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AN EXPERIMENTAL APPROACH TO OPTIMISE AND TEST PERTURBATIVE QCD TO O(α_s^2)*

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Abstract. Production rates of multijet hadronic final states observed in e^+e^- annihilation are compared with recent QCD calculations using renormalisation group improved perturbation theory to O(α_s^2). The scale parameter $\Lambda_{\overline{MS}}$ and the renormalisation scale μ^2 are adjusted in order to reproduce the observed 2-, 3- and 4-jet event production rates. Small scales of $\mu^2 \approx 0.002 \cdot E_{cm}^2$ provide a significantly better description of the overall 4-jet production rates and of 2- and 3-jet rates at small jet pair masses than calculations using $\mu^2 = E_{cm}^2$. The adjusted value of $\Lambda_{\overline{MS}}$ depends on the choice of μ^2. It decreases by a factor of two when going from $\mu^2 = E_{cm}^2$ to $\mu^2 = 0.002 \cdot E_{cm}^2$ and results in $\Lambda_{\overline{MS}} = 95\ MeV \pm 30\ MeV$ for $0.001 \leq \mu^2/E_{cm}^2 \leq 0.020$. The predictions of an Abelian vector theory complete to O(α_A^2) are not compatible with the observed dynamics of jet production. The experimental evidence for the energy dependence of α_s, obtained from 3-jet event production rates observed in the center of mass energy range between 22 GeV and 56 GeV, does not depend on the detailed choice of μ^2.

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Introduction

Quarks and gluons, which are also called partons, are the basic constituents of Quantum Chromodynamics (QCD), a renormalisable, nonabelian gauge theory of the strong interactions [**1**]. Within the framework of QCD, these quarks and gluons are confined by a coupling strength that increases at large distances or small momentum transfers, so that they cannot exist as free particles by themselves. Instead, they fragment and decay into the elementary particles which are observed in deep inelastic scattering experiments. This process of hadronisation cannot be described in terms of perturbative QCD, thus rendering the comparison of experimental data with the predictions of QCD rather difficult. At high enough energies, however, hadrons appear as jets of particles which reflect the dynamics of the initial quarks and gluons. Therefore analyses of multijet event production in hadronic final states of e^+e^- annihilations provide a powerful tool to test the theoretical predictions of QCD.

Recently, detailed studies of multijet event production rates in the e^+e^- annihilation center of mass energy range of 22 GeV to 56 GeV provided experimental evidence for the existence of two particular attributes predicted by QCD, namely the specific energy dependence of the strong coupling strength, α_s, [**2-7**] and signatures of the gluon self coupling [**6**]. Both these effects are manifestations of the nonabelian character of QCD, and the fact that these effects can be observed in the experimental data enlarges the confidence in the predictive power of this theory.

There is, however, not only perfect agreement between the data and perturbative QCD. Studies of jet production rates in e^+e^- annihilations also revealed that $O(\alpha_s^2)$ QCD models underestimate the production rates of 4-jet events [**8;5**]. The origin of this deficiency was shown to be caused by an apparent lack of 4-parton events in the underlying $O(\alpha_s^2)$ QCD calculations and could not be accounted for by varying the fragmentation parameters of these models. Recently, Kramer and Lampe pointed out that a more realistic description of the 4-jet rates can be achieved in $O(\alpha_2^2)$ if the calculations are carried out using rather small renormalisation scales $\mu^2 << E_{cm}^2$ instead of $\mu^2 = E_{cm}^2$ [**9;10**]. Since determinations of α_s and other QCD related experimental studies in e^+e^- annihilations mainly depend on $O(\alpha_s^2)$ QCD models, it is necessary to further investigate the origin and potential cures of known theoretical deficiencies.

In this paper, experimental jet production rates are compared to the most recent $O(\alpha_s^2)$ calculations of Kramer and Lampe [**9;10**]. The scale parameter $\Lambda_{\overline{MS}}$ and the renormalisation scale μ^2 are adjusted in order to optimise the description of the experimental jet rates. In order to demonstrate the sensitivity of this analysis to the specific properties of QCD, the data are also compared to an Abelian vector theory complete to $O(\alpha_A^2)$. These parts of the analysis are mainly based on the data collected with the Mark II detector at PEP (SLAC), recorded at 29 GeV center of mass energy [**7**]. Furthermore, 3-jet event production rates observed by several experiments in a wide range of center of mass energies will be studied in order to investigate to which degree the current experimental evidence for the energy dependence of α_s [2-7] depends on the definition of the renormalisation scale μ^2 in $O(\alpha_s^2)$ QCD calculations.

QCD and Jets in Finite Order Perturbation Theory.

In second order perturbation theory, the strong coupling strength, α_s, is given by

$$\alpha_s(\mu^2) \;=\; \frac{12\pi}{b_0 \cdot ln(\mu^2/\Lambda^2_{\overline{MS}}) \;+\; \frac{b_1}{b_0} \cdot ln\; ln(\mu^2/\Lambda^2_{\overline{MS}})}\;, \tag{1}$$

with $b_0 \;=\; (33-2N_f)$ and $b_1 \;=\; (918-114N_f)$. N_f is the number of quark flavours produced and is taken to be 5 throughout this analysis. $\Lambda_{\overline{MS}}$ is the QCD scale parameter which must be determined by experiment. Until recently, in e^+e^- annihilation the renormalisation scale μ^2 was usually chosen to be the center of mass energy of the hadronic system ($\mu^2 = E^2_{cm}$), leaving $\Lambda_{\overline{MS}}$ as the only free parameter. Theoretically, however, μ^2 is not determined at all and almost any choice of scale that may be defined in the physical process under consideration would be suitable. In the most general case, one uses a definition like

$$\mu^2 = f \cdot E^2_{cm}, \tag{2}$$

where f is a dimensionless factor defining the renormalisation scale. Renormalisation scales which do *not* scale with E^2_{cm} are not considered here since such a choice would render specific predictions of QCD, as e.g. the energy dependence of α_s, rather meaningless.

The renormalisation scale μ^2 is often called an "unphysical" parameter because it is a parameter of which the *true* result, if calculated to *all* orders in perturbation theory, is independent. Finite order calculations, however, depend on the detailed choice of μ^2, which has no direct physical interpretation but is an artifact of the incomplete perturbation series. It is thus appropriate to experimentally determine the renormalisation scale which provides the optimal description of the observables under study. From a theoretical point of view, the variation of the renormalisation scale is equivalent to modifying the contributions of higher-order corrections to a given perturbative order [10;11]. Therefore, in the theoretical language, "optimisation" of the renormalisation scale commonly means to minimise the (usually unknown) higher order corrections to a given, calculated order. Note that there is a certain difference between the experimental and theoretical motivation to optimise the renormalisation scale. Minimising higher order contributions to a given perturbative order (if it can reliably be done at all) must not automatically imply the best possible description of the data, and vice versa. For further discussions about the theoretical interpretation of renormalisation scales in finite order perturbation theory, see [10] and [11] and references quoted therein.

