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Mathematical Modeling of Language Learning

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for the degree of

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in Mathematics

by

Jacquelyn Leigh Rische

Dissertation Committee:
Professor Natalia L. Komarova, Chair
Professor Long Chen
Professor German A. Enciso Ruiz

2014

DEDICATION

To my family.

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ABSTRACT OF THE DISSERTATION

Mathematical Modeling of Language Learning

By

Jacquelyn Leigh Rische

Doctor of Philosophy in Mathematics

University of California, Irvine, 2014

Professor Natalia L. Komarova, Chair

When modeling language mathematically, we can look both at how an individual learns language, and at how language develops throughout a population. When considering individual learning, the fascinating ability of humans to modify the linguistic input and "create" a language has been widely discussed. In this thesis, we first look at two studies that have investigated language learning phenomena. We create two variants of a novel learning algorithm of the reinforcement-learning type which exhibits the patterns in Hudson Kam and Newport (2009) and Fedzechkina et al. (2012), and suggests ways to explain them. Hudson Kam and Newport (2009) explores the differences between adults and children when it comes to processing inconsistent linguistic input and making it more consistent. We introduce an asymmetry to our algorithm that sheds light on the differences between how children and adults regularize language. Fedzechkina et al. (2012) looks at how adults are able to restructure their linguistic input in order to improve communication. Finally, we look at mathematical modeling of language at the level of a population. We consider a scenario where language is a genetic mutation that has appeared in a population without language, and we study how language will develop in the population. We see that the language individuals have an advantage when they are able to communicate with each other and we find conditions that enable them to "invade" the population more quickly.

Introduction

The human ability to learn language is quite fascinating. As children we learn language without formal education. We simply hear sentences from our parents and others around us. Although the sentences we hear are not enough to recreate all the underlying grammatical rules of our language, we, nevertheless, are able to deduce the underlying grammatical rules and develop the same language as our parents (Komarova et al., 2001).

There are many ways to model language mathematically. For instance, it can be modeled (i) at the level of a population, by evolutionary methods, and (ii) by focusing on the process of individual learning. Evolutionary methods show how language emerges and develops in a population. Using evolutionary methods, we can model how many basic features of human language emerge (like words and grammar–see, for example, Nowak and Komarova (2001)).

We can also consider how individuals learn language. Instead of studying what is happening in a large population, we look at how we learn as an individual, sentence by sentence, and day by day. In this case, we focus on one specific aspect of the language, like the use of a determiner or the concept of case marking. In particular, the fascinating ability of humans to modify their linguistic input and "create" a language has been widely discussed. In this thesis, we first look at two studies that have investigated such language learning phenomena. We create a learning algorithm which exhibits the patterns reported in these studies and suggests ways to explain them.

In the work of Newport and colleagues, it has been demonstrated that both children and adults have some ability to process inconsistent linguistic input and "improve" it by making it more consistent. In Hudson Kam and Newport (2005) and Hudson Kam and Newport (2009), artificial miniature language acquisition from an inconsistent source was studied. It was shown that (i) children are better at language regularization than adults, and that (ii) adults can also regularize, depending on the structure of the input.

In Chapter 1, we create a learning algorithm of the reinforcement-learning type. We find that in order to capture the differences between children's and adults' learning patterns, we need to introduce a certain asymmetry in the learning algorithm. Namely, we have to assume that the reaction of the learners differs depending on whether or not the source's input coincides with the learner's internal hypothesis. We interpret this result in the context of a different reaction of children and adults to positive and negative evidence. We propose that a possible mechanism that contributes to the children's ability to regularize an inconsistent input is related to their heightened sensitivity to positive evidence rather than the (implicit) negative evidence. In our model, regularization comes naturally as a consequence of a stronger reaction of the children to evidence supporting their preferred hypothesis. In adults, their ability to adequately process implicit negative evidence prevents them from regularizing the inconsistent input, resulting in a weaker degree of regularization.

Newport and colleagues have also shown that adults have the ability to restructure linguistic input to facilitate better communication. In Fedzechkina et al. (2012), when learning an artificial language with inefficient case marking, the learners restructure their input to make the case marking more efficient, thus making the language easer to understand. In Chapter 2, we focus on a variant of our algorithm that models the pattens in Fedzechkina et al. (2012). In the study, there are four sentence types, each with different degrees of ambiguity. The meaning of an ambiguous sentence becomes clear when it is case-marked. Again, our learning algorithm is asymmetric, and we find that the learners (who are all adults) react

more strongly to implicit negative feedback. Also, the learners do not remember everything they learn. They forget a certain amount between each day of the experiment. In particular they forget more after the first day since what they learn is not reinforced with a test at the end of the first day (as it is at the end of each subsequent day). With these factors, the learners are able to restructure their input and make the language more efficient.

Finally, in Chapter 3, we look at language learning on a population level. We develop and study a model that looks at what would happen if language is a genetic mutation that appears in a population of individuals without language. Using computer simulations, we study how the individuals with language spread through a population of individuals without language. We consider a population without language on one- and two-dimensional grids. To study how the language group will grow, we focus on the effects of talking and movement. If two individuals with language are next to each other on the grid, they can communicate. We consider their ability to talk to be advantageous, giving them a higher reproduction rate. Individuals are also able to move around on the grid and reproduce within a certain radius, called the jump radius. We look at how these affect the time it takes for the individuals with language to invade the population. We find that, for a two-dimensional grid, a jump radius that is too small or too large will increase the time it takes to invade. However, this phenomenon is affected by the shape of the grid. For a one-dimensional grid, we do not see the same effect. The time to invasion decreases as the jump radius increases.

Chapter 1

Regularization of Languages by Learners: A Mathematical Framework

1.1 Introduction

Natural languages evolve over time. Every generation of speakers introduces incremental differences in their native language. Sometimes such gradual slow change gives way to an abrupt movement when certain patterns in the language of the parents differ significantly from those in the language of the children. The fascinating ability of humans to modify the linguistic input and "create" a language has been widely discussed. One example is the creation of the Nicaraguan Sign Language by children in the course of only several years (Senghas, 1995; Senghas et al., 1997; Senghas and Coppola, 2001). Other examples come from the creolization of pidgin languages (Andersen, 1983; Thomason and Kaufman, 1991; Sebba, 1997). It has been documented that in the time-scale of a generation, a rapid linguistic change occurs that creates a language from something that is less than a language (a limited pidgin language (Johnson et al., 1996), or a collection of home-signing systems in

the example of the Nicaraguan Sign Language).

Language regularization has been extensively studied in children, see e.g. work on the phenomenon of over-regularization in children (Marcus et al., 1992). Goldin-Meadow et al. (1984); Goldin-Meadow (2005); Coppola and Newport (2005) studied deaf children who received no conventional linguistic input, and found that their personal communication systems exhibited a high degree of regularity and language-like structure. The ability of adult learners to regularize has also been discussed (Klein and Perdue, 1993; Bybee and Slobin, 1982; Cochran et al., 1999).

Much attention in the literature is paid to statistical aspects of learning, showing that learners are able to extract a number of statistics from linguistic input with probabilistic variation (Gómez and Gerken, 2000; Saffran, 2003; Wonnacott et al., 2008; Griffiths et al., 2010). Identifying statistical regularities and extracting the underlying grammatical structure both seem to contribute to human language acquisition (Seidenberg et al., 2002).

In Reali and Griffiths (2009) it was demonstrated that in the course of several generations of learning, the speakers shift from a highly inconsistent, probabilistic language to a regularized, deterministic language. A mathematical description of this phenomenon was presented based on a Bayesian model for frequency estimation. This model demonstrated, much like in experimental studies, that while in the course of a single "generation" no bias toward regularization was observed, this bias became apparent after several generations. The same phenomenon was observed in the paper Smith and Wonnacott (2010). It was suggested that gradual, cumulative population-level processes is responsible for language regularity.

In this chapter we focus on a slightly different phenomenon. The work of Elissa Newport and colleagues demonstrates that language regularization can also happen within one generation. A famous example is a deaf boy Simon (see Singleton and Newport (2004)) who received all of his linguistic input from his parents, who were not fluent in American Sign Language (ASL).

Simon managed to improve on this inconsistent input and master the language nearly at the level of other children who learned ASL from a consistent source (e.g. parents, teachers, and peers fluent in ASL). Thus he managed to surpass his parents by a large margin, suggesting the existence of some innate tendency to regularization.

The work of Newport and her colleagues sheds light into this interesting phenomenon. In a number of studies, it has been demonstrated that both children and adults have the ability to process inconsistent linguistic input and "improve" it by making it more consistent. When talking about the usage of a particular rule, this ability was termed "frequency boosting," as opposed to "frequency matching." Let us suppose that the "teacher" (or the source of the linguistic input) is inconsistent, such that it probabilistically uses several forms of a certain rule. Frequency boosting is the ability of a language learner to increase the frequency of usage of a particular form compared to the source. Frequency matching happens when the learner reproduces the same frequency of usage as the source. In Hudson Kam and Newport (2005) and Hudson Kam and Newport (2009) it was shown that (i) children are better at frequency boosting than adults, and that (ii) adults can also frequency boost, depending on the structure of the input.

In this chapter we create an algorithm of the reinforcement-learning type, which is capable of reproducing the results reported in Hudson Kam and Newport (2009). It turns out that in order to capture the differences between children's and adults' learning patterns, we need to introduce a certain asymmetry in the learning algorithm. More precisely, we have to assume that the reaction of the learners differs depending on whether or not the sources' input coincides with the learner's internal hypothesis. We interpret this as learning from positive and implicit negative evidence. We therefore propose that the differences in adults' and children's abilities to regularize are related to the differences in their processing of positive and negative evidence.

This chapter is organized as follows. In Section 1.2 we introduce the mathematical model

used in this paper; it belongs to a wider class of reinforcement learning models. In Section 1.3 we report the results. We describe how our model can be fitted to the data of Hudson Kam and Newport (2009), and what parameters give rise to the observed differences between children and adults. In Section 1.4 we summarize our findings and discuss them in terms of processing positive and negative evidence in language acquisition.

1.2 Theory and calculations

1.2.1 Reinforcement learning in psychology and neuroscience

At the basis of our method is a mathematical model of learning which belongs to a larger class of reinforcement-learning models (Sutton and Barto, 1998; Norman, 1972; Narendra and Thathachar, 2012). Over the years, reinforcement models have played an important role in modeling many aspects of cognitive and neurological processes, see e.g. Maia (2009); Lee et al. (2012). Some of the most influential reinforcement learning algorithms have been created by Rescorla and Wagner (1972), see also Rescorla (1968, 1988). These works have given rise to a large number of papers in psychology and neuroscience, some of which are reviewed in Danks (2003); Schultz (2006), see also review Miller et al. (1995) for a detailed list of successes and failtures of the Rescorla-Wagner (RW) model.

In Schultz (2006), the neurophysiology of reward is studied. In particular, it explains that neurons "show reward activations only when the reward occurs unpredictably and fail to respond to well-predicted rewards, and their activity is depressed when the predicted reward fails to occur". These arguments at the the basis of Rescorla-Wagner models, and the models proposed below. Paper Gureckis and Love (2010) contrasts two broad classes of learning mechanisms: one based on transformations of an internal state (e.g., recurrent network architectures (Elman, 1990)), and the other based on learning direct associations (e.g. the

RW mechanism), and shows that, at least on shorter time scales, human sequence learning appears more consistent with a process based on simple, direct associations. Models of this kind have been used to study category learning (Love et al., 2004), learning of mathematical concepts (Schlimm and Shultz, 2009), and visual concepts (Shultz, 2006; Baayen et al., 2011).

What is especially relevant for this work is the usage of RW type models for language learning in humans, and even more specifically, language regularization.

Paper Ramscar and Yarlett (2007) proposes to use RW type modeling to study the process of learning regular and irregular plural nouns. It is further shown in Ramscar et al. (2013c) that incorporating expectation and prediction error into the model yields a surprising result that with time, the tendency of children to over-regularize irregular plurals can be reduced by exposing them to regular plurals.

Ramscar et al. (2013b) studied the problem of learning the meaning of words by adults and children, and found that the informativity of objects plays a more important role for children's learning than for adult learning.

Ramscar et al. (2013a) studies cognitive flexibility by looking response-conflict resolution behavior in children. The RW model is used to describe label-feature and feature-label learning processes and predict the very different testing results by children trained by the two methods. The success of the theory behind it is demonstrated in Ramscar et al. (2010), where the role of negative learning and cue competition is highlighted by modeling of two novel empirical studies. Ramscar et al. (2011) applies this theory to children's learning of small number words.

The theory presented below is not an attempt to explain the process of language acquisition in its entirety (which would be a formidable task). Following a tradition in mathematical linguistics and learning theory (see the papers cited above, as well as Steels (2000); Nowak et al. (2001); Niyogi (2006); Lieberman et al. (2007); Hsu et al. (2013)), we have deliberately

simplified the task of the learner to concentrate only on certain aspects of learning. For example, we have assumed that the learner is able to extract (segment) from all the utterances received, the correct mutually exclusive forms $1, \ldots, n$ of the word/rule under investigation. This in itself is a challenge studied in the literature, see e.g. Roy and Pentland (2002); Seidl and Johnson (2006); Monaghan et al. (2013). Once this pre-processing step has been achieved, the learner's task is obviously simplified.

1.2.2 The basic algorithm

Let us suppose that a certain rule has n variants (forms). A learner is characterized by a set of n positive numbers, $X = (x_1, x_2, \ldots, x_n)$ on a simplex: $\sum_{i=1}^{n} x_i = 1$. Each number x_i is the probability for the learner to use form i. We will call the numbers $\{x_i\}$ the frequencies of the learner. If for some $i = I$, $x_I = 1$, then the learner is deterministic and will consistently use form I. The learning process is modeled as a sequence of steps, which are responses of the learner to the input. The linguistic input (or source) emits a string of applications of the rule, and it is characterized by a set of constant frequencies, $\nu_1, \ldots, \nu_n$ (with $\sum_{i=1}^{n} \nu_i = 1$). At each iteration, the learner will update its frequencies in response to the input received from the source. If the source's input is form j, then the learner will update its frequencies according to the following update rules:

$$x_k \rightarrow x_k - F_k(X), \quad k \neq j, \tag{1.1}$$

$$x_j \rightarrow x_j + \sum_{k \neq j} F_k(X). \tag{1.2}$$

In this most general formulation, the function F_k can depend on any components of X. In the case of the linear reinforcement model, we have (Narendra and Thathachar, 2012)

$$F_k(X) = ax_k, \tag{1.3}$$

such that

$$x_k \rightarrow x_k - ax_k, \quad k \neq j, \tag{1.4}$$

$$x_j \rightarrow x_j + a(1 - x_j). \tag{1.5}$$

The RW model reduces to this update rule if we assume that (1) only one stimulus is presented at a time, and further if (2) the maximum conditioning produced by each stimulus and the rate parameters are the same for all stimuli (the latter assumption is made for example in Ramscar et al. (2010)).

Here we will use another version of reinforcement algorithm (1.1-1.2), whose basic form is given by

$$F_k(X) = \begin{cases} s/(n-1), & x_k > s/(n-1), \\ x_k, & x_k < s/(n-1), \end{cases} \tag{1.6}$$

where $k \neq j$, see (Mandelshtam and Komarova, 2014). Here, j is the signal emitted by the source, and the parameter $0 < s < 1$ defines the increment of a learning update. This simple update rule states that in response to form j produced by the source, the learner will increase its probability to use that form by a certain amount, and consequently the probabilities of all other forms will be reduced across the board. As in the RW model, the amount by which the strength of a certain rule is increased, depends on how “unpredictable"

the rule is for the current state of the learner. In RW model, the increment of learning is a linear function of the difference between the current state x_j and the maximum conditioning (which is one). In model (1.6), it is a nonlinear function that decreases when the strength of the rule approaches the maximum.

Learning algorithm (1.6) is a Markov process characterized by a stationary frequency distribution. That is, starting from any initial frequency vector, a learner will converge to a quasistationary state, where the values $\{x_1, \ldots, x_n\}$ fluctuate around fixed means. Figure 1.1 demonstrates this behavior.

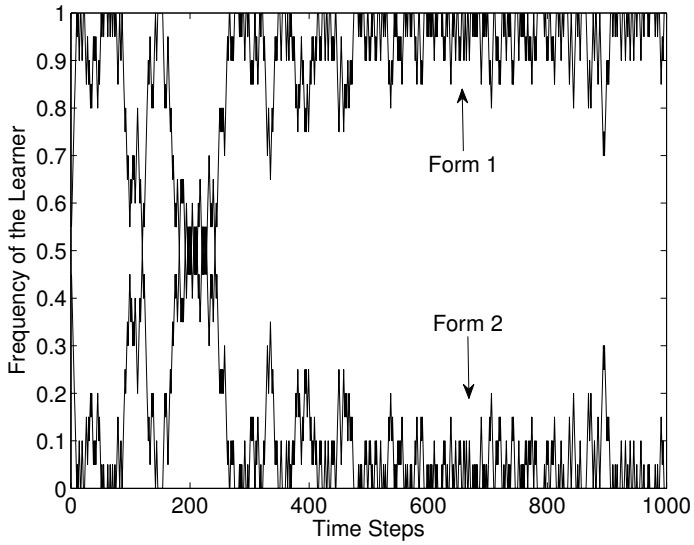


Figure 1.1: A plot of the frequency of the learner with respect to time. Here the frequency of the learner is characterized by two forms. The frequency of the source for form 1 is $\nu_1 = .6$. The increment of learning update is $s = .05$. Over time, the learner converges to a quasistationary state.

This algorithm possesses a source boosting property. If the (inconsistent) source is characterized by a dominant usage of a certain form, on the long run the learner will use the same form predominantly, and the frequency of usage for that form will be higher for the learner compared to the source. Let us suppose that $\nu_1 > 1/2$ is the highest source frequency for

$n = 2$. Then at quasisteady state, the learner will use form 1 with the expected frequency

$$\nu_1' = 1 - (1 - \nu_1)\left(\frac{s}{2\nu_1 - 1} + \frac{1 + s}{1 - \nu_1(1 + (\nu_1/(1 - \nu_1))^{1/s})}\right) > \nu_1, \tag{1.7}$$

see Appendix A.1 for details of the calculations. Figure 1.2 demonstrates the frequency boosting property by plotting the quasisteady state frequency ν_1' as a function of the source frequency, ν_1. Note that the more linear reinforcement model (1.3) does not have a frequency-boosting property, see Appendix A.2, but exhibits frequency-matching property.

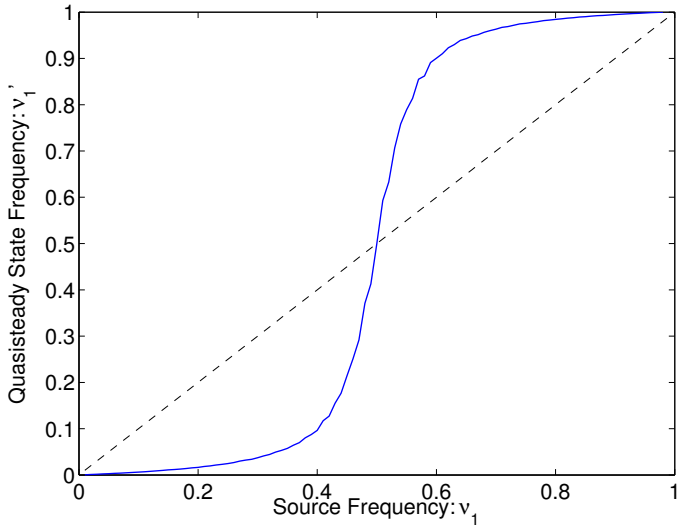


Figure 1.2: A plot of the quasisteady state frequency ν_1' as a function of the source frequency, ν_1 (the solid line). The dashed line is the line $\nu_1' = \nu_1$. When $\nu_1 > 1/2$, we can see the frequency boosting property ($\nu_1' > \nu_1$). For this plot, $s = 0.05$.

The frequency at which the dominant form is used by the source will affect the speed of convergence of algorithm (1.6): higher frequencies lead to faster convergence. The strength of the boosting depends on the increment of learning update, s: smaller values of s yield larger values of ν'. The value s also influences the speed of convergence: for higher s the algorithm converges faster, but the frequency at which the learner uses the dominant form decreases.

