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Updating the Common Ground

by

Mathias Böhm

A dissertation submitted in partial satisfaction of the

requirements for the degree of

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University of California, Berkeley

Committee in charge:

Professor John MacFarlane, Co-Chair
Professor Seth Yalcin, Co-Chair
Professor Amy Rose Deal

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Updating the Common Ground

Abstract

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by

Mathias Böhm

Doctor of Philosophy in Philosophy

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Professor John MacFarlane, Co-Chair

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Theories of discourse dynamics provide an account of the update or context change potentials of the sentences in a given fragment of language; how assertions of the sentences in question affect the context or the common ground of a conversation.

The literature on the discourse dynamics of various fragments falls roughly into two camps. On the one hand, we have a *non-semantic camp*. According to members of this camp we should strictly distinguish between theories of meaning and theories of discourse dynamics. On the other hand, we have a *semantic camp*. According to this camp meanings *are* context change potentials — that is, providing a theory of the discourse dynamics of a fragment of language just is to provide a theory of meaning for the fragment in question.

In this dissertation I hope to contribute to both the question of what the discourse dynamics of various fragments of language should look like as well as the question of whether we should side with the semantic or the non-semantic camp. With respect to the former question, I hope to defend a range of non-standard approaches to the discourse dynamics of conjunction, negation, conditionals and modals. With respect to the latter, my aim will be to convince the reader that, contra the popularity of dynamic approaches to the meaning of conditionals and modals, studying the discourse dynamics of such expressions provides an incentive to side with the non-semantic camp.

1

The main argument is found in Chapter 2. Here we will see that given some plausible constraints on the context change potentials of indicative conditionals, there is a prima facie conflict between the idea that meanings are context change potentials and the idea that meanings behave compositionally. As I will argue, on popular and widespread views about context change and what it is for a theory of meaning to be compositional, we have reasons to believe that context change potentials do not behave compositionally. So, given that meanings behave compositionally, meanings are not context change potentials. Moreover, we will see that we can build a non-compositional theory of the context change potentials of conditionals on the basis of a truth conditional approach to meaning that is compositional in the relevant sense.

In chapter 3, I focus on a purported advantage of a certain approach to the discourse dynamics of conjunction. According to the view in question, asserting a conjunction is like a successive assertion of its conjuncts. On this view the badness of certain conjunctions that are purportedly hard to explain on other accounts, reduces to the badness of certain discourses in an elegant way. However, I argue that this elegance is paid for with a range of implausible consequences. I conclude the chapter by discussing ways of giving up on the view and show that a version of the view developed in Chapter 2 does so in a particularly satisfying way.

Chapter 4 develops a positive proposal for a theory of discourse dynamics for 'might' and 'must' claims. Here I argue that 'must' claims are genuinely informative in the sense of corresponding to assertions that eliminate some ways the world might be but not others. 'Might' claims, however, are never informative in this sense. Still, sometimes 'might' claims are non-trivial, for they highlight possibilities that have not been highlighted before. The approach to the discourse dynamics developed here is non-compositional in the same sense as the view discussed in Chapter 2.

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i

Contents

iv

List of Figures

Chapter 1

Introduction

1.1 Conversation and the Common Ground

On a popular view about conversation, as developed by Stalnaker [1970, 1973, 1974, 1999, 2002], conversations take place on the background of a stock of information that is shared between the participants of the discourse: the *context* or the *common ground* of a conversation.[1]

On Stalnaker's picture, the common ground plays two roles. On the one hand, it is a *precondition* of conversation. On the other, it is what discourse participants *contribute to* when making an assertion. The common ground is a precondition of conversation, since if I assert

(1) It's raining in Berlin,

I do so assuming that we share the information that we all speak English, that the sentence asserted is in English, that my words are used in a standard way and so on. The common ground is what discourse participants contribute to, since if I assert (1) and none of my interlocutors objects to my assertion, what I said will be added to the stock of information that is shared between us.

It is helpful to think of the common ground as containing information that is, in some sense, *public* and distinguish it from information that is *private* to the participants of the conversation. In normal circumstance, if I assert something and none of my interlocutors objects, what I said does not only convey information that is learned by all the discourse participants

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[1] See Stalnaker [2014] for his latest views on the matter.

privately but the information conveyed is now public among all of us. The other way around, I have an incentive to assert a sentence if I privately have access to information that would be conveyed by asserting the sentence and the information in question is not yet public and relevant to our conversation. Making privately available information public enables the participants of the discourse to build on it at later stages of the conversation.

There are various ways of thinking about the public/private distinction and, hence, various ways of thinking about what constitutes the common ground. Stalnaker himself thinks of it in terms of some infinitely iterated notion of belief: roughly, something is common ground between the participants of a discourse if all discourse participants believe it, all believe that they believe it, all believe that all believe that all believe it and so on.[2] Other's think of it in terms of iterated knowledge [Yalcin, forthcoming] and yet others think about it in terms of some primitive notion of presupposition [Almotahari and Glick, 2010]. Finally, some deny that there is a fundamental distinction between public and private information in the first place [Hawthorne and Magidor, 2010, Lederman, 2017b]. But, even those who claim that there is no *fundamental* distinction between public and private information can (and I think should) accept that there is a distinction.[3]

In the following, I will remain neutral on these foundational questions. However, I will assume that we can distinguish between information that is and information that is not part of the common ground of a conversation and that when asserting a sentence we build on and contribute to this stock of in-

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[2]Such notions were introduced to philosophy of language by Lewis [1969] and have been extensively studied in the game theory literature [see Fagin et al., 1995]. [Greco, 2014] contains a helpful discussion of recent issues surrounding general epistemic iteration principles. Regarding Stalnaker's account of the common ground, note that I am ignoring some subtle issues about the exact kind of attitude at play here. The interested reader is referred to a more thorough discussion in [Stalnaker, 2014]. [Yalcin, forthcoming] contains a helpful exposition of Stalnaker's account.

[3]The sceptical positions developed by Lederman [2017b,a, 2018], for instance, do not deny that attitudes such as knowledge or belief can be iterated. Lederman denies that public information can be defined in terms of an *infinitely* iterated attitude. However, recent formal approaches to pragmatic reasoning, as put forward by Frank and Goodman [2012], are built on iterated attitudes but do not presuppose that agents ever have *infinitely* iterated attitudes [see Lassiter and Goodman, 2017, for a discussion of the connection of such approaches to Stalnaker's view of conversation]. On such views we can stipulate that some information is public just in case all participants of the discourse know or believe it to a sufficiently high but finite degree. Since 'sufficiently high' is vague, our distinction between public and private is vague. However, vague distinctions are still distinctions.

formation. So, I take it, the basic shape of Stalnaker's picture of conversation is correct.

1.2   Update and Meaning

Assuming Stalnaker's basic picture of conversation, we can start theorizing about how assertions of sentences affect the context or the common ground of a conversation. The way in which an assertion of a sentence affects the context of a conversation is typically referred to as the sentence's *context change potential* (sometimes also *update* or *information* change potential) and theories about sentences' context change potentials are typically called theories of *discourse dynamics*.

Now, Stalnaker thinks of theories of discourse dynamics as *pragmatic* theories that draw on some previously established theory of meaning. So, according to Stalnaker, we should distinguish between theorizing about meaning, on the one hand, and theorizing about context change, on the other. According to a recent trend in semantics, as pioneered by Heim [1982, 1992] and Kamp [1984], drawing this distinction is wrong headed. Our perspective on meaning should itself be *dynamic*. For instance, Groenendijk and Stokhof [1991] write:

> [...] the dynamic outlook on natural language interpretation, referred to as *dynamic semantics*, starts from a fundamentally different basic notion. Not the [truth-conditional] content, but the information change potential of a sentence is regarded as constituting its meaning. Consequently, the notion of the interpretation of a sentence [...] is given by a recursive definition of the result of updating an information state with the sentence. [Groenendijk and Stokhof, 1991, p.55]

What Groenendijk and Stokhof call an 'information state', in the above quote, can safely be thought of as the common ground of a conversation. Hence, they argue that what we do when we study the discourse dynamics of a particular fragment of language just is to study the meanings of the sentences in question; context change potentials, so the general idea, *are* meanings.

In light of these two approaches, let us distinguish between two perspectives on discourse dynamics: a *semantic* and a *non-semantic* perspective.

3

According to a non-semantic perspective we think about how assertions affect the common ground of a conversation in two steps. First, discourse participants use their semantic knowledge to figure out what the asserted sentence means. Second, they determine what is conveyed by the sentence in light of the meaning just computed.[4] Two remarks. First, I will not presuppose that a non-semantic approach to discourse dynamics is pragmatic. Depending on how we delineate the distinction between semantics and pragmatics, there is room for views according to which studying the discourse dynamics of a fragment of language is neither a pragmatic nor a semantic project.[5] Second, taking a non-semantic perspective does not presuppose the defense of a truth-conditional theory of meaning. As I will be using the term, all theories of meaning that do not give rise to a theory of discourse dynamics directly will be views that take a non-semantic perspective on discourse dynamics.

According to a semantic perspective on discourse dynamics there is no two step process. Each sentence comes equipped with what we called the sentence's *context change potential*. But, context change potentials tell us not only about the way in which an assertion of a sentence would update the context of a conversation but *also* about how the context change potential of a complex sentence is determined in terms of the context change potentials of its parts. Hence, any such theory delivers a theory about the compositional mechanisms and the discourse dynamics of the fragment of language we are interested in in one go.[6]

Now, here are two key questions surrounding these two perspectives on

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[4]Prominent members of this non-semantic camp are Robert Stalnaker [see, in particular, his Stalnaker, 1999, 2014] and Philipe Schlenker [see his Schlenker, 2009, 2010]. It is also worth pointing out that more traditional truth-conditional approaches to meaning — as developed in [Tarski, 1944, 1956], [Montague, 1973], and as they are found in classic introductory texts such as [Kratzer and Heim, 2000] or [Partee et al., 1990] — were not developed with a particular view about conversation in mind.

[5]For instance, we may use the term 'pragmatics' in a narrow sense, as the study of conversational implicatures: the study of how flouting various maxims of communication can be used by speakers to convey information [This is the classic picture developed in Grice, 1975]. Given this usage, none of the theories of discourse dynamics discussed in this dissertation are pragmatic theories. All of the models considered here capture processes where we take the discourse participants to adhere to and not flout maxims of conversation.

[6]As mentioned above such views were pioneered by Irene Heim [see her Heim, 1982, 1992] and Hans Kamp [Kamp, 1984] and have been developed further by many others [such as Veltman, 1985, Groenendijk et al., 1996, Gillies, 2004, Willer, 2014, to mention a few].

4

discourse dynamics: what is at issue between defenders of a semantic and defenders of a non-semantic perspective and are there reasons to prefer one perspective over the other? In my dissertation, I hope to contribute to both of these questions. But, before we get to some concrete contributions, let me take a look at one way we may try to approach the first question.

1.3 Updates as Proposition Adding

Let us be a little more concrete. A standard formal structure that can be used to describe or model a theory of discourse dynamics is the following:

Definition 1.3.1 (Models of Conversation). *A **model of conversation for a fragment of language** $\mathcal{L}$ is a pair $\langle C, \cdot[\cdot] \rangle$ consisting of a set of **contexts** C and an **update function** $\cdot[\cdot]$ mapping contexts $c \in C$ and sentences $\phi \in \mathcal{L}$ to contexts $c[\phi]$ in C.*[7]

C can be thought of as a set of states of information that model the common ground at possible stages of a conversation. The update function $\cdot[\cdot]$, on the other hand, can be seen as modelling what we have called the context change potentials of sentences.[8] Note that, given a model of conversation, $\cdot[\phi]$ is a function from contexts to contexts. So, any model of conversation will provide a model of how an assertion of ϕ will move the conversation from one context to another.

Assuming that such models help to deliver a theory of discourse dynamics for a fragment of language, taking the semantic perspective on discourse dynamics, or defending a dynamic approach to meaning, amounts to providing a model of conversation that describes the context change potentials of the fragment we are interested in in a compositional manner. Taking a non-semantic perspective amounts to coming up with a view about how our non-dynamic theory of meaning gives rise to an appropriate model of conversation.

While, as noted above, theories that take a non-semantic perspective on discourse dynamics are varied, let us take a look at the kind of non-dynamic theories mentioned in the above quote from [Groenendijk and Stokhof, 1991]:

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[7]This is what Rothschild and Yalcin [2016] call a *conversation system*.

[8]This is, of course, not the only interpretation of such a structure. After all mathematical structures can be interpreted in a myriad number of ways.

theories according to which meanings are thought of in terms of the truth-conditional content of a sentence. Further let us think of truth-conditional contents as propositions in a loose sense. In this loose sense, propositions may be any set of indices — sets of possible worlds or some more complex formal objects.

Given such a notion of meaning, we may wonder: doesn't any truth-conditional theory of meaning give rise to a theory of discourse dynamics in a trivial way? Given the propositions such theories associate with sentences, couldn't we take a context to be a proposition and just say that the discourse effect of a sentence is to 'add' the proposition associated with the sentence to the context of the conversation? If the answer is yes, then, contra [Groenendijk and Stokhof, 1991], context change potentials do not seem to be 'fundamentally' different from truth-conditional contents.

It turns out that, depending on certain abstract properties of contexts and context change, a model of conversation cannot be more than a general theory of proposition adding and, *vice versa*, given these properties, any dynamic theory of meaning will give rise to a propositional theory of meaning. Restricting attention to propositions as sets of possible worlds, van Benthem [1986] observes:

Proposition 1.3.1 (van Benthem). *Let W be a set of possible worlds. Given a model of conversation $\langle \mathcal{P}(W), \cdot[\cdot] \rangle$ for some fragment of language $\mathcal{L}$ we have*

> *__Intersectivity__: $c[\phi] = c \cap W[\phi]$*

if and only if we have

> *__Eliminativity__: $c \supseteq c[\phi]$,*

> *__Distributivity__: $\left(\bigcup_{i \in I} \{w_i\} \right)[\phi] = \bigcup_{i \in I} (\{w_i\}[\phi])$.*

That is, if we think of contexts as sets of possible worlds, we can think of $W[\phi]$ as the proposition expressed by ϕ. Then, if the model is Eliminative and Distributive, updating the common ground will always amount to adding the *same* proposition, the proposition expressed by ϕ, to the context of the conversation. Hence, given Eliminativity and Distributivity, any propositional theory of meaning gives rise to a theory of discourse dynamics

in a rather trivial manner and *vice versa*. Choosing one approach over the other does not seem to matter much.[9]

Similarly Rothschild and Yalcin [2016, 2017] observe

Proposition 1.3.2 (Rothschild and Yalcin). *Given a model of conversation $\langle C, \cdot [\cdot] \rangle$ for some fragment of language $\mathcal{L}$, $\langle C, \cdot [\cdot] \rangle$ is isomorphic to an intersective model if and only if for all $\phi, \psi \in \mathcal{L}$*

Idempotence: $c[\phi][\phi] = c[\phi]$,

Commutativity: $c[\phi][\psi] = c[\psi][\phi]$.

While isomorphic models of conversation do not have to be identical, Rothschild and Yalcin's result shows that any model of conversation that has an idempotent and commutative notion of update could, in principle, be spelled out in terms of some notion of proposition adding. (A more thorough discussion of this result will be provided in Chapter 3) That is, if our theory of discourse dynamics has a notion of update that is idempotent and commutative, it will give rise to some theory of propositional content, and any corresponding theory of propositional content translates to a theory of discourse dynamics in a rather trivial way. So the distinction between theories that take a truth-conditional vs. theories that take a dynamic perspective on meaning seems superficial.

Arguments against one of the above constraints can be taken to support a dynamic perspective. If we have good reasons to believe in corresponding violations of the constraints, we have good reasons to believe that a theory of discourse dynamics cannot simply be given in terms of a simple theory of proposition adding.

Indeed, as will become clear in the following Chapters, I think we *do* have good reasons to believe that adequate theories of discourse dynamics violate Rothschild and Yalcin's Commutativity constraint as well as van Benthem's Distributivity constraint. So, we *do* have a reason to think that providing a theory of discourse dynamics cannot be a simple theory of proposition adding. But does that show that we should think of meanings as context change potentials?

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[9]As Rothschild and Yalcin [2016] note, many take a failure of one of van Benthem's constraints to be a key indicator of dynamicness [see Groenendijk and Stokhof, 1991, van Benthem, 1996, Muskens et al., 1997, von Fintel and Gilles, 2007].

1.4 Contributions and Goals

One upshot of my dissertation is that, on a plausible way of interpreting the idea that meanings are context change potentials, the answer is 'no'. In particular, we can motivate theories of discourse dynamics on which at least one of the above mentioned constraints is violated but context change potentials are grounded in and not identical to meanings.[10] Moreover, in Chapter 2 we will see that we have an incentive to think that the discourse dynamics of 'if' *should* be grounded in a theory of meaning according to which meanings are not context change potentials. Here I will argue that we have a reason to reject the idea that the context change potentials of simple indicative conditionals behave compositionally (in a popular and widespread sense of the term). So, if meanings are compositional, then the context change potentials of such sentences are not meanings.

Finally, I hope to convince the reader that, whether we do or do not take a semantic perspective on discourse dynamics, providing theories of discourse dynamics that violate one of the above constraints should move us to reject some key ideas found among defenders of dynamic approaches to meaning.

For instance, Chapter 2 contains theories of discourse dynamics that violate the widespread idea that asserting a negated sentence, $\neg\phi$, has the effect of updating the common ground with the complement of the discourse effect of its prejacent sentence, ϕ. Chapter 3 argues that, given an adequate theory of the discourse dynamics of information sensitive expressions such as 'may', 'might', 'probably' and 'presumably', we have good reasons to reject the popular idea that asserting a conjunction has the same discourse effect as a successive assertion of its conjuncts. Finally, Chapter 4 contains a theory of the discourse dynamics of 'must' and 'might' claims that violates both of the just mentioned approaches to the discourse dynamics of conjunction and negation.

Dynamic approaches to meaning were developed with the hope of solving some difficult problems in semantics. As Hans Kamp puts it, in his [Kamp, 1984]: the "separation" of discourse dynamics from semantics "has become an obstacle to the development of semantic theory." [Kamp, 1984, p. 330].

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[10]That failures of Intersectivity may not entail that we have to adopt a dynamic approach to meaning in the sense I am using the term has been acknowledged in [Rothschild and Yalcin, 2017, section 7]. However, the positive proposals developed in this dissertation can be seen as empirically motivated theories of discourse dynamics which further support this observation.

Accordingly, the focus in the recent literature has been on equipping semantic theories for various fragments of language with notions of update and defining notions of entailment in terms of it. Indeed many such theories give rise to a range of predictions that are hard to get otherwise.

However, thinking of context change potentials as meanings, will restrict the range of possible theories of discourse dynamics we can provide. For, as mentioned above, such theories will have to deliver *both* a satisfying theory about the compositionality of meaning as well as a satisfying theory of context change. This, as we will see, may become an obstacle for the development of theories of discourse dynamics.

That said, let me note that this dissertation is written with a healthy respect for the progress we owe to proposals that have been developed with a dynamic perspective on meaning in mind.[11] Moreover, and this will be pointed out at various places in this dissertation, even members of the non-semantic camp can and should hold on to many of the key ideas developed in the dynamic semantics literature.

Finally, members of the non-semantic camp looking for knock-down arguments against dynamic approaches to meaning will be disappointed. Such arguments are rare in philosophy, and this dissertation will not add any. However, what I do hope to establish is that holding on to the idea that meanings are context change potentials is more demanding than typically appreciated and giving up on the idea may free us to provide theories of discourse dynamics that are otherwise hard to get.

1.5 Overview

While guided by a common theme, each of the chapters is a self-contained piece of work and can be read independently of the other chapters. Here is an overview.

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[11]The range of topics covered by defenders of such approaches is vast. Obviously we will only be able to cover a small fraction of them in this dissertation. But see [Beaver, 2001] and [Potts, 2015] for helpful overviews on dynamic contributions to the systematization of presuppositions, [von Fintel and Gilles, 2007] for an overview of dynamic contributions to epistemic discourse, and [Abbott, 2006] for an overview of dynamic approaches to definite and indefinite pronouns.

1.5.1   The Non-compositional Dynamics of 'If'

Irene Heim introduced the following account for the discourse dynamics of indicative conditionals [see her Heim, 2002].

$$c[\phi \to \psi] = c - (c[\phi] - c[\phi][\psi]).$$

The view, as I will argue, makes very plausible predictions about the discourse dynamics of plain indicative conditionals.

Unfortunately, it is well known that under certain standard, and independently plausible, assumptions about conjunction, negation and non-conditional sentences, the view makes terrible predictions about assertions of negated indicative conditionals.

The standard response to this problem is to propose an alternative view about the discourse dynamics of conditionals [see Gillies, 2004]. A view according to which the discourse dynamics of negated indicative conditionals will be

$$c[\neg(\phi \to \psi)] = \begin{cases} c, \text{ if } c[\phi][\psi] \neq c[\phi] \\ \varnothing, \text{ otherwise} \end{cases}$$

While I agree with defenders of this standard response that such views make much more plausible predictions about negated indicative conditionals, the view makes terrible predictions about plain conditionals.

So, it seems we are caught in a deadlock between Heim's view, a theory that makes great predictions about plain conditionals but bad ones for negated conditionals, and the above mentioned alternative which makes great predictions about negated conditionals but bad ones for plain conditionals.

I then show that the deadlock is easily broken, once we give up on the idea that meanings are context change potentials. Doing so allows us to provide a non-compositional theory of the discourse dynamics of 'if' that is based on a compositional theory of meaning. Our non-compositional theory of the discourse dynamics for 'if' has the advantage of combining the benefits and getting rid of the drawbacks of both of the just mentioned views.

1.5.2   Asserting Conjunctions, Asserting Conjuncts

According to a popular and widespread view about the discourse dynamics of conjunction, asserting a conjunction is like a successive assertion of its conjuncts (such a view was first suggested by Stalnaker [1974] and implemented

as a dynamic entry by Heim [2002]). Call such an account of the discourse effects of conjunction the *sequential view*.

What may be taken as an advantage of such a view is that the badness of assertions of some conjunctions reduces straightforwardly to the badness of discourses involving their conjuncts.

For instance, given the above view about assertions of conjunction and negation, the following sentences are predicted to be unassertable

(3) It is not raining and it might be raining,

(4) It is not raining but it is probably raining,

(5) It is not raining but suppose it is raining,

This is so, since in the following discourses Alice's second assertion sounds odd

(3') Alice: It is not raining.
      Alice: It might be raining.

(4') Alice: It is not raining
      Alice: It is probably raining.

(5') Alice: It is not raining.
      Alice: Suppose it is raining

While the explanation of the oddness of (3) to (5) in terms of the sequential view is elegant, I will argue in this paper that what seems like an advantage is actually a disadvantage.

My argument starts with a range of known challenges certain standard approaches to 'might' face. I argue that under fairly minimal assumptions, these challenges generalize to any view about so called information sensitive expressions that adopt the sequential view of conjunction. So the just mentioned problems are not due to the discourse effects of 'might' and related expressions but due to our account about the discourse effects of conjunction. I close the paper by discussing some ways in which the sequential view could be rejected and what its rejection entails for debates surrounding the question whether we should or should not use a dynamic perspective on meaning.

1.5.3 'Must' Be Informative, 'Might' Be Non-trivial

What do assertions of epistemic 'must' and 'might' claims add to the context of a conversation? According to a popular and widespread account neither 'must' nor 'might' claims add information to the context of a conversation.

In this paper, I argue that this claim is wrong with respect to epistemic 'must' claims but partially correct for 'might' claims. Still, while 'might' claims do not add information to the common ground, they are non-trivially asserted whenever they make a possibility salient that has not been salient before.

The literature contains views that do predict 'must' claims to be informative [see Santorio, 2022, for instance] as well as views according to which 'might' claims are salience makers [see Willer, 2013, for instance]. But, to my knowledge, there is no view that has both properties. This paper aims to provide further evidence in favor of such a view and to develop a positive proposal. The proposal, as we will see, results in a theory of discourse dynamics that is non-semantic in the sense of Chapter 2.

Chapter 2

The Non-compositional Dynamics of 'If'

2.1 Introduction

Discourse dynamics is about the update or context change potentials of sentences, how asserting a sentence would affect or update the context of the conversation. Approaches to discourse dynamics can be grouped into two camps: the *semantic* and the *non-semantic* camp. According to the semantic camp, update potentials are meanings. According to the non-semantic camp they are not: update potentials may be linked to meanings via bridge principles, but meanings may be something else entirely (such as a notion of truth at an index). While members of the semantic camp have to deliver theories of discourse dynamics that provide both an adequate notion of context change and a description of the compositional mechanisms that govern the fragment of language in questions, members of the non-semantic camp are free to provide a non-compositional theory of discourse dynamics on the basis of a compositional theory of meaning. In this paper, I wish to look at the discourse dynamics of simple indicative conditionals as a case study. I will argue that the discourse dynamics of such conditional sentences provides us with an incentive to give a non-compositional theory of update on the basis of a compositional theory of meaning. Hence, we have a reason to prefer a non-semantic perspective on discourse dynamics.

Here is an overview of the paper. We start in Section 2.2 by fixing some background notions needed to get the discussion going. In Section 2.3 we will

look at two ways to think about the meaning of simple indicative conditionals and two, related ways of thinking about theories of discourse dynamics for such sentences. In Section 2.4 I will argue that any theory of the discourse dynamics of simple indicative conditional sentences should satisfy a range of constraints, a range of constraints that are violated by a majority of extant proposals. In Section 2.5 we will see that any theory of discourse dynamics that satisfies these constraints will violate a popular and widespread notion of what it is for a theory of meaning to be compositional. We also introduce a non-semantic approach to discourse dynamics that delivers such a non-compositional theory on the basis of a compositional (non-dynamic) theory of meaning. While this suggests that our approach to the discourse dynamics of simple indicative conditional sentences should be non-semantic, in Section 2.6 we consider ways in which fans of dynamic approaches could try to adhere to the constraints provided in Section 2.5. We conclude with some final remarks in Section 2.7.

A caveat before we dive into the discussion. The literature on the semantics and discourse dynamics on indicative conditionals is vast and it will be impossible to do justice to all of the proposals which are found in the literature. This chapter should be of particular interest to those who defend versions of what we may call the *dynamic strict conditional view*, a view according to which an assertion of a conditional tests whether updating the common ground with the antecedent of the conditional results in a context that contains the information conveyed by the consequent. Others may be less convinced by the positive proposals considered in Section 2.5. However, I hope that the paper is of interest even to those readers. At a minimum, the accounts considered here illustrate a way in which meanings could fail to be context change potentials which has not received much attention in the literature on whether our approaches to meaning should be dynamic or not.

2.2 Background

Let us start by looking at some background notions needed to get the discussion going.

2.2.1 A Small Conditional Fragment

Throughout this chapter we will be looking at various formal approaches to the semantics and discourse dynamics of *if*. Hence, we need a formal language to work with. The fragment of language $\mathcal{L}$ I will be focusing on for the most part can be generated with the help of the following Backus-Naur form:

$(\mathcal{L})$ $\phi ::= \alpha \mid (\alpha \to \alpha) \mid \neg(\alpha \to \alpha),$

$\quad\quad \alpha ::= p \mid \neg\alpha \mid (\alpha \wedge \alpha),$

where p can be an arbitrary sentence taken from a countable set, `At`, of *atomic* sentences[1] Of course, our formal language will be seen as modelling a corresponding fragment of English where $\to, \neg$, and $\wedge$ stand for *if* (in the indicative mood), *not*, and *and*, respectively. We will use $\mathcal{B}$ to denote the set of all sentences in $\mathcal{L}_s$ that do not contain the conditional operator $\to$.

Note that the fragment of language is small. We only consider *simple* indicative conditional sentences with non-conditional antecedents and consequents such as

(1) If a member of the ground staff is the culprit, it is the driver,
$g \to d$.

(2) It is not the case that if a member of the ground staff is the culprit, it is the driver,
$\neg(g \to d)$.

Nested conditionals and compound conditionals such as

(3) If a member of the ground staff is the culprit, then if it is the driver, he did it at midnight,
$g \to (d \to m)$.

(4) It is not the case that if a member of the ground staff is the culprit, it is the driver, and if a member of the house staff did it, it was the butler,
$\neg(g \to d) \wedge (h \to b)$,

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[1]In a Backus-Naur form, everything on the right of '::=' is a possible substitution instance for an occurrence of the lower case greek letter on the left of '::='. The set $\mathcal{L}$ is the set of all strings not containing any greek letters which are substitution instances for ϕ.

will be ignored. The choice of this small fragment is intentional. While every account presented below will generalize to a full conditional fragment, the small fragment $\mathcal{L}$ is all we need to get the argument going.[2] Moreover, it will allow us to compare various approaches with respect to the sentences that matter for our argument while ignoring differences in prediction with respect to nested and compound conditionals such as (3) and (4).[3]

2.2.2 Modelling Conversation

Now that we have a fragment of language to work with, let us first look at the question of what it is to provide a theory of *discourse dynamics* for that fragment, regardless of whether we think of this as a semantic project or not. The key ideas go back to Stalnaker [1999]. According to his view, we should think of a conversation as taking place on the background of a stock of information that is shared between the participants of the discourse. This stock of informations is typically referred to as the *context* or the *common ground* of the conversation [see Stalnaker, 1999].[4] Assertions can then be seen as proposals to contribute to this shared stock of information. A theory about the discourse dynamics of some fragment of language can be thought

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[2]By *full conditional fragment* I mean the fragment

$$\phi := p \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \rightarrow \phi) \qquad (\mathcal{L}_\rightarrow)$$

with $p \in \mathtt{At}$.