In second order perturbation theory, the relative production rates R_n of 2-, 3- and 4-

parton events are quadratic functions of α_s:

$$\begin{aligned} R_2 &\equiv \frac{\sigma_2}{\sigma_{tot}} = 1 + C_{2,1} \cdot \alpha_s + C_{2,2} \cdot \alpha_s^2 \\ R_3 &\equiv \frac{\sigma_3}{\sigma_{tot}} = C_{3,1} \cdot \alpha_s + C_{3,2} \cdot \alpha_s^2 \\ R_4 &\equiv \frac{\sigma_4}{\sigma_{tot}} = C_{4,2} \cdot \alpha_s^2, \end{aligned} \tag{3}$$

where σ_{tot} is the total hadronic cross section and σ_n are the corresponding cross sections of n-parton event production. $C_{n,k}$ are the k−th order coefficients for n−jet production. While the leading order coefficients ($C_{2,1}$, $C_{3,1}$, $C_{4,2}$) are only functions of the jet resolution criteria chosen in the theoretical calculations, the next-to-leading order coefficients ($C_{2,2}$, $C_{3,2}$) are also functions of the scale factor f [**10;11**] defined in Eq. 2. It is this additional occurrence of the renormalisation scale, supplementary to the term $\mu^2/\Lambda^2_{\overline{MS}}$ in Eq. 1, which enables the optimisation of both μ^2 and $\Lambda_{\overline{MS}}$ in the next-to-leading order of perturbation theory. In leading order, a change of μ^2 could always be absorbed by readjusting $\Lambda_{\overline{MS}}$.

Criteria to define resolvable jets are introduced in order to calculate finite jet production cross sections. A commonly used method is to require the square of the scaled invariant mass of any pair of partons i and j,

$$y_{ij} = \frac{M^2_{ij}}{E^2_{cm}}, \tag{4}$$

to satisfy the relation

$$y_{ij} \geq y_{min}, \tag{5}$$

where y_{min} is the cut-off parameter defining *resolvable* partons.

Experimental Definition of Jets.

The above definition of resolvable partons has been adapted to an experimental jet finding algorithm [**8**], which works as follows. In each hadronic event, the squares of the scaled pair masses,

$$y_{kl} = \frac{M^2_{kl}}{E^2_{vis}}, \tag{6}$$

are calculated for all pairs of particles k and l, where E_{vis} is the visible energy of the event. Charged particles are usually assumed to be pions and neutrals to be photons. Those particles i and j with the smallest pair mass are replaced by a pseudoparticle with four-momentum ($\mathbf{p}_i$+ $\mathbf{p}_j$). The procedure is then repeated until the scaled masses of all particle or pseudoparticle pair-combinations exceed a certain threshold value y_{cut}:

$$y_{kl} \;\geq\; y_{cut}, \tag{7}$$

and the remaining clusters are called "jets". To calculate the pair mass M_{kl}, the expression

$$M^2_{kl} = 2 \cdot E_k \cdot E_l \cdot (1 - cos\Theta_{kl}) \tag{8}$$

is used. Studies with Monte Carlo generated events showed that this choice of M_{kl} provides close agreement between jet and parton multiplicities at comparable values of y_{cut}, the experimental cutoff in the jet finding algorithm, and y_{min}, the QCD cutoff parameter for massless partons in the perturbative QCD calculations [**3-7**].

This agreement was demonstrated for the Mark-II data at $E_{cm} = 29$ GeV over a wide range of invariant jet masses ($0.015 \leq y_{cut} \leq 0.140$) [**7**] and for the JADE data in a range of center of mass energies (22 GeV $\leq E_{cm} \leq$ 47 GeV) for $y_{cut} \geq 0.06$ [**3;4**]. Even in studies where neutral particles were not included in the jet finding algorithm [**5**], corrections for detector resolution and acceptance turned out to alter the experimental jet production rates by only a few percent. As a conclusion from these model studies, it is feasible to compare experimental jet production rates, defined by the jet algorithm as described above, directly with theoretical calculations of parton production rates. Possible corrections for fragmentation effects are small and can be absorbed in small readjustments ($\approx 6\%$) of α_s. These findings are also supported by the theoretical conjecture of "Local Parton Hadron Duality" [**12;13**].

The Experimental Optimisation of $\Lambda_{\overline{MS}}$ and μ^2 in O(α_s^2).

The relative production rates R_n of n-jet events, as reconstructed with the jet algorithm described above and presented by the Mark-II collaboration for their data sample at $E_{cm} = 29$ GeV [**7**], are shown in Fig. 1 as a function of the jet resolution parameter y_{cut}. The statistical errors of the data are omitted in the figure since they are smaller than the printed symbols. For reasons of clarity, the small rates of 5-jet events observed at $y_{cut} \leq 0.04$ are not included in Fig.1, but are, together with the production rates of 4-jet events and on a logarithmic scale, shown in Fig. 2. Monte Carlo studies and a software simulation of the Mark-II detector revealed that effects due to fragmentation, initial state photon radiation and the finite acceptance and resolution of the detector are sufficiently small such that the experimental n-jet event rates can directly be compared with theoretically calculated n-parton rates [**7**]. Therefore in Figs. 1 and 2 the data are compared to the second order QCD calculations of Kramer and Lampe [**9;10**] for three different choices of renormalisation scales: $\mu^2 = E_{cm}^2$, $\mu^2 = 0.005 \cdot E_{cm}^2$ and $\mu^2 = 0.0017 \cdot E_{cm}^2$. For these three choices, the remaining free parameter, $\Lambda_{\overline{MS}}$, was adjusted such that the 2- and 3-jet data above $y_{cut} = 0.04$ are well described. This results in $\Lambda_{\overline{MS}} = 213$ MeV, 86 MeV and 95 MeV, respectively, corresponding to $\alpha_s(\mu^2) =$ 0.145, 0.217 and 0.267 (c.f. Eq. 1). Assuming that the same renormalisation scale factor f can be applied to both R_2 and R_3, the theoretical expectations for R_2 and R_3 are calculated according to Eq. 3 and [**10**], while R_4 is determined from the O(α_s^2) unitarity requirement $R_4 = 1 - R_2 - R_3$. Comparing the theoretical curves with each other and with the data results in the following observations:

(1) For $y_{cut} > 0.06$, the relative 2- and 3-jet production rates seen in the data are well described in all three cases with χ^2 values around 1 per degree of freedom.

(2) The relative production rate of 4-jet events is largely underestimated in the case $\mu^2 =$

E_{cm}^2, as already reported in [**8**;**5**], but is well reproduced by the calculations with $\mu^2 = 0.0017 \cdot E_{cm}^2$.

(3) Below $y_{cut} \approx 0.06$, large differences in R_2 and R_3 between the three sets of theoretical curves are observed. While the case with $\mu^2 = E_{cm}^2$ does not even provide a coarse description of the data and reaches an "unphysical" region of negative 2-jet production rates at $y_{cut} \approx 0.01$, the case with $\mu^2 = 0.0017 \cdot E_{cm}^2$ provides an almost perfect description of R_2 and R_3 down to the lowest values of y_{cut}.

(4) The fitted values for $\Lambda_{\overline{MS}}$ depend on the choice of renormalisation scale: $\Lambda_{\overline{MS}}$ decreases by more than a factor of two when changing μ^2 from E_{cm}^2 to the lower values displayed in Fig. 1.

The production of 5-jet events is a process which is only described in third and higher order perturbation theory. Most recently, calculations of the $O(\alpha_s^3)$ Born cross section for 5-parton event production became available [**14**]. The predictions of these calculations are also presented in Fig. 2 for values of α_s that correspond to the $O(\alpha_s^2)$ calculations as shown in Figs. 1 and 2. For $\mu^2 = 0.0017 \cdot E_{cm}^2$ and $\Lambda_{\overline{MS}} = 95$ MeV, the values which reproduce the 2-, 3- and 4-jet rates, 5-parton event production rates are predicted which are about 50% smaller than the experimental rates of 5-jet events. Note that this can only be regarded as a rough estimate of the QCD predictions of R_5, since the application of a renormalisation scale, optimised in $O(\alpha_s^2)$, to an $O(\alpha_s^3)$ calculation may not be a meaningful procedure. Furthermore, model calculations show that nonperturbative fragmentation effects may also account for about 50% of the observed 5-jet events.

