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Author
Olander, Donald R.

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Donald R. Olander

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by
Donald R. Olander
Inorganic Materials Research Division of the Lawrence Berkeley Laboratory and the Department of Nuclear Engineering of the University of California, Berkeley, CA 94720

ABSTRACT

The interstage flows are determined for an ideal isotope separation cascade in which the enriched stream from a stage is fed two stages up and the depleted stream is delivered to the next lower stage. It is shown that this type of cascade configuration is superior to the conventional ideal cascade if the separating power of the units comprising the cascade increases with decreasing cut.

1. Introduction

Ideal cascades for isotope separation are usually constructed so that the heads stream from a stage furnishes part of the feed to the next stage up and the tails stream from the stage is fed to the adjacent lower stage. This symmetric arrangement is satisfactory if the separative power of the individual separating units of which the cascade consists is independent of the cut at which they are operated. However, some separating units operate more efficiently at a cut considerably less than one-half than they do at a cut in the neighborhood of one-half. The Becker separation nozzle is an example of an isotope separation device which exhibits this preference(1). Operation of these units in a cascade can be improved by carrying the enriched stream from each stage two stages forward instead of one stage forward. This modification permits each separative unit to operate at a lower cut than the one that would be required of it in a conventional cascade, yet maintains the condition of no-mixing of streams of different composition. The "two-up, one-down" ideal cascade has been briefly mentioned in the literature (2,3), but no detailed analysis has been reported. It is the purpose of this paper to provide such an analysis.

2. Enrichment

A schematic of a two-up, one-down ideal cascade is shown in Figure 1. Each separating unit in the cascade operates at the same throughput and cut; the heads and tails flow rates from each stage are permitted to vary in a manner which insures that the two streams which provide the feed to a stage have the same composition. The required variation of interstage flows with height in the cascade is accomplished by joining the appropriate number of separating units in parallel at each stage.

The relation between the heads and tails separation factors of the individual separating units can be obtained by considering the streams entering and leaving stage i+1 in Fig. 1. For simplicity, it is assumed that the desired isotope (e.g., U-235) is present in dilute concentrations, so that the abundance ratio is adequately approximated by the isotopic fraction. The condition of no-mixing is expressed by:

$$v_{i+2} = u_{i-1} \tag{1}$$

The heads separation factor is defined by:

$$\alpha = \frac{u_{i+1}}{u_{i-1}} = \frac{u_{i+1}}{u_i}\,\frac{u_i}{u_{i-1}} \qquad (2)$$

and the tails separation factor is:

$$\beta = \frac{u_{i-1}}{v_{i+1}} = \frac{u_i}{u_{i-1}} = \frac{u_{i+1}}{u_i} \qquad (3)$$

where the second equality in Eq. (3) is obtained by increasing the index i by one and using Eq. (1). The last equality is obtained by incrementing i by one. Because all of the separating units in the cascade are operated in an identical manner, α and β are independent of stage number. Combination of Eqs. (2) and (3) shows that $\alpha = \beta^2$, and the overall separation factor, $\alpha\beta$, is equal to β^3.

Equation (3) gives the enrichment gradient in the cascade in the integrated form $u_i = C\beta^i$, where C is a constant of integration. Using the boundary condition $u_1 = \beta^3 X_W$ obtained from the bottom stage, the heads stream concentration is given as a function of stage number by:

$$u_i = X_W\beta^{i+2} \qquad 1 \leq i \leq n \qquad (4)$$

and, using Eq. (1)

$$v_i = X_W\beta^{i-1} \qquad 1 \leq i \leq n \qquad (5)$$

The cut θ is determined by a material balance on stage i+1:

$$v_{i+2} = u_{i-1} = \theta u_{i+1} + (1-\theta)v_{i+1}$$

Upon dividing this equation by u_{i-2} and using Eqs. (1) and (3), we have:

$$\theta = \frac{\beta-1}{\beta^3-1} \qquad (6)$$

3. Interstage Flows

The two-up, one-down cascade provides two enriched streams rather than the single product obtained from a symmetric ideal cascade. In the latter, specification of the feed, product and waste compositions fixes the input and output flow rates (relative to, say, the product flow rate). When a cascade delivers two products, as in the present instance, specification of all external compositions does not fix the external flows (even the relative flow rates). Material balances around the cascade on both isotopes and on the desired isotope give:

$$F = P_1 + P_2 + W \tag{7}$$

$$X_F F = X_{p_1} P_1 + X_{p_2} P_2 + X_w W \tag{8}$$

when X_F, X_w, X_{p_1} and X_{p_2} $(=X_{p_1}/\beta)$ are specified, Eqs. (7) and (8), lead to a relationship between W/P_1 and P_2/P but do not uniquely determine either. This determination is possible only after the entire cascade analysis is complete.

In analyzing the cascade shown in Fig. 1, it is convenient to specify the total number of stages n, and the number of stages in the stripper, n_w, instead of the exit compositions X_w and X_{p_1}. By using the known feed concentration and Eqs. (4) and (5), values of n and n_w which provide upper product and waste compositions in the neighborhood of specified nominal values can be selected.

The tails separation factor β is presumed known. The throughput per separating unit which results in this value of β is also assumed to be known.

Material balances including stage j in the stripping section and the waste end of the cascade yield:

$$M_j + M_{j-1} + W = N_{j+1}$$

$$u_j M_j + u_{j-1} M_{j-1} + X_W W = v_{j+1} N_{j+1}$$

Combining these equations and eliminating the compositions in terms of β and j by use of Eqs. (4) and (5) provides the difference equation:

$$\beta^j(\beta^2-1)Y_j^s + \beta^j(\beta-1)Y_{j-1}^s = \beta^j - 1 \qquad (9)$$

where:

$$Y_j^s = M_j/W \qquad (10)$$

is the heads flow rate from stage j relative to the waste flow rate. Comparable material balances over stage 1 yield:

$$Y_1^s = \frac{1}{\beta(\beta+1)} \qquad (11)$$

Equation (9) is a first order, nonhomogeneous difference equation which has the general solution:

$$Y_j^s = k\left(-\frac{1}{\beta+1}\right)^{j-1} - \frac{\beta^{-j}}{(\beta-1)(2\beta+1)} + \frac{1}{(\beta-1)(\beta+2)} \qquad (12)$$

The constant of integration determined from Eq.(11) is:

$$k = \frac{1}{\beta(\beta+1)} + \frac{1}{\beta(\beta-1)(2\beta+1)} - \frac{1}{(\beta-1)(\beta+2)} \qquad (13)$$

Eq. (12) is valid for $1 \le j \le n_W$.

Material balances between stage i in the enriching section and the top of the cascade are:

$$M_i + M_{i-1} = P_1 + P_2 + N_{i+1}$$

$$u_i M_i + u_{i-1} M_{i-1} = X_{p_1} P_1 + X_{p_2} P_2 + v_{i+1} N_{i+1}$$

These equations and Eqs. (4) and (5) may be combined to yield:

$$(\beta^2-1)Y_i^E + (\beta-1)Y_{i-1}^E = (\beta^{n-i+2} - 1) + \gamma(\beta^{n-i+1} - 1) \quad (14)$$

where:

$$Y_i^E = M_i/P_1 \quad (15)$$

and:

$$\gamma = P_2/P_1 \quad (16)$$

is the as yet undetermined ratio of the upper and lower product flow rates. With the boundary condition:

$$Y_n^E = 1 \quad (17)$$

Eq. (14) has the solution:

$$Y_i^E = [-(\beta+1)]^{n-i} + \frac{\beta(\beta+\gamma)}{(\beta-1)(2\beta+1)} \{[-(\beta+1)]^{n-i} - \beta^{n-i}\}$$
$$+ \frac{1+\gamma}{(\beta-1)(\beta+2)} \{[-(\beta+1)]^{n-i} - 1\} \quad (18)$$

For the two stages nearest to the top stage, Eq. (18) is:

$$Y_{n-1}^E = \gamma \quad (19)$$

$$Y_{n-2}^E = \beta^2 + \beta + 1 = 1/\theta \quad (20)$$

Equation (18) is valid for $n_w \leq i \leq n$.