[3]Different predictions for nested conditionals will typically result from adding more structure to the semantics in terms of some kind of ordering of worlds as in [Stalnaker, 1975], [Van Fraassen, 1976] or [Lewis, 1981, 2001]. A full theory of the semantics and discourse dynamics of indicative conditionals will certainly have to say something about nested conditionals, as illustrated by the debate surrounding the import-export principle and Gibbard's collapse argument [see Allan Gibbard, 1980, Kratzer, 1986, 2012, Gillies, 2004, Khoo and Mandelkern, 2019, among others]. But my intent in this chapter is not to give a new theory of indicative conditionals. The negative arguments presented here are independent of these issues and candidate positive proposals, which we will look at later on, can be built on various approaches to the semantics of indicative conditionals.

[4]Note that this usage of 'context' is related but distinct from how we think of the context in truth conditional accounts of context-sensitive language. For instance Lewis says that '[a] context is a location [. . .] where a sentence is said.' [Lewis, 1980, p. 79]. Since information about relevant features of the context, in Lewis' sense, may or may not be shared between the participants of the discourse, the notions may come apart. The notion of context in Stalnaker's sense is closer to Lewis' notion of a *conversational score* [see his Lewis, 1979].

of as modelling this intuitive idea. The following kind of formal structure will help us to do so:[5]

Definition 2.2.1 (Models of Conversation)**.** *A **model of conversation for** $\mathcal{L}$ is a pair $\langle C, \cdot[\cdot] \rangle$ consisting of a set of **contexts** (sometimes also **information states**) C and an **update function** $\cdot[\cdot]$ mapping states $c \in C$ and sentences $\phi \in \mathcal{L}$ to states $c[\phi]$ in C.*

(This is Def. 1.3.1 from Chapter 1. In order to keep the chapter self-contained, it is repeated here.) The elements of C model, as the name suggests, the common ground or the context of a conversation at a respective stage of the discourse, whereas an *update function* is supposed to model the contribution of an assertion of a sentence to the context at the respective stage of the conversation.

In this chapter we will follow Stalnaker [1999] in thinking of contexts as sets of possible worlds: ways the world might be in light of the information shared at the respective stage of the conversation. The set of possible worlds W, in turn, will be thought of as a set of functions from atomic sentences to truth values (functions from At to $\{0, 1\}$). Sets of worlds will be called *propositions*. So C is just the set $\mathcal{P}(W)$ and propositions and contexts are the same type of formal object.

Here are three points to keep in mind when evaluating a model of conversation for adequacy. *First*, such models are supposed to capture *rational discourse* and should be thought of as abstracting away from cases in which discourse participants respond to assertions in an (epistemically) irrational way.[6] *Second*, the simple possible worlds models we will be focusing on for the most part are *limited*. They cannot capture all discourse effects we might want to have an account of. We will comment on more complex notions of context later on but, for the most part, simple possible worlds models will do. *Third*, models of conversation typically abstract away from so called

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[5]This is what Rothschild and Yalcin [2016] call a *conversation system* and and Bonnay and Westerståhl [2014] an *abstract frame*. Rothschild and Yalcin [2017] consider slightly more general structures.

[6]Some may be inclined to reject such models because they seem to rely on certain opacity assumptions that are unjustified [see Hawthorne and Magidor, 2010]. Others disagree with such objections [Almotahari and Glick, 2010, Stalnaker, 2009]. I consider such assumptions to be idealizing assumptions which may, if relevant, be dropped. However, as far as I can see, nothing substantial will depend on the issue for the questions I am interested in. Hence, to keep our models as simple as possible I will ignore the issue.

secondary changes [see Rothschild and Yalcin, 2017], changes in context not essentially due to the sentences asserted. For instance, asserting something *verbally* will change the information shared between the participants of the discourse in the sense that it will become common ground that the speaker just made noises with their mouth [see Stalnaker, 1999, for this example]. But this is a change in information not due to the *sentence* asserted, and hence will be ignored by a respective update function.

2.3 Discourse Dynamics and Meaning

The general idea of what it is to provide a theory about the discourse dynamics of some fragment of language is on the table. In this section we will look at two ways of thinking about the relationship between semantics and discourse dynamics.

2.3.1 Two Theories of Meaning for $\mathcal{L}$

Theories of meaning may be provided in terms of a recursively defined function from sentences and *indices* to truth values [A classic example of such a theory is Kaplan, 1977].[7] Here is one such approach for our fragment $\mathcal{L}$, a simplified version of a semantics for indicative conditionals found in many places [see Yalcin, 2007, Kolodny and MacFarlane, 2010, Gillies, 2010, for versions of this view]:[8]

Definition 2.3.1 (Truth conditional Semantics for $\mathcal{L}$). *Let an **index** be a pair $\langle w, s \rangle$ of a world, w, and a set of worlds, s, (an information state) such that $w \in s$. A notion of **truth at an index** for $\mathcal{L}$ is given in terms of a recursively defined function $[\![\cdot]\!]^{\cdot}$ from sentences $p \in \mathtt{At}$, $\alpha, \beta \in \mathcal{B}$, $\phi \in \mathcal{L}$ and indices to truth values as follows:*

(p_t) $[\![p]\!]^{\langle w,s \rangle} = 1$ *iff* $w(p) = 1$,

$(\neg_t)$ $[\![\neg \phi]\!]^{\langle w,s \rangle} = 1$ *iff* $[\![\phi]\!]^{\langle w,s \rangle} = 0$,

---

[7]Note that I will be using the terms *semantics* and *theory of meaning* interchangeably.

[8]The approaches cited above all differ with respect to the information state at which the consequent of a conditional is evaluated. These differences affect the evaluation of conditionals with modal or conditional antecedents. But the views all agree in predictions about the sentences of our small fragment $\mathcal{L}_s$.

$(\wedge_t)$ $[[\alpha \wedge \beta]]^{\langle w,s\rangle} = 1$ *iff* $[[\alpha]]^{\langle w,s\rangle} = 1$ *and* $[[\beta]]^{\langle w,s\rangle} = 1,$

$(\to_t)$ $[[\alpha \to \beta]]^{\langle w,s\rangle} = 1$ *iff for all* $w' \in \{w^* \in s \mid [[\alpha]]^{\langle w^*,s\rangle} = 1\},$
$$[[\beta]]^{\langle w',\{w^* \in s \mid [[\alpha]]^{\langle w^*,s\rangle}=1\}\rangle} = 1.$$

There are different ways of making philosophical sense of such a theory. For our purposes it suffices to note that the truth value of sentences containing indicative conditionals does not only depend on facts about the world w at which the claim is evaluated but also on facts about some contextually salient state of information s. Hence, our notion of truth is indexed to both parameters. Key, for our purposes, is that this approach to the meanings of the sentences in $\mathcal{L}$ has the following shape:

Definition 2.3.2 (Truth Conditional Theories of Meaning). *A theory of meaning for* $\mathcal{L}$ *is called* **truth conditional** *just in case it is provided in terms of a pair* $\langle I, [[\cdot]]^\cdot\rangle$ *consisting of a set of indices* I *and a function that maps sentences,* $\phi \in \mathcal{L}$, *and indices,* $i \in I$, *to truth values* $[[\phi]]^i \in \{0,1\}.$

The second way of looking at a theory of meaning relevant to us is dynamic. Let us look at an example of such a theory for the sentences in $\mathcal{L}$ [see Gillies, 2004, for instance]:

Definition 2.3.3 (Dynamic Semantics for $\mathcal{L}$). *Let (as above) a context c be a set of possible worlds. A notion of* **update** *for* $\mathcal{L}$ *is given in terms of a recursively defined function* $\cdot[\cdot]$ *from sentences* $p \in$ At, $\alpha, \beta \in \mathcal{B}$, $\phi \in \mathcal{L}$ *and contexts to contexts, as follows:*

(p_d) $c[p] = \{w \in c \mid w(p) = 1\},$

$(\neg_d)$ $c[\neg\phi] = c - c[\phi],$

$(\wedge_d)$ $c[\alpha \wedge \beta] = c[\alpha][\beta],$

$(\to_d)$ $c[\alpha \to \beta] = \{w \in c \mid c[\alpha][\beta] = c[\alpha]\}.$

This approach may be called the *dynamic strict conditional view*. According to it, the meaning of a sentence ϕ is given in terms of $\cdot[\phi]$, a function that models the sentences *update* or *context change potential*. Accordingly, such a theory provides a model of conversation (in the sense of Def. 2.2.1) for $\mathcal{L}$ directly. But, since our update function is defined recursively in a similar manner to that of our truth-conditional proposal from above, it does

not only deliver a theory of discourse dynamics but also tells us about the compositional mechanisms that govern the sentences in $\mathcal{L}$.

Notice that I am not yet presupposing a specific notion of compositionality. We will say more about this later. But on any reasonable notion of what it is for a theory of meaning to be compositional, if the former truth-conditional approach is compositional, so is the latter dynamic approach.

Generalizing from this example, let us adopt the following definition:

Definition 2.3.4 (Dynamic theories of meaning). *A theory of meaning for a fragment of language $\mathcal{L}$ is **dynamic** iff it is provided in terms of a recursively defined update function of a model of conversation for $\mathcal{L}$.*

This definition is intended to capture what we may call the *guiding slogan* of dynamic semantics: *meanings are context change potentials.* Since, as argued above, context change potentials can be modeled with the help of an update function, the definition seems adequate.

2.3.2 Two Theories of Discourse Dynamics for $\mathcal{L}$

We have just seen two theories of meaning. According to the first, meanings of sentences are modeled as functions from indices to truth values. According to the second, meaning of sentences are modeled as functions from contexts to context. Corresponding to these two approaches, we can provide two approaches to the discourse dynamics of $\mathcal{L}$.

The first is straightforward:

Definition 2.3.5 (Semantic approaches to discourse dynamics). *According to a **semantic approach to discourse dynamics**, to provide a theory of discourse dynamics for a fragment of language is to provide a dynamic theory of meaning for the fragment in question.*

If meanings are context change potentials, then to provide a theory of the latter is to provide a theory of the former.

However, a theory of discourse dynamics for a fragment of language does not have to be given in terms of a recursively defined update function and hence, it does not have to be given in terms of a dynamic theory of meaning. Truth-conditional approaches can easily be equipped with bridge principles that give rise to a model of conversation in terms of the previously defined notion of truth at an index.

Let us consider an example. The following bridge principle links the above truth-conditional approach to a notion of update for $\mathcal{L}$:[9]

$$c[\phi]_\dagger = \{w \in c \mid [\![\phi]\!]^{\langle w,c \rangle} = 1\}. \qquad \text{(Diagonal Updating)}$$

Treating $[\cdot]_\dagger$ as the update function of a model of conversation, this bridge principle enables a defender of a truth-conditional approach to provide a theory of discourse dynamics for the fragment of language we are interested in.

Generalizing from this example, we will adopt the following definition.

Definition 2.3.6 (Non-semantic approaches to discourse dynamics). *According to a **non-semantic approach to discourse dynamics**, a theory of discourse dynamics for a fragment of language is provided in terms of (i) a theory of meaning that does not model the discourse dynamics directly and (ii) a bridge principle that links that theory to a model of conversation, capturing the discourse dynamics for the fragment in question.*

Notice that this definition does not require our theory of meaning to be truth-conditional. What matters is that we distinguish between the compositional mechanisms that govern meanings and the update potentials that govern the effects of assertions of sentences in the fragment of language we care about.

Now, it is not difficult to verify (see Lemma A.1.2 in Appendix A below) that

$$\text{for all } \phi \in \mathcal{L} \text{ and } c \in C, c[\phi]_\dagger = c[\phi].$$

This means that the model of conversation for $\mathcal{L}$ we get by diagonal updating is exactly the same as the model provided by the dynamic approach from the previous section. So if, in the following, we are talking about the predictions of the dynamic strict conditional view, it does not matter whether we think about it in terms of the just mentioned semantic or non-semantic approach. Moreover, we should keep in mind that no empirical fact about the update

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[9]The name is taken from [Yalcin, 2007] and is inspired by the corresponding notion in [Stalnaker, 1999] according to which the diagonal content of a sentence at a context is the set of worlds of that context at which the asserted sentence is true.

potentials of sentences in $\mathcal{L}$ will distinguish between the two.[10]

However, it is worth pointing out that a defender of the above dynamic approach to indicative conditionals may rightly complain that providing a truth-conditional approach to the meaning of sentences in $\mathcal{L}$ is a roundabout way of doing things: why, the dynamic semanticist may ask, should we provide a semantics *and* a bridge principle to define a model of conversation for $\mathcal{L}$ if we can have both in one go? Hence, if you think that the dynamic strict conditional view is empirically adequate, dynamic approaches to meaning seem to have the upper hand. They can, while non-semantics approaches cannot, claim the virtue of simplicity.

Later on, I will argue that this gain in simplicity comes at a cost. Doing two things at once means that there are two things that can go wrong. In order to be empirically adequate, dynamic approaches should not only be able to adequately describe the compositional mechanisms governing the fragment of language we are interested in but make the right kind of predictions about the context change potentials of the sentences in question. As we will see in the next section, we have reason to believe that the dynamic strict conditional view is *not* empirically adequate. While there may be truth to the way it handles the compositional mechanisms of the language, it makes unwelcome predictions at the level of the discourse dynamics for the simple indicative conditional sentences we are interested in.

2.4 Constraints on a Discourse Dynamics of Simple Conditionals

In this section I will argue that the dynamic strict conditional view makes empirically inadequate predictions about the context change potentials of plain conditionals. We then look at an alternative view which does better

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[10]Here it is important to note that the equality, $c[\phi]_\dagger = c[\phi]$, would break down for the full conditional fragment $\mathcal{L}_\rightarrow$. It is not difficult to check, however, that it can be regained for $(\mathcal{L}_\rightarrow)$ if we replace the entry $(\wedge_t)$ by $(\wedge_t^a)$: $[\![\phi \wedge \psi]\!]^{\langle w,s \rangle} = 1$ iff $[\![\phi]\!]^{\langle w,s \rangle} = 1$ and $[\![\psi]\!]^{\langle w,s[\phi]_\dagger \rangle} = 1$, a notion of conjunction sometimes referred to as *asymmetric conjunction* [see Khoo and Mandelkern, 2019, for instance]. Alternatively, we could change the entry for conjunction in our dynamic view to $c[\phi \wedge \psi] = c[\phi] \cap c[\psi]$. Though interesting in its own right, I will ignore the discussion surrounding the interaction of *and* and *if* which pertain to choosing such an alternative notion of conjunction. What matters is that, even if we choose a larger conditional fragment, the *same* model of conversation can be defined in terms of a semantic and a non-semantics approach.

than the strict conditional view in some but not all respects and end the section by formulating some general constraints a theory of discourse dynamics for $\mathcal{L}$ should satisfy.

2.4.1 The Dynamic Strict Conditional View

Before we look at some concrete predictions of the dynamic strict conditional view, a word about the underlying mechanism of the view may be helpful. On this view, assertions of conditional sentences are sometimes thought of as *tests*: they test whether the context does possess a certain global property or not. The test in question is sometimes called the *Ramsey test*. According to it, we check whether updating the context with the antecedent results in a context that contains enough information to support the consequent of the conditional in question.[11] For plain conditionals we get the following predictions: if the context passes the Ramsey test, the assertion leaves the context untouched and is predicted to be felicitous. If the Ramsey test is not passed, the assertion is predicted to rule out all worlds from the context set and is, hence, predicted to be as infelicitous as an assertion of something contradictory or obviously false. For negated conditionals, it is the other way around. If the context does *not* pass the Ramsey test, they are predicted to be felicitous, if it is passed they are predicted to be as infelicitous as an assertion of something contradictory or obviously false.

Let us look at some concrete cases. Here is a vignette (slightly modified from [Gillies, 2004]) we may take as background for the cases discussed in the following.

> **The Mansion:** A crime has been committed at the mansion. Ann and Bob are investigating. It is common ground between Ann and Bob that the culprit acted alone and that there are three possible candidates: the butler (a member of the *house staff*), the gardener and the driver (both members of the *ground staff*).

First, a simplified but reasonable way of modelling the context or what is common ground between Ann and Bob would be as the set $\{w_b, w_g, w_d\}$,

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[11]The name *Ramsey Test* goes back to a comment of Ramsey about how to test whether a given indicative conditional is true or not [Ramsey, 1990, p. 155, fn. 1]. The view is widely accepted. However, see [Willer, 2010] for a discussion of some challenges.

consisting of, w_b, the world in which the *butler* did it, w_g, the world in which the *gardener* did it and w_d, the world in which the *driver* is the culprit.

Let us first look at what the dynamic strict conditional view predicts about an assertion of a negated conditional. Suppose Ann and Bob are investigating the crime scene together. Suppose she is the more experienced investigator and notices that the evidence is hard to read. It may, to the inexperienced investigator, seem that if a member of the ground staff committed the crime, it must have been the driver. In order to make sure that she and Bob are on the same page, she asserts

(1) It is not the case that if a member of the ground staff did it, it was the driver.

On the dynamic strict conditional view we get the following prediction:[12]

$$\{w_b, w_g, w_d\}[\neg(g \to d)] = \{w_b, w_g, w_d\}.$$

Ann's assertion leaves the context unchanged since

$$\{w_b, w_g, w_d\}[g] \neq \{w_b, w_g, w_d\}[g][d]$$

and hence the Ramsey test is not passed. While her assertion does not convey information to Bob, the prediction seems reasonable. The aim of an assertion need not be to contribute information to the common ground. Rather, we can think of Ann's assertion of (1) as a way of pointing out that we should keep the possibility that the gardener did it as a live option, even in light of the potentially misleading evidence.

Now, what is a reasonable way to think about assertions of negated indicative conditionals may not be a reasonable way of thinking about plain, non-negated conditionals. Let us look back at the above vignette but let

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[12]Let $c := \{w_b, w_g, w_d\}$ and g be true at w_d and w_g only whereas d is true at w_d only. We then have

$$
\begin{aligned}
c[\neg(g \to d)] &= c - c[g \to d], && \text{by Def. 2.4.1, } (\neg_d) \\
&= c - \{w \in c \mid c[g][d] = c[g]\}, && \text{by Def 2.4.1, } (\to_d) \\
&= c - \varnothing, && \text{since } c[g][d] \neq c[g] \\
&= c. && \text{by set theory}
\end{aligned}
$$

us suppose that this time the evidence points in a different direction. Let us suppose that the evidence does not rule out the butler but among the members of the ground staff, the driver but not the gardener is a potential culprit. Let us suppose further that Ann has looked at the evidence alone and wants to communicate her findings to Bob. To do so, she asserts

(2) If a member of the ground staff did it, it is the driver.

Since Bob did not look at the evidence, what is common ground between Ann and Bob is the same as above. Hence (2) should be evaluated at the same context as before. But then, we get

$$\{w_b, w_g, w_d\}[g \to d] = \varnothing.$$

So, Ann's assertion is predicted to be as infelicitous as asserting a contradiction or an obvious falsehood. That seems wrong.[13] In a situation like the one just described, Ann's assertion seems perfectly felicitous and adequate. While her assertion does seem informative, the prediction that it rules out *all* worlds from the context set is too strong. Rather, the prediction we should expect is this:

$$\{w_b, w_g, w_d\}[g \to d] = \{w_b, w_d\}.$$

In light of Ann's assertion we should expect that Ann and Bob coordinate on a context that no longer contains the driver as a possible culprit.

In conclusion, the dynamic strict conditional view seems to make reasonable predictions about assertions of negated indicative conditionals but its predictions about assertions of plain conditionals seem wrong.

2.4.2 Heim's View

We have seen above that the dynamic strict conditional view makes unreasonable predictions about assertions of plain indicative conditionals. Is there a way to do better? One way to do better with respect to assertions of plain

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[13]Note that this prediction is shared by a wide range of views which agree with the one presented here on the discourse dynamics of the sentences in the small fragment $\mathcal{L}$. On top of the above cited accounts of Gillies [2004] and Veltman [1985], see [Yalcin, 2012b] and [Moss, 2018], for a version which, on top of non-modal antecedents and consequents, can handle probabilistic sentences. For a version which explains the oddity of Sobel sequences, see [Willer, 2017].

indicative conditionals is to change the entry for the the conditional operator. Indeed, there is an early view about the discourse dynamics of indicative conditionals due to Heim [2002]. The view goes as follows.

Definition 2.4.1 (Heim's Dynamic Semantics for $\mathcal{L}$). *Let (as above) a context c be a set of possible worlds. **Heim's dynamic semantics** for $\mathcal{L}$ is given in terms of a recursively defined function $\cdot[\cdot]$ from contexts and sentences $p \in \texttt{At}$, $\alpha, \beta \in \mathcal{B}$, $\phi \in \mathcal{L}$ to contexts, as follows:*

(p_d) $c[p] = \{w \in c \mid w(p) = 1\}$,

$(\neg_d)$ $c[\neg\phi] = c - c[\phi]$,

$(\wedge_d)$ $c[\alpha \wedge \beta] = c[\alpha][\beta]$,

$(\rightarrow_h)$ $c[\alpha \rightarrow \beta] = c - (c[\alpha] - c[\alpha][\beta])$.

The entries for atomic sentence, negation and conjunction are the same as those found for the strict conditional view. But the entry for the conditional is different. It is not difficult to verify that Heim's view makes desirable predictions about assertions of plain conditionals. Indeed, on her view we get the desired prediction that

$$\{w_b, w_g, w_d\}[g \rightarrow d] = \{w_b, w_d\}.$$

Ann's assertion is predicted to be felicitous and informative in the way we would expect it to be.

Unfortunately, it is well known that that prediction comes at a cost. It has been observed that Heim's view is a dynamic version of the material conditional view, a view that has untenable consequences for assertions negated conditionals [see Gillies, 2004, for an argument like this]. Note that

$$
\begin{aligned}
c[\neg(\alpha \rightarrow \beta)] &= c - c[\alpha \rightarrow \beta], && \text{by } (\neg_d) \\
&= c - (c - (c[\alpha] - c[\alpha][\beta])), && \text{by } (\rightarrow_h) \\
&= c[\alpha] - c[\alpha][\beta], && \text{by set theory} \\
&= c[\alpha \wedge \neg\beta]). && \text{by } (\wedge_d) \ \& \ (\neg_d)
\end{aligned}
$$

Hence an assertion of the negated conditional (1) from the previous section is predicted to convey both of the following

(3) A member of the ground staff did it.

(4) It was not the driver.

But this seems too strong. An assertion of (1) seems certainly compatible with the Butler being a candidate culprit (hence (3) is not conveyed) and the same goes for the driver (hence (4) is not conveyed).

It is worth pointing out that these predictions about negated indicative conditionals are typically taken to be the key reason to prefer the dynamic strict conditional view over Heim's [see Gillies, 2004, for instance]. But in light of the strict conditional view's predictions about plain conditionals, choosing the strict conditional view over Heim's seems like throwing out the baby with the bath water. It seems that by adopting the dynamic strict conditional view we are trading in one bad prediction about negated conditionals for a bad prediction about plain conditionals. To my mind, an empirically adequate theory of the discourse dynamics of conditionals should combine the benefits and rid us of the drawbacks of both views.

2.4.3 Constraints on a Discourse Dynamics for $\mathcal{L}$

Since neither the dynamic strict conditional view nor Heim's view seem adequate, let us write down a wish list of constraints we want a theory of the discourse dynamics for $\mathcal{L}$ to satisfy.

First, note that all of the previously mentioned views agree on how we should handle the discourse dynamics of non-conditional sentences. Indeed, it is not difficult to prove that according to all of them we have (see Lemma A.1.1 in Appendix A below)

$$\text{for all } \alpha \in \mathcal{B}, \, c[\alpha] = c \cap \{w \in c \mid \mathcal{V}(w, \alpha)\}, \qquad \text{(Boolean Intersectivity)}$$

where $\mathcal{V}$ is a valuation function that extends our notion of truth at a world to all sentences in the non conditional fragment $\mathcal{B}$ of $\mathcal{L}$ in the standard way.[14] So, according to all of them, asserting a non-conditional sentence of $\mathcal{L}$ results in the removal of those worlds from the context at which the sentence is false. This is a standard assumption and hence we should be aiming for a view that is *conservative* in this regard.

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[14]By this I mean that $\mathcal{V}(w, p) = w(p)$, if $p \in \mathtt{At}$ and $\mathcal{V}(w, \neg \alpha) = 1$ iff $\mathcal{V}(w, \alpha) = 0$ as well as $\mathcal{V}(w, \alpha \wedge \beta) = 1$ iff $\mathcal{V}(w, \alpha) = 1$ and $\mathcal{V}(w, \beta) = 1$, for $\alpha, \beta \in \mathcal{B}$.

Second, in light of what we said about plain, non-negated conditionals it seems we should opt for a view that preserves the predictions of Heim's view but in light of what we said about negated conditionals, we should be aiming for a view that preserves the predictions of the dynamic strict conditional view. Let us give a name to possible worlds models that satisfy these three constraints. Let us call any such model of conversation *unorthodox*:

Definition 2.4.2 (Unorthodox Models for $\mathcal{L}$). *Let $\cdot[\cdot]$ be an update function that satisfies Boolean Intersectivity. An **unorthodox** model of conversation is a pair, $\langle \cdot[\cdot]^*, C \rangle$, such that $C := \mathcal{P}(W)$ and for all $c \in C$ and $\alpha, \beta \in \mathcal{B}$, $\cdot[\cdot]^*$ satisfies the following three constraints:*

> ***Conservativity***: $c[\alpha]^* = c[\alpha]$,
>
> ***Materiality***: $c[\alpha \rightarrow \beta]^* = c - (c[\alpha] - c[\alpha][\beta])$,
>
> ***Strict Negation***: $c[\neg(\alpha \rightarrow \beta)]^* = \{w \in c \mid c[\alpha] \neq c[\alpha][\beta]\}$.

The question now becomes whether and how we can provide such a model of conversation. We will investigate this question in Section 2.5. However, before we do so, let me end this section with some remarks about potential worries one may have about models that are unorthodox in the above sense.

First, Materiality says, basically, that assertions of simple non-negated conditionals behave like assertions of the material conditional. Note, however, that materiality is restricted to non-conditional antecedents and consequents. Hence, the constraint is perfectly compatible with views that treat nested conditionals or conditionals that embed modals in a different way. Second, semantically, the material conditional view has a bad reputation. But note that the constraint is intended to be read as a constraint about discourse dynamics and commitments about semantics presuppose a dynamic approach to meaning. Still, we may worry that certain arguments against adopting the material conditional view as a semantics for the conditional carry over to any view which adopts the view as a view about discourse dynamics.[15] While this is a worry to take seriously, the arguably strongest

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[15]One such worry is that indicative conditionals require that a plain conditional is assertable only if the antecedent is epistemically possible. Whether this should be implemented as a semantic constraint is questionable. Note, however that we could implement such a constraint while holding on to the spirit of Materiality as follows:

> **Weak Materiality**: $c[\alpha \rightarrow \beta]^* = c[\Diamond\alpha] \cap (c - (c[\alpha] - c[\alpha][\beta]))$

objection against the material conditional view is related to its predictions about negated conditionals. Unorthodox models are immune to this objection, since negated conditionals are treated differently.[16]

It is also worth pointing out that, as a view about plain conditionals, the view seems to experience a renaissance. Looking at the literature on Bayesian epistemology, for instance, it has recently been argued that updating one's credences in light of a conditional is best modeled in terms of updating on the corresponding material conditional [see Günther, 2018, Eva et al., 2020, for a defense of such views]. Of course, views about Bayesian conditionalization are not (or not necessarily) views about conversational updating. Still such views seem to lend additional plausibility to the constraint. Moreover, a recent proposal about the semantics of indicative conditionals found in [Santorio, 2022] comes paired with an update function that predicts the update potentials of indicative conditionals to behave like the material conditional (we will say more about Santorio's view and his notion of updating below). Hence, I am not alone in thinking that there is something to the Materiality constraint.[17]

Second, strict negation tells us that the update potentials of negated conditionals should be treated the same as the dynamic strict conditional view treats them. This might be unwelcome to some readers. Many think that, semantically, a conditional of the form $\neg(\alpha \to \beta)$ should be treated the same as $\alpha \to \neg\beta$. This goes under the label of Boethius' Thesis.[18] In its dynamic form it says:

$$\text{for all contexts } c \text{ s.t. } c[\alpha] \neq \varnothing, \ c[\neg(\alpha \to \beta)] = c[\alpha \to \neg\beta].$$

Given Conservativity and Materiality, Boethius' Thesis is in conflict with

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where $c[\Diamond\alpha] = \{w \in c \mid c[\alpha] \neq \varnothing\}$. All arguments provided below are compatible with such a weakening of the Materiality constraint. However, to keep the dialectic as simple as possible, I will confine a discussion of how respective arguments can be extended to this weakened version to footnotes.

