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Abstract

The Generalized Degrees of Freedom (GDoF) of the two user interference channel are characterized for all parameter regimes under the assumption of finite precision channel state information at the transmitters (CSIT), when a limited amount of cooperation is allowed between the transmitters in the form of π DoF of shared messages. In all cases, the number of over-the-air bits that each cooperation bit buys is shown to be equal to either $0, 1, 1/2$ or $1/3$.

1 Introduction

As distributed computing applications become increasingly practical there is renewed interest in fundamental limits of cooperative communication in robust settings. Partially overlapping message sets naturally arise as computing tasks are distributed with some redundancy, e.g., to account for straggling nodes and adverse channel conditions [?]. Studies of cellular communication with limited backhaul [1], unreliable cooperating links [2], and variable delay constrained messages [3] lead to similar scenarios as well. An elementary model for information theoretic analysis of such settings is an interference network with a limited amount of shared messages between the transmitters. While the body of literature on information theoretic benefits of cooperative communication is too vast to survey here (see [4]), it is notable that settings with limited cooperative capacities remain underexplored, especially with finite precision CSIT. Most closely related to this work are degrees of freedom (DoF) and generalized degrees of freedom (GDoF) studies in [5–11]. Connections to these prior works are explained in the remainder of this section.

Since exact capacity limits tend to be intractable, Generalized Degrees of Freedom (GDoF) studies have emerged as an alternative path to progress for understanding the fundamental limits of wireless networks. Robustness is enforced in GDoF studies by limiting the channel state information at the transmitters (CSIT) to finite precision. Until recently, a stumbling block for robust GDoF characterizations has been the difficulty of obtaining tight converse bounds under finite precision CSIT (cf. Lapidoth-Shamai-Wigger conjecture in [5] and the PN conjecture in [12]). However, the introduction of aligned images bounds in [6] has made it possible to circumvent this challenge. Building upon this opportunity, in this work we pursue the the GDoF of the interference channel under finite precision CSIT with limited cooperation between the transmitters.

Perhaps the most powerful regime for cooperative communication is the strong interference regime, because the sharing of messages allows essentially a re-routing of messages through stronger channels with potentially unbounded benefits. However, this regime turns out to be also the most challenging regime for information theoretic GDoF characterizations under finite precision CSIT. For example, in [7] the GDoF are characterized for the K user broadcast channel obtained by full transmitter cooperation in a K user symmetric interference channel with partial CSIT levels. Remarkably, while the GDoF are characterized for the weak interference regime, the strong interference regime remains open. More recently, the extremal GDoF benefits of transmitter cooperation under finite precision CSIT were characterized in [8] for large interference networks. The benefits of cooperation are shown to be substantial, but the extremal analysis is again limited to weak interference settings. Evidently the strong interference regime poses some challenges. To gauge the difficulty of robust GDoF characterizations in different parameter regimes with limited cooperation, especially the strong interference regime, in this work we explore the 2-user setting.

The main result of this work is the exact GDoF characterization of the 2 user interference channel under finite precision CSIT, when a limited amount of cooperation is allowed between the transmitters in the form of π DoF of shared messages. To place this work in perspective, let us note that the GDoF region for the 2-user broadcast channel (where all messages are shared) under finite-precision CSIT is found in [9], while the GDoF region of 2-user interference channel (where no messages are shared) under finite-precision CSIT is the same as that under perfect CSIT [10]. This work bridges the gap between these two extremes. Finally, let us recall that under perfect CSIT, Wang and Tse found in [11] that each bit of cooperation buys either $0, 1$ or $1/2$ bit over-the-air. In this work, with finite precision CSIT, for all parameter regimes we show that the number of over-the-air bits that each bit of transmitter cooperation buys is either $0, 1, 1/2$ or $1/3$. Remarkably, the $1/3$ factor shows up only in the strong interference regime. Indeed, while other regimes turn out to be relatively straightforward, the central contribution of this work, i.e., its most challenging aspect is the strong interference regime which requires the most sophisticated converse and achievability arguments.

Notations: The notation $(x)^+$ represents $\max(x, 0)$. Index set $\{1, 2, \ldots, n\}$ is represented as $[n]$. $f(x) = o(g(x))$ denotes that $\limsup_{x\to\infty} \frac{|f(x)|}{|g(x)|} = 0$. Define $\lfloor x \rfloor$ as the largest integer that is smaller than or equal to x when x is nonnegative. $\bar{x}$ is defined to be 1 if $x = 2$, and 2 if $x = 1$.

2 Preliminaries

Definition 1 (Power Levels)**.** *Consider the integer valued random variables X_i over alphabet $\mathcal{X}_{\lambda_i}$*

$$\mathcal{X}_{\lambda_i} \triangleq \{0, 1, 2, \cdots, \bar{P}^{\lambda_i} - 1\} \tag{1}$$

where $\bar{P}^{\lambda_i} \triangleq \lfloor \sqrt{P^{\lambda_i}} \rfloor$. We are primarily interested in limits as $P \to \infty$, where $P \in \mathbb{R}_+$ is denoted as power. The constant λ_i refers to the power level of X_i.

Definition 2. *For any nonnegative real numbers X, λ_1, λ_2, define $(X)_{\lambda_1}$, $(X)_{\lambda_1}^{\lambda_2}$ as*

$$(X)_{\lambda_1} \triangleq X - \bar{P}^{\lambda_1} \left\lfloor \frac{X}{\bar{P}^{\lambda_1}} \right\rfloor \tag{2}$$

$$(X)_{\lambda_1}^{\lambda_2} \triangleq \left\lfloor \frac{(X)_{\lambda_2}}{\bar{P}^{\lambda_1}} \right\rfloor \tag{3}$$

In other words, for any $X \in \mathcal{X}_{\lambda_1+\lambda_2}$, $(X)_{\lambda_1}$ is the bottom λ_1 power level of X, $(X)_{\lambda_1}^{\lambda_1+\lambda_2}$ retrieves the top λ_2 levels of X.

3 System Model

3.1 Interference Channel with Limited Cooperation

For GDoF studies, the 2-user interference channel with limited cooperation is described by the following input-output relationship.

$$Y_1(t) = \sqrt{P^{\alpha_{11}}} G_{11}(t) X_1(t) + \sqrt{P^{\alpha_{12}}} G_{12}(t) X_2(t) + Z_1(t) \tag{4}$$

$$Y_2(t) = \sqrt{P^{\alpha_{21}}} G_{21}(t) X_1(t) + \sqrt{P^{\alpha_{22}}} G_{22}(t) X_2(t) + Z_2(t) \tag{5}$$

During the t^{th} use of the channel, $Y_k(t) \in \mathbb{C}$ is the symbol observed by user k. $\sqrt{P}$ is a nominal parameter that approaches infinity to define the GDoF limit. $\alpha_{ki} \in \mathbb{R}^+$ is the coarse channel strength parameter between Transmitter i and Receiver k, and is known to both transmitters and receivers. $G_{ki}(t) \in \mathbb{C}$ are the corresponding channel coefficient values, known perfectly to receivers but only available to transmitters with finite precision. Recall that under finite precision CSIT [6], the transmitters are only aware of the probability density functions of the channel coefficients, and it is assumed that all joint and conditional probability density functions of channel coefficients exist and are bounded.

$X_i(t) \in \mathbb{C}$ is the symbol sent by Transmitter i and is normalized so it is subject to unit transmit power constraint. $Z_k(t) \sim \mathcal{N}_{\mathcal{C}}(0,1)$ is the zero mean unit variance additive white Gaussian noise (AWGN) at Receiver k. Message W_{ii} is the noncooperative message that originates at Transmitter i and is intended for Receiver i, while message W_{0i} is the cooperative message that is also intended for Receiver i, but is known to both transmitters through the limited conference link.

The definitions of probability of error, achievable rate tuples $(R_{11}, R_{22}, R_{01}, R_{02})$, codebooks and capacity region $\mathcal{C}$ are all in the usual Shannon-theoretic sense and will not be repeated [13]. The GDoF region is defined as

$$\begin{aligned}\mathcal{D} = \Bigg\{ & (d_{11}, d_{22}, d_{01}, d_{02}) : \\ & \exists((R_{11}(P), R_{22}(P), R_{01}(P), R_{02}(P)) \in \mathcal{C}(P) \\ & s.t. \quad d_{11} = \lim_{P\to\infty} \frac{R_{11}(P)}{\log(P)}, d_{22} = \lim_{P\to\infty} \frac{R_{22}(P)}{\log(P)}, \\ & \qquad d_{01} = \lim_{P\to\infty} \frac{R_{01}(P)}{\log(P)}, d_{02} = \lim_{P\to\infty} \frac{R_{02}(P)}{\log(P)} \Bigg\}\end{aligned} \tag{6}$$

Limited cooperation is modeled by the constraint,

$$d_{01} + d_{02} \leq \pi, \tag{7}$$

which may be interpreted as a half-duplex link between the transmitters, in which the transmission is one way at any time and the rate of the conference link is upper bounded. The sum-GDoF value for this channel, denoted $\mathcal{D}_{\Sigma,\text{ICLC}}$, is the maximum value of $d_{11} + d_{22} + d_{01} + d_{02}$. The encoding function of Transmitter i is $X_i(t) = f_{i,t}(W_{ii}, W_{01}, W_{02})$.