1.2.3 The asymmetric algorithm

The basic algorithm is characterized by a single parameter, s, which defines the increment of learning. Next we introduce a two-parametric generalization of this algorithm, where the update rules are different depending on whether the source's input value matches the highest-frequency value of the learner. Let us suppose that component x_m is the largest: $x_m = \max_i\{x_i\}$. We define form m to be the preferred form (or the preferred hypothesis) of the learner. Then, in responds to the source emitting form j, the update rule (1.1-1.2) will have the following increment function:

$$F_k(X) = \begin{cases} s/(n-1), & x_k > s/(n-1), \quad k = m, \\ x_k, & x_k < s/(n-1), \quad k = m, \\ p/(n-1), & x_k > p/(n-1), \quad k \neq m, \\ x_k, & x_k < p/(n-1), \quad k \neq m. \end{cases} \tag{1.8}$$

Here is another way to express the update rules. If the source emits form $j = m$ that matches the largest learner's frequency, the frequencies are updated as follows:

$$\begin{aligned} x_k &\rightarrow x_k - F_k, \quad F_k = \begin{cases} \frac{s}{n-1}, & x_k > \frac{s}{n-1} \qquad i \neq k \\ x_k \quad \text{otherwise} \end{cases} \\ x_j &\rightarrow x_k + \sum_{k \neq j} F_k \end{aligned} \tag{1.9}$$

If the source uses form $j \neq m$ different from the learner's preferred form, then the update is as follows:

$$\begin{aligned} x_k &\rightarrow x_k - F_k, \quad F_k = \begin{cases} \frac{p}{n-1}, & x_k > \frac{p}{n-1} \\ x_k & \text{otherwise} \end{cases} \qquad i \neq j \\ x_j &\rightarrow x_j + \sum_{k \neq j} F_k \end{aligned} \tag{1.10}$$

If $s = p$, then this algorithm is the same as equation (1.6). The novel feature of the two-parametric algorithm is that it tracks whether the source matches the learner's "preferred" hypothesis. If it does (that is, if the input is the form whose frequency is the largest for the learner), then the learner increases this frequency's value by amount defined by s. Otherwise, the increment is defined by p (see figure 1.4). For example, in the extreme case where $p \ll s$, the updates are only performed when the learner is reassured that its highest frequency form is used. Otherwise the frequencies are updated very little. The two increment values, s and p, will be refereed to as the "preferred increment" and the "non-preferred increment."

Note that this algorithm does not simply use different reward signals on positive and negative trials. Instead, it tracks whether the signal emitted by the source matches the current hypothesis of the learner. For example, let us suppose that with $n = 3$ forms, the current state of the learner is $(0.1, 0.7, 0.2)$ (and we further assume that $s, p < 0.05$). In this case, the preferred form of the learner is $m = 2$. If the source emits signal $j = 1$, which does not match the preferred form, the value of x_1 will receive an increment of p, and the other values (x_2 and x_3) will both decrease by $p/2$. If the source emits signal $j = 2$, which matches the preferred form, then the value of x_2 will receive an increment of s, and the other values (x_1 and x_3) will both decrease by $s/2$. We can see that that depending on whether the source's form is the same as the preferred form of the learner, the positive increment for that form will be different.

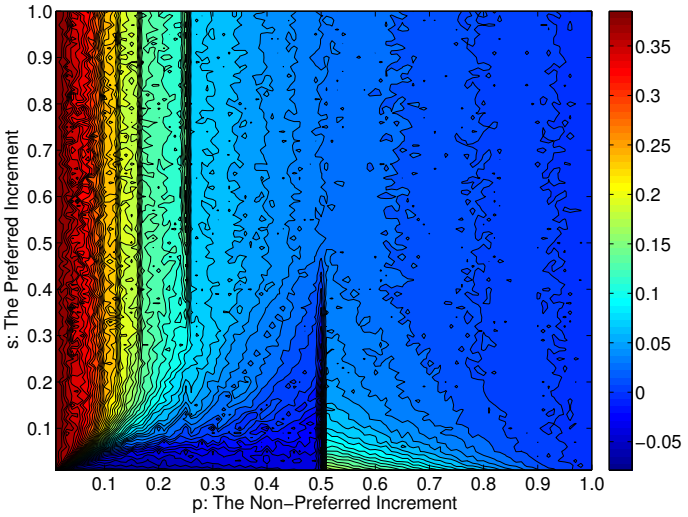


Figure 1.3: Contour plot of the value $\nu_1' - \nu_1$, which is the difference between the expected steady-state frequency of the learner and the frequency of the source. Here $n = 2$ and s and p are varied between 0.01 and 1 with step 0.01.

Figure 1.3 demonstrates the properties of the asymmetric learning algorithm depending on the increment values, s and p, which are varied between 0.01 and 1 with step 0.01. For the case $n = 2$, $\nu_1 = 0.6$, it presents the contour plot of the value $\nu_1' - \nu_1$, which is the difference between the expected frequency of the learner at steady-state (equation (1.7)) and the frequency of the source. All positive values correspond to the existence of the boosting property, and the larger the value (denoted by the red colors), the stronger is the boosting effect. We can see that the strongest boosting property is observed when the non-preferred increment, p, is the smallest. On the other hand, the boosting effect disappears entirely if p is significantly larger than s (the dark blue regions).

1.3 Results

In this paper we create a mathematical framework to describe adults' and children's learning from an inconsistent source. We will use the results of Hudson Kam and Newport (2009) to

test and parametrize our model (see also Appendix A.3 for an application of our model to the data from Hudson Kam and Newport (2005)). In Hudson Kam and Newport (2009), the authors expose adults and children to miniature artificial languages. The participants learn the language by listening to sentences of the language, which are presented in an inconsistent fashion (allowing for a probabilistic usage of several forms). The structure and complexity of the probabilistic input varies from experiment to experiment. The goal is to assess what kinds of input are most consistent with the tendency of adults and children to regularize. The authors also evaluate the differences in the learning patterns between adult learners and children.

1.3.1 Regularization by adult learners

In one of the experiments performed in Hudson Kam and Newport (2009) with only adult participants, the authors used five different types of inconsistent input. In the control case (termed 0ND), the sentences with the "correct" (most frequent) form are given 60% of the time, and 40% of the time sentences with an alternative form are given. In the other four conditions, the most frequent form was also uttered 60% of the time, but different numbers of alternative forms were used. In the 2ND case two alternative forms are each used 20% of the time. Similarly, in each of the conditions iND with $i = 2, 4, 8, 16$, i alternative forms were used $(40/i)$% of the time each. It was found that the adults in the control case did not boost the frequency but rather frequency-matched the 60% of the more frequent forms. Interestingly, as the complexity of the input increased, the learners in each of the conditions produced the most frequent form of the language more often. That is, the frequency-boosting increased with the number of alternative forms used.

We constructed two types of reinforcement models that describe the learning process as a sequence of iterations. The input is a string of applications of a rule, that uses different forms

with certain (constant) frequencies. The learner is characterized by frequencies of usage of each of the possible forms. After each application of the rule by the source, based on the input, the learner updates the probabilities for each of the forms. The frequency of the form uttered by the source increases, and all the other frequencies decrease. Figure 1.4(a) provides a graphical illustration of this algorithm with the example of 2 alternative forms. In the first, symmetric model, the increment is constant no matter what form is used by the source, figure 1.4(a). In an asymmetric generalization of this model, the increments are different depending on whether the form uttered by the source matches the most frequent form of the learner. If the form uttered by the source is the same as the "preferred" (most frequent) form of the learner, the corresponding frequency receives a "preferred boost," s. Otherwise, a "non-preferred boost" p is used. Two cases, $s > p$ and $s < p$, are illustrated in figures 1.4(b,c).

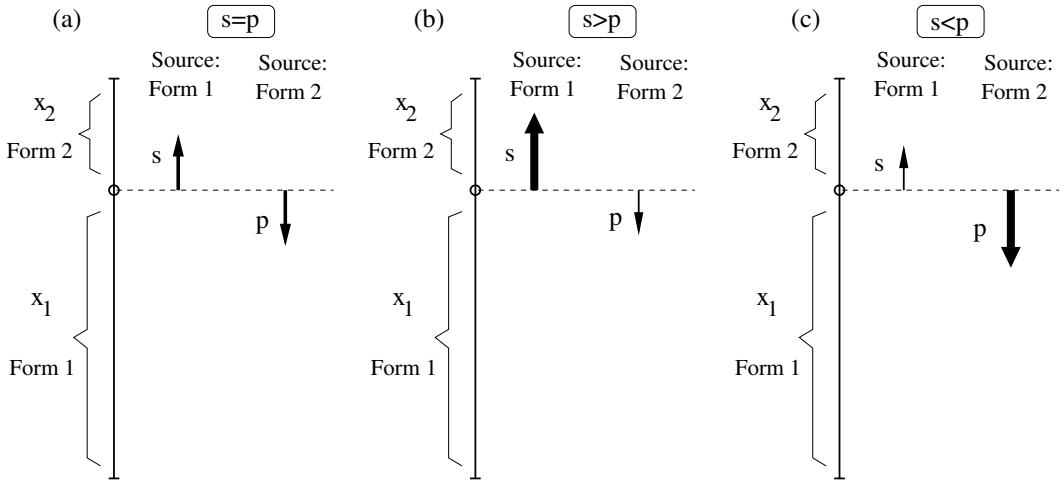


Figure 1.4: A schematic illustration of the reinforcement learning algorithm proposed here, in the case of two forms of the rule. The vertical bar represents the state of the learner: the small circle splits the bar into two parts, x_1 and x_2, with $x_1 + x_2 = 1$, such that x_1 is the probability of the learner to use form 1, and x_2 is the probability of the learner to use form 2. In this example, $x_1 > x_2$, that is, form 1 is the preferred form of the learner. The black arrows show the change in the state of the learner following an utterance of the source. If the source produces form 1, the value of x_1 increases by amount s. If the source produces form 2, the value of x_1 decreases by amount p. Three cases are presented: (a) $s = p$ (the symmetric learner), (b) $s > p$, and (c) $s < p$.

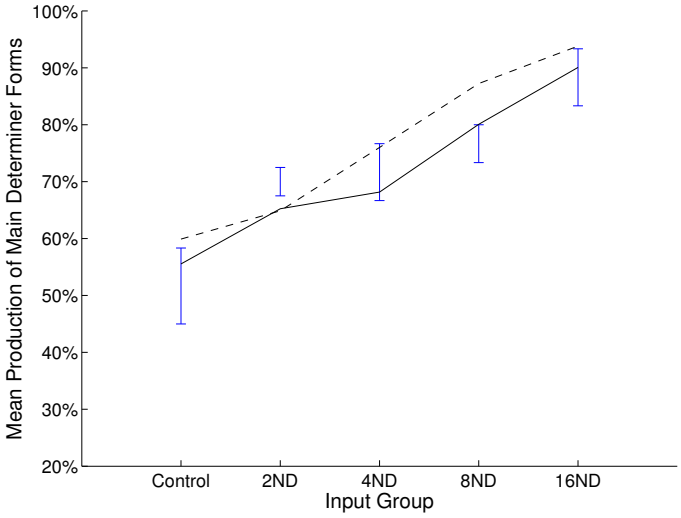


Figure 1.5: Comparison of the results from Hudson Kam and Newport (2009) (the vertical intervals) with the results from our model (the lines), adult learners only. The dashed line is for the best fitting symmetric reinforcement model with $s = 1$. The solid line is the best-fitting asymmetric model corresponding to $s = .19$ and $p = .50$.

We performed computer simulations to study whether our models can predict the observed patterns. For each condition, for each value of the increments s and p, we have run each model 100 times for 1500 time steps. After each run, we noted the probability that the learner uses the "correct" (most frequent) form by averaging its frequency found at each time step (starting at time step 500 to give the algorithm time to converge), and averaged these probabilities. We used these averages to calculate the least squares error compared to the results from Hudson Kam and Newport (2009), to determine the values s and p that best match the experimental results.

Applying the simple reinforcement model (1.6) to describe the data of Hudson Kam and Newport (2009) we found that no parameter s gives a satisfactory fit. The best fit was obtained for the value $s = 1$ and is depicted by the dashed line in figure 1.5. We can see that this model overestimates the amount of frequency boosting compared to the experiment. The averages that we find are (except for the 2ND group) too high when compared with the results from the paper. Also, with $s = 1$ the model predicts the behavior of the learner to be

very unstable, characterized by frequent switchings between 0% and 100% for the frequency of a form.

We then turned to the asymmetric model. Figure 1.6 gives a heat plot of the least square error computed for all pairs (s, p). The dark blue areas give the best overall error. We found that the parameters $s = 0.19$ and $p = 0.50$ give the best match. This is represented by the solid line in figure 1.5. Therefore, for the adults, the best match comes when the "non-preferred increment" is larger than the "preferred increment." A similar result was obtained when we used the data from Hudson Kam and Newport (2005), see Appendix A.3.

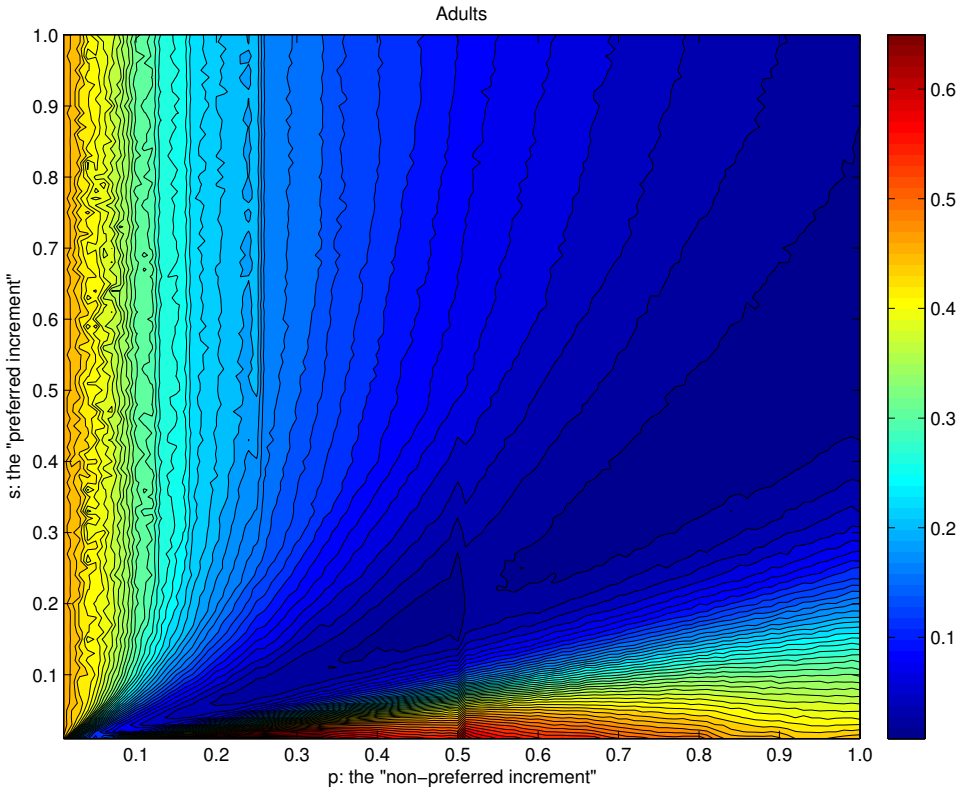


Figure 1.6: Heat plot of least squares error of the asymmetric model compared to the results from Hudson Kam and Newport (2009). s and p vary between 0.01 and 1 with a step size of 0.01. The dark blue areas indicate where the smallest error occurs.

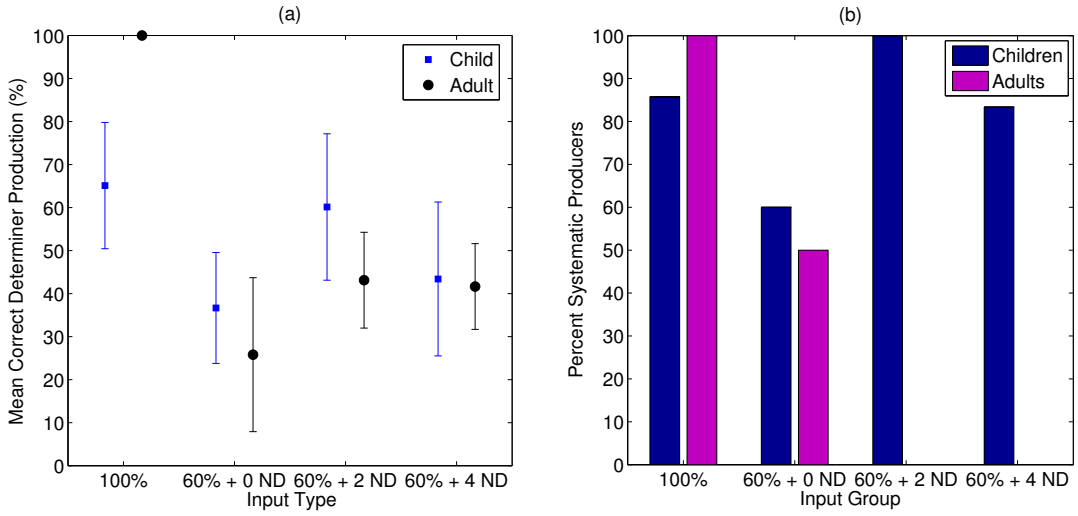


Figure 1.7: Results from the experiment with children and adults in Hudson Kam and Newport (2009). (a) Correct form production. (b) Percent of systematic users broken down by children and adults for each input group.

1.3.2 Regularization by children and adults - a comparison

In order to investigate the differences in frequency boosting between children and adults, Hudson Kam and Newport (2009) performed a similar experiment, both with children and adult participants, by using a simpler artificial language. First, the "correct" form of the language was used 100% of the time, and then three more conditions, 0ND, 2ND, and 4ND were explored. Figure 1.7(a) presents the data by plotting the mean and the standard deviation of the most frequent form production by adults and children. It was found that children perform worse than the adults in the 100% case and similar to the adults in the other cases.

The graphs in figure 1.7(b) contain some additional information. The authors of Hudson Kam and Newport (2009) went a step further and measured the number of systematic users (as opposed to "correct" users) for adults and children in each condition. Systematic users were defined as learners that always used the same form–even if it is not the right form. It turned out that although the number of "correct" usages (that is, usages of the most

frequent form of the source) in children was similar to that of the adults, the children used their forms significantly more systematically (although their form did not always match the most frequent form of the source). This is represented in figure 1.7(b). We can see that for the 2 ND and 4 ND groups, the children were almost always systematic learners, while the adults were never systematic learners.

In order to explain these data, we chose the following approach. First of all, we note that children did not always produce the "correct" form in the 100% case. This suggests that there was a noise factor at play here. Children did not always pay sufficient attention to the input, which we included in the model as a noise parameter r, the probability that, although form i was used, the child heard form j (with $j \neq i$). We fitted the data on the "correct" usage (figure 1.7(a)) assuming different values of r ($r = 0.2, 0.4, 0.6$, and 0.8) and varying the parameters s and p.

Figure 1.8 shows a heat-plot of the error obtained by using different values of the noise parameter, and varying the increments. We can see that the noise level of $r = 0.4$ gives the best match (it contains the most dark blue regions, which correspond to the regions with the smallest error). However, we can see that there are two separate regions of dark blue which correspond to two different types of learning. One is on the left side of the plot and corresponds to very small values of p (with $s > p$). The other region corresponds to values of s and p where $p > s$. The error estimate given by parameters from the two regions is similar. Given the noise in the data, it is difficult to decide which parameter regime is more consistent with reality simply on the basis of the error. Instead, we turn to the data on systematic production.