[16]Arguments against the view which are about the interaction of indicative conditionals with epistemic modals and probability operators [Yalcin, 2012b] can be ignored for the same reason.

[17]Let me note also that [as pointed out in Goldstein, 2019] views satisfying Materiality do, while the strict conditional view does not, constitute a theory of conditional assertion in the spirit of [Quine, 1982]. So fans of such a view have an incentive to defend a model of conversation that satisfies the Materiality constraint.

[18]See [Wansing, 2023] for a helpful discussion of Boethius' Thesis.

strict negation.[19] Hence we cannot hold on to both views about negated conditionals. Which of the treatments of negated indicative conditionals is to be preferred will be left open in this paper. For the sake of simplicity I will stick to views that validate strict negation. But nothing of substance depends on this choice. As argued in greater detail in Appendix A, the arguments I am interested in go through no matter which view of negated conditionals we adopt as long as it disagrees with Heim's view on some contexts.

2.5 The Non-compositional Dynamics of 'If'

In this section we point out that on a widespread notion of what it is for a theory of meaning to be compositional, unorthodox models of conversation are not compositional. We then show that we can provide a bridge principle for the above mentioned truth conditional semantics for $\mathcal{L}$ that delivers a non-compositional theory of discourse dynamics that is unorthodox on the basis of a compositional theory of meaning. Hence, anyone who thinks that meanings are compositional and agrees with me that the context change potentials for the sentence in $\mathcal{L}$ are unorthodox, has a reason to adopt such a non-semantic approach to the discourse dynamics of $\mathcal{L}$.

2.5.1 Compositionality for Unorthodox Models?

In Section 2.3.1 I defined truth-conditional theories of meaning in terms of a recursively defined function from indices to truth values and dynamic theories of meaning in terms of a recursively defined update function of a model of conversation. Here, the notion of 'recursion' was used loosely in the sense of functions that are defined on the basis of the inductive structure of the underlying fragment of language. This, you may think, adequately captures what it is for such theories to count as capturing the *compositionality* of

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[19]To see this, consider:

$$\{w_b, w_g, w_d\}[\neg(g \to d)]^* = \{w_b, w_g, w_d\}[(g \to \neg d)]^*,$$
$$= \{w_b, w_g, w_d\} - (\{w_b, w_g, w_d\}[g]^* - \{w_b, w_g, w_d\}[g]^*[\neg d]^*),$$
$$= \{w_b, w_g\}.$$

The first equality holds by the dynamic version of Boethius' Thesis, the second by Materiality and the last by Conservativity and set theory. Since $\{w_b, w_g\} \neq \{w_b, w_g, w_d\}$, we have a conflict with Strict Negation.

meanings. However, typically, semanticists have a slightly stronger notion of compositionality in mind.

Let us fix an arbitrary fragment of language $\mathcal{L}^*$. Given three sentences $\phi, \psi, \chi \in \mathcal{L}^*$, we will us the notation $\chi^{\phi/\psi}$ in order to denote the sentence which is just like χ except the all occurrences of ϕ are replaced by ψ. Given this, a standard way of thinking about the compositionality of meanings is, in its theory neutral form, captured by the following principle:

> **Compositionality (neutral)**: If ϕ and ψ have the same meaning, χ and $\chi^{\phi/\psi}$ have the same meaning.

The principle is at the core of what we may call *arguments from embedding behavior*. While all of this is well known, it may be helpful to see the principle at work in one such argument. A classic argument is the standard argument for why Moore paradoxical sentences are not contradictions.

(5) I believe it is raining but it is not raining (ϕ),

(6) I believe that it is raining and it is not the case that I believe that it is raining (ψ),

(7) Suppose I believe it is raining but it is not raining (χ),

(8) Suppose I believe that it is raining and it is not the case that I believe that it is raining ($\chi^{\phi/\psi}$).

(5) sounds as odd as (6).[20] However (8) sounds odd while (7) sounds fine. Hence, (7) and (8) must differ in meaning. But this, together with the above compositionality principle entails that (5) must differ in meaning from (6), even though a difference in meaning may not be detectable by looking at the unembedded (5) and (6) alone.

Given all this, let us be explicit and write down what the above compositionality constraint amounts to, both in the truth-conditional as well as the dynamic setting:

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[20]The oddity of such sentences was first observed by Moore [1942]. [Wittgenstein, 2000] contains an early discussion of the embedding behaviour of such sentences. The oddity of sentences such as (5) is typically explained pragmatically [Sorensen, 1988, Williamson, 2000]. A notable non-pragmatic explanation is found in [Gillies, 2001].

Compositionality (truth conditional) If for every index $\langle w, s \rangle$, $[\![\phi]\!]^{\langle w,s \rangle} = [\![\psi]\!]^{\langle w,s \rangle}$, then, for every index $\langle w, s \rangle$, $[\![\chi]\!]^{\langle w,s \rangle} = [\![\chi^{\phi/\psi}]\!]^{\langle w,s \rangle}$.

Compositionality (dynamic) If for every context c, $c[\phi] = c[\psi]$, then, for every context c, $c[\chi] = c[\chi^{\phi/\psi}]$.

[Compare Rothschild and Yalcin, 2016, section 4, for an equivalent definition of compositionality for dynamic approaches]. Now, both the truth conditional and the dynamic approach to sentences in our small fragment $\mathcal{L}$ from Section 2.3 satisfy the respective version of the constraint and so do the great majority of views found in the literature. An unorthodox model of conversation, however, will not be compositional in this sense. To be more precise, let us call a possible worlds model of conversation *non-trivial*, if it is defined on a set of worlds that contains at least three worlds. We then have

Proposition 2.5.1. *There is no non-trivial unorthodox model of conversation that satisfies the above compositionality constraint.*

Given the examples we have seen above, Proposition 2.5.1 should not come as a surprise. Still a proof and a discussion of a slightly stronger version is found in Appendix A below.[21]

Proposition 2.5.1 tells us that we have two options. First, we reject the idea that meanings are context change potentials and deliver a non-compositional theory of discourse dynamics on the basis of a compositional theory of meaning. Second, we hold on to the idea that meanings are context change potentials. But choosing this latter option we either have to capture the spirit of an unorthodox model in some other way or reject the idea that meanings are compositional in the sense just described.

The rest of this section is dedicated to exploration of the first option. The second option is discussed in Section 2.6.

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[21]The first (equality) holds since, by (Materiality), we have $c[g \to d]^* = c - (c[\alpha] - c[\alpha][\beta])$ but

$$
\begin{aligned}
c[\neg(\alpha \wedge \neg\beta)]^* &= c[\neg(\alpha \wedge \neg\beta)], &\text{by (Conservativity)} \\
&= c - c[\alpha \wedge \neg\beta], &\text{by } (\neg_d) \\
&= c - c[\alpha][\neg\beta], &\text{by } (\wedge_d) \\
&= c - (c[\alpha] - c[\alpha][\beta]). &\text{by } (\neg_d)
\end{aligned}
$$

The second (inequality) holds by what we have seen above and since by (Conservativity) $\{w_b, w_g, w_d\}[\neg\neg(g \wedge \neg d)]^*$ would mirror the predictions of the material conditional view about negated indicatives – i.e. $\{w_b, w_g, w_d\}[\neg\neg(g \wedge \neg d)]^* = \{w_g\}$.

2.5.2   Non-semantic Accounts for an Unorthodox Discourse Dynamics of $\mathcal{L}$

Is there a way to provide a non-compositional theory of discourse dynamics on the basis of a compositional theory of meaning? In this section we will see that the answer is yes.

The idea behind the account I wish to consider in this section is inspired by a discussion about update due to Yalcin [2007] and formally close to a recent proposal discussed in [Santorio, 2022]. The key idea I wish to build on is that we can think about conversational updates in terms of our truth-conditional semantics for $\mathcal{L}$ and an alternative bridge principle: a bridge principle that is defined in terms of a notion of *support*. With respect to our truth-conditional theory of meaning for $\mathcal{L}$ such a notion can be defined as follows:

Definition 2.5.1 (Support). *As above, let a state of information s be a set of possible worlds. We say that s* **supports** ϕ *just in case for all* $w \in s$, $[[\phi]]^{\langle w,s \rangle} = 1$.

What the support relation tells us is that, given the information contained in s, ϕ will be true no matter how the world might turn out to be in light of s.[22]

Now, contexts are states of information, the information that is shared between the participants of the discourse. Moreover, it is reasonable to suppose that, if ϕ is asserted by a speaker, discourse participants may operate under the assumption that the speaker has enough information available to them to support ϕ — i.e. from the perspective of the information available to the speaker ϕ is true no matter how the world might turn out to be. Hence, discourse participants may exclude all worlds as candidate possibilities from the context that are incompatible with any way of supporting ϕ.

Formally, this corresponds to the following definition [See Santorio, 2022, Appendix, for a similar definition that is based on a different notion of support]:

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[22]This is a standard notion found all over the literature on epistemic modals and indicative conditionals. Sometimes it is described as a notion of *acceptance*. I prefer 'support' in order to avoid confusion with the *attitude* of acceptance. Analogues of the above definition are found in [Yalcin, 2007] and [Kolodny and MacFarlane, 2010], for instance. In the dynamic semantics literature such a notion is sometimes defined in terms of an update function [see Veltman, 1996, for instance].

Definition 2.5.2 (Informational Updating)**.** *Let the support relation be defined as in Def. 2.5.1. Then the pair* $\langle C, \cdot[\cdot]^i \rangle$ *with* $C := \mathcal{P}(W)$ *and* $\cdot[\cdot]^i$ *defined as*

$$c[\phi]^i := \bigcup \big(\{ c' \subseteq c \mid c' \, supports \; \phi \} \big),$$

is a model of conversation for $\mathcal{L}$*. We will refer to the respective notion of update as* **informational updating***.*

Here is one consequence of the view that is worth highlighting. First, for any sentence ϕ in our small conditional fragment $\mathcal{L}$, $c[\phi]^i$ supports ϕ. Since the removal of worlds from the context corresponds to adding information to it, this means that for all sentences $\phi \in \mathcal{L}$, there is a unique way of adding a minimal amount of information to the common ground so that the asserted sentence is supported.

However, moving to larger fragments of language, this is not true in general. Once we move to a fragment in which conditionals may scope below disjunction, for instance, there may be updates which do not have such a unique supporting substate. To see this, consider

(9) Either, if a member of the ground staff did it, it is the driver or if a member of the ground staff did it, it is the caretaker.

Let us suppose that disjunction is defined in terms of conjunction and negation in the usual way (so that its discourse effects are $c[\phi \vee \psi] = c[\neg(\neg\phi \wedge \neg\psi)]$). Further, let us assume that (9) is asserted at a context that is just like $\{w_b, w_g, w_d\}$ except that we add one world w_c in which a third member of the ground staff, the caretaker, is the culprit. Finally, let g and d be as above and let c be true at w_c but false in all other wolds. We then get,

$$\{w_b, w_g, w_d, w_c\}[(g \to d) \vee (g \to c)] = \{w_b, w_d, w_c\}.$$

Now, $\{w_b, w_d, w_c\}$ does not support the sentence $(g \to d) \vee (g \to c)$. The sentence is supported by either $\{w_b, w_g\}$ or $\{w_b, w_c\}$ but not their union [see Santorio, 2022, Appendix, for a discussion of a similar sentence involving epistemic necessity modals].

Does that constitute a problem for the view? I do not think so. First, native speakers I consulted with seem to confirm that the context change potential of (9) is that of

(10) If a member of the ground staff did it, it is the driver or the caretaker,

a sentence that is reasonably modeled by $g \to (d \lor c)$. But on our view we have

$$\{w_b, w_g, w_d, w_c\}[g \to (d \lor c)] = \{w_b, w_d, w_c\}.$$

A prediction that seems reasonable. Moreover, $\{w_b, w_d, w_c\}$ *does* support $g \to (d \lor c)$. So the view seems to make the right kind of predictions about the update potential of both (9) and (10). It is interesting to note that many judge (9) to be slightly odd and prefer (10) instead. On the above view, the relative oddity (9) vs. (10) can be explained by the fact that for (10) there is but for (9) there is no unique way of adding a minimal amount of information to the common ground so that the resulting state supports (9). A thorough discussion of these predictions is beyond the scope of this chapter and something I hope to come back to in future work. For now, it suffices to note that, in terms of its predictions about the update potentials of conditionals in larger fragments, the view seems reasonable.

Now, here is the key consequence of thinking about update in terms of our notion of informational updating:

Proposition 2.5.2. *The model of conversation* $\langle C, \cdot[\cdot]^i \rangle$ *for* $\mathcal{L}$ *with* $C :=$ $\mathcal{P}(W)$ *and* $\cdot[\cdot]^i$ *as in Def. 2.5.2 satisfies*

> ***Conservativity:*** $c[\alpha]^i = c[\alpha]$,

> ***Materiality:*** $c[\alpha \to \beta]^i = c - (c[\alpha] - c[\alpha][\beta])$,

> ***Strict negation:*** $c[\neg(\alpha \to \beta)]^i = \{c \in w \mid c[\alpha][\beta] \neq c[\alpha]\}$,

where, as above, $\cdot[\cdot]$ *is an update function that satisfies Boolean Intersectivity.*

So, the view gives rise to an unorthodox model of conversation, a model of conversation that, in light of Proposition 2.5.1, is non-compositional. But since our notion of update is defined in terms of the above truth-conditional semantics for $\mathcal{L}$ which *does* satisfy our compositionality principle, we have a non-compositional theory of discourse dynamics that is defined in terms of a compositional theory of meaning.

Let me highlight that, while on this view meanings and context change potentials come apart, context change potentials may still be taken to inform us about meaning. We still have:

> **Update to Meaning** For all ϕ and ψ in $\mathcal{L}$, if there is a context c s.t. $c[\phi]^i \neq c[\psi]^i$, then $[\![\phi]\!]^{\langle w,s \rangle} \neq [\![\psi]\!]^{\langle w,s \rangle}$ for some index $\langle w, s \rangle$.

The converse principle, however, fails.

As noted above, the just introduced notion of update is similar in spirit to one found in Santorio. However, his approach is based on a new approach to the semantics of indicative conditionals he refers to as path semantics. Introducing this approach goes beyond the scope of this paper. But it is interesting to note that Santorio's proposal follows a pattern similar to the approach just introduced. His approach too can be divided into a truth-conditional semantic part and a dynamic part. On the level of discourse dynamics Santorio's approach validates a principle analogous to Materiality, but for negated conditionals his approach validates a dynamic version of Boethius' Thesis. Moreover, like the approach just sketched, Santorio's truth conditional semantics satisfies our standard compositionality constraint, while his notion of update does not. Hence, for fans of Boethius' Thesis, Santorio's approach may be a promising alternative.

2.6 Prospects for Dynamic Responses

Both the approach just sketched as well as the approach found in [Santorio, 2022] fit the pattern of a theory according to which a non-compositional theory of discourse dynamics is given on the basis of a compositional theory of meaning. Such theories have the advantage of explaining the rationale behind standard semantic theorizing (viz. arguments from embedding behaviour) while, at the same time, explaining the non compositional dynamics of the sentences in $\mathcal{L}$. But both of the views reject the guiding slogan of dynamic semantics. Context change potentials are not meanings on either view. So, in this section I wish to discuss some of the most promising ways in which fans of dynamic approaches to meaning may respond to the challenge raised in the previous section.

I will start by looking at two responses that aim at preserving the spirit of an unorthodox model while holding on to a compositional theory of discourse dynamics and then discuss a response that will result in giving up on the above notion of compositionality.

2.6.1 Pragmatics and Contexts

The first response builds on the fact that, as noted above, the dynamic strict conditional view does satisfy our notion of compositionality. The draw-

back of the view was that it makes unwelcome predictions about the context change potentials of plain conditionals. A defender of a dynamic approach to meaning may argue that the dynamic strict conditional view *does* model the meanings of the sentences in $\mathcal{L}$ but, in order to circumvent the objection from plain conditionals, should be equipped with an extra-semantic principle (pragmatic, presupposition theoretic or postsemantic) that delivers the right kind of predictions with respect to plain conditionals.

If successful, we seem to be able to hold on to a dynamic theory of meaning that satisfies our compositionality principle from above but, with the help of some extra semantic reasoning, delivers better predictions about how assertions of conditionals affect the context of a conversation.

While such a story could certainly be told, I find it unsatisfying. If some additional, extra semantic mechanism is needed to get the right kind of predictions about how assertions of the sentences in ϕ change the context of a conversation, it is unclear what the update function of the underlying model of conversation is supposed to model in the first place. Moreover, the advantage of a dynamic approach to meaning is supposed to be that it takes care of *both* the compositional mechanism as well as a notion of context change for the sentences we are interested in. Postulating an extra semantic principle to get accurate predictions about the context change potentials of plain conditionals seems to undermine this advantage.

A more promising response to the non-compositionality of unorthodox models starts by noting that the application of Proposition 2.5.1 is limited. It applies to *possible worlds* models of conversation only. But, as mentioned above, there are discourse effects such models cannot capture. Hence, a defender of a semantic approach to discourse dynamics may hope that, once we move to more involved notions of context and context change, we can regain compositionality on the level of discourse dynamics while capturing the spirit of the constraints on the basis of which our unorthodox models were defined.

Indeed the literature contains a range of proposals that are generalizations of possible worlds models of conversation. To mention a view, Yalcin [2007] considers contexts to consist of both a set of possible worlds and a (set of) probability functions, Moss [2018] considers contexts as sets of probability spaces, defenders of inquisitive approaches to meaning [see Ciardelli et al., 2019] may think of contexts as downward closed sets of sets of possible worlds and, as mentioned above, Santorio [2022] considers contexts to be ordered sets of possible worlds. These more involved notions of context

are intended to model changes of the conversational scoreboard that capture more than simple changes in information content. So, on such views we may be able to devise a notion of context change for the sentences in $\mathcal{L}$ where Materialty holds if we restrict attention to the pure information content of the assertion, but fails if we consider additional parameters of the conversational scoreboard.

A response along these lines would certainly be interesting. However, at the moment, I do not see how adding an additional parameter to the conversational scoreboard could help to arrive at an account that does at least as well as our unorthodox view, while satisfying the above notion of compositionality.

2.6.2 Non-Compositional Bilateral Approaches

Let us turn to a third kind of response. It starts with the observation that key to our above discussion is the interaction of simple indicative conditionals with negation. Hence, one may wonder whether there is a more flexible way of thinking about negation that may provide a model of conversation that captures our unorthodox constraints.

Indeed, there is a recent proposal for the discourse dynamics of 'if' due to Willer [2022] that involves an alternative treatment of negation. He proposes what he refers to as a *bilateral approach* to dynamic semantics. According to this view, the discourse effects of assertions are modeled with the help of a notion of coming to accept a sentence. However, negations get a special treatment. According to Willer [2022], coming to accept a negated sentence $\neg\phi$, is coming to reject its embedded sentence ϕ. Unfortunately, Willer's own bilateral approach treats plain conditionals in the same way the dynamic strict conditional view treats them. Hence, this approach too suffers from the objection discussed in Section 2.4.1. However, we can use the flexibility of such an approach to write down a bilateral semantics that gives rise to an unorthodox model of conversation.

Here is one way of doing so:

Definition 2.6.1 (Unorthodox Bilateral Semantics)**.** *An unorthodox bilateral semantics for $\mathcal{L}$ is a model of conversation $\langle C, \cdot[\cdot]^+ \rangle$ where $C := \mathcal{P}(W)$ and $\cdot[\cdot]^+$ is recursively defined with the help of $\cdot[\cdot]^-$ as follows:*

$(p)_+$ $c[p]^+ = \{w \in c \mid w(p) = 1\}$,
$(p)_-$ $c[p]^- = \{w \in c \mid w(p) = 0\}$,

$$(\neg)_+ \ c[\neg\alpha]^+ = c[\alpha]^-,$$
$$(\neg)_- \ c[\neg\alpha]^- = c[\alpha]^+,$$

$$(\wedge)_+ \ c[\alpha \wedge \beta]^+ = c[\alpha]^+ \cap c[\beta]^+,$$
$$(\wedge)_- \ c[\alpha \wedge \beta]^- = c[\alpha]^- \cup c[\beta]^-,$$

$$(\rightarrow)_+ \ c[\alpha \rightarrow \beta]^+ = c[\alpha]^- \cup c[\beta]^+,$$
$$(\rightarrow)_- \ c[\alpha \rightarrow \beta]^- = \{w \in c \mid c[\alpha]^+[\beta]^+ \neq c[\alpha]^+\}.$$

Note that, since this is a recursive definition of a model of conversation, this counts as a semantic approach to discourse dynamics in the loose sense of Def. 2.3.4.

However, any such approach will come at a cost. First, note that, on this view, the function $\cdot[\cdot]^-$ does not govern any empirically observable speech act of *rejecting* a sentence. The notion of conversational update we ultimately care about is $\cdot[\cdot]^+$. Willer's main motivation for postulating the existence of $\cdot[\cdot]^-$ is indirect. As he shows, paired with an appropriate notion of consequence, such an approach can give rise to an interesting logic for conditionals. While this may be so, it is not clear to me that providing such an indirect argument is enough evidence to postulate the existence of a dedicated notion of update as rejection.

Second, the view is only compositional in the loose sense of Section 2.3.1. Since the model $\langle C, \cdot[\cdot]^+ \rangle$ is a possible worlds model that satisfies Materiality and Conservativity as well as Strict negation, Proposition 2.5.1 applies.[23] Hence $\cdot[\cdot]^+$ is not compositional in the sense of section 2.5.1. So the success of such an approach depends on whether there is a plausible weakening of our compositionality constraint. But since, as argued above, our compositionality constraint is at the heart of arguments from embedding behaviour, I am not hopeful that we can finding an appropriate weakening that preserves standard methodology in semantics.

2.7 Conclusion

Hans Kamp starts his [Kamp, 1984] with the remark that the "separation" of discourse dynamics from semantics "has become an obstacle to the devel-

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[23]Note that the applicability of Proposition 2.5.1 is not affected by the presence of $\cdot[\cdot]^-$ in the definition of the model $\langle C, \cdot[\cdot]^+ \rangle$. The sole purpose of $\cdot[\cdot]^-$ is to define how $\cdot[\cdot]^+$ treats negated sentences.

opment of semantic theory." [Kamp, 1984, p. 330]. I think that blurring the distinction between discourse dynamics and semantics may become an obstacle to the development of theories of discourse dynamics. As I have argued, there is evidence that a theory of discourse dynamics for simple conditional sentences is non-compositional. Moreover, we can provide such a non-compositional theory on the basis of a compositional theory of meaning, if we are willing to give up on the idea that meanings are context change potentials. In light of this, I think we have good grounds to reconsider dynamic approaches to meaning.

However, rejecting the guiding idea of dynamic semantics does not entail that we have to reject all of the innovations we owe to such approaches. First, a central idea of dynamic theories of meaning is that a notion of update may play a role at the level of compositional semantics. But, as illustrated by the truth conditional approaches to conditionals and epistemic modals found in [MacFarlane, 2008, 2014], [Yalcin, 2007] and [Santorio, 2022], this is entirely compatible with rejecting the idea that context change potentials are meanings. All of these theories make use of a notion of update in their truth-conditional theories about the meanings of conditionals and epistemic modals and all such theories could be (and in the case of Santorio are) equipped with a notion of conversational update that is non-compositional in our sense. Second, it has been argued that an essential feature of dynamic theories of discourse dynamics is that either one of the following principles about update fails [see Rothschild and Yalcin, 2017, 2016].

Commutativity $c[\phi][\psi] = c[\psi][\phi]$,

Idempotence $c[\phi][\phi] = c[\phi]$.

Again, this is entirely compatible with rejecting the idea that meanings are context change potentials. Our notion of informational updating, is non-compositional but violates Commutativity. So, we can hold on to dynamicness at the discourse level while rejecting dynamicness at the level of meaning.

Chapter 3

Asserting Conjunctions, Asserting Conjuncts

3.1 Introduction

Consider the following discourse

    D1: **Alice**: (1) It isn't raining.
              Alice: (2) It might be raining.

Assuming that Alice's assertion of (1) is felicitous and none of the discourse participants objected to her assertion, Alice's second assertion of (2) is odd.[1] A plausible story about why this is so, is a story that is fairly standard in the literature about the discourse dynamics of expression such as 'might'. First, we provide an account about the discourse effects of negated sentences that ensures that Alice's assertion of (1) makes it the case that rain is no longer compatible with the information shared between the participants of the discourse. According to one such view [going back to Heim, 1992], asserting a negated sentence, $\neg\phi$, is like asserting something which is the complement of the discourse effect of its prejacent ϕ. Let us call such an account of the discourse dynamics for 'not' *negation as complementation*.

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[1]Alternatively, consider

    D1′: **Alice**: (1) It isn't raining.
            Bob: (2) It might be raining.

Here Bob's assertion will sound like a disagreement with what Alice says.

Second we provide an account of 'might' claims that predicts them to be assertable only if their prejacent is compatible with what is common ground between the participants of the discourse. Given such a view, Alice's second assertion is bound to be unassertable in light of the discourse effects of (1).

Now, consider

D2: Bob: (3) It isn't raining and it might be raining.

Asserting this sentence is as odd as Alice's assertion of (2) in the first discourse. Indeed, the consensus in the recent literature is that, whatever your account of the discourse effects of conjunction, negation and the epistemic modal 'might', it had better predict that the sentence is unassertable at every context [see Groenendijk et al., 1996, Yalcin, 2007, Gillies, 2018, Mandelkern, 2019, among many others].

One popular explanation of what is going on is this: according to a view about conjunction (first discussed by Stalnaker [1974] and formally implemented by Heim [1992]) the discourse effects of assertions of conjunctions reduce to the discourse effects of successive assertions of their conjuncts. Let us refer to this view about conjunction as the *sequential view*. If the sequential view is right, the badness of Bob's assertion of (3) simply reduces to the badness of Alice's assertion of (2) in the first discourse: the first conjunct in (3) establishes what Alice's assertion of (1) establishes where the second conjunct corresponds to Alice's assertion of (2) [see Groenendijk et al., 1996].

This is an elegant explanation of what is going on.[2] Hence, observations like this seem to provide an argument in favor of negation as complementation, the just mentioned view about 'might' and, in particular, the sequential view of conjunction.

The sequential view is especially popular among defenders of so called dynamic approaches to meaning — views according to which meanings are context change potentials — hence the sequential view is sometimes referred to as *dynamic conjunction*.[3] While I will discuss dynamic approaches to meaning later on, I wish to avoid the name 'dynamic conjunction' in order to highlight the main focus of this paper. The focus is on the discourse effects of

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[2]The explanation is not only elegant but it has been observed in many places [see Yalcin, 2007, for instance] that such predictions are hard to get with a classic Kratzerian approach to natural language modals [as developed in Kratzer, 1977, 1981, 1991]

[3]Dynamic theories of meaning were pioneered by Heim [1982] and Kamp [1984] and later developed further by Veltman [1996], Gillies [2004], Willer [2017] and many others.

conjunctions, regardless of whether meanings are context change potentials or not. The focus is on the fact that the sequential view of conjunction makes a strong claim: asserting a conjunction is *always* like a successive assertion of its conjuncts.

In this paper I wish to defend the claim that, contra the argument just presented, if we want to hold on to an adequate account of the discourse dynamics of information sensitive expressions such as 'might', 'probably', 'presumably' and others, there are reasons to *reject*, not to *accept* the sequential view. The above explanation of what is going on in D2 may be elegant, but this elegance is paid for with a range of consequences that are not.

Here is an overview of my paper: After reviewing some background (Section 3.2) I will start by discussing some characteristic features of information sensitive language (in Section 3.3). I then turn to the main argument. It proceeds in three steps. I first review some well known challenges dynamic approaches to 'might' face and argue that they generalize to all accounts of information sensitive expressions that are paired with the above mentioned accounts of negation and conjunction (Section 3.4). Hence, if, like me, you think that some expressions are information sensitive, it is not 'might' but our accounts of negation and conjunction that are to blame. Second (Section 3.5) I argue that conjunction alone has consequences that are objectionable and that giving up on sequential conjunction will resolve many of the problems discussed in Section 3.4. Hence, the sequential view is the main culprit. Finally, I discuss a response to the challenges that would enable us to hold on to the accounts of conjunction and negation and argue that it is unsatisfying. I close the discussion (in Section 3.6) by exploring possible routes we could take if we want to hold on to the claim that some expressions are information sensitive but reject sequential conjunction. I conclude with some final remarks in Section 3.7.

3.2   Background

Let us build up some background. Section 3.2.1 introduces some general formal structures typically used to provide a theory of discourse dynamics. In Section 3.2.2 we will provide a more rigorous version of the argument in favor of sequential conjunction sketched in the introduction and introduce what we may call the orthodox approach to the discourse dynamics of 'might',

'and' and 'not'.

