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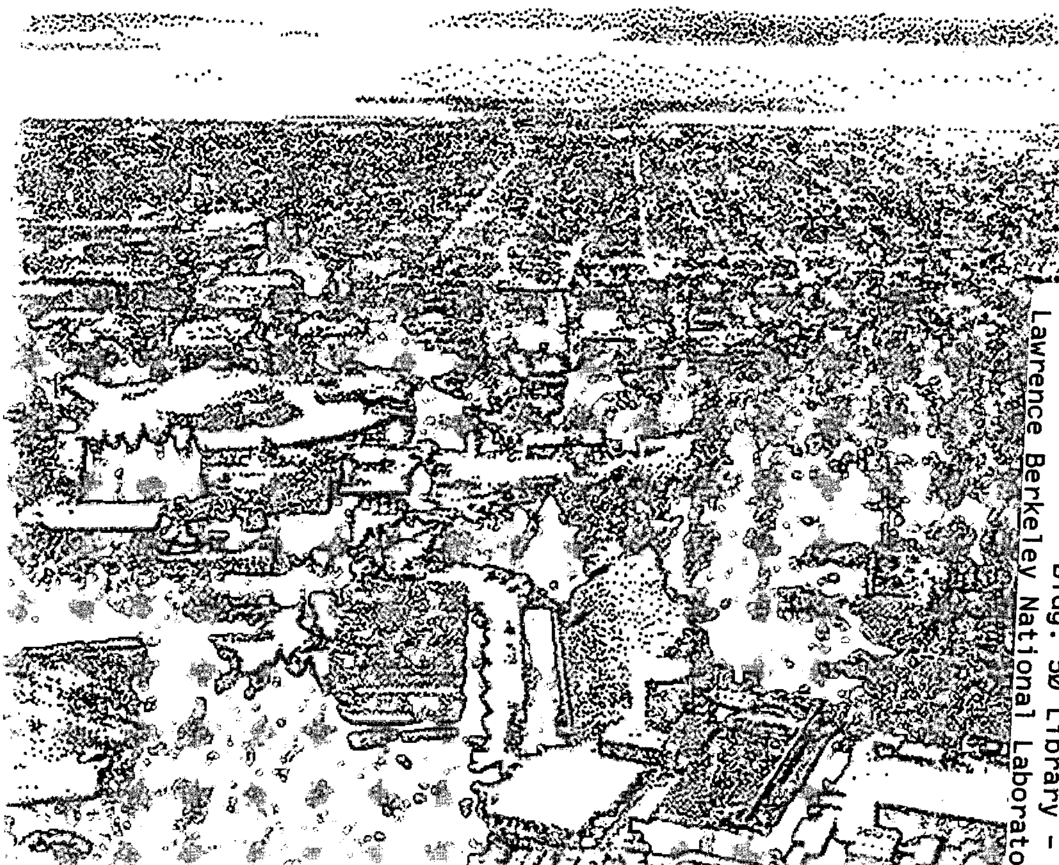

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Quantum-Induced Soft Supersymmetry Breaking In Supergravity*

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Abstract

We calculate the one-loop quantum contributions to soft supersymmetry breaking terms in the scalar potential in supergravity theories regulated à la Pauli-Villars. We find "universal" contributions, independent of the regulator masses and tree level soft supersymmetry breaking, that contribute gaugino masses and A-terms equal to the "anomaly mediated" contributions found in analyses using spurion techniques, as well as a scalar mass term not identified in those analyses. The universal terms are in general modified – and in some cases canceled – by model-dependent terms. Under certain restrictions on the couplings we recover the one-loop results of previous "anomaly mediated" supersymmetry breaking scenarios. We emphasize the model dependence of loop-induced soft terms in the potential, which are much more sensitive to the details of Planck scale physics then are the one-loop contributions to gaugino masses. We discuss the relation of our results to previous analyses.

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There has been considerable interest recently in soft supersymmetry breaking induced by quantum corrections, starting with the observation [1, 2] that there are several "anomaly mediated" contributions: a gaugino mass term proportional to the β-function, an A-term proportional to the chiral multiplet (matrix-valued) γ-function, and a scalar mass term proportion to the derivative of the γ-function, arising first at two-loop level. This contribution to the gaugino masses has been confirmed in subsequent calculations [3, 4]. The result in [3] was found by an analysis of the relevant loop-induced superfield operator in Kähler $U(1)$ superspace [5], and also by an explicit Pauli-Villars (PV) calculation. The "anomaly mediated" A-term contribution has also been confirmed by a Pauli-Villars calculation given in [6] as an illustrative application of PV regularization of supergravity. Here we extend the PV calculation to obtain the one-loop contribution to scalar masses. We also display our result in the form of a superfield operator, and indicate the origin of this operator as a superspace integral. We work with the standard chiral formulation of supergravity in Kähler $U(1)$ superspace, with the Einstein term canonically normalized. The full contribution to gaugino masses in string-derived supergravity models with the dilaton in a linear supermultiplet, and including a Green-Schwarz term and string threshold effects, was presented in [3]. A general parameterization of all the soft supersymmetry breaking terms in the context of superstring-derived supergravity will be given elsewhere [7].

The points we wish to emphasize in the PV calculation given here are 1) the presence in general of $O(m_{\tilde{G}})$ contributions to the scalar masses that are proportional to the chiral supermultiplet gamma-function (rather than its derivative, which is a two-loop effect), and 2) the difference between gaugino masses and soft terms in the scalar potential with respect to dependence on the details of Planck-scale physics. To this end we will present our calculations under the simplifying assumption that the Pauli-Villars squared-mass matrix commutes with other operators that are relevant to quantum corrections. The full PV mass-dependence in the general case will be indicated in the final result. We further restrict our analysis to one-loop order and retain only terms of lowest order in $m_{\tilde{G}}/m_P$, where $m_{\tilde{G}}$ and m_P are the gravitino mass and the reduced Planck mass, respectively. We then use our results to address the issue of anomaly-mediated supersymmetry breaking [1, 2, 8, 9].

The one-loop logarithmic divergences of standard chiral supergravity were determined in [10], and it was shown in [6, 11] that they can be regulated[1] by a set of Pauli-Villars chiral superfields Φ^A. As in these references we denote the light superfields by Z^i, and introduce covariant derivatives

[1] The full regulation of gravity loops requires the introduction of Abelian gauge superfields as well; these play no role here and we ignore them.

