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STRUCTURAL ENGINEERING,
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CALREL
USER MANUAL

by

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CHAPTER 1
INTRODUCTION

1.1 Introduction

CALREL (CAL-RELiability) is a general-purpose structural reliability analysis program. It is designed to compute probability integrals of the form

$$P_f = \int_F f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x} \tag{1}$$

where $\mathbf{X}$ is a vector of random variables with joint probability density function $f_{\mathbf{X}}(\mathbf{x})$ and F is the failure domain defined by

$$F \equiv \{g(\mathbf{x}) < 0\} \tag{2}$$

or

$$F \equiv \{\bigcup_{k=1}^{ncs} \bigcap_{i \in C_k} g_i(\mathbf{x}) \leq 0\} \tag{3}$$

in which $g(.)$ and $g_i(.)$ are limit-state functions. The form in Eq. 2 corresponds to a *component* reliability problem, whereas the form in Eq. 3 corresponds to a *system* reliability problem. In the latter, C_k and *ncs* refer to the k-th cut set and the number of cut sets, respectively, where each cut set is defined as an intersection of componental failure events. When each cut set has only one element, the system is referred to as a *series system*; otherwise, it is referred to as a *general system.* In addition to the above, CALREL computes the *generalized reliability index* defined by

$$\beta_g = \Phi^{-1}(1 - P_f) \tag{3}$$

where $\Phi^{-1}(.)$ denotes the inverse of the standard normal cumulative probability, and the sensitivities of estimates of P_f and β_g with respect to deterministic parameters defining the probability distribution or the limit-state functions.

CALREL incorporates four techniques for computing the above quantities:

(1) First-order reliability method (FORM), where the limit-state surfaces ($g(\mathbf{x}) = 0$ and $g_i(\mathbf{x}) = 0$) are replaced by tangent hyperplanes at *design* points in a transformed standard normal space;

(2) Second-order reliability method (SORM), where the limit-state surfaces are replaced by hyperparaboloids fitted at the design points in the standard normal space;

(3) Directional simulation with exact or approximate surfaces; and

(4) Monte Carlo simulation.

FORM and SORM are applicable to component reliability analysis, FORM is applicable to series system reliability, directional simulation in conjunction with FORM or SORM is applicable to component or system reliability analysis, and Monte Carlo simulation is applicable to all classes of problems.

A detailed technical review of the above methods is beyond the scope of this manual. The reader is referred to the bibliography in Appendix A for the relevant information. In particular, Refs. 6, 8, 9, 10 and 17 are recommended for the FORM, Refs. 3, 5, 11, 17 and 20 for the sensitivity analysis, Refs. 4, 7, 12, 13, 18 and 19 for the SORM, Refs. 2 and 13 for the directional simulation method, and Ref. 1 for the Monte Carlo simulation method.

CALREL has a large library of probability distributions that can be used to define $f_{\mathbf{X}}(\mathbf{x})$ for independent as well as dependent random variables. Joint distributions can be defined either in terms of a sequence of conditional distributions, or in terms of marginal distributions and correlation coefficients (Nataf family of distributions, Ref. 14). Table 2.1 of Chapter 2 lists the probability distributions that are currently available. These distributions can be used both as marginal or conditional. Additional distributions can be included through a user-defined subroutine, UDD.

CALREL requires the definition of limit-state functions through a user-defined subroutine, UGFUN. In addition, the user may define the gradients of the limit-state functions (required for FORM and SORM analyses) through a user-defined subroutine, UDGX. However, if desired, CALREL will automatically compute the gradients by the finite-difference method and the subroutine UDGX can be left blank. The definition of limit-state functions and their gradients through the subroutines UGFUN and UDGX may involve call statements to other subroutines or an entire subprogram. As an example, Fig. 1 shows a block diagram of CALREL where the finite-element program FEAP has been used to define the limit-state functions and their gradients (Ref. 16). In such applications, it is necessary that CALREL stay in control of the execution of the combined program. The user-provided subroutines UGFUN, UDGX and UDD may contain a collection of g functions, their derivatives and distributions, so that a single executable version of CALREL can solve a variety of problems.

As shown in Fig. 1, CALREL has a modular structure with each group of analysis routines contained in a separate module. To run CALREL, it is necessary to compile the user-defined subroutines UGFUN, UDGX and UDD (and other user-provided routines called by these subroutines) and link them with the object modules of CALREL. If necessary, certain object modules that are not needed for a particular problem can be replaced with dummy modules to

reduce the required computer space. The file README included in the installation diskette of CALREL contains instruction for this purpose. The format of the user-defined subroutines is described at the end of this chapter.

CALREL has a restart option that allows continuation of an analysis after termination of a previous run, provided the macro command CALRel is not re-issued. Data-base files created by the program should not be removed if the restart option is to be used at a later time.

CALREL executes the above reliability analyses in accordance to a sequence of macro commands issued in an input file. The following is a list of macro commands currently available in CALREL. Note that only the first four characters are needed to identify each macro command:

CALRel	defines problem size;
RESTart	restarts a previous run;
DATA	reads input data;
END	ends input data;
FORM	performs FORM analysis;
BOUNd	computes first-order bounds for series systems;
PNET	computes PNET approximation for series systems;
SENSitivity	computes sensitivities of first-order probabilities;
SORM	performs SORM analysis;
DIRS	performs directional simulation;
MONT	performs Monte Carlo simulation;
EXIT	terminates execution.

A full description of the above commands and the input data, which are to be provided between macro commands DATA and END, is described in Chapter 2. The output from the program for each command is described in Chapter 3, and example runs are presented in Chapter 4 and the installation diskette of CALREL.

CALREL has been developed on a virtual-memory computer, Micro VAX, in FORTRAN-77 language. It is also available on IBM-PC and compatible computers with at least 640K RAM. For implementation on a PC, read the README file provided on the floppy diskette containing the object code of CALREL.

1.2 User-Defined Subroutines

The following lines describe the required format of the user-defined subroutines UGFUN, UGDX and UDD. For each subroutine, the user-provided executable portion is marked by two lines of u's. Table 1.1 lists several CALREL library subroutines that can be used the in user-defined subroutines.

```
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
c
c     subroutine ugfun(g,x,tp,ig)
c
c     function: defines limit-state functions in the original space.
c
c     input:
c           x : vector of basic random variable.
c           tp: vector of deterministic parameters defining limit-state
c               functions.
c           ig: current limit-state function.
c
c     output:
c           g: value of the ig-th limit-state function at point x.
c
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

      subroutine ugfun(g,x,tp,ig)
      implicit real*8 (a-h,o-z)
      dimension x(1),tp(1)
c     uuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuu

            User to define limit-state function here for each ig.

c     uuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuu
      return
      end

ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccoc
c
c     subroutine udgx(dgx,x,tp,ig)
c
c     function: computes the gradient vector of the limit-state function
c               with respect to x when igr>0 is specified;  for igr=0,
c               this subroutine can be left blank; the central-difference
c               method will be used to compute the gradients.
c
c     input:
c           x : vector of basic random variables.
c           tp: vector of deterministic parameters defining limit-state
c               functions.
c           ig: current limit-state function.
c
c     output:
c           dgx: gradient vector.
c
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

      subroutine udgx(dgx,x,tp,ig)
      implicit real*8 (a-h,o-z)
      dimension x(1),dgx(1),tp(1)
c     uuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuu

            User to define the gradient vector here for each ig.

c     uuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuu
      return
      end
```

```
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
c
c     subroutine udd(x,par,sg,ids,cdf,pdf,bnd,ib)
c
c     function: defines user-defined probability distributions.
c
c     input:
c           x  : the basic random variable vector in the original space.
c           par: statistical parameters of user-defined distributions.
c           ids: current user-defined distribution function. ids.ge.51.
c
c     output:
c           sg : estimated standard deviation of user-defined distribution.
c           cdf: CDF of user-defined distribution.
c           pdf: PDF of user-defined distribution.
c           bnd: lower and upper bounds of user-defined distribution.
c           ib : code for distribution bounds:
c                =0, no bounds;
c                =1, bounded from below;
c                =2, bounded from above;
c                =3, bounded from below and above.
c
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

      subroutine udd(x,par,sg,ids,cdf,pdf,bnd,ib)
      implicit real*8 (a-h,o-z)
      dimension x(1),par(4),bnd(2)
c     uuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuu

           User to define sg, pdf, cdf, bnd and ib here for each ids.

c     uuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuuu
      return
      end
```

Table 1.1 CALREL Library Subroutines Which Can Be Used by User

Distribution	CDF	Inverse CDF
Normal	dnorm (y, p) Purpose: compute $p = \Phi(y)$	dnormi (y, p, *ier*) Purpose: compute y in $p = \Phi(y)$ where *ier* = error index.
Gamma	dchis (xt, pp, p, *ier*) Purpose: compute $p = \frac{\Gamma(k, \lambda x)}{\Gamma(k)}$ where $xt = 2\lambda x$ $pp = 2k$	dchisi (xt, pp, p, *ier*) Purpose: compute x in $p = \frac{\Gamma(k, \lambda x)}{\Gamma(k)}$
Beta	dbeta (xt, q, r, p, *ier*) Purpose: compute $p = F_X(x)$ where $xt = \frac{(x - a)}{(b - a)}$ q, r, a, b as defined in Table 2.1	dbetai (xt, q, r, p, *ier*) Purpose: compute x in $p = F_X(x)$
Chi-square	dchis (xt, rn, p, *ier*) Purpose: compute $p = \chi^2_{rn}(x^2) = \frac{\Gamma(\frac{rn}{2}, \frac{x}{2})}{\Gamma(\frac{rn}{2})}$ where $xt = x^2$ rn = degrees of freedom	dchisi (xt, rn, p, *ier*) Purpose: compute x in $p = \chi^2_{rn}(x^2)$
dgama (x) Purpose: evaluate the gamma function, $\Gamma(x)$, of a double precision argument x. Usage: result = dgama (x)		

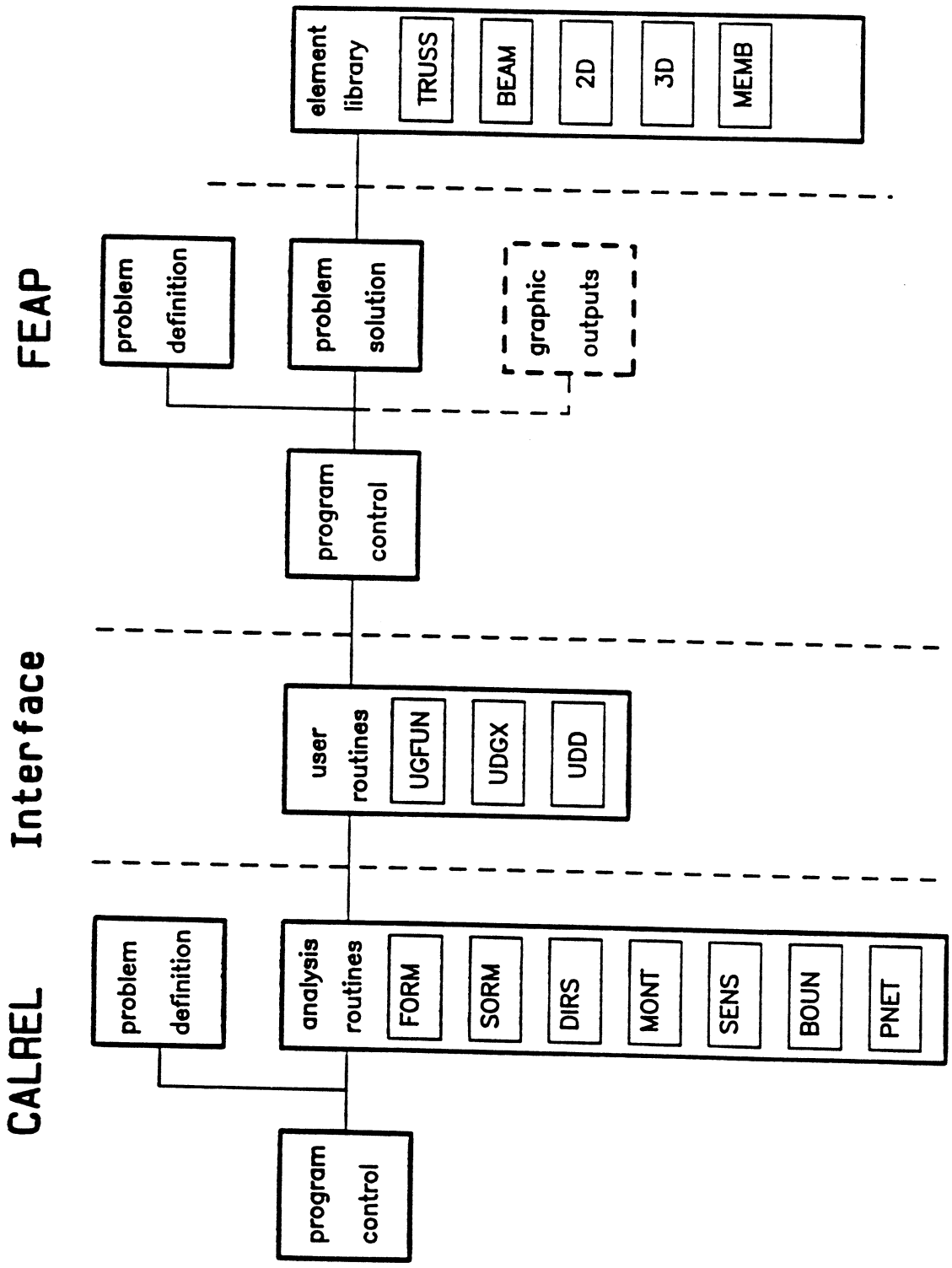

Figure 1. Block Diagram of CALREL-FEAP

CHAPTER 2
MACRO COMMANDS AND INPUT DATA

2.1 Introduction

To run CALREL, the user should provide an input file containing a list of macro commands that execute the required operations and one or more sets of data that define the parameters of the problem or specify the solution scheme. The input file should always begin with either the CALRel or RESTart command and terminate by the command EXIT. All other commands can be repeated and their order in the file is flexible. However, certain commands are executable only after other commands have been issued. For example, the SENSitivity and SORM commands can be executed only after the command FORM. A detailed description of the CALREL macro commands is presented in the following section.