In order to quantify the significance of observations (1) to (4) described above, both $\Lambda_{\overline{MS}}$ and a common scale factor f (see Eq. 2) are calculated from jet production rates at single values of y_{cut}. This is possible in regions where at least three separate jet classes are resolved, since otherwise the number of degrees of freedom is less than two and the determination of two parameters is not possible. For the data at $y_{cut} = 0.06$, this determination results in $f = 0.0017 \pm 0.0003$ and $\Lambda_{\overline{MS}} = (95 \pm 5)$ MeV. Correlations between both parameters are taken into account in the error calculations. Within these errors, determinations at other values of y_{cut} always agree with these results and with each other, even in the region of $y_{cut} < 0.04$, where a description of the data by pure second order QCD calculations would not necessarily be expected. This is demonstrated in Fig. 3, where the results for $\Lambda_{\overline{MS}}$ and f are shown as a function of the jet resolution parameter y_{cut}. Note that both $\Lambda_{\overline{MS}}$ and f are well determined even at $y_{cut} = 0.08$, where only 0.4 % of the data are classified as 4-jet events (c.f. Fig. 2) but the possibility of faking 4-jet events by other processes essentially vanishes [**2**;**5**;**8**].

Although the renormalisation scale which provides an optimal description of the experimental jet rates can be determined quite accurately, it is, from a theoretical point of view, not obvious that this scale also minimises the unknown higher order contributions to the $O(\alpha_s^2)$ calculations. It is therefore important to evaluate how experimental determinations of $\Lambda_{\overline{MS}}$, the "physical" parameter of QCD, depend on the specific choice of the renormalisation scale in the underlying $O(\alpha_s^2)$ calculations. In Figure 4 the results for $\Lambda_{\overline{MS}}$ and the corresponding χ^2, determined in a one-parameter fit to the jet production rates observed at $y_{cut} = 0.06$, are shown as a function of the scale factor f. While $\Lambda_{\overline{MS}}$ strongly depends on f around $f = 1$, which was the standard choice for experimental determinations of $\Lambda_{\overline{MS}}$ and α_s in the past, it decreases by more than 50% and reaches a relatively stable region for $0.001 < f < 0.01$, where χ^2 also reaches zero at $f = 0.0017$. The sharp discrimination in χ^2 is mainly determined by the rate of 4-jet events observed in the data, while the actual value of $\Lambda_{\overline{MS}}$ is most sensitive to the ratio of 3- and 2-jet event rates. In further studies it was found that $\Lambda_{\overline{MS}}$, even if it is determined at y_{cut} above 0.10 where no 4-jet events are resolved, follows the same universal curve as a function of f as seen in Fig. 4, although f can no longer be optimised by means of minimal χ^2 in such cases.

Up to this point, the assumption was made that both R_2 and R_3 obey the same renormalisation scale. Theoretically, however, different physical observables may require different renormalisation scales in order to minimise the (usually unknown) higher order contributions to the respective finite order calculations [**10**;**11**]. To evaluate this possibility experimentally, simultaneous determinations of $\Lambda_{\overline{MS}}$ and two separate scale factors f_2 and f_3 for 2-jet and 3-jet production rates, respectively, have been performed. Since this study involves the adjustment of three free parameters, it is necessary to perform the fits to experimental jet rates in ranges of y_{cut} rather than at single y_{cut} values. Using the jet rates observed for $0.04 \leq y_{cut} \leq 0.08$ and assuming unitarity of the sum of 2-, 3- and 4-jet rates as given in $O(\alpha_s^2)$, the results are[1] $\Lambda_{\overline{MS}} = (97 \pm 2)$ MeV, $f_2 = 0.00172 \pm 0.00005$ and $f_3 = 0.00170 \pm 0.00004$. From these and similar results obtained when selecting other ranges of y_{cut} it is concluded that, from an experimental point of view, the renormalisation scales for 2-jet and 3-jet event production in $O(\alpha_s^2)$ are similar in size and are probably identical within 10%.

So far, the analysis has been carried out by comparing experimental jet production rates directly with calculated jet cross sections, assuming that nonperturbative fragmentation effects and corrections due to initial state radiation, detector acceptance and resolution are small and can be neglected [**7**]. In order to estimate the contribution of systematic uncertainties imposed by this procedure, adjustments of $\Lambda_{\overline{MS}}$ and the scale factor f were also performed using data samples which were explicitly corrected for these effects according to various QCD and fragmentation model calculations. As an example, the experimental jet rates at $y_{cut} = 0.06$, uncorrected as well as corrected for fragmentation effects, initial state radiation and the acceptance of the Mark-II detector, are listed in Table 1 together with the corresponding fit results of $\Lambda_{\overline{MS}}$ and f. The corrections were derived from the following model calculations: (A) the Lund QCD shower model [**15**] as discussed in [**7**] and (B) a version of the Lund $O(\alpha_s^2)$ model [**15**] which was modified to incorporate the jet cross sections of Kramer and Lampe [**10**], with $\Lambda_{\overline{MS}} = 95$ MeV and $f = 0.0017$. As can be seen in Table 1,

[1]**The statistical fit errors shown are probably underestimated since correlations between jet rates at different values of y_{cut} are not accounted for in this error calculation.**

the different model corrections affect the data only slightly, and the results of $\Lambda_{\overline{MS}}$ and f are compatible with each other. The same conclusions are derived at other values of y_{cut}. Thus the results for $\Lambda_{\overline{MS}}$ and f, adjusted in $O(\alpha_s^2)$ to describe the experimental jet production rates, are quoted as

$$f = 0.0020 \pm^{0.0025}_{0.0009}$$

$$\Lambda_{\overline{MS}} = 95\ MeV\ \pm 18\ MeV.$$

The errors include the statistical errors, the variation of the results when determined for various jet mass regions and an estimate of systematic uncertainties from corrections for fragmentation and detector acceptance effects. Systematic errors due to theoretical uncertainties in the $O(\alpha_s^2)$ calculations are not included yet, but will be further discussed in the summary.

Experimental Tests of the Nonabelian Nature of QCD

The nonabelian structure of QCD manifests itself in the existence of the gluon self coupling and in the prediction that α_s logarithmically decreases with increasing energies. While evidence for the specific energy dependence of α_s [**2-7**] and first signatures of the gluon self coupling [**6**] in e^+e^- annihilations have been reported only recently, comparisons of data with the predictions of an abelian vector theory date back to the year 1977 [**16**]. The abelian vector theory, as an alternative to QCD, does not include the process of gluon self coupling and is therefore similar to Quantum Electrodynamics (QED), the theory of the electromagnetic interactions, where the effective coupling constant *in*creases with energy. In order to further evaluate the experimental evidence for the specific predictions and the validity of QCD, the implications of choosing and optimising the renormalisation scale μ^2 in both perturbative QCD and "QED" will be explored in the following paragraphs.

In 1982, the JADE collaboration presented a first comparison of differential 3-jet distributions with the predictions of QCD and an abelian vector theory, both calculated to $O(\alpha_s^2)$ and to $O(\alpha_A^2)$, respectively [**18**]. While QCD provided a good description of the data, the abelian vector theory predicted production cross sections for 3-jet events which were an order of magnitude too small or even negative. However, in the theoretical calculations used in that analysis, certain terms to $O(\alpha_s^2)$ have been neglected [**19**] and the renormalisation scale μ^2 was fixed to $\mu^2 = E_{cm}^2$.