of the superpotential $W(Z^i)$ as follows:

$$\begin{aligned} A &= e^K W, \quad A_i = D_i A = \partial_i A, \quad A_{ij} = D_i D_j A = \partial_i \partial_j A - \Gamma^k_{ij} A_k, \\ \bar{A}^i &= K^{i\bar{m}} \bar{A}_{\bar{m}}, \quad etc., \quad K_{i\bar{m}} = \partial_i \partial_{\bar{m}} K(Z^i, \bar{Z}^{\bar{m}}), \quad \partial_i = \frac{\partial}{\partial Z^i}, \end{aligned} \tag{1}$$

where Γ^k_{ij} is the affine connection associated with the Kähler metric $K_{i\bar{m}}$ and its inverse $K^{i\bar{m}}$. In the regulated theory, the one-loop correction to the chiral multiplet Kähler potential is given by [6] the superfield operator (up to a Weyl transformation necessary to put the Einstein term in canonical form[2])

$$\Delta L = -\frac{1}{32\pi^2} \int d^4\theta E e^{-K} \sum_{AB} \eta_A \bar{A}^{AB} A_{AB} \ln(m_A^2/\mu^2) = \int d^4\theta E \delta K(Z^i, \bar{Z}^{\bar{m}}), \tag{2}$$

where $A_{AB}(Z^i, \bar{Z}^{\bar{m}})$ is defined as in (1), with the light field indices i, j replaced by PV indices A, B, $\eta_A = \pm 1$ is the PV signature, m_A is the (supersymmetric) PV mass, and μ is the (scheme-dependent) normalization point. The wave function renormalization matrix is given by

$$\begin{aligned} \gamma_i^j &= \left\langle K^{j\bar{n}} D_{\bar{n}} D_i \frac{\partial}{\partial \ln \mu^2} \delta K \right\rangle = \frac{1}{32\pi^2} \left\langle D^j D_i \sum_{AB} \eta_A (e^{-K} \bar{A}^{AB} A_{AB}) \right\rangle \\ &= \frac{1}{32\pi^2} \left\langle e^{-K} \sum_{AB} \eta_A \bar{A}^{jAB} A_{iAB} \right\rangle + \cdots, \end{aligned} \tag{3}$$

where here and throughout ellipses represent terms of higher dimension.

The regulation of matter and Yang-Mills loop contributions to the matter wave function renormalization requires the introduction of PV chiral superfields $\Phi^A = Z^I, Y_I, \varphi^a$, which transform according to the chiral matter, anti-chiral matter and adjoint representations of the gauge group and have signatures $\eta_A = -1, +1, +1$, respectively. These fields couple to the light fields through the superpotential[3]

$$W(\Phi^A, Z^i) = \frac{1}{2} W_{ij}(Z^k) Z^I Z^J + \sqrt{2} \varphi^a Y_I (T_a Z)^i + \cdots, \tag{4}$$

where T_a is a generator of the gauge group, and their Kähler potential takes the form

$$K(\Phi^A, \bar{\Phi}^{\bar{A}}) = K_{i\bar{m}} Z^I \bar{Z}^{\bar{M}} + K^{i\bar{m}} Y_I \bar{Y}_{\bar{M}} + g_a^{-2} e^K |\varphi^a|^2 + \cdots, \tag{5}$$

[2]This brings in terms with factors of V_{tree} that we neglect since if $< V_{tree} >= 0$, they can at most give small corrections to the tree level soft terms.

[3]Full regulation of the theory requires several copies of fields with the same gauge quantum numbers, and the coupling parameters and signatures given here actually represent weighted average values.

where g_a is the (possibly field-dependent) gauge coupling constant for the gauge subgroup $\mathcal{G}_a$. With these choices, the ultraviolet divergences cancel, and, for the leading (lowest dimension) contribution, one obtains the standard result for the matter wave function renormalization in the supersymmetric gauge [12]

$$\gamma_i^j = \frac{1}{32\pi^2} e^K \sum_{AB} \eta_A W_{iAB} \overline{W}^{jAB} = \frac{1}{32\pi^2} \left[4\delta_i^j \sum_a g_a^2 (T_a^2)_i^i - e^K \sum_{kl} W_{ikl} \overline{W}^{jkl} \right]. \tag{6}$$

The matrix (6) is diagonal in the approximation in which generation mixing is neglected in the Yukawa couplings; in practice only the $T^c Q_3 H_u$ Yukawa coupling is important. We will make this approximation in the following, and set

$$\begin{aligned} \gamma_i^j &\approx \gamma_i \delta_i^j, \quad \gamma_i = \gamma_i^W + \gamma_i^g, \quad \gamma_i^W = \sum_{jk} \gamma_i^{jk}, \quad \gamma_i^g = \sum_a \gamma_i^a, \\ \gamma_i^a &= 4 g_a^2 (T_a^a)_i^i, \quad \gamma_i^{jk} = -(g_i g_j g_k)^{-1} \left| W_{ijk} \right|^2, \end{aligned} \tag{7}$$

where for gauge-charged fields Z^i the Kähler metric is

$$K_{i\bar{j}} = g_i(Z^n)\delta_{ij} + O|Z^i|^2, \tag{8}$$

with the Z^n gauge singlets.

The Lagrangian (2) generally contains soft supersymmetry (SUSY) breaking terms, displayed below, that are proportional to those of the tree-level Lagrangian. What are usually referred to as "anomaly mediated" soft SUSY-breaking terms are finite contributions that are not remnants, like (2), of the ultraviolet divergences. To evaluate such terms in the framework of PV regularization, we must retain all contributions that do not vanish in the limit $m_A^2 \to \infty$. Here we are interested in the scalar potential, given by

$$\begin{aligned} \mathcal{L} &= \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \mathrm{STr}\eta \ln\left(p^2 - m^2 - H\right) \\ &= -\frac{1}{32\pi^2} \mathrm{STr}\eta \left[\left(hm^2 + \frac{1}{2} g^2 \right) \ln(m^2) + \frac{1}{2} h^2 \ln\left(\frac{m^2}{\mu^2}\right) + \frac{H^3}{6m^2} - \frac{H^4}{24m^4} \right] \\ &\quad + O\left(\frac{1}{m^2}\right), \end{aligned} \tag{9}$$

where H is the effective field-dependent squared mass with the supersymmetric PV mass matrix m^2 separated out:

$$H_{PV} = H + m^2, \quad H = h + g, \quad h \sim m^0, \quad g \sim m^1. \tag{10}$$

The terms in (9) proportional to $\ln(m^2/\mu^2)$ are the bosonic part of (2). The first term in (9), proportional to m^2, is the remnant of the quadratically divergent contribution [11]. It is completely controlled by Plank scale physics, and can be made to vanish with appropriate conditions on m^2. If it is present, it contains A-terms and scalar masses proportional to the tree potential soft terms, with coefficients suppressed by $1/32\pi^2$; we neglect it in the following.

The PV loops contribute soft SUSY breaking terms to the light field effective Lagrangian if the PV tree Lagrangian contains such terms. In the presence of SUSY breaking one generally expects the matrix g in (10) which is linear in m to contain "B-terms". Indeed it is these B-terms that generate the "anomaly mediated" contributions to the gaugino masses and the A-terms of the light theory[4] that have been discussed in the literature [1, 2, 8, 9]. As we shall see below, there are two contributions to SUSY-breaking scalar masses that arise from a double B-term insertion in a Feynman diagram. These two contributions cancel, resulting in the assertion [1, 8, 9] that there is no anomaly mediated contribution to scalar masses at one loop. However, there can in general be soft masses and A-terms in the matrix h in (10). In leading order in $m_{\tilde{G}}^2/\mu^2$, A-terms are present in the PV part of h only if there are dimension-three soft SUSY-breaking operators in the tree Lagrangian. Soft PV mass terms, which in leading order contribute only to scalar masses, are not similarly restricted by the low energy theory. Specifically, if the regulator masses are constant there are always soft squared-mass terms in the PV sector.