3.2 Interference Channel

The interference channel corresponds to the setting with no cooperation, i.e., $\pi = 0$, so there are are no cooperative messages W_{01}, W_{02}. In [10], the GDoF region of the interference channel is characterized under perfect CSIT. As noted in [14], for the 2-user interference channel, GDoF under finite precision CSIT are the same as that under perfect CSIT. The sum-GDoF value, denoted $\mathcal{D}_{\Sigma,\text{IC}}$ is found to be,

$$\begin{aligned}\mathcal{D}_{\Sigma,\text{IC}} = \min\Big(&\max(\alpha_{11} - \alpha_{21}, \alpha_{12}) + \max(\alpha_{22} - \alpha_{12}, \alpha_{21}),\\ &\max(\alpha_{11}, \alpha_{12}) + (\alpha_{22} - \alpha_{12})^+,\\ &\max(\alpha_{21}, \alpha_{22}) + (\alpha_{21} - \alpha_{11})^+,\\ &\alpha_{11} + \alpha_{22}\Big) \end{aligned} \tag{8}$$

3.3 Broadcast Channel

The broadcast channel corresponds to unlimited cooperation, e.g., $\pi \to \infty$, so that only cooperative messages, W_{01}, W_{02} are needed for the sum-GDoF characterization. The sum-GDoF value, denoted $\mathcal{D}_{\Sigma,\text{BC}}$ under finite-precision CSIT is found in [9] as,

$$\begin{aligned}\mathcal{D}_{\Sigma,\text{BC}} = \min\big(&\max(\alpha_{11}, \alpha_{12}) + \max(\alpha_{21} - \alpha_{11}, \alpha_{22} - \alpha_{12})^+,\\ &\max(\alpha_{21}, \alpha_{22}) + \max(\alpha_{11} - \alpha_{21}, \alpha_{12} - \alpha_{22})^+\big)\end{aligned} \tag{9}$$

Note that unlike the interference channel, the broadcast channel suffers a loss in GDoF due to finite precision CSIT as compared to perfect CSIT.

4 Results

Our main result appears in the following theorem.

Theorem 1. *If* $\max(\alpha_{11}, \alpha_{22}) \geq \min(\alpha_{12}, \alpha_{21})$, *then*

$$\mathcal{D}_{\Sigma,\text{ICLC}} = \min\Big(\mathcal{D}_{\Sigma,\text{IC}} + \pi, \mathcal{D}_{\Sigma,\text{BC}}\Big). \tag{10}$$

Otherwise, if $\max(\alpha_{11}, \alpha_{22}) < \min(\alpha_{12}, \alpha_{21})$, *then we say the channel is in the strong interference regime, and*

$$\mathcal{D}_{\Sigma,\text{ICLC}} = \min\Big(\mathcal{D}_{\Sigma,\text{IC}} + \pi, \frac{\mathcal{D}_{2e} + \pi}{2}, \frac{\mathcal{D}_{3e} + \pi}{3}, \mathcal{D}_{\Sigma,\text{BC}}\Big) \tag{11}$$

where $\mathcal{D}_{2e} = \alpha_{12} + \alpha_{21}$, *and* $\mathcal{D}_{3e} = \min(\alpha_{21} - \alpha_{22}, \alpha_{11}) + 2\max(\alpha_{21} - \alpha_{11}, \alpha_{22}) + \alpha_{12} + \max(\alpha_{12} - \alpha_{22}, \alpha_{11})$.

Corollary 1. *Let π^* denote the minimum cooperation GDoF needed to achieve the broadcast channel bound. If $\alpha_{22} < \alpha_{11} < \min(\alpha_{12}, \alpha_{21})$, then $\pi^* > \mathcal{D}_{\Sigma,\mathrm{BC}} - \mathcal{D}_{\Sigma,\mathrm{IC}}$, and its value is given below*

$$\pi^* = \begin{cases} M - 2\alpha_{11} & \alpha_{12}, \alpha_{21} \leq M, N \leq M + \alpha_{11} \\ 2N - 4\alpha_{11} - \alpha_{22} & \alpha_{12}, \alpha_{21} \leq M, N \geq M + \alpha_{11} \\ \alpha_{12} + 2\alpha_{21} - 3\alpha_{11} & \alpha_{12} \geq M, \alpha_{21} \leq M \\ 2\alpha_{12} + \alpha_{21} - 3\alpha_{11} & \alpha_{12} \leq M, \alpha_{21} \geq M \\ N + \alpha_{22} - 2\alpha_{11} & \alpha_{12} \geq M, \alpha_{21} \geq M \end{cases} \tag{12}$$

where $M = \alpha_{11} + \alpha_{22}, N = \alpha_{12} + \alpha_{21}$. In all other parameter regimes, $\pi^* = \mathcal{D}_{\Sigma,\mathrm{BC}} - \mathcal{D}_{\Sigma,\mathrm{IC}}$.

5 Proof of Converse (upperbound)

The bound $\mathcal{D}_{\Sigma,\mathrm{ICLC}} \leq \mathcal{D}_{\Sigma,\mathrm{BC}}$ is trivial because full cooperation cannot reduce GDoF. The bound $\mathcal{D}_{\Sigma,\mathrm{ICLC}} \leq \mathcal{D}_{\Sigma,\mathrm{IC}} + \pi$ is also trivial because $d_{11} + d_{22} \leq \mathcal{D}_{\Sigma,\mathrm{IC}}$ and $d_{01} + d_{02} \leq \pi$ by assumption. These bounds hold in all regimes. Next we prove the bound $\mathcal{D}_{\Sigma,\mathrm{ICLC}} \leq (\mathcal{D}_{2e} + \pi)/2$ that holds in the regime $\max(\alpha_{11}, \alpha_{22}) \leq \min(\alpha_{12}, \alpha_{21})$. For compact notation, throughout this section we will suppress conditioning on all channel coefficients that is assumed to be present in all entropies and mutual information terms. We will also suppress $o(\log(P))$ terms that are inconsequential for GDoF. Starting from Fano's inequality,

$$\begin{aligned} nR_{11} + nR_{01} &\leq I(W_{11}, W_{01}; Y_1^{[n]}) && (13)\\ &\leq I(W_{11}, W_{01}; Y_1^{[n]} \mid W_{02}) && (14)\\ &\leq H(Y_1^{[n]} \mid W_{02}) - H(W_{22}) && (15)\\ &\leq n \max(\alpha_{11}, \alpha_{12}) \log(P) - nR_{22} && (16)\\ &\leq n\alpha_{12} \log(P) - nR_{22} && (17) \end{aligned}$$

where (15) holds because $\alpha_{12} \geq \alpha_{22}$, and (17) holds because in the strong interference regime $\alpha_{12} \geq \alpha_{11}$. In the GDoF sense, (17) produces the bound $d_{11} + d_{22} + d_{01} \leq \alpha_{12}$. Similarly, we have the bound $d_{11} + d_{22} + d_{02} \leq \alpha_{21}$, and adding these bounds along with the bound (7) we obtain $\mathcal{D}_{\Sigma,\mathrm{ICLC}} \leq (\mathcal{D}_{2e} + \pi)/2$.

Finally, we prove the remaining bound for the strong interference regime, $\mathcal{D}_{\Sigma,\mathrm{ICLC}} \leq (\mathcal{D}_{3e} + \pi)/3$, for which we will need Aligned Images inequalities. Here we need the deterministic model of [6] whose GDoF bound the GDoF of the original channel model from above. Since this bound only works in the regime $\max(\alpha_{11}, \alpha_{22}) \leq \min(\alpha_{12}, \alpha_{21})$, the deterministic model can be simplified as:

$$\bar{Y}_1(t) = \lfloor \sqrt{P^{\alpha_{11} - \alpha_{21}}} G_{11}(t) \bar{X}_1(t) \rfloor + \lfloor G_{12}(t) \bar{X}_2(t) \rfloor \tag{18}$$

$$\bar{Y}_2(t) = \lfloor G_{21}(t) \bar{X}_1(t) \rfloor + \lfloor \sqrt{P^{\alpha_{22} - \alpha_{12}}} G_{22}(t) \bar{X}_2(t) \rfloor \tag{19}$$

where $\bar{X}_i(t) = \bar{X}_{iR}(t) + j\bar{X}_{iI}(t)$, $i \in [2]$, and $\bar{X}_{1R}, \bar{X}_{1I} \in \{0, 1, 2, \cdots, \lceil\sqrt{P^{\alpha_{21}}}\rceil\}$, while $\bar{X}_{2R}, \bar{X}_{2I} \in \{0, 1, 2, \cdots, \lceil\sqrt{P^{\alpha_{12}}}\rceil\}$.