As we know from studying the properties of the model, learners with $s > p$ possess a larger boosting property. This is explained graphically in figure 1.9, which assumes that there are $n = 2$ forms and that form 1 is the more frequent one ($\nu_1 > \nu_2$ for the source). The value of x_1, the learner's frequency for form 1, is presented by a small circle in a unit interval,

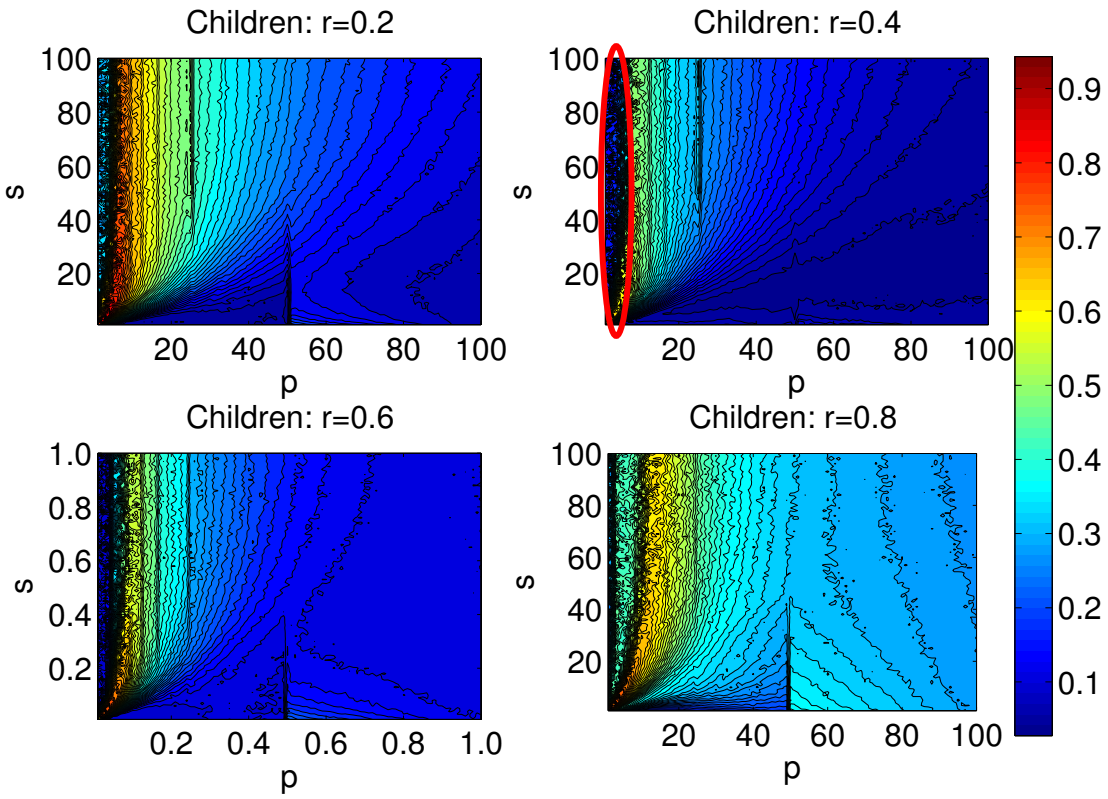


Figure 1.8: Error for children with four different noise rates: $r = 0.2, r = 0.4, r = 0.6$, and $r = 0.8$. For each contour plot, the values of s and p range between 0.01 and 1 with step 0.01.

similar to figure 1.4. If this circle is closer to the right border of the interval, this means that form 1 is the learner's preferred form. Otherwise form 2 is the learner's preferred form. The arrows on the diagram show the direction and the relative size of the update followed by the source's utterance. The black arrows are the updates if the source utters form 1, and the light green arrows are the updates if the source utters form 2.

We can see that if $s > p$, then the arrows pushing the dot towards the edges (zero and one) are larger than the ones pushing it toward the middle (for $s < p$, this is reversed). This means that if the learner is characterized by a stronger response to the source when the utterance coincides with its preferred form, then the boosting tendency is observed.

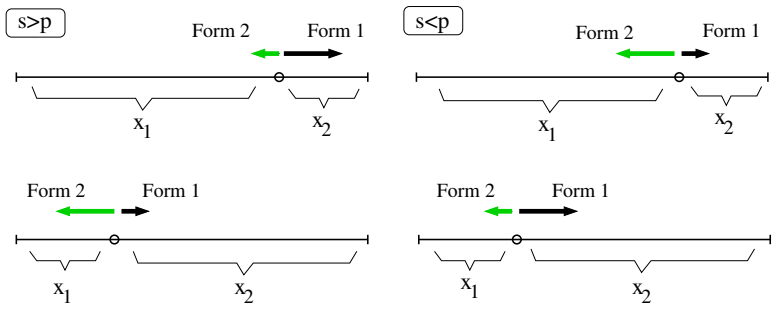


Figure 1.9: Boosting tendency occurs when $s > p$. The value of x_1 is the learner's frequency for form 1. The arrows on the diagram show the direction and the relative size of the update following the source's utterance. The black arrows are the updates if the source utters form 1, and the light green arrows are the updates if the source utters form 2.

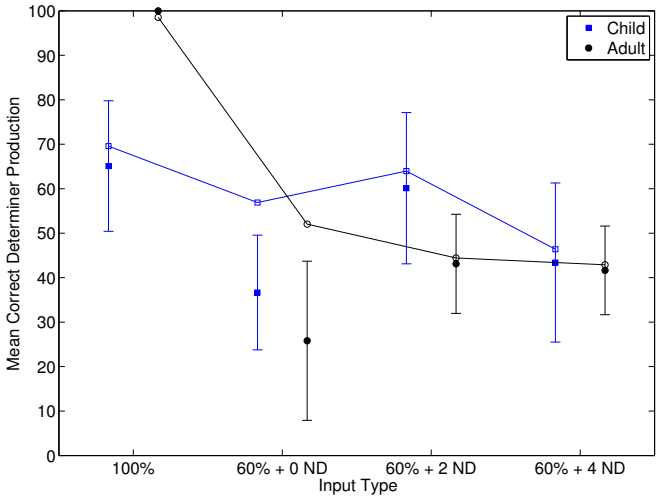


Figure 1.10: Best fit for the children. The line gives the best fit, which corresponds to $s = 1.0$, $p = 0.01$, and $r = 0.52$.

From these arguments, it is clear that in order to be consistent with the data, the parameters in the child learning model must satisfy $s > p$. This corresponds to the circled area in figure 1.8. Figure 1.10 presents the result of the best fit for the children. For that choice of parameters, the learners demonstrate a large degree of consistency in their choices, even though the choice is not always the right one. This underlies the difference between the adult and child behavior in the experiments by Hudson Kam and Newport (2009).

Finally we note that for consistency, the noise parameter was also included in the fitting of the data for the adults, see Appendix A.4. We find that the best fit for the adults occurs with a very small noise parameter, which is consistent with the fact that the adults performed well in the 100% case.

1.4 Discussion

In this paper we created a mathematical model of learning that is able to reproduce the results of Hudson Kam and Newport (2009). The following summarizes our findings.

- The proposed learning algorithm is a reinforcement-type, two-parametric algorithm (the third parameter is used to include noise). Differences in the parameters account for the observed differences in learning patterns between adults and children. In the algorithm, children are characterized by a stronger response to positive evidence, while adults - by a stronger response to negative evidence.

- In our algorithm, both "adults" and "children" can demonstrate frequency-boosting behavior, depending on the structure of the inconsistent input. The strength of frequency boosting increases as the number of alternative forms increases. This is consistent with the data of Hudson Kam and Newport (2009).

- When fitted to the data, the children have a heightened ability to regularize. Most of the child learners become "consistent users," even if their preferred form differs from the most frequent form of the source. Again, this is consistent with the findings of Hudson Kam and Newport (2009).

Language regularization in children manifests itself in a number of ways, for example, children often have difficulties learning exceptions to rules (Marcus et al., 1992). Regularization of

linguistic input by children has been related to overmatching or maximizing (Bever, 1982). It has also been reported that children invent and use their own patterns (Craig and Myers, 1963).

1.4.1 Mechanisms of frequency matching and frequency boosting

The sources and mechanisms of frequency-boosting behavior have been extensively discussed in the literature. The Language Bioprogram Hypothesis (Bickerton, 1984) has been proposed to explain the ease with which children regularize inconsistent linguistic input when exposed to reduced communication systems. It was argued that children utilize some innate language-specific constraints that contribute to the process of language acquisition (DeGraff, 2001). Overregularization in children learning grammatical rules has been explained by means of a dual-mechanism model (Marcus, 1995), or an alternative connectionist model (Marchman et al., 1997; Plunkett and Juola, 1999). In Wonnacott et al. (2008); Wonnacott (2011) it is proposed that both children and adult learners use distributional statistics to make inferences about when generalization is appropriate. The "less-is-more" hypothesis of Newport (1990) suggests that the differences in adults' and children's language learning abilities can be ascribed to children's limited cognitive capacities. In Hudson Kam and Newport (2009), the less-is-more hypothesis is used to explain the children's remarkable ability to regularize.

Interestingly, the tendency of children to "maximize" has been observed in non-linguistic contexts (see Derks and Paclisanu (1967)), where participants were asked to guess which hand a candy is in, when the two hands contained candy at the 25:75 ratio. It was observed that young chidren frequency boosted, and frequency matching behavior began to emerge after the age of 4, becoing stronger in older participants. In Thompson-Schill et al. (2009); Ramscar and Gitcho (2007) it has been suggested that, while both adults and children have a natural tendency to regularize, the adults use their well-developed prefrontal-cortex-

mediated control system to override this. Adults implement their highly efficient machinery for cognitive control and conflict processing, which has evolved for performance. In learning, however, this may be considered an impediment as it makes regularization harder.

In this paper we explore this phenomenon further and propose an additional possible explanation for the differences between adults' and children's abilities to generalize. This explanation is rooted in the fundamentally different way by which adults and children deal with negative feedback. In van Leijenhorst et al. (2006), it is suggested that while children and adults recruit similar brain regions during risk-estimation and feedback processing, there are some key differences between the age groups. For example, it appears that children's decision-making under uncertainty is associated with a high degree of response conflict, and further, children may find negative feedback more aversive than adults do. The former factor was proposed to be responsible for the differences in learning patterns by Ramscar and Gitcho (2007). Here we concentrate on the latter factor, the response to negative feedback, and propose that it may be related to the observed differences in the regularization behavior exhibited by adults and children.

By comparing simulations with data from Hudson Kam and Newport (2009), we found that the best fitting reinforcement models were not symmetric with respect to the learning increments. That is, the increments following the source's utterance must be different depending on whether or not the source's utterance coincides with the learner's most frequent ("preferred") form. For the child's best fitting model, the increment following the utterance that coincides with the learner's most frequent form is the largest. For adults, it is the smallest. We hypothesize that this is related to the two forms of feedback, positive and negative.

1.4.2 Negative feedback

The learning algorithm proposed in this paper is characterized by two parameters, s and p, see figure 1.4. The value of s is the amount of change in the state of the learner following the source's input if it coincides with the learner's present preferred hypothesis. The value of p is the amount of change following the source's input if it differs from the learner's most preferred form. Here we argue that one can regard the value s as the reaction of the learner to positive feedback, and p as the reaction to negative feedback.

The notion of "negative evidence" or "negative feedback" is often defined as "information about which structures are not allowed by the grammar" (Marcus, 1993; Seidenberg, 1997; Leeman, 2003). In Chouinard and Clark (2003), negative evidence is specifically "information that identifies children's errors as errors during acquisition." Most generally, negative evidence is defined as "feedback that involves an incorrect form" (see e.g. Kang (2010), and also in the context of second language acquisition (Doughty, 2003)). The first two definitions are not applicable in the context of the language learning experiments of Hudson Kam and Newport (2009). Even the latter, most general definition, cannot be used directly in the context of learning from an inconsistent source. According to the most general definition above, any input involving an incorrect form is negative evidence. In our setting however, the source itself is highly probabilistic, and therefore we cannot regard the source's utterances of the less frequent form as "negative feedback."

In the context of learning from an inconsistent source, it makes sense to define negative feedback as **the source's utterances that do not coincide with the learner's "idea" of the correct form**. A similar notion of implicit negative evidence has been proposed in the literature, see e.g. Marcotte (2004). In Rohde and Plaut (1999) it is suggested that "... one way the statistical structure of a language can be approximated is through the formulation and testing of implicit predictions. By comparing one's predictions to what actually occurs,

feedback is immediate and negative evidence derives from incorrect predictions." In the illustration presented in figure 1.4, the learner's preferred form is form 1 (because $x_1 > x_2$). Thus negative feedback corresponds to the source's usage of form 2.

Here we postulate that the source's input that does not coincide with the learner's own hypothesis can be considered negative feedback. The applicability of this definition is similar to considering "recasts" a form of negative feedback (Nicholas et al., 2001). In first language (L1) acquisition, recasts do not occur universally, but they have been observed in Western middle class culture, when an adult understands a child perfectly well, but chooses to reformulate the child's utterance into a more adult-like form nevertheless (Tomasello, 1992). In L1 research it has been proposed (Saxton, 1997) that this type of feedback leads to the perception of a contrast between the original form and the adult form, which then may facilitate the child's eventual rejection of the incorrect form. Recasts are common in second language (L2) classroom situations. In L2 research, it has been proposed that recasts help focus the learner's attention on the form (as both forms convey the same meaning), and have been considered an effective learning tool (Schmidt, 1990; Long and Robinson, 1998; Doughty, 2001).

Although some researchers have casted doubt about the usefulness of recasts as corrective feedback, or assumed that they simply provide positive evidence (Schwartz, 1993), much of the literature classifies recasts as "implicit negative feedback" (Long and Robinson, 1998; Long et al., 1998; Nicholas et al., 2001). They are thought to provide negative evidence because they expose the gap between the learner's form and the source's form, by juxtaposing target and non-target forms (Long, 2006; Adams et al., 2011). The learner compares her own original form with the source's input and then realizes that her language differs from the target language. This process is called "cognitive comparison" (Nelson, 1987). The comparison may signal that the original utterance was erroneous, thus providing the learner with implicit negative evidence, and triggering cognitive processes that may lead to the

restructuring of the learner's language (Nassaji, 2013).

1.4.3 Negative feedback in adults and children

Differences in children's and adults' processing of positive and negative evidence in language acquisition have been widely discussed in the literature. It is recognized that children preferentially utilize positive evidence while adults are relatively more successful in their processing of negative evidence than children, see (Pinker, 1989; Birdsong, 1989; Carroll and Swain, 1993) for language acquisition literature, and (Crone et al., 2004, 2008; Huizinga et al., 2006) in a more general context. In the paper by Van Duijvenvoorde et al. (2008), the authors study the neural mechanisms involved in the reaction of adults and children to positive and negative feedback. In the experiment, the participants were shown pictures and they had to respond whether or not the pictures followed a rule by clicking one of two buttons. If they were correct, a plus sign was shown (to represent positive feedback) and if they were incorrect, an "x" was shown (to represent negative feedback). The trials were conducted in pairs. First came the "guess trial," where the participant did not know the rule. Next came the "repetition trial," where the participants used the feedback from the first trial to inform their response. The authors analyzed the participants' MRI brain scans obtained in the course of the experiment.

The authors looked at three different age groups: 8-9 years, 11-13 years, and 18-25 years. Looking at the fMRI images, they found that there was activation in specific regions of the brain. This activation was larger after negative feedback than after positive feedback for the 18-25 years old group. However, this was reversed for the young children (8-9 years old), who had a larger activation after positive feedback. For the 11-13 years old group, the activation amount was about the same for the positive and negative feedback. The authors concluded that the young children had a harder time learning from negative feedback than the adults.

The adults were better able to process the negative feedback.

By matching our learning model with the data in Hudson Kam and Newport (2009), we found that the best fit for the children corresponded to the learning algorithm where p was very small and s was larger, illustrated in figure 1.4(b) (the best fit overall was for $s = 1.0$ and $p = 0.01$). This is consistent with the finding that children have a harder time processing negative feedback. Receiving evidence that the current hypothesis is correct (which happens when the source's input coincides with the learner's preferred form) evokes a strong positive response, making the preference for that form even stronger. On the other hand, receiving a piece of "negative" evidence (when the source's input does not coincide with the learner's preferred form) is followed by a weaker response.

On the other hand, the best fit for the adults in our model occurred when $s = 0.19$ and $p = 0.50$ (illustrated schematically in figure 1.4(c)). The adults are quick to weaken their current hypothesis in response to the negative evidence. This is consistent with the fact that the regions of the adult groups' brains were activated more in reaction to negative feedback (Van Duijvenvoorde et al., 2008).

Therefore we propose that a possible mechanism that contributes to the children's ability to "frequency boost" and regularize an inconsistent input is related to their heightened sensitivity to positive evidence rather than the (implicit) negative evidence, in the sense defined here. In our model, regularization comes naturally as a consequence of a stronger reaction of the children to evidence supporting their preferred hypothesis. For adults, their ability to adequately process implicit negative evidence prevents them from regularizing the inconsistent input, resulting in a weaker degree of regularization.

Chapter 2

Restructuring of Languages by Learners: A Mathematical Framework

2.1 Introduction

One way that languages get shaped is for efficient communication. In fact, many languages have unexpected properties that make communication more efficient (see, for example, Zipf (1949), Aylett and Turk (2004), Florian Jaeger (2010), Piantadosi et al. (2011), Qian and Jaeger (2012), Van Son and Van Santen (2005), Piantadosi et al. (2012), Manin (2006), andMaurits et al. (2010)). However, how these properties come about is not known.

One idea is that this shaping can happen over time, where adults might subtly change the input for the next generation (Bates and MacWhinney (1982)). Another idea is that can also happen as we learn, with learners changing the input they receive during language acquisition (Florian Jaeger (2010)). In, Fedzechkina et al. (2012), Elissa Newport and colleagues look at the later case. They consider a miniature artificial language that does not have efficient case marking, and they want to see if the learners will make the language more efficient by

restructure the language as they learn it.

Consider the following two sentences: “The man the wall hit” and “The man the woman hit.” Each sentence has a subject, an object, and a verb, but what is the subject of each sentence, and what is the object? The meaning of the first sentence is more clear. “The man” is the object and “the wall” is the subject. However, in the second sentence, this is not so clear. Did the man hit the woman, or did the woman hit the man? This is an example of where the addition of case marking would make the meaning of the sentence clear. Case marking is the addition of marker in nouns to indicate which noun is the subject and which noun is the object.

In Fedzechkina et al. (2012), the authors consider differential case marking systems (see, for example, Mohanan (1994) and Aissen (2003)). In language that use differential case marking systems, sentiences with inanimate subjects and animate objects are always case-marked (since this combination is not typical). More typical combinations (such as animate subjects and inanimate objects) are not marked.

2.2 Materials and Methods

2.2.1 The Language Structure of Experiment 1

In Fedzechkina et al. (2012), learners are taught an artificial language with inefficient case marking. The artificial language consists of simple sentences with a subject, an object, and a verb. In the first experiment, the subjects are always animate, and the objects can be animate or inanimate. The order of the sentences can be Subject-Object-Verb (SOV) or Object-Subject-Verb (OSV). The sentences are harder to understand when both the subject and the object are animate. They are also harder to understand when the OSV form is used.

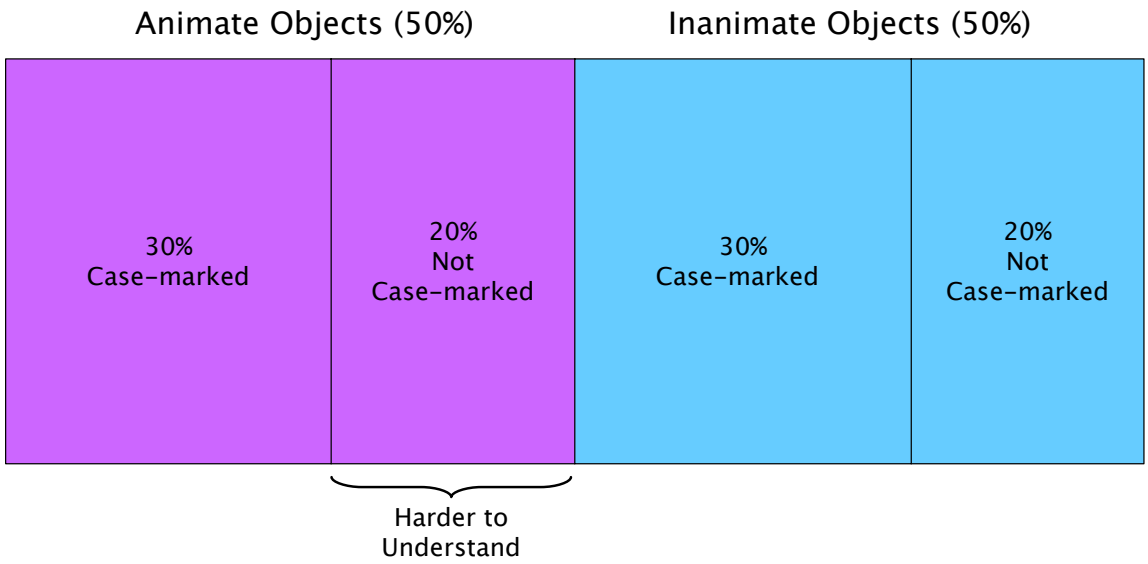


Figure 2.1: The breakdown of the sentences in terms of their object: either animate or inanimate.