3.2.1 Conversations and Information Structure

When studying the discourse effects of a fragment of language, $\mathcal{L}$, we typically rely on a formal structure we may call a model of conversation.[4]

Definition 3.2.1 (Model of Conversation). *A model of conversation for a formal language $\mathcal{L}$ is a pair $\langle C, \cdot[\cdot] \rangle$, where C is some set, the set of contexts, and $\cdot[\cdot]$ an update function, mapping contexts $c \in C$ and sentences $\phi \in \mathcal{L}$ to contexts $c[\phi] \in C$.*

(This is Def. 1.3.1 from Chapter 1. To keep each chapter self-contained, it is repeated here.) The members of C, the set of contexts, may be all kinds of formal objects. The choice of which particular notion of context we should work with will depend on what kind of fragments of language we are looking at and what kind of discourse effects we are interested in. There may be certain empirical constraints on what can and what cannot count as an appropriate model of a context, but these need not concern us here.

One popular and particularly simple notion of a context is found in [Stalnaker, 1999]. According to this notion, a context is just a set of possible worlds: *ways the world might be in light of the information shared between the participants of the discourse.*

To illustrate, let us consider a toy model. Suppose, as is fairly standard in the literature, that the discourse effects of simple non-modal sentences (sentences that could be reasonably modelled as an atomic sentence of some propositional language $\mathcal{L}$) are treated as follows

$$c[p] = \{w \in c \mid w(p) = 1\}. \qquad (p)$$

So, asserting a simple non-modal sentence results in the removal of those worlds at which the sentence is false. Here is a way of depicting the various discourse effects of such a view. Let p be an atomic sentence and a and b worlds such that p is true at a but false at b. Then, the entry just given, would give rise to a model of conversation as depicted in Figure 3.1.[5]

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[4]Compare the notion of an *abstract frame* in Bonnay and Westerståhl [2014] and the slightly more general notion of a conversational model in Rothschild and Yalcin [2016, 2017].

[5]Formally, for the simple language $\mathcal{L} = \{p\}$, the model is $\langle \mathcal{P}(\{a, b\}), \cdot[\cdot] \rangle$, where $\mathcal{P}(S)$ denotes the power set of a set S.
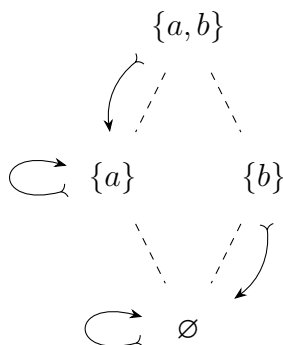
Figure 3.1: A simple model

The nodes of the graph in Figure 3.1 are arranged as the nodes of a Hasse diagram of the subset relation over $\mathcal{P}(\{a,b\})$. So a move downward in the diagram amounts to the elimination of worlds from the context and hence to the addition of information: fewer ways the world might be correspond to more information about it. Notice also that, at the very bottom, we have $\varnothing$. It is the most informative context in a defective sense. The information contained in it rules out *any* way the world might be in light of the participants of a discourse. Notice, also that this explains why an assertion of p at $\{b\}$ is bad. Accepting an assertion like it would result in a defective context that is incompatible with any way the world might be. Assuming that discourse participants want to avoid ending up at a defective context, such assertions would either trigger some Gricean implicature or would not be accepted.

Again, contexts may not be sets of possible worlds. But the above provides a good example to illustrate a minimal set of assumptions that will be needed to model the discourses we are interested in. While more assumptions may be added later on, here are three assumptions we will make throughout this chapter:

Information Order: C is partially ordered by some relation $\geq$.

Eliminativity: $c \geq c[\phi]$ for any $c \in C$ and $\phi \in \mathcal{L}$.

Defective Context: There is a context $\bot$ which is a minimal element in C with respect to $\geq$ so (in light of Eliminativity) it satisfies $\bot[\phi] = \bot$

for any $\phi \in \mathcal{L}$.

Think of $\geq$ as an order generalizing the subset relation $\supseteq$ on contexts in possible worlds models of conversation.[6] In analogy to what we said about the subset relation above, we may think of $c \geq c'$ as telling us that c' contains more information than c. Similarly, we can think of $\bot$ as a generalization of $\varnothing$, the defective context of a possible worlds model of conversation. The assumption that there is such a context will help us to think about the discourse effect of contradictions or assertions that sound false at a particular stage of a conversation. Finally, Eliminativity corresponds to the constraint that no assertion will have the affect of causing information loss. It is a property that holds for all models of conversation I am aware of and all the concrete proposals discussed below.[7] Whether the principle is plausible for all fragments of language is a question that has to be discussed elsewhere. For the fragments of language I will be considering, the principle is plausible.

3.2.2 Sequential Conjunction and Negation as Complementation

Now that we have a formalism to work with, let us give a more detailed presentation of the argument in favor of sequential conjunction presented above. Hence let us consider a model of conversation for the language

$$\alpha ::= p \mid \neg\alpha \mid (\alpha \wedge \alpha) \mid \Diamond\alpha, \qquad\qquad (\mathcal{L}_{\Diamond})$$

where p is supplied by a finite set of atomic sentences, $\mathtt{At}$ and $\Diamond$, $\wedge$ and $\neg$ model the epistemic possibility modal 'might', conjunction and negation respectively.

With the help of $\mathcal{L}_{\Diamond}$, let us consider the discourse from the introduction (repeated):

D1: **Alice**: (1) It isn't raining.
       Alice: (2) It might be raining.

In terms of a model of conversation for $\mathcal{L}_{\Diamond}$, this discourse is reasonably

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[6]See [Veltman, 1996] for an example of someone considering very general information structures in terms of such an ordering.

[7]See [Rothschild and Yalcin, 2016, 2017] for discussions on this constraint.

modelled as

$$c[\neg r][\Diamond r]. \tag{1.1}$$

In light of the fact that we judge a discourse like D1 to be defective, a sensible prediction would be that for any context c,

$$c[\neg r][\Diamond r] = \bot. \tag{1.2}$$

That is, asserting $\Diamond r$ after $\neg r$ has been asserted is like asserting something that sounds contradictory or false at such a context. This is predicted on possible worlds models of conversation which accept the entry for atomic sentences, (p), from above as well as

$(\neg_h)$ $c[\neg \phi] = c - c[\phi]$,

$(\Diamond)$ $c[\Diamond \phi] = \{w \in c \mid c[\phi] \neq \varnothing\}$.

Now, according to the sequential view of conjunction, asserting a conjunction is like a successive assertion of the conjunctions, i.e for any $c \in C$ and $\phi, \psi \in \mathcal{L}_O$:

$$c[\phi \wedge \psi] = c[\phi][\psi]. \qquad \text{(Sequential Conjunctions)}$$

So for Bob's assertion of (3), (1.2) and Sequential Conjunction straightforwardly leads to

$$c[\neg r \wedge \Diamond r] = c[\neg r][\Diamond r] = \bot.$$

So any account that predicts the discourse D1 to be bad in the sense of (1.2), will predict any assertion of (3) to be bad, if the sequential view is adopted. A welcome result.

In what follows, we will refer to any possible worlds model that adopts the entries for the sentences in $\mathcal{L}_\Diamond$ discussed so far as an *orthodox model*. To have everything in one place, let us record this as a definition:

Definition 3.2.2 (Orthodox dynamics for $\mathcal{L}_\Diamond$). *Let $\langle \mathcal{P}(W), \cdot[\cdot] \rangle$ be a model of conversation for $\mathcal{L}_\Diamond$ such that for all $p \in \mathtt{At}$ and $\phi, \psi \in \mathcal{L}_\Diamond$,*

(p) $\{w \in c \mid w(p) = 1\}$,

$(\neg)$ $c[\neg \phi] = c - c[\phi]$,

($\wedge$) $c[\phi \wedge \psi] = c[\phi][\psi]$,

($\diamondsuit$) $c[\diamondsuit \phi] = \{w \in c \mid c[\phi] \neq \varnothing\}$.

What we have seen so far speaks in favor of sequential conjunction in the context of an orthodox model for 'might'.

3.3   Information Sensitivity

Now, 'might' is one member of a larger class of expressions that are sensitive to information that is available at a particular stage of a conversation. Such expressions are typically called *information sensitive*. In this section we will provide some general characteristics of such expressions and arguments in favor of their existence.

3.3.1   Characterizing Information Sensitivity

While for some expressions, membership in the class of information sensitive expressions is controversial, here is a (non-exhaustive) list of expressions which have been argued to be information sensitive in the sense I am interested in.

might	Groenendijk et al. [1996] and many others
must	Groenendijk et al. [1996] and many others
able to	Willer [2021]
probably	Yalcin [2012b], and others
presumably	Veltman [1996]
if (indicative)	Gillies [2004], and many others
if (counterfactual)	Gillies [2007]
ought	Kolodny and MacFarlane [2010]
therefore/hence	Kocurek and Pavese [2022]
suppose	Veltman [1996]

What makes an expression information sensitive? To get a feeling for what the phenomenon is about, let us start by looking at some abstract properties of an orthodox model.

Given an orthodox model of conversation for $\mathcal{L}_\diamondsuit$, it is not difficult to verify that

$$c[\phi] = c \cap W[\phi] \qquad \text{(Intersectivity)}$$

holds, whenever ϕ is a non modal sentence. Since $W[\phi]$ is a proposition (a set of worlds), Intersectivity amounts to the claim that such sentences always add the same proposition to the common ground no matter the context at which the sentence is asserted.

However, Intersectivity fails for some modal sentences. This is easily seen by realizing that for any non-contradictory non-modal sentence ϕ, $W[\Diamond\phi] = W$. Whenever ϕ is in conflict with the information shared at c, however, (i.e. whenever $c[\phi] = \varnothing$), we have $c[\Diamond\phi] = \varnothing$. But such contexts may be non-empty and hence $c[\Diamond\phi] \neq c \cap W[\phi]$.

To sum up, on the just mentioned approach, updates with non-modal sentences can and updates with modal sentences sometimes cannot be spelled out in terms of some notion of 'proposition adding'. In what follows, I will take failures of Intersectivity as a hallmark property of information sensitive expressions. In general, I will say that

>**Information Sensitivity**: A model of conversation for $\mathcal{L}$ has room for information sensitivity, if there are sentences $\phi \in \mathcal{L}$ whose context change potential cannot be spelled out in terms of some notion of proposition adding.

Moreover, let us call an *expression* information sensitive, if it requires a model of conversation that has room for information sensitivity in the above sense.[8] Let me highlight that, in the following, I will use the term 'proposition' loosely as sets of 'points in logical space'. Such 'points' may require a more complex formal representation than simple possible worlds. This makes the above definition quite weak. All that matters for a model of conversation to make room for information sensitivity is that it cannot be spelled out in terms of a notion of proposition adding for *some* notion of a proposition.

What about systems on which contexts are not sets of possible worlds? One key observation is due to Rothschild and Yalcin [2016, 2017].[9]

Proposition 3.3.1 (Rothschild and Yalcin)**.** *A model of conversation* $\langle C, \cdot[\cdot] \rangle$ *for a language* $\mathcal{L}$ *has an update function that satisfies*

>***Idempotence:*** $c[\phi][\phi] = c[\phi]$,

---

[8]See Neth [2019] and Kolodny and MacFarlane [2010] for slightly different but related characterizations of information sensitivity

[9]For the sake of readability of general principles such as Idempotence and Commutativity, universal quantification over contexts and sentences is sometimes left implicit.

Commutativity: $c[\phi][\psi] = c[\psi][\phi]$,

if and only if it is isomorphic to a model of conversation $\langle C', \cdot[\cdot]' \rangle$ such that there is an $s \in C'$ with

$$c[\phi]' = c \cap s[\phi]'$$

for all $c \in C'$ and $\phi \in \mathcal{L}$.[10]

Note that the result does not presuppose that C is a set of sets of possible worlds and while isomorphic models do not need to be identical, Rothschild and Yalcin's result shows that any model of conversation that satisfies Idempotence and Commutativity could, in principle, be spelled out in terms of some notion of proposition adding.[11] Hence, models of conversation that make room for information sensitivity, in the above sense, will violate at least one of these properties. In light of this, we may take failures of either Idempotence or Commutativity as indicators of the phenomenon.

A second property is worth mentioning. Bonnay and Westerståhl [2014] consider

> **Discourse Persistence**: $c[\phi_1] \ldots [\phi_n][\psi] = c[\phi_1] \ldots [\phi_n]$,
> whenever $\psi = \phi_i$ for some $1 \leq i \leq n$.

Discourse Persistence says that whenever a sentence ϕ has been asserted at some point of the conversation, a second assertion, later on, will not change the context of conversation further.

It is not difficult to see that Idempotence and Commutativity entail Discourse Persistence while the converse, as Bonnay and Westerståhl [2014] observe, is not true. But since failures of Discourse Persistence entail either a failure of Idempotence or Commutativity, we can take failures of Discourse Persistence to indicate information sensitivity as well.

All the approaches mentioned in the above table come equipped with a notion of update that violates Idempotence, Commutativity or Discourse Persistence. Hence, all of them count as making room for information sensitivity in the sense I am interested in.

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[10]We say that two models of conversation $\langle C, \cdot[\cdot] \rangle$ and $\langle C', \cdot[\cdot]' \rangle$ for $\mathcal{L}$ are *isomorphic* iff there is a bijections f from C to C' such that $f([c[\phi]) = f(c)[\phi]'$ for all $\phi \in \mathcal{L}$ and all $c \in C$.

[11]The 'points' in C' could be seen as but may not necessarily be possible worlds. But as noted above, I am using the notion of a proposition in a 'loose' sense: a sense in which its elements may but do not have to be possible worlds.

3.3.2 There are Information Sensitive Expressions

From what has been said above, it is clear that our orthodox approach to the discourse dynamics of $\mathcal{L}_\diamond$ *predicts* that 'might' is information sensitive. Moreover, the approaches listed above predict the same for all the other expressions mentioned in the left column of the table. But since the existence of information sensitive expressions is crucial for the arguments to follow, let me argue that these predictions are warranted *independently* of whether you agree with the concrete models of conversation mentioned above.

In light of the above discussion, let us look at the following three discourses (D1 is repeated from above [see also Veltman, 1996, p. 223, for a discussion of versions of D2 and D3]):

> **D1** **Alice**: (1) It is not raining.
> **Alice**: (2) It might be raining
>
> **D2** **Alice**: (2) It might be raining.
> **Bob**: (1) It is not raining.
>
> **D3** **Alice**: (1) It might be raining.
> **Alice**: (2) It is not raining.
> **Bob**: (1) It might be raining.

As already argued above, D1 seems to be a defective discourse. Alice's assertion of (2) seems like the assertion of an obvious falsehood at a context at which what she said before is common ground between the participants of the conversation. Accordingly, any reasonable model of conversation should predict that $c[\neg r][\diamond r] = \bot$ for any context c. Note however, that D2 is fine (at least in some instances). At some stage of the conversation Alice may suspect that it is raining and assert (2) in order to give voice to that possibility. Bob, leaving Alice's assertion unchallenged, may receive additional information later on and share it with Alice by asserting (1).

Let me highlight that there may be readings of D2 where Bob's assertion of (1) in D2 is plausibly taken to give voice to a disagreement with Ann. That reading becomes salient if we imagine Bob asserting (1) with a focus on 'not'. While sometimes an assertion of (1) may indicate disagreement with a previous assertion of (2), D2 does not *have* to be read that way.

It is not easy to spell out how disagreements about epistemic modal claims work, and there is a rich literature on the topic. In what follows, I will

remain neutral on what is going in such cases.[12] All that matters for the arguments below is that there are discourses in which Bob's assertion of (1) when asserted some time after (2) does not crash the context.[13] In light of this, I maintain that any model of conversation for $\mathcal{L}_\Diamond$ should make room for there being contexts c such that

$$c[\Diamond r][\neg r] \neq \bot.$$

But if this is right, D1 and D2 illustrate that updates do not commute. Hence, any such model will have to make room for information sensitivity in the above sense.

Let us look at D3. Note that D3 starts like D2. By the argument just given, there should be a context c such that $c[\Diamond r][\neg r] \neq \bot$. So all is well after Alice's second assertion in D3. However, Bob's assertion of (1) should crash whatever context we are in, since, by the above argument, we are in a context at which $\neg r$ is already established. Hence the prediction we should expect is

$$c[\Diamond r][\neg r][\Diamond r] = \bot.$$

But since $c[\Diamond r][\neg r] \neq \bot$, we have a counterexample to Discourse Persistence. Again, we have a reason to believe that an empirically adequate model for $\mathcal{L}_\Diamond$ should make room for information sensitivity.

Veltman considers discourse like D2 and D3 [Veltman, 1996, p. 223] as counterexamples to a related principle

$$c \geq c' \ \& \ c[\phi] = c, \ \text{then } c'[\phi] = c'. \qquad \text{(Persistence)}$$

Note that Persistence and Discourse Persistence are not, in general, equivalent.[14] While on all systems I am aware of, the above discourses are also counterexamples to Persistence, and failures of that principle are certainly interesting, what matters in the following is that they are counterexamples to Discourse Persistence.

Finally, note that the above judgments about the discourses remain the

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[12]Both [MacFarlane, 2008] and [Willer, 2013] contain a helpful discussions of this issue.

[13]Note also, that, even if Bob's assertion is read as a disagreement with Ann's assertion of (1), this does not entail that our model of conversation *should* predicts $c[\Diamond r][\neg r] = \bot$.

[14]They are equivalent on systems that are idempotent and satisfy

$$c \geq c' \ \text{iff} \ \exists \psi_n, \ldots \psi_n \in \mathcal{L}; c' = c[\psi_1] \ldots c[\psi_n]. \qquad \text{(Path Connectedness)}$$

same, if we replace (2) in the above discourses with either of the following:

(4) It is probably raining,

(5) Presumably, it is raining.

Hence, any account of a fragment like $\mathcal{L}_\diamond$ in which $\diamond$ is replaced by a suitable operator for 'probably' or 'presumably' will have to make room for information sensitivity as well. Indeed the model for 'presumably' provided in Veltman [1996] and the model for 'probably' provided in Yalcin [2012b], treat the discourses D1, D2 and D3 in exactly the way we just discussed. So all of these models confirm our intuitions about these cases.[15]

Let me highlight that, throughout this paper I wish to remain neutral on the question of whether any of the other entries in our table from the previous section provide additional evidence for the existence of information sensitive expressions. All that matters is that the just mentioned predictions about D1, D2 and D3 are adequate and that there are at least three cases of expressions that give rise to information sensitivity by violating Commutativity and Discourse Persistence for discourses analogous to D1, D2 and D3.

3.4 Challenges

In this section we review some well known challenges our orthodox dynamic approach to 'might' faces and argue that these challenges generalize to any model of conversation that has room for information sensitivity, if the above entries for negation and conjunction are assumed.

Note that the notion of negation as complementation only makes sense on models of conversation on which there is some notion of complementation with respect to the set of contexts. Indeed, in the following we will assume

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I will argue below that we should aim for models of conversation that are idempotent (but may violate Discourse Persistence and Commutativity). And since we take our systems to be eliminative and we typically ignore discourses that require an infinite number of assertions, Path Connectedness seems a reasonable assumption. Since the principles are equivalent under these assumptions, I take it that the name *Discourse Persistence* is adequate.

[15]A range of accounts which make use of slightly different notions of context or treat the information sensitive expressions under consideration in a slightly different way confirm these predictions as well [see Gillies, 2018, Willer, 2013, for instance].

that we are working in a model of conversation which is equipped with a notion of complementation '−' that satisfies

$$\text{for all } c \in C; \ c - c[\phi] = \bot \Rightarrow c = c[\phi]. \qquad \text{(Complementation)}$$

While this is satisfied in all possible worlds models of conversation where '−' is interpreted as set theoretic complementation, the requirement is compatible with a much wider class of possible notions of contexts.[16]

3.4.1   Challenges from Idempotence Failures

The first challenge is that standard approaches to the discourse dynamics of some information sensitive expressions predict that not all sentences of the form $\phi \wedge \neg \phi$ crash the context of a conversation to the defective context.

For instance, if we let $\phi = \Diamond p \wedge \neg p$, then our above system predicts that for some contexts c,

$$c[\phi \wedge \neg \phi] \neq \varnothing.$$

Mandelkern [2020] observes that on our orthodox approach to $\mathcal{L}_\Diamond$ this is the case for all sentences $\phi \in \mathcal{L}_\Diamond$ that violate Idempotence.

Indeed, this is the case for all models of conversation on which we accept the above notions of conjunction and negation. To see this, let us assume

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[16]The principle is satisfied whenever $\langle C, \geq \rangle$ is an orthomodular lattice. The reason is that an orthomodular lattice is a complete lattice that satisfies

$$\forall c, c' \in C, \ \text{if } c \geq c', c \wedge (c')^* = \bot \text{ then } c = c',$$

where $\wedge$ is the join operation of the lattice $\langle C, \geq \rangle$ and $(c)^*$ the complement of c in C. Since, $c - c'$ is an abbreviation for $c \wedge (c')^*$ and since we assume Eliminativity, we have $c \geq c[\phi]$ and thus, by orthomodularity,

$$c - c[\phi] = \bot \Rightarrow c = c[\phi],$$

for all $c \in C$. Let me mention that orthomodularity is rejected on recent algebraic semantics for the epistemic modal 'might' [see Holliday and Mandelkern, forthcoming]. But note that on such a proposal the goal is not to provide a theory of discourse dynamics for 'might'. As far as I can see, rejecting orthomodularity on the level of semantics is compatible with accepting it on the level of discourse dynamics. Moreover, we will see later on that, while the challenges discussed here depend on the principle, giving up on negation as complementation will not enable us to hold on to sequential conjunction.

that ϕ is not idempotent. Hence we can find a context $c \in C$ such that

$$c[\phi][\phi] \neq c[\phi].$$

By complementation, we have

$$c[\phi] - c[\phi][\phi] \neq \bot,$$

but by the above notions of negation and conjunction we have

$$c[\phi \wedge \neg\phi] = c[\phi][\neg\phi] = c[\phi] - c[\phi][\phi].$$

So, $c[\phi \wedge \neg\phi] \neq \bot$ and hence there must be a context c at which an assertion of $\phi \wedge \neg\phi$ does not crash the context to $\bot$.

This is an odd consequence, since any sentence of the form $\phi \wedge \neg\phi$ seem to look and feel like a contradiction. Accordingly we would expect it to crash any context to the defective context. If our model predicts failures of Idempotence, this expectation cannot be met whenever sequential conjunction and our notion of negation as complementation is assumed.

It is worth pointing out at this stage that it is hard to find intuitive counterexamples to Idempotence. And while the discourses considered in Section 3.3.2 seem to be intuitive counterexamples to commutativity and Discourse Persistence, the claim that our model of conversation makes room for information sensitivity is compatible with the claim that its notion of update satisfies Idempotence.[17] So one may hope to save sequential conjunction from the challenge by trying to find a model of conversation that is idempotent but still captures the discourses considered in Section 3.3.2.

Since Idempotence is a property that is of interest in its own right, we will come back to this later on. We will see that the hope of meeting the just mentioned challenge by adopting a notion of update that is idempotent is in vain, if sequential conjunction is accepted. But, for now, let us bracket this discussion and observe that any view that makes room for information sensitivity by violating Idempotence, accepts negation as complementation and sequential conjunction, faces the challenge of predicting that some contradictory looking sentences do not crash every context of conversation.

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[17]This will become clear by considering the systems discussed in Section 3.5.2 and Section 3.6.2. The models of conversation for $\mathcal{L}$ discussed there satisfy Idempotence but violate both Commutativity and Weak Persistence.

3.4.2 Challenges From Commutativity Failures

The second challenge is straightforward and concerns Heim's notion of conjunction only. Above we considered (repeated),

(3) It is not raining and it might be raining,

and judged this sentence to be unassertable at any context. As seen above, this is predicted on an orthodox approach to the discourse dynamics of $\mathcal{L}_\diamond$ since $c[\neg r \wedge \diamond r]$ reduces $c[\neg r][\diamond r]$, a discourse that is predicted to be defective.

However, in Section 3.3.2 we have argued that updates do not commute, since

$$c[\diamond r][\neg r] \neq \bot,$$

for some contexts c.

But if this is right, sequential conjunction entails that at the same context

$$c[\diamond r \wedge \neg r] \neq \bot.$$

This, it seems to me, is an unwelcome prediction. Most agree that

(6) It might be raining and it is not raining.

is as unassertable as (3) at any context (as we will see, even defenders of the orthodox approach agree on this).[18] Notice that this prediction would carry over to any account that violates commutativity of updates in the same way as D1 and D2 from Section 3.3.2 do. In particular, we get that

(7) It is probably raining and it is not raining.

(8) Presumably it is raining, and it is not raining.

do not crash any context to the defective context $\bot$, if sequential conjunction is assumed.

We will consider possible responses to this below. For now let us record that this is a challenge to anyone who wants to make room for information sensitivity by accepting commutativity failures such as those observed in D1 and D2 but wants to hold on to our notions of negation as complementation

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[18]That sentences like (3) and (6) should be treated exactly the same way is most forcefully argued for in [Mandelkern, 2019]. More on that in Section 3.6.1.

and sequential conjunction. Note that failures of Commutativity are compatible with holding on to Idempotence. Hence the challenge is independent of whether our notion of update is idempotent or not.

3.4.3 Challenges from Discourse Persistence Failures

A third challenge stems from 'long' contradictions, by which I mean sentences of the form

$$\psi_1 \wedge \cdots \wedge \psi_n \wedge \neg\phi$$

such that $\psi_i = \phi$ for some $1 \leq i \leq n - 1$.[19] For instance, it is not difficult to see that on our orthodox approach to the discourse dynamics of $\mathcal{L}_\Diamond$

$$c\big[\Diamond r \wedge \neg r \wedge \neg \Diamond r\big] \neq \varnothing$$

for some non-defective contexts c. Again, this is an odd prediction. We seem to have a conjunction that contains conjuncts that are negations of each other: a sentence that looks and feels like a contradiction. Hence, we would expect an assertion of such a sentence to crash the context to the defective context at any possible stage of a conversation.

But note that such a prediction is not possible on any model of conversation that violates Discourse Persistence but accepts both sequential conjunction and negation as complementation.

Here is why. Suppose that $\psi_1 \wedge \cdots \wedge \psi_n \wedge \neg\phi$ is such that $\psi_i = \phi$ for some $1 \leq i \leq n - 1$. Let c be an arbitrary context, we then have

$$c\big[\psi_1 \wedge \cdots \wedge \psi_n \wedge \neg\phi\big] = \bot, \tag{3.1}$$

$$\Leftrightarrow c[\psi_1]\ldots[\psi_n][\neg\phi] = \bot, \tag{3.2}$$

$$\Leftrightarrow c[\psi_1]\ldots[\psi_n] - c[\psi_1]\ldots[\psi_n][\phi] = \bot, \tag{3.3}$$

$$\Leftrightarrow c[\psi_1]\ldots[\psi_n] = c[\psi_1]\ldots[\psi_n][\phi]. \tag{3.4}$$

(3.1) holds by assumption. (3.2) holds by Sequential Conjunction. (3.3) holds by Negation as Complementation and, finally, (3.4) by Complementation.

---

[19]It is worth pointing out that all notions of conjunction considered in this paper are associative in the sense that $\cdot[(\phi \wedge (\psi \wedge \chi))] = \cdot[((\phi \wedge \psi) \wedge \chi)]$. Hence, we omit parentheses. In particular, sequential conjunction is associative since, on this view, conjunctions reduce to a composite function of the updates associated with its conjuncts. Since function composition is always associative, so is the respective notion of conjunction.

But since c was arbitrarily chosen, the last line is just Discourse Persistence. Hence, failures of Discourse Persistence entail that some assertions of long contradictions are predicted to not crash a context to the defective context.[20]

Notice that in Section 3.3.2 we have seen concrete evidence that 'might', 'presumably' and 'probably' violate Discourse Persistence. Hence, no account that makes room for information sensitivity by allowing Discourse Persistence to fail can adopt the above notions of conjunction and negation without accepting that some long contradictions fail to crash the context of a conversation. Note also that, since failures of Discourse Persistence are compatible with holding on to Idempotence, this challenge remains even if we find an Independent notion of update. So, an idempotent notion of update may help to circumvent the challenge from Section 3.4.1 but we would still have to deal with predicting the assertability of long contradictions.

3.5 Meeting the Challenges?

We have seen that there are several challenges to accounts that both make room for information sensitivity and accept sequential conjunction as well as negation as complementation. So, if, like me, you think that some models of conversation should predict failures of Commutativity and Discourse Persistence, you have a reason to reject either a standard view about the discourse dynamics of negation or a popular view about the discourse dynamics of conjunction. Which one should go? In this section I argue that the main culprit is our notion of sequential conjunction.

3.5.1 Giving Up Negation as Complementation Does Not Suffice

Idempotence violations on models such as the orthodox view will give rise to the assertability of sentences of the form $\phi \wedge \neg\phi$. But we also pointed out that, while our orthodox approach to $\mathcal{L}_\diamond$ *predicts* failures of Idempotence, it is hard to find intuitively plausible cases.