Let us provide the proof for the case $\alpha_{12} \geq \alpha_{21}$, and the alternative setting of $\alpha_{12} \leq \alpha_{21}$ will follow similarly. For ease of notation, define

$$A = \begin{cases} (\bar{X}_1^{[n]})_{\alpha_{22}}^{\alpha_{21}} & \alpha_{21} \leq \alpha_{11} + \alpha_{22} \\ (\bar{X}_1^{[n]})_{\alpha_{21}-\alpha_{11}}^{\alpha_{21}} & \alpha_{21} \geq \alpha_{11} + \alpha_{22} \end{cases} \tag{20}$$

$$B = \begin{cases} (\bar{X}_1^{[n]})_{\alpha_{21}-\alpha_{11}}^{\alpha_{22}} & \alpha_{21} \leq \alpha_{11} + \alpha_{22} \\ 0 & \alpha_{21} \geq \alpha_{11} + \alpha_{22} \end{cases} \tag{21}$$

$$C = (\bar{X}_2^{[n]})_{\alpha_{12}-\alpha_{22}}^{\alpha_{12}} \tag{22}$$

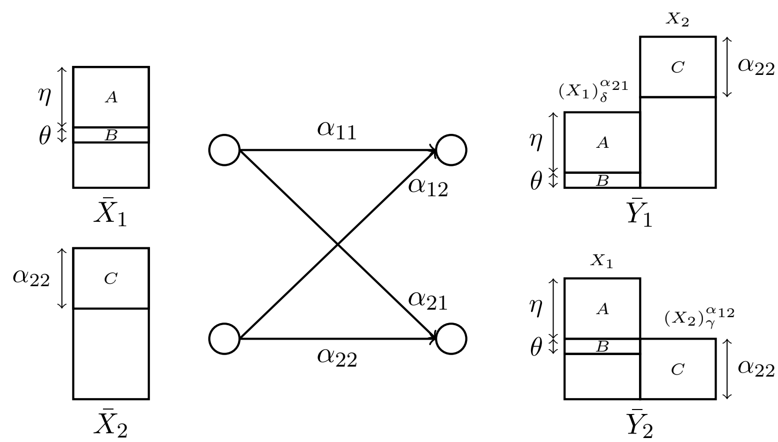

Figure 1: *Power level partitions* A, B, C *where* $\eta = \alpha_{21} - \alpha_{22}, \theta = \alpha_{11} + \alpha_{22} - \alpha_{21}, \delta = \alpha_{21} - \alpha_{11}, \gamma = \alpha_{12} - \alpha_{22}, \bar{X}_1 \in \mathcal{X}_{\alpha_{21}}, \bar{X}_2 \in \mathcal{X}_{\alpha_{12}}$ *and* $\alpha_{21} \leq \alpha_{11} + \alpha_{22}$.

Figure 1 shows the definition under $\alpha_{21} \leq \alpha_{11} + \alpha_{22}$, where the notation $[n]$ is omitted for simplicity. The case $\alpha_{21} \geq \alpha_{11} + \alpha_{22}$ can be shown similarly. Note that if $\alpha_{21} \leq \alpha_{11} + \alpha_{22}$, then A represents the top $\alpha_{21} - \alpha_{22}$ power levels of $\bar{X}_1^{[n]}$, and B represents the remaining power level partition of $\bar{X}_1^{[n]}$ that appears above the noise floor at Receiver 1. Otherwise A represents the top α_{11} levels of $\bar{X}_1^{[n]}$ and B is zero. The combination of A, B is the partition of $\bar{X}_1^{[n]}$ that is heard by Receiver 1 above the noise floor. Note that in both case, A represents a partition of $\bar{X}_1^{[n]}$ that is heard clearly above the signal due to $\bar{X}_2^{[n]}$ at Receiver 2, i.e.,

$$H(A \mid \bar{Y}_2^{[n]}) = no(\log(P)) \tag{23}$$

C represents the top α_{22} power levels of $\bar{X}_2^{[n]}$, which is what Receiver 2 is able to hear from Transmitter 2. Note that the sum of power levels of A and C is always less than α_{12}, which is important when applying the sum-set inequality.

Because C is a function of W_{22}, W_{01}, W_{02},

$$\begin{aligned} & H(C|W_{22}, W_{02}) \\ & = I(C; W_{01}|W_{22}, W_{02}) && (24) \\ & \leq I(A, C; W_{01} \mid W_{22}, W_{02}) \\ & = I(A; W_{01}|W_{22}, W_{02}) + I(C; W_{01}|W_{22}, W_{02}, A) && (25) \\ & \leq I(A; W_{01}|W_{22}, W_{02}) + H(C|W_{22}, W_{02}, A) && (26) \end{aligned}$$

Thus,

$$I(A; W_{01}|W_{22}, W_{02}) \geq H(C|W_{22}, W_{02}) - H(C|W_{22}, W_{02}, A) \quad (27)$$

At the same time, we also have the following bound,

$$\begin{aligned} & H(C|W_{22}, W_{02}) \\ & \geq H(C|W_{02}) - H(W_{22}|W_{02}) && (28) \\ & = I(C; W_{11}, W_{01}|W_{02}) && (29) \\ & = I(C, \bar{Y}_1^{[n]}; W_{11}, W_{01}|W_{02}) - I(\bar{Y}_1^{[n]}; W_{11}, W_{01}|C, W_{02}) && (30) \\ & \geq I(\bar{Y}_1^{[n]}; W_{11}, W_{01}) - H(\bar{Y}_1^{[n]}|C) && (31) \\ & \geq nR_{11} + nR_{01} - H(\bar{Y}_1^{[n]}|C) && (32) \end{aligned}$$

Then from Fano's inequality, we have

$$\begin{aligned} & nR_{11} + nR_{01} \\ & \leq I(W_{11}, W_{01}; \bar{Y}_1^{[n]}) && (33) \\ & \leq H(\bar{Y}_1^{[n]}) - H(A, C|W_{11}, W_{01}) && (34) \\ & \leq H(\bar{Y}_1^{[n]}) - H(A|W_{11}, W_{01}) - H(C|W_{11}, W_{01}, W_{02}) && (35) \\ & = H(\bar{Y}_1^{[n]}) - H(A|W_{11}, W_{01}) - nR_{22} && (36) \\ & = H(\bar{Y}_1^{[n]}) - I(A; W_{22}, W_{02}|W_{11}, W_{01}) - nR_{22} && (37) \\ & \leq H(\bar{Y}_1^{[n]}) - I(A; W_{22}, W_{02}) - nR_{22} && (38) \end{aligned}$$

where (34) is due to the sumset inequality (Theorem 1 in [15]). Rearranging the above inequality we get

$$I(A; W_{22}, W_{02}) \leq H(\bar{Y}_1^{[n]}) - n(R_{11} + R_{22} + R_{01}) \quad (39)$$

Next, applying Fano's inequality at Receiver 2, we have

$$
\begin{aligned}
& nR_{22} + nR_{02} \\
&\quad \leq I(W_{22}, W_{02}; \bar{Y_2}^{[n]}) && (40) \\
&\quad \leq I(W_{22}, W_{02}; \bar{Y_2}^{[n]}, A) && (41) \\
&\quad = I(W_{22}, W_{02}; A) + I(W_{22}, W_{02}; \bar{Y_2}^{[n]}|A) && (42) \\
&\quad = I(W_{22}, W_{02}; A) + H(\bar{Y_2}^{[n]}|A) - H(\bar{Y_2}^{[n]}|A, W_{22}, W_{02}) && (43) \\
&\quad = I(W_{22}, W_{02}; A) + H(\bar{Y_2}^{[n]}|A) - H(\bar{Y_2}^{[n]}|W_{22}, W_{02}) \\
&\qquad\quad + I(\bar{Y}_2^{[n]}; A \mid W_{22}, W_{02}) && (44) \\
&\quad \leq I(W_{22}, W_{02}; A) + H(\bar{Y}_2^{[n]}|A) - H(A, C|W_{22}, W_{02}) \\
&\qquad\quad + I(\bar{Y}_2^{[n]}; A \mid W_{22}, W_{02}) && (45) \\
&\quad \leq I(W_{22}, W_{02}; A) + H(\bar{Y_2}^{[n]}|A) - H(C|A, W_{22}, W_{02}) && (46)
\end{aligned}
$$

where in (45), we used sum-set inequality from Theorem 1 in [15]. Combining (39) and (46),

$$
\begin{aligned}
H(C|A, W_{22}, W_{02}) \leq\ & H(\bar{Y}_1^{[n]}) + H(\bar{Y}_2^{[n]}|A) \\
& - n(R_{11} + 2R_{22} + R_{01} + R_{02}) && (47)
\end{aligned}
$$

Combining (27), (32), (47), we have,

$$
\begin{aligned}
I(A; W_{01}|W_{22}, W_{02}) \geq\ & n(2R_{11} + 2R_{22} + 2R_{01} + R_{02}) \\
& - H(\bar{Y}_1^{[n]}|C) - H(\bar{Y}_1^{[n]}) - H(\bar{Y}_2^{[n]}|A) && (48)
\end{aligned}
$$