The stripper and enricher solutions, Eqs. (12) and (18), are matched by requiring that the heads flow rate from the last stripper stage, M_{n_w}, be the same when calculated from either formula, or:

$$Y^E_{n_w} = Y^s_{n_w}(W/P_1) \tag{21}$$

Equation (21) represents a relation between $\gamma = P_2/P_1$ and W/P_1. An independent relation between these two quantities may be obtained from balances over the entire cascade. When the external concentrations are eliminated by use of the enrichment equations (Eqs. (4) and (5)), Eqs. (7) and (8) become:

$$\frac{W}{P_1} = \frac{\beta + \gamma - (1+\gamma)/\beta^{n-n_w}}{\dfrac{1}{\beta^{n-n_w}} - \dfrac{1}{\beta^{n+1}}} \tag{22}$$

Eliminating W/P_1 between Eqs. (21) and (22) permits γ to be determined by:

$$\gamma = \frac{r - \beta s - p + t}{q + s - t} \tag{23}$$

where:

$$p = \left(\frac{\beta^{n-n_w+1} - 1}{1 - 1/\beta^{n_w+1}}\right) Y^s_{n_w} \tag{24}$$

$$q = \left(\frac{\beta^{n-n_w} - 1}{1 - 1/\beta^{n_w+1}}\right) Y^s_{n_w} \tag{25}$$

$$r = \left[-(\beta+1)\right]^{n-n_w} \tag{26}$$

$$s = \frac{\beta\{[-(\beta+1)]^{n-n_w} - \beta^{n-n_w}\}}{(\beta-1)(2\beta+1)} \tag{27}$$

$$t = \frac{[-(\beta+1)]^{n-n_w} - 1}{(\beta-1)(\beta+2)} \tag{28}$$

All external interstage flows have now been determined by specification of β, n and n_w. The cascade design is complete.

Table 1 shows the heads concentrations and heads flow rates from each stage in an ideal two-up, one-down cascade. In this example, there are 7 stages, of which 3 are in the stripping section, and the tails separation factor is 1.3.

4. Total Number of Separating Units in the Cascade

The total number of separating units per unit upper product flow rate is a good measure of the cost of isotope separation. The number of separating units in stage i is given by:

$$c_i = M_i/\theta L \qquad (29)$$

where θL is the heads flow rate from a single separating unit. Expressing M_i in terms of Y_i^S or Y_i^E by Eqs. (10) and (15) and summing over all stages in the cascade yields:

$$\frac{\sum_1^n c_i}{P_1} = \frac{(W/P_1) \sum_1^{n_w} Y_j^S + \sum_{n_w+1}^{n} Y_i^E}{\theta L} \qquad (30)$$

The sums in Eqs. (30) may be determined from Eqs. (12) and (18) in analytical form, if desired. The quantity $L\sum c_i/P_1$ is a known function of β, n_w, and n. It is of interest to ascertain whether the same result is obtained by dividing the separative duty of the cascade by the separative power of the individual separating units, or by:

$$\frac{\sum_{1}^{n} c_i}{P_1} = \frac{\Delta U/P_1}{\delta U} \tag{31}$$

where:

$$\Delta U = P_1V(X_{P_1}) + P_2V(X_{P_2}) + WV(X_W) - FV(X_F) \tag{32}$$

$$\delta U = L[\theta V(u) + (1-\theta)V(v) - V(w)] \tag{33}$$

In these formulae, $V(x)$ is the value function according to Cohen (4):

$$V(x) = (2x-1)\ln[x/(1-x)] \approx -\ln x \tag{34}$$

and u, v, and w are the heads, tails, and feed compositions of a separating unit operated at throughput L and cut θ.