Repeating similar calculations but using the complete second order QCD calculations of Kramer and Lampe [**9;10**], with the group constants modified according to the abelian vector theory[1], results in 3-jet cross sections similar to those presented in [**18**] if the scale μ^2 is fixed to E_{cm}^2. As in the case of QCD, however, μ^2 is an unphysical parameter which is not theoretically determined. In fact, if μ^2, in addition to adjusting the abelian coupling constant

[1] A "QED" like theory for the *strong* interactions can easily be obtained from the corresponding QCD calculations by replacing the group constants of SU(3) (QCD) by the ones of U(1) ("QED"): $C_F = 1$, $N_C = 0$ and $T_R = 6 \cdot T_R^{QCD}$ [**16;17**].

α_A, is also allowed to vary, the experimental 2-, 3- and 4-jet rates for fixed values of y_{cut} can be exactly described. For example, in Fig. 5 the jet production rates observed by Mark-II at 29 GeV center of mass energy are compared to both QCD and "QED" calculations in second order perturbation theory. The QCD parameters $\Lambda_{\overline{MS}}$ and μ^2 and the "QED" parameters α_A and μ^2 are adjusted such that the jet rates at $y_{cut} = 0.04$ are described, resulting in $\Lambda_{\overline{MS}} = (95 \pm 4)$ MeV, $\mu^2 = 0.0016 \pm 0.0002$ and $\alpha_A = 0.315 \pm 0.008$, $\mu^2 = 0.0025 \pm 0.0002$, respectively. In the case of "QED", however, the 2- and 3-jet data are only described for this particular value of y_{cut}, while the prediction over ranges of y_{cut} fails to reproduce the data. The same observations can be done when adjusting the theoretical parameters at other values of y_{cut}: while QCD always provides a good description of the data as shown in Fig. 5, "QED" is not compatible with the data over *ranges* of y_{cut}.

It is thus concluded that the abelian vector theory in second order perturbation theory in the $\overline{MS}$ renormalisation scheme, even if the renormalisation scale is treated as an additional free parameter, is not adequate to fully reproduce the experimental data. Furthermore, the differences between QCD and "QED" observed in Fig. 5 show that experimental studies of jet production rates are indeed sensitive to the specific structure of QCD. The good agreement that can be achieved between data and QCD is therefore unlikely to be accidental.

As a further field of direct tests of QCD, investigations of jet production rates make it possible to study the energy dependence of the strong coupling strength without determining explicit values of α_s [**2-7**]. According to Eq. 3, the energy dependence of jet production rates for constant y_{cut} is determined only by the energy evolution of α_s (see Eq. 1). The 3-jet event production rates observed by JADE [**4**], TASSO [**5**], AMY [**6**] and Mark-II [**7**] in the center of mass energy range from 22 GeV to 56 GeV for various values of y_{cut}, are shown in Fig. 6. The data are compared to the calculations of Kramer and Lampe for two renormalisation scales with $f = 1$ and $f = 0.0017$. The corresponding fit values for $\Lambda_{\overline{MS}}$ are optimised using all the available data at $y_{cut} \geq 0.06$, with the exception of data at $E_{cm} =$ 22 GeV where fragmentation effects could already bias the jet rates [**3-5**]. The theoretical curves are displayed for the central values of the fit results, $\Lambda_{\overline{MS}} = (205 \pm 13)$ MeV and (91 ± 5) MeV for $f = 1$ and 0.0017, respectively.

The data are compatible with each other and with the theoretical expectations based on an energy dependent coupling strength. While both QCD calculations describe the energy dependence as well as the absolute normalisation of the data well for all $y_{cut} \geq 0.06$, only the calculation with the lower renormalisation scale also provides a simultaneous description at $y_{cut} = 0.04$. The assumption of an energy independent coupling strength, however, is not compatible with the data: in this case, R_3 is expected not to depend on the center of mass energy[1].

The following fit results, obtained at $y_{cut} = 0.08$ where the most data are available, quantitatively express the significance of these observations. The hypothesis of an energy

[1]The validity of this expectation was verified by QCD and fragmentation model calculations.

independent coupling constant results in $\chi^2 = 29.7$ for 7 degrees of freedom corresponding to a confidence level (CL) of 10^{-4}, while $\chi^2 = 6.1$ and 5.5 (CL = 0.5 and 0.6) for QCD with $\mu^2 = E_{cm}^2$ and $\mu^2 = 0.0017 \cdot E_{cm}^2$, respectively. The preference for calculations based on low renormalisation scales is not very obvious from the energy dependence of 3-jet production, but the case of an energy *in*dependent α_s is clearly ruled out. Note that the abelian vector theory, in contrast to QCD, predicts that the coupling strength, α_A, *in*creases with increasing center of mass energy. While quantitative calculations about the expected increase of α_A are still to be done, it is obvious that the experimental evidence against a coupling constant that rises with energy is more significant than for the case of an energy independent coupling as described above.

So far, the experimental evidence for the running of α_s apparently does not depend on the detailed choice of μ^2 in the $O(\alpha_s^2)$ calculations. In the previous section it was shown, however, that the 2- and 3-jet rates for $y_{cut} \leq 0.04$ and the 4-jet rates in general, observed at $E_{cm} = 29$ GeV, could only be consistently described with $\mu^2 \approx 0.002 \cdot E_{cm}^2$. While some of these effects are visible already in Fig. 6 for the 3-jet data at $y_{cut} = 0.04$, it should now be possible also to compare the energy dependence of the observed 4-jet rates with the expectations of QCD. Therefore in Fig. 7 the production rates of 4-jet events, observed by JADE [4] and Mark-II [7] in the center of mass energy range between 22 GeV and 46.7 GeV, are shown together with the $O(\alpha_s^2)$ QCD expectations, calculated for $\mu^2 = 0.0017 \cdot E_{cm}^2$ and $\Lambda_{\overline{MS}} = 91$ MeV. The 4-jet rates significantly decrease with increasing center of mass energy and are well described by the theoretical expectations, where the rate of 4-parton events is proportional to α_s^2. The significance for the running α_s from 4-jet production is comparable to the results derived from the energy dependence of R_3. Note that for the case of $\mu^2 = E_{cm}^2$ a simultaneous description of both R_3 and R_4 is not possible at all, since $\Lambda_{\overline{MS}} > 2$ GeV would be required to match the 4-jet rates while $\Lambda_{\overline{MS}} = 205$ MeV in order to describe the 3-jet rates.

Summary and Discussion.

The comparison of jet production rates, observed at $E_{cm} = 29$ GeV for a large range of minimum scaled jet masses, with theoretical jet production cross sections, calculated in complete second order QCD perturbation theory, reveals a number of new observations. Optimisations of $\Lambda_{\overline{MS}}$ and the renormalisation scale μ^2 to describe the observed 2-, 3- and 4-jet production rates result in $\mu^2 = (0.0020^{+0.0025}_{-0.0009}) \cdot E_{cm}^2$ and $\Lambda_{\overline{MS}} = 95\ MeV\ \pm 18\ MeV$. With these parameters, the observed ratios of 2-, 3- and 4-jet events, down to the smallest jet masses analysed, are consistently described by the $O(\alpha_s^2)$ calculations. This cannot be achieved with calculations using $\mu^2 = E_{cm}^2$, where the deficiency of 4-jet events is an especially serious problem which could not be accounted for within the current $O(\alpha_s^2)$ QCD and fragmentation models. The abelian vector theory in $O(\alpha_A^2)$ also results in more realistic jet cross sections if the renormalistion scale is chosen to be $\mu^2 \approx 0.0025 \cdot E_{cm}^2$, but ultimately cannot consistently describe the experimental jet rates for ranges of jet pair masses.