Additionally, using the same sum-set inequality to get

$$
H(\bar{Y}_2^{[n]}|W_{22}, W_{02}) \geq H(A, B|W_{22}, W_{02}) \quad (49)
$$

Combining (49) with (44), and rearranging the terms we get

$$
\begin{aligned}
H(B|W_{22}, W_{02}, A) \leq\ & H(A) + H(\bar{Y}_2^{[n]}|A) - n(R_{22} + R_{02}) \\
& - H(A|W_{22}, W_{02}) && (50)
\end{aligned}
$$

Message W_{11} can only be transmitted through A, B because it needs to be successfully decoded by User 1. Therefore,

$$
\begin{aligned}
nR_{11} &= H(A, B|W_{22}, W_{02}, W_{01}) && (51) \\
&= H(A|W_{22}, W_{02}, W_{01}) + H(B \mid W_{22}, W_{02}, W_{01}, A) && (52) \\
&\leq H(A|W_{22}, W_{02}, W_{01}) + H(B|W_{22}, W_{02}, A) && (53)
\end{aligned}
$$

Combining (50) and (53), we get

$$I(A; W_{01}|W_{02}, W_{22}) \leq \\ H(A) + H(\bar{Y}_2^{[n]}|A) - n(R_{11} + R_{22} + R_{02}). \quad (54)$$

Because (54) and (48) are upper and lower bound on the same mutual information, combining them we have

$$3n(R_{11} + R_{22}) + 2n(R_{01} + R_{02}) \leq \\ H(A) + 2H(\bar{Y}_2^{[n]}|A) + H(\bar{Y}_1^{[n]}) + H(\bar{Y}_1^{[n]}|C) \quad (55)$$

The following bounds hold, with $o(\log(P))$ terms omitted.

$$H(A) \leq n \min(\alpha_{21} - \alpha_{22}, \alpha_{11}) \log(P) \quad (56)$$

$$H(\bar{Y}_2^{[n]}|A) \leq n \max(\alpha_{21} - \alpha_{11}, \alpha_{22}) \log(P) \quad (57)$$

$$H(\bar{Y}_1^{[n]}) \leq n\alpha_{12} \log(P) \quad (58)$$

$$H(\bar{Y}_1^{[n]}|C) \leq n \max(\alpha_{12} - \alpha_{22}, \alpha_{11}) \log(P) \quad (59)$$

Thus, (55) yields the GDoF bound,

$$3d_{11} + 3d_{22} + 2d_{01} + 2d_{02} \leq D_{3e} \quad (60)$$

Combining it with (7), we get the bound

$$\mathcal{D}_{\Sigma,\text{ICLC}} \leq \frac{D_{3e} + \pi}{3}. \quad (61)$$

Proceeding similarly, the same bound is obtained for $\alpha_{21} \geq \alpha_{12}$.

6 Proof of Achievability (lowerbound)

The achievability is relatively simpler for weak and mixed interference regimes, so those cases are relegated to the full paper. Here we focus on the strong interference regime. Without loss of generality, we will assume $\alpha_{11} \geq \alpha_{22}$.

6.1 Weak Interference Regime: $\alpha_{11} \geq \alpha_{12}, \alpha_{22} \geq \alpha_{21}$

Considering the weak interference regime firstly. In the regime $\alpha_{11} \geq \alpha_{12} \geq \alpha_{22} \geq \alpha_{21}$, $\mathcal{D}_{\Sigma,\text{IC}} = \mathcal{D}_{\Sigma,\text{BC}} = \alpha_{11}$. And in the regime $\max(2\alpha_{12} + \alpha_{21}, 2\alpha_{21} + \alpha_{12}) \geq \alpha_{11} + \alpha_{22}$, $\mathcal{D}_{\Sigma,\text{IC}} = \mathcal{D}_{\Sigma,\text{BC}} = \min(\alpha_{11} + \alpha_{22} - \alpha_{12}, \alpha_{11} + \alpha_{22} - \alpha_{21})$. So there is no cooperation gain in both regimes. Next, we will consider the achievable scheme in three subcases in

$\alpha_{11} \geq \alpha_{22} \geq \max(\alpha_{12}, \alpha_{21})$, which means the strengths of direct links is always greater than that of cross links. In the achievable scheme, the cooperative messages W_{01}, W_{02} act as the common message, which can be decoded by both users, so let us define $W_0^c = (W_{01}, W_{02})$. In the weak interference regime, we assume $\pi \leq \mathcal{D}_{\Sigma,\text{BC}} - \mathcal{D}_{\Sigma,\text{IC}}$. The achievability for $\pi \geq \mathcal{D}_{\Sigma,\text{BC}} - \mathcal{D}_{\Sigma,\text{IC}}$ is the same as that for $\pi = \mathcal{D}_{\Sigma,\text{BC}} - \mathcal{D}_{\Sigma,\text{IC}}$, because the upperbound of $\mathcal{D}_{\Sigma,\text{BC}}$ is achieved without need for further cooperation.

Case 1: $\alpha_{11} \geq \alpha_{12} + \alpha_{21}, \alpha_{22} \geq \alpha_{12} + \alpha_{21}$

This is the TIN regime, in which treating interference as noise is shown in [16] to be optimal even under finite precision CSIT [8]. In this regime, $\mathcal{D}_{\Sigma,\text{IC}} = \alpha_{11} + \alpha_{22} - \alpha_{12} - \alpha_{21}$ and $\mathcal{D}_{\Sigma,\text{BC}} = \min(\alpha_{11} + \alpha_{22} - \alpha_{12}, \alpha_{11} + \alpha_{22} - \alpha_{21})$. The achievable scheme for this bound is as follows: W_{11}, W_{22} carry $\alpha_{11} - \alpha_{21}, \alpha_{22} - \alpha_{12}$ GDoF, respectively and they are encoded into independent Gaussian codebooks X_{11}, X_{22}, with powers $\text{E}|X_{11}|^2 = P^{-\alpha_{21}}, \text{E}|X_{22}|^2 = P^{-\alpha_{12}}$ so they arrive at the noise floor at their undesired receivers. W_0^c carries π GDoF and it is encoded to a vector Gaussian codebook $X_0^c = (X_{01}^c, X_{02}^c)$ with power covariance matrix $\texttt{Diag}(1-P^{-\alpha_{21}}, 1-P^{-\alpha_{12}})$. The transmitted symbols are $X_1 = X_{01}^c + X_{11}, X_2 = X_{02}^c + X_{22}$. Suppressing the time index for clarity, the received signals are:

$$Y_1 = \sqrt{P^{\alpha_{11}}} G_{11}(X_{01}^c + X_{11}) + \sqrt{P^{\alpha_{12}}} G_{12}(X_{02}^c + X_{22}) + Z_1$$
$$Y_2 = \sqrt{P^{\alpha_{21}}} G_{21}(X_{01}^c + X_{11}) + \sqrt{P^{\alpha_{22}}} G_{22}(X_{02}^c + X_{22}) + Z_2$$

For decoding X_0^c at User 1, the desired signal power is $\sim P^{\alpha_{11}}$ while the interference power is $\sim P^{\alpha_{11}-\alpha_{21}}$, so that the SINR is $\sim P^{\alpha_{21}}$. Similarly, for decoding X_0^c at User 2, the desired signal power is $\sim P^{\alpha_{22}}$ while the interference power is $\sim P^{\alpha_{22}-\alpha_{12}}$, so that the SINR is $\sim P^{\alpha_{12}}$. Since X_0^c carries $\pi \leq \min(\alpha_{12}, \alpha_{21})$ GDoF, it is successfully decoded by both users. After decoding W_0^c, both receivers are able to reconstruct the codeword X_0^c and subtract its contribution from received signals. Then Receiver i decodes X_{ii} for message W_{ii}, while treating the remaining signals as noise. For the decoding of W_{11} by User 1, the desired signal power is $\sim P^{\alpha_{11}-\alpha_{21}}$ while the interference power is $\sim P^0$. Since W_{11} carries only $\alpha_{11} - \alpha_{21}$ GDoF, it is successfully decoded by User 1. Similarly, W_{22} is successfully decoded by User 2.