---

[20]Bonnay and Westerståhl [2014] prove that a popular notion of dynamic consequence is classical just in case Discourse Persistence is satisfied. The above shows that Discourse Persistence, together with Heim's notions of conjunction and negation, is also at the heart of another notion of classicality: the idea that a classic treatment of conjunctions that contain conjuncts that are negations of each other are treated the same as contradictions.

Of course, we sometimes do have reasons to make an assertion several times in a row. For instance, a pilot may have reason to shout 'Mayday', 'Mayday', 'Mayday'. However, 'Mayday' is not asserted three times in a row because the pilot aims to convey something different the second and the third time. The pilot asserts it several times, because she wants her message to be received, even if the communications channel is defective. But models of conversation abstract away from the quality of our communications channel. Hence, no such example supports failures of Idempotence.

Moreover, even if there are fragments of language in which there are counter examples to Idempotence, I maintain that the fragments of language we are interested in do not contain any. So absent intuitive counterexamples to the property in the fragments of language we consider here, it seems to me that our default attitude to Idempotence should be to accept it and failures of the principle should be met with suspicion.

Now, it is not difficult to see that anyone who agrees with me on the judgments regarding discourses D1 and D2, cannot hold on to sequential conjunction without accepting failures of Idempotence. Remember, our intuitions about D2 supported that

$$c[\Diamond r][\neg r] \neq \bot$$

for some context c. Hence, by sequential conjunction,

$$c[\Diamond r \wedge \neg r] \neq \bot$$

for some context c. But our intuitions about D1 supported

$$c[\neg r][\Diamond r] = \bot$$

for any context c. So, we have

$$
\begin{aligned}
& c[\Diamond r][\neg r][\Diamond r] = \bot, && \text{by the last observation} \\
\Leftrightarrow\ & c[\Diamond r][\neg r][\Diamond r][\neg r] = \bot, && \text{by Eliminativity} \\
\Leftrightarrow\ & c[\Diamond r \wedge \neg r][\Diamond r \wedge \neg r] = \bot. && \text{by Sequential Conjunction}
\end{aligned}
$$

Hence,

$$c[\Diamond r \wedge \neg r][\Diamond r \wedge \neg r] \neq c[\Diamond r \wedge \neg r]$$

for some context c. So, accepting sequential conjunction is incompatible

with finding an idempotent notion of update that supports our intuitions about D1 and D2. So, sequential conjunction does not only give rise to some odd predictions about short contradictions of the form '$\phi \land \neg\phi$ but prevents us from adopting an idempotent model in the first place. Moreover, since, for the fragments we are interested in, Idempotence seems to be plausible, we seem to have a reason to reject sequential conjunction independently of whether we do or do not accept negation as complementation.

Second, note that in Section 3.4.3 we used negation as complementation to show that predicting long contradictions to crash any context, entails Discourse Persistence, if our above notions of conjunction and negation are assumed. While this should be relevant to anyone who accepts negation as complementation as well as failures of Discourse Persistence, we get our unwelcome predictions without assuming much about the discourse effects of negation. To see this, consider the following discourse

D4 **Alice**:   (1) It might be raining.
        Bob: (2) It is not raining.
        Clem: (9) It is not the case that it might be raining.

Note that Clem's assertion is odd or funny sounding not because it sounds false. It is funny sounding because asserting (9) at this stage of the conversation seems like stating the obvious. So, what we should expect is that an assertion of (9) after (2) has been asserted, does not do anything to change the context further:

$$c[\Diamond r][\neg r][\neg \Diamond r] = c[\Diamond r][\neg r].$$

Indeed, this is a prediction that is confirmed by the orthodox view of 'might' and similar predictions hold on the accounts for 'probably' and 'presumably' cited above. So, judging D2 to be a non-defective discourse, D4 should not be interpreted as a discourse in which Clem's assertion crashes the context of the conversation either. But accepting this and sequential conjunction entails accepting that

$$c[\Diamond r \land \neg r \land \neg \Diamond r] \neq \bot$$

for some non-defective context c. Hence, if you found the argument in Section 3.4.3 worrisome, the key assumption that causes the worry is Sequential Conjunction.

3.5.2 Giving Up on Sequential Conjunction Does (Almost) Suffice

While the challenges discussed in Section 3.4 of this paper rely on negation as complementation, giving up on such a view does not take us very far. As long as we accept sequential conjunction we still have to deal with unmotivated counterexamples to Idempotence and the prima facie assertability of some long contradictions.

To get a better grasp on the role sequential conjunction plays in the above challenges, let us consider the easiest way to give up on sequential conjunction. So, let us replace the entry for conjunction in our orthodox approach to the discourse dynamics for $\mathcal{L}_\diamond$ with

$$c[\phi \wedge \psi] = c[\phi] \cap c[\psi]. \qquad \text{(Intersective Conjunction)}$$

Call such an approach to the discourse dynamics of $\mathcal{L}_\diamond$ the *simple unorthodox approach*. Notice that this view would make the same predictions as the orthodox approach about the discourses D1, D2, D3 and D4 since all these discourses involve sentences that are conjunction free.

So, in particular we get

$$c[\diamond r][\neg r][\neg \diamond r] \neq \varnothing.$$

However, it is not difficult to see that the view predicts

$$c[\diamond r \wedge \neg r \wedge \neg \diamond r] = \varnothing,$$

for any context c. So, on the simple unorthodox view, sentences that look and feel like contradictions crash the context at every possible stage of a conversation.[21] Moreover, the case illustrates that, on the simple unorthodox view, asserting a conjunction is not always like a successive assertion of its conjuncts.[22]

---

[21]We can prove by induction on the complexity of ϕ that $c[\phi \wedge \neg\phi] = \varnothing$ for all $\phi \in \mathcal{L}_\diamond$. Since set intersection commutes and intersecting any set with the empty set will result in the empty set, all long and short contradictions are predicted to crash the context of conversation to the empty set.

[22]Some defenders of a dynamic approach to meaning consider Intersective Conjunction. See [Willer, 2017] for instance. So the issues discussed above are not essential to whether you do or do not choose a dynamic approach to meaning.

Finally, it is not difficult to show that our simple unorthodox model of conversation is idempotent while making room for information sensitivity since, in light of its predictions about D1, D2 and D3, it violates both Commutativity and Discourse Persistence. All of this seems to be good news. We can hold on to a view that makes room for reasonable predictions about the discourse effects of information sensitive expressions while circumventing the challenges pointed out in Sections 3.4 and 3.5.1.

There is one catch, however. The simple unorthodox approach predicts neither $\Diamond p \wedge \neg p$ nor $\neg p \wedge \Diamond p$ to be crashing the context of conversation at every possible stage of a conversation. We will say more about this in the next section. For now, let us note that the view has almost all of the features we deemed desirable above.

3.5.3 Unassertability by Coherence Constraints

Defenders of the orthodox approach have a response that may be used to rescue sequential conjunction from the challenges discussed so far. The response starts with the following definitions.

Let us call a sentence ϕ *inconsistent* if asserting ϕ crashes every context of conversation to $\bot$. Formally:

$$\phi \text{ is inconsistent iff } \forall c \in C, c[\phi] = \bot. \qquad \text{(Inconsistency)}$$

Let us call a sentence ϕ *incoherent* if $c[\phi] = c$ entails $c = \bot$ [see Groenendijk et al., 1996, for such a definition].

$$\phi \text{ is incoherent iff } c[\phi] = c \Rightarrow c = \bot. \qquad \text{(Incoherence)}$$

Now any sentence that is inconsistent is incoherent but the converse is not true. Here is a list of examples that illustrate this. All of the following are incoherent but not inconsistent:

(i) $\Diamond r \wedge \neg r$,

(ii) $(\Diamond r \wedge \neg r) \wedge \neg (\Diamond r \wedge \neg r)$,

(iii) $\Diamond r \wedge \neg r \wedge \neg \Diamond r$.

But these are all the problem cases we discussed above!

So, defenders of the orthodox approach could try to defend sequential conjunction by adopting the following constraint:

The Coherence Constraint: ϕ can only be rationally asserted if ϕ is coherent.

And indeed, the constraint is used by Groenendijk et al. [1996] to explain the unassertability of (i). The idea seems to be that adopting the Coherence Constraint may help us to explain why none of the predictions we discussed above are problematic: it may be, so the argument would go, that some long and some short contradictions as well as $\Diamond r \wedge \neg r$ do not crash every context. But, all of the problematic cases are incoherent and, hence, they could never be rationally asserted in the first place.

First, I think that this response is (at a minimum) *ad hoc*. To see this, let us go back to the simple unorthodox view, the view which is just like the orthodox view but sequential conjunction is replaced by intersective conjunction. As observed above, the view gives rise to a notion of update that is idempotent, predicts long and short contradictions to crash any context of conversation and agrees with our judgments about the discourses D1, D2, D3 and D4. The only drawback of the view was that neither $\Diamond p \wedge \neg p$ nor $\neg p \wedge \Diamond p$ are predicted to crash the context at every possible stage of a conversation.

However, observe that on that view both $\Diamond p \wedge \neg p$ and $\neg p \wedge \Diamond p$ are incoherent! To see this, let us suppose that $c[\Diamond r \wedge \neg r] = c$. By intersective conjunction, this is the case exactly if

$$c[\Diamond r] \cap c[\neg r] = c. \qquad (\star)$$

But since the account is eliminative, $(\star)$ holds if and only if both $c[\neg r] = c$ and $c[\Diamond r] = c$. But $c[\Diamond r] = c$ holds only if,

$$c = \varnothing \text{ or } c[r] \neq \varnothing.$$

However, since $c[\neg r] = c$ we have $c[r] = \varnothing$ and hence $c = \varnothing$. So $c[\Diamond r \wedge \neg r] = c$ only if $c = \varnothing$. Since on an intersective view about conjunction, conjunction commutes, the same is true for $\neg r \wedge \Diamond r$.

The point is that, if we accept the Coherence Constraint, we loose what motivated adopting a sequential view of conjunction in the first place. Why shouldn't we explain the unassertability of *both* $\neg r \wedge \Diamond r$ and $\Diamond r \wedge \neg r$ in terms of the constraint? Moreover, the explanations for why the various sentences are unassertable on the simple unorthodox view are uniform. Unlike the orthodox view, sentences which look like contradictions (conjunctions that contain conjuncts that are negations of each other) are all predicted to be

inconsistent. Sentences of the form $\neg r \wedge \Diamond r$ and $\Diamond r \wedge \neg r$ are predicted to be incoherent. This, it seems to me provides an abductive reason in favor of the simple unorthodox view.

Second, I think that there is a further reason for why the Coherence Constraint does not save sequential conjunction. For, on all views that agree with our above judgments about D4, we would get that for all contexts c,

$$c[\Diamond p \wedge \neg p \wedge \neg \Diamond p] = c[\Diamond p][\neg p].$$

So any such view predicts that the discourse effects of an assertion of $\Diamond p \wedge \neg p \wedge \neg \Diamond p$ should match the predictions about what our context would look like at the end of D2. But if our update function is supposed to capture how assertions of the sentences in $\mathcal{L}_\Diamond$ would affect the context of a conversation, this remains an odd prediction. Whatever context change an assertion of $\Diamond p \wedge \neg p \wedge \neg \Diamond p$ brings about, it is not the context we are in at the end of of D2.

Notice that a similar objection could be voiced against the simple unorthodox view. On that view, contexts that are not crashed by $\neg r \wedge \Diamond r$ are contexts where asserting such a sentence is like asserting $\neg r$. But again, whether incoherent or not, that just does not seem to be the way in which an assertion of $\neg r \wedge \Diamond r$ affects the context of a conversation. Asserting 'it is not raining but it might be raining' is not just a roundabout way to convey that it is raining.

To sum up. We have argued that, at a minimum, the orthodox as well as the simple unorthodox view are on a par, given the Coherence Constraint. Since one does and the other does not adopt the sequential view, adopting the coherence constraint to save sequential conjunction is *ad hoc*. Moreover, if models of conversation are supposed to model the ways in which assertions of the sentences we are interested in, the predictions of the orthodox approach seem to be inadequate. Hence, we need an alternative.

3.6 Routes to Non-sequential Conjunction

My main argument is now on the table. I hope I have convinced the reader that the sequential view faces a range of issues and that a popular attempt to deal with them is not only ad hoc but does not do much to provide an adequate account of the context change potentials of conjunctions containing

information sensitive vocabulary.

In this final section I will discuss one more argument that, if successful, may sway us to accept the unwelcome consequences discussed above. The argument, as we will see, is not successful. Rather, it points to one general way of rejecting sequential conjunction. A second, independently motivated way of rejecting the sequential view is discussed in Section 3.6.2. While such positive proposals have to be developed and defended elsewhere, it may be worthwhile to convey a taste of what a view rejecting sequential conjunction may look like.

3.6.1 Meanings, Context Change and Embedding Behavior

Let me start with a comment on one more argument that, if successful, may move us to accept sequential conjunction despite the consequences discussed in Section 3.4.

As argued above, on the orthodox view, our explanations for why $\neg r \wedge \Diamond r$ and $\Diamond r \wedge \neg r$ are unassertable are not uniform. The first is unassertable because it is inconsistent the second is consistent but incoherent. The same goes for some sentences that look like contradictions.

Sequential conjunction is especially popular among dynamic *semanticists*. According to defenders of such a view, context change potentials are *meanings*. So a theory of discourse dynamics is not just a theory about how assertions of sentences affect the context of a conversation but also a theory about the compositional mechanisms that govern the fragment of language in question. Note that on our orthodox approach the context change potentials of $\neg r \wedge \Diamond r$ and $\Diamond r \wedge \neg r$ differ. Hence, from a dynamic perspective on meaning, the sentences are predicted to differ in *meaning*. So, if we could point to an independent reason to believe that $\neg r \wedge \Diamond r$ and $\Diamond r \wedge \neg r$ differ in meaning, we may have a reason to accept an explanation of their unassertability which is not uniform. While we would have to provide similar arguments for all other sentences who are incoherent but not inconsistent, let us focus on $\neg r \wedge \Diamond r$ and $\Diamond r \wedge \neg r$ for now.

Now, a classic way of arguing that two sentences differ in meaning, is to show that their compositional contributions come apart. That is, if we can find uniform embeddings of the sentences in question that differ in meaning, we have a reason to believe that the sentences embedded must differ in

meaning as well.

Unfortunately, there is a growing consensus in the literature that $\neg r \wedge \diamond r$ and $\diamond r \wedge \neg r$ have exactly the same embedding behaviour [This claim is most forcefully defended in Mandelkern, 2019]. Consider

(9) a. Hank believes that it is not raining and it might be raining.

b. Hank believes that it might be raining and it is not raining.

(10) a. Suppose it is not raining and it might be raining.

b. Suppose that it might be raining and it is not raining.

So $\neg r \wedge \diamond r$ seems to be as unbelievable and as unsupposable as $\diamond r \wedge \neg r$. And while it is impossible to check all possible embeddings, I doubt that any such argument could establish a view that would move us to accept the consequence of adopting a sequential view of conjunction we discussed above. So even if, you adopt a dynamic perspective on meaning, there seems to be no additional reason which would motivate adopting the sequential view.

Indeed, Mandelkern [2019], argues that the embedding behaviour of $\neg r \wedge \diamond r$ and $\diamond r \wedge \neg r$ support views according to which such sentences pattern with contradictions and the same seems to hold for all conjunctions that contain conjuncts which are negations of each other. This is easily explained on views on which all of these sentences mean the same. Moreover, a plausible minimal constraint on the relationship between meanings and context change potentials seems to be

> **Contradiction to Crash**: If ϕ is a contradiction, $c[\phi] = \bot$, for any context c.

In light of our intuitions about D4, any such view would have to reject sequential conjunction. Note however, that any such view would also reject the simple unorthodox approach, since on such a view, $\neg r \wedge \diamond r$ is a counterexamples to the just mentioned principle.

3.6.2   Informational Updating

Let me look at one more proposal that would make predictions about the context change potentials of the sentences in $\mathcal{L}_\diamond$ that are more desirable than the predictions of both the orthodox and the simple unorthodox view. The idea behind the proposal is that we should think about updates as determined

by some non-dynamic notion of meaning. And while the key ideas behind the view are compatible with many approaches to the meaning of the sentences in $\mathcal{L}_\diamond$, let me introduce a simplified version in terms of a formalism already available to us.

For the purposes of this section, think of the simple unorthodox view not as a view about the discourse dynamics of $\mathcal{L}_\diamond$ but as a view about the propositions the sentences in $\mathcal{L}_\diamond$ express at a given context c. So the 'update' function $\cdot[\cdot]$ is not seen as modelling how assertions of the sentences affect the context of a conversation but just as a function from contexts and sentences to propositions.

Given this perspective, let us provide an alternative notion of update (the notion of update, Def. 2.5.2, explored in Section 2.5.2 of Chapter 2)

$$c[\phi]^* = \bigcup\{c' \subseteq c \mid c'[\phi] = c'\}. \qquad \text{(Informational Updating)}$$

So the update potential is not provided by $\cdot[\cdot]$ but by the union of all those propositions that carry no less information than is already contained in c and which match the proposition expressed by ϕ at that context. Notice that this view validates Contradiction to Crash. The reason is that, as argued above, our simple unorthodox model maps all sentences that look like contradictions to the empty set. So, since any sentences that expresses the empty proposition at every context has a context change potential that is the union of some empty sets and hence is the empty set itself. Moreover, the view predicts all sentences that are incoherent with respect to $\cdot[\cdot]$ (in the sense of section 4.3) to crash the context of a conversation. For it will update a given context by the union of contexts that satisfy $c'[\phi] = c'$ which, by definition of an incoherent sentence, has to be $\varnothing$. Accordingly, both $\neg r \wedge \diamond r$ and $\diamond r \wedge \neg r$ are predicted to crash the context of a conversation at every possible stage.

So, on this view the Coherence Constraint does not do any additional work. Moreover, the update potentials of the sentences in question is uniform. All sentences of the form $\neg r \wedge \diamond r$ and $\diamond r \wedge \neg r$ and all contradictory looking sentences are treated the same.

Finally, it is not difficult to check that the view is idempotent and that it agrees with both the orthodox as well as the unorthodox view about the discourses D1, D2, D3 and D4. Thus it retains all the properties we deemed desirable for an adequate account of the discourse dynamics of 'might' discussed above.

One last consequence is worth pointing out. Informational Updating re-

jects negation as complementation. In particular

$$c[\neg \Diamond \alpha]^* \neq c - c[\Diamond \alpha]^*$$

for some contexts c. The view predicts that an assertion of 'it is not the case that it might be raining' conveys that it is not raining. Which, it seems to me, is a desirable prediction. But this is not the place to provide a motivation of the view and a more thorough motivation has been given elsewhere (see Chapter 2 and 4). For the purposes of this part, it suffices to note that there is an account about the discourse dynamics of $\mathcal{L}_\Diamond$ which rejects the sequential view but gives us all the predictions we deemed desirable without relying on the Coherence Constraint.

3.7   Conclusion

I have argued that theories of discourse dynamics for fragments of language that could reasonably be modelled with the help of $\mathcal{L}_\Diamond$ or analogous fragments for 'probably' and 'presumably' do not provide a reason to accept sequential conjunction. Rather, we have good reasons to think that adequate theories of the discourse dynamics for such expressions provide us with a reasons to reject such a view.

As mentioned above, the sequential view is particularly popular among defenders of a dynamic approach to meaning. Let me highlight, however, that unlike the arguments found in [Mandelkern, 2020], for instance, I do not claim that the problem cases considered above rule out dynamic approaches to meaning. On the face of it, dynamic semanticists may accept a non-sequential account of conjunction as well.

However, one consequence of my argument is that defenders of a dynamic approaches to meaning have a particularly strong incentive to give up on sequential conjunction. For, it has been argued that an essential feature of such approaches is that they give rise to theories of discourse dynamics that violate Idempotence, Commutativity or both constraints [see Rothschild and Yalcin, 2017, 2016]. As seen above, violations of these constraints, together with the sequential view, give rise to untenable consequences for the discourse dynamics of conjunction. Hence, anyone who defends a dynamic approach to meaning has a reason to look for an alternative view about the discourse effects of 'and'.

Chapter 4

'Must' Be Informative, 'Might' Be Non-trivial

4.1 Introduction

Consider a conversation between Alice and Bob. Suppose Alice asserts

(1) The gardener is the culprit.

In normal circumstances, Bob learns and it will become common ground between Alice and Bob that the actual world is not among those in which the gardener is innocent.

Now, consider

(2) The gardener might be the culprit.

What does an assertion of (2) contribute to the common ground between Alice and Bob? Many think that in typical cases 'might'-claims do not tell us anything about what the actual world is like. Rather such claims concern the information we have about it. A popular way of spelling this out is in terms of what we may call the *test view* about assertions of epistemic modal claims [see Veltman, 1996, Groenendijk et al., 1996, Gillies, 2004, 2018, for various versions of the view]. On that view, a felicitous assertion of a sentence such as (2) does not contribute information but *tests* whether what is common ground between the discourse participants is compatible with the gardener being the culprit.

This chapter is about two puzzles surrounding the test view. Both concern explaining the contrast between trivial and non trivial assertions of epistemic

69

modal claims. To get a feel for the puzzles let us look at the contrast for non-modal sentences. Plausibly, a felicitous assertion of (1) is non-trivial, if the information that the gardener did it is not yet shared between Alice and Bob. But, if it is already clear that the gardener is the culprit and this is common ground between Alice and Bob, asserting (1) is like stating the obvious. All views about the discourse dynamics of non-modal sentences I am aware of can give an explanation along those lines.

The first puzzle I wish to engage with is the following. Just like non-modal claims, epistemic possibility claims such as (2) may be trivial or not (examples will be discussed below). But, if such assertions do not add information to the common ground, what explains the difference? While I do think that there is some truth to the test view in that such assertions do not genuinely add information to the common ground, the test view is not well equipped to capture the contrast [see Groenendijk et al., 1996, for an observation along those lines]. Hence we need an alternative.

The second puzzle concerns an asymmetry between the discourse effects of epistemic possibility and epistemic necessity claims. Consider an assertion of

(3) The gardener must be the culprit.

Suppose Bob does not know who did it. After a thorough investigation of the crime scene, Alice asserts (3). It seems that Alice's assertion is a perfectly natural way for Alice to convey that the gardener did it. So we would expect that Alice's assertion of (3) is not only felicitous but Bob learns and it will become common ground between Alice and Bob that the gardener did it. Hence, it seems natural to explain a contrast between trivial and non-trivial assertions of epistemic necessity claims analogously to non-modal claims. They are non-trivial, if they add information to the common ground and trivial otherwise. However, according to the test view, epistemic necessity claims like (3) are treated similar to epistemic possibility claims. They too are seen as not contributing information to the common ground when used felicitously. This, it seems to me, is wrong. Hence, we need an alternative to the test view.

Versions of the first puzzle are well known and a plausible approach is mentioned but not developed in [Groenendijk et al., 1996]. According to this idea, 'might' claims can be seen as salience makers. They make possibilities salient that have not been salient before. So, the difference between trivial

and nontrivial uses of *might* claims can be seen as a difference in whether or not a possibility was made salient that has not been salient before.

Willer [2013] contains an implementation of such a view and while I agree that it provides a solution to the first puzzle mentioned above, I will argue that its predictions for 'must' claims are odd. In particular, it does not predict the above mentioned asymmetry between the discourse effects of 'might' and 'must' claims. Moreover, I will argue that his view is based on an unsatisfactory notion of what it is for a possibility to be salient.[1] Hence, we need an alternative.

While, to my knowledge, the second puzzle has not received much attention, Santorio [2022] recently put forward a notion of update that, as he observes, predicts that necessity claims are informative in the same sense as non-modal sentences are. However, his proposal is developed in the context of devising a non-standard approach to the semantics of indicative conditionals and the predictions about epistemic necessity claims are not motivated independently. Moreover, Santorio's view does not answer our first puzzle. Hence, we need an alternative.

The goal of this chapter is to build on the insights and ideas of Groenendijk et al. [1996], Willer [2013] and Santorio [2022] with the goal of developing a view that solves both of the above mentioned puzzles; a view according to which 'must' claims are informative in the same way non-modal claims are and 'might' claims are non-trivial, if they make a possibility salient that have not been salient before. Finally, the test view, as we will see, comes in two flavours: a dynamic and a purely semantic flavour. As I will argue,

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[1]A slightly different but related question is discussed in [Ciardelli et al., 2015]. My paper will be about the question in what sense an assertion is informative *at a context*. They investigate the question whether there is an *absolute* sense in which 'might' claims are informative. The idea is roughly that a sentence is informative in the absolute sense, if its content is not tautologous. They then show that a new, more fine grained, notion of content helps to spell out a sense in which epistemic possibility claims are not tautologous. Dever and Schiller [2021] introduce a notion of update building on [Ciardelli et al., 2015]. The focus of their paper, however, is on how updates with epistemic modals affect our credences. Here I will omit a thorough comparison to these views. The main reason is that I am not only interested in the dynamics of epistemic possibility but also the dynamics of epistemic necessity claims. Unfortunately, neither [Ciardelli et al., 2015] nor [Dever and Schiller, 2021] discuss epistemic necessity claims for both predict negated 'might' claims to be on a par with contradictions. In comparison, my view is standard in the sense that 'must' claims are taken to be duals of 'might' claims (and, hence, essentially treated as negated 'might' claims).

we can hold on to the core ideas of the test view, if we treat it as a purely semantic proposal. But as a view about the discourse dynamics of 'must' and 'might' claims, it has to be rejected.

Here is an overview of the paper. In Section 4.2 I will introduce the test view in its dynamic (Section 4.2.1) and in its purely semantic flavour (Section 4.2.2) and discuss some of their predictions (Section 4.2.3). I then discuss a plausible version of Grice's maxim of quantity due to Stalnaker (Section 4.2.4) and show how a general way of thinking about the contrast of trivial and non-trivial assertions of epistemic modal claims fails with respect to this way of thinking about the contrast (Section 4.2.5). In Section 4.3 I develop my positive proposal. The discussion starts with an exploration of the general mechanisms that predict the asymmetry of the discourse effects of epistemic possibility and epistemic necessity claims (Section 4.3.1). I then discuss Willer's view as one way of looking at the distinction between salient and non-salient possibilities and show that the view does not captures the just mentioned asymmetry. Moreover, I argue that it is built on an odd notion of what it is for a possibility to be salient (Section 4.3.2). I introduce an alternative account of salience (in Section 4.3.3) and conclude with a presentation of my positive proposal (Section 4.3.4). In Section 4.4 I address some open questions, extensions and potential worries one may have with respect to the positive view developed here. I close the paper with some final remarks in Section 4.5.

4.2 Tests and Information Exchange

Readers familiar with the standard dynamic as well as standard truth- conditional accounts of epistemic modal claims are invited to skim sections 4.2.1 and 4.2.2 in which these views are introduced. However, it may be helpful to keep in mind that views presented later in the paper depend on the notions defined here. Sections 4.2.3, 4.2.4 and 4.2.5 will cover arguments that motivate the positive view to be developed in Section 4.3 of this chapter.

4.2.1 The Test view, a Dynamic Perspective

Let us start with a formal language. For the most part we will focus on natural language sentences that can reasonably be modelled with the help of the following formal language:

$$\mathcal{L}_s \quad \phi ::= \alpha \mid \Diamond\alpha \mid \Box\alpha \mid \neg\Diamond\alpha \mid \neg\Box\alpha,$$

$$\alpha ::= p \mid \neg\alpha \mid (\alpha \wedge \alpha).$$

Above p is supplied by a finite set of atomic sentences At. As usual $\neg$ and $\wedge$ model negation and conjunction and $\Diamond$ and $\Box$ model epistemic possibility and necessity claims respectively. Notice that the fragment is small (hence the index s) in the sense that it is a proper subset of the fragment:

$$\mathcal{L}_\Diamond \quad \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \Box\phi.$$

For instance, the latter, but not the former, contains conjunctions of epistemic modal sentences and not just bare epistemic modals and their negations. While we will comment on sentences in the larger fragment $\mathcal{L}_\Diamond$, we will look at various accounts that agree in predictions with respect to $\mathcal{L}_s$ but may disagree on sentences in $\mathcal{L}_\Diamond$ that are not in $\mathcal{L}_s$. Since the main arguments of this chapter concern sentences that can be modelled with the help of $\mathcal{L}_s$, differences in predictions outside of $\mathcal{L}_s$ do not matter for the core arguments of this chapter.