The PV mass for each superfield Φ^A is generated by coupling it to a field Φ^α in the representation of the gauge group conjugate to that of Φ^A through the superpotential term

$$W_m = \sum_{(A,\alpha)} \mu_{A\alpha}\Phi^A\Phi^\alpha, \tag{11}$$

where $\mu_{A\alpha} = \mu_{A\alpha}(Z^i)$ can in general be a holomorphic function of the light superfields. If the Kähler potential for the PV fields is

$$K_{PV} = \sum_X g_X(z)|\Phi^X|^2, \quad X = A, \alpha, \tag{12}$$

then the PV masses are

$$m_A^2 = m_\alpha^2 = f_A\mu_{A\alpha}^2, \quad f_A = e^K g_A^{-1} g_\alpha^{-1}. \tag{13}$$

[4]There may also be B-terms generated at one loop in the light theory if there are quadratic holomorphic terms in its tree-level superpotential or Kähler potential. These contributions were considered in [6]; we ignore them and use the expression "B-term" to designate the B-term proportional to the PV mass.

The general field-dependent matrix H in (9) has been evaluated in [10]. Denoting by H^χ and H^ϕ the matrices H in (10) for fermions and bosons, respectively, we have, with constant background fields,

$$\begin{aligned} H^\chi &= h^\chi + \frac{r}{4}, \quad (h^\chi)^A_B = e^{-K} A_{BC}\bar{A}^{AC}, \quad (h^\chi)^\alpha_\beta = 0, \\ (h^\chi)^\alpha_D &= K^{\alpha\bar{\beta}}\mu_B K^{\bar{B}C} A_{CD} = g^\alpha_D = e^{-K} f_A \mu_{A\alpha} A_{AD}, \\ (h^\chi)^A_\beta &= \bar{A}^{AB}\mu_B = g^A_\beta, \quad H^\phi = h^\chi + \hat{V} + m^2_{\tilde{G}} + R, \end{aligned} \tag{14}$$

where $m^2_{\tilde{G}} = e^K|W|^2$, $\hat{V} = e^{-K}A_j\bar{A}^j - 3m^2_{\tilde{G}}$. The last term in (14) depends on the curvature of the PV metric:

$$R^a_b = R^a_{b\bar{m}i} e^{-K} A^{\bar{m}}\bar{A}^i = -\delta_{ab} F^i \bar{F}^{\bar{m}} \partial_{\bar{m}} \left(g^{-1}\partial_i g_a\right) = -\delta_{ab} F^i \bar{F}^{\bar{m}} \partial_{\bar{m}} \partial_i \ln g_a, \quad a = A, \alpha, \tag{15}$$

where $F^i = -e^{-K/2}\bar{A}^i$ is the auxiliary field of the supermultiplet Z^i. Terms involving the space-time curvature r are replaced by terms proportional to the tree potential V after a Weyl transformation that restores the one-loop corrected Einstein term to canonical form. We assume throughout a vanishing cosmological constant, $< V >= 0$, so we can drop them. Similarly, we can drop $\hat{V}$ if D-terms vanish in the vacuum: $V = \hat{V} + \mathcal{D}$, $\mathcal{D} = \frac{1}{2}g^2\sum_a D^2_a$, $(T^a z)^i K_i$, $z^i = Z^i|$, $\langle D_a \rangle = 0$. Terms containing only powers of h^χ cancel in the supertrace, so we get contributions only from scalar trace terms that include the scalar mass term $m^2_{\tilde{G}} + R$ in (14) or factors of $H_{XY} = K_{X\bar{X}} H^{\bar{X}}_Y$:

$$\begin{aligned} H_{AB} &= h_{AB} = e^{-K}\left(\bar{A}^i D_i A_{AB} - A_{AB}\bar{A}\right), \quad h_{\alpha\beta} = 0, \\ H_{A\beta} &= g_{A\beta} = e^{-K}\bar{A}^i D_i\left(e^K W_{A\beta}\right) - \bar{A}W_{A\beta} = -\delta_{A\beta}\mu_{A\alpha}\left(\bar{A} - \bar{A}^i\partial_i \ln f_A\right). \end{aligned} \tag{16}$$

$H_{A\alpha}$ is the B-term mentioned above, and the part of h_{AB} linear in $z^i - \langle z^i\rangle$ is the A-term. Neglecting B-terms in the tree Lagrangian, the leading contribution to W_{AB} is linear in a gauge nonsinglet field Z^i. Explicitly expanding H_{AB} gives

$$H_{AB} = e^K K^{i\bar{j}}\overline{W}_{\bar{j}} W_{iAB} - e^{K/2} F^n \partial_n \ln(g_i g_A g_B e^{-K}) z^i W_{iAB}. \tag{17}$$

The second term in (17) is the A-term where $\langle F^n\rangle \neq 0$ with Z^n a gauge singlet in the SUSY-breaking sector. The tree-level A-terms are given by

$$\langle \partial_i\partial_j\partial_k V\rangle = a_{ijk}\left\langle e^{K/2} W_{ijk}\right\rangle, \quad a_{ijk} = \left\langle F^n\partial_n \ln(g_i g_j g_k e^{-K})\right\rangle, \tag{18}$$

and the tree-level gaugino masses are given by

$$m_a = \frac{1}{2}\left\langle F^n \partial_n \ln(g_a^{-2})\right\rangle. \tag{19}$$

Using the couplings given in (5), we have

$$F^n \partial_n \ln(g_i g_{Z^J} g_{Z^K} e^{-K}) = a_{ijk}, \quad F^n \partial_n \ln(g_i g_{Y_I} g_{\varphi^a} e^{-K}) = 2m_a. \tag{20}$$

Then we obtain

$$\begin{aligned} \mathrm{Tr}\eta h^2 &\ni 2\sum_{AB} \eta_A h_{AB} h^{AB} \ni -2e^{3K/2} W_j z^i \sum_{AB} \eta_A W_{iAB} \overline{W}^{jAB} F^n \partial_n \ln(g_j g_A g_B e^{-K}) + \text{h.c.} \\ &= -64\pi^2 e^{K/2} W_i z^i \left[\sum_{jk} \gamma_i^{jk} a_{ijk} + 2\sum_a \gamma_a^i m_a\right] + \text{h.c.}, \end{aligned} \tag{21}$$

for the leading contribution to the A-term from the second term in (9), *i.e.*, the contribution from the shift in the potential due to the shift in the Kähler potential. The leading order contribution to the "anomaly-induced" A-term arises from a PV loop diagram with one B-term insertion:

$$\begin{aligned} \mathrm{Tr}\eta \frac{H^3}{6m^2} &\ni \mathrm{Tr}\eta \frac{hg^2}{2m^2} \ni \sum_{AB} \frac{\eta_A}{2m_A^2} h_{\bar{B}}^A \left(g_\gamma^{\bar{B}} g_A^\gamma + g_{\bar{\gamma}}^{\bar{B}} g_A^{\bar{\gamma}}\right) + \text{h.c.} \\ &\ni -32\pi^2 e^{K/2} W_i z^i \left[\gamma_i m_{\tilde{G}} + F^n \partial_n \left(\sum^a \gamma_i^a \ln f_{ia} + \sum_{jk} \gamma_i^{jk} \ln f_{jk}\right)\right] + \text{h.c.}, \\ f_{AB} &= \sqrt{f_A f_B}, \quad f_{jk} = \sqrt{f_{Z^J} f_{Z^K}}, \quad f_{ia} = \sqrt{f_{\varphi^a} f_{Y_I}}, \end{aligned} \tag{22}$$

which reduces to the "anomaly mediated term" found in [2] provided that $\langle F^n \partial_n \ln f_A \rangle = 0$. We discuss below the circumstances under which this is the case. The full leading-order A-term Lagrangian is

$$\begin{aligned} \mathcal{L}_A &= e^{K/2} W_i z^i \Big[\gamma_i m_{\tilde{G}} + \sum_a \gamma_i^a \left(2m_a \ln(m_{ia}^2/\mu^2) + F^n \partial_n \ln f_{ia}\right) \\ &\quad + \sum_{jk} \gamma_i^{jk} \left(a_{ijk} \ln(m_{jk}^2/\mu^2) + F^n \partial_n \ln f_{jk}\right)\Big] + \text{h.c.} + \cdots, \\ m_{AB}^2 &= m_A m_B, \quad m_{jk}^2 = m_{Z^J} m_{Z^K}, \quad m_{ia}^2 = m_{\varphi^a} m_{Y_I}. \end{aligned} \tag{23}$$

Scalar masses get a contribution from the term quartic in H:

$$\begin{aligned} \mathrm{Tr}\eta \frac{H^4}{24m^2} &\ni \mathrm{Tr}\eta \frac{g^4}{24m^2} \ni \sum \frac{\eta_A}{12m_A^2} \left[2\left(g_\beta^A g_C^\beta g_{\bar{\delta}}^C g_A^{\bar{\delta}} + g_B^\alpha g_\gamma^B g_{\bar{D}}^\gamma g_\alpha^{\bar{D}}\right) + g_\beta^A g_{\bar{C}}^\beta g_{\bar{\delta}}^{\bar{C}} g_A^{\bar{\delta}} + g_B^\alpha g_{\bar{\gamma}}^B g_{\bar{D}}^{\bar{\gamma}} g_\alpha^{\bar{D}}\right] \\ &\quad + \text{h.c.} \ni e^{-K} \sum_{AB} \eta_A |m_{\tilde{G}} + F^i \partial_i \ln f_A|^2 A_{AB} \bar{A}^{AB}, \end{aligned} \tag{24}$$

which corresponds to two B-term insertions in the PV loop. The contribution from the cubic term is

$$
\begin{aligned}
\mathrm{Tr}\eta\frac{hg^2}{2m^2} \ni\; & \frac{1}{2}\sum_{AB}\eta_A h^A_{\bar{B}}\left[g^{\bar{B}}_{\beta}g^{\beta}_{A}\frac{1}{m^2_B}+g^{\bar{B}}_{\bar{\alpha}}g^{\bar{\alpha}}_{A}\frac{1}{m^2_A}\right] \\
& +\frac{1}{2}m^2_{\tilde{G}}\sum_{AB}\eta_A g^{A}_{\beta}g^{\beta}_{A}\left(\frac{1}{m^2_B}+\frac{1}{m^2_A}\right) \\
& -\frac{1}{2}\sum_{AB}\eta_A g^{A}_{\beta}g^{\beta}_{A}\left(\frac{1}{m^2_B}F^n\bar{F}^{\bar{m}}\partial_n\partial_{\bar{m}}\ln g_\beta+\frac{1}{m^2_A}F^n\bar{F}^{\bar{m}}\partial_n\partial_{\bar{m}}\ln g_A\right) \\
& +\frac{1}{2}\sum_{B}\eta_B m^{-2}_B g_{B\beta}g^{B\beta}h^B_B+\mathrm{h.c.} \\
\ni\; & \frac{1}{2}e^{-K}\sum_{AB}\eta_A\left[F^n\partial_n\ln\left(g_i g_A g_B e^{-K}\right)\right]\left(m_{\tilde{G}}+\bar{F}^{\bar{m}}\partial_{\bar{m}}\ln f_B\right)z^i A_{iAB}\bar{A}^{AB} \\
& +\frac{1}{2}e^{-K}\sum_{AB}\eta_A A_{AB}\bar{A}^{AB}\left(F^n\bar{F}^{\bar{m}}\partial_n\partial_{\bar{m}}\ln f_A-m^2_{\tilde{G}}\right) \\
& +\frac{1}{2}e^{-K}\sum_{AB}\eta_A A_{AB}\bar{A}^{AB}|m_{\tilde{G}}+\bar{F}^{\bar{m}}\partial_{\bar{m}}\ln f_B|^2+\mathrm{h.c.},
\end{aligned}
\tag{25}
$$

where we used the vacuum condition

$$\hat{V}=K_{n\bar{m}}F^n\bar{F}^{\bar{m}}-3m^2_{\tilde{G}}=0. \tag{26}$$

The last term in (25) is a double B-term insertion; it cancels (24) in the Lagrangian (9). The first term on the right hand side of (25) corresponds to one B-term and one A-term insertion, and the second term corresponds to a PV soft squared-mass insertion. Explicitly,

$$F^n\bar{F}^{\bar{m}}\partial_n\partial_{\bar{m}}\ln f_A-m^2_{\tilde{G}}=\mu^2_A+\mu^2_\alpha, \tag{27}$$

where

$$\mu^2_a=m^2_{\tilde{G}}-F^n\bar{F}^{\bar{m}}\partial_n\partial_{\bar{m}}\ln g_a,\quad a=A,\alpha \tag{28}$$

is the soft SUSY-breaking squared mass of the field Φ^a. For $a=A$ the masses are determined by the SUSY-breaking masses of the tree Lagrangian

$$\mu^2_{Z^I}=\mu^2_{z^i}\equiv\mu^2_i,\quad \mu^2_{Y_I}=2m^2_{\tilde{G}}-\mu^2_i,\quad \mu^2_{\varphi^a}=-2m^2_{\tilde{G}}-m^2_a, \tag{29}$$

so these terms give no contribution if $\mu_i=m_a=0$. However, even if no soft SUSY-breaking masses are present in the tree Lagrangian, one cannot *a priori* exclude such terms in the theory above the effective cut-off, that could be reflected in soft SUSY breaking masses μ_α^2 in the PV sector that parameterizes the underlying Planck scale physics. Finally we have