Case 2: $\alpha_{11} \geq \alpha_{12} + \alpha_{21}, \alpha_{22} \leq \alpha_{12} + \alpha_{21}$

In this regime, $\mathcal{D}_{\Sigma,\text{IC}} = \alpha_{11}$, $\mathcal{D}_{\Sigma,\text{BC}} = \min(\alpha_{11} + \alpha_{22} - \alpha_{12}, \alpha_{11} + \alpha_{22} - \alpha_{21})$. (1) If $\alpha_{12} \geq \alpha_{21}$, W_{11} is split into the private message and common message W_{11}^p and W_{11}^c, which carry $\alpha_{11} - \alpha_{21}, \alpha_{12} + \alpha_{21} - \alpha_{22}$ GDoF respectively. W_{22} carries $\alpha_{22} - \alpha_{12}$ and the cooperative message W_0^c carries π GDoF. Messages $W_{11}^p, W_{11}^c, W_{22}$ are encoded into independent Gaussian codebooks $X_{11}^p, X_{11}^c, X_{22}$, with powers $\text{E}|X_{11}^p|^2 = P^{-\alpha_{21}}, \text{E}|X_{11}^c|^2 = 1 - P^{-\alpha_{21}} - P^{\alpha_{22}-\alpha_{12}-\alpha_{21}}, \text{E}|X_{22}|^2 = P^{-\alpha_{12}}$ Message W_0^c is encoded to a vector Gaussian codebook $X_0^c = (X_{01}^c, X_{02}^c)$ with power covariance matrix $\texttt{Diag}(P^{\alpha_{22}-\alpha_{12}-\alpha_{21}}, 1 - P^{-\alpha_{12}})$. (2) If $\alpha_{12} \leq \alpha_{21}$, $W_{11}^p, W_{11}^c, W_{22}$ carry $\alpha_{11} - \alpha_{21}, 2\alpha_{21} - \alpha_{22}, \alpha_{22} - \alpha_{21}$ GDoF respectively.

Their codewords $X_{11}^p, X_{11}^c, X_{22}$ have powers $\mathrm{E}|X_{11}^p|^2 = P^{-\alpha_{21}}, \mathrm{E}|X_{11}^c|^2 = 1 - P^{-\alpha_{21}} - P^{\alpha_{21}-\alpha_{22}}, \mathrm{E}|X_{22}|^2 = P^{-\alpha_{21}}$. Message W_0^c carries π GDoF and is encoded to a vector Gaussian codebook $X_0^c = (X_{01}^c, X_{02}^c)$ with power covariance matrix $\mathtt{Diag}(P^{2\alpha_{21}-\alpha_{22}}, 1-P^{-\alpha_{21}})$. The transmitted symbols are $X_1 = X_{11}^c + X_{01}^c + X_{11}^p, X_2 = X_{02}^c + X_{22}$. When decoding, User 1 decodes W_{11}^c firstly while treating everything else as noise, then subtracts its contribution from received signals. After this, it decodes X_0^c for message W_0^c. After decoding, reconstructing and subtracting the contribution of X_0^c, it can successfully decode W_{11}^p. User 2 proceeds similarly by decoding X_0^c, X_{11}^c, X_{22} successively.

Case 3: $\alpha_{11} \le \alpha_{12} + \alpha_{21}, \alpha_{22} \le \alpha_{12} + \alpha_{21}$

In this regime, $\mathcal{D}_{\Sigma,\mathrm{IC}} = \min(\alpha_{12}+\alpha_{21}, \alpha_{11}+\alpha_{22}-\alpha_{12}, \alpha_{22}+\alpha_{11}-\alpha_{21})$, $\mathcal{D}_{\Sigma,\mathrm{BC}} = \min(\alpha_{11}+\alpha_{22}-\alpha_{12}, \alpha_{11}+\alpha_{22}-\alpha_{21})$. Only when $2\alpha_{12}+\alpha_{21} \le \alpha_{11}+\alpha_{22}$ and $2\alpha_{21}+\alpha_{12} \le \alpha_{11}+\alpha_{22}$, there is a cooperation gain. We only discuss the order $\alpha_{12} \ge \alpha_{21}$ because the other direction is similar. In the achievable scheme, noncooperative messages W_1 and W_2 are both split, e.g., $W_{11} = (W_{11}^c, W_{11}^p)$, $W_{22} = (W_{22}^c, W_{22}^p)$. W_{11}^c, W_{11}^p, W_{22}^c, W_{22}^p carry $\alpha_{12}+\alpha_{21}-\alpha_{22}$, $\alpha_{11}-\alpha_{21}$, $\alpha_{12}+\alpha_{21}-\alpha_{11}$, $\alpha_{22}-\alpha_{12}$ GDoF respectively. $W_{11}^p, W_{11}^c, W_{22}^p, W_{22}^c$ are encoded into independent Gaussian codebooks $X_{11}^p, X_{11}^c, X_{22}^p, X_{22}^c$ with powers $\mathrm{E}|X_{11}^p|^2 = P^{-\alpha_{21}}, \mathrm{E}|X_{11}^c|^2 = 1 - P^{-\alpha_{21}} - P^{-(\alpha_{12}+\alpha_{21}-\alpha_{22})}, \mathrm{E}|X_{22}^p|^2 = P^{-\alpha_{12}}, \mathrm{E}|X_{22}^c|^2 = 1 - P^{-\alpha_{12}} - P^{-(\alpha_{12}+\alpha_{21}-\alpha_{11})}$. Message W_0^c carries π GDoF and is encoded to vector Gaussian codebook $X_0^c = (X_{01}^c, X_{02}^c)$ with power covariance matrix $\mathtt{Diag}(P^{-(\alpha_{12}+\alpha_{21}-\alpha_{22})}, P^{-(\alpha_{12}+\alpha_{21}-\alpha_{11})})$. The transmitted symbols are $X_1 = X_{11}^c + X_{01}^c + X_{11}^p, X_2 = X_{22}^c + X_{02}^c + X_{22}^p$. For decoding, User 1 (resp. User 2) decodes $X_{11}^c, X_0^c, X_{22}^c, X_{11}^p$ (resp. $X_{22}^c, X_0^c, X_{11}^c, X_{22}^p$) while treating others as noise and then subtracting their reconstructed codewords successively.

6.2 Mixed Interference Regime: $\alpha_{11} \le \alpha_{12}, \alpha_{22} \ge \alpha_{21}$ or $\alpha_{11} \ge \alpha_{12}, \alpha_{22} \le \alpha_{21}$

In the mixed interference regime with additional contraint $\alpha_{11} + \alpha_{22} \le \alpha_{12} + \alpha_{21}$, there exists three subcases where cooperation could provide a gain. Their achievability are shown in the following. Here we also assume $\pi \le \mathcal{D}_{\Sigma,\mathrm{BC}} - \mathcal{D}_{\Sigma,\mathrm{IC}}$ due to the same reason as the weak interference regime.

Case 1: $\alpha_{21} \le \alpha_{22} \le \alpha_{11} \le \alpha_{12}$

In this regime, User 1 is strictly stronger than User 2. $\mathcal{D}_{\Sigma,\mathrm{IC}} = \alpha_{11}+\alpha_{22}-\alpha_{12}, \mathcal{D}_{\Sigma,\mathrm{BC}} = \alpha_{21}$. W_{11}, W_{22}, W_{01} are carrying $\alpha_{11}-\alpha_{21}, \alpha_{22}, \pi$ GDoF respectively and they are encoded to independent Gaussian codebooks X_{11}, X_{22}, X_{01} with powers $\mathrm{E}|X_{11}|^2 = P^{-\alpha_{21}}, \mathrm{E}|X_{22}|^2 = 1 - P^{-\alpha_{22}}, \mathrm{E}|X_{01}|^2 = P^{-\alpha_{22}}$. $X_1 = X_{11}, X_2 = X_{22} + X_{01}$. User 1 decodes X_{22} for W_{22} firstly, while treating everything else as noise. The rate that is supported for this message is:

$$
\begin{aligned}
&= \log\Big(\frac{P^{\alpha_{12}}|G_{12}|^2(1-P^{-\alpha_{22}})}{1+P^{\alpha_{11}}|G_{11}|^2P^{-\alpha_{21}}+P^{\alpha_{12}}|G_{12}|^2P^{-\alpha_{22}}}\Big)\\
&\geq \alpha_{22}\log(P)+o(\log(P)) \qquad (62)
\end{aligned}
$$

which gives us the GDoF value $d_{22} = \alpha_{22}$. After decoding W_{22}, Receiver 1 is able to reconstruct the codeword X_{22} and subtract its contribution from the received signal and then decodes the codeword X_{01} for its desired message W_{01}, while treating the remaining signal as noise. The rate is:

$$
\begin{aligned}
&\log\Big(\frac{P^{\alpha_{12}}|G_{12}|^2P^{-\alpha_{22}}}{1+P^{\alpha_{11}}|G_{11}|^2P^{-\alpha_{21}}}\Big)\\
&\geq (\alpha_{12}+\alpha_{21}-\alpha_{11}-\alpha_{22})\log(P)+o(\log(P)) \qquad (63)
\end{aligned}
$$

The GDoF value is $\alpha_{12}+\alpha_{21}-\alpha_{11}-\alpha_{22}$. Since π is less than it, W_{01} can be successfully decoded. After reconstructing and subtracting the contribution of codeword X_{01}, User 1 decodes X_{11} for its desired message W_{11}, while treating the remaining signal as noise. The rate is

$$
\begin{aligned}
&\log\Big(\frac{P^{\alpha_{11}}|G_{11}|^2P^{-\alpha_{21}}}{1}\Big)\\
&\geq (\alpha_{11}-\alpha_{21})\log(P)+o(\log(P)) \qquad (64)
\end{aligned}
$$

which gives us the GDoF value $d_{11} = \alpha_{11}-\alpha_{21}$, such that message W_{11} can be successfully decoded. Receiver 2 is able to decode X_{22} by treating everything else as noise.