The macro commands DATA and END indicate the beginning and end of the data-input mode, respectively. In between these commands, the input data is provided in several data sections as described in Section 2.3. An input file to CALREL may contain any number of data sets marked by the commands DATA and END.

2.2 Macro Commands of CALREL

The format and functions of CALREL macro commands are listed below. Only the first four characters are required to identify each macro command. Most commands are followed by a list of modifiers. Modifiers inside brackets have default values and are optional.

CALRel NRX=*nrx* [NGF=*ngf*] [NIG=*nig*] [NTP=*ntp*]

CALRel defines the size of a problem, where

nrx : total number of basic random variables.

ngf : number of limit-state functions, default = 1.

nig : number of independent groups of basic random variables, default = 1.

ntp : number of deterministic parameters in limit-state functions, default=0.

This command should be the first line of the input file.

RESTart

RESTart restarts a previous CALREL run. All problem data are read from the files saved in the previous run. The user should skip command CALRel if RESTart is issued. This command should be the first line of the input file.

DATA

DATA reads the input data of a problem. The user should provide the problem data following this command. The format of the input data is described in the next section. This command can be skipped in a restart analysis.

END

END stops data input mode and evokes the macro command mode.

FORM [INI=*ini*] [IST=*ist*] [IGF=*igf*] [NPR=*npr*]

FORM performs first-order reliability analyses on individual limit-state functions, where

ini : initialization flag, default = 0.
- *ini* = −1, start from the design point of the previous run;
- *ini* = 0, start from the mean point;
- *ini* = 1, start from the point specified by the user.

ist : restart flag, default = 0.
- *ist* = 0, analyze a new problem;
- *ist* = 1, continue an unconverged problem.

igf : limit-state function to be analyzed, default = 1.
igf applies to component analysis only.

npr : print interval, default = 0.
npr < 0, print no first-order results;
npr = 0, print the final results of FORM;
npr > 0, print the results after every *npr* steps.

BOUNd [IBT=*ibt*]

BOUNd calculates first-order failure probability bounds for series systems, where
ibt : type of bounds required, default = 0.
ibt = 1, unimodal bounds;
ibt = 2, relaxed bimodal bounds;
ibt = 3, bimodal bounds;
ibt = 0, all of the above.

Command FORM must have been executed before this command.

PNET [RHO=*rho*]

PNET estimates the first-order failure probability by the PNET method (see Ref. 1), where
rho : correlation coefficient threshold, default = 0.6.

Command FORM must have been executed before this command.

SENSitivity [ISC=*isc*] [ISV=*isv*] [IGF=*igf*]

SENSitivity performs first-order sensitivity analysis with respect to deterministic parameters defining probability distributions and/or limit-state functions, where

isc : type of sensitivity analysis required, default = 1.
- *isc* = 1, sensitivity analysis at component level;
- *isc* = 2, sensitivity analysis for a series system.

isv : type of parameters for sensitivity analysis, default = 0.
- *isv* = 1, sensitivity analysis with respect to distribution parameters;
- *isv* = 2, sensitivity analysis with respect to limit-state parameters;
- *isv* = 0, both the above.

igf: the same as in FORM.

Command FORM must have been executed before this command, and so must command BOUNd if *isc* = 2.

SORM [ISO=*iso*] [ITG=*itg*] [IGF=*igf*] [INP=*inp*] [ITM=*itm*] [NPK=*npk*] [EPA=*epa*] [EPB=*epb*] [EPC=*epc*] [SEE=*see*] [IST=*ist*]

SORM performs second-order reliability analysis on individual limit-state functions, where

iso : type of second-order analysis to be performed, default = 1.
- *iso* = 1, point-fitting method;
- *iso* = 2, curvature-fitting method;
- *iso* = 3, both the above.

itg : type of integration scheme used; default = 2 for the point-fitting method and default = 1 for the curvature-fitting method.
- *itg* = 1, improved Breitung formula;
- *itg* = 2, improved Breitung formula and Tvedt formula.

igf: the same as in FORM.

inp : maximum number of iterations for each fitting point, default = 4.

itm : type of method to compute principal curvatures; only needed when *iso* > 1; default = 0.
- *itm* = 0, using second-derivative matrix;
- *itm* = 1, gradient-based method (see Ref. 19);

npk : number of principal curvatures to be computed; for *itm* = 1 only; default = all.

epa : smallest $|\beta\kappa_i|$ value to be computed; for *itm* = 1 only; default = 0.05.

epb : convergence rate parameter; for *itm* = 1 only; default = 0.30.

epc : perturbation length parameter for starting point; for *itm* = 1 only; default = 0.50.

see : seed for starting point; for *itm* = 1 only; default = 1000.

ist : restart flag; for *itm* = 1 only; default = 0.

ist = 0, analyze a new problem;

ist = 1, continue a previous problem;

Command FORM must have been executed before this command.

DIRS [IFS=*ifs*] [IST=*ist*] [NSM=*nsm*] [NPR=*npr*] [COV=*cov*] [STP=*stp*] [IGF=*igf*]

DIRS performs directional simulation, where

ifs : type of surface used in simulation, default = 2 if SORM has been executed, otherwise default = 1.

ifs = 0, exact limit-state surface;

ifs = 1, first-order approximation surface;

ifs = 2, second-order approximation surface.

ist : restart flag, default = 0.

ist = 0, perform a new directional simulation;

ist = 1, continue a previous simulation.

nsm : total number of trials, default = 10000.

npr : print interval for simulation results, default = *nsm*/50.

cov : threshold for coefficient of variation of probability estimate, default = 0.05.

stp : seed for random number generation; should lie in the range [1,2147483647]. If not specified, *stp* is generated by the program.

igf : the same as in FORM.

Simulation stops at *nsm* trials or after the sample coefficient of variation falls below *cov*, whichever arrives first. If *ifs* = 1 (2), command FORM (SORM) must have been executed.

MONT [IST=*ist*] [NSM=*nsm*] [NPR=*npr*] [COV=*cov*] [STP=*stp*] [RAD=*rad*] [IGF=*igf*]

MONT performs the Monte Carlo simulation, where *ist, nsm, npr, cov, stp,* and *igf* are the same as defined in DIRS, and

rad : radius of β-sphere within which the limit-state surface is known to be positive, default = 0.

Simulation stops at *nsm* trials or after the sample coefficient of variation falls below *cov*, whichever arrives first.

EXIT

EXIT stops program execution.

2.3 Input Data of CALREL

The input data of CALREL is composed of several data sections. Each section starts with a title describing the type of data to follow. Only the first four characters of each section title need to be specified. All data sections are optional, except in a non-restart job where the STATistics and PARAmeter (if $ntp > 0$) data sections must be input at least once. The order of data sections is arbitrary. All input data are in free format, with data items separated by spaces, commas, or tabs. In the following, superposed numbers refer to notes at the end of each data section.

TITLe

n

text

TITLe reads the description of the problem, where

n : the number of text lines that follow.

text : *n* lines of text describing the problem.

FLAG

icl,igr

FLAG reads analysis flags, where

icl : problem type, default = 1.

icl = 1, component reliability;

icl = 2, series system reliability;

icl = 3, general system reliability.

igr : flag for gradient computation, default = 0.

igr = 0, gradient computed by finite difference;

igr = 1, gradient provided by the user in subroutine UDGX.

OPTImization

iop,ni1,ni2,tol,op1,op2,op3

OPTImization reads optimization parameters, where

iop : type of optimization scheme used, default = 2.

iop = 1, the HL-RF method;

iop = 2, the modified HL-RF method;

iop = 3, the gradient projection method;

iop = 4, the sequential quadratic programming method.

ni1 : maximum number of iteration cycles, default = 100, maximum = 100.

ni2 : maximum number of steps in line search, default = 4, maximum = 4.

tol : convergence tolerance, default = 0.001, minimum = 0.001.

op1 : step size reduction factor in line search, default = 1.0 for *iop* = 1, default = 0.5 for *iop* = 2, 3 or 4.

op2 : optimization parameter.

iop = 2, parameter c in the merit function, default = 10;

iop = 3, 4 convergence tolerance for line search, default = *tol*.

op3 : optimization parameter.

iop = 3, 4 Maximum step size in line search, default = 4.

To override the restrictions $ni1 \leq 100$, $ni2 \leq 4$ and $tol \geq 0.001$, input negative $ni1$, $ni2$ and tol values. Their absolute values will be used in the analysis regardless of the limits. See Ref. 15 for a detailed description of each optimization algorithm and the associated parameters.

STATistics

igt,nge,ngm

xn,nv,id,p1,p2,p3,p4,x0

ro

...

STATistics reads statistical data of the basic random variables. The variables are divided into *nig* statistically independent groups. For each group, the user should provide:

igt : group type.

igt = 1, all variables in the group are statistically independent;

igt = 2, variables described by their marginal distributions and correlation matrix (Nataf family of distributions);

igt = 3, variables described by conditional and marginal distributions and correlation matrix.

nge : number of variables in the group.

ngm : number of variables in a type 3 group which are described by their marginal distributions.

For each of the nge variables, provide: [1]

xn : variable name. [2]

nv : variable number.

id : *ids* = |*id*| = distribution type. [3]

ids = 0, deterministic;

ids = 1, normal;

ids = 2, lognormal;

ids = 3, gamma;

ids = 4, shifted exponential;

ids = 5, shifted Rayleigh;
ids = 6, uniform;
ids = 7, beta;
ids = 11, type-I largest value;
ids = 12, type-I smallest value;
ids = 13, type-II largest value;
ids = 14, type-III smallest value;
ids > 50, user-defined distributions.

*p*1 : distribution parameter 1. [4]
id > 0, *p*1 : mean value;
id < 0, *p*1 : as defined in Table 2.1.

*p*2 : distribution parameter 2. [4]
id > 0, *p*2 : standard deviation;
id < 0, *p*2 : as defined in Table 2.1.

*p*3 : distribution parameter 3. [4]

*p*4 : distribution parameter 4. [4]

*x*0 : initial value of the variable, needed when *ids* > 50 or *ini* = 1 (see command FORM).

If *igt* = 2, or *igt* = 3 with *ngm* > 0, input the lower triangle of the correlation matrix for variables described by their marginal distributions; read after the distribution data of the variables. Otherwise, skip *ro* and continue to input the distribution data for the next group.

ro : lower triangle of the correlation matrix excluding the diagonals. Read it row-wise and in a triangular form.

Repeat the above input data for all groups.

Notes:

(1) If a type 3 group contains variables described by marginal distributions, i.e. *ngm* > 0, then such variables must be defined before the variables described by conditional distributions.

(2) The first four characters of the line is treated as the variable name. The user should provide *nv, ids, p*1, *p*2, *p*3, *p*4, and *x*0 from the fifth character on. No comma should be inserted between *xn* and *nv*.

(3) See Tables 2.1 and 2.2 for a description of probability distributions available in CALREL.

(4) For variable groups with *igt* = 3, entries for parameters *p*1, *p*2, *p*3, and *p*4 can be real or integer numbers. If real, then the parameter is assigned the entered value. If an integer ip_k is entered for parameter p_k, then p_k is set equal to the ip_k-th basic random variable in the

group. The ip_k-th variable must have been defined earlier in the same group. This allows assignment of distributions whose parameters are themselves random variables.

PARAmeter

tp

PARAmeter reads the deterministic parameters defining the limit-state functions, where

tp : *ntp* deterministic parameters.

CUTSets

ncs,ntl

cs

CUTSets reads the cut set data, where

ncs : number of cut sets, default = *ngf*.

ntl : total number of components in all cut sets, default = *ncs*.

cs : components of cut sets. Use zero to indicate the end of each cut set. Use a negative sign to indicate the complement of a limit-state function. E.g., for cut sets (1,2,4) and (-2,5,7), *ncs* = 2, *ntl* = 6, and *cs* = 1 2 4 0 –2 5 7 0.

If CUTS is skipped in a system problem, the system will be treated as a series system with the first *ngf* (which is specified through the macro command CALRel) user-provided limit-state functions as its components.

Table 2.1 CALREL Probability Distribution Library

distribution name	*id*	PDF, $f_x(x)$	CDF, $F_x(x)$	Parameters				note
				p_1	p_2	p_3	p_4	
normal	1	$\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}(\frac{x-\mu}{\sigma})^2\right]$	$\Phi\left[\frac{x-\mu}{\sigma}\right]$	μ	$0<\sigma$			1
lognormal	2	$\frac{1}{\sqrt{2\pi}\zeta x}\exp\left[-\frac{1}{2}(\frac{\ln x-\lambda}{\zeta})^2\right]$, $0<x$	$\Phi\left[\frac{\ln x-\lambda}{\zeta}\right]$	λ	$0<\zeta$			1
gamma	3	$\frac{\lambda(\lambda x)^{k-1}}{\Gamma(k)}\exp(-\lambda x)$, $0\le x$	$\frac{\Gamma(k,\lambda x)}{\Gamma(k)}$	$0<\lambda$	$0<k$			2,3
shifted exponential	4	$\lambda\exp[-\lambda(x-x_0)]$, $x_0\le x$	$1-\exp(-\lambda(x-x_0))$	$0<\lambda$	x_0			
shifted Rayleigh	5	$\frac{(x-x_0)}{\alpha^2}\exp\left[-\frac{1}{2}\left(\frac{x-x_0}{\alpha}\right)^2\right]$, $x_0\le x$	$1-\exp\left[-\frac{1}{2}\left[\frac{x-x_0}{\alpha}\right]^2\right]$	α	x_0			
uniform	6	$\frac{1}{b-a}$, $a\le x\le b$	$\frac{x-a}{b-a}$	a	b			
beta	7	$\frac{(x-a)^{q-1}(b-x)^{r-1}}{B(q,r)(b-a)^{q+r-1}}$, $a\le x\le b$		$0<q$	$0<r$	a	b	4
type I largest value	11	$\alpha_u\exp[-\alpha_u(x-u_u)-\exp(-\alpha_u(x-u_u))]$	$\exp[-\exp(-\alpha_u(x-u_u))]$	u_u	$0<\alpha_u$			5
type I smallest value	12	$\alpha_1\exp[\alpha_1(x-u_1)-\exp(\alpha_1(x-u_1))]$	$1-\exp[-\exp(\alpha_1(x-u_1))]$	u_1	$0<\alpha_1$			
type II largest value	13	$\frac{k}{u_u}\left(\frac{u_u}{x}\right)^{k+1}\exp\left[-\left(\frac{u_u}{x}\right)^k\right]$, $0<x$	$\exp\left[-\left(\frac{u_u}{x}\right)^k\right]$	u_u	$0<k$			
type III smallest value	14	$\frac{k}{u_1-\epsilon}\left(\frac{x-\epsilon}{u_1-\epsilon}\right)^{k-1}\exp\left[-\left(\frac{x-\epsilon}{u_1-\epsilon}\right)^k\right]$, $\epsilon\le x$	$1-\exp\left[-\left(\frac{x-\epsilon}{u_1-\epsilon}\right)^k\right]$	u_1	$0<k$			6

note 1. $\Phi(x)=(2\pi)^{-1}\int_{-\infty}^{x}\exp(-u^2/2)\,du$ is the standard normal cumulative probability.