Combining the 3-jet event production rates observed by several experiments in the center of mass energy range between 22 GeV and 56 GeV, evidence for the energy dependence of α_s, in good agreement with the predictions put forward by the nonabelian nature of QCD, is obtained. This evidence does not depend on the detailed choice of renormalisation scales in $O(\alpha_s^2)$ QCD. A consistent description of the energy dependence of both 3-jet and 4-jet production rates, however, is only possible with $\mu^2 \approx 0.002 \cdot E_{cm}^2$ and $\Lambda_{\overline{MS}} \approx 95$ MeV, which agrees with the results obtained at $E_{cm} = 29$ GeV as given above.

The question remains, however, whether renormalisation scales as low as $0.002 \cdot E_{cm}^2$, corresponding to $\mu = 1.3$ GeV at $E_{cm} = 29$ GeV, are compatible with and expected by perturbation theory. While an accurate answer to this question requires knowledge about the behaviour of the higher ($\geq 3^{rd}$) order terms in the perturbation series, which is not available so far, some intuitive arguments towards compatibility and theoretical expectations can be made.

In general, confidence in the validity of perturbative QCD calculations requires that the relative size of the higher order terms for the respective observable are "reasonably" small [**11**]. The definition of "reasonably small" is, to some extend, a matter of personal taste which may not be completely satisfied by the current results in $O(\alpha_s^2)$. Nevertheless, the next-to-leading order contributions to 3-jet production ($C_{3,2}$ in Eq. 3) are smaller for the optimised renormalisation scale than for the case $\mu^2 = E_{cm}^2$, as is demonstrated in Table 2. One should keep in mind, however, that smaller second order correction factors do not automatically imply that the contributions of all the unknown higher orders are also small.

Theoretical attempts to optimise the renormalisation scale for jet production in order $O(\alpha_s^2)$ QCD calculations also lead to small values of μ^2. According to Stevenson's *Principle of Minimal Sensitivity* [**20**], Kramer and Lampe determined the optimised renormalisation scale as $\mu^2 \approx 0.005 \cdot E_{cm}^2$ [**10**]. As was demonstrated in Fig. 4, the experimental results of $\Lambda_{\overline{MS}}$ are indeed least sensitive to small changes of $ln(\mu^2)$ in this region, while otherwise $\Lambda_{\overline{MS}}$ strongly depends on the choice of μ^2. A more sophisticated approach was proposed by Brodsky, Lepage and Mackenzie [**11**]. It is based on the requirement of completely absorbing terms which depend on the number of fermions in the theory, N_f, into the coupling constant α_s, such that the next-to-leading order coefficient of the observable under study (as $C_{2,2}$ and $C_{3,2}$ in Eq. 3) is independent of N_f. This procedure determines the renormalisation scale appropriate for the particular process. Applied to the jet cross sections of Kramer and Lampe, the method of Brodsky et al. leads to optimised renormalisation scales of $\mu_{opt}^2 \approx 0.10 \cdot y_{min} \cdot E_{cm}^2$ and $\mu_{opt}^2 \approx 0.09 \cdot y_{min} \cdot E_{cm}^2$ for 2- and 3-jet production, respectively. Again, this is compatible with the experimental results on the scale factor f and with the observation that the renormalisation scales for both jet classes are identical within 10%. An explicit dependence of f on y_{min} is however not observed.

Another important observation is that the experimental value of $\Lambda_{\overline{MS}}$ explicitly depends on the choice of renormalisation scale in the $O(\alpha_s^2)$ calculations. This implies that a rather large systematic uncertainty, of the order of a factor of two in $\Lambda_{\overline{MS}}$, must be accounted for in determinations of $\Lambda_{\overline{MS}}$ if the renormalisation scale could not be constrained at all. Note that theoretical uncertainties due to unknown renormalisation scales have not been considered in experimental determinations of $\Lambda_{\overline{MS}}$ in e^+e^- annihilations so far [**21**]. However, the data themselves were shown to give a definite answer on the preferred renormalisation scale in $O(\alpha_s^2)$. According to the results of this analysis and the theoretical predictions of Brodsky et al. [**11**], a reasonable constraint on the renormalisation scale optimised for 2- and 3-jet event cross sections in $O(\alpha_s^2)$ is $0.001 \leq \mu^2/E_{cm}^2 \leq 0.020$. This translates into a theoretical uncertainty of $\Lambda_{\overline{MS}}$ by $\pm 12\%$ (see Fig. 4). Further theoretical uncertainties, which were not addressed in this analysis but should generally be accounted for, originate from different approximations in the second order QCD expression for α_s ($\pm$ 6%) [**21**] and from the choice of parton dressing schemes in $O(\alpha_s^2)$ calculations ($\pm$ 20%) [**22**]. The experimental and theoretical errors on the determination of $\Lambda_{\overline{MS}}$ described above are listed in Table 3. Adding them in quadrature, the final result of $\Lambda_{\overline{MS}}$, obtained from the 2-, 3- and 4-jet production rates observed at E_{cm} = 29 GeV [**7**], is quoted as

$$\Lambda_{\overline{MS}} \;=\; 95\ MeV\ \pm\ 30\ MeV,$$

assuming the production of five quark flavours[1] ($N_f = 5$) and $O(\alpha_s^2)$ renormalisation scales of $\mu^2 = (0.001 \text{ to } 0.020) \cdot E_{cm}^2$. This value of $\Lambda_{\overline{MS}}$ is about 50% smaller than the corresponding result obtained when using $\mu^2 = E_{cm}^2$. Note that the error of $\Lambda_{\overline{MS}}$ includes theoretical uncertainties which usually were not accounted for in determinations of α_s in e^+e^- annihilations [**21**] so far. While this error seems to be relatively small, the corresponding systematic uncertainty on the value of α_s, determined for E_{cm} = 29 GeV and using Eq. 1 with $\Lambda_{\overline{MS}}$ and μ^2 as given above, is much larger: $0.17 < \alpha_s(\mu^2) < 0.33$. The large systematic uncertainty in α_s predominantly depends on the range of μ^2 considered in the calculation.

Concluding the experimental approach to optimise the renormalisation scale μ^2 in $O(\alpha_2^2)$ QCD calculations of jet production, the advantages of utilising small scales like $\mu^2 \approx 0.002 \cdot E_{cm}^2$ rather than the usual choice of $\mu^2 = E_{cm}^2$ are rather obvious. In this respect, the results of this investigation unveil several positive aspects for present and future studies of QCD in e^+e^- annihilations: over a large range of center of mass energies, $O(\alpha_s^2)$ perturbative QCD calculations are able to describe the dynamics of hard jet production but may also well extend into the region of soft gluon radiation and fragmentation. The successful parametrisation of jet rates at the present center of mass energies enlarges the confidence in predictions for future experiments at higher energies where, for example, detailed knowledge about the QCD-"background" is an important pre-requisite to search for new, heavy particles. The strong dependence of some experimental observables and of the experimental value of $\Lambda_{\overline{MS}}$ on variations of an "unphysical" parameter like μ^2, however, should cause at least a reasonable

[1]Repeating the determination of μ^2 and $\Lambda_{\overline{MS}}$ for $N_f = 4$ and $N_f = 3$ results in $\Lambda_{\overline{MS}}$ = 130 MeV and $\Lambda_{\overline{MS}}$ = 170 MeV, respectively, while μ^2 shows no dependence on N_f within the statistical fit errors.

level of caution in the comparison and interpretation of certain experimental results, as for instance determinations of α_s. In order to improve this situation, calculations of jet rates to complete third or even higher order perturbation theory are necessary.