$$
\begin{aligned}
\mathrm{Tr}h^2 &\ni 2\sum_{AB}\eta_A\left(H^A_B H^B_A + H_{AB}H^{AB}\right) \\
&\ni 2e^{-K}\sum_{AB}\eta_A\Big[2\mu_A^2\overline{W}^{AB}W_{AB} \\
&\quad + z^i\bar{z}^{\bar{\jmath}}F^n\bar{F}^{\bar{m}}\left(\partial_n\ln(g_i g_A g_B e^{-K})\right)\left(\partial_{\bar{m}}\ln(g_j g_A g_B e^{-K})\right)\overline{W}^{AB}_{\bar{\jmath}}W_{iAB}\Big], \qquad (30)
\end{aligned}
$$

for the part of the renormalization of the Kähler potential that contributes to scalar masses. Using (20) and (27)–(29), the full scalar mass term is

$$
\begin{aligned}
\mathcal{L}_m &= -\sum_i |z^i|^2\Bigg(m_{\tilde{G}}^2\gamma_i \\
&\quad -\sum_a \gamma_i^a\left\{\left(m_a^2+\mu_i^2\right)\ln(m_{ia}^2/\mu^2) + F^n\bar{F}^{\bar{m}}\partial_{\bar{m}}\partial_n\ln f_{ia} + m_a\left[\left(\bar{F}^{\bar{m}}\partial_{\bar{m}} + F^n\partial_n\right)\ln f_{ia} + m_{\tilde{G}}\right]\right\} \\
&\quad -\sum_{jk}\gamma_i^{jk}\Big\{\left(\mu_j^2+\mu_k^2+\frac{1}{2}a_{ijk}^2\right)\ln(m_{jk}^2/\mu^2) + F^n\bar{F}^{\bar{m}}\partial_{\bar{m}}\partial_n\ln f_{jk} \\
&\quad +\frac{1}{2}a_{ijk}\left[\left(\bar{F}^{\bar{m}}\partial_{\bar{m}} + F^n\partial_n\right)\ln f_{jk} + m_{\tilde{G}}\right]\Big\}\Bigg) + \cdots. \qquad (31)
\end{aligned}
$$

In the absence of tree-level soft SUSY breaking, this expression reduces to the first ("universal") term if $\langle F^n\bar{F}^{\bar{m}}\partial_n\partial_{\bar{m}}f_A\rangle = 0$.

The above results hold in the general case of a noncommuting squared-mass matrix, with the replacements

$$
\begin{aligned}
\ln m_{AB}^2 &= q(m_A^2, m_B^2) = \frac{m_A^2\ln(m_A^2/\mu^2) - m_B^2\ln(m_B^2/\mu^2)}{m_A^2 - m_B^2} - 1, \\
\partial_{\bar{m}}\ln f_{AB} &= \partial_{\bar{m}}q(m_A^2, m_B^2), \\
\partial_n\partial_{\bar{m}}\ln f_{AB} &= \partial_n\partial_{\bar{m}}q(m_A^2, m_B^2). \qquad (32)
\end{aligned}
$$

The soft SUSY-breaking Lagrangian for the canonically normalized scalars ϕ_R is obtained by making the substitution $\phi^i = g_i^{-\frac{1}{2}}\phi_R^i$ in $\mathcal{L}_A + \mathcal{L}_m$.

The Lagrangian $\mathcal{L}_A + \mathcal{L}_m$ is the bosonic part of the superfield Lagrangian

$$
\mathcal{L}_1 = \frac{1}{32\pi^2}\int d^4\theta E\sum_{AB}\eta_A e^{K/2}\overline{W}^{AB}\left[\ln\Box_\chi - \bar{R}\frac{6}{\Box_\chi}R - q(m_A^2, m_B^2)\right]e^{K/2}W_{AB} + \cdots, \qquad (33)
$$

where [5] R is a superfield constructed from elements of the super-Riemann tensor, $\langle 2R| \rangle = m_{\tilde{G}}$, and

$$\Box_\chi = \frac{1}{16}\left(\bar{\mathcal{D}}^2 - 8R\right)\left(\mathcal{D}^2 - 8\mathcal{R}\right) \tag{34}$$

is the chiral superfield propagator given in [13]. Provided the Käher metric is defined from the full Kähler potential including the PV part, K_{AB} is covariantly constant, and $\phi = e^{K/2}W_{AB}$ is a chiral superfield [5] of chiral weight +2:

$$\langle \Box_\chi \phi \rangle = \left\langle \left(\Box + \frac{1}{2}R\mathcal{D}^2 + 4R\bar{R}\right)\Phi \right\rangle, \tag{35}$$

where we used the vacuum condition (26). The superfield $f(\Box_\chi)\phi$ is also a chiral superfield of chiral weight 2, since $(\bar{\mathcal{D}}^2 - 8R)$ is the weight-zero chiral projection operator. Evaluating (33) with the methods of [5]:

$$\int d^4\theta\Phi = \frac{1}{16}\left(\mathcal{D}^2 - 24\bar{R}\right)\left(\bar{\mathcal{D}}^2 - 8R\right)\Phi\Big| + \text{gravitino terms}, \tag{36}$$

and expanding in inverse powers of the d'Alembertian, $\langle \Box \rangle = \mu^2$, we recover the scalar potential given in (23), (31) and (32), up to corrections of order $m^2_{\tilde{G}}/\mu^2$. To understand the origin of the expression (33), consider the tree-level superfield Lagrangian for quantum fluctuations $\hat{Z}$ around canonically normalized background superfields Z:

$$\mathcal{L}_0 = \int d^4\theta E\left[\sum_i |\hat{Z}^i|^2 + e^{K/2}\left(\frac{1}{4R}\hat{Z}^i\hat{Z}^j W_{ij}(Z) + \text{h.c.}\right)\right]. \tag{37}$$

Variation of the action $S = \int d^4x\mathcal{L}_0$ with respect to the unconstrained superfields ρ^I, defined by

$$\rho^I = \begin{pmatrix} \rho^i \\ \rho^{\bar{\imath}} \end{pmatrix}, \quad \hat{Z}^i = \frac{1}{4}\left(\bar{\mathcal{D}}^2 - 8R\right)\rho, \quad \rho^{\bar{\imath}} = \left(\rho^i\right)^\dagger, \tag{38}$$

gives the inverse superfield propagator

$$\begin{aligned} \Delta^{-1}_{IJ}(y,y') &= \frac{\partial^2 S}{\partial\rho^I(y)\partial\rho^J(y')}, \\ \Delta^{-1}(y,y') &= \begin{pmatrix} \Box_{\bar{\chi}} & -\frac{1}{4}\phi\left(\mathcal{D}^2 - 8\bar{R}\right) \\ -\frac{1}{4}\bar{\phi}\left(\bar{\mathcal{D}}^2 - 8R\right) & \Box_\chi \end{pmatrix}_y \delta^8(y-y'), \end{aligned} \tag{39}$$

where $y = x^m, \theta_\mu, \bar{\theta}_{\dot{\mu}}$, and $\phi = e^{K/2}W_{ij}$ is a weight-2 chiral superfield. In the flat superspace limit with $\phi = W_{ij} = \frac{1}{2}m_{ij}$, (39) reduces to the inverse of the Wess-Bagger free superfield propagator [14].