2. $\Gamma(k)=\int_0^{\infty}e^{-u}u^{k-1}du$ is the gamma function. For integer k, $\Gamma(k)=(k-1)!$.

3. $\Gamma(k,x)=\int_0^{x}e^{-u}u^{k-1}du$ is the incomplete gamma function with $\Gamma(k,\infty)=\Gamma(k)$.

4. $B(q,r)=\Gamma(q)\Gamma(r)/\Gamma(q+r)$ is the beta function.

5. This distribution is also known as the *Gumbel* distribution.

6. For $\epsilon=0$, this distribution is known as the *Weibull* distribution.

Table 2.2 Mean and Standard Deviation of CALREL Library Distributions

distribution name	mean	standard deviation
normal	μ	σ
lognormal	$\exp\left(\lambda + \frac{\zeta^2}{2}\right)$	$\exp\left(\lambda + \frac{\zeta^2}{2}\right)[\exp(\zeta^2) - 1]^{1/2}$
gamma	$\frac{k}{\lambda}$	$\frac{\sqrt{k}}{\lambda}$
shifted exponential	$\nu_0 + \frac{1}{\lambda}$	$\frac{1}{\lambda}$
shifted Rayleigh	$\nu_0 + \left(\frac{\pi}{2}\right)^{1/2} \alpha$	$\left(2 - \frac{\pi}{2}\right)^{1/2} \alpha$
uniform	$\frac{a+b}{2}$	$\frac{b-a}{2\sqrt{3}}$
beta	$a + \frac{q(b-a)}{q+r}$	$\frac{b-a}{q+r}\left(\frac{qr}{q+r+1}\right)^{1/2}$
type I largest value	$u_n + \frac{0.5772}{\alpha_n}$	$\frac{\pi}{\sqrt{6}\alpha_n}$
type I smallest value	$u_1 - \frac{0.5772}{\alpha_1}$	$\frac{\pi}{\sqrt{6}\alpha_1}$
type II largest value	$u_n\,\Gamma(1 - \frac{1}{k})$	$u_n\left[\Gamma(1 - \frac{2}{k}) - \Gamma^2(1 - \frac{1}{k})\right]^{1/2}$
type III smallest value	$\epsilon + (u_1 - \epsilon)\Gamma(1 + \frac{1}{k})$	$(u_1 - \epsilon)\left[\Gamma(1 + \frac{2}{k}) - \Gamma^2(1 + \frac{1}{k})\right]^{1/2}$

CHAPTER 3
CALREL OUTPUT

3.1 Introduction

This chapter describes the results produced and the notation used in the CALREL output for each executable macro command. Familiarity with the underlying concepts and definitions is assumed. The unfamiliar reader must consult the references in Appendix A for a more complete description of the terms used in this chapter.

3.2 Description of CALREL Output

In addition to echoing all the input data, CALREL prints the following results for each micro command listed:

FORM computes and prints first-order probability estimates for each limit-state function, where

iter	number of iterations required for convergence;
β	first-order reliability index;
P_{f1}	first-order failure probability estimate;
$g(\mathbf{x})$	value of limit-state function;
$\mathbf{x}^*$	coordinates of design point in original space;
$\mathbf{u}^*$	coordinates of design point in standard normal space;
α	sensitivity of β with respect to $\mathbf{u}^*$;
γ	scaled and normalized sensitivity of β with respect to $\mathbf{x}^*$ (see Ref. 5);
δ	scaled sensitivity of β with respect to the mean of each basic random variable, $\delta = \left\{\sigma_i \frac{\partial \beta}{\partial \mu_i}\right\}$;
η	scaled sensitivity of β with respect to the standard deviation of each basic random variable, $\eta = \left\{\sigma_i \frac{\partial \beta}{\partial \sigma_i}\right\}$.

BOUNd computes and prints first-order failure probability bounds for series systems,

where, depending on the method of analysis requested, the printed results may include:

β_i reliability indices for individual failure modes;

p_i first-order failure probabilities for individual failure modes;

α_i outward normal unit vector for mode i;

ρ_{ij} matrix of cross-modal correlation coefficients, $\rho_{ij} = \alpha_i \alpha_j^T$ (see Ref. 8);

$[p_{ij}]_{lower}^{upper}$ matrix of lower and upper bounds on joint-modal probabilities (see Ref. 8);

p_{ij} matrix of joint-modal probabilities computed by integration over the cross-modal correlation coefficient (see Ref. 17);

$[\beta_g]_{lower}^{upper}$ upper and lower first-order bounds on system generalized reliability index, and

$[P_{f1}]_{lower}^{upper}$ lower and upper bounds on first-order system failure probability:

- $ibt = 1$ by use of unimodal probabilities, p_i (see Ref. 17);
- $ibt = 2$ by use of unimodal probabilities, p_i, and bounds on joint-modal probabilities p_{ij} (see Ref. 8);
- $ibt = 3$ by use of unimodal probabilities, p_i, and joint-modal probabilities p_{ij} (see Ref. 8);
- $ibt = 0$ all three methods.

PNET estimates the first-order failure probability by the PNET method (see Ref. 1), where

β_g generalized reliability index;

P_{f1} first-order probability estimate;

rm representative modes.

SENSitivity computes and prints first-order sensitivities for components and series systems (upper bound failure probability), where the printed results include partial derivatives of β_g and P_{f1} with respect to:

μ_i mean of each variable (with σ_i, p_3, and p_4 fixed);

σ_i standard deviation of each variable (with μ_i, p_3, and p_4 fixed);

p_k k-th distribution parameter of each variable (with p_j, $j \neq k$ fixed);

t_i limit-state function parameters.

See Refs. 11 and 17 for component reliability sensitivities and Ref. 3 for series system reliability sensitivities.

SORM computes and prints second-order probability estimates for each limit-state function, where

For the point-fitting method (see Ref. 7),

u'_i coordinate of fitting point along i-th axis in rotated standard space;

u'_n coordinate of fitting point along n-th axis in rotated standard space;

$G(\mathbf{u})$ value of limit-state function at fitting point;

a'_i weighted average main curvature for i-th axis;

P_{f2} second-order failure probability estimate, and

β_g generalized reliability index by:

itg = 1 improved Breitung formula (see Refs. 4 and 12)

itg = 2 the above and Tvedt integral formula (see Ref. 18).

for the curvature-fitting SORM method (see Ref. 7),

κ_i main curvatures in $(n-1)\times(n-1)$ space;

P_{f2} second-order failure probability estimate, and

β_g generalized reliability index by:

itg = 1 improved Breitung formula (see Refs. 4 and 12)

itg = 2 the above and Tvedt integral formula (see Ref. 18).

DIRS computes and prints:

nos number of simulations;

$\hat{E}[P_f]$ mean of simulated probability;

$\hat{\beta}_g$ estimate of β_g based on $\hat{E}[P_f]$;

$\hat{cov}(P_f)$ coefficient of variation of the estimated probability.

MONT computes and prints:

nos number of simulations;

$\hat{E}[P_f]$ mean of simulated probability;

$\hat{\beta}_g$ estimate of β_g based on $\hat{E}[P_f]$;

$\hat{cov}(P_f)$ coefficient of variation of the estimated probability.

CHAPTER 4
EXAMPLES

4.1 Introduction

This chapter describes eight examples designed to demonstrate the capabilities and use of CALREL. For each example, listed are the limit-state function(s), distribution information on the random variables, values of the deterministic parameters, and a list of the analysis performed. The installation diskette of CALREL contains the complete list of the user-provided subroutines and the input and output files for each example in the subdirectory EXAMPLE. The following notation is used throughout this chapter:

$g(\mathbf{X})$	limit-state function,
t_i	deterministic parameters defining the limit-state function.
μ	mean value,
σ	standard deviation,
$\mathbf{R}$	correlation coefficient matrix,
$F_X(x)$	cumulative distribution function,
$f_X(x)$	probability density function,
$\|$	condition sign.

For a description of distribution parameters other than μ and σ see Tables 2.1 and 2.2 of Chapter 2.

Example 1 -- Component reliability with independent random variables.

Limit-state function:

$$g(\mathbf{X}) = X_1^{t_1} - X_2 t_2 + t_3$$

Distributions:

X_1 : uniform ($a = 0, b = 100$)

X_2 : shifted exponential ($\lambda = 0.05, x_0 = 0$)

Deterministic parameters:

$t_1 = 1, t_2 = 1, t_3 = 0$

Analysis performed: FORM, SENS, SORM, MONT.

INPUT FILE:

```
==========================================================================

CALRel nrx=2 ntp=3
DATA
TITL nline title
1
Example 1 -- Component reliability with independent random variables.
FLAG icl,igr
1 1
OPTI iop,ni1,ni2,tol,op1,op2,op3
1,20,4,-0.0001
STAT igt(i),nge,ngm nv,ids,ex,sg,p3,p4,x0
1 2
x1   1,-6,0.,100.,0.,0.,23.52
x2   2,-4,0.05,0.,0.,0., 23.52
PARA tp
1.,1.,0.
END
FORM ini=1
SENS isc=1 isv=0
SORM iso=3 itg=2
MONT nsm=5000 npr=1000 cov=0.05 stp=621320774.
EXIT

==========================================================================
```

OUTPUT FILE:

```
================================================================================

>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval .........................npr=      0
  npr<0 .........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag .....................ini=      1
  ini=0 .......................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ............................ist=      0
  ist=0 .......................analyze a new problem
  ist=1 ............continue an unconverged problem

limit-state function     1
--------------------------------------------------------------------------------
iteration number ..............iter=         3
value of limit-state function..g(x)=-5.894e-07
reliability index .............beta=    0.8776
probability ....................Pf1= 1.901e-01
var          design point                        sensitivity vectors
           x*             u*           alpha     gamma     delta      eta
x1      2.407e+01    -7.041e-01      -0.8022   -0.8022    0.7438   -0.6681
x2      2.407e+01     5.240e-01       0.5970    0.5970   -0.5153   -0.1048
--------------------------------------------------------------------------------

>>>> SENSITIVITY ANALYSIS AT COMPONENT LEVEL <<<<

type of parameters for sensitivity analysis
.........................................isv=      0
  isv=1 ......................distribution parameters
  isv=2 ...................limit-state fcn parameters
  isv=0 ..distribution and limit-state fcn parameters

sensitivity with respect to distribution parameters

limit-state function    1
--------------------------------------------------------------------------------
d(beta)/d(parameter) :
var      mean       std dev      par 1       par 2       par 3       par 4
x1     2.576e-02 -2.314e-02  1.956e-02  6.201e-03
x2    -2.576e-02 -5.242e-03  1.240e+01 -2.576e-02

d(Pf1)/d(parameter) :
var      mean       std dev      par 1       par 2       par 3       par 4
x1    -6.993e-03  6.282e-03 -5.310e-03 -1.683e-03
x2     6.993e-03  1.423e-03 -3.366e+00  6.993e-03
--------------------------------------------------------------------------------

sensitivity with respect to limit-state function parameters

limit-state function    1
--------------------------------------------------------------------------------
par   d(beta)/d(parameter)      d(Pf1)/d(parameter)
 1          1.973e+00               -5.354e-01
 2         -6.201e-01                1.683e-01
 3          2.576e-02               -6.993e-03
--------------------------------------------------------------------------------
```

```
>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used .....................itg=    2
  itg=1 ............................improved Breitung formula
  itg=2 ............................improved Breitung formula
        ............................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function    1
-------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)          u'i    u'n     G(u)          a'i
   1 1.033   0.839  9.277e-06    -1.010  0.866  1.732e-10    -2.3900e-02

                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         0.8527              0.8533
probability                          Pf2 =     1.969e-01           1.968e-01
-------------------------------------------------------------------------


>>>> SECOND-ORDER RELIABILITY ANALYSIS  -- CURVATURE FITTING <<<<

type of integration scheme used .....................itg=    2
  itg=1 ............................improved Breitung formula
  itg=2 ............................improved Breitung formula
        ............................& Tvedt's exact integral

limit-state function    1
-------------------------------------------------------------------------
main curvatures in (n-1)x(n-1) space

       1
 1 -2.127e-02
                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         0.8556              0.8560
probability                          Pf2 =     1.961e-01           1.960e-01
-------------------------------------------------------------------------


>>>> MONTE CARLO SIMULATION <<<<

 print interval ............................npr=    1000
 number of simulations .....................nsm=    5000
 threshold for coef. of variation .....cov=  5.000e-02
 seed ............................stp=   621320774.00000

limit-state function     1
-------------------------------------------------------------------------
 trials       Pf-mean        betag-mean     coef of var of Pf
  1000    1.9900000e-01    8.4519878e-01    6.3475624e-02
  2000    1.9950000e-01    8.4340878e-01    4.4802559e-02

=========================================================================
```

Example 2 -- Component reliability with dependent random variables defined by marginal distributions and correlation matrix.