Given the above formal language, let us put a version of what we have called the test view on the table, a version of the view which models the discourse effects of epistemic modal claims directly.[2]

Definition 4.2.1 (The Test View (Dynamic Version)). *Let a context c be a set of possible worlds. A theory of the discourse dynamics for the sentences in $\mathcal{L}_s$ is given in terms of a function $\cdot[\cdot]$ from contexts c and sentences $\phi \in \mathcal{L}_s$ to contexts $c[\phi]$, where a context is a set of possible worlds. The function $\cdot[\cdot]$ is recursively defined as follows:*

(p_d) $c[p] = \{w \in c \mid w(p) = 1\},$ $(\Diamond_d)$ $c[\Diamond\alpha] = \{w \in c \mid c[\alpha] \neq \varnothing\},$
$(\neg_d)$ $c[\neg\alpha] = c - c[\alpha],$ $(\Box d)$ $c[\Box\alpha] = \{w \in c \mid c[\alpha] = c\}.$
$(\wedge_d)$ $c[\alpha \wedge \beta] = c[\alpha][\beta],$

(Except for the last entry, this corresponds to Def. 3.2.2 from Chapter 3. In order to keep this chapter self-contained, it is repeated here.) Here, worlds, w, are treated as functions from At to truth-values in $\{0,1\}$. Since

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we assume `At` to be finite, we will, in the following take the set W of all possible worlds to be finite.

Following Stalnaker [1999], the context, c, should be seen as encoding the information that is shared between the participants of the discourse. Since, according to the above view, c is a set of possible worlds, we may think about it as *ways the world might be in light of the information shared between the participants of the discourse.* Note that a function like $\cdot[\cdot]$ provides a theory of the discourse dynamics of the sentences in $\mathcal{L}_s$ *directly* for $c[\phi]$ can reasonably be interpreted as the context the context c evolves to in light of an assertion of ϕ. Hence, $\cdot[\cdot]$ should be seen as modelling the *context change potentials* of the sentences we are interested in.

Before we look at some concrete predictions and the question of why the view deserves the name *test* view, let me mention a purely semantic version of the view, a version that does not provide a theory of the discourse dynamics of $\mathcal{L}_\diamond$ directly.

4.2.2 The Test view, a Truth-conditional Perspective

The approach below is due to MacFarlane [2008] and is given in terms of a notion of truth at an index.[3]

Definition 4.2.2 (The test view (Truth conditional version)). *Let an index be a pair of a possible world w and a set of possible worlds s such that $w \in s$. Truth relative to an index for sentences $p \in$ `At` and $\alpha, \beta \in \mathcal{L}_s$ is then defined as follows:*

(p_t) $[[p]]^{\langle w,s \rangle} = 1$ *iff* $w(p) = 1$,
$(\neg_t)$ $[[\neg\alpha]]^{\langle w,s \rangle} = 1$ *iff* $[[\alpha]]^{\langle w,s \rangle} = 0$,
$(\wedge_t)$ $[[\alpha \wedge \beta]]^{\langle w,s \rangle} = 1$ *iff* $[[\alpha]]^{\langle w,s \rangle} = 1$ *and* $[[\beta]]^{\langle w,s \rangle} = 1$.
$(\diamond_t)$ $[[\diamond\alpha]]^{\langle w,s \rangle} = 1$ *iff* $\exists w' \in c; [[\phi]]^{\langle w,s \rangle} = 1$,
$(\square_t)$ $[[\square\alpha]]^{\langle w,s \rangle} = 1$ *iff* $\forall w' \in c; [[\phi]]^{\langle w,s \rangle} = 1$.

Here s should be seen as standing for a *state of information* of which a context of conversation is a special case.[4] Details about how exactly we should interpret a view like this do not matter for our purposes [the interested

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[3]This semantics first occurred in print in [Yalcin, 2007] but is based on a manuscript later published as [MacFarlane, 2008] [see Yalcin, 2007, fn 10].

[4]Other states of information may be states of belief or knowledge.

reader is referred to MacFarlane, 2008, 2014, Yalcin, 2007]. What matters is that, on this view, the truth of an epistemic modal claim depends on a state of information s in a similar manner as the context change potential of an epistemic modal claim depends on the context of conversation c on the above dynamic version of the test view. And while this truth-conditional approach to the semantics of $\mathcal{L}_s$ does not give rise to a theory of discourse dynamics directly, the connection to the above dynamic view can be drawn out by considering the following bridge principle:

$$c[\phi]_\dagger = \{w \in c \mid [\![\phi]\!]^{\langle w,c \rangle} = 1\}. \qquad \text{(Diagonal Updating)}$$

According to this principle, the conversational effect an assertion of ϕ has on the common ground is to eliminate all those worlds from the context c at which the sentence ϕ is false.[5]

Given this update function, we can show that the truth-conditional and the dynamic version of the test view are related. At least for the sentence in $\mathcal{L}_s$ the views make exactly the same predictions about their discourse effects.[6] That is, for all contexts c and $\phi \in \mathcal{L}_s$ we have

$$c[\phi]_\dagger = c[\phi].$$

Now, going back to the dynamic version of the view, note that assertions of epistemic modal claims have only two possible outcomes. Such assertions either leave the context c unchanged, or they 'crash' the context to the empty set $\varnothing$; a context at which any way the world might be is ruled out. Which possibility is realized depends entirely on whether the context in question passes a certain test. Epistemic possibility claims test whether updating the context with the prejacent sentence crashes the context. Epistemic necessity claims test whether updating the context with the prejacent would change it at all. Now, given our notion of diagonal updating as well as the fact that on $\mathcal{L}_s$ the predictions about update are the same, it is not difficult to verify

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[5]The name is inspired by Stalnaker's notion of a diagonal proposition. However, the similarity in nomenclature should not be taken literally. For the purposes at hand, it does not matter whether the update function does or does not capture what Stalnaker had in mind. This bridge principle is also discussed in Chapter 2.

[6]It is worth pointing out that the views come apart for some sentences in $\mathcal{L}_\diamond$. Equality could be regained by changing the entry for conjunction of either view. I will come back to conjunction in Section 4.4.2 of this chapter. But for now, the difference need not concern us here.

that the entries for *must* and *might* could equivalently be given as

$(\diamond_t)$ $[\![\diamond\alpha]\!]^{\langle w,c\rangle} = 1$ iff $c[\alpha]_\dagger \neq \varnothing$, $\qquad$ $(\square_t)$ $[\![\square\alpha]\!]^{\langle w,c\rangle} = 1$ iff $c[\alpha]_\dagger = c$.

Hence, given diagonal updating, MacFarlane's semantics can be seen as a truth-conditional version of the test view.[7]

4.2.3 Updates as Tests

Let us look at Alice's assertions from the introduction (repeated):

(1) The gardener is the culprit,

(2) The gardener might be the culprit,

(3) The gardener must be the culprit.

Let us think of p, $\diamond p$ and $\square p$ as modelling (1), (2) and (3), respectively. Moreover let a be a world in which the gardener is the culprit (and thus $a(p) = 1$) and b a world in which someone else did it (hence $b(p) = 0$). The predictions of the view about these sentences, can now be depicted with the help of the diagrams in Figure 4.1. Think of the sets at the nodes of the diagrams in Figure 4.1 as possible contexts of a conversation. Indeed, each diagram contains all possible context that can be built out of the worlds a and b (i.e. all contexts in $\mathcal{P}(\{a,b\})$, the power set of $\{a,b\}$). Note that at the bottom we have the empty context. As already indicated, this can be seen as the *defective* context, a context at which *any* way the world might be is ruled out.

The context sets are arranged as the nodes of a Hasse diagram: the dashed edges indicate that the set on the lower end of the edge is a subset of the above and that there is no subset of the above set that lies between the two. The arrows depict the predictions about the discourse effect of the views discussed above in the sense that an arrow from c to c' indicates that $c[\phi] = c'$ for the respective sentence ϕ.

Finally, note that plausibly an assertion of ϕ at context c adds information to the context of the conversation if $c[\phi] \subsetneq c'$: the fewer ways the actual world might be in light of the information shared between the participants

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[7]MacFarlane's semantics, together with diagonal updating, gives rise to what we have called the simple unorthodox view in Chapter 3.
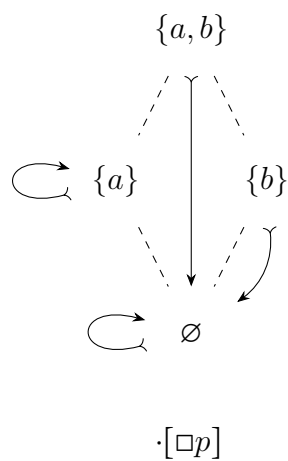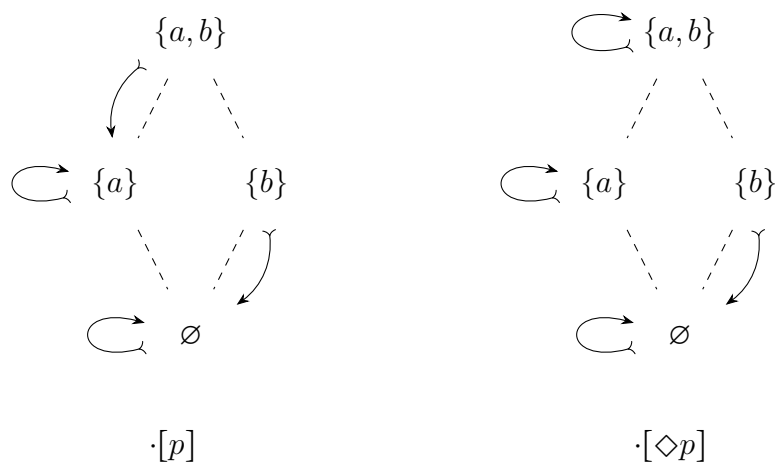
Figure 4.1: Some predictions of the test view

of the discourse, the more information is shared about it. Hence, a reflexive arrow on the left of some node in the diagram indicates that, at that context, the assertion does not contribute new information to the context of the conversation. A downward arrow, in turn, indicates that an assertion of the sentence does contribute information to the common ground. Note that, in this sense, updates that point to the empty set are informative in that they add too much information to the common ground: they add so much information that the resulting state is incompatible with any way the world might be. So, while defective, the empty set can be seen as the most informative state.

4.2.4 Quantity and Stalnaker's Principle of Rational Communication.

Now, Stalnaker suggests the following principle of rational communication [see Stalnaker, 1999, p.88].

> **Informativity**: A rational assertion at context c always eliminates some but not all possible worlds from c.

As seen above, the removal of worlds corresponds to a change in information content of the context in question. Hence, I take it that the principle is a weak version than Grice's maxim of quantity, according to which you ought to make your contribution as informative but not more informative as required [see Grice, 1975, p. 45]. *Ceteris paribus*, an assertion that does not contribute anything to the common ground, is not as informative as required and an assertion that contributes so much information to the common ground that any way the world might be is ruled out is certainly more informative than required.[8] Note also that, with respect to the diagrams of Figure 4.1, Informativity is satisfied at exactly those contexts where we have a downward pointing arrow that does not point to the empty set. Informativity is violated at all other contexts.

Now, note that the test view, together with Stalnaker's informativity maxim, predicts that assertions of non-modal sentences are odd, if they are asserted at a context at which the information conveyed by the sentence is

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[8]It is a weak version of the principle, since there may be assertions that do not violate Informativity but still do not count as 'as informative as required', such as answering 'I have a bike or a car' to the question, 'do you have a car?'.

either already shared or already taken to be ruled out by the participants of the discourse. However, the test view, together with our Informativity maxim would also predict that *any* assertion of a bare epistemic modal claim is odd. For any assertion of such a sentence is bound to violate the maxim at any context (see the second and the third graph in Figure 4.1 for an illustration of this point).

Let me highlight that the fact that assertions of a certain class of sentences are bound to violate a maxim of rational communication is not problematic in and of itself. However, what is problematic is that we seem to be quite competent at detecting a contrast between trivial and non-trivial assertions of the sentences in $\mathcal{L}_\diamond$, even among those sentences that are modal. For non-modal sentences, the contrast is easily explained: non-trivial assertions do not violate Informativity, trivial ones do. But since epistemic modal claims are bound to violate Informativity in every possible context, an analogous explanation cannot be given for epistemic 'might' and 'must' claims.

4.2.5 Modal Assertions, Trivial and Non-trivial

For concreteness and to fix ideas about how to arrive at a view that can capture the contrast between trivial and non-trivial epistemic modal claims, let us look at some data.

Everybody should agree that an assertion of (3) (repeated)

(3) The gardener must be the culprit,

sounds plainly wrong at a context at which the gardener is already ruled out as a candidate culprit. The predictions of the test view seem correct in this regard (such a case is reasonably modelled as an assertion of $\Box p$ at $\{b\}$ as in the graph at the bottom of Figure 4.1).

What about the difference between trivial and non-tivial assertions of epistemic necessity claims? Consider the following two cases.

> *Case 1.a*: Alice and Bob are discussing the crime scene. Bob knows that Alice thoroughly investigated the case. Excited to hear what she found out, he asks Alice about her results. Alice looks up from the photos taken at the crime scene and asserts (3) (repeated)

> (3) The gardener must be the culprit.

Case 1.b: Alice and Bob are discussing the crime scene. Bob knows that Alice thoroughly investigated the case. Excited to hear what she found out, he asks Alice about her results. Alice looks up from the photos taken at the crime scene. At that moment the gardener walks in and exclaims "I did it!". Alice looks at Bob and asserts (3) (repeated)

(3) The gardener must be the culprit.

Observe first that in neither case does Alice's assertion of (3) sound like an assertion of something false. However, in Case 1.b an assertion of (3) is certainly funny sounding while in Case 1.a this is not so. Why is that?

The test view together with our informativity maxim from the previous section is in a position to explain Case 1.b. Initially, the context of Alice and Bob's conversation can be modelled in terms of the set $\{a, b\}$. However, the gardener's confession plausibly resulted in a context shift from $\{a, b\}$ to $\{a\}$. As illustrated in Figure 4.1, the test view predicts that an assertion of (3) at $\{a\}$ will result in a violation of our informativity maxim. However, as observed in the previous section, the test view cannot but violate the informativity maxim. So, what is going on in Case 1.a.?

Here I think the blame lies with the test view and not with the informativity maxim. As in Case 1.b, $\{a, b\}$ seems to be a natural way to interpret the context of conversation between Alice and Bob before Alice's assertion. But then, the test view predicts that Alice's assertion is on a par with an assertion of something obviously false. An unwelcome prediction. A reasonable alternative prediction would be that (3) affects a context change from $\{a, b\}$ to $\{a\}$. Any such alternative view would, together with Stalnaker's informativity maxim, straightforwardly predict the difference between Case 1.a and 1.b.

Let us turn to assertions of epistemic possibility claims. I think everybody should agree that an assertion of (3) (repeated)

(2) The gardener might be the culprit,

sounds odd at a context at which the gardener is already ruled out as a candidate culprit. Hence, the test view together with Informativity gives us the right predictions in such a case (consider context $\{b\}$ in the second graph of Figure 4.1).

Now, let us look at the following two cases:

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Case 2.a: Alice and Bob are discussing the crime scene. Bob knows that Alice thoroughly investigated the case. Excited to hear what she found out, he asks Alice about her results. Alice looks up from the photos taken at the crime scene and asserts (2) (repeated)

(2) The gardener might be the culprit.

Case 2.b: Alice and Bob are discussing the crime scene. Bob notes "Given the evidence, it seems it was the gardener or some other staff member." Alice consults the photos taken at the crime scene and asserts (2) (repeated)

(2) The gardener might be the culprit.

Again, in neither cases does Alice's assertion sound like an assertion of an obvious falsehood. However, in the second but not the first case, Alice's assertion seems odd. Why is that?

In the case of epistemic necessity claims we concluded that Stalnaker's Informativity maxim seems to explain what is going on, if we change our view about the context change potentials of such claims. In the case of epistemic possibility claims, however, a change of the underlying view about context change does not seem to suffice. Here my intuitions lie with the test view insofar as it is implausible to hold that assertions of epistemic possibility claims remove worlds from the context of a conversation. After all, such claims tell us about a way the world *might* be, not a way the world is.[9] One way to capture the contrast is in terms of the above mentioned idea of Groenendijk et al. [1996]. 'Might' claims, according to this idea, make possibilities salient that have not been salient before. For note that in Case 2.b but not 2.a the possibility of the gardener being the culprit was already 'on the table'.

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[9]If anything, an assertion of an epistemic possibility claim adds a possibility to the context set. However, just adding possible worlds to a given context cannot be the solution. First, just like a removal of worlds from a given context corresponds to information gain, an addition of worlds will correspond to information loss. But, certainly, Alice's assertion will not cause the removal of information from the common ground. Second, if epistemic might claims would cause the addition of worlds to a context, why do we deem Alice's assertion of (2) to be odd at a context where the gardener is already ruled out as a candidate culprit?

What is needed for such a view to work, is a notion of context that captures the contrast between possibilities that are merely compatible with the information shared at a stage of the conversation and possibilities that are salient. Moreover, we need a notion of context *change* according to which a context may change in terms of what possibilities are salient without a change in what is compatible with the information shared between participants of the discourse. Unfortunately, neither the test view nor our Informativity maxim can deal with such changes.

To conclude, in light of our observations regarding epistemic necessity claims, it seems we need to look for an alternative view that makes 'must' claims informative in the sense discussed above. However, in light of our observations regarding epistemic possibility claims, we need both a different principle of rational communication as well as an alternative view about the discourse dynamics of such claims. We need a principle of rational communication that does not deem assertions of epistemic possibility claims irrational just because they do not remove ways the world might be from the context of conversation, and we need a notion of update that explains how 'might' claims can act as salience makers.

4.3   'Must' Be Informative, 'Might' Be Non-trivial

We now have two items on our to-do list. The first is to provide an account of conversational updating that makes 'must' informative. The second is to provide an account of conversational updating that explains what the difference between trivial and non-trivial 'might' claims is. In Section 4.3.1 we will start with the first item. In Section 4.3.2 we discuss Willer's approach to non-trivial epistemic possibility claims and turn to my own view in sections 4.3.3 and 4.3.4.

4.3.1   'Must' Be Informative

Why is 'must' informative? The reason, as I will argue, is not a semantic one but one that has to do with a particular notion of update.

Our story starts with the above mentioned idea that contexts, like states of belief or states of knowledge, are *states of information.* Such states contain information that settle some but not necessarily all sentences of a fragment

of language. For instance, if you know that the gardener did it, the state of information corresponding to your state of knowledge settles the sentence 'The gardener did it'. If you do not know who did it, the corresponding state settles neither the sentence nor its negation.

A semantics for a fragment of language typically gives rise to an account of what it is for a state of information, in particular a context of conversation, to settle a sentence. A natural way of providing such a notion in terms of MacFarlane's semantics from Section 4.2.2 is as follows:

Definition 4.3.1 (Support). *Let c be a context (a set of possible worlds). We say that c supports $\phi \in \mathcal{L}_\diamond$ just in case*

$$\forall w \in c; [\![\phi]\!]^{\langle w,c\rangle} = 1,$$

where $[\cdot]^{\cdot}$ is provided by MacFarlane's semantics (as in Def.4.2.2).

(This is Def. 2.5.1 from Chapter 2. In order to keep this chapter self-contained, it is repeated here.) Such a notion of support is found in many places in the literature on the semantics and discourse dynamics of epistemic discourse.[10] It tells us that, with respect to a supporting context c, ϕ will be true in light of the information available at c, no matter which of the possibilities compatible with c will be realized.

Note that the way in which our notion of diagonal updating, $\cdot[\cdot]_\dagger$, was defined, our notion of support could be equivalently defined as

$$c \text{ supports } \phi \text{ iff } c[\phi]_\dagger = c.$$

So all the states that have a reflexive arrow on the left in Figure 4.1 support the respective sentence.[11]

A notion of support puts us in the position to define an alternative notion of update. The idea of thinking about conversational update in terms of

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[10]See the definition of *acceptance* in [Yalcin, 2007, p. 464] and the notion of being *true throughout* in [Kolodny and MacFarlane, 2010, p. 136]. Assuming diagonal updating, these definitions are equivalent to the definition of *support* as it is found in [Gillies, 2004, p. 600], for instance.

[11]Of course, since $c[\phi] = c[\phi]_\dagger$ for all c and $\phi \in \mathcal{L}_s$, we also have that c supports ϕ just in case

$$c[\phi] = c.$$

Hence, this notion of support agrees with the dynamic notion for all sentences in $\mathcal{L}_s$.

support goes back to Yalcin [2007] though the function I will present below comes closest to a definition recently put forward by Santorio [see his Santorio, 2022, Appendix]. Since he introduces his update function with a notion of context that is formally different from ours, a direct comparison goes beyond the scope of this chapter. However, I take it that any motivation for the view provided here will count as a motivation for Santorio's view (and *vice versa*).

The way in which I would like to introduce our new update function goes as follows. Listeners may reasonably operate under the assumption that a speaker asserts ϕ only if the information available to them supports the sentence asserted.[12] So, in light of the just mentioned assumption, listeners are in a position to rule out all those possibilities from the context that are incompatible with any way of settling the sentence asserted. The resulting context is, so the idea behind this notion of update, the new context of conversation.

Formally, the idea can be spelled out as follows:

$$c[\phi]_{\ddagger} := \bigcup \{c' \subseteq c \mid c' \text{ supports } \phi\}. \qquad \text{(Informational Updating)}$$

(Compare Def. 2.5.2 from Chapter 2.) Given this definition, let us compute all the updates for the sentences (1), (2) and (3) with respect to the contexts in $\mathcal{P}(\{a, b\})$. As above, we will visualize the predictions with the help of some diagrams in Figure 4.2.

Comparing the two graphs in the upper left corner of Figure 4.1 and Figure 4.2, we can note that $\cdot[\cdot]_{\ddagger}$ makes exactly the same predictions about the discourse effects of non-modal sentences and epistemic possibility claims as the test view. But the two approaches differ with respect to their predictions about the discourse effects of epistemic necessity claims (compare the graphs at the bottom of Figure 4.1 and Figure 4.2). In particular note that at context $\{a, b\}$, (repeated),

(3) The gardener must be the culprit,

is predicted to be informative. Hence, at a context where it is not yet common ground between Alice and Bob that the gardener did it, our new view predicts that an assertion of (3) satisfies Stalnaker's Informativity maxim. A welcome prediction.

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[12]This could be justified further by a knowledge or a certainty norm of assertion [as defended in Williamson, 1996, 2000, Hawthorne et al., 2016].
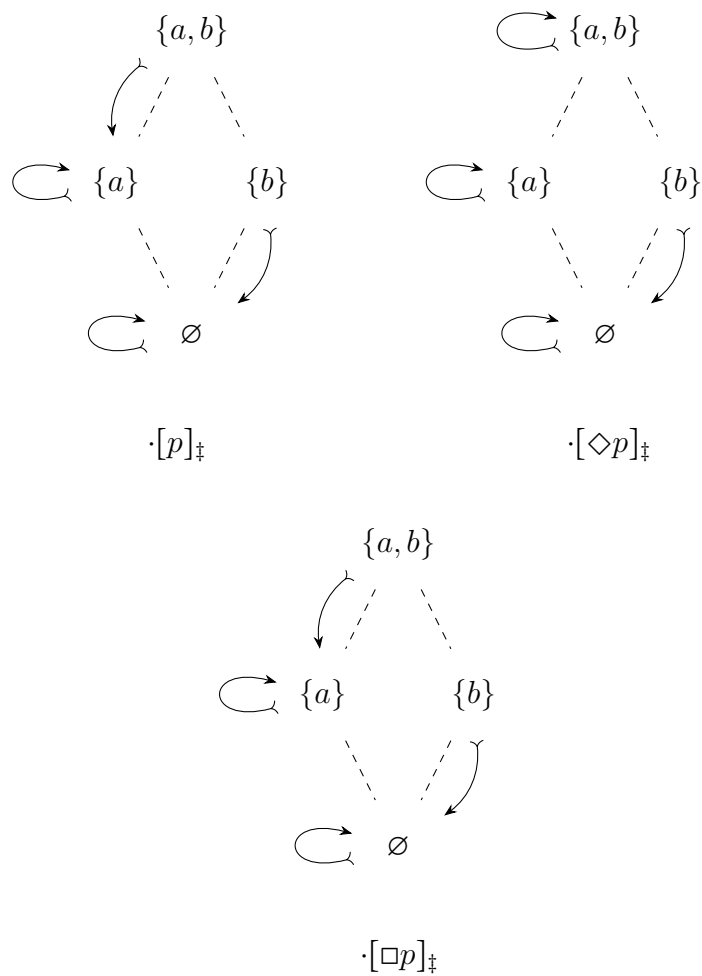
Figure 4.2: Informational updating

Note that bare epistemic necessity claims and bare epistemic possibility claims are treated differently. Epistemic possibility claims are still predicted to violate Informativity at any context. While, initially, this may seem like a drawback of the view, we will see in Section 4.3.4 and 4.3.5 that it is an advantage. Once we have implemented the idea that some possibilities compatible with the information shared at a context are salient while others are not, we can provide a view which captures the differences in update effects of epistemic possibility and epistemic necessity claims observed above.

There is much more to be said about the view just sketched and I will come back to some consequences and potential worries in Section 4.4. For now, let us move on.

4.3.2 Salient Possibilities (Supervaluationism)

We have seen above that shifting from diagonal updating to informational updating will make 'must' informative and hence, with the help of Stalnaker's principle we are able to explain the difference between trivial and non-trivial assertions of epistemic necessity claims. However, we have not yet explained the difference between trivial and non-trivial epistemic possibility claims. This is the second item on our to-do list.

As mentioned above, an attractive way of providing such a view builds on the idea found in [Groenendijk et al., 1996] that epistemic possibility claims make possibilities salient that have not been salient before. The difficulty of providing such a proposal lies in providing an account that enables us to spell out what a salient possibility is.

To see this, note that in both cases we discussed above (Case 2.a and b) the gardener being the culprit is plausibly compatible with the information shared between Alice and Bob. That seems to be the reason why in neither case does Alice's assertion sounds wrong. Hence, epistemic possibility modals do not *just* highlight compatibility. Rather, we should aim for a model in which we are able to distinguish between three kinds of contexts:

Type I) Contexts in which p is *incompatible* with the information shared between the participants of the discourse.

Type II) Contexts in which p is *compatible* with the information shared between the participants of the discourse but *not salient.*

Type III) Contexts in which p is *both compatible* with the information shared between *and salient* to the participants of the discourse.

The idea would be that a more adequate approach to the discourse dynamics of epistemic possibility claims predicts an assertion of $\Diamond p$ to be wrong sounding at a context of type I, felicitous and natural sounding at a context of type II and odd or trivial sounding at a context of type III. Unfortunately, on the simple possible worlds picture we have been working with so far, we do not have enough structure to capture all three types of context. In particular, we cannot distinguish contexts of type II and type III.

One view which can provide a model for contexts of all three types is found in [Willer, 2013]. Willer suggests that the difference between the three types should be modelled with the help of a *supervaluationist* approach. On this approach contexts are not just modelled in terms of a set of possible worlds, c, but in terms of a set of sets of possible worlds, $\mathfrak{c} = \{c_1, c_2, \ldots, c_n\}$. To illustrate Willer's idea, let us focus on atomic sentences p (that can be evaluated for truth or falsity at a possible world). On Willer's view, p is *compatible* with a context $\{c_1, c_2, \ldots, c_n\}$, if there is some context c_i containing a world at which p is true and incompatible otherwise. p is *salient* with respect to a context $\{c_1, c_2, \ldots, c_n\}$, if every nonempty c_i in $\{c_1, c_2, \ldots, c_n\}$ contains a world at which p is true. Finally, update, on Willer's account, corresponds to individually updating all the sets c_i at a context $\{c_1, c_2, \ldots, c_n\}$ in terms of the update function of the test view. Let us refer to this way of updating a Willer-style context as *point-wise diagonal updating*:[13]

$$\mathfrak{c}[\phi]_p = \{c \mid c = c'[\phi]_\dagger \text{ for } c' \in \mathfrak{c}\}. \qquad \text{(Point-Wise Diagonal Updating)}$$

This view seems to get the right predictions with respect to assertions of epistemic possibility claims. To illustrate, let the worlds a, b and the sentence p be as above, we then have

$$\{\{b\}, \{a, b\}\}[\Diamond p]_p = \{\varnothing, \{a, b\}\}.$$

Note that p is salient at $\{\varnothing, \{a, b\}\}$ but not at $\{\{b\}, \{a, b\}\}$.[14] So on Willer's

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[13]So, the view is reminiscent of views about updating in imprecise credence settings [see Joyce, 2010, for a helpful overview]. Yalcin [2012a] and Moss [2018] consider similar notions of update.

[14]Here are examples illustrating all three types of contexts and their respective updates with $\Diamond p$:

view, 'might' claims are salience makers.

Unfortunately, there are two issues with the view. The first has to do with what the view predicts about assertions of epistemic necessity claims. To see this, consider Case 1.a again. To recap the case: Alice and Bob are discussing a crime scene. Bob asks Alice about what she found out while it is common ground between Alice and Bob that the butler and the gardener are the only candidate culprits. Let us suppose in addition that both of the possibilities are salient at the context of conversation. Taking b to be a world in which the butler is the culprit, the context $\{\{a,b\}\}$ seems to adequately capture the respective context of conversation. Above, we judged Alice's assertion of (3) as unproblematic and informative. But Willer's view predicts

$$\{\{a,b\}\}[\Box p] = \{\varnothing\}.$$

That is, Willer's view inherits the bad prediction of the test view. Alice's assertion is predicted to crash the context of conversation while, intuitively, it does not. (Note that in Willer's setting $\{\varnothing\}$ is the analogue of the defective context $\varnothing$). This seems implausible.