The effective one-loop action is determined by evaluating STr ln Δ in superspace. Writing

$$\begin{aligned} \Delta^{-1} &= \Delta_0^{-1}(1+\delta), \quad \Delta_0^{-1}(y,y') = \begin{pmatrix} \Box_{\bar{\chi}} & 0 \\ 0 & \Box_{\chi} \end{pmatrix}_y \delta^8(y-y'), \\ \delta(y,y') &= -\frac{1}{4}\begin{pmatrix} 0 & \Box_{\bar{\chi}}^{-1}\bar{\phi}\,(\mathcal{D}^2 - 8\bar{R}) \\ \Box_{\chi}^{-1}\bar{\phi}\,(\bar{\mathcal{D}}^2 - 8R) & 0 \end{pmatrix}_y \delta^8(y-y'), \\ \mathrm{STr}\ln\Delta &= \mathrm{STr}\left[\ln\Delta_0 + \ln(1+\delta)\right], \end{aligned} \tag{40}$$

we are interested in the term proportional to δ^2 in the expansion of the log. Consider first the flat superspace limit: $R=0$, $\Box_\chi \to \Box$. Since $\delta(\theta) = \theta$, one factor of $\delta^4(\theta-\theta')$ is removed by integration over $d^4\theta'$, and the other is removed by the spinorial derivative operators $\mathcal{D}^2, \bar{\mathcal{D}}^2$. The x' integration can be performed by replacing $\delta^4(x-x')$ by its Fourier transform, yielding the flat space limit of the first term[5] in (33). In the curved superspace generalization of that term, the superdeterminant E of the supervielbein appears as the Jacobian relating tangent space to Einstein superspace coordinates. Additional spinorial derivatives appear in the expansion of $\Box_\chi^{-1}$ in powers of $\Box^{-1}$ [*c.f.* (35)]. For example, there is a contribution in which two θ factors in δ^2 are removed by $\mathcal{D}^2$ in the numerator, and two others by a $\bar{\mathcal{D}}^2$ in the expansion of the denominator, resulting in a term proportional to the second term in (33). Other contributions to this term arise from the superspace curvature implicit in the definition [5] of the tangent space spinorial covariant derivative $\mathcal{D}_\alpha$. The last term in (33) is obtained by replacing $\Box_\chi \to \Box_\chi - m^2$ in the expression (40) for δ and dropping terms of order m^{-2}.

For completeness and comparison we give the result for the one-loop induced (left-handed) gaugino mass [3, 4] under the same assumptions used here to calculate $\mathcal{L}_A + \mathcal{L}_m$:

$$\begin{aligned} \Delta m_a &= -\frac{g_a^2(\mu)}{16\pi^2}\left[(3C_a - C_a^M)m_{\tilde{G}} + \sum_X \eta_X C_a^X F^n\partial_n \ln f_X\right] + \cdots \\ &= -\frac{g_a^2(\mu)}{16\pi^2}\left[(3C_a - C_a^M)m_{\tilde{G}} + C_a F^n\partial_n K - \sum_i F^n C_a^i \partial_n \ln f_i\right] + \cdots, \\ f_i &= g_i^{-2}e^K, \end{aligned} \tag{41}$$

where C_a, C_a^X, C_a^i are the quadratic Casimirs in the adjoint of the gauge subgroup $\mathcal{G}_a$ and in the representations of Φ^X, Z^i, respectively, with $C_a^M = \sum_i C_a^i$. The second equality in (41) follows

[5] This is obvious for the contribution proportional to γ_i^W, since $W_{Z^I Z^J} = W_{ij}$. The contribution proportional to γ_i^g does not arise from superpotential couplings, but it must be of the same form since the same result is obtained from loops of massless fields with the same superpotential couplings (4) as φ^a, Y_I.

from the requirements of finiteness [11] and supersymmetry [15] of the chiral/conformal anomaly proportional to the squared gauge field strength. In contrast to the results in (23) and (31), the leading one-loop contribution to the gaugino masses is completely determined by the low energy theory. In this case all gauge-charged PV fields Φ^X contribute, their mass matrix is block diagonal and commutes with the relevant operators, and the gauge-charge weighted masses are constrained to give the second equality in (41). On the other hand, only a subset of charged PV fields contribute to the renormalization of the Kähler potential. While the Kähler metrics of the fields Φ^A that appear in $W(\Phi^A, Z^i)$, Eq. (4), are determined as in (5) by the finiteness requirement, the metrics of the fields Φ^α to which they couple in W_m, Eq. (11), are arbitrary. Since the conformal anomaly associated with the renormalization of the Kähler potential is a D-term, it is supersymmetric by itself and there is no constraint analogous to the conformal/chiral anomaly matching in the case of gauge field renormalization with an F-term anomaly. As a consequence the "nonuniversal" terms appearing in $\mathcal{L}_A + \mathcal{L}_m$ cannot be determined precisely in the absence of a detailed theory of Planck scale physics.

As a check of our results, consider the "no-scale" model defined by

$$K = k(S) + G, \quad G = -3\ln(T + \bar{T} - \sum_i |\Phi^i|^2), \quad W = W(\Phi^i) + W(S), \tag{42}$$

which has no soft SUSY breaking in the observable sector Φ^i at tree level. If we regulate the theory so as to preserve the no-scale structure, we have $g_\alpha = g_A$, and

$$f_A = f_i = h_i(s)e^{G/3}. \tag{43}$$

Then

$$\partial_t \ln f_A = \frac{1}{3}G_t, \quad \partial_{\bar{t}}\partial_t \ln f_A = \frac{1}{3}G_{t\bar{t}}. \tag{44}$$

Vanishing vacuum energy at tree level requires $F^s = 0$, so if $< \phi^i >= 0$, $\phi^i = \Phi^i|$, the no-scale Kähler potential satisfies

$$-F^n G_n = e^{K/2} W G_{\bar{t}} K^{\bar{t}t} G_t = 3m_{\tilde{G}}, \quad F^n \bar{F}^{\bar{m}} G_{n\bar{m}} = e^{K/2} |W|^2 G_{\bar{t}} K^{\bar{t}t} G_t = 3m_{\tilde{G}}^2, \tag{45}$$

and all the soft SUSY-breaking terms, (23), (31) and (41), cancel,[6] in agreement with explicit calculations [16] and nonrenormalization theorems [17] in the context of this model. In the context of string theory however, the "no-scale" regularization is unacceptable, because it leads to a

[6]The vanishing of the one-loop contribution to the gaugino mass in this model was noted by Randall and Sundrum, private communication.

loop-corrected Lagrangian that is anomalous under modular transformations, which are known to be perturbatively unbroken in string theory. Therefore one must restore modular invariance by including, for example, a modular covariant field dependence in the PV mass parameters in (11), $\mu_{A\alpha} = \mu_{A\alpha}(T)$, which might reflect string loop threshold corrections; since these necessarily break the no-scale structure of the theory soft SUSY parameters would be generated since $F^T \neq 0$ in this toy model. Alternatively, as shown in [6], this theory can be regulated with field-independent masses for the PV fields that contribute to the renormalization of the Kähler potential: $\partial_n f_A =$ constant, in which case the A-terms are precisely those found previously [1, 2, 8, 9], and scalar masses are also generated at one loop: $\Delta\mu_i^2 = \gamma_i m_{\tilde{G}}^2$. Gauginos remain massless, since their masses are insensitive to the specific choice of PV regulator masses.