Limit-state function:

$$g(\mathbf{X}) = t_1 - \frac{X_2}{1000X_3} - \left(\frac{X_1}{200X_3}\right)^2$$

Distributions:

X_1, X_2, X_3 defined in terms of marginals and correlation matrix:

X_1 : lognormal ($\mu = 500$, $\sigma = 100$)

X_2 : lognormal ($\mu = 2000$, $\sigma = 400$)

X_3 : uniform ($\mu = 5$, $\sigma = 0.5$)

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.3 & 0.2 \\ 0.3 & 1.0 & 0.2 \\ 0.2 & 0.2 & 1.0 \end{bmatrix}$$

Deterministic parameters:

$t_1 = 1.0$

Analysis performed: FORM not converged.

The input file for this example specifies $ni\,1 = 10$ maximum number of iterations in the optimization algorithm for finding the design point. The selected algorithm does not converge in 10 cycles. The use of the RESTart option is demonstrated in Example 3 to complete the solution of this problem.

INPUT FILE:

```
==========================================================================

CALREL nrx=3 nig=1 ntp=1
DATA
TITLE nline title
2
Example 2 -- Component reliability with dependent random variables
             defined by marginal distributions and correlation matrix.
```

```
FLAG icl,igr
1 0
OPTIMIZATION iop,ni1,ni2,tol,op1,op2,op3
1,10,4,0.001
STATISTICS igt(i),nge,ngm nv,ids,ex,sg,p3,p4,x0
2 3
x1  1,2,500.,100.
x2  2,2,2000.,400.
x3  3,6,5.,0.5
0.3
0.2 0.2
para
1.0
END
FORM npr=1 igf=2
SENS isc=1 isv=0 igf=2
SORM iso=3 itg=2 igf=2
MONT igf=2 nsm=500000 cov=0.03 npr=2000 stp=621319411.
EXIT
```

==========

OUTPUT FILE:

==========

```
>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval .........................npr=      1
  npr<0 .........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag ....................ini=      0
  ini=0 ......................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ...........................ist=      0
  ist=0 ......................analyze a new problem
  ist=1 ............continue an unconverged problem

limit-state function     2
-----------------------------------------------
iteration number ..............iter=         1
reliability index .............beta=    0.1268
value of limit-state function..g(x)= 3.500e-01
var       linearization point       unit normal
            x                 u          alpha
x1       5.000e+02      9.902e-02         0.5911
x2       2.000e+03      7.233e-02         0.3440
x3       5.000e+00     -3.243e-02        -0.7295
-----------------------------------------------
```

```
.
.
.
-----------------------------------------------
iteration number ..............iter=          6
reliability index .............beta=     1.7696
value of limit-state function..g(x)= 1.155e-03
var       linearization point        unit normal
             x                u            alpha
x1      6.273e+02      1.244e+00         0.7343
x2      2.307e+03      4.641e-01         0.2787
x3      4.499e+00     -1.170e+00        -0.6190
-----------------------------------------------
iteration number ..............iter=          7
reliability index .............beta=     1.7714
value of limit-state function..g(x)= 4.197e-04
var       linearization point        unit normal
             x                u            alpha
x1      6.343e+02      1.301e+00         0.7160
x2      2.328e+03      4.937e-01         0.2681
x3      4.544e+00     -1.096e+00        -0.6446
-----------------------------------------------
iteration number ..............iter=          8
reliability index .............beta=     1.7720
value of limit-state function..g(x)= 1.541e-04
var       linearization point        unit normal
             x                u            alpha
x1      6.303e+02      1.269e+00         0.7273
x2      2.315e+03      4.750e-01         0.2742
x3      4.516e+00     -1.142e+00        -0.6292
-----------------------------------------------
iteration number ..............iter=          9
reliability index .............beta=     1.7723
value of limit-state function..g(x)= 5.648e-05
var       linearization point        unit normal
             x                u            alpha
x1      6.329e+02      1.289e+00         0.7205
x2      2.323e+03      4.860e-01         0.2704
x3      4.532e+00     -1.115e+00        -0.6386
-----------------------------------------------
iteration number ..............iter=         10
reliability index .............beta=     1.7723
value of limit-state function..g(x)= 2.075e-05
var       linearization point        unit normal
             x                u            alpha
x1      6.314e+02      1.277e+00         0.7246
x2      2.318e+03      4.792e-01         0.2727
x3      4.522e+00     -1.132e+00        -0.6329
ERROR 21: convergence not achieved in nil steps for failure mode    2

================================================================================
```

Example 3 -- Use of RESTart option for Example 2

In this example, we use the restart option to continue the analysis of Example 2 with a larger value of *ni* 1. There is no need to provide the statistical data again.

Analysis performed: FORM, SENS, SORM, MONT.

```
INPUT FILE:

==========================================================

REST
DATA
TITLE nline title
1
Example 2.1 -- Use of RESTart option for Example 2.
OPTIMIZATION iop,ni1,ni2,tol,op1,op2,op3
1,20,4,0.001
END
FORM ist=1 npr=1 igf=2
SENS isc=1 isv=0 igf=2
SORM iso=3 itg=2 igf=2
MONT igf=2 nsm=500000 cov=0.03 npr=2000 stp=621319411.
EXIT

==========================================================

 OUTPUT FILE:

 ==========================================================

 >>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

 print interval .........................npr=      1
   npr<0 ..........no first order results are printed
   npr=0 ........print the final step of FORM results
   npr>0 ........print the results of every npr steps
 initialization flag ....................ini=      0
   ini=0 .......................start from mean point
   ini=1 ..........start from point specified by user
   ini=-1 ....start from previous linearization point
 restart flag ...........................ist=      1
   ist=0 .......................analyze a new problem
   ist=1 ............continue an unconverged problem

 limit-state function     2
 ------------------------------------------------
 iteration number ..............iter=        11
 reliability index .............beta=    1.7724
 value of limit-state function..g(x)= 7.619e-06
 var       linearization point       unit normal
             x                u            alpha
 x1      6.323e+02      1.284e+00         0.7221
 x2      2.321e+03      4.833e-01         0.2713
 x3      4.528e+00     -1.122e+00        -0.6363
 ------------------------------------------------
```

```
.
.
.
-----------------------------------------------
iteration number .............iter=        14
reliability index ............beta=    1.7724
value of limit-state function..g(x)= 3.775e-07
var       linearization point       unit normal
            x                u            alpha
x1      6.319e+02      1.281e+00         0.7233
x2      2.320e+03      4.814e-01         0.2719
x3      4.526e+00     -1.126e+00        -0.6347
-------------------------------------------------------------------------
iteration number .............iter=        15
value of limit-state function..g(x)= 1.387e-07
reliability index ............beta=    1.7724
probability ..................Pf1= 3.816e-02
var          design point                          sensitivity vectors
           x*             u*           alpha      gamma      delta      eta
x1      6.320e+02      1.282e+00      0.7230     0.6960    -0.5902    -0.7868
x2      2.320e+03      4.820e-01      0.2717     0.3660    -0.3434    -0.2485
x3      4.526e+00     -1.125e+00     -0.6352    -0.6178     0.6300    -0.5968
-------------------------------------------------------------------------


>>>> SENSITIVITY ANALYSIS AT COMPONENT LEVEL <<<<

type of parameters for sensitivity analysis
.........................................isv=      0
  isv=1 .....................distribution parameters
  isv=2 ....................limit-state fcn parameters
  isv=0 ..distribution and limit-state fcn parameters

sensitivity with respect to distribution parameters

limit-state function    2
-------------------------------------------------------------------------
d(beta)/d(parameter) :
var      mean       std dev      par 1       par 2       par 3       par 4
x1   -5.902e-03 -7.868e-03 -3.738e+00 -4.791e+00
x2   -8.585e-04 -6.213e-04 -1.966e+00 -1.669e+00
x3    1.260e+00 -1.194e+00  9.746e-01  2.854e-01

d(Pf1)/d(parameter) :
var      mean       std dev      par 1       par 2       par 3       par 4
x1    4.895e-04  6.526e-04  3.100e-01  3.974e-01
x2    7.121e-05  5.153e-05  1.630e-01  1.384e-01
x3   -1.045e-01  9.900e-02 -8.083e-02 -2.367e-02
-------------------------------------------------------------------------

sensitivity with respect to limit-state function parameters

limit-state function    2
-------------------------------------------------------------------------
par   d(beta)/d(parameter)     d(Pf1)/d(parameter)
 1          3.834e+00              -3.180e-01
-------------------------------------------------------------------------
```

```
>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used .....................itg=    2
  itg=1 ............................improved Breitung formula
  itg=2 ............................improved Breitung formula
        ..............................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function    2
-------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)         u'i    u'n     G(u)          a'i
   1 1.683  1.858 -2.034e-08    -1.306  2.139 -6.713e-12     1.0658e-01
   2 1.772  1.685  5.332e-07    -1.772  1.715  7.659e-08    -2.3251e-02

                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         1.8347              1.8374
probability                          Pf2 =    3.328e-02           3.308e-02
-------------------------------------------------------------------------


>>>> SECOND-ORDER RELIABILITY ANALYSIS  -- CURVATURE FITTING <<<<

type of integration scheme used .....................itg=    2
  itg=1 ............................improved Breitung formula
  itg=2 ............................improved Breitung formula
        ..............................& Tvedt's exact integral

limit-state function    2
-------------------------------------------------------------------------
main curvatures in (n-1)x(n-1) space

       1           2
 1  1.710e-01 -4.490e-02

                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         1.8491              1.8542
probability                          Pf2 =    3.222e-02           3.186e-02
-------------------------------------------------------------------------


>>>> MONTE CARLO SIMULATION <<<<

 print interval ..........................npr=    2000
 number of simulations ...................nsm=  500000
 threshold for coef. of variation .....cov=  3.000e-02
 seed ........................stp=   621319411.00000

limit-state function     2
-------------------------------------------------------------------------
 trials       Pf-mean        betag-mean     coef of var of Pf
  2000    3.9500000e-02    1.7565184e+00     1.1029193e-01
  4000    3.7000000e-02    1.7866138e+00     8.0674555e-02
 .
 .
 .
 24000    3.5541667e-02    1.8049452e+00     3.3626082e-02
 26000    3.5153846e-02    1.8099237e+00     3.2491091e-02
 28000    3.4785714e-02    1.8146913e+00     3.1480393e-02
 30000    3.4300000e-02    1.8210456e+00     3.0635196e-02
 32000    3.4187500e-02    1.8225279e+00     2.9712865e-02

=========================================================================
```

Example 4 -- Component reliability with two independent groups of dependent random variables defined by marginal distributions and correlation matrix.

Limit-state function:

$$g(\mathbf{X}) = t_1 - \frac{X_2}{1000X_3} - \left(\frac{X_1}{200X_3}\right)^2 - \frac{X_5}{1000X_6} - \left(\frac{X_4}{200X_6}\right)^2$$

Distributions:

Group 1:

X_1, X_2, X_3 defined in terms of marginals and correlation matrix:

X_1 : lognormal ($\mu = 500$, $\sigma = 100$)

X_2 : lognormal ($\mu = 2000$, $\sigma = 400$)

X_3 : uniform ($\mu = 5$, $\sigma = 0.5$)

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.3 & 0.2 \\ 0.3 & 1.0 & 0.2 \\ 0.2 & 0.2 & 1.0 \end{bmatrix}$$

Group 2:

X_4, X_5, X_6 defined in terms of marginals and correlation matrix:

X_4 : lognormal ($\mu = 450$, $\sigma = 90$)

X_5 : lognormal ($\mu = 1800$, $\sigma = 360$)

X_6 : uniform ($\mu = 4.5$, $\sigma = 0.45$)

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.3 & 0.2 \\ 0.3 & 1.0 & 0.2 \\ 0.2 & 0.2 & 1.0 \end{bmatrix}$$

Deterministic parameters:

$t_1 = 1.7$

Analysis performed: FORM, SENS, SORM, MONT.

INPUT FILE:

```
==========================================================================
CALREL nrx=6 nig=2 ntp=1
DATA
TITLE nline title
```

```
3
Example 4 --  Component reliability with two independent groups of
              dependent random variables defined by marginal
              distributions and correlation matrix
FLAG icl,igr
1 0
OPTIMIZATION iop,ni1,ni2,tol,op1,op2,op3
2,20,4,0.001,0.5
STATISTICS igt(i),nge,ngm nv,ids,ex,sg,p3,p4,x0
2 3
x1  1,2,500.,100.
x2  2,2,2000.,400.
x3  3,6,5.,0.5
0.3
0.2 0.2
2 3
x4  4,2,450.,90.
x5  5,2,1800.,360.
x6  6,6,4.5,0.45
0.3
0.2 0.2
para
1.7
END
FORM igf=4
SENS isc=1 isv=0 igf=4
SORM iso=3 itg=2 igf=4
MONT igf=4 nsm=50000 npr=1000 stp=62120774.
EXIT
```

==========

OUTPUT FILE:

==========

>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

```
print interval .........................npr=      0
  npr<0 ..........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag .....................ini=      0
  ini=0 .......................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ............................ist=      0
  ist=0 .......................analyze a new problem
  ist=1 .............continue an unconverged problem

limit-state function     4
-------------------------------------------------------------------------
iteration number ..............iter=         6
value of limit-state function..g(x)= 1.059e-06
reliability index .............beta=    1.6213
probability ...................Pf1= 5.247e-02
```

var	design point x*	u*	alpha	sensitivity vectors gamma	delta	eta
x1	5.697e+02	7.577e-01	0.4678	0.4482	-0.4324	-0.2637
x2	2.189e+03	3.403e-01	0.2102	0.2802	-0.2823	-0.1050
x3	4.635e+00	-7.902e-01	-0.4868	-0.4697	0.4253	-0.3108
x4	5.127e+02	7.577e-01	0.4678	0.4482	-0.4324	-0.2637
x5	1.970e+03	3.403e-01	0.2102	0.2802	-0.2823	-0.1050
x6	4.171e+00	-7.902e-01	-0.4868	-0.4697	0.4253	-0.3108