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	raw data	corrected (A)	corrected (B)
$R_2(\%)$	67.2 ± 0.5	67.0 ± 0.8	65.2 ± 0.9
$R_3(\%)$	31.5 ± 0.5	31.7 ± 0.8	33.6 ± 0.9
$R_4(\%)$	1.31 ± 0.13	1.34 ± 0.20	1.18 ± 0.20
$\Lambda_{\overline{MS}}$(MeV)	$94.5^{+5.2}_{-4.9}$	$96.9^{+8.1}_{-7.4}$	$107.6^{+7.7}_{-6.7}$
f	$0.0017^{+.0004}_{-.0003}$	$0.0017^{+.0007}_{-.0004}$	$0.0029^{+.0030}_{-.0010}$

Table 1. Relative n-jet production rates R_n at $y_{cut} = 0.06$ from Mark-II [**7**] and fit results of $\Lambda_{\overline{MS}}$ and of the energy scale factor f, for the uncorrected data as well as data corrected for fragmentation, initial state radiation and detector acceptance effects. The corrections are calculated using two different types of model calculations; for details see text.

y_{min}	$C_{3,1}$	$C_{3,2}$ ($f = 1$)	$C_{3,2}$ ($f = 0.0017$)
0.02	3.886	8.075	-7.045
0.04	2.266	6.390	-2.412
0.06	1.516	4.675	-1.254
0.08	1.074	3.384	-0.799
0.10	0.783	2.456	-0.577
0.12	0.578	1.785	-0.452

Table 2. O(α_s^2) QCD coefficients for 3-jet production, $R_3 = C_{3,1} \cdot \alpha_s + C_{3,2} \cdot \alpha_s^2$, calculated for two different energy scales, $\mu^2 = f \cdot E_{cm}^2$; from Kramer and Lampe [**9**;**10**].

Error source	relative error
statistical error	10%
fragm. and detector corrections	15%
α_s-formula in $O(\alpha_s^2)$	6%
scale uncertainty in $O(\alpha_s^2)$	12%
parton dressing scheme	20%
overall uncertainty	30%

Table 3. Experimental and theoretical uncertainties in the determination of $\Lambda_{\overline{MS}}$ as described in the text.

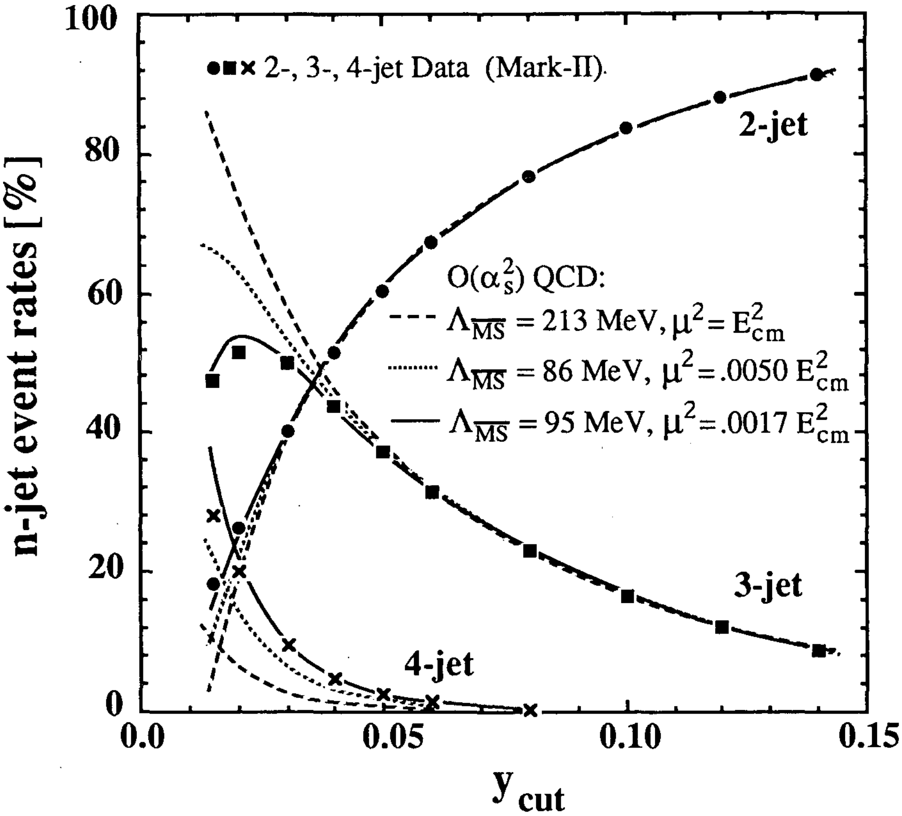

Fig. 1. Two-, three- and four-jet event rates observed at $E_{cm} = 29$ GeV as a function of the jet resolution parameter y_{cut}, compared with the $O(\alpha_s^2)$ QCD calculations by Kramer and Lampe for different renormalisation scales μ^2.

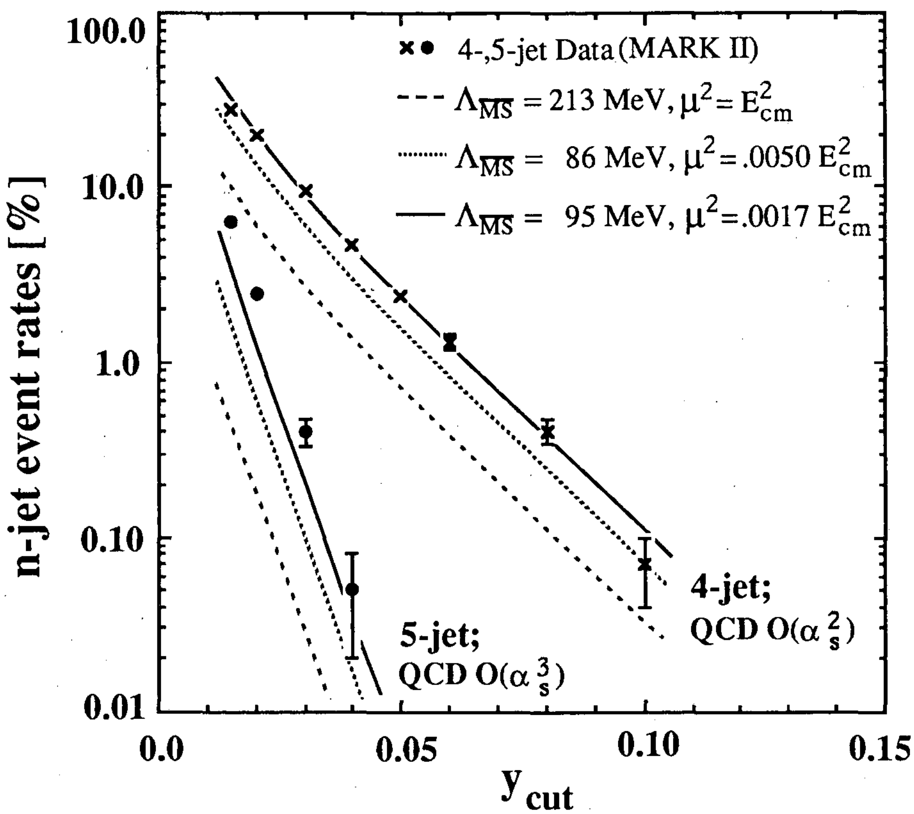

Fig. 2. Four- and five-jet event rates observed at $E_{cm} = 29$ GeV, compared with the $O(\alpha_s^2)$ QCD calculations as shown in Fig. 1 and with recent calculations on 5-jet production to $O(\alpha_s^3)$.