Randall and Sundrum [1] considered a class of models defined by a Kähler potential

$$K = -3\ln\left[1 - \sum_i |\Phi^i|^2 - f(Z^n, \bar{Z}^{\bar{m}})\right], \tag{46}$$

where the Φ^i represent gauge-charged matter, and the Z^n are in a hidden sector where SUSY is broken: $\langle\Phi^i\rangle = \langle F^i\rangle = 0$, $\langle F^n\rangle \neq 0$. For these models

$$\langle K_{i\bar{j}}\rangle = \left\langle e^{K/3}\delta_{ij}\right\rangle = g_i\delta_{ij}, \quad \mu_i^2 = a_{ijk} = 0, \tag{47}$$

from the definitions (18) and (28) and the vacuum condition (26). In addition there is no dilaton: $g_a =$ constant, $m_a = 0$, so there is no soft SUSY breaking in the tree Lagrangian. If we *assume* $g_\alpha = g_A$, there are also no soft SUSY breaking parameters in the PV Lagrangian. Then the scalar masses vanish at one loop, and we obtain:

$$\begin{aligned}
\ln f_{jk} &= g_a^{-1}\ln f_{ia} = \ln f_i = K/3, \\
\mathcal{L}_A &= -\frac{1}{32\pi^2}e^{K/2}W_i z^i\gamma_i\left(m_{\tilde{G}} + \frac{1}{3}F^n K_n\right) + \text{h.c.} + \cdots, \\
\Delta m_a &= -\frac{g_a^2(\mu)}{16\pi^2}(3C_a - C_a^M)\left(m_{\tilde{G}} + \frac{1}{3}F^n K_n\right) + \cdots.
\end{aligned} \tag{48}$$

To determine the model-dependent contribution proportional to $\langle F^n K_n\rangle$, we study the vacuum conditions $\langle V\rangle = \langle V_z\rangle = 0$ for the potential $V(z = Z|)$ derived from $W(Z)$ and $K(Z) = -3\ln[1 - f(Z)]$, with the gauge-charged fields Φ set to zero. This potential is classically invariant under the Kähler transformation

$$K(Z) \to K^\xi(Z) = K(Z) + \xi\ln|W(Z)|^2, \quad f^\xi = (1-f)|W|^{\frac{2\xi}{3}}, \quad W(Z) \to W^{1-\xi}(Z). \tag{49}$$

If one imposes a "separability" condition [1] on the superpotential

$$W = W(\Phi^i) + W(Z^n), \tag{50}$$

the redefinition (49) is not a classical invariance of the full theory with $\Phi \neq 0$, but rather defines a one-parameter family of models of the general form defined by (46) and (50), with the same vacuum, but with different couplings of the hidden sector to gauge-charged matter.

If $f(Z,\bar{Z}) = f(|Z|^2)$ and $\langle z\rangle = \langle Z|\rangle = 0$, then $\langle K_z\rangle = \langle 3f'(z)\bar{z}\rangle = 0$, and the "anomaly mediated" results are recovered in the dimension-three soft operators (48). An example of this type is given in [1] for the case of a single hidden sector field Z. For the family of models generated from that one by the redefinitions (49), we obtain for the coefficient of the soft terms in (48):

$$B^\xi = m_{\tilde{G}} + \frac{1}{3}\left\langle F^z K^\xi_z\right\rangle = m_{\tilde{G}} + \frac{\xi}{3}\left\langle F^z \frac{W_z}{W}\right\rangle = (1-\xi)m_{\tilde{G}}, \tag{51}$$

since $\langle F^z \bar{F}^{\bar{z}} K_{z\bar{z}}\rangle = -\langle F^z(K_z W + W_z)\rangle = 3m^2_{\tilde{G}}$ is invariant under (49). As a second example, consider the simpler Kähler potential, $f(Z) = |Z|^2$, with $W(Z) = \lambda(1+Z)^3$, which for $\Phi = 0$ is classically equivalent, by a field redefinition and a Kähler transformation, to the no-scale theory defined by (42) with $f(T) = 1 - T - \bar{T}$, $T = (1-Z)/[2(1+Z)]$, $W_T = 0$. In this case we find

$$B^\xi = m^2_{\tilde{G}}\left(1 - h^\xi\right), \quad \xi \leq h^\xi = \left\langle \frac{\bar{z} + \xi + |z|^2(1-\xi)}{1+z}\right\rangle \leq 1, \tag{52}$$

if $0 \leq \xi \leq 1$, since $\langle z\rangle$ is undetermined in this no-scale model, but satisfies $|z| \leq 1$. For $\langle z\rangle = \xi = 0$, we have $\langle F^z K_z\rangle = 0$, giving the standard result [1, 2, 8, 9] $B^\xi = m_{\tilde{G}}$. For $\xi = 1$, this model is precisely the one defined by (42), with $B^\xi = 0$. Quite generally, if $\langle F^n W_n\rangle = 0$ in the class of models defined by the separability conditions (46), (50) and $g_\alpha = g_A$ for the PV fields, the soft SUSY-breaking terms all vanish, since in this case $\langle F^n K_n\rangle = \langle F^n \bar{F}^m K_{i\bar{m}}\rangle / m_{\tilde{G}} = -3m_{\tilde{G}}$ by the vacuum condition (26).

To summarize, we have found that the "anomaly mediated" results for soft SUSY breaking rest on the separability assumptions stated above, but also on more specific assumptions on the form of the hidden sector potential. We now address the question as to why these same results were obtained by spurion analyses.[7] In its original incarnation [8], these techniques of deriving observable sector soft SUSY breaking terms were applied solely to models in flat superspace (such as models

[7]The authors of [9] also pointed out that these results are correct only if $\langle F^n K_n\rangle$ can be neglected.

of gauge-mediated SUSY breaking). In these cases the Kähler potential and superpotential obeyed the separability conditions between observable and hidden sectors

$$K_{\rm tot} = K_{\rm obs}\,(\Phi,\bar{\Phi}) + K_{\rm hid}\,(Z,\bar{Z})\,, \quad W_{\rm tot} = W_{\rm obs}\,(\Phi) + W_{\rm hid}\,(Z)\,. \tag{53}$$

Furthermore, the observable sector Kähler potential was of a minimal variety: $K_{\rm obs}\,(\Phi,\Phi) = \sum_i |\Phi^i|^2$.