>>>> SENSITIVITY ANALYSIS AT COMPONENT LEVEL <<<<

type of parameters for sensitivity analysis
.....isv= 0
isv=1 .....distribution parameters
isv=2 .....limit-state fcn parameters
isv=0 ..distribution and limit-state fcn parameters

sensitivity with respect to distribution parameters

limit-state function 4

d(beta)/d(parameter) :

var	mean	std dev	par 1	par 2	par 3	par 4
x1	-4.324e-03	-2.637e-03	-2.426e+00	-1.838e+00		
x2	-7.058e-04	-2.626e-04	-1.517e+00	-8.412e-01		
x3	8.506e-01	-6.216e-01	6.047e-01	2.459e-01		
x4	-4.805e-03	-2.930e-03	-2.426e+00	-1.838e+00		
x5	-7.842e-04	-2.918e-04	-1.517e+00	-8.412e-01		
x6	9.451e-01	-6.907e-01	6.719e-01	2.732e-01		

d(Pf1)/d(parameter) :

var	mean	std dev	par 1	par 2	par 3	par 4
x1	4.634e-04	2.826e-04	2.600e-01	1.970e-01		
x2	7.564e-05	2.814e-05	1.625e-01	9.016e-02		
x3	-9.117e-02	6.662e-02	-6.481e-02	-2.635e-02		
x4	5.149e-04	3.140e-04	2.600e-01	1.970e-01		
x5	8.404e-05	3.127e-05	1.625e-01	9.016e-02		
x6	-1.013e-01	7.402e-02	-7.202e-02	-2.928e-02		

sensitivity with respect to limit-state function parameters

limit-state function 4

par	d(beta)/d(parameter)	d(Pf1)/d(parameter)
1	3.211e+00	-3.442e-01

>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used .....itg= 2
itg=1 .....improved Breitung formula
itg=2 .....improved Breitung formula
.....& Tvedt's exact integral
max. number of iterations for each fitting point ..inp= 4

limit-state function 4

coordinates and ave. main curvatures of fitting points in rotated space

axis	u'i	u'n	G(u)	u'i	u'n	G(u)	a'i
1	1.621	1.480	4.511e-07	-1.547	1.693	-6.946e-09	-1.6497e-02
2	1.621	1.556	5.978e-08	-1.621	1.571	1.230e-08	-2.2058e-02
3	1.621	1.621	-9.916e-07	-1.322	1.873	3.601e-08	6.1579e-02
4	1.621	1.524	3.041e-07	-1.621	1.564	5.162e-09	-2.9572e-02
5	1.621	1.563	4.927e-08	-1.621	1.577	9.859e-09	-1.9479e-02

		improved Breitung	Tvedt's EI
generalized reliability index	betag =	1.5840	1.5865
probability	Pf2 =	5.660e-02	5.631e-02

```
>>>> SECOND-ORDER RELIABILITY ANALYSIS  -- CURVATURE FITTING <<<<

type of integration scheme used .....................itg=    2
  itg=1 ............................improved Breitung formula
  itg=2 ............................improved Breitung formula
        ..........................& Tvedt's exact integral

limit-state function    4
-------------------------------------------------------------------------
main curvatures in (n-1)x(n-1) space

       1           2           3           4           5
 1  8.223e-02 -2.591e-02  9.495e-02 -1.050e-01 -3.430e-02
                                     improved Breitung      Tvedt's EI
generalized reliability index betag =         1.5704            1.5892
probability                      Pf2 =     5.816e-02         5.601e-02
-------------------------------------------------------------------------


>>>> MONTE CARLO SIMULATION <<<<

 print interval ..........................npr=    1000
 number of simulations ...................nsm=   50000
 threshold for coef. of variation .....cov=  5.000e-02
 seed ............................stp=   62120774.000000

limit-state function     4
-------------------------------------------------------------------------
 trials       Pf-mean        betag-mean     coef of var of Pf
  1000    6.3000000e-02    1.5300681e+00    1.2201600e-01
  2000    6.1000000e-02    1.5464336e+00    8.7752901e-02
  3000    5.8666667e-02    1.5660658e+00    7.3145528e-02
  4000    6.0000000e-02    1.5547741e+00    6.2591102e-02
  5000    6.0000000e-02    1.5547741e+00    5.5981784e-02
  6000    5.9833333e-02    1.5561748e+00    5.1178953e-02
  7000    6.0571429e-02    1.5499949e+00    4.7073878e-02

=========================================================================
```

Example 5 -- Component reliability with dependent random variables defined by a mixture of marginal and conditional distributions.

Limit-state function:

$$g(\mathbf{X}) = X_3 - X_4$$

Distributions:

X_1, X_2 defined in terms of marginals and correlation matrix:

X_1 : normal ($\mu = 95.39$, $\sigma = 8.42$)

X_2 : gamma ($\lambda = 22474$, $k = 11.24$)

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.2 \\ 0.2 & 1.0 \end{bmatrix}$$

X_3, X_4 defined in terms of conditional distributions:

$X_3 | X_1 = x_1$: normal ($\mu = x_1$, $\sigma = 20$)

$X_4 | X_2 = x_2$: $F_{X_4|X_2}(x_4|x_2) = 1 - \exp(-x_2 x_4^2)$,

$$f_{X_4|X_2}(x_4|x_2) = 2x_2 x_4 \exp(-x_2 x_4^2), \quad 0 \le x_4$$

Analysis performed: FORM, SENS, SORM, MONT.

INPUT FILE:

```
==========================================================================

calr  nrx=4
data
title
2
Example 5 -- Component reliability with dependent random variables def
             by a mixture of marginal and conditional distributions.
flag
1 1
opti
1,100,4,0.001
stat
3,4,2
m   1,1,95.39,8.42,0.,0.,95.
b   2,-3,22474.,11.24,0.,0.,0.0005
r   3,1, 1,20.,0.,0.,95.
s   4,51,2,0.,0.,0.,0.0005
0.2
end
```

```
form ini=1 igf=5
sens isc=1 isv=1 igf=5
sorm iso=3 itg=2 igf=5
mont igf=5 nsm=100000 npr=2000 cov=0.05 stp=621320774.
exit
```

==========

OUTPUT FILE:

==========

```
>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval .........................npr=      0
  npr<0 .........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag ....................ini=      1
  ini=0 .......................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ...........................ist=      0
  ist=0 .......................analyze a new problem
  ist=1 ..............continue an unconverged problem

limit-state function     5
-------------------------------------------------------------------------
iteration number ..............iter=         6
value of limit-state function..g(x)=-1.190e-05
reliability index .............beta=    1.6575
probability ....................Pf1= 4.871e-02
var          design point                        sensitivity vectors
           x*             u*          alpha      gamma      delta      eta
m       9.145e+01    -4.674e-01    -0.2820     0.         0.2194    -0.1025
b       4.044e-04    -5.027e-01    -0.3034     0.         0.3612    -0.2446
r       7.417e+01    -8.640e-01    -0.5212    -0.5734     0.5212    -0.4503
s       7.417e+01     1.237e+00     0.7462     0.8193
-------------------------------------------------------------------------

>>>> SENSITIVITY ANALYSIS AT COMPONENT LEVEL <<<<

type of parameters for sensitivity analysis
.........................................isv=      1
  isv=1 .....................distribution parameters
  isv=2 ..................limit-state fcn parameters
  isv=0 ..distribution and limit-state fcn parameters

sensitivity with respect to distribution parameters

limit-state function    5
-------------------------------------------------------------------------
d(beta)/d(parameter) :
var      mean        std dev      par 1       par 2       par 3       par 4
m      2.606e-02  -1.218e-02   2.606e-02  -1.218e-02
b      2.421e+03  -1.640e+03  -4.300e-05   9.686e-02
r                 -2.252e-02              -2.252e-02
s                                          0.000e+00   0.000e+00   0.000e+00
```

```
d(Pf1)/d(parameter) :
var      mean       std dev      par 1       par 2       par 3       par 4
m     -2.632e-03  1.230e-03 -2.632e-03  1.230e-03
b     -2.446e+02  1.656e+02  4.343e-06 -9.783e-03
r                 2.274e-03             2.274e-03
s                                       0.000e+00  0.000e+00  0.000e+00
-------------------------------------------------------------------------

>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used ..................itg=    2
  itg=1 .........................improved Breitung formula
  itg=2 .........................improved Breitung formula
        .........................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function
-------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)        u'i    u'n     G(u)          a'i
   1 1.651  1.664 -2.909e-07   -1.653  1.662 -6.877e-08     2.0364e-03
   2 1.658  1.655  5.921e-10   -1.658  1.656  1.128e-08    -8.4722e-04
   3 1.658  1.605  4.555e-07   -1.658  1.592  1.490e-06    -2.1525e-02

                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         1.6361              1.6364
probability                      Pf2 =     5.091e-02           5.088e-02
-------------------------------------------------------------------------

>>>> SECOND-ORDER RELIABILITY ANALYSIS  -- CURVATURE FITTING <<<<

type of integration scheme used ..................itg=    2
  itg=1 .........................improved Breitung formula
  itg=2 .........................improved Breitung formula
        .........................& Tvedt's exact integral

limit-state function    5
-------------------------------------------------------------------------
main curvatures in (n-1)x(n-1) space

       1           2           3
 1  1.750e-02  5.366e-06 -3.811e-02

                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         1.6328              1.6339
probability                      Pf2 =     5.125e-02           5.113e-02
-------------------------------------------------------------------------

>>>> MONTE CARLO SIMULATION <<<<

 print interval ..........................npr=    2000
 number of simulations ...................nsm=  100000
 threshold for coef. of variation .....cov=  5.000e-02
 seed ........................stp=   621320774.00000

limit-state function     5
-------------------------------------------------------------------------
 trials       Pf-mean        betag-mean     coef of var of Pf
  2000    5.2000000e-02    1.6257639e+00    9.5498401e-02
  4000    4.9750000e-02    1.6472829e+00    6.9110923e-02
  6000    5.3500000e-02    1.6118256e+00    5.4305521e-02
  8000    5.4000000e-02    1.6072484e+00    4.6798382e-02

=========================================================================
```

Example 6 -- Component reliability with dependent variables; repeated runs for different values of the deterministic parameter.

Limit-state function:

$$g(\mathbf{X}) = \frac{X_2 - X_4}{(X_1 + X_3)^{1/2}} - t_1$$

Distributions:

$$X_1: \quad F_{X_1}(x_1) = 1 - \frac{\Gamma\left(4.5, \frac{1400}{x_1}\right)}{\Gamma(4.5)}, \quad f_{X_1}(x_1) = \left(\frac{1400}{x_1}\right)^{5.5} \frac{\exp\left(-\frac{1400}{x_1}\right)}{1400\,\Gamma(4.5)}, \quad 0 \le x_1$$

$$X_2 | X_1 = x_1: \quad \text{normal}\,[\mu = 100, \sigma = (10 x_1)^{1/2}]$$

$$X_3: \quad F_{X_3}(x_3) = 1 - \frac{\Gamma\left(4.5, \frac{350}{x_3}\right)}{\Gamma(4.5)}, \quad f_{X_3}(x_3) = \left(\frac{350}{x_3}\right)^{5.5} \frac{\exp\left(-\frac{350}{x_3}\right)}{350\,\Gamma(4.5)}, \quad 0 \le x_3$$

$$X_4 | X_3 = x_3: \quad \text{normal}\,[\mu = 40, \sigma = (10 x_3)^{-1/2}]$$

Deterministic parameters:

t_1 = 2.5 and 3.0

Analysis performed: FORM, SENS, SORM.

INPUT FILE:

```
==========================================================================

calrel ngf=1 nig=2 nrx=4 ntp=1
data
title
2
Example 6 -- Component reliability with dependent variables; repeated
             runs for different values of the limit-state parameter.
flag icl,igr
1 0
optimization iop,ni1,ni2,tol,op1,op2,op3
2 90 4 0.001 0.5 10 0
```

```
statistics igt(i),nge,ngm
3 2 1
v1    1,52,1400.,4.5,0.,0.,810.
m1    2,53,100.,1,10.,0.,100.
3 2 1
v2    3,54,350.0,4.5,0.,0.,89.
m2    4,55,40.,3,10.,0.,40.
para
2.5
end
form ini=1 igf=6
sens igf=6
sorm iso=3 itg=2 igf=6
data
para
3.0
end
form ini=-1 npr=-1 igf=6
sorm iso=1 itg=2 igf=6
exit
```

==========

OUTPUT FILE:

==========

```
>>>> INPUT DATA <<<<

deterministic parameters in limit-state function:
  tp (  1) =  2.500e+00


>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval .........................npr=      0
  npr<0 .........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag ....................ini=      1
  ini=0 .......................start from mean point
  ini=1 .........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ...........................ist=      0
  ist=0 .....................analyze a new problem
  ist=1 ............continue an unconverged problem

limit-state function     6
------------------------------------------------------------------------
iteration number ..............iter=         4
value of limit-state function..g(x)=-5.140e-07
reliability index .............beta=    0.7314
probability ...................Pf1= 2.323e-01
var          design point                        sensitivity vectors
           x*            u*         alpha     gamma     delta      eta
v1      4.870e+02     7.217e-01    0.9866    0.9866
m1      1.000e+02    -7.442e-04   -0.0010   -0.0010
v2      8.895e+01     1.190e-01    0.1632    0.1632
m2      4.000e+01     1.743e-03    0.0024    0.0024
------------------------------------------------------------------------
```

```
>>>> SENSITIVITY ANALYSIS AT COMPONENT LEVEL <<<<

type of parameters for sensitivity analysis
.....................................isv=      0
  isv=1 .....................distribution parameters
  isv=2 ....................limit-state fcn parameters
  isv=0 ..distribution and limit-state fcn parameters

sensitivity with respect to distribution parameters

limit-state function    6
-------------------------------------------------------------------------
d(beta)/d(parameter) :
var     mean       std dev      par 1       par 2       par 3       par 4
v1                          -1.287e-03  5.155e-01  0.000e+00  0.000e+00
m1                           1.276e-02             3.788e-08  0.000e+00
v2                          -9.405e-04  8.111e-02  0.000e+00  0.000e+00
m2                          -5.726e-02             2.077e-07  0.000e+00

d(Pf1)/d(parameter) :
var     mean       std dev      par 1       par 2       par 3       par 4
v1                           3.930e-04 -1.574e-01  0.000e+00  0.000e+00
m1                          -3.896e-03            -1.157e-08  0.000e+00
v2                           2.871e-04 -2.476e-02  0.000e+00  0.000e+00
m2                           1.748e-02            -6.340e-08  0.000e+00
-------------------------------------------------------------------------

sensitivity with respect to limit-state function parameters

limit-state function    6
-------------------------------------------------------------------------
par   d(beta)/d(parameter)     d(Pf1)/d(parameter)
 1         -1.705e+00                 5.206e-01
-------------------------------------------------------------------------


>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used ....................itg=    2
  itg=1 ..........................improved Breitung formula
  itg=2 ..........................improved Breitung formula
        ..........................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function    6
-------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)        u'i    u'n     G(u)          a'i
   1 1.000  0.731 -8.145e-08   -1.000  0.731 -7.387e-08    -2.3471e-06
   2 1.000  0.731 -4.801e-08   -1.000  0.731 -5.130e-08    -1.4996e-06
   3 1.068  0.628  7.497e-09   -1.034  0.682  8.684e-08    -6.8973e-02

                                      improved Breitung      Tvedt's EI
generalized reliability index betag =        0.6536             0.6596
probability                      Pf2 =     2.567e-01          2.547e-01
-------------------------------------------------------------------------


>>>> SECOND-ORDER RELIABILITY ANALYSIS  -- CURVATURE FITTING <<<<

type of integration scheme used ....................itg=    2
  itg=1 ..........................improved Breitung formula
```

```
  itg=2 .........................improved Breitung formula
        .........................& Tvedt's exact integral

limit-state function    6
-------------------------------------------------------------------------
main curvatures in (n-1)x(n-1) space

       1           2           3
 1 -1.631e-06  4.937e-07 -6.160e-02

                                        improved Breitung       Tvedt's EI
generalized reliability index betag =          0.6629              0.6675
probability                      Pf2 =       2.537e-01           2.522e-01
-------------------------------------------------------------------------


>>>> INPUT DATA <<<<


deterministic parameters in limit-state function:
  tp (  1) =  3.000e+00


>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval .........................npr=     -1
  npr<0 ..........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag .....................ini=     -1
  ini=0 .......................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ............................ist=      0
  ist=0 .......................analyze a new problem
  ist=1 ..............continue an unconverged problem

limit-state function     6


>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used ...................itg=    2
  itg=1 .........................improved Breitung formula
  itg=2 .........................improved Breitung formula
        .........................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function    6
-------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)          u'i    u'n     G(u)          a'i
   1-1.000 -0.120  1.421e-07     1.000 -0.120  1.439e-07    -1.5481e-05
   2-1.000 -0.120  1.351e-07     1.000 -0.120  1.337e-07    -1.4549e-05
   3-0.989 -0.193 -1.079e-06     0.976 -0.248 -3.564e-07    -1.0377e-01

                                        improved Breitung       Tvedt's EI
generalized reliability index betag =         -0.2124             -0.2205
probability                      Pf2 =       5.841e-01           5.873e-01
-------------------------------------------------------------------------

=========================================================================
```

Example 7 -- Series system reliability with dependent random variables.

Limit-state functions:

$$g_1(\mathbf{X}) = X_1 + X_2 + X_4 + X_5 - X_6 t_1$$
$$g_2(\mathbf{X}) = X_1 + 2X_3 + 2X_4 + X_5 - X_6 t_1 - X_7 t_1$$
$$g_3(\mathbf{X}) = X_2 + 2X_3 + X_4 - X_7 t_1$$

Distributions:

Group 1:

$X_1, \ldots, X_5$ defined in terms of marginals and correlation matrix:

X_1 : Weibull ($\mu = 134, \sigma = 23$)

X_2 : Weibull ($\mu = 134, \sigma = 23$)

X_3 : Weibull ($\mu = 160, \sigma = 35$)

X_4 : Weibull ($\mu = 150, \sigma = 30$)

X_5 : Weibull ($\mu = 150, \sigma = 30$)

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.4 & 0.2 & 0.2 & 0.2 \\ 0.4 & 1.0 & 0.4 & 0.2 & 0.2 \\ 0.2 & 0.4 & 1.0 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0.4 & 1.0 & 0.4 \\ 0.2 & 0.2 & 0.2 & 0.4 & 1.0 \end{bmatrix}$$

Group 2:

X_6, X_7 defined in terms of marginals and correlation matrix:

X_6 : uniform ($\mu = 65, \sigma = 20$)

X_7 : uniform ($\mu = 50, \sigma = 15$)

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.4 \\ 0.4 & 1.0 \end{bmatrix}$$

Deterministic parameters:

$t_1 = 5$

Analysis performed: BOUN, PNET, SENS, SORM, DIRS, MONT.

INPUT FILE:

```
==========================================================================

CALREL ngf=3 nig=2 nrx=7 ntp=1
DATA
TITLE
1
Example 7 -- Series system reliability with dependent random variables.
FLAG icl,igr
2 0
CUTS ncs,ntl
3,3
7,0,8,0,9,0
OPTI iop,ni1,ni2,tol,op1,op2,op3
4,40,4,0.001,0.25,0.001,4
STAT igt(i),nge,ngm nv,ids,ex,sg,p3,p4,x0
2 5
x1  1,14,134.,23.,0.,0.,0.
x2  2,14,134.,23.,0.,0.,0.
x3  3,14,160.,35.,0.,0.,0.
x4  4,14,150.,30.,0.,0.,0.
x5  5,14,150.,30.,0.,0.,0.
0.4
0.2 0.4
0.2 0.2 0.4
0.2 0.2 0.2 0.4
2,2
x6  6,6,65.,20.,0.,0.,0.
x7  7,6,50.,15.,0.,0.,0.
0.4
PARA tp
5.
END
FORM
BOUN ibt=3
PNET
SENS isc=2
SORM iso=1
DIRS ifs=0 nsm=1000 npr=500 cov=0.05 stp=619944257.
DIRS ifs=1 nsm=1000 npr=500 cov=0.05 stp=619944257.
DIRS ifs=2 nsm=1000 npr=500 cov=0.05 stp=619944257.
MONT nsm=4000 npr=2000 cov=0.05 stp=619944257.
EXIT

==========================================================================
```

OUTPUT FILE:

```
==========================================================================

>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval ..........................npr=      0
  npr<0 ..........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag .....................ini=      0
  ini=0 .......................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
```

```
restart flag ..........................ist=      0
  ist=0 ....................analyze a new problem
  ist=1 ............continue an unconverged problem

limit-state function     7
-------------------------------------------------------------------------
iteration number ..............iter=         7
value of limit-state function..g(x)= 3.490e-06
reliability index .............beta=    1.9340
probability ....................Pf1= 2.656e-02
var          design point                        sensitivity vectors
           x*            u*          alpha      gamma      delta      eta
x1      1.126e+02    -9.390e-01    -0.4856    -0.2947     0.2191   -0.2165
x2      1.126e+02    -6.108e-01    -0.3159    -0.2947     0.2191   -0.2165
x3      1.370e+02    -2.966e-01    -0.1534     0.         0.0000   -0.0000
x4      1.169e+02    -7.810e-01    -0.4038    -0.3810     0.2847   -0.3235
x5      1.169e+02    -5.686e-01    -0.2940    -0.3810     0.2847   -0.3235
x6      9.180e+01     1.210e+00     0.6256     0.7321    -0.9414   -1.2616
x7      6.001e+01     0.000e+00     0.         0.         0.        0.
-------------------------------------------------------------------------

limit-state function     8
-------------------------------------------------------------------------
iteration number ..............iter=         6
value of limit-state function..g(x)= 5.595e-05
reliability index .............beta=    1.5142
probability ....................Pf1= 6.499e-02
var          design point                        sensitivity vectors
           x*            u*          alpha      gamma      delta      eta
x1      1.250e+02    -4.589e-01    -0.3030    -0.1536     0.1163   -0.0569
x2      1.248e+02    -3.070e-01    -0.2027     0.         0.0000   -0.0000
x3      1.291e+02    -7.449e-01    -0.4919    -0.4840     0.3540   -0.3360
x4      1.236e+02    -5.460e-01    -0.3606    -0.4209     0.3029   -0.2867
x5      1.328e+02    -2.236e-01    -0.1477    -0.2039     0.1524   -0.1024
x6      8.714e+01     9.136e-01     0.6034     0.5639    -0.4968   -0.5499
x7      6.549e+01     5.001e-01     0.3305     0.4531    -0.3726   -0.3847
-------------------------------------------------------------------------

limit-state function     9
-------------------------------------------------------------------------
iteration number ..............iter=         7
value of limit-state function..g(x)= 1.209e-04
reliability index .............beta=    2.6986
probability ....................Pf1= 3.481e-03
var          design point                        sensitivity vectors
           x*            u*          alpha      gamma      delta      eta
x1      1.175e+02    -7.528e-01    -0.2790     0.         0.0000   -0.0000
x2      9.807e+01    -1.269e+00    -0.4704    -0.2780     0.1880   -0.2861
x3      7.561e+01    -1.797e+00    -0.6659    -0.7338     0.5040   -1.0563
x4      9.904e+01    -7.153e-01    -0.2650    -0.3470     0.2410   -0.3876
x5      1.271e+02     1.085e-05     0.         0.         0.        0.
x6      7.791e+01     4.854e-01     0.1799     0.        -0.0000   -0.0000
x7      6.967e+01     1.062e+00     0.3936     0.5137    -0.6190   -0.8115
-------------------------------------------------------------------------


>>>> FIRST-ORDER BOUNDS FOR SERIES SYSTEM <<<<

type of bounds ........................ibt=      3
  ibt=1 ..........................unimodal bounds
  ibt=2 ......................relaxed bimodal  bounds
  ibt=3 ............................bimodal  bounds
  ibt=0 ..........................all of the above
```

```
modal correlation coefficient matrix
        7           8           9
 7   1.000e+00   8.531e-01   6.058e-01
 8   8.531e-01   1.000e+00   8.417e-01
 9   6.058e-01   8.417e-01   1.000e+00

joint-modal probabilities by numerical integration
        7           8           9
 7   2.656e-02   2.068e-02   1.543e-03
 8   2.068e-02   6.499e-02   3.342e-03
 9   1.543e-03   3.342e-03   3.481e-03

bimodal bounds:                          lower bound  upper bound
  probability ..................... 1.469e+00 ,  1.468e+00
  generalized reliability index..... 7.087e-02 ,  7.101e-02


>>>> PNET APPROXIMATION <<<<

threshold for coef. of correlation ..rho0= 6.000e-01

generalized reliability index ......betag= 1.514e+00
failure probability ..................pf= 6.499e-02
representative modes:
      8


>>>> SENSITIVITY ANALYSIS FOR SERIES SYSTEM <<<<

type of parameters for sensitivity analysis
.........................................isv=      0
  isv=1 .....................distribution parameters
  isv=2 ....................limit-state fcn parameters
  isv=0 ..distribution and limit-state fcn parameters


sensitivity measures based on bimodal upper bound of Pf1
----------------------------------------------------------------------

sensitivity with respect to distribution parameters
d(betag)/d(parameter) :
var      mean       std dev      par 1       par 2       par 3       par 4
x1     5.895e-03 -3.693e-03   4.916e-03   1.689e-02
x2     1.755e-03 -2.135e-03   1.298e-03   8.067e-03
x3     8.440e-03 -8.534e-03   6.054e-03   6.345e-02
x4     1.014e-02 -9.398e-03   7.652e-03   5.666e-02
x5     5.909e-03 -4.619e-03   4.616e-03   2.919e-02
x6    -2.904e-02 -3.271e-02 -5.077e-03 -2.396e-02
x7    -2.082e-02 -2.318e-02 -3.720e-03 -1.710e-02

d(Pf1)/d(parameter) :
var      mean       std dev      par 1       par 2       par 3       par 4
x1    -8.002e-04  5.013e-04 -6.674e-04 -2.293e-03
x2    -2.382e-04  2.898e-04 -1.762e-04 -1.095e-03
x3    -1.146e-03  1.159e-03 -8.218e-04 -8.613e-03
x4    -1.377e-03  1.276e-03 -1.039e-03 -7.692e-03
x5    -8.022e-04  6.271e-04 -6.267e-04 -3.962e-03
x6     3.942e-03  4.440e-03  6.893e-04  3.253e-03
x7     2.827e-03  3.147e-03  5.051e-04  2.322e-03
```

```
sensitivity with respect to limit-state function parameters
par   d(betag)/d(parameter)     d(Pf1)/d(parameter)
 1         -7.869e-01                1.068e-01
-----------------------------------------------------------------


>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used .....................itg=    2
  itg=1 .............................improved Breitung formula
  itg=2 .............................improved Breitung formula
        .............................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function    7
-----------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)          u'i    u'n     G(u)          a'i
   1 1.499  2.288  1.240e-07   -1.652  2.180 -9.887e-05     1.2139e-01
   2 1.853  2.012 -2.054e-05   -1.860  2.005 -8.484e-04     2.1670e-02
   3 1.917  1.951 -1.094e-05   -1.917  1.951 -6.226e-05     4.6508e-03
   4 1.842  2.021 -1.709e-05   -1.841  2.022 -9.629e-04     2.5929e-02
   5 1.930  1.938 -3.991e-07   -1.894  1.973 -2.788e-04     5.9639e-03
   6 1.934  1.934 -2.700e-07   -1.934  1.934 -2.700e-07     4.5382e-09

                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         2.0817              2.0872
probability                      Pf2 =     1.868e-02           1.843e-02
-----------------------------------------------------------------------------

limit-state function    8
-----------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)          u'i    u'n     G(u)          a'i
   1 1.391  1.628 -8.715e-05   -1.441  1.584 -2.178e-04     4.6029e-02
   2 1.490  1.538 -4.194e-05   -1.495  1.533 -6.760e-05     9.4512e-03
   3 1.457  1.569 -9.155e-05   -1.473  1.554 -1.895e-04     2.2024e-02
   4 1.514  1.503  1.255e-06   -1.514  1.510  9.727e-08    -3.3801e-03
   5 1.514  1.497  2.492e-06   -1.514  1.504  6.380e-07    -6.1054e-03
   6 1.232  1.752  1.536e-05   -1.356  1.657 -7.177e-05     1.1442e-01

                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         1.6668              1.6739
probability                      Pf2 =     4.778e-02           4.707e-02
-----------------------------------------------------------------------------

limit-state function    9
-----------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)          u'i    u'n     G(u)          a'i
   1 2.312  3.036 -1.495e-05   -2.355  3.003 -6.177e-05     5.8974e-02
   2 2.330  3.022 -2.008e-05   -2.361  2.999 -7.106e-05     5.6694e-02
   3 2.444  2.932 -4.879e-05   -2.430  2.943 -2.339e-04     4.0161e-02
   4 2.699  2.686  2.669e-06   -2.600  2.793 -3.187e-04     5.9083e-03
   5 2.699  2.699 -1.737e-05   -2.699  2.699 -1.737e-05     1.3530e-07
   6 2.499  2.884 -1.395e-04   -2.517  2.869 -7.392e-04     2.8322e-02

                                        improved Breitung      Tvedt's EI
generalized reliability index betag =         2.8618              2.8665
probability                      Pf2 =     2.106e-03           2.075e-03
-----------------------------------------------------------------------------
```

```
>>>> DIRECTIONAL SIMULATION <<<<

 print interval ..........................npr=    500
 number of simulationts ..................nsm=   1000
 threshold for coef. of variation .....cov= 5.000e-02
 seed ............................stp=   619944257.00000
 trials       Pf-mean       betag-mean     coef of var of pf
   500    5.7031022e-02    1.5801963e+00    8.4795575e-02
  1000    5.3227368e-02    1.6143357e+00    6.2514367e-02


>>>> FIRST-ORDER DIRECTIONAL SIMULATION <<<<

 print interval ..........................npr=    500
 number of simulationts ..................nsm=   1000
 threshold for coef. of variation .....cov= 5.000e-02
 seed ............................stp=   619944257.00000

trials       Pf-mean       betag-mean     coef of var of pf
   500    7.2771819e-02    1.4554543e+00    7.2591446e-02
  1000    6.9878991e-02    1.4766933e+00    5.2352926e-02


>>>> SECOND-ORDER DIRECTIONAL SIMULATION <<<<

 print interval ..........................npr=    500
 number of simulationts ..................nsm=   1000
 threshold for coef. of variation .....cov= 5.000e-02
 seed ............................stp=   619944257.00000

trials       Pf-mean       betag-mean     coef of var of pf
   500    5.4094307e-02    1.6063888e+00    8.8488712e-02
  1000    5.1734063e-02    1.6282682e+00    6.3741221e-02


>>>> MONTE CARLO SIMULATION <<<<

 print interval ..........................npr=    2000
 number of simulations ...................nsm=    4000
 threshold for coef. of variation .....cov=  5.000e-02
 seed ............................stp=   619944257.00000
 trials       Pf-mean       betag-mean     coef of var of Pf
  2000    5.8500000e-02    1.5674914e+00    8.9727555e-02
  4000    5.6000000e-02    1.5892681e+00    6.4925646e-02

================================================================================
```