The language is broken down in two ways: (i) 50% of the sentences have an animate object and 50% of the sentences have an inanimate object, and (ii) 60% of the sentences are SOV and 40% of the sentences are OSV. The objects are case-marked 60% of the time, with both animate and inanimate objects being equally likely to be case-marked. Figures 2.1 and 2.2 diagram the two ways that the language is broken up.

The language has four types of sentence: (i) animate and SOV, (ii) animate and OSV, (iii) inanimate and SOV, and (iv) inanimate and OSV. Each of these sentences will either be case-marked or not. The way the language is presented, the case marking is not efficient. A sentence that is harder to understand, such as an animate, OSV sentence might not be case-marked, while an easier to understand sentence (inanimate, SOV, for example) is case-marked.

The experiment went on for 4 days and had 20 people participating. Each day, they were taught 80 sentences, and then tested, with the exception that they were not tested at the end of the first day. Therefore, the paper only has data on the proportion of object case-marking

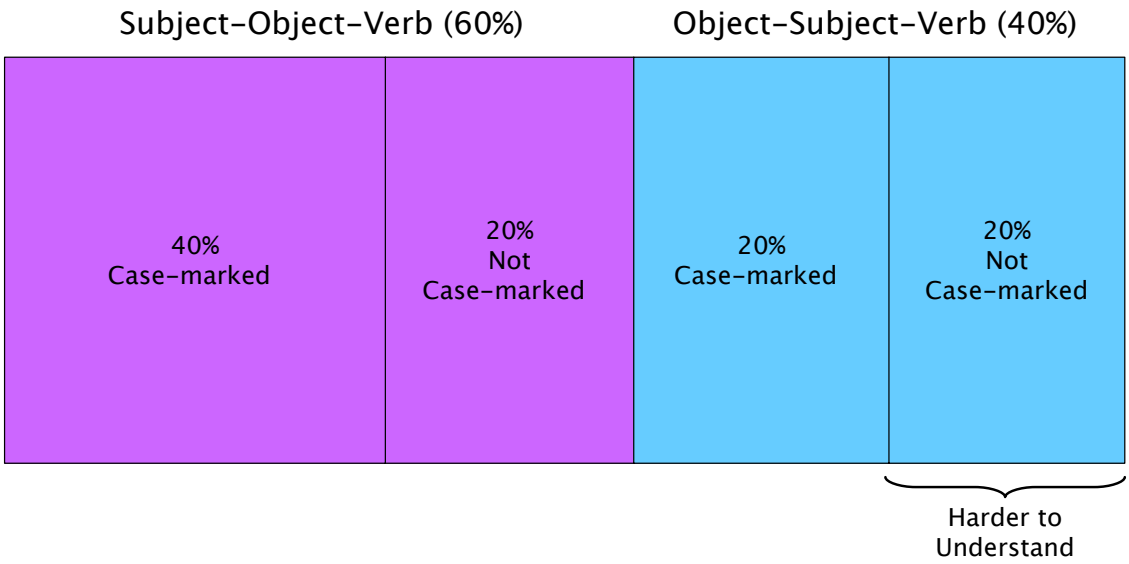


Figure 2.2: The breakdown of the sentences in terms of their word order: either subject-object-verb (SOV) or object-subject-verb (OSV).

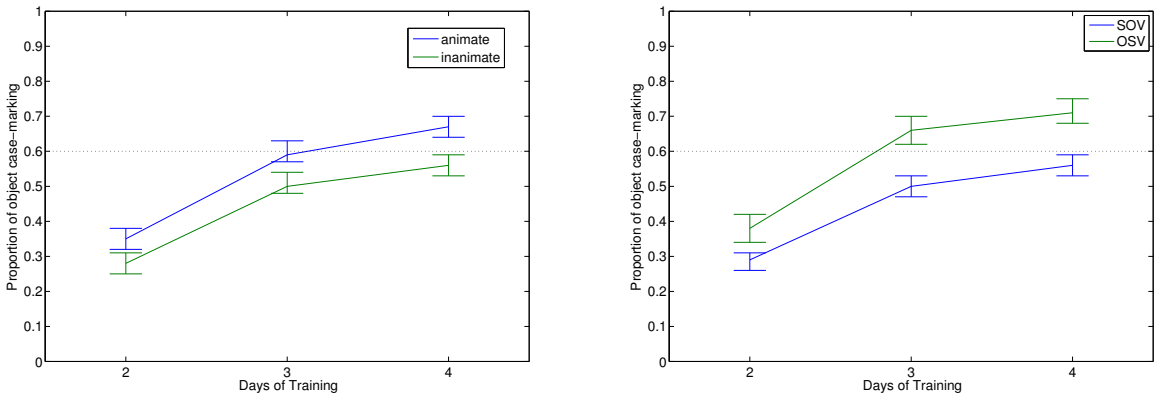
from days 2 through 4.

When the objects are animate, the sentences are harder to understand, so we expect the learners to case-mark the animate objects more than the inanimate objects, which they do (see figure 2.3a). Also, since the OSV sentences are harder to understand, we expect that the learners will case-mark the objects more often in these than they do in the SOV sentences. As we can see in figure 2.3b, they do.

2.2.2 The Language Structure of Experiment 2

In the second experiment, the input language is the complement of the language in the first experiment. Now the objects are always inanimate, and the subjects can be either animate or inanimate (50% of the subjects are animate and 50% are inanimate). The other aspects of the language were all the same.

Newport and her colleagues were testing two things in the second experiment. First, they



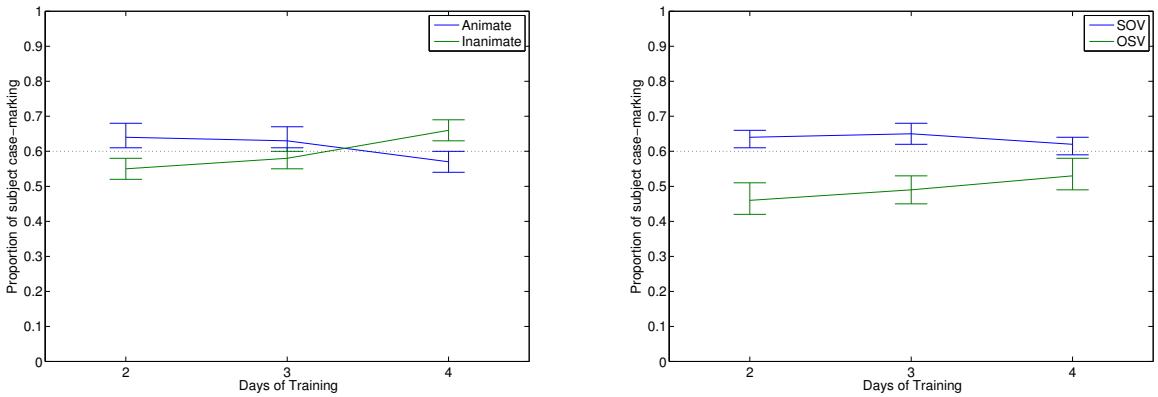
(a) Animate versus Inanimate

(b) SOV versus OSV

Figure 2.3: The results from experiment 1 in Fedzechkina et al. (2012). The black dashed line give the frequency of case marking of the source.

wanted to know if, in the first experiment, the preference to case-mark animate objects came from a bias to case-mark the atypical (if this was the case, then in the second experiment, they would expect to see the inanimate subjects being case-marked more often) or if it came from a bias toward animate things (if this was the case, then they would expect to see the animate sentences case-marked the most in both experiments). They found that both biases were at play. At first, the learners case-marked animate sentences more often, but by day three, they were case-marking the inanimate sentences more (see figure 2.4a).

With regards to the SOV and OSV sentences, the authors wanted to see if the bias was to case-mark to avoid miscommunication (if this was the case, then we should see OSV sentences case-marked more often again) or if it was "a bias to mention disambiguating information as early as possible in the sentence" (if this was the case, then the SOV should be case-marked more often). Again, they find that both biases seemed to be at play. SOV sentences are case-marked more often than OSV, but by day 4, the rate of case-marking SOV sentences was decreasing, while the rate of case-marking OSV sentences was increasing (see figure 2.4b).



(a) Animate versus Inanimate

(b) SOV versus OSV

Figure 2.4: The results from experiment 2 in Fedzechkina et al. (2012). The black dashed line give the frequency of case marking of the source.

2.2.3 Memory

In our model, we will keep track of the learners' frequencies to case-mark the four types of sentences as they learn each day. We would like to include discounted learning into our model to simulate the learners not remembering everything that they learned from the day before. At the beginning of day 2, the learners' frequencies to case-mark the four types of sentences will be multiplied by d_1, where d_1 will represent the percent that they remember from their first day of learning. Starting with day 3, we will instead multiply their frequencies by d_2, where d_2 represents the amount that they remember from the day before (after they have already been learning for at least 2 days).

The reasoning behind two memory parameter is that after only one day of learning this artificial language with case marking (with which the english speaking learners are not familiar with), the learners will not remember much of what they learned. However, after learning for two days, they amount that they remember will increase.

We can also justify having two parameters because the learners are not tested at the end day 1, but they are tested at the end of each subsequent day. In Spitzer (1939) the author gave

reading assignments to various groups of 6th graders, and then tested their retention after various time intervals. The students were tested on the material a total of three times over a 63 day period. Some students were tested right after they read the article, and then again some time later, while others were not tested right away. The students who were tested right away did better on the subsequent tests than the other students. They found that "... more is forgotten in one day without recall than is forgotten in sixty-three days with the aid of recall."

2.2.4 The Learning Algorithm

Suppose each individual is characterized by probabilities

$$P_{an,so}, \quad P_{an,os}, \quad P_{in,so}, \quad P_{in,os},$$

where $P_{an,so}$ is the probability of the learner to case-mark an animate, SOV sentence; $P_{an,os}$ is the probability of the learner to case-mark an animate, OSV sentence; $P_{in,so}$ is the probability of the learner to case-mark an inanimate, SOV sentence; and $P_{in,os}$ is the probability of the learner to case-mark an inanimate, OSV sentence. These are numbers between 0 and 1. In the data, these are not measured. Instead, the quantities P_{an}, P_{in}, P_{so}, and P_{os} were presented. These are the probabilities to case-mark animate, inanimate, SOV, and OSV sentences, respectively. Given the breakdown of the sentences (see figures 2.1 and 2.2), they are connected with our variables as follows:

$$P_{an} = 3/5P_{an,so} + 2/5P_{an,os}, \quad P_{in} = 3/5P_{in,so} + 2/5P_{in,os}, \tag{2.1}$$

$$P_{so} = 1/2P_{an,so} + 1/2P_{in,so}, \quad P_{os} = 1/2P_{an,os} + 1/2P_{in,os}. \tag{2.2}$$

Note that the latter quantity is a linear combination of the first three, because

$$1/2P_{an} + 1/2P_{in} = 3/5P_{so} + 2/5P_{os}.$$

An input string can be of 8 kinds: it is characterized by a pair such as an, so, and whether the sentence is marked or unmarked. The updates happen in the following way. Suppose a marked an, so sentence is received. Then, we have

$$P_{an,so} \rightarrow \begin{cases} P_{an,so} + \Delta^{+}_{an,so}, & P_{an,so} > 1/2, \\ P_{an,so} + \Delta^{-}_{an,so} & P_{an,so} < 1/2. \end{cases}$$

If we receive an unmarked an, so sentence, then we have

$$P_{an,so} \rightarrow \begin{cases} P_{an,so} - \Delta^{-}_{an,so}, & P_{an,so} > 1/2, \\ P_{an,so} - \Delta^{+}_{an,so} & P_{an,so} < 1/2. \end{cases}$$

This is a variation of our model (1.9-1.10) from Chapter 1. Again, we have two different increments that are used depending on whether or not the learner's internal hypothesis matches the source. For example, if the source gives a case-marked an, so sentence, and the learner prefers to case-mark ($P_{an,so} > 1/2$), we update with the "+" increment. If the sentence from the source does not match the learner's hypothesis, we use the "−" increment. (See Appendix B.1 for an alternate variation on this algorithm that also models the date from Fedzechkina et al. (2012).)

In the simplest case, this is the only update that takes place. However, even in this simplest case with only one sentence type updating, we have 8 variables, not including the memory variables. To simplify, one could imagine that some of the variables are equal to each other.

The sentences in the experiment differ by the degree of ambiguity. Denoting "ambiguous"

by A and "unambiguous" by U, we can classify each sentence as type AA, AU, UA, or UU. In the first experiment, animate and OSV are ambiguous, so an, so sentences are classified as AU, as, os sentences are AA, in, so sentences are UU, and in, os sentences are UA. In the second experiment, inanimate and OSV are ambiguous, so an, so sentences are classified as UU, as, os sentences are UA, in, so sentences are AU, and in, os sentences are AA.

Let us consider three increment types: Δ_{AA}, Δ_{AU}, and Δ_{UU} (with each of they types having a "+" and "−" increment). Δ^+_{AA} and Δ^-_{AA} will correspond to AA sentences, Δ^+_{AU} and Δ^-_{AU} will correspond to AU and UA sentences, and Δ^+_{UU} and Δ^-_{UU} will correspond to UU sentences. This gives us six increment parameters and two memory parameters, for a total of eight parameters. We can reduce the number of parameter further by assuming that

$$\Delta^+_{AU} = \frac{\Delta^+_{AA} + \Delta^+_{UU}}{2} \quad \text{and} \quad \Delta^-_{AU} = \frac{\Delta^-_{AA} + \Delta^-_{UU}}{2}.$$

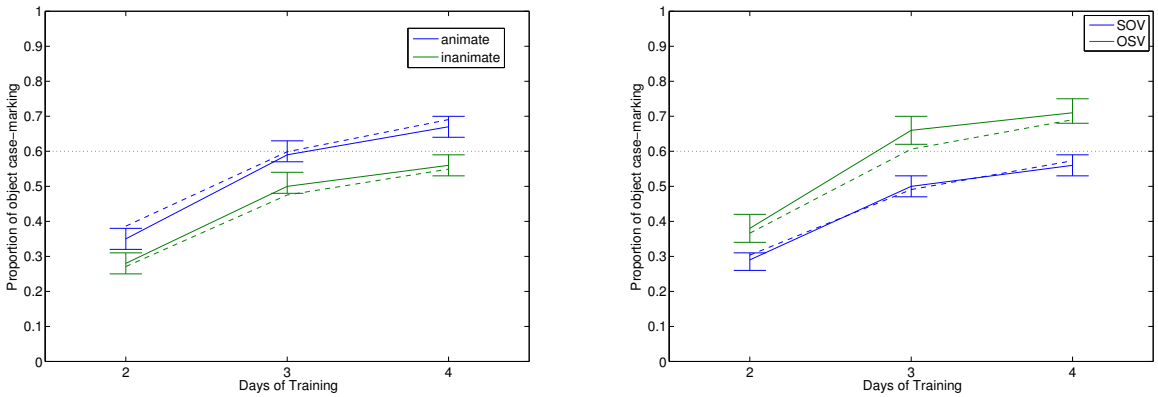
This leaves us with a total of six parameters: $\Delta^+_{AA}, \Delta^-_{AA}, \Delta^+_{UU}, \Delta^-_{UU}, d_1$, and d_2.

As in the experiments in Fedzechkina et al. (2012), we run the simulations for four days, and the learners hear 80 sentences each day. We run the simulations for 100 learners while varying the values of the parameters. Using least squares average, we find the parameters that give us the best fit to the data in Fedzechkina et al. (2012).

2.3 Results

For experiment 1, the best fit occurs when $\Delta^+_{AA} = 0.080, \Delta^-_{AA} = 0.089, \Delta^+_{UU} = 0.001, \Delta^-_{UU} = 0.017, \Delta^+_{AU} = 0.0405, \Delta^+_{AU} = 0.0530, d_1 = 0.01$, and $d_2 = 0.97$ (see figure 2.5).

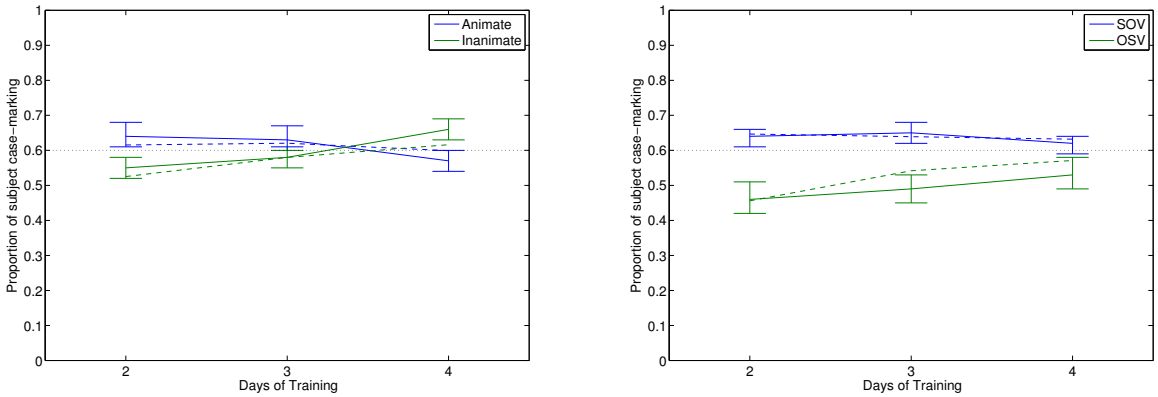
For experiment 2, the best fit occurs when $\Delta^+_{AA} = 0.013, \Delta^-_{AA} = 0.049, \Delta^+_{UU} = 0.100, \Delta^-_{UU} = 0.200, \Delta^+_{AU} = 0.0565, \Delta^+_{AU} = 0.1245, d_1 = 0.10$, and $d_2 = 1.00$ (see figure 2.6).



(a) Animate versus Inanimate

(b) SOV versus OSV

Figure 2.5: The best fit for experiment 1. It occurs when $\Delta^+_{AA} = 0.080, \Delta^-_{AA} = 0.089, \Delta^+_{UU} = 0.001, \Delta^-_{UU} = 0.017, \Delta^+_{AU} = 0.0405, \Delta^+_{AU} = 0.0530, d_1 = 0.01$, and $d_2 = 0.97$.



(a) Animate versus Inanimate

(b) SOV versus OSV

Figure 2.6: The best fit for experiment 2. It occurs when $\Delta^+_{AA} = 0.013, \Delta^-_{AA} = 0.049, \Delta^+_{UU} = 0.100, \Delta^-_{UU} = 0.200, \Delta^+_{AU} = 0.0565, \Delta^+_{AU} = 0.1245, d_1 = 0.10$, and $d_2 = 1.00$.

In both experiments, we see that the "+" increments are smaller than the "−" increments. This agrees with our findings in Chapter 1. The adults adjust more after the inherent negative feedback that they receive when the source's sentence does not match their internal hypothesis. Also in both experiments, $d_2 > d_1$, which agrees with the findings of Spitzer (1939), who showed that we remember more of what we learn if we are tested on it right away. We also see that in the first experiment, $\Delta_{AA} > \Delta_{UU}$. However, in the second experiment, the reverse happens, and $\Delta_{UU} > \Delta_{AA}$.

2.4 Discussion

During experiment 1, Fedzechkina et al. (2012) found that the learners did restructure the language for better communication by case-marking the ambiguous (animate and OSV) sentences more often than the unambiguous (inanimate and SOV) sentences. Although the sentences were broken down into animate versus inanimate and SOV versus OSV, in the experiment each sentence was both either animate or inanimate, and either SOV or OSV. This means that the learners could hear four types of sentences: an, so, an, os, in, so, and in, os.

We categorized these according to their ambiguity, using A for ambiguous and U for unambiguous. Therefore, an, os sentences are characterized as AA, in, so sentences are characterized as UU, and an, so and in, os sentences are characterized as AU and UA, respectively. With this characterization of the sentences, we have three increment types (with a "+" and "−" increment for each type). These are Δ_{AA}, Δ_{AU}, and Δ_{UU}, where $\Delta_{AU} = \dfrac{\Delta_{AA} + \Delta_{UU}}{2}$.

In our best fit for experiment 1, we found that $\Delta_{AA} > \Delta_{UU}$. This means that the learners adjust more when they hear an ambiguous sentence than when they hear an unambiguous sentence. This agrees with the finding of the experiment.