The key properties of models in which the leading contributions to soft terms arise from the conformal anomaly were enumerated by Randall and Sundrum and are encapsulated in the form of the Kähler potential (46). This Kähler potential was the result of demanding separability in the function $\Omega = -3e^{-K/3}$:

$$\Omega_{\rm tot} = -3 + \Omega_{\rm obs}\,(\Phi,\bar{\Phi}) + \Omega_{\rm hid}\,(Z,\bar{Z})\,, \quad W_{\rm tot} = W_{\rm obs}\,(\Phi) + W_{\rm hid}\,(Z)\,, \tag{54}$$

where the factors -3 ensure the canonical normalization of the Einstein term in the supergravity Lagrangian. The separability condition (54) and the requirement that $\Omega_{\rm obs} \propto \sum_i |\Phi^i|^2$ (necessary to ensure vanishing tree level soft SUSY breaking in the visible sector) give rise to (46). Of course in the flat space limit Kähler separability (53) and separability in Ω (54) are equivalent statements. Thus the Kähler potential assumed in (46) is of precisely the limited class of potentials for which the flat-space spurion techniques can be imported into a supergravity context, as in Refs. [1] and [9], without complication. This intimate connection between (46) and the canonical flat space of the spurion technique is not surprising as the ansatz of (54) represents a set of models with very special conformal properties, as we will elucidate below.

For dimension-three soft terms the distinction between curved and flat superspace is irrelevant, and the dependence of the anomaly-induced soft terms (48) on the auxiliary multiplet of supergravity is fixed by the conformal properties of the operators involved [2]. The complete anomaly contribution for the dimension-two soft terms given in (31) not proportional to the normal logarithmic running can in fact be obtained from the spurion technique by use of the following construction. We promote the wave function renormalization coefficient Z to a spurion superfield $\mathcal{Z}$ as in [8]. However, this field is not only dependent on the chiral compensator $\eta = 1 + F_\eta\theta^2$ and its Hermitian conjugate, but also on a *real* superfield. Using the PV soft term definitions in (20) we can see that this spurion is given schematically by

$$V = 1 - (a + 2m_a)\theta^2 - (a + 2m_a)\bar{\theta}^2 + \mu_a^2\theta^2\bar{\theta}^2, \tag{55}$$

where here a and $2m_a$ generically represent the tree-level A-terms of the Pauli-Villars sector that correspond, respectively, to the tree-level A-terms a and gaugino masses m_a of the light field sector, and μ_a represents the soft scalar masses of the Pauli-Villars sector. The functional dependence of the superfield $\mathcal{Z}$ on these spurions is given by

$$\mathcal{Z} = \mathcal{Z}\left(\frac{\mu V}{(\eta\bar{\eta})^{1/2}}\right) \tag{56}$$

with μ the renormalization point as before, in analogy with Ref. [8].

We can now perform a Taylor series expansion of this expression about $\theta = 0$ to obtain

$$\begin{aligned}\ln \mathcal{Z} &= \ln Z(\mu) + \frac{\partial \ln Z(\mu)}{\partial \ln \mu}\left[-\left(a + 2m_a + \frac{F_\eta}{2}\right)\theta^2 - \left(a + 2m_a + \frac{\bar{F}_\eta}{2}\right)\bar{\theta}^2\right] \\ &+ \frac{\partial \ln Z(\mu)}{\partial \ln \mu}\left[\left(\mu_a^2 + \frac{(a+2m_a)F_\eta}{2} + \frac{(a+2m_a)\bar{F}_\eta}{2} + \frac{|F_\eta|^2}{4}\right)\theta^2\bar{\theta}^2\right] + \text{two} - \text{loop}. \end{aligned} \tag{57}$$

Now the *chiral* field redefinition

$$\eta \to \eta' = Z(\mu)^{1/2} \exp\left(-\frac{1}{2}\frac{\partial \ln Z(\mu)}{\partial \ln \mu} F_\eta \theta^2\right)\eta \tag{58}$$

can be performed as usual in the spurion derivation to eliminate the one-loop contribution to soft masses arising from the supergravity auxiliary field and generate the one-loop contribution to the A-terms. This process is equivalent to the cancellation of the double B-term insertions mentioned above (27). Note the importance of the assumptions of (46), in particular the fact that the Kähler potential for the observable sector is minimal to lowest order in $1/m_P$, for the rotation (58) to be performed.

This same chiral rotation cannot be performed on the real superfield contributions of the Pauli-Villars soft SUSY breaking terms. This real superfield is not itself the product of a chiral and anti-chiral superfield. The terms proportional to θ^2 are thus irrelevant provided that there is no SUSY breaking in the observable sector. This is a result of the celebrated holomorphy that underlies the spurion technique. The scalar masses are then read off from the $\theta^2\bar{\theta}^2$ component of (57). Use of (27) and the equation of motion for the auxiliary field F_η then leads to identification with (31).

The question remains, why do flat-space spurion techniques imply the vanishing of the Pauli-Villars tree-level soft SUSY breaking parameters independent of the specific nature of the Kähler potential and superpotential? The answer can again be found in the special class of supergravity theories for which these techniques can be applied. Specifically, as mentioned above a "sequestered" sector model is really nothing more than a model on an Einstein-Kähler manifold, of which the no-scale models are a particular subset [18]. These spaces are defined by the fact that the curvature is proportional to the metric. The constant of proportionality determines the normalization of the Einstein term in the supergravity lagrangian. In flat space these are empty statements, but in curved space a properly normalized Einstein term in an Einstein-Kahler manifold will cause the scalar mass term $R + m^2_{\tilde{G}}$ in (14) to be proportional to $\hat{V}$. Hence in a space with vanishing cosmological constant and Kähler potential given by (46) (which is equivalent to using the spurion technique in flat space) the PV tree-level soft masses are identically zero. It follows that no one-loop scalar masses will be generated in these theories by the conformal anomaly.

In conclusion, we have shown that, even in the absence of soft SUSY breaking at tree level, the loop-induced soft SUSY breaking operators are not uniquely determined by anomalies. In particular, the hidden sector Kähler potential and superpotential must be separately specified. This is closely related to the well-known fact[8] that Kähler invariance of supergravity is broken at the quantum level, as is manifest in the expressions (48); F^n is Kähler covariant while K_n is not. Once the full low energy Lagrangian is specified, including any hidden sector, the one-loop gaugino masses are completely determined by the requirements of finiteness and supersymmetry of the Kähler anomaly. However the soft terms in the scalar potential depend on the details of Planck scale physics, since the corresponding PV couplings are not sufficiently constrained. In particular, scalars can acquire masses at one loop in the absence of tree-level soft SUSY breaking. This is the case in the no-scale model when the PV couplings are chosen so that the renormalized Kähler potential does not break Kähler invariance. Kähler invariance is necessarily broken by gauge coupling renormalization (unless specific constraints are imposed on the low energy theory) because there is no similar freedom to adjust the relevant PV couplings. In the context of string-derived supergravity, field theory anomalies for Kähler transformations associated with the exact perturbative symmetries of string theory must be canceled, for example by the introduction of a Green-Schwarz counterterm [20] in the case of gauge coupling renormalization. This breaks the no-scale structure of the untwisted matter sector, and there are generally soft SUSY terms at tree level [21], with supersymmetry broken in the dilaton (S) sector. (In fact if modular invariance is not broken by string nonperturbative effects, the moduli are stabilized at self-dual points with $F^T = 0$.) One-loop effects can nevertheless be important, especially for gaugino masses [22]. A general analysis of soft supersymmetry breaking in these models is in progress [7].

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