The validity of Eq. (31) has been demonstrated only for cascades consisting of symmetrically operated separating units (4); it has not been proven for cascades comprised of asymmetric separating units with large separation factors.

$\Delta U/P_1$ is determined by substituting Eqs. (4) and (5) into Eq. (32) and using the form of the value function given by Eq. (34) for $x \ll 1$. Similarly, using Eq. (34) in Eq. (33) and noting that $u/w=\beta^2$ and $w/v = \beta$, the separative power of a single unit may be obtained. Eq. (31) then becomes:

$$\frac{\sum_{i}^{n} c_i}{P_1} = \frac{(n_W+1)(W/P_1) - (n-n_W)(P_1/P_2) - (n-n_W+1)}{L(1 - 3\theta)} \tag{35}$$

Upon comparing $L\sum c_i/P_1$ calculated from Eqs. (30) and (35), we find that the two are numerically equivalent. It appears that Eqs. (30) and (35) are mathematically identical, although we have not proven this analytically.

Despite the applicability of Eq. (31) to the two-up, one-down cascade, the usefulness of this formula as a convenient measure of isotope separation costs is diminished by the need to know the flow rate ratios W/P_1 and P_2/P_1 in order to use it. These quantities cannot simply by obtained from external material balances over the cascade as can the comparable quantities for a symmetric cascade that delivers a single product stream. In the two-up, one-down ideal cascade, determination of the external flows requires calculation of the interstage flows.

Because Eq. (31) is applicable to both the two-up, one-down cascade as well as to the conventional symmetric cascade, it provides a method of deciding when the former arrangement is preferable to the latter. Suppose, for example, that $\delta U/L$ of a particular separating unit is 0.05 and independent of the cut θ. If the separating units were installed in a two-up, one-down cascade, the cut and the tails separation factor are related by Eq. (6) and the separative power is given by:

$$\frac{\delta U}{L} = (1 - 3\theta)\ \ln\beta \tag{36}$$

For $\delta U/L = 0.05$, simultaneous solution of Eqs. (6) and (36) yields $\theta = 0.261$ and $\beta = 1.257$. For the conventional cascade of symmetrically operated units ($\alpha=\beta$):

$$\frac{\delta U}{L} = (1 - 2\theta)\ln\alpha \tag{37}$$

and

$$\theta = \frac{1}{\alpha+1} \tag{38}$$

For $\delta U/L = 0.05$, these equations yield $\theta = 0.422$, $\alpha = \beta = 1.372$. Since δU is the same at $\theta = 0.261$ as it is at $\theta = 0.422$, the number of separating units required for a specific separative duty ΔU is the same for the two-up, one-down cascade as it is for the symmetric ideal cascade. The choice between the two types of cascade arrangements depends upon the desirability of producing two products in the asymmetric modification.

On the other hand, consider a separating unit for which the separative power at a fixed throughput varies as:

$$\frac{\delta U}{L} = 0.08(1-\theta) \tag{39}$$

The operating conditions for a two-up, one-down ideal cascade, obtained by simultaneous solutions of Eqs. (6), (36), and (39), are:

$$\beta = 1.285$$
$$\theta = 0.254$$
$$\delta U/L = 0.0596$$

For the conventional symmetric cascade, Eqs. (37) - (39) yield:

$$\alpha = \beta = 1.356$$
$$\theta = 0.424$$
$$\delta U/L = 0.0460$$

In this example, 30% fewer separating units are needed to produce the same cascade separative duty in the two-up, one-down arrangement as in the conventional symmetric mode of operation.

