau^- \to \mu^-\mu^+\mu^-)$, $BR(\mu^- \to e^-\gamma)$ and the flavor changing decay modes [#7] of Z^0 which can be tested with the 10^7 Z^0's expected at LEP. The predictions on these quantities are likewise given in table 3. Again, if one takes eq. (33) literally, one finds from the required strength of $|\hat{g}^{\rm L,R}_{\rm sd}|$ (of table 2) for B^0_d–$\bar{B}^0_d$ mixing

$$M \simeq (275 \pm 66)\ {\rm GeV} \times (f_{\rm B}/0.15\ {\rm GeV}), \tag{36}$$

[#6] The charge $-1/3$ quark mass matrix of the form of eq. (24) with $\alpha = 0$ for the two family case is well known to give the phenomenologically successful relation $\sin\theta_C \simeq \sqrt{m_d/m_s} = \sqrt{1/20} = 0.22$, where θ_C is the Cabibbo angle. A similar mass matrix for the neutrinos gives rise to the well known seesaw mechanism [15]. For three families of quarks, a generalization of this matrix gives phenomenologically successful relations [4,16] between their mixing angles and masses, where the entire set of KM angles are expressible in terms of the square roots of the quark mass ratios. For the four family case, see ref. [6].

[#7] These flavor changing Z^0 decay mode branching ratios are much larger than the ones expected [17] in the standard model.

Table 3
Summary of the predictions for future experiments.

Mediating boson	Present experimental upper limits	Predictions for future experiments	
		$p=1/2$	$p=1$
Higgs scalar H^0	D^0–$\bar{D}^0$ mixing: $r_D \leqslant 5.6\times10^{-3}$	D^0–$\bar{D}^0$ mixing of O(10%) of present upper limit	D^0–$\bar{D}^0$ mixing of O(0.1%) of present upper limit
	$BR(\mu^-\to e^-\gamma)\leqslant 4.9\times10^{-11}$	$(1.1\pm0.6)\times10^{-12}$	$(1.1\pm0.8)\times10^{-13}$
Z^0	$BR(\mu^-\to e^-e^+e^-)\leqslant 1.0\times10^{-12}$	any value in the neighborhood of present upper limit	$(1.7\pm0.8)\times10^{-15}$
	D^0–$\bar{D}^0$ mixing: $r_D \leqslant 5.6\times10^{-3}$	D^0–$\bar{D}^0$ mixing of O(1–10)% of present upper limit	O(0.01–0.1)% of present upper limit
	$BR(B_s^0\to\mu^+\mu^-X)\leqslant 1.2\times10^{-3}$	$(8.5\pm4.2)\times10^{-5}$	$(1.6\pm0.7)\times10^{-2}$
	$BR(\tau^-\to\mu^-\mu^+\mu^-)\leqslant 2.9\times10^{-5}$	$(8.8\pm4.8)\times10^{-8}$	$(3.8\pm1.8)\times10^{-7}$
	$BR(\mu^-\to e^-\gamma)\leqslant 4.9\times10^{-11}$	$(2.5\pm2.0)\times10^{-14}$	$(2.4\pm2.0)\times10^{-15}$
	Flavor changing Z^0 decay mode branching ratios		
	$BR(Z^0\to b\bar{s}+s\bar{b})\times10^7$	14 ± 7	270 ± 120
	$BR(Z^0\to t\bar{c}+c\bar{t})\times10^7$	$(1500\pm700)(m_t/60\ \mathrm{GeV})$	$O(10^4)$
	$BR(Z^0\to b'\bar{b}+b\bar{b}')\times10^7$	$(4800\pm2300)(m_{b'}/50\ \mathrm{GeV})$	$O(10^5)$
	$BR(Z^0\to\mu^-\tau^++\mu^+\tau^-)\times10^7$	3.6 ± 1.8	15 ± 7
	$BR(Z^0\to\tau'\bar{\tau}+\bar{\tau}'\tau)\times10^7$	$(1300\pm600)(m_{\tau'}/50\ \mathrm{GeV})$	$O(10^4)$

for $p=1/2$, while $p=1$ gives $M\simeq(7.5\pm0.9)$ GeV$\times\sqrt{f_B}/0.15$ GeV. The latter value of M (=the scale of the heavy exotic fermion masses) is, again, rather too low to be realistic. It is interesting to note that the observed strength of B_d^0–$\bar{B}_d^0$ mixing implies that the values of M_H in eq. (35) and of M in eq. (36) to be O(v=250 GeV), the scale of the electroweak symmetry breaking; this may not be a numerical coincidence.

6. Summary and conclusion

In this article, we investigated the implications of the possibility that the observed B_d^0–$\bar{B}_d^0$ mixing is due to small flavor changing couplings of the Higgs scalar, H^0, or the Z^0, induced by new physics at an energy scale beyond the standard model. The implications are rich and the predictions for future experiments are summarized in table 3. Moreover, the scale associated with the new physics, M, and/or the mass of the Higgs scalar seem to coincide with the Higgs vacuum expectation value, v=250 GeV. This may not be a coincidence but may indicate that this new region will indeed show up at a mass scale of 250 GeV.

Although our discussions were made in the context of a model of ordinary fermions mixing with heavy, exotic ones, the general structure of these flavor changing couplings should be valid in a broader class of theories [#8]. *CP* violation could be included in such a class of models; as the GIM mechanism is violated, an electric dipole moment could be induced at the one-loop level. Likewise these effects would become stronger with increasing quark mass. We plan to report on these effects in a future publication [18].

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[#8] We believe that some composite models of quarks and leptons and some technicolor models can give rise to similar flavor changing couplings.

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