In experiment 2, we still categorize the sentences as AA, AU, UA, and UU. However, the sentences that correspond to each of these categories is different than in the first experiment. In the second experiment, the sentences with inanimate subjects are now the ambiguous sentences, with OSV being ambiguous in both experiments. So now, AA corresponds to in, os, UU corresponds to an, so, and AU/UA corresponds to in, so and an, os.

However, Fedzechkina et al. (2012) were looking a few things in the second experiment. They wanted to know whether, in the first experiment, the learners case-marked animate sentences more often because they were ambiguous, or because the learners had a preference

for animate things. If it was the former, then we would expect them to case-mark inanimate sentences more in the second experiment (since now the inanimate sentence are ambiguous). If it was the latter, then we would expect them to case-mark animate sentences more often again (even though these are now unambiguous). As seen in figure 2.4a, the learner do case-mark animate sentence more at first, but they begin to case-mark inanimate sentences more often on day 4. In our results, we found that the best fit occurs when $\Delta_{UU} > \Delta_{AA}$. This indicates that the learner's preference towards animate sentences is stronger than their preference to case-mark for clarity.

When comparing SOV and OSV case-marking in the second experiment, Fedzechkina et al. (2012) were also looking at two factors. First, they were looking to see if there was a preference towards case-marking the more ambiguous sentence (OSV), or if there was a preference to case-mark right away. In the first experiment, the objects were case-marked and the OSV sentences were case-marked more often. In the second experiment, the subjects were case-marked, so they wanted to see if the SOV sentences be case-marked more often even though they were less ambiguous. They found that the SOV sentences were indeed case-marked more often, although by the end of day 4, it appeared that the proportion of OSV case-marking was increasing. Again, our finding that $\Delta_{UU} > \Delta_{AA}$ supports this. The learner's preference to case-mark early in the sentence is stronger than their preference to case-mark the more ambiguous sentences.

Given the learner's biases towards animate sentences and case-marking early in the sentence (and thus towards SOV sentences) in the second experiment, the learners adjust more for the unambiguous sentences. This is because, in the second experiment, the unambiguous sentences are the sentences that they are more biased towards.

We have also seen that our asymmetric learning algorithm from Chapter 1 (model 1.9-1.10) is versatile. We are able to use it to reproduce the results of Hudson Kam and Newport (2005), Hudson Kam and Newport (2009) and Fedzechkina et al. (2012). We find that in all

the experiments, the adults' increments are larger when their internal hypothesis does not match the source. The adults adjust more from the inherent negative feedback they receive in this case.

Chapter 3

Language as a Genetic Mutation

3.1 Introduction

We will now shift our focus to language modeled at the level of a population by evolutionary methods. For example, Nowak and Komarova (2001) construct a mathematical framework to develop an evolutionary theory of language. Natural selection is integrated through a "reward" for successful communication. Using the concept of biological fitness, when individuals are able to communicate, they receive a "payoff." This comes in the form of higher reproductive success. Those that can communicate have a better chance at survival and will therefore have more offspring. Evolutionary theory can show how arbitrary signals become associated with specific referents, words are formed, syntax is developed, and grammars evolve.

We would like to consider what happens when language first develops in a society. We will look at a simplified case, where we have a group of individuals without language, and we will suppose that language is a trait that appears though genetic mutation. If the offspring of individuals without language have this mutation, they will be able to speak. We would

like to see how language will spread through the population, and then find conditions that enable it to spread more quickly.

In particular, we would like to see how the ability to communicate with others will help the individuals with language spread through the population. By being able to talk, our individuals with language will be able to cooperate with each other. "Cooperation in hunting, making plans, coordinating activities, task sharing, social bonding, manipulation and deception all benefit from an increase in expressive power," (Nowak and Komarova (2001)).

3.2 Materials and Methods

We use a spatial model, where the group of individuals are set up on a grid. We look at one- and two-dimensional grids. The one-dimensional grids consist of 625 spots in a line. For the two-dimensional grids, we consider 25x25 grids and 50x50 grids. When we start, the grid is randomly half filled with individuals without language. The rest of the spots on the grid are empty. At each time step of the program, we randomly pick a spot on the grid. If the spot is empty, we do nothing and move on to the next time step. If the spot is filled, then the occupant has the opportunity to reproduce with probability l. They will reproduce only if there is an empty spot near them for their offspring to go. Then they will die with probability d. Finally, if the individual does not die, they has the opportunity to move (with probability m) to an open spot near them (if there is an open spot).

3.2.1 Jump Radius

To define a spot to be near a given spot, we consider all the spots within a set "jump radius" of our given spot. For a one-dimensional grid, if we look at all the spots within jump radius r of our given spot, then we are looking at the r spots to the left and the r spots to the

right of our given spot (see figure 3.1a). For a two-dimensional grid, we look at the square of spots around our given spot. If the jump radius is 1, then this is the 8 spots around our given spot (see figure 3.1b). Jump Radius 2 gives the square of spots around jump radius 1, and so on. For both the one- and two-dimensional grids, a jump radius of infinity is the entire grid. We consider our boundaries as being geographic barriers (see figure 3.2).

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(a) One-dimensional grid.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(b) Two-dimensional grid.

Figure 3.1: An example of our chosen spot (in red) and the spots within jump radius 1 of it (in blue).

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a) One-dimensional grid.

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(b) Two-dimensional grid.

Figure 3.2: An example when our our chosen spot (in red) is near the edge of the grid. The spots within jump radius 1 are in blue.

We use this jump radius for both movement and reproduction. Before an individual reproduces or moves we consider all open spots within the given jump radius of them. We then randomly pick one of those spots for the individual to reproduce or move to. If there are no open spots, then the individual cannot reproduce or move.

3.2.2 Mutations

When an individual without language reproduces, their offspring will have a q probability of also not having language. Therefore, there is a $1 - q$ probability of the offspring having language. When $q = 1$, there are no mutations. When we consider cases without mutations, in order to have individuals with language, we start with one individual with language at the center of the grid.

3.2.3 Reproduction Rates

We have two reproduction rates in this model, l and $L > l$. All the individuals without language will reproduce with the smaller rate l. When the individuals with language have an advantage, they will reproduce with the larger rate L.

3.2.3.1 Talking

We will consider two cases. In the first case, all individuals with language will have an advantage, and their reproduction rate will be L. We will also consider a case where the individuals with language are only advantaged if they can communicate with other individuals with language (if there is no one near them to talk to, the ability to talk is not helpful). We say that two individuals with language can talk when they are "near" (i.e. within jump radius 1) of each other. In this case, when an individual with language is selected to reproduce, they will reproduce with probability l (the same as those without language) when there is no one with language to "talk" to near them. Where there is someone to talk to, they will reproduce with probability $L > l$. To study how talking will affect the spread of language, we will compare the models with and without talking.

3.2.4 Time to Invasion

We define the time to invasion as the number of time steps it takes for the individuals with language to "invade" the grid. We define "invasion" to be when the individuals without language to die out completely. We are careful to make sure that the individuals with language have taken over the grid (if everyone has died out, it is not an invasion).

With mutation, as long as everyone does not die out, if the individuals with language have a higher reproduction rate, they will eventually invade. If $d \geq 0.5$ or if $d \geq l$, the population will die out eventually, however, when $l > 2d$, the population does not die out.

The reason that language will always invade is because language individuals will keep appearing on the grid through mutation. Thus even if the language individuals die out initially, more will appear on the grid eventually. When the individuals with language have a higher reproduction rate, their number will eventually start to grow. Therefore, we want to look at how the time to invasion changes as we vary the jump radius.

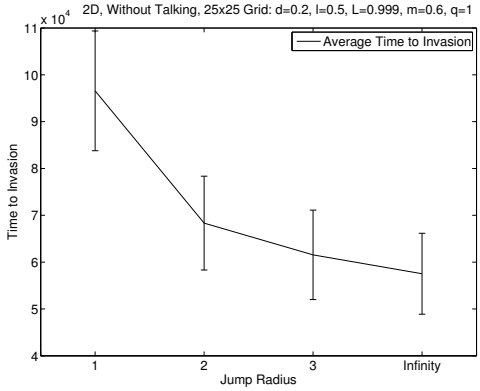
If we consider runs without mutations, then we start with a language individual on the grid. It is possible for the language individuals to die out in this case (and without mutations, they will not reappear), so if they do die out, we throw out that run and start again.

We run our simulation 1000 times, noting the time to invasion at the end of each run. We then find the mean and standard deviation of the times to invasion. We will consider jump radius 1, 2, 3, and infinity. We would like to see how the time to invasion varies for the different jump radii with and without talking. In all the results to follow, $d = 0.2, l = 0.5, L = 0.999$, and $m = 0.6$.

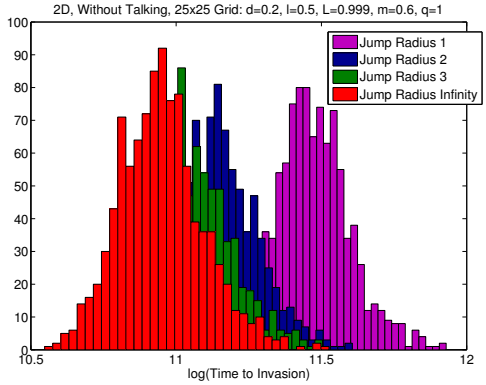
3.3 Results

3.3.1 Two-Dimensional Grid without Talking

In the two-dimensional grid, without talking, we see that a larger jump radius leads to a shorter times to invasion (see figure 3.3 for the 25x25 grid, and figure 3.4 for the 50x50 grid). This is expected, since without talking all the individuals with language have a higher reproduction rate. Therefore, with larger jump radii, there will be more spots available to reproduce into, and therefore more opportunity to reproduce. Therefore, they invade more quickly.

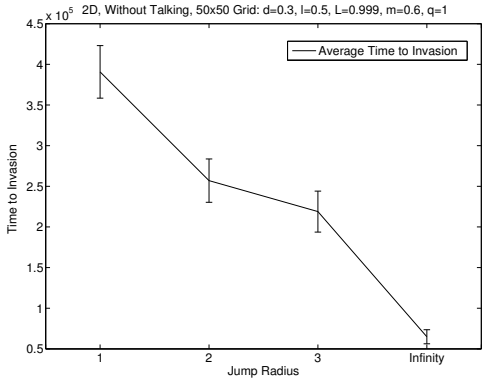


(a) Average time to invasion, plus and minus the standard deviation, for a 2D, 25x25, grid without talking and without mutations.

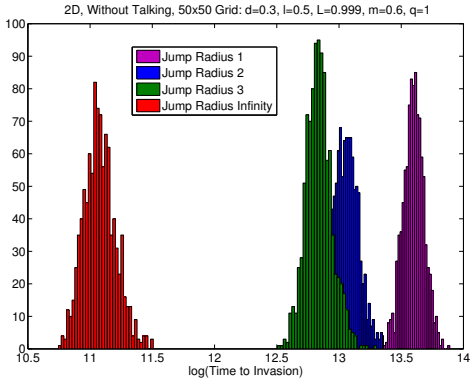


(b) Histograms of the natural log of the times to invasion for each jump radius.

Figure 3.3: 2D, 25x25, grid without talking and without mutations.



(a) Average time to invasion, plus and minus the standard deviation, for a 2D, 50x50, grid without talking and without mutations.



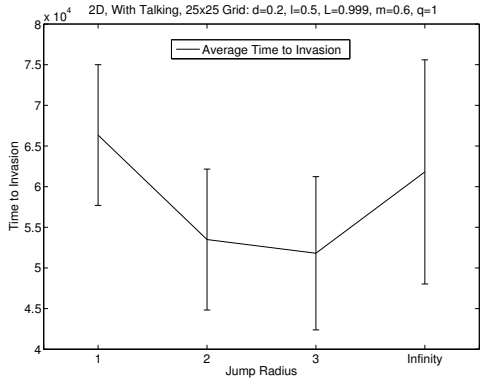
(b) Histograms of the natural log of the times to invasion for each jump radius.

Figure 3.4: 2D, 50x50, grid without talking and without mutations.

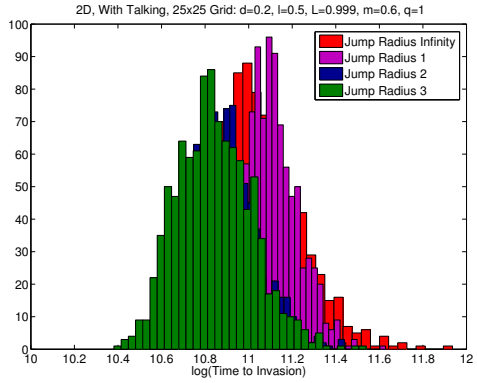
3.3.2 Two-Dimensional Grid with Talking

When we look at the model with talking, we expect to see a different picture. With smaller jump radii, the offspring of individuals with language will be placed close by. Therefore, there is more opportunity for talking to occur, and thus for the language individuals to reproduce at the higher rate. However, with smaller jump radii, the available spots to reproduce into will quickly fill up, leading to less reproduction. When the jump radius is too large, the individuals with language will place their offspring farther away, so that the individuals with language will be too far apart to talk. So although they will be able to reproduce more, they will not be doing so with the higher rate.

We want to find the ideal jump radii such that that the offspring are not placed too far away, but such that there are still more spots available to place their offspring. We can see this "dip" best when we look at jump radii 1, 2, 3, and infinity. This dip is present when we do not have mutations or when we have a very small mutation rate (see figures 3.5 and 3.6 for $q = 1$, and figure 3.7 is $q = .999$). As the mutation rate increases, more individuals with language appear throughout the grid due to mutation. Therefore, even when the offspring of individuals with language are placed farther away, they are still near enough to someone to talk to, and the dip disappears (see figure 3.8 for $q = .99$).

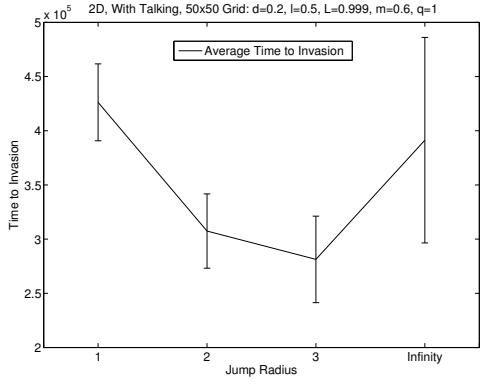


(a) Average time to invasion, plus and minus the standard deviation, for a 2D, 25x25, grid with talking and without mutations.

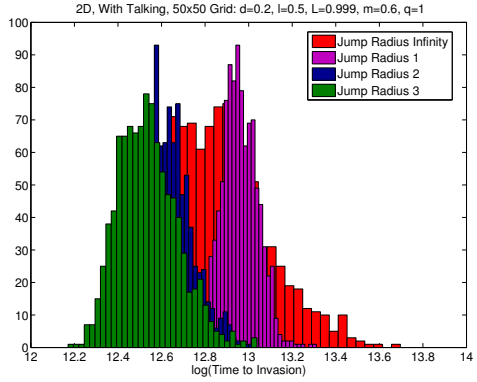


(b) Histograms of the natural log of the times to invasion for each jump radius.

Figure 3.5: 2D, 25x25, grid with talking and without mutations.

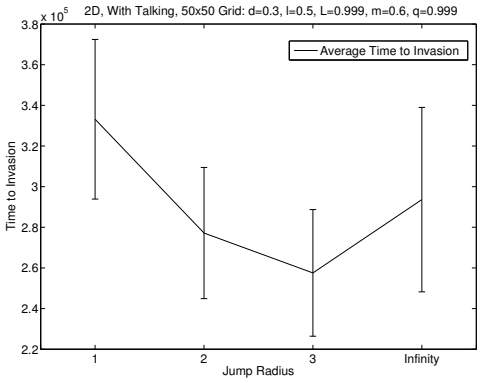


(a) Average time to invasion, plus and minus the standard deviation, for a 2D, 50x50, grid with talking and without mutations.

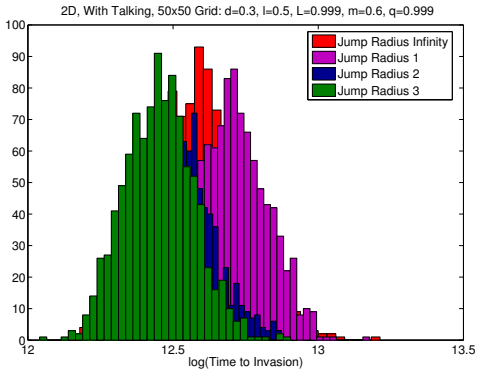


(b) Histograms of the natural log of the times to invasion for each jump radius.

Figure 3.6: 2D, 50x50, grid with talking and without mutations.

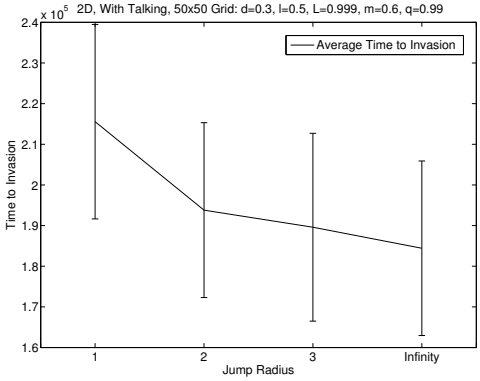


(a) Average time to invasion, plus and minus the standard deviation, for a 2D, 50x50, grid with talking and with mutation rate 0.001.

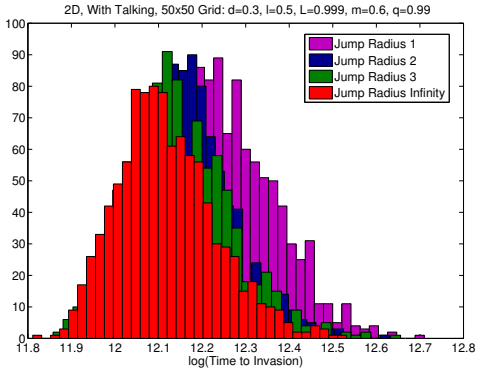


(b) Histograms of the natural log of the times to invasion for each jump radius.

Figure 3.7: 2D, 50x50, grid with talking and with mutation rate 0.001.



(a) Average time to invasion, plus and minus the standard deviation, for a 2D, 50x50, grid with talking and with mutation rate 0.01.

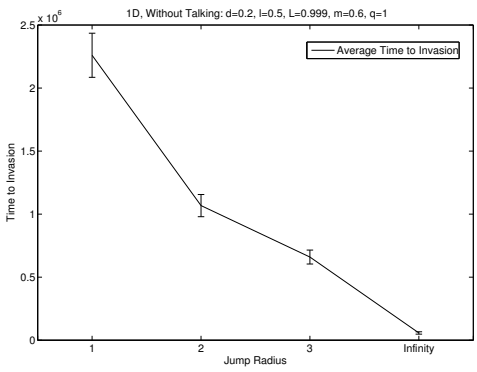


(b) Histograms of the natural log of the times to invasion for each jump radius.

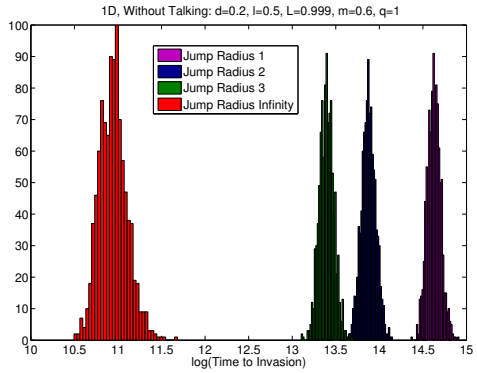
Figure 3.8: 2D, 50x50, grid with talking and with mutation rate 0.01.

3.3.3 One-Dimensional Grid without Talking

In the one-dimension grid, without talking, we see that a higher jump radius leads to a smaller time to invasion, as expected (see figure 3.9). Therefore, without talking the one- and two-dimensional grids behave similarly.



(a) Average time to invasion, plus and minus the standard deviation, for a 1D grid without talking and without mutations.