Example 8 -- General system reliability with dependent random variables.

Failure domain:

$$F \equiv [\{(g_1 \le 0)(g_4 \le 0)\} \cup \{(g_1 \le 0)(g_5 \le 0)\} \cup \{(g_1 \le 0)(g_6 \le 0)\} \cup \{(g_2 \le 0) g_7 \le 0)\} \cup \{(g_2 \le 0)(g_5 \ge 0)\} \cup \{(g_2 \le 0)(g_8 \le 0)\} \cup \{(g_3 \le 0)(g_9 \le 0)\}]$$

Limit-state functions:

$$g_1(\mathbf{X}) = X_1 - 4X_3 - 4X_4$$
$$g_2(\mathbf{X}) = X_1 - 2X_3 - 7X_4$$
$$g_3(\mathbf{X}) = X_2 - 0.2X_3 - 0.7X_4$$
$$g_4(\mathbf{X}) = 2.5X_1 - 10X_3 - 5X_4$$
$$g_5(\mathbf{X}) = X_1 - 10X_4$$
$$g_6(\mathbf{X}) = X_2 - 0.05X_1 - 0.5X_4$$
$$g_7(\mathbf{X}) = 2X_1 - 5X_3 - 10X_4$$
$$g_8(\mathbf{X}) = -X_1 + 10X_2$$
$$g_9(\mathbf{X}) = X_1 - 10X_3 - 20X_4$$

Distributions:

Group 1:

X_1, X_2 defined in terms of marginals and correlation matrix:

X_1 : lognormal ($\mu = 100, \sigma = 20$)

X_2 : lognormal ($\mu = 15, \sigma = 3$)

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.2 \\ 0.2 & 1.0 \end{bmatrix}$$

Group 2:

X_3, X_4 defined in terms of marginals and correlation matrix:

X_3 : type II largest value ($\mu = 7, \sigma = 2.1$)

X_4 : type II largest value ($\mu = 5, \sigma = 1.5$)

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.3 \\ 0.3 & 1.0 \end{bmatrix}$$

Analysis performed: FORM, SORM, DIRS, MONT.

INPUT FILE: (part I: not complete in FORM analysis)

```
==========================================================================

CALREL ngf=9 nig=2 nrx=4
DATA
TITLE nline title
1
Example 8 -- General system reliability with dependent random variables.
FLAG icl,igr
3,0
CUTSET ncs,ntl
7,14
11,14,0,11,15,0,11,16,0,12,17,0,12,-15,0,12,
18,0,13,19,0
OPTIMIZATION iop,ni1,ni2,tol,op1,op2,op3
1,50,4,-0.0001,0.5
STATISTICS igt(i),nge,ngm nv,ids,ex,sg,p3,p4,x0
2 2
x1  1,2,100.,20.,0.,0.,0.
x2  2,2,15.,3.,0.,0.,0.
0.2
2 2
x3  3,13,7.,2.1,0.,0.,0.
x4  4,13,5.,1.5,0.,0.,0.
0.3
END
FORM
SORM iso=1
DIRS ifs=0 nsm=5000 npr=500 cov=0.05 stp=619949844.
DIRS ifs=1 nsm=5000 npr=500 cov=0.05 stp=619949844.
DIRS ifs=2 nsm=5000 npr=500 cov=0.05 stp=619949844.
MONT nsm=100000 cov=.05 npr=5000 stp=619949855.
EXIT

==========================================================================
```

OUTPUT FILE: (part I: not complete in FORM analysis)

```
==========================================================================

>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval .........................npr=      0
  npr<0 .........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag ....................ini=      0
  ini=0 .......................start from mean point
  ini=1 .........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ...........................ist=      0
  ist=0 .......................analyze a new problem
  ist=1 .............continue an unconverged problem

limit-state function    11
--------------------------------------------------------------------------
iteration number ..............iter=        32
value of limit-state function..g(x)=-7.654e-08
reliability index .............beta=    2.4260
probability ....................Pf1= 7.634e-03
var          design point                    sensitivity vectors
```

	x*	u*	alpha	gamma	delta	eta
x1	7.808e+01	-1.151e+00	-0.4742	-0.5175	0.6044	-0.6272
x2	1.404e+01	3.753e-11	0.	0.	0.	0.
x3	1.260e+01	2.014e+00	0.8300	0.7840	-0.2618	-0.6742
x4	6.918e+00	7.123e-01	0.2936	0.3427	-0.1968	-0.1930

---

limit-state function 12

---

iteration number .....iter= 22
value of limit-state function..g(x)= 1.169e-05
reliability index .....beta= 2.2811
probability .....Pf1= 1.127e-02

var	design point		sensitivity vectors			
	x*	u*	alpha	gamma	delta	eta
x1	8.008e+01	-1.022e+00	-0.4482	-0.4703	0.5599	-0.5364
x2	1.412e+01	3.843e-08	0.	0.	0.	0.
x3	8.567e+00	1.027e+00	0.4503	0.1647	-0.1264	-0.0631
x4	8.993e+00	1.762e+00	0.7723	0.8670	-0.3017	-0.7744

---

limit-state function 13

---

iteration number .....iter= 23
value of limit-state function..g(x)=-1.869e-07
reliability index .....beta= 3.0843
probability .....Pf1= 1.020e-03

var	design point		sensitivity vectors			
	x*	u*	alpha	gamma	delta	eta
x1	9.394e+01	-2.164e-01	-0.0702	0.	0.	0.
x2	1.191e+01	-1.043e+00	-0.3381	-0.3566	0.4343	-0.4278
x3	9.359e+00	1.287e+00	0.4172	0.1011	-0.0658	-0.0549
x4	1.434e+01	2.593e+00	0.8406	0.9288	-0.0864	-1.1834

---

limit-state function 14

---

iteration number .....iter= 22
value of limit-state function..g(x)= 1.325e-05
reliability index .....beta= 2.7860
probability .....Pf1= 2.668e-03

var	design point		sensitivity vectors			
	x*	u*	alpha	gamma	delta	eta
x1	7.974e+01	-1.044e+00	-0.3747	-0.3876	0.4697	-0.4564
x2	1.410e+01	3.843e-08	0.	0.	0.	0.
x3	1.673e+01	2.569e+00	0.9221	0.9156	-0.1621	-1.0493
x4	6.406e+00	2.704e-01	0.0970	0.1073	-0.0760	-0.0506

---

limit-state function 15

---

iteration number .....iter= 20
value of limit-state function..g(x)= 5.977e-05
reliability index .....beta= 2.0174
probability .....Pf1= 2.182e-02

var	design point		sensitivity vectors			
	x*	u*	alpha	gamma	delta	eta
x1	8.315e+01	-8.328e-01	-0.4128	-0.4128	0.5003	-0.4173
x2	1.422e+01	1.537e-07	0.	0.	0.	0.
x3	7.650e+00	6.525e-01	0.3234	0.	0.0000	0.0000
x4	8.315e+00	1.718e+00	0.8515	0.9108	-0.3922	-0.7779

---

limit-state function 16
ERROR 21: convergence not achieved in ni1 steps for failure mode 16

==========

INPUT FILE: (part II: continuous analysis of part I)

```
==========================================================================

REST
DATA
TITLE nline title
1
Example 8.1 --Continue to do analysis of Example 8.
OPTIMIZATION iop,ni1,ni2,tol,op1,op2,op3
1,100,4,-0.0001,0.5
END
FORM
SORM iso=1
DIRS ifs=0 nsm=5000 npr=500 cov=0.05 stp=619949844.
DIRS ifs=1 nsm=5000 npr=500 cov=0.05 stp=619949844.
DIRS ifs=2 nsm=5000 npr=500 cov=0.05 stp=619949844.
MONT nsm=100000 cov=.05 npr=5000 stp=619949855.
EXIT

==========================================================================
```

OUTPUT FILE: (part II: continuous analysis of part I)

```
==========================================================================

>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval .........................npr=      0
  npr<0 ..........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag ....................ini=      0
  ini=0 .....................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ...........................ist=      0
  ist=0 .....................analyze a new problem
  ist=1 .............continue an unconverged problem

limit-state function    16
--------------------------------------------------------------------------
iteration number ..............iter=        55
value of limit-state function..g(x)=-1.032e-08
reliability index .............beta=    2.9613
probability ....................Pf1= 1.532e-03
var          design point                      sensitivity vectors
            x*             u*          alpha     gamma     delta      eta
x1      1.100e+02      5.815e-01     0.1963    0.3104   -0.3017   -0.1214
x2      1.061e+01     -1.805e+00    -0.6096   -0.5986    0.8543   -1.1280
x3      8.001e+00      8.075e-01     0.2727    0.        0.         0.
x4      1.022e+01      2.126e+00     0.7184    0.7384   -0.2108   -0.8120
--------------------------------------------------------------------------

limit-state function    17
--------------------------------------------------------------------------
iteration number ..............iter=        33
value of limit-state function..g(x)=-2.837e-07
reliability index .............beta=    2.7013
probability ....................Pf1= 3.454e-03
var          design point                      sensitivity vectors
```

	x*	u*	alpha	gamma	delta	eta
x1	7.775e+01	-1.172e+00	-0.4337	-0.4696	0.5546	-0.5826
x2	1.403e+01	1.876e-11	0.	0.	0.	0.
x3	9.993e+00	1.464e+00	0.5420	0.2908	-0.1579	-0.1775
x4	1.055e+01	1.944e+00	0.7198	0.8336	-0.1956	-0.8357

limit-state function 18

iteration number .....iter= 20
value of limit-state function..g(x)= 9.794e-05
reliability index .....beta= 1.6218
probability .....Pf1= 5.242e-02

var	design point		sensitivity vectors			
	x*	u*	alpha	gamma	delta	eta
x1	1.201e+02	1.024e+00	0.6312	0.7071	-0.6717	-0.6414
x2	1.201e+01	-1.258e+00	-0.7756	-0.7071	0.9898	-0.9490
x3	6.498e+00	6.045e-07	0.	0.	0.	0.
x4	4.641e+00	4.171e-07	0.	0.	0.	0.

limit-state function 19

iteration number .....iter= 21
value of limit-state function..g(x)=-7.472e-05
reliability index .....beta= -1.8797
probability .....1 - Pf1= 3.007e-02

var	design point		sensitivity vectors			
	x*	u*	alpha	gamma	delta	eta
x1	1.332e+02	1.547e+00	-0.8227	-0.8587	0.6133	1.0879
x2	1.565e+01	7.685e-08	0.	0.	0.	0.
x3	5.551e+00	-8.134e-01	0.4327	0.3055	-0.6302	0.3699
x4	3.885e+00	-6.929e-01	0.3686	0.4116	-0.8938	0.5570