Case 2: $\alpha_{12} \leq \alpha_{22} \leq \alpha_{11} \leq \alpha_{21}$

In this regime, $\mathcal{D}_{\Sigma,\mathrm{IC}} = \alpha_{11}+\alpha_{22}-\alpha_{12}, \mathcal{D}_{\Sigma,\mathrm{BC}} = \alpha_{21}$. Messages W_{11}, W_{22}, W_{02} are carrying $\alpha_{11}, \alpha_{22}-\alpha_{12}, \pi$ GDoF respectively. They are encoded to independent codewords X_{11}, X_{22}, X_{02} with powers $\mathrm{E}|X_{11}|^2 = 1-P^{-\alpha_{11}}, \mathrm{E}|X_{22}|^2 = P^{-\alpha_{12}}, \mathrm{E}|X_{02}|^2 = P^{-\alpha_{11}}$. $X_1 = X_{11}+X_{02}, X_2 = X_{22}$. This case is symmetric to $\alpha_{21} \leq \alpha_{22} \leq \alpha_{11} \leq \alpha_{12}$.

Case 3: $\alpha_{22} \leq \alpha_{12} \leq \alpha_{11} \leq \alpha_{21}$

In this regime, $\mathcal{D}_{\Sigma,\mathrm{IC}} = \alpha_{11}, \mathcal{D}_{\Sigma,\mathrm{BC}} = \alpha_{21}$. W_{11}, W_{02} carry α_{11}, π GDoF respectively. They are encoded to codewords X_{11}, X_{02} with powers $\mathrm{E}|X_{11}|^2 = 1-P^{-\alpha_{11}}, \mathrm{E}|X_{02}| = P^{-\alpha_{11}}$. $X_1 = X_{11}+X_{02}$. User 1 decodes X_{11} for its desired message W_{11}, while treating everything else as noise. User 2 decodes X_{11} firstly, then subtracts its contribution from the received signal. After that, Receiver 2 can decode X_{02} for message W_{02}.

6.3 Strong Interference Regime: $\max(\alpha_{11}, \alpha_{22}) \leq \min(\alpha_{12}, \alpha_{21})$

Even though we say the strong interference regime as $\max(\alpha_{11}, \alpha_{22}) \leq \min(\alpha_{12}, \alpha_{21})$, a more general setting is $\alpha_{22} \leq \alpha_{21}, \alpha_{11} \leq \alpha_{12}$. So here we cover the achievability for the regime $\alpha_{22} \leq \alpha_{21} \leq \alpha_{11} \leq \alpha_{12}$, which is the gap between the general setting and our setting. In this regime, $\mathcal{D}_{\Sigma,\text{ICLC}} = \min(\mathcal{D}_{\Sigma,\text{IC}} + \pi, \mathcal{D}_{\Sigma,\text{BC}}) = \min(\alpha_{11} + \pi, \alpha_{12})$. The achievability scheme is quite trivial: Message W_{11} carries α_{11} GDoF while W_{01} carries π GDoF for $\pi \leq \alpha_{12} - \alpha_{11}$. It act as a multiple access channel to User 1, which can decode both W_{11} and W_{01}.

Let us begin with an illustrative example where $\alpha_{11} = \alpha_{22} = 2, \alpha_{12} = 5, \alpha_{21} = 3$. For this setting, $\mathcal{D}_{\Sigma,\text{BC}} = 6$ according to [9] and $D_{\Sigma,\text{IC}} = 3$ according to [10]. Let us consider how much cooperation is needed in this case to achieve $\mathcal{D}_{\Sigma,\text{BC}}$. The achievable scheme of [9] summarized in Figure 2, requires $\pi = 6$ GDoF of cooperation, i.e., all messages must be shared between the two transmitters. This is because in order to take advantage of the strong interference links, the private messages of Users 1 and 2, are sent from opposing transmitters, i.e., Transmitters 2 and 1, respectively. These are[1] messages W_{01}^p, W_{02}^p in Fig. 2. The common message W_o^c that is decoded by both users is sent from both transmitters, so it is shared as well. However, as shown in Theorem 1 in this paper, the sum-GDoF of limited

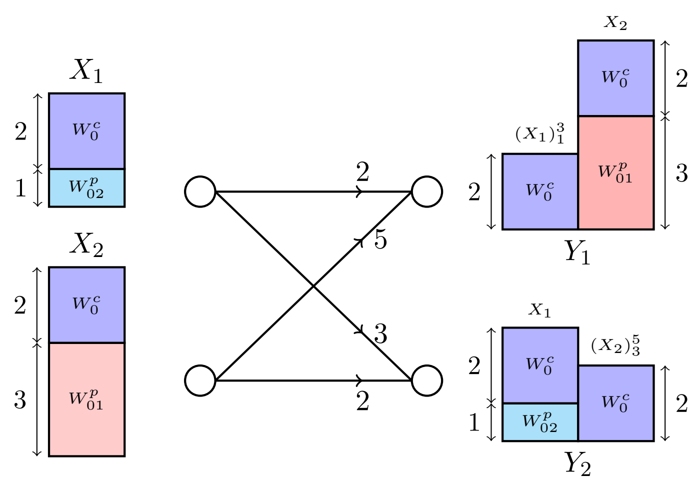

Figure 2: *The scheme from [9] requires $\pi = 6$ GDoF of cooperation to achieve the broadcast channel bound.*

[1]In the description of the achievable scheme, we partition messages into sub-messages, and in labeling these sub-messages we use subscripts to indicate transmitter cooperation, while the superscripts are associated with the decodability of the message. Specifically, if the subscript contains a 0 then that part of the message is shared between the two transmitters, otherwise it is not. Similarly, if the superscript is a p then that part of the message is private, i.e., only decodable at its desired receiver, otherwise it is common, i.e., decodable by both receivers.

cooperation interference channel for this example is $D_{\Sigma,\text{ICLC}} = \min(3+\pi, \frac{8+\pi}{2}, \frac{13+\pi}{3}, 6)$. Therefore, $\pi^* = 5$ is the minimum value of cooperative GDoF needed to achieve the BC bound. The optimally efficient scheme is shown in Figure 3. The improvement in efficiency come from the observation that part of the common message (in this case, W_{22}) can be transmitted from only one transmitter (in this case, Transmitter 2), and therefore requires no cooperation.

The achievable scheme is as follows: Messages W_{01}, W_{02} are split into the cooperative common message W_0^c and the cooperative private messages W_{01}^p, W_{02}^p. Messages $W_{22}, W_0^c, W_{01}^p, W_{02}^p$ carry $1, 1, 3, 1$ GDoF respectively such that $\pi = 5$. $W_{22}, W_{01}^p, W_{02}^p$ are encoded into independent Gaussian codebooks $X_{22}, X_{01}^p, X_{02}^p$ respectively with powers $\mathrm{E}|X_{22}|^2 = 1 - P^{-1}$, $\mathrm{E}|X_{01}^p|^2 = P^{-2}$, $\mathrm{E}|X_{02}^p|^2 = P^{-2}$, Message W_0^c carries 1 GDoF and is encoded to a vector Gaussian codebook $X_0^c = (X_{01}^c, X_{02}^c)$ with power covariance matrix $\mathtt{Diag}(1 - P^{-2}, P^{-1} - P^{-2})$. The transmitted symbols are $X_1 = X_{01}^c + X_{02}^p, X_2 = X_{22} + X_{02}^c + X_{01}^p$. Suppressing the time index for clarity, the received signals are:

$$Y_1 = \sqrt{P^2} G_{11}(X_{01}^c + X_{02}^p) + \sqrt{P^5} G_{12}(X_{22} + X_{02}^c + X_{01}^p) + Z_1$$
$$Y_2 = \sqrt{P^3} G_{21}(X_{01}^c + X_{02}^p) + \sqrt{P^2} G_{22}(X_{22} + X_{02}^c + X_{01}^p) + Z_2$$

When decoding, User 1 first decodes X_{22} for W_{22} while treating everything else as Gaussian noise. Since X_{22} is received at power level $\sim P^5$ while all other signals are received with power levels $\sim P^4$ or lower, the SINR for decoding W_{22} is $\sim P^1$, which gives us the GDoF value $d_{22} = 1$. After decoding W_{22}, Receiver 1 is able to reconstruct codeword X_{22} and subtract its contribution from the received signal. After this, Receiver 1 decodes the codeword X_0^c for message W_0^c, while treating the remaining signals as Gaussian noise. Since the desired signal for this decoding is received with power level P^4 while all other signals are received with power levels P^3 or less, the SINR for decoding W_0^c is P^1 which gives GDoF value $d_0^c = 1$. Then Receiver 1 subtracts the contribution of X_0^c and decodes message W_{01}^p while treating all other remaining signals as Gaussian noise. As evident from Fig. 3, the SINR for this decoding is P^3 which gives us GDoF value $d_{01}^p = 3$. Receiver 2 proceeds similarly by successively decoding W_0^c, W_{22}, W_{02}^p.

In general, there are 5 subcases in the strong interference regime, which cover all possibilities.