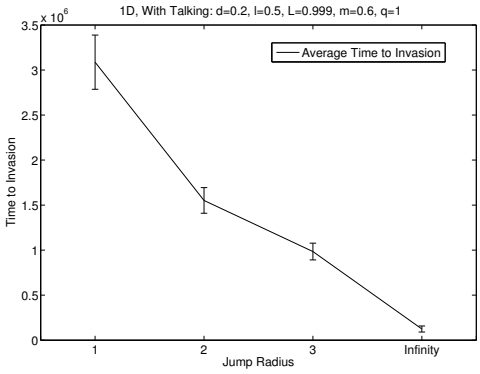


(b) Histograms of the natural log of the times to invasion for each jump radius.

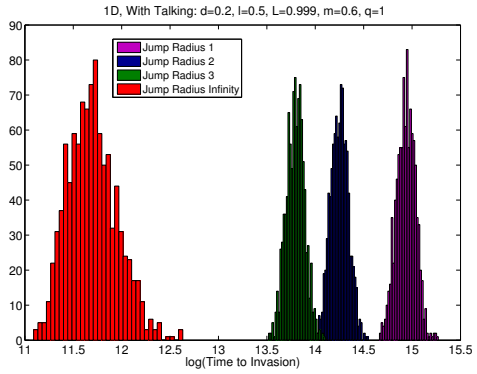
Figure 3.9: 1D grid without talking and without mutations.

3.3.4 One-Dimensional Grid with Talking

When we add talking to the one-dimensional grid, we can see in figure 3.10 that we do not get the same "dip" as we do in the two-dimensional case (see figure 3.6). The overall time to invasion does increase when we add talking, but we see that, even with talking, the time to invasion decreases as we increase the jump radius. We get a similar graph when we add in mutations (see figure 3.11).

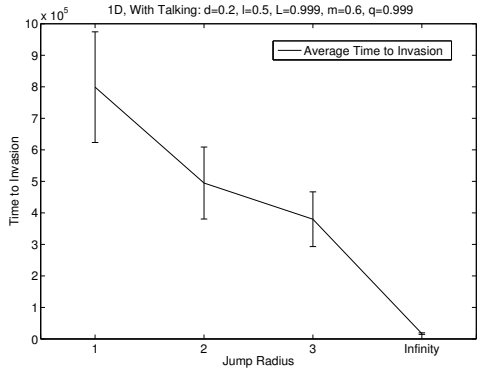


(a) Average time to invasion, plus and minus the standard deviation, for a 1D grid with talking and without mutations.

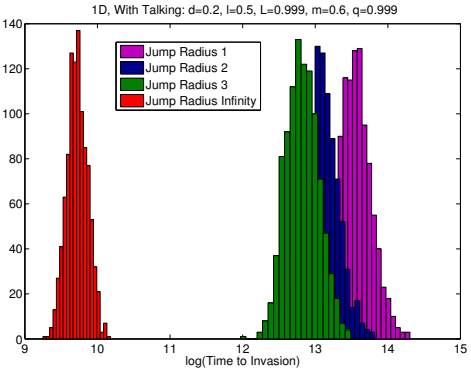


(b) Histograms of the natural log of the times to invasion for each jump radius.

Figure 3.10: 1D grid with talking and without mutations.



(a) Average time to invasion, plus and minus the standard deviation, for a 1D grid with talking and with mutation rate 0.001.



(b) Histograms of the natural log of the times to invasion for each jump radius.

Figure 3.11: 1D grid with talking and with mutation rate 0.001.

3.4 Discussion

We have seen that if we consider all the individuals with language to have an advantage, then, the larger the jump radius, the faster the time to invasion. This makes sense, because with a larger jump radius, there will be more reproduction, as there are more open spots for offspring to be placed. When all the language individuals have an advantage, their reproduction rate is larger than those without language, so they produce more offspring.

However, the idea that everyone with language would have an advantage is not realistic. Language is beneficial because it allows cooperate and solve problems in parallel (see Pinker and Bloom (1990) and Pinker (2010)). Therefore, we look at the case where the individuals with language only have an advantage if they can talk with other individuals with language. Thus, only the individuals with language who are able to talk to others reproduce at the higher rate. Those who do not have anyone to talk to (as well as those without language) reproduce at the lower rate.

We are able to see the effects of talking clearly in the two-dimensional grid. We find that we need the jump radius to be small enough to keep those with language close enough together so that they can talk. If the jump radius is too large, then the offspring are placed farther away, and they do not have anyone to speak with. However, if the jump radius is too small, then the open spots available for the offspring to be placed quickly fill, and without a place for the offspring to go, reproduction slows. We find that the optimal jump radii are jump radius 2 and jump radius 3. These radii are small enough that the language offspring are kept close together so that they can talk. These radii are also large enough so that there will be more open spots for the offspring to be placed. We find that these jump radii lead to a faster time to invasion, both without mutations and will small mutation rates.

We do not see the same patterns in the one-dimensional grid. In these grids, even with talking, a larger jump radius leads to a faster time to invasion. This shows a difference between the behavior of the one- and two-dimensional grids. In the one-dimensional case, for jump radius infinity, individuals with language appear all over the grid very quickly. This gives the individuals with language more chances to talk to others and helps to speed up their invasion. In the two-dimensional grid, it takes much longer for the individuals with language to establish themselves on the grid. Once a cluster starts to develop, they will spread quickly, but we need this cluster to appear before that will happen. This gives us an inherent difference between the one- and two-dimensional grids.

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Appendix A

Appendix for Chapter 1

A.1 A teacher-learner pair as a Markov walk

Consider the reinforcement learning algorithm with $n = 2$ forms, and suppose that the source is characterized by the values $\nu_1 = \nu$ and $\nu_2 = 1 - \nu$, with $0 < \nu < 1$. Let us describe the reinforcement learner algorithm as a Markov walk in an interval $[0, L]$, where the state space consists of integer numbers in $[0, L]$, and state i corresponse to the frequency of variant 1 being $x_1 = i/L$. The increment of learning is denoted by Δ and is also an integer (in this case, increment $s = \Delta/L$ will give the increment on the interval $[0, 1]$). Then transition matrix,$P = \{p_{ij}\}$, is given by the following:

$$
\begin{aligned}
p_{i,i+\Delta} &= \nu, \quad \text{for} \quad 0 \le i \le L - \Delta, \\
p_{i,i-\Delta} &= 1 - \nu, \quad \text{for} \quad s \le i \le L, \\
p_{i,i+\Delta} &= 1 - \nu, \quad p_{L-i,L} = \nu \quad \text{for} \quad 0 \le i \le \Delta - 1;
\end{aligned}
$$

the rest of the entries in this matrix being zero. The stationary probability distribution is given by the equation $qP = q$, where q is a string vector of probabilities. This vector has a simple expression if the value Δ is a divisor of L. In this case we have (for the non-normalized eigenvector)

$$q = \begin{cases} \left(\frac{1-\nu}{\nu}\right)^{L/\Delta - i}, & i = ks, \quad k = 0, 1, \ldots, L/\Delta, \\ 0, & \text{otherwise.} \end{cases}$$

The mean value for form 1 is given by

$$\begin{aligned} \nu' &= \frac{\sum_{i=0}^{L} q_i i}{L \sum_{i=0}^{L} q_i} \\ &= 1 - (1-\nu)\left(\frac{s}{2\nu - 1} + \frac{1+s}{1 - \nu(1 + (\nu/(1-\nu))^{1/s})}\right). \end{aligned}$$

This is a monotonically increasing function of ν with $\nu'(0) = 0$ and $\nu'(1) = 1$. We also have the following:

$$\nu'(\nu) < \nu \text{ for } \nu < 1/2, \quad \nu'(\nu) > \nu \text{ for } \nu > 1/2.$$

In other words, if the source uses form 1 predominantly ($\nu > 1/2$), then the learner will be using form 1 more often than the source, on average. The function ν' depends on s in the following way. If $\nu < 1/2$, then $\partial\nu'/\partial s > 0$, and if $\nu > 1/2$, then $\partial\nu'/\partial s < 0$. In other words, if the source uses form 1 predominantly, then increasing s will decrease the performance of the learner.

A.2 The dynamical systems approach

Suppose there are only two forms, and the frequency of the source is given by ν. Let us denote by X_1 the learner's frequency of the first form and let s be the learning increment. The rules of the original model then can be written as follows:

If input 1 (prob. ν):

$$X_1 \to X_1 + A(X_1), \quad A(X_1) = \begin{cases} s, & X_1 < 1 - s, \\ 1 - X_1, & X_1 \geq 1 - s. \end{cases} \tag{A.1}$$

If input 2 (prob. $1 - \nu$):

$$X_1 \to X_1 - B(X_1), \quad B(X_1) = \begin{cases} s, & X_1 > s, \\ X_1, & X_1 \leq s. \end{cases} \tag{A.2}$$

We would like to compare the reinforcement learning algorithm with another learning model, the linear reward-penalty model (1.3), see also Narendra and Thathachar (2012).

For the linear reward-penalty model with two forms, we have

$$\text{If input 1 (prob. } \nu\text{): } X_1 \to X_1 + s(1 - X_1), \tag{A.3}$$

$$\text{If input 2 (prob. } 1 - \nu\text{): } X_1 \to X_1 - sX_1. \tag{A.4}$$

We can see that the increment of X_1 is a nonlinear function of X_1 in the first model, and it is a linear function of X_1 in the linear reward-penalty model. Therefore, the analysis of the latter model is easier.

The mean increment in the linear reward-penalty model is given by

$$\langle \Delta X_1 \rangle = \langle \nu s(1 - X_1) - (1 - \nu) s X_1 \rangle = \langle s(\nu - X_1) \rangle = s(\nu - \langle X_1 \rangle).$$

Therefore, at steady state, we have

$$\langle X_1 \rangle = \nu.$$

For the linear reward-penalty model with n forms, we have

$$\begin{aligned} &\text{If input 1 (prob. } \nu\text{):} \\ X_1 \rightarrow &\begin{cases} X_1 + s(1 - X_1), & \max\{X_j\} = X_1, \\ X_1 - sX_1, & \max\{X_j\} \neq X_1 \end{cases} \qquad \text{(A.5)} \\ &\text{If input } i \text{ (prob. } \tfrac{1-\nu}{n-1}\text{):} \\ X_1 \rightarrow &\begin{cases} X_1 - sX_1, & \max\{X_j\} = X_1, \\ X_1 + s\left(\frac{1}{n-1} - X_1\right), & \max\{X_j\} \neq X_1 \end{cases} \qquad \text{(A.6)} \end{aligned}$$

where $1 < i \leq n$ and $1 \leq j \leq n$.

So if $\max\{X_j\} = X_1$

$$\begin{aligned} \langle \Delta X_1 \rangle &= \left\langle \nu s(1 - X_1) - (n-1)\frac{1-\nu}{n-1} s X_1 \right\rangle \\ &= \langle s(\nu - X_1) \rangle = s(\nu - \langle X_1 \rangle). \end{aligned}$$

as above in the two form case. Therefore, at steady state, we have

$$\langle X_1 \rangle = \nu.$$

If $\max\{X_j\} \neq X_1$

$$\langle \Delta X_1 \rangle = \left\langle \nu(-sX_1) + (n-1)\frac{1-\nu}{n-1} s\left(\frac{1}{n-1} - X_1\right) \right\rangle$$

$$= \left\langle s\left(\frac{1-v}{n-1} - X_1\right) \right\rangle = s\left(\frac{1-\nu}{n-1} - \langle X_1 \rangle\right).$$

Therefore, at steady state, we have

$$\langle X_1 \rangle = \frac{1-\nu}{n-1} < \nu$$

for $v > 1/2$ and $n > 2$. We conclude that the linear reward-penalty model does not possess a frequency-boosting property.

In the original model, the above calculation does not hold because the operation of taking the mean cannot be applied to the argument of a nonlinear function. Therefore, we have to use the full Markov process calculation to find the mean value of X_1. It is not uncommon however to model the mean dynamics of such systems by using quasispecies equations. In discrete time, we have

$$X_1(t+1) = \nu(X_1 + A(X_1(t))) + (1-\nu)(X_1 - B(X_1(t))),$$

where the variables have the meaning of the expected values. This can also be expressed in terms of a quasispecies-type ODE,

$$\dot{X}_1 = \nu(X_1 + A(X_1)) + (1-\nu)(X_1 - B(X_1)) - X_1.$$

We have three cases to consider when analyzing this equation: X_1 is not near 0 or 1, X_1 is near 0, and X_1 is near 1. When X_1 is not near 0 or 1, the increment is given by s, so we get

that

$$\dot{X}_1 = (2\nu - 1)s.$$

Therefore, when $\nu > 1/2$, we have linear growth of X_1.

When X_1 is near 0, $A(X_1) = s$ and $B(X_1) = X_1$, so

$$\dot{X}_1 = (s + X_1)\nu - X_1.$$

At steady state,

$$X_1 = \frac{a\nu}{1 - \nu} > \nu$$

for $\nu > 1 - s$.

When X_1 is near 1, $A(X_1) = 1 - X_1$ and $B(X_1) = s$, so that

$$\dot{X}_1 = (1 - X_1 + s) - s.$$

Analyzing for steady states, we find that

$$X_1 = 1 - s\left(\frac{1}{\nu} - 1\right) > \nu$$

for $\nu > s$.

Combining these cases together, we have a boosting property when

$$\nu > s > 1 - \nu,$$

so

$$2\nu > 1.$$

In other words, we see a boosting property as long as $\nu > 1/2$. Given these conditions, at steady state,

$$X_1 = 1 - s\left(\frac{1}{\nu} - 1\right).$$

Note that the value obtained by this method does not coincide with the mean value obtained by the Markov chain calculation. However, in the regime where ν is close to 1, both methods give

$$X_1 \approx 1 - s(1-\nu).$$

This is because for such high frequencies, the function $A(X_1)$ is almost always linear with X_1, and the quasispecies equations describe the dynamics correctly (that is, the mean increment is given by a linear function of the mean value of X_1).

For n forms, we can see in equation (1.6) that the rules of the model are:

If input 1 (prob. ν):

$$X_1 \to X_1 + A(X_1), \quad A(X_1) = \begin{cases} s, & X_1 < 1 - s, \\ 1 - X_1, & X_1 \geq 1 - s. \end{cases} \tag{A.7}$$

If input i, $1 < i \leq n$ (prob. $\frac{1-\nu}{n-1}$):

$$X_1 \to X - B(X_1), \quad B(X_1) = \begin{cases} s, & X_1 > s, \\ X_1, & X_1 \leq s. \end{cases} \tag{A.8}$$

Then in discrete time, we have

$$\begin{aligned} X_1(t+1) &= \nu(X_1 + A(X_1(t))) + (n-1)\frac{1-\nu}{n-1}(X_1 - B(X_1(t))) \\ &= \nu(X_1 + A(X_1(t))) + (1-\nu)(X_1 - B(X_1(t))), \end{aligned}$$

where the variables have the meaning of the expected values. When we express these in terms of an ODE, we get

$$\begin{aligned} \dot{X}_1 &= \nu(X_1 + A(X_1)) + (n-1)\frac{1-\nu}{n-1}(X_1 - B(X_1)) - X_1 \\ &= \nu(X_1 + A(X_1)) + (1-\nu)(X_1 - B(X_1)) - X_1 \end{aligned}$$

Both of these equations are the same as above, so with the same analysis, we get the for $\nu > 1/2$.

$$X = 1 - s\left(\frac{1}{\nu} - 1\right).$$

A.3 Another application of the model

In Hudson Kam and Newport (2005), Newport and her colleagues conduct experiments similar to those from Hudson Kam and Newport (2009). In the first experiment, they create an artificial language that is divided into two groups. The "count/mass nouns" group has the nouns of the language divided into two classes on basis of meaning. The "gender condition" has the nouns divided into two classes in an arbitrary fashion. Each of these is then divided into four groups, where the determiner is used k% of the time (with $k = 45$ (the low group), $k = 60$ (the mid group), $k = 75$ (the high group), and $k = 100$ (the perfect group)), and

one incorrect form is used otherwise. Altogether, there are eight groups of adults, with five learners in each group. In this experiment, the adults were not able to regularize the language. Their mean determiner production was almost always less than the input.

We applied model (1.9-1.10) to fit the data presented in Hudson Kam and Newport (2005). As seen in figure A.1, for the count/mass groups, the best fit occurs when $s = 0.17$ and $p = 0.50$. For the gender groups, the best fit occurs when $s = 0.05$, and $p = 0.50$. These results are similar to what we found in experiment 1 from Hudson Kam and Newport (2009) (see figure 1.5). We again find that $s < p$.

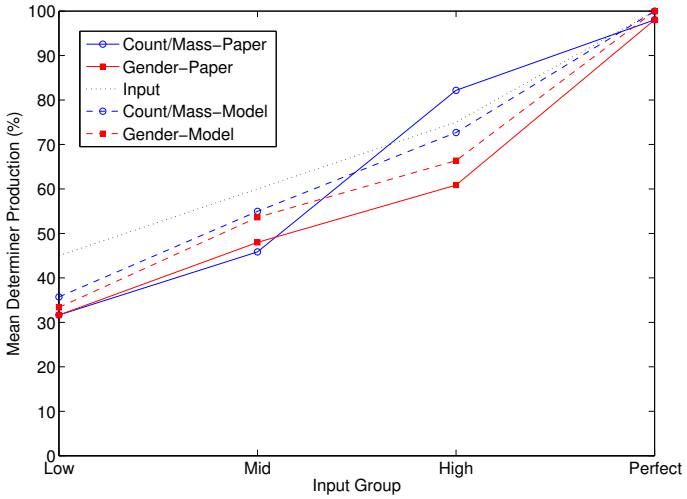


Figure A.1: The best fit of the model compared to the results of Hudson Kam and Newport (2005). The solid lines give the frequency of production observed in the paper (blue with circles for the count/mass groups, and red with squares for the gender groups). The dashed lines show the best fit for each group by using model (1.9-1.10). The dashed black line give the frequency of the input for each group. For the count/mass groups, the best fit occurs when $s = 0.17$ and $p = 0.50$. For the gender groups, the best fit occurs when $s = 0.05$, and $p = 0.50$.

In the second experiment of Hudson Kam and Newport (2005), both children and adults were taught a simpler artificial language. In this experiment, there were 15 children and 8 adults. The children and adults were each divided into two groups. In one group, the determiner was used 100% of the time. In the other, it was used 60% of the time. These groups

correspond to the 100% and 60% + 0ND groups in the second experiment of Hudson Kam and Newport (2009), and their results were the same in both. The adults performed better than the children in the 100% group, and the adults and children performed about the same in the 60% group. However, the children were much more likely to be systematic learners. Although it is not possible to fit these data because there are only two data points for each group, the experimental results reported in Hudson Kam and Newport (2005) correspond to our findings in section 1.3.2, where we modeled the second experiment of Hudson Kam and Newport (2009).

A.4 Adding a noise parameter to the adult experiments

In figure 1.5 we found the best fit for the adults in experiment 1. Adding noise to this model (as we did in the children's experiment), we get a very similar best fit. With noise, the best fit occurs when $s = 0.19, p = 0.50$, and $r = 0.01$, as shown in figure A.2.

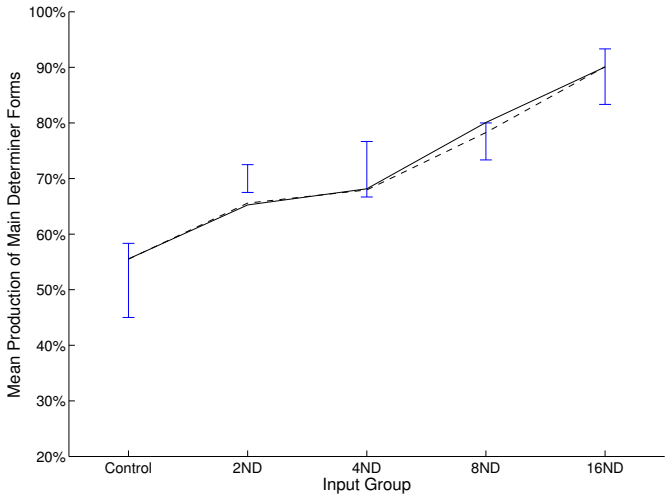


Figure A.2: The best fits of the model with and without noise, compared to the results of the paper. The solid black line gives the best fit without noise, and corresponds to $s = 0.19$ and $p = 0.50$. The dashed black line gives the best fit with noise, and corresponds to $s = 0.19, p = 0.50$, and $r = 0.01$.