Now, one response could be to switch from a notion of point-wise diagonal updating to a notion of point-wise *informational* updating:

$$\mathfrak{c}[\phi]_{pi} = \{c \mid c = c'[\phi]_{\ddagger} \text{ for } c' \in \mathfrak{c}\}. \qquad \text{(Point-Wise Informational Updating)}$$

This would make 'must' informative (as discussed in the previous section) but preserve Willer's predictions about the discourse effects of epistemic possibility claims. Unfortunately, there is a second issue with Willer's view which would not be resolved by switching to point-wise informational updating.

To see the issue, consider Case 2.a again. In this case Alice tells Bob (2), that the gardener might be a culprit. So the possibility that the gardener did it will become salient at that stage of the conversation. As mentioned above, (2) does not rule out any other candidate culprits and hence it seems that the possibility of the gardener *not* being the culprit also becomes salient at that stage of the conversation.

Considerations like this support the following general principle:

Type I	$\{\{b\}\}$	$\{\{b\}\}[\Diamond p]_p = \{\varnothing\}$
Type II	$\{\{b\}, \{a,b\}\}$	$\{\{b\}, \{a,b\}\}[\Diamond p]_p = \{\varnothing, \{a,b\}\}$
Type III	$\{\{a\}, \{a,b\}\}$	$\{\{a\}, \{a,b\}\}[\Diamond p]_p = \{\{a\}, \{a,b\}\}$

Complementation: Let α be a non-modal sentence. If α is a salient possibility and $\neg\alpha$ is not ruled out at the respective context of conversation, then $\neg\alpha$ is a salient possibility at that context.[15]

Unfortunately, Willer's view allows for contexts that violate Complementation, such as

$$\{\{a,b\},\{a\}\}.$$

At this context p is a salient possibility. However $\neg p$ is neither salient nor ruled out. So it seems that Willer's view over-generates contexts of conversation.

4.3.3 Salient Possibilities (Partitions)

The two issues just discussed tell us that we not only need a different notion of update (that seems to be a consequence of the first issue) but also a different notion of context, a notion of context that satisfies Complementation. And indeed, there is a popular alternative to Willer's notion of context which does satisfy Complementation.

On the view I have in mind, a context is not just a set of possible worlds but a partition $\pi(s)$ of a set of possible worlds s [see Roberts, 2012].[16] Indeed, partitions of sets of worlds have been explored from various angles in semantics and epistemology.[17] This is not the place to draw connections to these various ways of interpreting partitions of worlds [But see Yalcin, 2016, for a helpful overview of these various proposals]. Instead, let me focus on how partitions help to provide a plausible account of what it is for a possibility to be salient at a context of conversation.

Definition 4.3.2 (Salience relative to a partition)**.** *Let s be a set of worlds and $\pi(s)$ a partition of s modelling the context of a conversation. For non-modal sentences α we say that*

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[15]While conceptually related, this principle should not be confused with the way in which the term 'complementation' was used in Chapter 3.

[16]Remember, a partition of a set c is a set of disjoint sets whose union is c. So $\{\{a,b\},\{a\}\}$, for instance, is not a partition of the set $\{a,b\}$ while $\{\{a\},\{b\}\}$ is.

[17]Lewis [2008] and later Yablo [2014] use partitions to model the subject matter of a sentence, Groenendijk and Stokhof [1984] explore such structures to provide an account about the logic of questions and how to think about questions under discussion in conversation [see Roberts, 2012, 2006]. Yalcin [2016] and Schaffer [2008] build on this previous work to provide accounts of belief and knowledge.

i) α is **compatible** with the information shared at context $\pi(s)$, if it is true at some world in s,

ii) α is **salient** at the context $\pi(s)$, if there are cells of the partition, $c_1, \ldots c_n \in \pi(s)$, such that the union of $c_1, \ldots c_n$ is the set of worlds in s at which α is true.

In what follows, we will write $c \sqsubseteq \pi(s)$, if $c = c_1 \cup \cdots \cup c_n$ for some $c_1, \ldots c_n \in \pi(s)$. Accordingly, α is salient at a context $\pi(s)$ if for some $c \sqsubseteq \pi(s)$, c is the set of worlds in s at which α is true. Also, let us write π for arbitrary partitions and $\pi(s)$ if the set of worlds s underlying the partition matters.

To illustrate the definition, assume that p and the worlds a and b are as above (i.e. p is true at a but false at b), then p is compatible with both of the contexts $\{\{a, b\}\}$ and $\{\{a\}, \{b\}\}$ but p is a salient possibility at the latter context but not at the former. Note that, by definition, any possibility that is salient at a context is also compatible with it but not the other way around. Note also that, in virtue of the fact that a context is a partition, if p is salient at a context π and $\neg p$ is not incompatible with π, then $\neg p$ is also salient at the context. Hence, unlike Willer's supervaluationist account, thinking about salience in the above way results in a view that does satisfy Complementation.

What remains to be done is to provide an account of update for our new notion of context. Let me sketch the rough idea in this section and provide the details in the next.

To set things up, let us refer to contexts seen as sets of possible worlds as *world contexts* and contexts seen as partitions over sets of possible worlds as *partitional contexts*. Now Figure 4.3 contains two graphs. The left graph depicts all possible world contexts that can be built out of the worlds a and b (again, this is just the power set, $\mathcal{P}(\{a, b\})$) ordered by the subset-relation). The right graph depicts all possible partitional contexts that can be built out of the worlds a and b, which is the set $\{\pi \subseteq \mathcal{P}(\mathcal{P}(\{a, b\})) \mid \pi$ is a partition of some set $s \in \mathcal{P}(\{a, b\})\}$.

However, instead of the subset relation, the contexts on the right graph in Figure 4.3 are ordered by a notion of *refinement*. To define the particular relation I have in mind, we need one technical notion.

Definition 4.3.3 (Down set). *Let S be a set of sets of possible worlds. The down set $\downarrow S$ of S is defined as the smallest set that satisfies:*

i) $S \in \downarrow S$,

$$\{\{a,b\}\}$$

$$\{a,b\}$$

$$\{\{a\},\{b\}\}$$

$$\{a\} \qquad \{b\}$$

$$\{\{a\}\} \qquad \{\{b\}\}$$

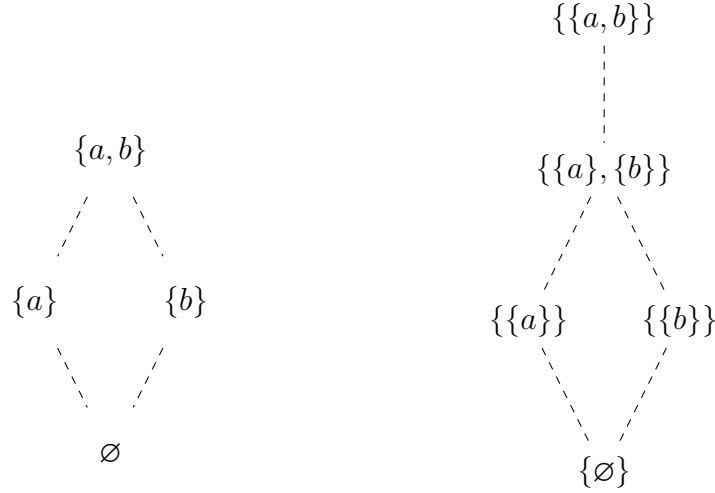$$\varnothing$$

$$\{\varnothing\}$$

Figure 4.3: Subsets and refinements

ii) if $x \in\ \downarrow S$ and $y \subseteq x$, then $y \in\ \downarrow S$.

For instance, $\{\{a,b\},\{a\},\{b\},\varnothing\}$ is $\downarrow \{\{a,b\}\}$, the down set of $\{\{a,b\}\}$. Given this notion, we can define our notion of refinement, $\leq$, as

$$\pi' \leq \pi \text{ iff } \downarrow \pi' \subseteq\ \downarrow \pi. \qquad \text{(Refinement)}$$

So, one partition is a refinement of the other just in case the down set of the one is a subset of the down set of the other.[18] Let us call π a proper refinement of π' if $\pi \leq \pi'$ but $\pi \neq \pi'$. According to this definition both $\{\{a\},\{b\}\}$ and $\{\{a\}\}$ are proper refinements of $\{\{a,b\}\}$. In general, if π is a proper refinement of π', π makes more possibilities salient, contains less worlds, or both.[19]

While the subset relation gives rise to a partial order on the set of all worlds contexts, our relation of refinement gives rise to a partial order on the

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[18]Similar orders are studied in inquisitive semantics [Ciardelli et al., 2019]. However, the objects studied here are down sets of *arbitrary* sets of possible worlds (sets of sets of worlds which do not necessarily form a partition).

[19]But note that not any context which contains less worlds is is a refinement of a context that contains more worlds. For instance, $\{\{a,b\}\}$ contains less worlds but does not refine $\{\{a\},\{b\},\{c\}\}$.

set of all partitional contexts.[20] So, the Hasse diagram on the right in Figure 4.3 is a depiction of the refinement relation on all possible contexts that can be built out of the worlds a and b.

Given our refinement relation, a natural generalization of Stalnaker's Informativity maxim is the following:

> **Non-triviality:** A rational assertion at a context π always results in a proper refinement of π that is not a refinement to the deviant context $\{\varnothing\}$.

That is, a rational assertion will always make a possibility salient that has not been salient before, eliminate some possible worlds or both. The conjecture is that an appropriate notion of update for the sentences in our small fragment $\mathcal{L}_s$ together with our new Non-triviality maxim explains the difference between the various cases discussed in section 4.2.5. That is, we will end up with a view according to which non-modal and epistemic necessity claims satisfy Non-triviality because they are genuinely informative: they will eliminate possibilities from the context of conversation. Epistemic possibility claims, on the other hand, will never eliminate possible worlds from the context. They are non-trivial because they partition a context further into more cells. But, in either case, updates that result in a 'move' downward which is not a 'move' to $\{\varnothing\}$ in our Hasse diagram will be updates that satisfy Non-triviality.

4.3.4 'Might' Be Non-trivial

To introduce our new update function, we need to say more about salient truths for all sentences in our fragment of language $\mathcal{L}_\Diamond$. A natural way to do so is to lift MacFarlane's truth conditional semantics to our new setting. Remember, on his account a sentence is thought of as true relative to an index, and an index was thought of as a pair consisting of a context c and a world w such that $w \in c$. A natural extension to our new setting would be to evaluate sentences for truth at a pair consisting of a context $\pi(s)$ and a cell c such that $c \in \pi(s)$. So, the role played by a possible world in our earlier setting is now played by a cell of a partition. The role played by the

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[20]A partial order is a reflexive, transitive and anti-symmetric relation. Note that $\leq$ is reflexive, transitive and anti-symmetric, since the subset relation is reflexive, transitive and anti-symmetric even if restricted to down sets of partitions.

information state, thought of as a set of worlds, is now played by a partition (a set of cells). Given this we can extend our definition of salience to all sentences in $\mathcal{L}_\diamond$ as follows.

Definition 4.3.4 (Lifted test view (truth-conditional version)). *Let an index be a pair of a cell c and a context $\pi(s)$ such that $c \in \pi(s)$. Truth relative to an index for sentences $p \in$ At and $\alpha, \beta \in \mathcal{L}_s$ is then defined as follows:*

(p_s) $\qquad [[p]]^{\langle c, \pi(s) \rangle} = 1,$ *if* $\exists c' \sqsubseteq \pi(s); c' = \{w \in s \mid w(p) = 1\}$
$\qquad\qquad\qquad$ *and* $c \subseteq \{w \in s \mid w(p) = 1\},$

$\qquad [[p]]^{\langle c, \pi(s) \rangle} = 0,$ *if* $\exists c' \sqsubseteq \pi(s); c' = \{w \in s \mid w(p) = 0\}$
$\qquad\qquad\qquad$ *and* $c \subseteq \{w \in s \mid w(p) = 0\},$

$\qquad [[p]]^{\langle c, \pi(s) \rangle} = \#$ *otherwise,*

$(\neg_s)$ $\qquad [[\neg\alpha]]^{\langle c, \pi(s) \rangle} = 1,$ *if* $[[\alpha]]^{\langle c, \pi(s) \rangle} = 0,$

$\qquad [[\neg\alpha]]^{\langle c, \pi(s) \rangle} = 0,$ *if* $[[\alpha]]^{\langle c, \pi(s) \rangle} = 1,$

$\qquad [[\neg\alpha]]^{\langle c, \pi(s) \rangle} = \#$ *otherwise,*

$(\wedge_s)$ $\qquad [[\alpha \wedge \beta]]^{\langle c, \pi(s) \rangle} = 1,$ *if* $[[\alpha]]^{\langle c, \pi(s) \rangle} = 1$ *and* $[[\beta]]^{\langle c, \pi(s) \rangle} = 1,$

$\qquad [[\alpha \wedge \beta]]^{\langle c, \pi(s) \rangle} = 0,$ *if* $[[\alpha]]^{\langle c, \pi(s) \rangle} = 0$ *or* $[[\beta]]^{\langle c, \pi(s) \rangle} = 0,$

$\qquad [[\alpha \wedge \beta]]^{\langle c, \pi(s) \rangle} = \#,$ *otherwise,*

$(\diamond_s)$ $\qquad [[\diamond\alpha]]^{\langle c, \pi(s) \rangle} = 1,$ *if* $\exists c' \in \pi(s); [[\alpha]]^{\langle c', \pi(s) \rangle} = 1,$

$\qquad [[\diamond\alpha]]^{\langle c, \pi(s) \rangle} = 0,$ *if* $\forall c' \in \pi(s); [[\alpha]]^{\langle c', \pi(s) \rangle} = 0,$

$\qquad [[\diamond\alpha]]^{\langle c, \pi(s) \rangle} = \#,$ *otherwise,*

$(\Box_s)$ $\qquad [[\Box\alpha]]^{\langle c, \pi(s) \rangle} = 1,$ *if* $\forall c' \in \pi(s); [[\alpha]]^{\langle c', \pi(s) \rangle} = 1,$

$\qquad [[\Box\alpha]]^{\langle c, \pi(s) \rangle} = 0,$ *if* $\exists c' \in \pi(s); [[\alpha]]^{\langle c', \pi(s) \rangle} = 0,$

$\qquad [[\Box\alpha]]^{\langle c, \pi(s) \rangle} = \#,$ *otherwise.*

While different in detail, I see the above semantics as similar in spirit to the informal proposal found in [Yalcin, 2016] and the semantics provided in

[Moss, 2015, 2018].[21] To see the connection to our notion of salience from the previous section, note that according to the atomic case, (p_s), of our definition, an atomic sentence p counts as (saliently) true at a pair $\langle c, \pi(s) \rangle$, if p is salient at the context $\pi(s)$ and true throughout c. It is (saliently) false if p is false throughout c and its falsity is salient at $\pi(s)$. p is undefined otherwise. That is, p is neither true nor false throughout c, nor salient at $\pi(s)$ or both. The entries for conjunction and negation are just the Kleene entries reformulated for our setting [Kleene, 1938]. The (salient) truth of epistemic possibility claims, in turn, depends on whether there exists a cell so that the prejacent sentence is a salient truth relative to the index consisting of the cell and the partition at which the modal sentence is evaluated.

Given our lifted semantics, we can provide an account of what it is for an information state to (saliently) support a sentence in analogy to our notion of support from Section 4.3.1.

Definition 4.3.5 (Salient Support). *Let π be a context, we say that $\pi(s)$ saliently supports or s-supports $\phi \in \mathcal{L}_{\Diamond}$ just in case*

$$\forall c \in \pi(s), [\![\phi]\!]^{\langle c, \pi(s) \rangle} = 1,$$

where $[\cdot]$ is provided by our lifted version of MacFarlane's semantics (as in Def. 4.3.4).

Analogously to our notion of support from above, s-support tells us that a ϕ that is s-supported by $\pi(s)$ will be true, given the information available at $\pi(s)$, and regardless of which of the possibilities that are salient at $\pi(s)$ will be realized.

One more ingredient before we can provide our update function. Above, we defined informational updating for worlds contexts in terms of the union

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[21]The similarities lie in the fact that on both of these accounts the truth or falsity of (or the proposition expressed by) an epistemic modal claim depends on whether a possibility is salient in the sense of Def. 4.3.2. However, on Moss' account, partitions are provided by an index associated with each modal. In Def. 4.3.4, partitions are fixed globally. Moreover, Moss gives her semantics in terms of functions from a set of probability spaces to a set of probability spaces. This can be interpreted as a notion of update that is similar to Willer's proposal discussed above. Epistemic possibility claims, for instance, are seen as ruling out all those spaces which assign a 0-probability to the prejacent of the modal. So her view is a version of the notion of point-wise diagonal updating (with possible worlds contexts replaced by probability spaces) discussed above [see also Yalcin, 2012a, for a similar account of updating].

of a set of supporting contexts. While the union of a set of sets is typically defined as the set that contains all the elements of the elements in the given set of sets, the union operation may equivalently be defined as the least upper bound of the sets in the given set with respect to the subset relation $\subseteq$. That is, given some set of sets S, $\bigcup S$ is a least upper bound of S in the sense that

(i) for all $s \in S, s \subseteq \bigcup S$
($\bigcup S$ is an upper bound of S),

(ii) for all T such that $s \subseteq T$ for all $s \in S$, $\bigcup S \subseteq T$
(any upper bound, T, of S is greater or equal to $\bigcup S$).

Accordingly, we can provide an analogous operation $\bigvee$, the *join* of a set of partitions P, defined in terms of our refinement relation.[22] According to this definition $\bigvee$ is defined in terms of the following two properties:

(i) for all $\pi \in P, \pi \leq \bigvee P$,

(ii) for all π' such that $\pi \leq \pi'$ for all $\pi \in P$, $\bigvee P \leq \pi'$.

Given this operation, we can define update in analogy to the definition given in Section 4.3.1:

$$\pi[\phi]_\partial = \bigvee \{\pi' \leq \pi \mid \pi' \text{ s-supports } \phi\}. \qquad \text{(Informational Updating II)}$$

So contexts are replaced by partitions, the subset relation is replaced by our generalization of that relation to partitions, the refinement relation, and the union operation is replaced by its generalization to spaces of partitions (the join operation).

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[22]Here is a sketch proof that the join exists in our setting. Let π and π' be partitions over some subsets of some finite set of worlds W. Let U be an arbitrary partition of a subset of W such that $\pi \leq U$ and $\pi' \leq U$. Now the down set $\downarrow U$ of U must contain the sets in $\pi \cup \pi'$. Note that $\pi \cup \pi'$ is not necessarily a partition. But since U is a partition its down set must contain all the cells in $\pi \cup \pi'$ that do not overlap and unions of cells in $\pi \cup \pi'$ that do overlap. So, let $p(\pi \cup \pi')$ be the set that results from $\pi \cup \pi'$ by keeping all cells in $\pi \cup \pi'$ that do not overlap, and unioning all cells that do overlap. So the down-set of $p(\pi \cup \pi')$ must be a subset of the down-set of U. But, by the way $p(\pi \cup \pi')$ was constructed, it is a partition of some subset of W and thus $p(\pi \cup \pi') \leq U$. Finally, since the down set of $\pi \cup \pi'$ is a subset of the down set of $p(\pi \cup \pi')$ and since the down set of $\pi \cup \pi'$ is a superset of the down sets of π and π', we have $\pi, \pi' \leq p(\pi \cup \pi')$. So $p(\pi \cup \pi')$ is an upper bound of $\{\pi, \pi'\}$ and, since U was arbitrarily chosen, it is a least upper bound of $\{\pi, \pi'\}$.

As before, the idea is that discourse participants operate under the assumption that the speaker asserts ϕ only if the information available to them settles the sentence asserted. In light of this, discourse participants are in a position to move to the least refined context that does not preclude them from arriving at one that settles ϕ.

4.3.5 Predictions

Let us look at the predictions of our new view with respect to the sentences (1), (2) and (3), modelled in terms of p, $\Diamond p$ and $\Box p$. As above, let us look at what our new update function tells us with respect to all contexts that can be built out of the worlds a and b. The corresponding diagrams are found in Figure 4.4.

To illustrate the view, let us look at an assertion of (2) at a context where the gardener being the culprit is not yet salient between the participants of the discourse. A situation like this can reasonably be modelled as

$$\{\{a,b\}\}[\Diamond p]_{\partial}.$$

Now, according to our new update rule, we should look at all the refinements of the state $\{\{a,b\}\}$ that s-support $\Diamond p$. The corresponding states are marked in boldface in Figure 4.4. Next, we need to compute the join of these states, which, in this case, is the state $\{\{a\},\{b\}\}$ (it is the least upper bound of the states highlighted in boldface). Hence, like Willer's view, our view predicts that might-claims are salience makers: they put possibilities on the table that have not been on the table before. With the help of our generalized informativity maxim, our view, like Willer's, is able to model the difference between Case 2.a and 2.b.

Note, however, that the view preserves the predictions of our first notion of informational updating presented in Section 4.3.1. As above, the graph in the upper left corner and the graph at the bottom of Figure 4.4 are the same. This illustrates that, again, the distinction between trivial and non-trivial epistemic necessity claims can be piggybacked on our earlier explanation for why non-modal claims are trivial or not. Case 1.a is a case which is plausibly modelled with a state like $\{\{a,b\}\}$ or $\{\{a\},\{b\}\}$, depending on what possibilities are taken to be salient. In each case Alice's assertion of (3) is non-trivial because it is informative in the sense of eliminating some world from the context. Her assertion in Case 1.b is plausibly modelled as
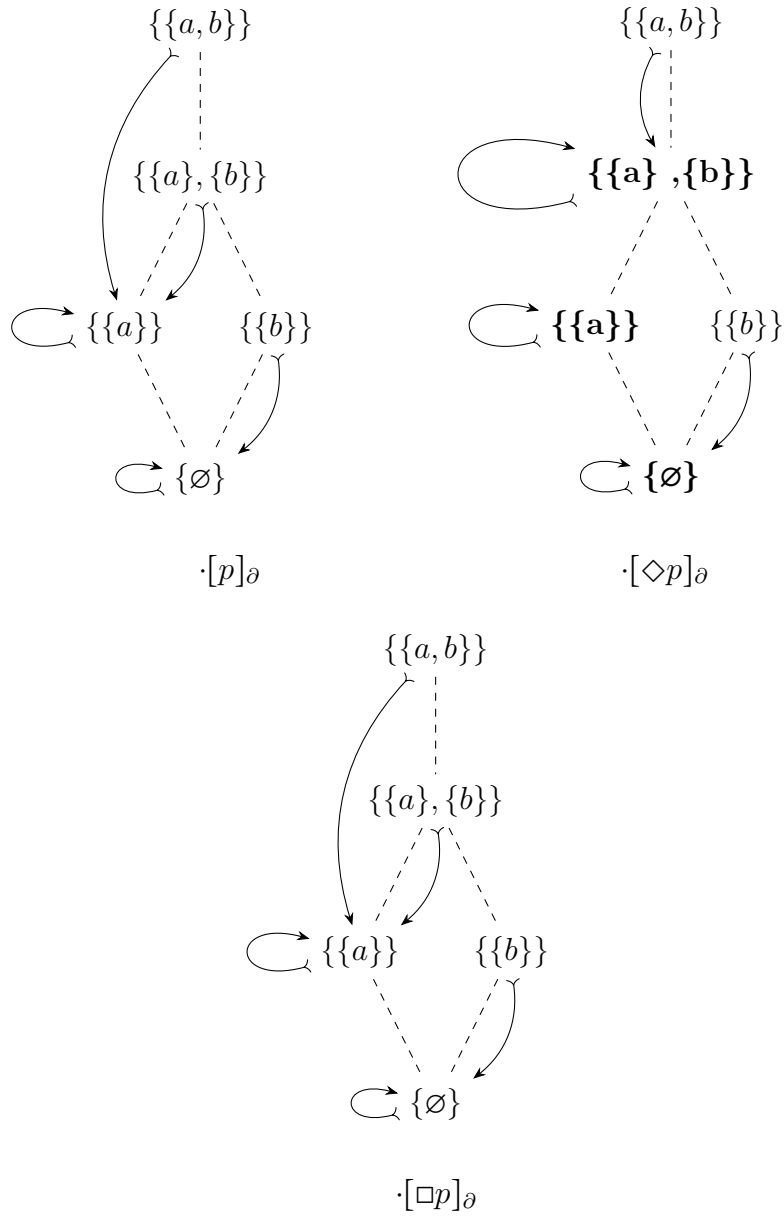
Figure 4.4: Informational updating on partitions

taking place at a context like $\{\{a\}\}$. As before, it is predicted to be trivial according to our Non-triviality maxim.

To sum up, felicitous assertions of 'must' claims are non-trivial, if they are genuinely informative. Felicitous assertions of 'might' claims are non-trivial whenever they introduce salient possibilities that have not been salient before.

4.4 Open Questions, Objections and Replies

The core of the positive view is now on the table. In this section I wish to address some potential worries and open questions. While most of the questions are ultimately left open, I hope to convey a sense of how the view can be modified, adjusted and extended without changing the core predictions about the sentences in the fragment $\mathcal{L}_s$ this chapter is about.

4.4.1 'Must', Again

Our view predicts that for non-modal sentences α, $\Box\alpha$ and α have the same update potential, i.e. $\pi[\alpha]_\partial = \pi[\Box\alpha]_\partial$ (for any $\alpha \in \mathcal{B}$, and context π). While, as argued above, I hold this to be a desirable feature of the view, there are two worries one may have about this prediction.

First, at context where α is not yet common ground between the participants of the discourse, there is a felt weakness of assertions of $\Box\alpha$ relative to α [see Karttunen, 1972, for this observation]. Flat out asserting that the gardener did it, sounds much more confident than claiming that he *must* be the culprit. Hence, one may worry that our predictions about the update potentials for bare epistemic necessity modals are too strong. Second, if sentences of the form α and $\Box\alpha$ have the same update potential, why would we ever have a reason to assert the more complex $\Box\alpha$ instead of the simpler α? A full response to these worries is beyond the scope of this chapter and something I hope to develop in future work. But, to provide a taste of how such a response could go, let us look at a standard explanation of the felt weakness of 'must'.

To explain the felt weakness of bare epistemic necessity claims, one may conjecture that 'must' is on a par with something like being 'relatively certain'. However, most believe that a view like this cannot be right [see von Fintel and Gillies, 2010, for instance]. To see this, compare

(4) I am relatively certain that the butler is the culprit though he might not be the culprit.

(5) The butler must be the culprit though he might not be the culprit.

The first sounds perfectly felicitous while the second sounds false. Hence, $\Box\alpha$ does not seem to be weaker than α. Moreover, a plausible explanation for the felt weakness of 'must' can be given in terms of what may be called an *evidentiality effect*: an assertion of $\Box\alpha$ instead of α sometimes conveys that the speaker has only indirect evidence supporting α [see von Fintel and Gillies, 2010, for such an explanation]. While a discussion of how these pragmatic effects come about is beyond the scope of this chapter, let me note that I do not see a principled reason why the view developed here should be incompatible with such an explanation. Our approach to the discourse dynamics models how the context of conversation changes in terms of its information content and what possibilities are taken to be salient. Nothing should prevent us from adding pragmatic principles that explain the evidentiality effects just mentioned.

If such an extension is possible, the first of the above mentioned worries can be addressed by pointing to the fact that the felt weakness is perfectly compatible with $\Box\alpha$ and α having the same update potential. The felt weakness of the former is not due to the fact that it conveys less information than α but due to pragmatic effects conveying something about the kind of evidence available to the speaker. The second worry about speakers' reasons for asserting the more complex $\Box\alpha$ instead of α can be addressed by the speakers' desire to trigger pragmatic reasoning on the part of the listeners: reasoning that helps them to figure out what kind of evidence is available to the speaker.

4.4.2 Unsupported Updates

A second worry concerns the update potentials of certain complex sentences in $\mathcal{L}_\Diamond$ (note that, up until now, we only looked at sentences in our small fragment $\mathcal{L}_s$). One tempting way to look at our proposal is in terms of a view about the *aim* of assertion due to Yalcin [2007]. According to this view, the aim of asserting a sentence is to convince the discourse participants to switch to a context that supports the asserted sentence. Since we defined update in terms of support, one may conjecture that our update function
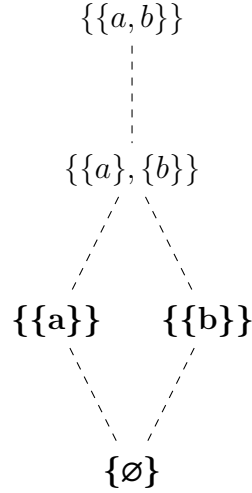
$$\{\{a,b\}\}$$

$$\{\{a\},\{b\}\}$$

$$\mathbf{\{\{a\}\}} \qquad \mathbf{\{\{b\}\}}$$

$$\{\varnothing\}$$

Figure 4.5: Unsupporting contexts

captures Yalcin's thesis about the aim of assertion in the sense that the following holds for any sentence $\phi \in \mathcal{L}_\Diamond$ and any context π:

$$\pi[\phi]_\partial \ \textit{s-supports } \phi. \qquad\qquad \text{(Update to Support)}$$

In words, an assertion of a sentence ϕ at a context π will result in a context that s-supports ϕ. While it can be proven (by induction over the sentences in $\mathcal{L}_s$) that our update function satisfies Update to Support for all sentences in $\mathcal{L}_s$, this is not so, if we consider assertions of sentences in the larger fragment $\mathcal{L}_\Diamond$.