Case 1: $\alpha_{12} \leq \alpha_{11} + \alpha_{22}, \alpha_{21} \leq \alpha_{11} + \alpha_{22}, \alpha_{12} + \alpha_{21} \geq 2\alpha_{11} + \alpha_{22}$

In this regime, the sum-GDoF, as characterized in (11), is:

$$\mathcal{D}_{\Sigma,\text{ICLC}} = \min\Big(\min(\alpha_{12}, \alpha_{21}) + \pi, \frac{\alpha_{12} + \alpha_{21} + \pi}{2}, \frac{\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22} + \pi}{3}, \alpha_{12} + \alpha_{21} - \alpha_{11}\Big) \tag{65}$$

If $2\alpha_{11} + 2\alpha_{22} - \alpha_{12} - \alpha_{21} \leq \pi \leq 2\alpha_{12} + 2\alpha_{21} - 4\alpha_{11} - \alpha_{22}$, the third bound is tight. $W_{11}, W_{22}, W_{01}^p, W_{02}^p$ carry $(2\alpha_{21} - \alpha_{12} + 2\alpha_{11} - \alpha_{22} - \pi)/3, (2\alpha_{12} - \alpha_{21} + 2\alpha_{22} - \alpha_{11} - \pi)/3, (\alpha_{11} + \alpha_{22} + \alpha_{12} - 2\alpha_{21} + \pi)/3, (\alpha_{11} + \alpha_{22} + \alpha_{21} - 2\alpha_{12} + \pi)/3$ GDoF respectively.

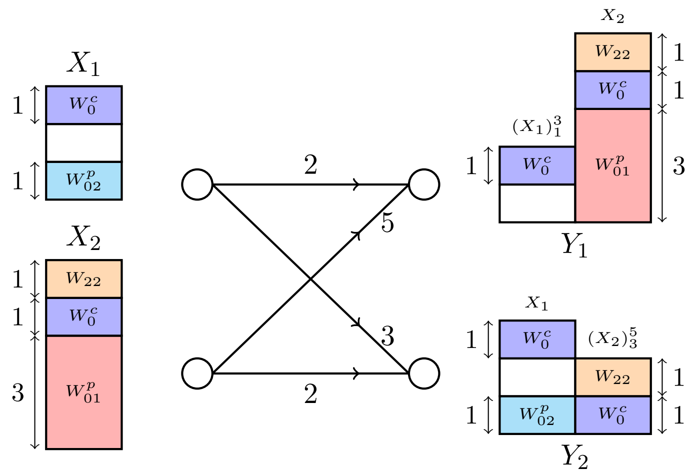

Figure 3: *The optimally efficient achievable scheme achieves the broadcast channel bound with only $\pi = 5$ GDoF of cooperation.*

They are encoded into independent Gaussian codebooks $X_{11}, X_{22}, X_{01}^p, X_{02}^p$ with powers $\mathrm{E}|X_{11}|^2 = 1 - P^{-d_{11}}, \mathrm{E}|X_{22}|^2 = 1 - P^{-d_{22}}, \mathrm{E}|X_{01}^p|^2 = P^{(\alpha_{11}+\alpha_{22}-2\alpha_{12}-2\alpha_{21}+\pi)/3}, \mathrm{E}|X_{02}^p|^2 = P^{(\alpha_{11}+\alpha_{22}-2\alpha_{12}-2\alpha_{21}+\pi)/3}$, where d_{11}, d_{22} are the GDoFs of W_{11}, W_{22} respectively. W_0^c carries $(\alpha_{12}+\alpha_{21}-2\alpha_{11}-2\alpha_{22}+\pi)/3$ GDoF and it is encoded to a vector Gaussian codebook $X_0^c = (X_{01}^c, X_{02}^c)$ with power covariance matrix $\texttt{Diag}(P^{-d_{11}} - P^{(\alpha_{11}+\alpha_{22}-2\alpha_{12}-2\alpha_{21}+\pi)/3}, P^{-d_{22}} - P^{(\alpha_{11}+\alpha_{22}-2\alpha_{12}-2\alpha_{21}+\pi)/3})$. The transmitted symbols are $X_1 = X_{11} + X_{01}^c + X_{02}^p, X_1 = X_{11} + X_{02}^c + X_{01}^p$. When decoding, User 1 decodes X_{22} for W_{22} while treating everything else as noise. The desired power is $\sim P^{\alpha_{12}}$ while the interference power is $\sim P^{-d_{22}+\alpha_{12}}$, so that the SINR is $\sim P^{d_{22}}$. Therefore W_{22} can be successfully decoded. After this, User 1 subtracts the contribution of reconstructed codeword X_{22} from received signals and decodes X_0^c for W_0^c. The desired signal power is $\sim P^{-d_{22}+\alpha_{12}}$ while the interference power is $\sim P^{\alpha_{11}}$. Since W_0^c carries $(\alpha_{12}+\alpha_{21}-2\alpha_{11}-2\alpha_{22}+\pi)/3 = -d_{22}+\alpha_{12}-\alpha_{11}$ GDoF, it is successfully decoded by User 1. After subtracting the reconstructed codeword X_0^c, User 1 decodes X_{11} for W_{11}. The desired power for W_{11} is $\sim P^{\alpha_{11}}$ while the interference power is $\sim P^{(\alpha_{11}+\alpha_{22}+\alpha_{12}-2\alpha_{21}+\pi)/3}$, so that the SINR is $P^{2\alpha_{21}-\alpha_{12}+2\alpha_{11}-\alpha_{22}}$. Hence W_{11} can be successfully decoded. Finally, User 1 decodes X_{01}^p for W_{01}^p. The desired signal power is $\sim P^{(\alpha_{11}+\alpha_{22}+\alpha_{12}-2\alpha_{21}+\pi)/3}$ while the interference power is $\sim P^0$. Since W_{01}^p carries $(\alpha_{11}+\alpha_{22}+\alpha_{12}-2\alpha_{21}+\pi)/3$ GDoF, it is successfully decoded. User 2 proceeds similarly by successively decoding $W_{11}, W_0^c, W_{22}, W_{02}^p$. See Figure 4 for an illustration.

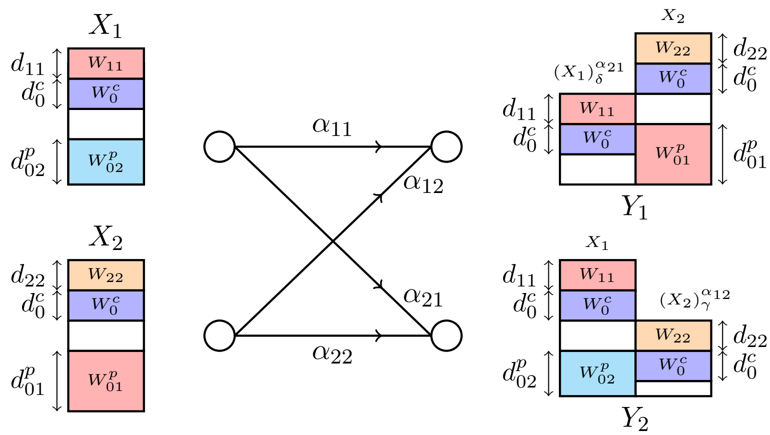

Figure 4: *Signal partition in the regime* $\alpha_{12}, \alpha_{21} \leq \alpha_{11} + \alpha_{22}, \alpha_{12} + \alpha_{21} \geq 2\alpha_{11} + \alpha_{22}$, *where* $\delta = \alpha_{21} - \alpha_{11}, \gamma = \alpha_{12} - \alpha_{22}$.

Case 2: $\alpha_{12} \geq \alpha_{11} + \alpha_{22}, \alpha_{21} \leq \alpha_{11} + \alpha_{22}$

In this regime, the sum-GDoF is

$$\mathcal{D}_{\Sigma,\text{ICLC}} = \min(\alpha_{21} + \pi, \frac{2\alpha_{12} + \alpha_{21} + \pi}{3}, \alpha_{12} + \alpha_{21} - \alpha_{11}) \tag{66}$$

If $\alpha_{12} - \alpha_{21} \leq \pi \leq \alpha_{12} + 2\alpha_{21} - 3\alpha_{11}$, the second bound is tight. The achievable scheme is as follows: $W_{11}, W_{22}, W_{01}^p, W_{02}^p$ are carrying $(2\alpha_{21} + \alpha_{12} - 3\alpha_{22} - \pi)/3, (3\alpha_{22} + \alpha_{12} - \alpha_{21} - \pi)/3, (2\alpha_{12} - 2\alpha_{21} + \pi)/3, (\alpha_{21} - \alpha_{12} + \pi)/3$ respectively. They are encoded into independent Gaussian codebooks $X_{11}, X_{22}, X_{01}^p, X_{02}^p$ with powers $\mathrm{E}|X_{11}|^2 = 1 - P^{-d_{11}}, \mathrm{E}|X_{22}|^2 = 1 - P^{-d_{22}}, \mathrm{E}|X_{01}^p|^2 = P^{-\alpha_{22}}$, $\mathrm{E}|X_{02}^p|^2 = P^{-(2\alpha_{21}+\alpha_{12}-\pi)/3}$. W_0^c carries $(\alpha_{21} - \alpha_{12} + \pi)/3$ GDoF and is encoded into a vector Gaussian codebook with power covariance matrix $\texttt{Diag}(P^{-d_{11}} - P^{-(2\alpha_{21}+\alpha_{12}-\pi)/3}, P^{-d_{22}} - P^{-\alpha_{22}})$. The transmitted symbols are $X_1 = X_{11} + X_0^c + X_{02}^p, X_2 = X_{22} + X_0^c + X_{01}^p$. When decoding, User 1 decoded W_{22}, W_0^c successively while treating everything else as noise. After subtracting the reconstructed codeword X_{22}, X_0^c, Receiver 1 jointly decodes W_{11} and W_{01}^p while treating the remaining signal as noise. User 2 proceeds similarly.