Acknowledgment

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Table 1

Concentrations and Interstage Flows in a Two-up, One-down Ideal Cascade with $n=7$, $n_w=3$, $\beta=1.3(\theta=0.25)$

$P_2/P_1 = 2.868$ $W/P_1 = 12.479$ $X_w = 0.246$

Stage Number	% U-235 in Heads	M_i/P_1
1	.545	4.160
2	.710	5.546
3	.925	7.400
4	1.205	8.473
5	1.570	4.000
6	2.045	2.868
7	2.664	1.000

Figure Caption

1. Interstage Flows in a two-up, one-down cascade.

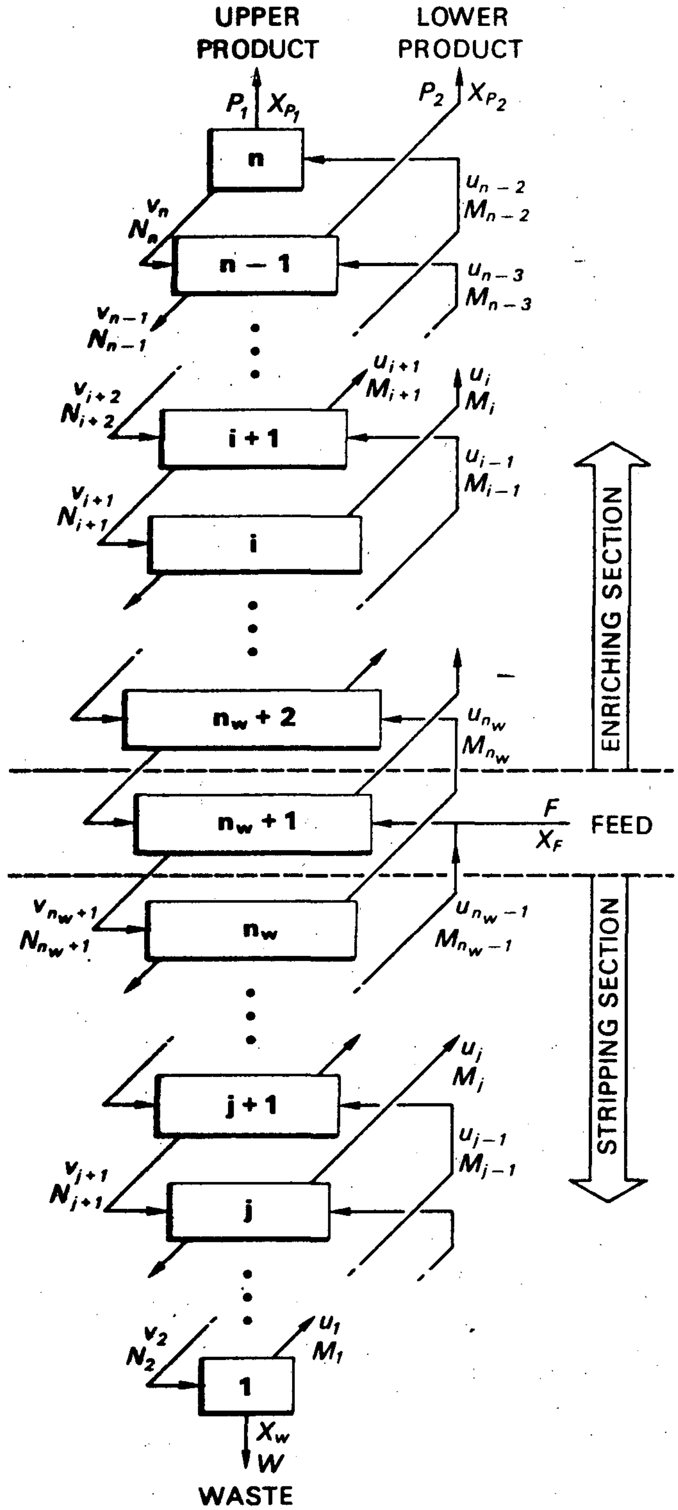

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Fig. 1