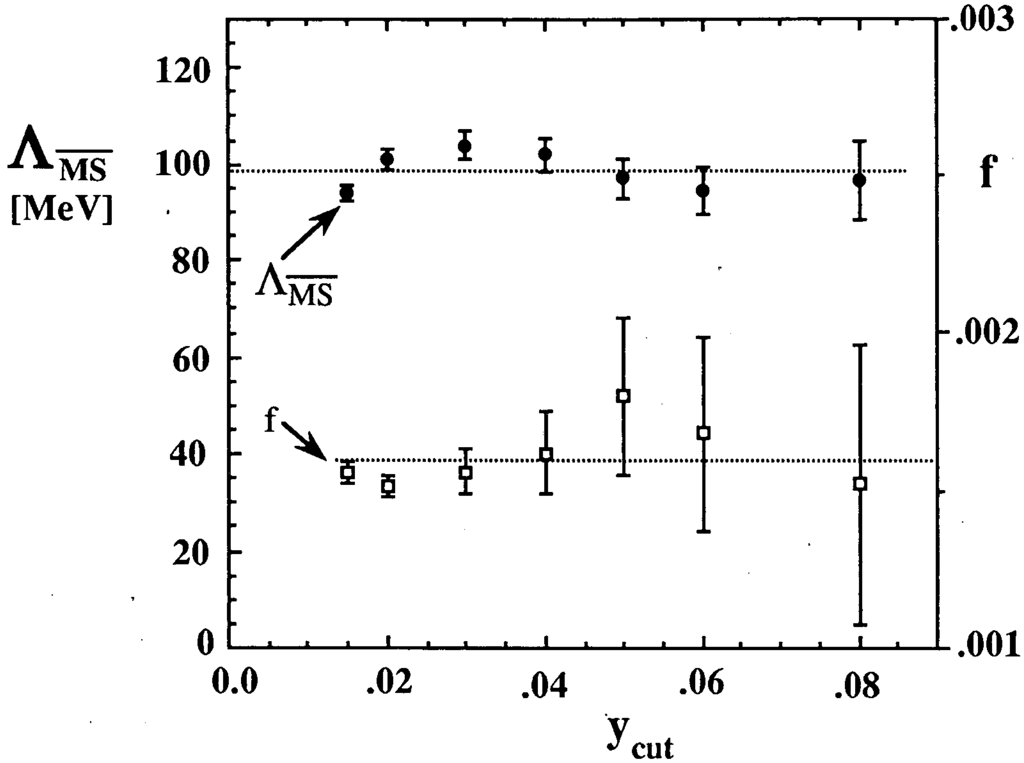

Fig.3. Results for $\Lambda_{\overline{MS}}$ and the scale factor f, simultaneously determined from the relative production rates of 2-, 3- and 4-jet events at $E_{cm} = 29$ GeV, for different values of the jet resolution parameter y_{cut}. The lines are drawn to guide the eye.

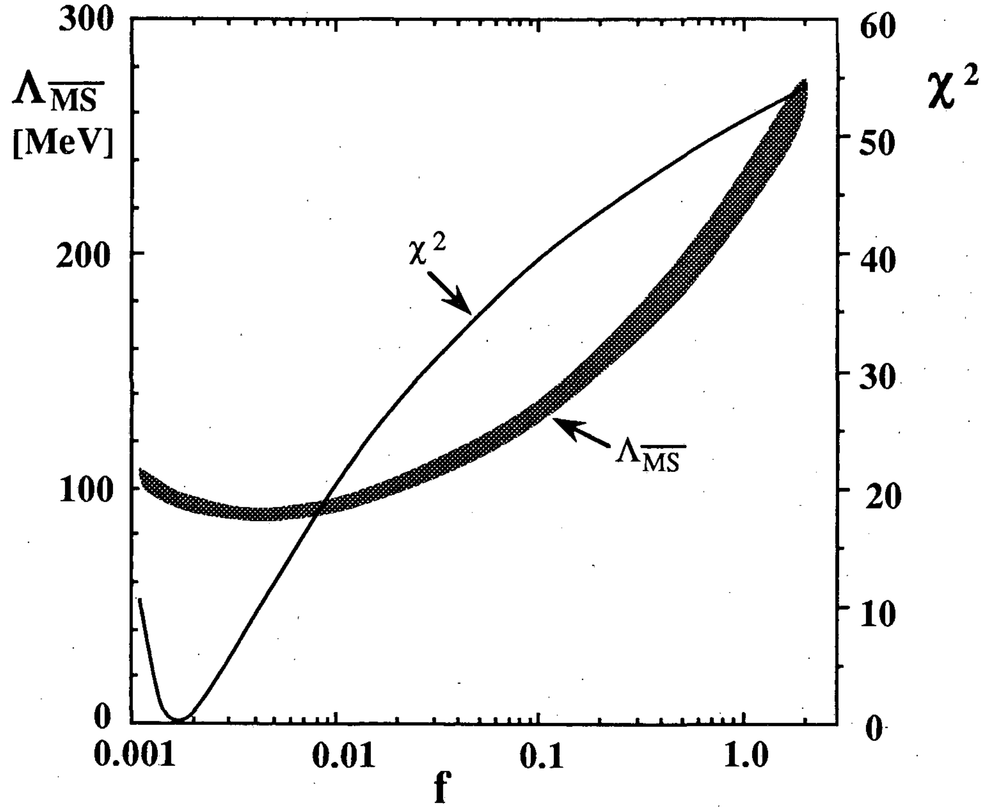

Fig.4. $\Lambda_{\overline{MS}}$ and the corresponding χ^2, determined from jet production rates observed at $E_{cm} = 29$ GeV for $y_{cut} = 0.06$, as a function of the renormalisation scale factor f in $O(\alpha_s^2)$. The width of the $\Lambda_{\overline{MS}}$ curve corresponds to the statistical fit error at fixed values of f.