>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used .....itg= 2
itg=1 .....improved Breitung formula
itg=2 .....improved Breitung formula
.....& Tvedt's exact integral
max. number of iterations for each fitting point ..inp= 4

limit-state function 11

coordinates and ave. main curvatures of fitting points in rotated space

axis	u'i	u'n	G(u)	u'i	u'n	G(u)	a'i
1	2.426	1.937	8.489e-03	-2.426	2.174	2.932e-03	-6.5164e-02
2	2.426	2.426	3.946e-08	-2.426	2.426	3.946e-08	-7.3395e-10
3	2.426	2.083	3.882e-02	-2.426	1.959	7.311e-04	-6.9376e-02

	improved Breitung	Tvedt's EI
generalized reliability index betag =	2.2529	2.2673
probability Pf2 =	1.213e-02	1.168e-02

limit-state function 12

coordinates and ave. main curvatures of fitting points in rotated space

axis	u'i	u'n	G(u)	u'i	u'n	G(u)	a'i
1	2.281	2.109	2.052e-02	-2.281	2.023	6.382e-04	-4.1627e-02
2	2.281	2.281	3.841e-08	-2.281	2.281	3.841e-08	6.2313e-08
3	2.281	1.881	4.638e-03	-2.281	2.144	3.098e-03	-5.4541e-02

	improved Breitung	Tvedt's EI
generalized reliability index betag =	2.1677	2.1735
probability Pf2 =	1.509e-02	1.487e-02

limit-state function 13

coordinates and ave. main curvatures of fitting points in rotated space

axis	u'i	u'n	G(u)	u'i	u'n	G(u)	a'i
1	3.000	3.075	4.107e-05	-3.000	3.074	2.314e-05	-1.0490e-03
2	3.000	2.936	2.686e-02	-3.000	2.834	1.437e-03	-2.2352e-02
3	3.000	2.380	5.454e-03	-3.000	2.945	5.470e-03	-5.3372e-02

	improved Breitung	Tvedt's EI
generalized reliability index betag =	2.9918	2.9955
probability Pf2 =	1.387e-03	1.370e-03

limit-state function 14

coordinates and ave. main curvatures of fitting points in rotated space

axis	u'i	u'n	G(u)	u'i	u'n	G(u)	a'i
1	2.786	2.279	8.172e-02	-2.786	2.662	3.485e-02	-4.4013e-02
2	2.786	2.786	7.164e-07	-2.786	2.786	7.164e-07	1.5675e-08
3	2.786	2.615	3.141e-01	-2.786	2.492	1.315e-02	-3.0258e-02

	improved Breitung	Tvedt's EI
generalized reliability index betag =	2.7001	2.7030
probability Pf2 =	3.466e-03	3.436e-03

limit-state function 15

coordinates and ave. main curvatures of fitting points in rotated space

axis	u'i	u'n	G(u)	u'i	u'n	G(u)	a'i
1	2.017	1.925	6.021e-03	-2.017	1.885	6.489e-04	-2.7691e-02
2	2.017	2.017	6.114e-08	-2.017	2.017	6.114e-08	3.6773e-07
3	2.017	2.015	2.390e-06	-2.017	2.015	1.739e-06	-6.5962e-04

	improved Breitung	Tvedt's EI
generalized reliability index betag =	1.9869	1.9875
probability Pf2 =	2.347e-02	2.343e-02

limit-state function 16

coordinates and ave. main curvatures of fitting points in rotated space

axis	u'i	u'n	G(u)	u'i	u'n	G(u)	a'i
1	2.961	2.415	4.835e-06	-2.961	2.653	1.548e-03	-4.9908e-02
2	2.961	2.504	1.032e-02	-2.961	2.267	4.100e-07	-6.7018e-02
3	2.961	2.913	3.492e-05	-2.961	2.905	3.700e-06	-5.9268e-03

```
                                                  improved Breitung      Tvedt's EI
generalized reliability index betag =                    2.8034             2.8141
probability                          Pf2 =            2.529e-03          2.446e-03
-------------------------------------------------------------------------------

limit-state function   17
-------------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)          u'i    u'n     G(u)          a'i
   1 2.701  2.441  1.865e-01    -2.701  2.315  2.930e-03    -4.4693e-02
   2 2.701  2.701  1.829e-07    -2.701  2.701  1.829e-07    -9.0511e-10
   3 2.701  1.842  1.422e-02    -2.701  2.381  4.621e-02    -9.0860e-02

                                                  improved Breitung      Tvedt's EI
generalized reliability index betag =                    2.5123             2.5328
probability                          Pf2 =            5.997e-03          5.658e-03
-------------------------------------------------------------------------------

limit-state function   18
-------------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)          u'i    u'n     G(u)          a'i
   1 1.622  1.622 -1.456e-05    -1.622  1.622 -4.831e-06     1.2270e-06
   2 1.622  1.622 -9.918e-06    -1.622  1.622 -9.918e-06     1.2274e-06
   3 1.622  1.622 -9.918e-06    -1.622  1.622 -9.918e-06     1.2274e-06

                                                  improved Breitung      Tvedt's EI
generalized reliability index betag =                    1.6218             1.6218
probability                          Pf2 =            5.242e-02          5.242e-02
-------------------------------------------------------------------------------

limit-state function   19
-------------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)          u'i    u'n     G(u)          a'i
   1-1.734 -2.015  1.604e-06     1.682 -2.059  3.807e-04    -5.3858e-02
   2-1.880 -1.880  7.069e-06     1.880 -1.880  7.069e-06    -6.5365e-07
   3-1.755 -1.997  1.797e-06     1.695 -2.048  3.541e-04    -4.8035e-02

                                                  improved Breitung      Tvedt's EI
generalized reliability index betag =                   -1.9698            -1.9723
probability                          Pf2 = 1-  2.443e-02          1-  2.429e-02
-------------------------------------------------------------------------------


>>>> DIRECTIONAL SIMULATION <<<<

 print interval ..........................npr=    500
 number of simulationts ..................nsm=   5000
 threshold for coef. of variation .....cov= 5.000e-02
 seed ............................stp=   619949844.00000
 trials       Pf-mean        betag-mean     coef of var of pf
   500    8.2079832e-03    2.3995344e+00    8.6705509e-02
  1000    8.4004281e-03    2.3910377e+00    5.8434920e-02
  1500    8.2462306e-03    2.3978319e+00    4.8416416e-02
```

```
>>>> FIRST-ORDER DIRECTIONAL SIMULATION <<<<

 print interval ..........................npr=    500
 number of simulationts ..................nsm=   5000
 threshold for coef. of variation .....cov= 5.000e-02
 seed ............................stp=   619949844.00000

trials        Pf-mean        betag-mean     coef of var of pf
   500    7.2987396e-03    2.4422149e+00    9.0713187e-02
  1000    7.2747946e-03    2.4434009e+00    6.2124435e-02
  1500    7.3250836e-03    2.4409140e+00    5.1423315e-02
  2000    7.1967816e-03    2.4472890e+00    4.4401185e-02


>>>> SECOND-ORDER DIRECTIONAL SIMULATION <<<<

 print interval ..........................npr=    500
 number of simulationts ..................nsm=   5000
 threshold for coef. of variation .....cov= 5.000e-02
 seed ............................stp=   619949844.00000

trials        Pf-mean        betag-mean     coef of var of pf
   500    9.9602245e-03    2.3278435e+00    7.9756131e-02
  1000    9.9453798e-03    2.3284028e+00    5.4873717e-02
  1500    9.9608204e-03    2.3278211e+00    4.5518781e-02


>>>> MONTE CARLO SIMULATION <<<<

 print interval ..........................npr=    5000
 number of simulations ...................nsm=  100000
 threshold for coef. of variation .....cov=  5.000e-02
 seed ............................stp=   619949855.00000
 trials        Pf-mean        betag-mean     coef of var of Pf
  5000    1.1200000e-02    2.2835166e+00    1.3289347e-01
 10000    1.1100000e-02    2.2869291e+00    9.4392267e-02
 15000    1.0466667e-02    2.3091854e+00    7.9392571e-02
 20000    9.7500000e-03    2.3358327e+00    7.1263308e-02
 25000    1.0000000e-02    2.3263485e+00    6.2929789e-02
 30000    1.0266667e-02    2.3164576e+00    5.6687980e-02
 35000    1.0228571e-02    2.3178568e+00    5.2581400e-02
 40000    1.0300000e-02    2.3152370e+00    4.9012698e-02

================================================================================
```

USER PROVIDED SUBROUTINES FOR EXAMPLES 1 ~ 8:

==========

***** subroutine UGFUN *****

```
      subroutine ugfun(g,x,tp,ig)
      implicit real*8 (a-h,o-z)
      dimension x(1),tp(1)
      if(ig.ge.7.and.ig.le.9) go to 70
      if(ig.ge.11.and.ig.le.19) go to 80
      go to(10,20,20,40,50,60) ig
c---- Examples 1
  10  g = x(1)**tp(1)-x(2)*tp(2)+tp(3)
      return
c---- Example 2 and Example 2.1
  20  g = tp(1) - x(2)/1000.d0/x(3) - (x(1)/200.d0/x(3))**2
      return
c---- Example 4
  40  g =tp(1) - x(2)/1000.d0/x(3) - (x(1)/200.d0/x(3))**2-x(5)/1000.d
     *    /x(6)-(x(4)/200.d0/x(6))**2
      return
c---- Example 5
  50  g=x(3)-x(4)
      return
c---- Example 6
  60  g=(x(2)-x(4))/dsqrt(x(1)+x(3))-tp(1)
      return
c---  Examples 7 ,7.1, 7.2
  70  go to (71,72,73) ig-6
   71 g = x(1)+x(2)+x(4)+x(5)-x(6)*tp(1)
      return
   72 g = x(1)+2.*x(3)+2.*x(4)+x(5)-x(6)*tp(1)-x(7)*tp(1)
      return
   73 g = x(2)+ 2.*x(3)+x(4)-x(7)*tp(1)
      return
c---  Examples 8, 8.1, 8.2, 8.3
  80  go to (81,82,83,84,85,86,87,88,89) ig-10
   81 g= x(1)-4.d0*x(3)-4.d0*x(4)
      return
   82 g= x(1)-2.d0*x(3)-7.d0*x(4)
      return
   83 g= x(2)-0.2d0*x(3)-0.7d0*x(4)
      return
   84 g=2.5d0*x(1)-10.d0*x(3)-5.d0*x(4)
      return
   85 g=x(1)-10.d0*x(4)
      return
   86 g=x(2)-0.05d0*x(1)-0.5d0*x(4)
      return
   87 g=2.d0*x(1)-5.d0*x(3)-10.d0*x(4)
      return
   88 g=-x(1)+10.d0*x(2)
      return
   89 g=x(1)-10.d0*x(3)-20.d0*x(4)
      return
      end
```

***** subroutine UDGX *****

```
      subroutine udgx(dgx,x,tp,ig)
      implicit real*8 (a-h,o-z)
      dimension x(1),dgx(1),tp(1)
      if(ig.ge.7.and.ig.le.9) go to 70
      if(ig.ge.11.and.ig.le.19) go to 80
      go to(10,20,20,40,50,60) ig
c---- Example 1
  10  dgx(1)=tp(1)*x(1)**(tp(1)-1.d0)
      dgx(2)=-tp(2)
      return
c---- Example 2 and Example 2.1
  20  return
c---- Example 4
  40  return
c---- Example 5
  50  dgx(1)=0.d0
      dgx(2)=0.d0
      dgx(3)=1.d0
      dgx(4)=-1.d0
      return
c---- Example 6
  60  return
c---- Examples 7, 7.1, 7.2
  70  go to (71,72,73) ig-6
   71 return
   72 return
   73 return
c---- Examples 8, 8.1, 8.2, 8.3
  80  return
      end
```

***** subroutine UDD *****

```
      subroutine udd(x,par,sg,ids,cdf,pdf,bnd,ib)
      implicit real*8 (a-h,o-z)
      dimension x(1),par(4),bnd(2)
      go to(10,20,30,40,50,60) ids-50
c---  Example 5
  10  if (x(4) .lt. 0.) then
          cdf=0.1d-14
          pdf=0.1d-14
      else
          cdf=1.d0-dexp(-par(1)*x(4)**2.)
          pdf=2.d0*par(1)*x(4)*dexp(-par(1)*x(4)**2.)
      end if
      bnd(1)=0.d0
      ib=1
      sg=21.d0
      return
c---  Example 6
   20 alambda=par(1)
      ak=par(2)
      sg=alambda/ak/dsqrt(ak-1.d0)
      if(x(1).le.0) then
        pdf=0.
        cdf=0.
```

```
        return
      endif
      ak1=ak+1.
      gamk=dgama(ak)
      alx=alambda/x(1)
      pdf=alx**ak1*dexp(-alx)/alambda/gamk
      xt=2.*alx
      pp=2.*ak
      call dchis(xt,pp,p,ier)
      cdf=1.-p
      ib=1
      bnd(1)=0.
      return
   30 sg=1./dsqrt(par(3)*par(2))
      xx=(x(2)-par(1))/sg
      call dnorm(xx,cdf)
      pdf=0.39894228/sg*dexp(-0.5*xx*xx)
      ib=0
      return
   40 alambda=par(1)
      ak=par(2)
      sg=alambda/ak/dsqrt(ak-1.d0)
      if(x(3).le.0.) then
        pdf=0.
        cdf=0.
        return
      endif
      ak1=ak+1.
      gamk=dgama(ak)
      alx=alambda/x(3)
      pdf=alx**ak1*dexp(-alx)/alambda/gamk
      xt=2.*alx
      pp=2.*ak
      call dchis(xt,pp,p,ier)
      cdf=1.-p
      ib=1
      bnd(1)=0.
      return
   50 sg=1./dsqrt(par(3)*par(2))
      xx=(x(4)-par(1))/sg
      call dnorm(xx,cdf)
      pdf=0.39894228/sg*dexp(-0.5*xx*xx)
      ib=0
      return
c---  Example 13, 18
   60 if (x(4) .lt. 0.) then
          cdf=0.1d-14
          pdf=0.1d-14
      else
          cdf=1.d0-dexp(-par(1)*x(4)**2.)
          pdf=2.d0*par(1)*x(4)*dexp(-par(1)*x(4)**2.)
      end if
      bnd(1)=0.d0
      ib=1
      sg=21.d0
      return
      end
```

==========

APPENDIX A
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