Case 3: $\alpha_{12} \leq \alpha_{11} + \alpha_{22}, \alpha_{21} \geq \alpha_{11} + \alpha_{22}$

This is symmetric to $\alpha_{12} \geq \alpha_{11} + \alpha_{22}, \alpha_{21} \leq \alpha_{11} + \alpha_{22}$. Therefore the achievability is also similar.

Case 4: $\alpha_{12} \geq \alpha_{11} + \alpha_{22}, \alpha_{21} \geq \alpha_{11} + \alpha_{22}$

In this regime, we have

$$\mathcal{D}_{\Sigma,\text{ICLC}} = \min\Big(\alpha_{11} + \alpha_{22} + \pi, \frac{2\alpha_{12} + 2\alpha_{21} - \alpha_{11} - \alpha_{22} + \pi}{3}, \alpha_{12} + \alpha_{21} - \alpha_{11}\Big) \tag{67}$$

If $\alpha_{12} + \alpha_{21} - 2\alpha_{11} - 2\alpha_{22} \leq \pi \leq \alpha_{12} + \alpha_{21} + \alpha_{22} - 2\alpha_{11}$, the second bound in (67) is tight. It is achieved as follows: $W_{11}, W_{22}, W_{01}^p, W_{02}^p$ carry $(\alpha_{12} + \alpha_{21} - 2\alpha_{22} + \alpha_{11} - \pi)/3, (\alpha_{12} + \alpha_{21} - 2\alpha_{11} + \alpha_{22} - \pi)/3, (2\alpha_{12} + \pi - \alpha_{11} - \alpha_{22} - \alpha_{21})/3, (2\alpha_{21} + \pi - \alpha_{11} - \alpha_{22} - \alpha_{12})/3$ GDoF respectively and are encoded into independent Gaussian codewords $X_{11}, X_{22}, X_{01}^p, X_{02}^p, X_0^c$. with powers $\text{E}|X_{11}|^2 = 1 - P^{-d_{11}}, \text{E}|X_{22}|^2 = 1 - P^{-d_{22}}, \text{E}|X_{01}^p|^2 = P^{-\alpha_{22}}, \text{E}|X_{02}^p|^2 = P^{-\alpha_{11}}$. W_0^c carries $(2\alpha_{11} + 2\alpha_{22} + \pi - \alpha_{12} - \alpha_{21})/3$ GDoF and is encoded into a vector Gaussian codebook X_0^c with power covariance matrix $\texttt{Diag}(P^{-d_{11}} - P^{-\alpha_{11}}, P^{-d_{22}} - P^{-\alpha_{22}})$. The transmitted symbols are $X_1 = X_{11} + X_0^c + X_{02}^p, X_2 = X_{22} + X + X_0^c + X_{01}^p$. User 1 decodes W_{22}, W_0^c successively while treating everything else as noise. After subtracting the contribution of X_{22}, X_0^c, it jointly decodes W_{11} and W_{01}^p while treating the remaining signals as noise. User 2 proceeds similarly. The receiver signal depiction for $\pi = \pi^*$ is shown in Figure 5.

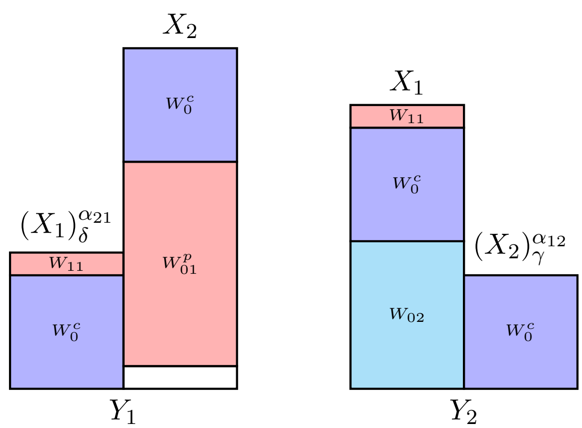

Figure 5: Receiver's signal depiction of $\alpha_{12}, \alpha_{21} \geq \alpha_{11} + \alpha_{22}$, $\pi = \pi^*$, $\delta = \alpha_{21} - \alpha_{11}, \gamma = \alpha_{12} - \alpha_{22}$.

Case 5: $\alpha_{12} \leq \alpha_{11} + \alpha_{22}, \alpha_{21} \leq \alpha_{11} + \alpha_{22}, \alpha_{12} + \alpha_{21} \leq 2\alpha_{11} + \alpha_{22}$

The sum-GDoF value in this case is characterized as:

$$\mathcal{D}_{\Sigma,\text{ICLC}} = \min\Big(\min(\alpha_{12}, \alpha_{21}) + \pi, \frac{\alpha_{12} + \alpha_{21} + \pi}{2}, \alpha_{12} + \alpha_{21} - \alpha_{11}\Big) \tag{68}$$

We assume $\alpha_{12} \geq \alpha_{21}$. The other direction can be achieved similarly. If $\alpha_{12} - \alpha_{21} \leq \pi \leq N - 2\alpha_{11}$. The second bound is tight and is achieved by the following: $W_{11}, W_{22}, W_{01}, W_{02}$ are encoded into independent Gaussian codewords $X_{11}, X_{22}, X_{01}, X_{02}$ with powers $\mathrm{E}|X_{11}|^2 = 1 - P^{-(\alpha_{12}+\alpha_{21}-\pi)/2}, \mathrm{E}|X_{22}|^2 = 1 - P^{-(\alpha_{12}+\alpha_{21}-\pi)/2}, \mathrm{E}|X_{01}|^2 = P^{-(\alpha_{12}+\alpha_{21}-\pi)/2}, \mathrm{E}|X_{02}|^2 = P^{-(\alpha_{12}+\alpha_{21}-\pi)/2}$. $X_1 = X_{11} + X_{02}, X_2 = X_{22} + X_{01}$. In addition, $W_{11}, W_{22}, W_{01}, W_{02}$ carry $(\alpha_{21} + 2\alpha_{11} - \alpha_{12} - \pi)/2, \alpha_{12} - \alpha_{11}, (\alpha_{12} - \alpha_{21} + \pi)/2, (\alpha_{21} - \alpha_{12} + \pi)/2$ GDoF respectively. User 1 decodes X_{22}, X_{11}, X_{01} successively, User 2 jointly decodes X_{11} and X_{22} while treating everything else as noise, after this User 2 subtracts X_{11} and X_{22} and decodes X_{02}. The receiver signal depiction is shown in Figure 6.

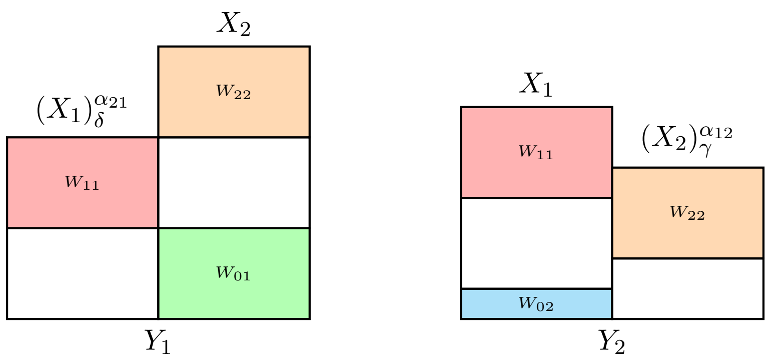

Figure 6: Receiver's signal depiction in the regime $\alpha_{12} \leq \alpha_{11} + \alpha_{22}, \alpha_{21} \leq \alpha_{11} + \alpha_{22}, \alpha_{12} + \alpha_{21} \leq 2\alpha_{11} + \alpha_{22}, \pi = \pi^*, \delta = \alpha_{21} - \alpha_{11}, \gamma = \alpha_{12} - \alpha_{22}$.

7 Conclusion

The aligned image sets approach of [6], and the sum-set inequalities of [15] are utilized to characterize the sum-GDoF of two user interference channel with limited cooperation, which bridges the gap between the interference channel and broadcast channel. The sum-GDoF value are characterized for arbitrary parameter regimes.

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