Appendix B

Appendix for Chapter 2

B.1 An Additional Variant of the Learning Algorithm

In what follows, we present another variant of our learning algorithm that is capable of exhibiting the results from Fedzechkina et al. (2012). In the sections that follow, we will start with a simple version of our learning algorithm, and then increase its complexity until we are able to exhibit the patterns seen in Fedzechkina et al. (2012). With our model that works, along with the simpler attempts that do not, we will be able to see what factors contribute to learners restructuring the language.

B.1.1 2 Parameters

Let P_{an} be the learner's frequency of the learner to case-mark an animate sentence, P_{in} be the learner's frequency of the learner to case-mark an inanimate sentence, P_{so} be the learner's frequency to case-mark a SOV sentence, and P_{os} be the learner's frequency to case-mark an OSV sentence. All the participants are native English speakers, and do not know any other

languages, so when we start, these frequencies will all be zero.

Given that all the sentences in the experiment are split up in two ways: animate/inanimate (50%/50%) and SOV/OSV (60%/40%), we have the following constraint:

$$\frac{1}{2}P_{an} + \frac{1}{2}P_{in} = \frac{3}{5}P_{so} + \frac{2}{5}P_{os} \tag{B.1}$$

At each time step of the algorithm, the learner receives a sentence that is either animate or inanimate, either SOV or OSV, and either case-marked or not (according to the probabilities given in the paper). If the sentence is case-marked, then their frequency to case-mark will increase by an increment. If it is not marked, then their frequency to case-mark will decrease by an increment. The increments are given by:

Δ_{an} for animate sentences

Δ_{in} for inanimate sentences

Δ_{so} for SOV sentences

Δ_{os} for OSV sentences.

These increments also need to follow the constraint:

$$\frac{1}{2}\Delta_{an} + \frac{1}{2}\Delta_{in} = \frac{3}{5}\Delta_{so} + \frac{2}{5}\Delta_{os} \tag{B.2}$$

Let us suppose that the learner hears an animate, SOV, case-marked sentence. Then we

adjust as follows:

$P_{an} \to P_{an} + \Delta_{an}$

$P_{so} \to P_{so} + \Delta_{so}$

and P_{in} and P_{os} do not change. If they hear an animate, SOV, not marked sentence, then we adjust as follows:

$P_{an} \to P_{an} - \Delta_{an}$

$P_{so} \to P_{so} - \Delta_{so}$

and P_{in} and P_{os} do not change.

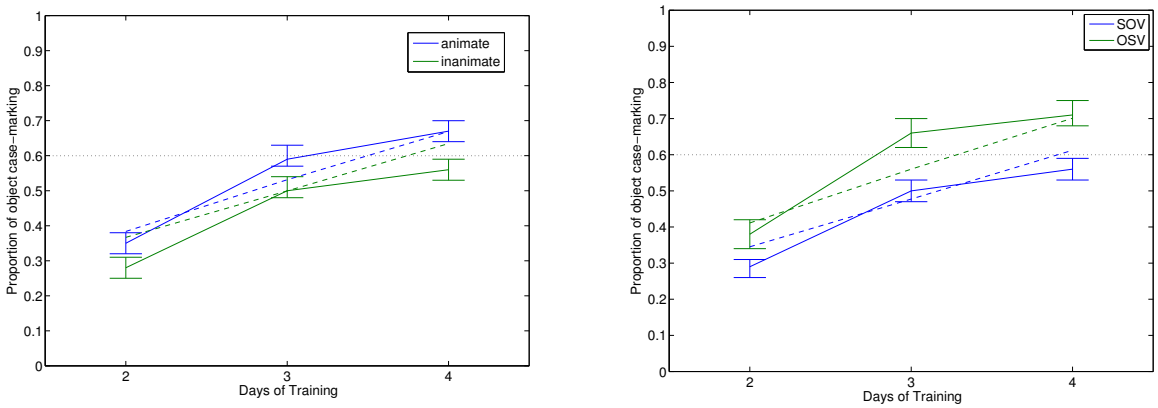
With similar adjustments for the other types of sentences, and given (B.2), we get the following equations:

$$\begin{aligned}
\frac{1}{2}\Delta_{an} &= \frac{3}{5}\Delta_{so} \\
\frac{1}{2}\Delta_{an} &= \frac{2}{5}\Delta_{os} \\
\frac{1}{2}\Delta_{in} &= \frac{3}{5}\Delta_{so} \\
\frac{1}{2}\Delta_{in} &= \frac{2}{5}\Delta_{os}
\end{aligned}$$

Which we can solve to get:

$$\begin{aligned}
\Delta_{an} &= \frac{4}{5}\Delta_{os} \\
\Delta_{in} &= \frac{4}{5}\Delta_{os} \\
\Delta_{so} &= \frac{2}{3}\Delta_{os}
\end{aligned}$$

Therefore, Δ_{an}, Δ_{in}, and Δ_{so} all depend just on Δ_{os}.



(a) Animate versus Inanimate

(b) SOV versus OSV

Figure B.1: Two parameter model: the best fit (given by the dashed lines) occurs when $\Delta_{an} = 0.02, \Delta_{in} = 0.02, \Delta_{so} = 0.01667, \Delta_{os} = 0.0425, and d = 0.96$..

We also include a memory parameter to simulate the learners not remembering everything that they learned from the day before. At the beginning of each day, the learner's frequencies are replaced by d times their frequencies from the day before. Therefore, in total, we have two parameters: Δ_{os} and d. As we see in figure B.1, this model is too linear and does not give a good fit for the paper.

B.1.2 3 Parameters

Next, we consider two different types of increments: $\Delta^{\uparrow}$ to be used if the sentence is case-marked, and $\Delta^{\downarrow}$ if the sentence is not marked. Then, for example, if the learner hears a animate, SOV, case-marked sentence they update by:

$P_{an} \to P_{an} + \Delta^{\uparrow}_{an}$

$P_{so} \to P_{so} + \Delta^{\uparrow}_{so}$

and P_{in} and P_{os} do not change.

If they receive an animate, SOV, not marked sentence, then:

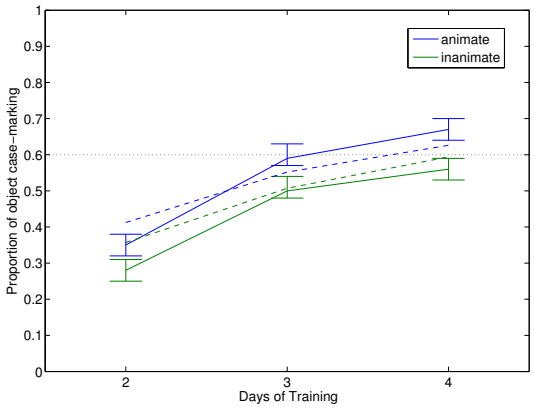
$P_{an} \rightarrow P_{an} - \Delta^{\downarrow}_{an}$

$P_{so} \rightarrow P_{so} - \Delta^{\downarrow}_{so}$

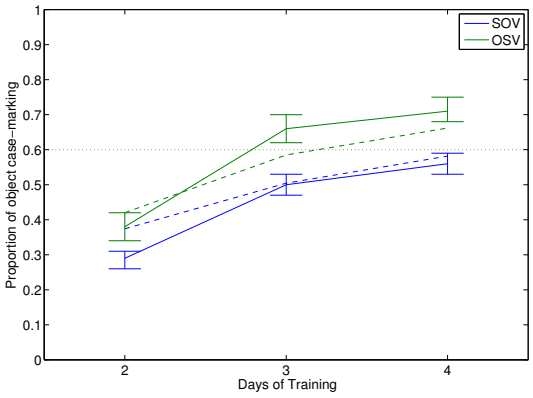
and P_{in} and P_{os} do not change.

This gives us the following equations:

$$
\begin{aligned}
\frac{1}{2}\Delta^{\uparrow}_{an} &= \frac{3}{5}\Delta^{\uparrow}_{so} \\
\frac{1}{2}\Delta^{\uparrow}_{an} &= \frac{2}{5}\Delta^{\uparrow}_{os} \\
\frac{1}{2}\Delta^{\uparrow}_{in} &= \frac{3}{5}\Delta^{\uparrow}_{so} \\
\frac{1}{2}\Delta^{\uparrow}_{in} &= \frac{2}{5}\Delta^{\uparrow}_{os} \\
\frac{1}{2}\Delta^{\downarrow}_{an} &= \frac{3}{5}\Delta^{\downarrow}_{so} \\
\frac{1}{2}\Delta^{\downarrow}_{an} &= \frac{2}{5}\Delta^{\downarrow}_{os} \\
\frac{1}{2}\Delta^{\downarrow}_{in} &= \frac{3}{5}\Delta^{\downarrow}_{so} \\
\frac{1}{2}\Delta^{\downarrow}_{in} &= \frac{2}{5}\Delta^{\downarrow}_{os}
\end{aligned}
$$



(a) Animate versus Inanimate



(b) SOV versus OSV

Figure B.2: Three parameter model: the best fit (given by the dashed lines) occurs when $\Delta^{\downarrow}_{os}$, and d. The best fit occurs when $\Delta^{\uparrow}_{an} = 0.0328, \Delta^{\downarrow}_{an} = 0.0392, \Delta^{\uparrow}_{in} = 0.0328, \Delta^{\downarrow}_{in} = 0.0392, \Delta^{\uparrow}_{so} = 0.02733, \Delta^{\downarrow}_{so} = 0.03266, \Delta^{\uparrow}_{os} = 0.041, \Delta^{\downarrow}_{os} = 0.049, and d = 0.91$.

Which we can solve to get:

$$\begin{aligned}
\Delta^{\uparrow}_{an} &= \frac{4}{5}\Delta^{\uparrow}_{os} \\
\Delta^{\uparrow}_{in} &= \frac{4}{5}\Delta^{\uparrow}_{os} \\
\Delta^{\uparrow}_{so} &= \frac{2}{3}\Delta^{\uparrow}_{os} \\
\Delta^{\downarrow}_{an} &= \frac{4}{5}\Delta^{\downarrow}_{os} \\
\Delta^{\downarrow}_{in} &= \frac{4}{5}\Delta^{\downarrow}_{os} \\
\Delta^{\downarrow}_{so} &= \frac{2}{3}\Delta^{\downarrow}_{os}
\end{aligned}$$

So we now have have three parameters: $\Delta^{\uparrow}_{os}$, $\Delta^{\downarrow}_{os}$, and d. The best fit can be seen in figure B.2, however, it is still too linear. We do find, though, that the best fit occurs when $\Delta^{\uparrow}_{os} < \Delta^{\downarrow}_{os}$. This indicated that the learners are somewhat resistant to case marking, which makes sense, given that it is a new concept to them.

B.1.3 4 parameters

So far, when the learner hears a sentence, we update the two sentence types contained in that sentence, but we do nothing to the other two types. Let us consider a case where there is a connection between the sentence types. If, for example, the learner hears an animate, SOV, case-marked sentence, they will adjust as follows:

$P_{an} \to P_{an} + \Delta^{\uparrow}_{an}$

$P_{in} \to P_{in} + c\Delta^{\uparrow}_{an}$

$P_{so} \to P_{so} + \Delta^{\uparrow}_{so}$

$P_{os} \to P_{os} + c\Delta^{\uparrow}_{so}$,

and if they hear an animate, SOV, not marked sentence, they will adjust as follows:

$P_{an} \to P_{an} - \Delta^{\downarrow}_{an}$

$P_{in} \to P_{in} - c\Delta^{\downarrow}_{an}$

$P_{so} \to P_{so} - \Delta^{\downarrow}_{so}$

$P_{os} \to P_{os} - c\Delta^{\downarrow}_{so}$.

The parameter c represents how much of a connection there is between the sentence types, and we assume that the connection amount is proportional to the amount that the main type increased (i.e. after hearing an animate sentence, P_{an} increases by $\Delta^{\uparrow}_{an}$, and P_{in} increases by $c\Delta^{\uparrow}_{an}$).

With this proportional connection, our equations become:

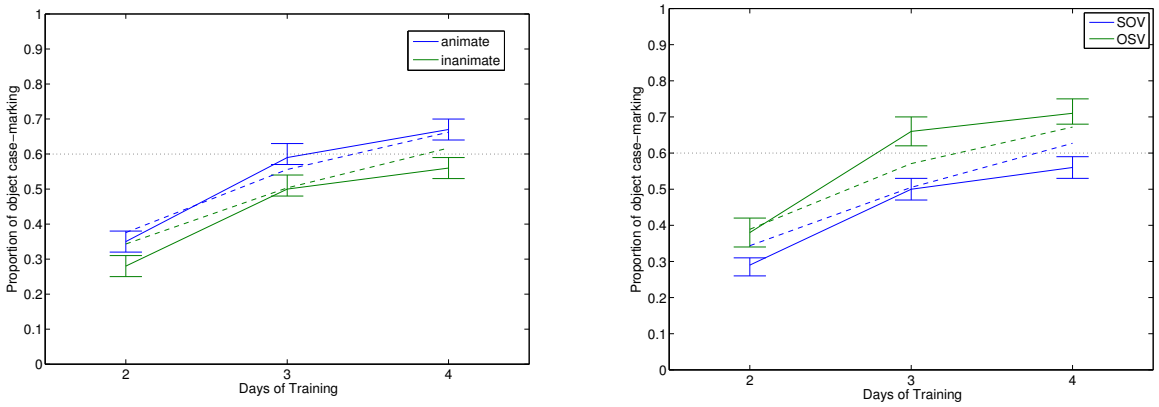
$$\begin{aligned}
\frac{1}{2}\Delta^{\uparrow}_{an} + \frac{1}{2}c\Delta^{\uparrow}_{in} &= \frac{3}{5}\Delta^{\uparrow}_{so} + \frac{2}{5}c\Delta^{\uparrow}_{os} \\
\frac{1}{2}\Delta^{\uparrow}_{an} + \frac{1}{2}c\Delta^{\uparrow}_{in} &= \frac{3}{5}c\Delta^{\uparrow}_{so} + \frac{2}{5}\Delta^{\uparrow}_{os} \\
\frac{1}{2}c\Delta^{\uparrow}_{an} + \frac{1}{2}\Delta^{\uparrow}_{in} &= \frac{3}{5}\Delta^{\uparrow}_{so} + \frac{2}{5}c\Delta^{\uparrow}_{os} \\
\frac{1}{2}c\Delta^{\uparrow}_{an} + \frac{1}{2}\Delta^{\uparrow}_{in} &= \frac{3}{5}c\Delta^{\uparrow}_{so} + \frac{2}{5}\Delta^{\uparrow}_{os} \\
\frac{1}{2}\Delta^{\downarrow}_{an} + \frac{1}{2}c\Delta^{\downarrow}_{in} &= \frac{3}{5}\Delta^{\downarrow}_{so} + \frac{2}{5}c\Delta^{\downarrow}_{os} \\
\frac{1}{2}\Delta^{\downarrow}_{an} + \frac{1}{2}c\Delta^{\downarrow}_{in} &= \frac{3}{5}c\Delta^{\downarrow}_{so} + \frac{2}{5}\Delta^{\downarrow}_{os} \\
\frac{1}{2}c\Delta^{\downarrow}_{an} + \frac{1}{2}\Delta^{\downarrow}_{in} &= \frac{3}{5}\Delta^{\downarrow}_{so} + \frac{2}{5}c\Delta^{\downarrow}_{os} \\
\frac{1}{2}c\Delta^{\downarrow}_{an} + \frac{1}{2}\Delta^{\downarrow}_{in} &= \frac{3}{5}c\Delta^{\downarrow}_{so} + \frac{2}{5}\Delta^{\downarrow}_{os}
\end{aligned}$$

Solving these, we get the same equations as in the 3 parameter case:

$$\begin{aligned}
\Delta^{\uparrow}_{an} &= \frac{4}{5}\Delta^{\uparrow}_{os} \\
\Delta^{\uparrow}_{in} &= \frac{4}{5}\Delta^{\uparrow}_{os} \\
\Delta^{\uparrow}_{so} &= \frac{2}{3}\Delta^{\uparrow}_{os} \\
\Delta^{\downarrow}_{an} &= \frac{4}{5}\Delta^{\downarrow}_{os} \\
\Delta^{\downarrow}_{in} &= \frac{4}{5}\Delta^{\downarrow}_{os} \\
\Delta^{\downarrow}_{so} &= \frac{2}{3}\Delta^{\downarrow}_{os}
\end{aligned}$$

Now our four parameters are: $\Delta^{\uparrow}_{os}$, $\Delta^{\downarrow}_{os}$, c, and d.

The best fit occurs when: $\Delta_{os}^{\uparrow} = 0.035, \Delta_{os}^{\downarrow} = 0.042, c = 0.19, andd = 0.95$ (see figure B.3). Again, the down increments are larger than the up increments. If we consider that the learners are all native English speakers and are not familiar with case-marking, it makes sense that they would adjust more for the form that they are more familiar with.



(a) Animate versus Inanimate

(b) SOV versus OSV

Figure B.3: Four parameter model: the best fit occurs when $\Delta_{an}^{\uparrow} = 0.028, \Delta_{an}^{\downarrow} = 0.0336, \Delta_{in}^{\uparrow} = 0.028, \Delta_{in}^{\downarrow} = 0.0336, \Delta_{so}^{\uparrow} = 0.02333, \Delta_{so}^{\downarrow} = 0.028, \Delta_{os}^{\uparrow} = 0.035, \Delta_{os}^{\downarrow} = 0.042, c = 0.19, andd = 0.95$.

B.1.4 5 Parameters

Let us consider the same model as in Section B.1.3, but with two memory parameters as follows:

At the beginning of day 2, the learners' frequencies to case-mark the four types of sentences (animate, inanimate, SOV, and OSV) will be multiplied by d_1, where d_1 will represent the percent that they remember from their first day of learning. Starting with day 3, we will instead multiply their frequencies by d_2, where d_2 represents the amount that they remember from the day before (after they have already been learning for at least 2 days).

The idea is that after only one day of learning this completely new, artificial language with case marking (which the english speakers are not familiar with), the learners will not

remember much of what they learned. However, after learning for two days, they amount that they remember will increase. We can also justify this change because the learners are not tested at the end day 1, but they are tested at the end of each subsequent day. In Spitzer (1939) the author gave reading assignments to 6th grades, and then tested their retention after various time intervals. The students were tested on the material a total of three times over a 63 day period. Some students were tested right after they read the article, and then again some time later, while others were not tested right away. The students who were tested right away did better on the subsequent tests than the other students. They found that "...more is forgotten in one day without recall than is forgotten in sixty-three days with the aid of recall."

Our model is otherwise the same is in Section B.1.3. Recall, for example, if the learner hears an animate/SOV, case-marked sentence, they will adjust as follows:

$P_{an} \to P_{an} + \Delta_{an}^{\uparrow}$

$P_{in} \to P_{in} + c\Delta_{an}^{\uparrow}$

$P_{so} \to P_{so} + \Delta_{so}^{\uparrow}$

$P_{os} \to P_{os} + c\Delta_{so}^{\uparrow}$,

and if they hear an animate/SOV, not marked sentence, they will adjust as follows:

$P_{an} \to P_{an} - \Delta_{an}^{\downarrow}$

$P_{in} \to P_{in} - c\Delta_{an}^{\downarrow}$

$P_{so} \to P_{so} - \Delta_{so}^{\downarrow}$

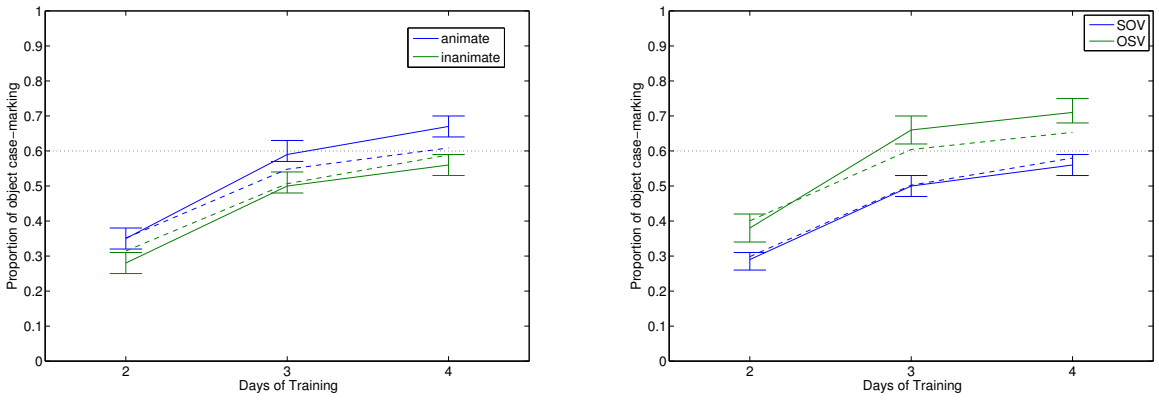
$P_{os} \to P_{os} - c\Delta_{so}^{\downarrow}$.