Here is a simple counterexample. Let the sentence p and the worlds a and b be as before (that is, p is true at a and false at b). Moreover, suppose we have a sentence q that is true at b and false at a. We can then show that the following is false

$$\{\{a\},\{b\}\}[\neg(\Diamond p \wedge \Diamond q)]_\partial \ \textit{s-supports } \neg(\Diamond p \wedge \Diamond q).$$

To see this, consider the diagram in Figure 4.5. Here, the contexts that s-support $\neg(\Diamond p \wedge \Diamond q)$ are in boldface. So their join is the context $\{\{a\},\{b\}\}$ right above. Hence, $\{\{a\},\{b\}\}[\neg(\Diamond p \wedge \Diamond q)]_\partial = \{\{a\},\{b\}\}$. But note that $\{\{a\},\{b\}\}$ does not s-support $\neg(\Diamond p \wedge \Diamond q)$ since both conjuncts, $\Diamond p$ and $\Diamond q$,

are true at any $c \in \{\{a\}, \{b\}\}$.

Two remarks. First, my account is by no means the only account for which counterexamples to principles like Update to Support exist. In particular, the dynamic version of the test view introduced in Section 4.2.1 does not satisfy such a principle.[23] Second, I do not think that cases like these result in empirically inadequate predictions. Readers are invited to consult their intuitions about assertions of a sentence that share the logical form of the above, such as

(6) It is not the case that both the gardener might be the culprit and the butler might be the culprit.

To my ears this sentence would not add any information to a context in which both the possibility of the gardener being the culprit and the possibility of the butler being the culprit are salient possibilities. Since this is confirmed by the above derivation, the predictions of our view seem adequate in this regard.

But, what about Yalcin's idea about the aim of assertions? Note that, while our view is inspired by Yalcin's idea, it is based on the weaker claim of thinking about update in terms of support. No claim is made about what speakers aim to achieve. Further, it is worth pointing out that our predictions may not be in conflict with Yalcin's idea after all. It may well be that the speakers *aim* of asserting $\neg(\Diamond p \wedge \Diamond q)$ at context $\{\{a\}, \{b\}\}$ is to convince the discourse participants to change the context to one that supports it (i.e. either $\{\{a\}\}$ or $\{\{b\}\}$). But just because an agent aims to ϕ by χing, does not entail that the agent will succeed to ϕ. In general, χ might just be the wrong means to the envisioned end.

While sentences like (6) do not seem to sound contradictory or false, I, for one, am not able to find contexts at which such a sentence sounds perfectly natural. If this is so for any sentence that violates Update to Support, that would speak in favor of the combination of Yalcin's thesis and my account. For it would predict that asserting $\neg(\Diamond p \wedge \Diamond q)$ at a context like $\{\{\{a\}, \{b\}\}\}$ would be as odd as aiming to go from point A to B by flapping your arms (in certain special cases it may work but, typically, it does not).

I am sympathetic to Yalcin's thesis about the aim of assertion. But

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[23]The reason is that on such views support is typically defined as follows: c supports ϕ just in case $c[\phi] = c$. As discussed in Chapter 3, it is well known that there are sentences such that $c[\phi][\phi] \neq c[\phi]$ for some contexts c. Examples are sentences of the form $\Diamond p \wedge \neg p$.

whether the claim is true in general and what constraints on assertion follow from it, has to be explored elsewhere.

4.4.3 Wittgenstein Sentences

One topic in the literature on epistemic modal sentences has been to explain why sentences of the form $p \wedge \Diamond \neg p$ or $\Diamond \neg p \wedge p$ such as

(7) The gardener is the culprit and he might not be the culprit,

(8) The gardener might not be the culprit and he is the culprit,

sound false at any context. Let us follow Mandelkern [2019] and call such sentences *Wittgenstein sentences*.[24] Further, let us reserve the term *epistemic contradiction* [Yalcin, 2007] for sentences which are of the form of (7) only.

It is interesting to note that the test view in its dynamic implementation only predicts epistemic contradictions (only sentences of the form $p \wedge \Diamond \neg p$) to be on a par with asserting something false at every context. Sentences of the form $\Diamond \neg p \wedge p$ are predicted to be felicitous at some contexts. Note also that MacFarlane's truth-conditional version of the test view, together with the notion of diagonal updating, predicts that for all Wittgenstein sentences there are contexts at which they have the same context change potential as an assertion of p.[25] This, it seems to me, is an unwelcome prediction.

Our view predicts all Wittgenstein sentences to be on a par with asserting something contradictory or false at every context. The reason is simply that no context other than $\{\varnothing\}$ supports a Wittgenstein sentence. For fans of MacFarlane's semantics, it seems to me, this prediction should lend further support to the view about update presented here.

However, Mandelkern [2019], for instance, argues that such sentences are not only unassertable but should be treated as *contradictions* in a semantic sense. That is, semantically, they should always evaluate as a false. This is not predicted by the versions of MacFarlane's semantics presented above.

While I do not wish to take a stance on Mandelken's arguments here, let me note that some of his ideas may easily be implemented without changing

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[24]As Mandelkern points out, an early discussion of such sentences is found in Wittgenstein [2000].

[25]To illustrate this point, let p be true at a and false at b Def. 4.2.2 tells us that $[p \wedge \Diamond \neg p]^{\langle a, \{a,b\}\rangle} = 1$. So, according to our notion of diagonal updating, as introduced in section 4.2.2, we have $\{a,b\}[p \wedge \Diamond \neg p]_\dagger = \{a\}$.

the core predictions about the sentences in $\mathcal{L}_s$ of the view presented above. For, on Mandelkern's own proposal, a change in the semantics of conjunction will do the trick. Using the formalism available to us, we can give an entry for 'and' that mirrors Mandelkern's ideas in our setting. The entry would look like this:

$(\wedge_m)$ $[\![\alpha \wedge \beta]\!]^{\langle c,\pi(s)\rangle} = 1$, if $[\![\alpha]\!]^{\langle c,\pi(s)[\beta]_\partial\rangle} = 1$ and $[\![\beta]\!]^{\langle c,\pi(s)[\alpha]_\partial\rangle} = 1$,

$\qquad [\![\alpha \wedge \beta]\!]^{\langle c,\pi(s)\rangle} = 0$, if $[\![\alpha]\!]^{\langle c,\pi(s)[\beta]_\partial\rangle} = 0$ or $[\![\beta]\!]^{\langle c,\pi(s)[\alpha]_\partial\rangle} = 0$,

$\qquad [\![\alpha \wedge \beta]\!]^{\langle c,\pi(s)\rangle} = \#$, otherwise.

Note that, on this view, both conjuncts are evaluated for truth or falsity not at the original context $\pi(s)$ but at contexts that are updated with the opposite conjunct respectively. On this view of conjunction, both sentences of the form $\Diamond\neg p \wedge p$ and $p \wedge \Diamond\neg p$ will evaluate as false at every index.

Whether such a notion of conjunction is indeed needed, is a question with an answer beyond the scope of this chapter. For the purposes of this chapter, the observation that we *could* adopt such a notion of conjunction suffices.

4.4.4 'Or'

Finally, let us look at another class of sentences that are outside our small fragment $\mathcal{L}_s$ (and arguably outside of $\mathcal{L}_\Diamond$). Note that, up until now, we have only looked at sentences that contain modals, conjunction and negation. But what about 'or'?

One way to treat disjunction is to think of it as a derived operator, defined in terms of conjunction and negation. That is, we stipulate

$(\vee_l)$ $[\![\alpha \vee_l \beta]\!]^{\langle c,\pi(s)\rangle} = [\![\neg(\neg\alpha \wedge \neg\beta)]\!]^{\langle c,\pi(s)\rangle}$.

Let us refer to the operator, $\vee_l$ as *logical disjunction*. Now, consider an assertion of

(9) The gardener is the culprit or the butler is the culprit,

at a context where neither the possibility of the gardener being the culprit nor the possibility of the butler being the culprit is salient to all the participants of the discourse. Accordingly, let the sentences p, q and the worlds a and b be as above, then $\{\{a,b\}\}$ is a reasonable model of such a context. Now,

treating 'or' as logical disjunction, the sentence $p \vee_l g$ models (9). Our view predicts:

$$\{\{a, b\}\}[p \vee_l q]_\partial = \{\{a, b\}\}.$$

That seems odd. What we should want is

$$\{\{a, b\}\}[(9)]_\partial = \{\{a\}, \{b\}\}.$$

An assertion of a disjunction like (9) seems to have the effect of making its disjuncts salient. But, as just seen, if we treat 'or' as logical disjunction, this is not what we get.

Is this an objection against our notion of informational updating? I do not think so. The literature on natural language disjunction, especially on how disjunctions interact with modals, knowledge and belief operators is vast and this is not the place to dive into this topic. But many agree that logical disjunction is not an adequate model for treating natural language 'or' in the first place.[26] Hence, we may hope that an alternative semantics for 'or' may result in the right kind of predictions. As a proof of concept let us give an entry for 'or' that would produce the right results. To keep the formalism simple, I will restrict myself to non-modal disjuncts.

Let me start with a bit of terminology. First, if s is a set and $\mathfrak{s}$ a set of sets (the elements of a partition, for instance), we say that $\mathfrak{s}$ is a *cover* of s if and only if

$$\bigcup \mathfrak{s} = s.$$

Second, let $\mathfrak{s}$ and $\mathfrak{s}'$ be sets of sets. We say that $\mathfrak{s}'$ is a *point-wise cover* of $\mathfrak{s}$ if and only if

$$\mathfrak{s}' \text{ covers } \bigcup \mathfrak{s} \text{ and } \forall x \in \mathfrak{s}, \exists y \subseteq \mathfrak{s}' \text{ s.t. } y \text{ covers } x.$$

Notice that for two partitions π and π', if π' is a point-wise cover of π, then π is a refinement of π, i.e. $\pi \leq \pi'$. But notice that neither π nor π' need to be partitions.

Now, taking inspiration from Simons [2005], according to which the truth value of a disjunctions is dependent on such a covering relation, for non-

---

[26]Non logical notions of disjunction have been considered as responses to Kamp's paradox of free choice permission [see Kamp, 1973]. [Lewis, 1997] contains an early discussion disjunction in the context of knowledge ascriptions. See also [Aloni, 2003] and [Fusco, 2021] for a discussion of the interaction of 'or' with epistemic and ability modals.

modal α, let us write $[\![\alpha]\!]^s$ for the set of worlds in s at which α is true. Then, for non-modal α and β we have:

$(\vee_i)$ $[\![\alpha \vee_i \beta]\!]^{\langle c, \pi(s) \rangle} = 1$, if $\pi(s)$ is a point wise cover of $\{[\![\alpha]\!]^s, [\![\beta]\!]^s\} - \varnothing$, and $c \subseteq [\alpha]^s \cup [\beta]^s$,

$[\![\alpha \vee_i \beta]\!]^{\langle c, \pi(s) \rangle} = 0$, if both $s - [\![\alpha]\!]^s$ and $s - [\![\beta]\!]^s$ are covered by some $c \sqsubseteq \pi(s)$ and $c \subseteq s - [\![\alpha]\!]^s \cup [\![\beta]\!]^s$,

$[\![\alpha \vee_i \beta]\!]^{\langle c, \pi(s) \rangle} = \#$, otherwise.

In words, a disjunction is true at an index if the disjuncts that are compatible with the context are salient possibilities relative to the information state of the index and the logical disjunction of the disjuncts is true throughout the cell of the index. It is false if the negation of both disjuncts are salient possibilities at the context and the logical disjunction of the disjuncts false throughout the cell. A disjunction is undefined otherwise.

Together with our notion of update, this view of disjunction produces the desired predictions. It agrees, both on the level of semantics as well as at the level of discourse dynamics, with logical disjunction on all contexts where the disjuncts are either salient or incompatible with the information state parameter. But the notions come apart if we consider contexts at which at least one disjunct is not salient.

Again, this is not the place to settle the semantics of disjunction. For the purposes of this chapter, it suffices to note that some generalization of our notion of logical disjunction may give us what we want.

4.5 Conclusion

I have argued that we should provide a notion of update for epistemic modal claims that makes 'must' informative and 'might' claims non-trivial, if they make possibilities salient that have not been salient before. While many open questions remain, I hope I have convinced the reader that, at least for the fragment of language considered here, we can arrive at a satisfactory view by defending a notion of informational updating. It is worth pointing out that, as I have argued elsewhere, such a notion of update gives rise to a view according to which update is not a compositional notion while the

underlying notion of (salient) truth at an index is.[27] Hence, if meanings are compositional, meanings are not context change potentials but functions from indices to truth values. While the costs and benefits of dynamic vs. non-dynamic approaches to meaning has to be explored elsewhere, I believe that the explanatory benefits of the view developed above provide further evidence that we should reconsider the idea that context change potentials are meanings. Separating our notion of context change from the compositional mechanisms that govern the fragment of language we are interested in might be the way to go, if we want to arrive at an explanatorily adequate theory of discourse dynamics for 'must' and 'might'.

---

[27]Note that $\Box p$ and p have the same context change potential but may differ in truth value at the same index. Moreover $\neg \Box p$ and $\neg p$ do *not* have the same context change potential. Hence, at the level of discourse dynamics, our account is not compositional in the sense of Chapter 2.

Chapter 5

Concluding Remarks: Are Context Change Potentials Meanings?

I hope I have convinced the reader that the question asked in the title of this conclusion is a substantial and philosophically interesting one. For the above chapters illustrate that there are interesting theories of discourse dynamics on which context change potentials are not meanings.

I also hope that the dissertation provides a shift in perspective. Dynamic theories of meaning should not only be judged by what they tell us about the compositional behaviour of a fragment of language but also by their notion of context change. If we take meanings to be context change potentials, such a theories would have to be adequate both as a theory of the former as well as a theory of the latter. Looking at classic dynamic approaches to modals and conditionals we can see that at the level of discourse dynamics we can and should do better.

Finally, I hope that the message of this dissertation is freeing. Giving up on the idea that context change potentials are meanings can free us from the burden of doing too many things at once. Dynamic approaches to meaning seem to unneccessarily restrict the space of possible theories of context change we can give. Holding on to a standard notion of what it is for a theory of meaning to be compositional rules out non-compositional theories of discourse dynamics that seem to make more adequate predictions than the standard approaches to discourse dynamics found in the literature. If meanings are not context change potentials, we no longer have to aim for a

theory that delivers both an empirically adequate theory of discourse dynamics as well as a semantic theory that captures the compositional mechanisms governing the fragment of language we are interested in.

But, giving up on the idea that context change potentials are meanings does not entail that context change potentials cannot inform us about meanings. On all the theories considered in this dissertation a difference in context change potentials indicates a difference in meaning. Moreover, giving up on the idea that context change potentials are meanings does not entail that we cannot learn from and build on the insights gained by defenders of dynamic approaches to meaning. For instance, all theories of discourse dynamics defended in this dissertation exhibit dynamicness on the discourse level. All of them violate van Benthem's distributivity property and Rothschild and Yalcin's commutativity principle. Still, the theories defended are not theories of meaning, at least not, if we like to hold on to a theory of meaning that vindicates the compositionality constraint from Chapter 2.

Time will tell whether the proposals defended in this dissertation will hold up to scrutiny and maybe it will turn out that our overall best theory of meaning is dynamic. But until we know more, such a claim should not be a presupposition but the conclusion of an investigation that takes the compositionality of meanings and the notion of context change equally seriously.

Appendix A

On the Non-compositional Dynamics of 'If'

A.1 Some Lemmata

Remark A.1.1. *All possible worlds models of conversation considered in Chapter 2 are eliminative in the sense that the update function $\cdot[\cdot]$ satisfies*

$$\forall \phi \in \mathcal{L}; \ c \supseteq c[\phi].$$

Proof. In each case the proof is a straightforward proof by induction. □

Lemma A.1.1 (Boolean Intersectivity). *Let W be a set of possible worlds and $\mathcal{V}$ be a valuation function such that for $p \in \mathtt{At}$, and $\alpha, \beta \in \mathcal{B}$ (the boolean sub-fragment of $\mathcal{L}$)*

(i) $\mathcal{V}(w, p) = w(p)$,

(ii) $\mathcal{V}(w, \neg\alpha) = 1$ *iff* $\mathcal{V}(w, \alpha) = 0$,

(iii) $\mathcal{V}(w, \alpha \wedge \beta) = 1$ *iff* $\mathcal{V}(w, \alpha) = 1$ *and* $\mathcal{V}(w, \beta) = 1$.

Finally, let $\cdot[\cdot]$ be an update function as in Def. 2.4.1 from section 2.3. We then have

$$\text{for all } \alpha \in \mathcal{B}, \ c[\alpha] = c \cap \{w \in c \mid \mathcal{V}(w, \alpha) = 1\}.$$

Proof. The proof is by induction on the complexity of formula in $\mathcal{B}$. The base (atomic) case is straightforward and the same goes for the case of negation.

Here we only proof the case for conjunction. (IH) Let the claim be true for α and β, we then have

$$
\begin{aligned}
c[\alpha \wedge \beta] &= c[\alpha][\beta] && \text{by } (\wedge_d) \\
&= c[\alpha] \cap \{w \in c[\alpha] \mid \mathcal{V}(w, \beta) = 1\} && \text{by (IH)} \\
&= c[\alpha] \cap \{w \in c \mid \mathcal{V}(w, \beta) = 1\} && \text{by } c \supseteq c[\alpha]) \\
&= c \cap \{w \in c \mid \mathcal{V}(w, \alpha) = 1\} \cap \{w \in c \mid \mathcal{V}(w, \beta) = 1\} && \text{by (IH))} \\
&= c \cap \{w \in c \mid \mathcal{V}(w, \alpha) = 1 \ \& \ \mathcal{V}(w, \beta) = 1\} && \text{by set theory} \\
&= c \cap \{w \in c \mid \mathcal{V}(w, \alpha \wedge \beta) = 1\} && \text{by (iii)}
\end{aligned}
$$

This completes the proof. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $\square$

Lemma A.1.2. *For all $\phi \in \mathcal{L}$ and $c \in C$, $c[\phi]_\dagger = c[\phi]$.*

Proof. For sentences in $\mathcal{B} \subseteq \mathcal{L}$ this is a simple induction on the complexity of formula in $\mathcal{B}$.[1] It then suffices to show that the claim holds for sentences of the form $\alpha \to \beta$ (for sentences of the form $\neg(\alpha \to \beta)$, we repurpose the induction step for negated sentences). To show this last claim, consider the following:

$$
\begin{aligned}
&c[\alpha \to \beta]_\dagger \\
&= \{w \in c \mid \forall w' \in c[\alpha]_\dagger, [\![\beta]\!]^{\langle w', c[\alpha]_\dagger \rangle} = 1\} && \text{by Def. } \cdot[\cdot]_\dagger \ \& \ (\to_t) \\
&= \{w \in c \mid \{w' \in c[\alpha]_\dagger \mid [\![\beta]\!]^{\langle w', c[\alpha]_\dagger \rangle} = 1\} = c[\alpha]_\dagger\} && \text{by Def. } \cdot[\cdot]_\dagger \ \& \ \text{set theory} \\
&= \{w \in c \mid c[\alpha]_\dagger[\beta]_\dagger = c[\alpha]_\dagger\} && \text{by Def. } \cdot[\cdot]_\dagger \\
&= \{w \in c \mid c[\alpha][\beta] = c[\alpha]\} && \text{by } \cdot[\alpha]_\dagger = \cdot[\alpha] \text{ for } \alpha \in \mathcal{B} \\
&= c[\alpha \to \beta]. && \text{by } (\neg_d)
\end{aligned}
$$

---

[1]For illustration, here is one induction step for the case of negation (where the induction hypothesis, (IH), is that α is such that $c[\alpha] = c[\alpha]_\dagger = \{w \in c \mid [\![\alpha]\!]^{\langle w, s \rangle} = 1\}$):

$$
\begin{aligned}
c[\neg\alpha]_\dagger &= \{w \in c \mid [\![\neg\alpha]\!]^{\langle w, s \rangle} = 1\}, && \text{by Def. } \cdot[\cdot]_\dagger \\
&= \{w \in c \mid [\![\alpha]\!]^{\langle w, s \rangle} \neq 1\}, && \text{by } (\neg_t) \\
&= c - \{w \in c \mid [\![\alpha]\!]^{\langle w, s \rangle} = 1\}, && \text{by set theory} \\
&= c - c[\alpha], && \text{by (IH)} \\
&= c[\neg\alpha]. && \text{by } (\neg_d)
\end{aligned}
$$

This completes the proof. □

A.2   Proof of Proposition 2.5.1

Let us repeat Proposition 2.5.1 from Chapter 2.

Proposition 2.5.1 *There is no non-trivial unorthodox model of conversation for $\mathcal{L}$ that satisfies the above compositionality constraint*

Here we consider the slightly stronger claim

Proposition A.2.1. *There is no non-trivial trivial possible worlds model of conversation for $\mathcal{L}$ that satisfies Materiality, Conservativity and either Strict Negation or the dynamic version of Boethius' Thesis.*

Proof. The proof mirrors the example we discussed above. Let W contain at least three worlds w and w' and w''. Then there is a context $c \in \mathcal{P}(W)$ such that $c := \{w, w', w''\}$. Now pick a sentence α that is true at both w' and w'' (and false at w) as well as a sentence β that is true at w'' only. Let us consider the conditional $\alpha \to \beta$. By Conservativity and Materiality we have for any context c,

$$c[\alpha \to \beta]^* = c[\neg(\alpha \wedge \neg\beta)]^*.$$

Moreover, by Conservativity we have $\{w, w', w''\}[\neg\neg(\alpha \wedge \neg\beta)] = \{w'\}$. Now, Strict Negation entails

$$\{w, w', w''\}[\neg(\alpha \to \beta)]^* = \{w, w', w''\}$$

and Boethius' Thesis entails

$$\{w, w', w''\}[\neg(\alpha \to \beta)]^* = \{w, w', w''\}[\alpha \to \neg\beta]^*$$
$$= \{w, w'\}$$

Thus, in light of Materiality and Conservativity $\alpha \to \beta$ and $\neg(\alpha \wedge \neg\beta)$ have the same context potential. But the context change potentials of $\neg(\alpha \to \beta)$ and $\neg\neg(\alpha \wedge \neg\beta)$ come apart, no matter whether an unorthodox or an unorthodox Boethian model is chosen. □

111

Note that key to this argument is that Materiality and Conservativity entail that, assuming our compositionality principle from Section 2.5.1, we are forced to accept

$$c[\neg(\alpha \to \beta)] = c[\alpha \wedge \neg\beta],$$

for any context c and any non conditional α and β. Since, as argued above, any theory of update for indicative conditionals should disagree with this prediction, we have to give up our compositionality principle, if we want to hold on to Materiality and Conservativity.

Now, as noted above, some may find the Materiality constraint too strong. For some think that conditionals with epistemically impossible antecedents are unassertable. Would a corresponding weakening of the Materiality constraint help to regain compositionality? Let me provide one reason to think that the answer is 'no'.

Let $\mathcal{L}_\Diamond$ be the fragment of language which is just like $\mathcal{L}$ except that it is closed under a one place operator $\Diamond$ which should be taken to model epistemic possibility claims. Here is the standard dynamic treatment of such a modal

$$c[\Diamond\alpha] = \{w \in c \mid c[\alpha] \neq \varnothing\}.$$

Above we mentioned

Weak Materiality: $c[\alpha \to \beta]^i = c[\Diamond\alpha] \cap \big(c - (c[\alpha] - c[\alpha][\beta])\big),$

A constraint which would account for the unassertability of conditionals with epistemically impossible antecedents. But, as the next corollary of our above argument shows, Weak Materiality will not help to regain compositionality.

Corollary A.2.1. *There is no non-trivial trivial possible worlds model of conversation for $\mathcal{L}_\Diamond$ that satisfies Conservativity, Weak Materiality, the standard dynamic entry for $\Diamond$ and either Strict Negation or Boethious' Thesis.*

Proof. According to weak materiality we have:

$$c[\alpha \to \beta] = c[\Diamond\alpha] \cap (c - (c[\alpha] - c[\alpha][\beta]))$$

By Conservativity

$$(c - (c[\alpha] - c[\alpha][\beta])) = c[\neg(\alpha \wedge \neg\beta)]$$

Since Boolean Intersectivity applies, Weak Materiality is thus equivalent to

$$c[\alpha \to \beta] = c[\Diamond \alpha \wedge \neg(\alpha \wedge \neg\beta)]$$

As above, let W contain at least three worlds w and w' and w''. Then there is a context $c \in \mathcal{P}(W)$ such that $c := \{w, w', w''\}$. Now pick a sentence α that is true at both w' and w'' (and false at w) as well as a sentence β that is true at w'' only.

   Then

$$\{w, w', w''\}[\neg(\Diamond \alpha \wedge \neg(\alpha \wedge \neg\beta))] = \{w, w', w''\} - \{w, w', w''\}[\Diamond \alpha \wedge \neg(\alpha \wedge \neg\beta)]$$
$$= \{w'\}.$$

But, as in the proof of the above proposition, updating the same context with $\neg(\alpha \to \beta)$ will result in $\{w, w', w''\}$, if Strict Negation is used and $\{w, w'\}$, if Boethius' Thesis is used.   □

A.3   Proof of Proposition 2.5.2

Let us repeat Proposition 2.5.2 from Chapter 2.

Proposition 2.5.2 *A model of conversation $\langle \mathcal{P}(W), \cdot[\cdot]^i \rangle$ for $\mathcal{L}$ and $\cdot[\cdot]^i$ as in Def. 2.5.2 satisfies for all $\alpha, \beta \in \mathcal{B}$ (the non conditional fragment of $\mathcal{L}_s$)*

   Conservativity: $c[\alpha]^i = c[\alpha]$,

   Materiality: $c[\alpha \to \beta]^i = c - (c[\alpha] - c[\alpha][\beta])$,

   Strict negation: $c[\neg(\alpha \to \beta)]^i = \{c \in w \mid c[\alpha][\beta] \neq c[\alpha]\}$,

where, as above, $\cdot[\cdot]$ is an update function that satisfies Boolean Intersectivity.

Proof. It is not difficult to see that by the way we defined our support relation in Def. 2.5.1 as well as our notion of Diagonal Updating we have

$$\text{c supports } \phi \text{ iff } c[\phi]_\dagger = c$$

So by lemma A.1.1 and lemma A.1.2 we have

$$\text{c supports } \alpha \text{ iff } c \cap \{w \in c \mid \mathcal{V}(w, \alpha) = 1\} = c$$

for any $\alpha \in \mathcal{B}$.

Now, $c \cap \{w \in c \mid \mathcal{V}(w, \alpha) = 1\}$ will be the largest subset of worlds in c that supports α. Hence,

$$c[\phi]^i = \bigcup \{c' \subseteq c \mid c' \text{ supports } \alpha\} = c \cap \{w \in c \mid \mathcal{V}(w, \alpha) = 1\}$$

for any $\alpha \in \mathcal{B}$.

So informational updating as defined in Def 2.5.2, satisfies Conservativity. *Materiality* follows since for $\alpha, \beta \in \mathcal{B}$:

$$\begin{aligned}
c[\alpha \to \beta]_\dagger = c \text{ iff } c[\alpha \to \beta] &= c, &&\text{by Lemma A.1.2}\\
\text{iff } c[\alpha][\beta] &= c[\alpha], &&\text{by (SC)}\\
\text{iff } c &= c - (c[\alpha] - c[\alpha][\beta]). &&\text{by set theory}
\end{aligned}$$

But since $c - (c[\alpha] - c[\alpha][\beta]) = c[\neg(\alpha \wedge \neg\beta)]$ and $\neg(\alpha \wedge \neg\beta) \in \mathcal{B}$

$$c[\alpha \to \beta]^i = c - (c[\alpha] - c[\alpha][\beta])$$

follows from Conservativity.

Strict Negation holds, trivially, whenever $c = \varnothing$. For non.empty contexts c, c either does contain an α-world that is not a β-world or it does not. By Lemma A.1.1 and Lemma A.1.2 we know that

$$c \text{ supports } \neg(\alpha \to \beta) \text{ iff } c[\neg(\alpha \to \beta)] = c,$$

where $\cdot[\cdot]$ is the update function of the dynamic strict conditional view. Hence, in the first case we know that c itself is the (unique) largest sub-context of c that supports $\neg(\alpha \to \beta)$ while in the latter it tells us that it must be $\varnothing$. Hence, in the former case we have $\bigcup \{c' \subseteq c \mid c' \text{ supports } \neg(\alpha \to \beta)\} = c$ while in the latter we have $\bigcup \{c' \subseteq c \mid c' \text{ supports } \neg(\alpha \to \beta)\} = \varnothing$. So *Strict Negation* holds in each case.

$\square$

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