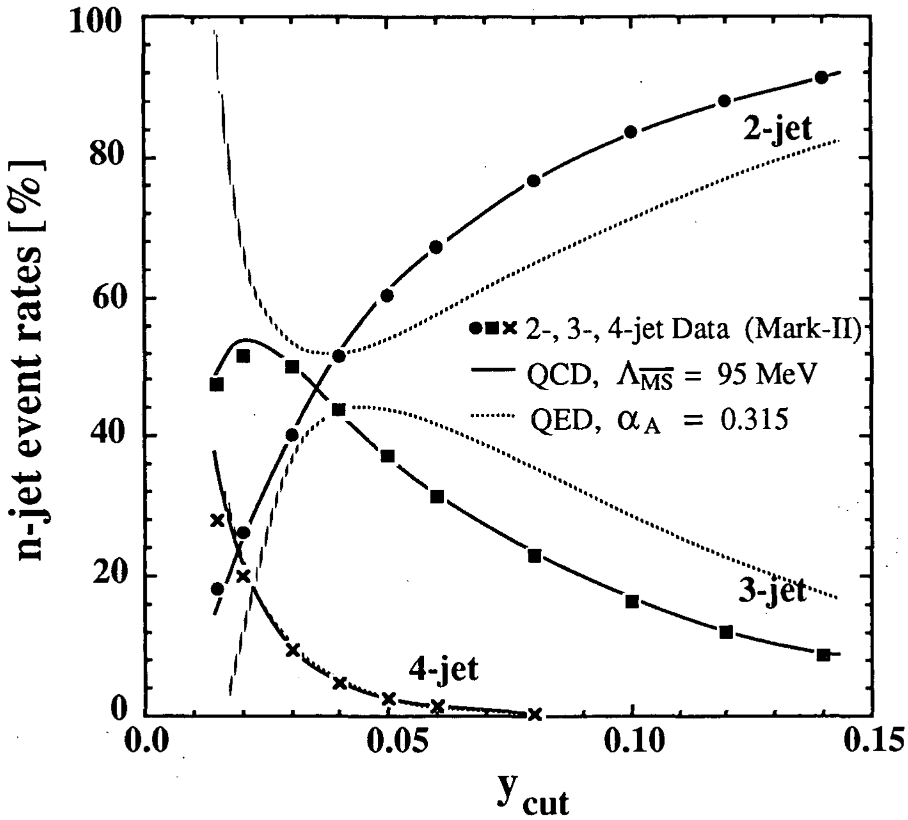

Fig. 5. Two-, three- and four-jet event rates observed at $E_{cm} = 29$ GeV as a function of the jet resolution parameter y_{cut}, compared with $O(\alpha_s^2)$ QCD and an $O(\alpha_A^2)$ abelian vector theory, adjusted to describe the jet rates at $y_{cut} = 0.04$.

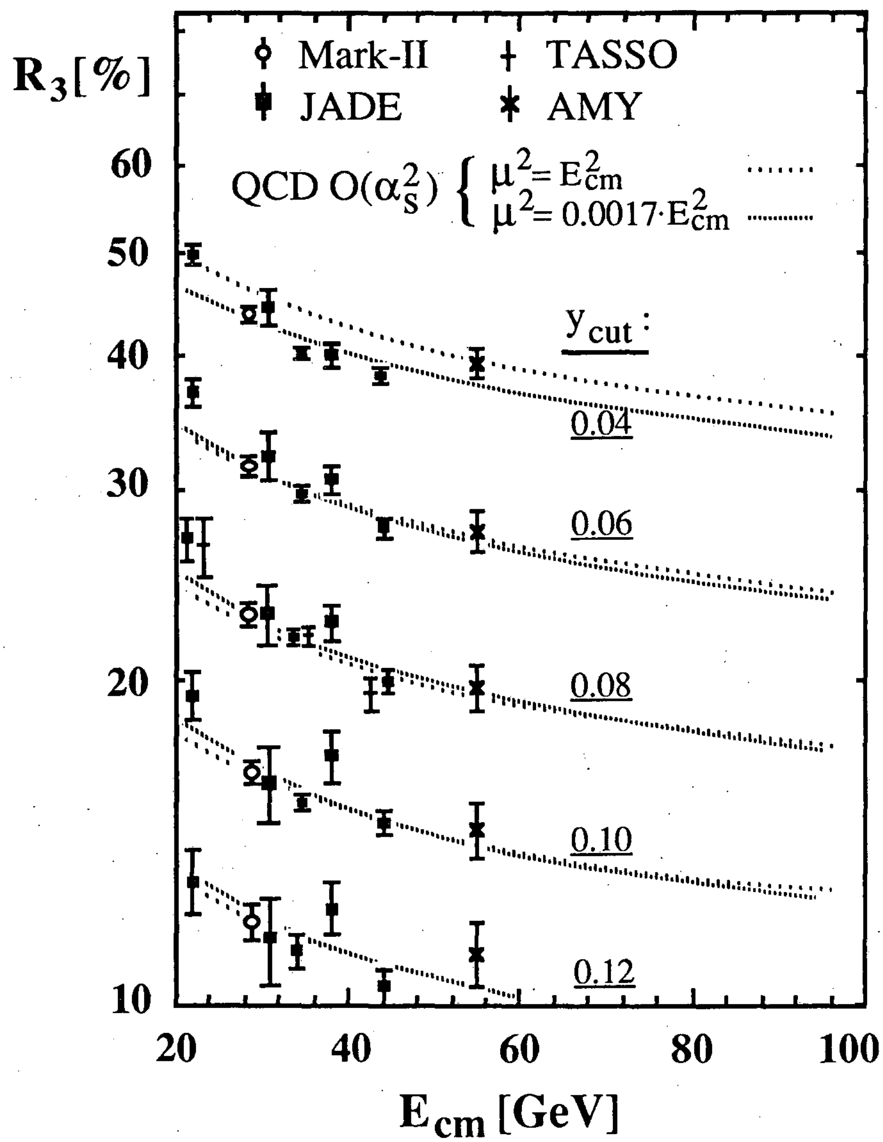

Fig.6. Three-jet event production rates observed at different center of mass energies, compared to $O(\alpha_s^2)$ calculations with two different renormalisation scales μ^2.

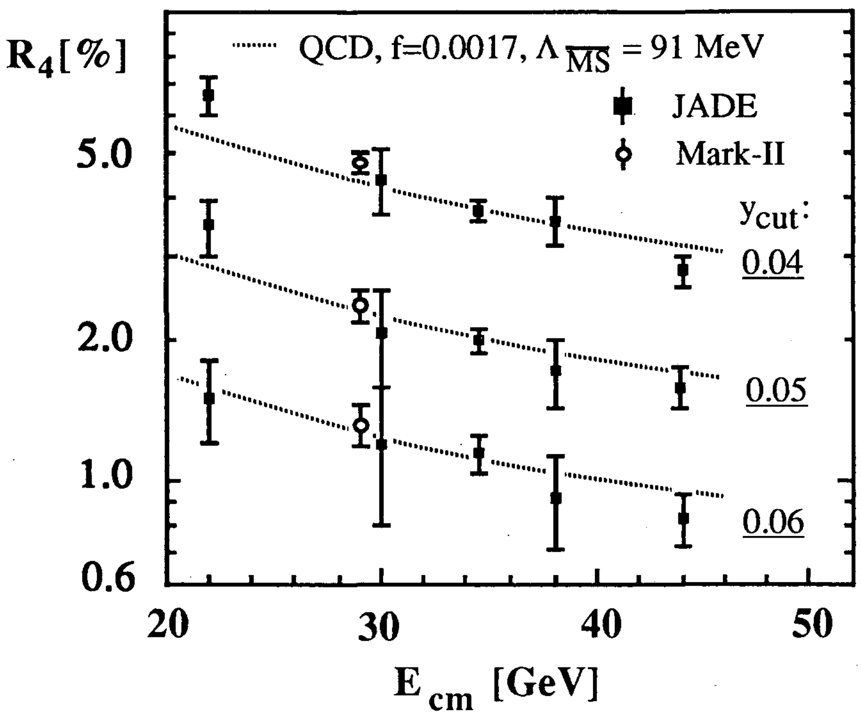

Fig.7. Four-jet event production rates at different center of mass energies, compared to an $O(\alpha_s^2)$ calculation with $\mu^2 = 0.0017 \cdot E_{cm}^2$.

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