The parameter c represents how much of a connection there is.

So we again wind up with:

$$\begin{aligned}
\Delta^{\uparrow}_{an} &= \frac{4}{5}\Delta^{\uparrow}_{os} \\
\Delta^{\uparrow}_{in} &= \frac{4}{5}\Delta^{\uparrow}_{os} \\
\Delta^{\uparrow}_{so} &= \frac{2}{3}\Delta^{\uparrow}_{os} \\
\Delta^{\downarrow}_{an} &= \frac{4}{5}\Delta^{\downarrow}_{os} \\
\Delta^{\downarrow}_{in} &= \frac{4}{5}\Delta^{\downarrow}_{os} \\
\Delta^{\downarrow}_{so} &= \frac{2}{3}\Delta^{\downarrow}_{os}
\end{aligned}$$

So now we have five parameters: $\Delta^{\uparrow}_{os}$, $\Delta^{\downarrow}_{os}$, c, d_1, and d_2. The best fit occurs when $\Delta^{\uparrow}_{os} = 0.054, \Delta^{\downarrow}_{os} = 0.065, c = 0.045, d_1 = 0.11$, and $d_2 = 0.95$. We can see that in figure B.4, that the fit is better, but we are still missing something.



(a) Animate versus Inanimate

(b) SOV versus OSV

Figure B.4: Five parameter model: the best fit occurs when $\Delta^{\uparrow}_{an} = 0.0432\Delta^{\downarrow}_{an} = 0.052, \Delta^{\uparrow}_{in} = 0.432, \Delta^{\downarrow}_{in} = 0.052, \Delta^{\uparrow}_{so} = 0.036, \Delta^{\downarrow}_{so} = 0.0433, \Delta^{\uparrow}_{os} = 0.054, \Delta^{\downarrow}_{os} = 0.065, c = 0.045, d_1 = 0.11$, and $d_2 = 0.95$.

B.1.5 5 Parameters with Non-proportional Connections

Now let us consider a case with 5 parameters, but where the connections between the sentence types is not proportional to the other type's update as it was before. Let Δ_c be the connection increment. Then, if, for example, the learner hears an animate, SOV, case-marked sentence, they will adjust as follows:

$P_{an} \rightarrow P_{an} + \Delta_{an}^{\uparrow}$

$P_{in} \rightarrow P_{in} + \Delta_c$

$P_{so} \rightarrow P_{so} + \Delta_{so}^{\uparrow}$

$P_{os} \rightarrow P_{os} + \Delta_c$,

and if they hear an animate, SOV, not marked sentence, they will adjust as follows:

$P_{an} \rightarrow P_{an} - \Delta_{an}^{\downarrow}$

$P_{in} \rightarrow P_{in} - \Delta_c$

$P_{so} \rightarrow P_{so} - \Delta_{so}^{\downarrow}$

$P_{os} \rightarrow P_{os} - \Delta_c$.

Now we get the following equations:

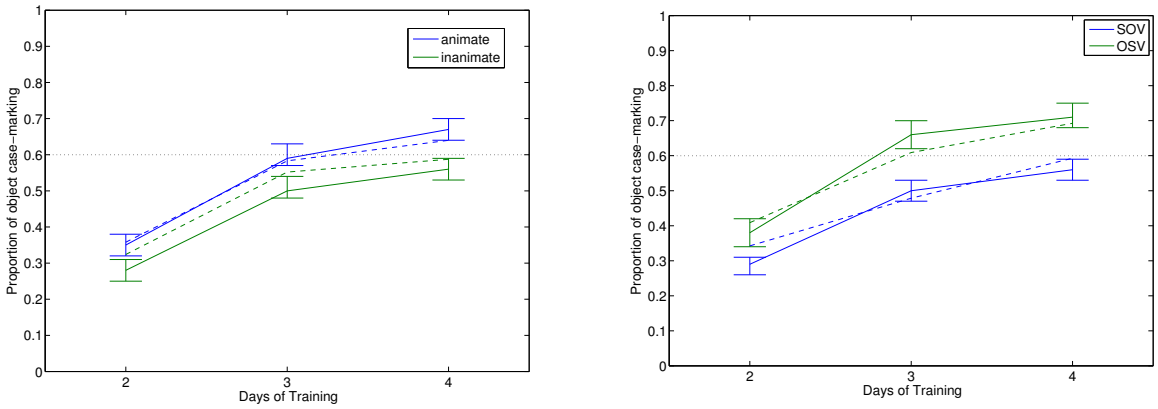
$$\begin{aligned}
\frac{1}{2}\Delta_{an}^{\uparrow} + \frac{1}{2}\Delta_c &= \frac{3}{5}\Delta_{so}^{\uparrow} + \frac{2}{5}\Delta_c \\
\frac{1}{2}\Delta_{an}^{\uparrow} + \frac{1}{2}\Delta_c &= \frac{3}{5}\Delta_c + \frac{2}{5}\Delta_{os}^{\uparrow} \\
\frac{1}{2}\Delta_c + \frac{1}{2}\Delta_{in}^{\uparrow} &= \frac{3}{5}\Delta_{so}^{\uparrow} + \frac{2}{5}\Delta_c \\
\frac{1}{2}\Delta_c + \frac{1}{2}\Delta_{in}^{\uparrow} &= \frac{3}{5}\Delta_c + \frac{2}{5}\Delta_{os}^{\uparrow} \\
\frac{1}{2}\Delta_{an}^{\downarrow} + \frac{1}{2}\Delta_c &= \frac{3}{5}\Delta_{so}^{\downarrow} + \frac{2}{5}\Delta_c \\
\frac{1}{2}\Delta_{an}^{\downarrow} + \frac{1}{2}\Delta_c &= \frac{3}{5}\Delta_c + \frac{2}{5}\Delta_{os}^{\downarrow} \\
\frac{1}{2}\Delta_c + \frac{1}{2}\Delta_{in}^{\downarrow} &= \frac{3}{5}\Delta_{so}^{\downarrow} + \frac{2}{5}\Delta_c \\
\frac{1}{2}\Delta_c + \frac{1}{2}\Delta_{in}^{\downarrow} &= \frac{3}{5}\Delta_c + \frac{2}{5}\Delta_{os}^{\downarrow}
\end{aligned}$$

Solving these, we again get:

$$\begin{aligned}
\Delta_{an}^{\uparrow} &= \frac{4}{5}\Delta_{os}^{\uparrow} + \frac{1}{5}\Delta_c \\
\Delta_{in}^{\uparrow} &= \frac{4}{5}\Delta_{os}^{\uparrow} + \frac{1}{5}\Delta_c \\
\Delta_{so}^{\uparrow} &= \frac{2}{3}\Delta_{os}^{\uparrow} + \frac{1}{3}\Delta_c \\
\Delta_{an}^{\downarrow} &= \frac{4}{5}\Delta_{os}^{\downarrow} + \frac{1}{5}\Delta_c \\
\Delta_{in}^{\downarrow} &= \frac{4}{5}\Delta_{os}^{\downarrow} + \frac{1}{5}\Delta_c \\
\Delta_{so}^{\downarrow} &= \frac{2}{3}\Delta_{os}^{\downarrow} + \frac{1}{3}\Delta_c
\end{aligned}$$

So now we have five parameters: $\Delta_{os}^{\uparrow}$, $\Delta_{os}^{\downarrow}$, Δ_c, d_1, and d_2. The best fit occurs when $\Delta_{os}^{\uparrow} = 0.011, \Delta_{os}^{\downarrow} = 0.032, \Delta_c = 0.042, d_1 = 0.23$, and $d_2 = 0.92$, but again the fit is not great.

The problem comes from the way that our equations simplify with this simple connection parameter. We get that $\Delta_{an}^{\uparrow} = \Delta_{in}^{\uparrow}$ and $\Delta_{an}^{\downarrow} = \Delta_{in}^{\downarrow}$.



(a) Animate versus Inanimate

(b) SOV versus OSV

Figure B.5: Five parameter model with non-proportional connections: the best fit occurs when $\Delta_{an}^{\uparrow} = 0.0172, \Delta_{an}^{\downarrow} = 0.034, \Delta_{in}^{\uparrow} = 0.0172, \Delta_{in}^{\downarrow} = 0.034, \Delta_{so}^{\uparrow} = 0.02133 \Delta_{so}^{\downarrow} = 0.03533, \Delta_{os}^{\uparrow} = 0.011, \Delta_{os}^{\downarrow} = 0.032, \Delta_c = 0.042, d_1 = 0.23$, and $d_2 = 0.92$.

B.1.6 6 parameters with New Connection Parameters

We would like to consider a model where each sentence type has its own connection increment, however, this will give us too many parameters. So instead, we will have two parameters for the connections: Δ_c^1 and Δ_c^2. Δ_c^1 will correspond to the sentence types that are harder to understand: animate and OSV, while Δ_c^2 will correspond to the inanimate and SOV sentence types.

If, for example, the learner hears an animate/SOV, case-marked sentence, they will adjust as follows:

$P_{an} \rightarrow P_{an} + \Delta_{an}^{\uparrow}$

$P_{in} \rightarrow P_{in} + \Delta_c^2$

$P_{so} \rightarrow P_{so} + \Delta_{so}^{\uparrow}$

$P_{os} \rightarrow P_{os} + \Delta_c^1$,

and if they hear an animate/SOV, not marked sentence, they will adjust as follows:

$P_{an} \rightarrow P_{an} - \Delta^{\downarrow}_{an}$

$P_{in} \rightarrow P_{in} - \Delta^{2}_{c}$

$P_{so} \rightarrow P_{so} - \Delta^{\downarrow}_{so}$

$P_{os} \rightarrow P_{os} - \Delta^{1}_{c}$.

Or if, for example, the learner hears an inanimate/OSV, case-marked sentence, they will adjust as follows:

$P_{an} \rightarrow P_{an} + \Delta^{1}_{c}$

$P_{in} \rightarrow P_{in} + \Delta^{\uparrow}_{in}$

$P_{so} \rightarrow P_{so} + \Delta^{2}_{c}$

$P_{os} \rightarrow P_{os} + \Delta^{\uparrow}_{os}$,

and if they hear an animate/SOV, not marked sentence, they will adjust as follows:

$P_{an} \rightarrow P_{an} - \Delta^{1}_{c}$

$P_{in} \rightarrow P_{in} - \Delta^{\downarrow}_{in}$

$P_{so} \rightarrow P_{so} - \Delta^{2}_{c}$

$P_{os} \rightarrow P_{os} - \Delta^{\downarrow}_{os}$,

So P_{an} and P_{os} both adjust by Δ^{1}_{c} when they are not the type that was heard, and P_{in} and P_{so} both adjust by Δ^{2}_{c} when they are not the type that was heard.

Now we get the following equations:

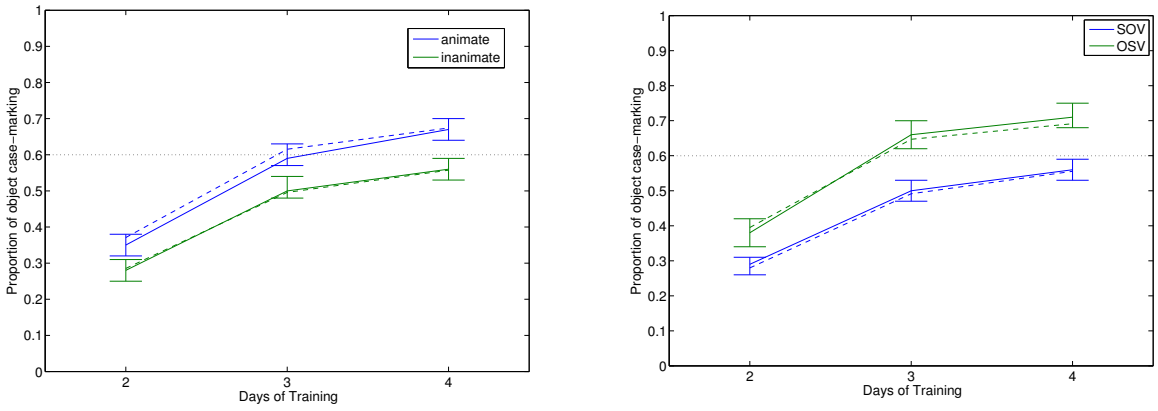
$$\begin{aligned}
\frac{1}{2}\Delta^{\uparrow}_{an} + \frac{1}{2}\Delta^{2}_{c} &= \frac{3}{5}\Delta^{\uparrow}_{so} + \frac{2}{5}\Delta^{1}_{c} \\
\frac{1}{2}\Delta^{\uparrow}_{an} + \frac{1}{2}\Delta^{2}_{c} &= \frac{3}{5}\Delta^{2}_{c} + \frac{2}{5}\Delta^{\uparrow}_{os} \\
\frac{1}{2}\Delta^{1}_{c} + \frac{1}{2}\Delta^{\uparrow}_{in} &= \frac{3}{5}\Delta^{\uparrow}_{so} + \frac{2}{5}\Delta^{1}_{c} \\
\frac{1}{2}\Delta^{1}_{c} + \frac{1}{2}\Delta^{\uparrow}_{in} &= \frac{3}{5}\Delta^{2}_{c} + \frac{2}{5}\Delta^{\uparrow}_{os} \\
\frac{1}{2}\Delta^{\downarrow}_{an} + \frac{1}{2}\Delta^{2}_{c} &= \frac{3}{5}\Delta^{\downarrow}_{so} + \frac{2}{5}\Delta^{1}_{c} \\
\frac{1}{2}\Delta^{\downarrow}_{an} + \frac{1}{2}\Delta^{2}_{c} &= \frac{3}{5}\Delta^{2}_{c} + \frac{2}{5}\Delta^{\downarrow}_{os} \\
\frac{1}{2}\Delta^{1}_{c} + \frac{1}{2}\Delta^{\downarrow}_{in} &= \frac{3}{5}\Delta^{\downarrow}_{so} + \frac{2}{5}\Delta^{1}_{c} \\
\frac{1}{2}\Delta^{1}_{c} + \frac{1}{2}\Delta^{\downarrow}_{in} &= \frac{3}{5}\Delta^{2}_{c} + \frac{2}{5}\Delta^{\downarrow}_{os}
\end{aligned}$$

Solving these, we get:

$$\begin{aligned}
\Delta^{\uparrow}_{an} &= \frac{4}{5}\Delta^{\uparrow}_{os} + \frac{1}{5}\Delta^{2}_{c} \\
\Delta^{\uparrow}_{in} &= \frac{4}{5}\Delta^{\uparrow}_{os} - \Delta^{1}_{c} + \frac{6}{5}\Delta^{2}_{c} \\
\Delta^{\uparrow}_{so} &= \frac{2}{3}\Delta^{\uparrow}_{os} - \frac{2}{3}\Delta^{1}_{c} + \Delta^{2}_{c} \\
\Delta^{\downarrow}_{an} &= \frac{4}{5}\Delta^{\downarrow}_{os} + \frac{1}{5}\Delta^{2}_{c} \\
\Delta^{\downarrow}_{in} &= \frac{4}{5}\Delta^{\downarrow}_{os} - \Delta^{1}_{c} + \frac{6}{5}\Delta^{2}_{c} \\
\Delta^{\downarrow}_{so} &= \frac{2}{3}\Delta^{\downarrow}_{os} - \frac{2}{3}\Delta^{1}_{c} + \Delta^{2}_{c}
\end{aligned} \tag{B.3}$$

So we have six parameters: $\Delta^{\uparrow}_{os}, \Delta^{\downarrow}_{os}, \Delta^{1}_{c}, \Delta^{2}_{c}, d_1$, and d_2. The best fit occurs when: $\Delta^{\uparrow}_{os} =$

$0.015, \Delta_{os}^{\downarrow} = 0.032, \Delta_{c}^{1} = 0.038, \Delta_{c}^{2} = 0.033, d_1 = 0.20$, and $d_2 = 0.88$, and can be seen in figure B.6



(a) Animate versus Inanimate

(b) SOV versus OSV

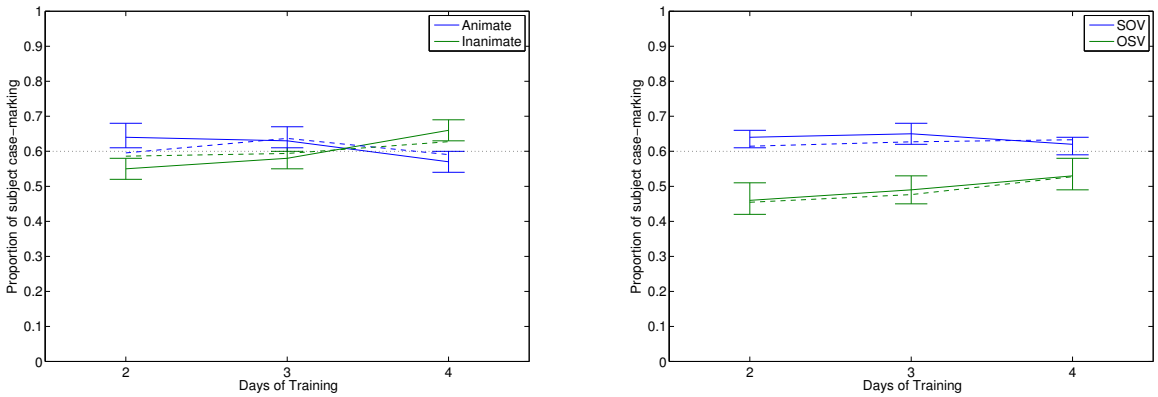
Figure B.6: Six parameter model: the best fit occurs when: $\Delta_{an}^{\uparrow} = 0.0186, \Delta_{an}^{\downarrow} = 0.0322, \Delta_{in}^{\uparrow} = 0.0136, \Delta_{in}^{\downarrow} = 0.0272, \Delta_{so}^{\uparrow} = 0.017667, \Delta_{so}^{\downarrow} = 0.0290, \Delta_{os}^{\uparrow} = 0.015, \Delta_{os}^{\downarrow} = 0.032, \Delta_{c}^{1} = 0.038, \Delta_{c}^{2} = 0.033, d_1 = 0.20$, and $d_2 = 0.88$.

B.1.7 Experiment 2

Using the same model as in section B.1.6, and assuming that the memory parameters should be near what they were in experiment 1 we get the that the best fit occurs when: $\Delta_{os}^{\uparrow} = 0.14, \Delta_{os}^{\downarrow} = 0.211, \Delta_{c}^{1} = 0.005, \Delta_{c}^{2} = 0.630, d_1 = 0.17$, and $d_2 = 0.87$ (see figure B.7).

B.2 Discussion

We find that our six parameter model in B.1.6 is able to exhibit the patterns seen in both the experiments in Fedzechkina et al. (2012). We need different increments for when the sentences were case-marked, then for when they are not marked. We also need two connection parameters, and 2 memory parameters.



(a) Animate versus Inanimate

(b) SOV versus OSV

Figure B.7: The best fit occurs when: $\Delta_{an}^{\uparrow} = 0.238, \Delta_{an}^{\downarrow} = 0.2948, \Delta_{in}^{\uparrow} = 0.863, \Delta_{in}^{\downarrow} = 0.9198, \Delta_{so}^{\uparrow} = 0.72, \Delta_{so}^{\downarrow} = 0.76733, \Delta_{os}^{\uparrow} = 0.14, \Delta_{os}^{\downarrow} = 0.211, \Delta_{c}^{1} = 0.005, \Delta_{c}^{2} = 0.630, d_1 = 0.17$, and $d_2 = 0.87$.

We find that "up" parameters that correspond to case marking are smaller than the "down" parameters. This indicates that the learners are somewhat resistant to case marking, and react more strongly when they hear a sentence that is not marked. We also find that the connection parameter that corresponds to the harder to understand sentences is larger than the other connection parameter. This indicates that the learners adjust their frequencies more for the more difficult sentences. Finally, our memory parameter for the beginning of day 2 is much smaller than the memory parameter for the subsequent parameters. We attribute this to the fact that the learners are not tested at the end of day 1, and therefore, do not remember much about what they learned from that day. Once they start getting tested at the end of each day, they are able to retain more of what they learn.