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THE HIERARCHY PROBLEM IN SUPERSYMMETRIC GRAND UNIFIED THEORIES*

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Abstract

Supersymmetric models are discussed with a view to solving the hierarchy problem in grand unified theories. Several types of light scalars are obtained and classified. A definition of naturalness is given for supersymmetric grand unified models, and a semi-realistic $O(10)$ model is presented.

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The possibility that the world is supersymmetric above a scale M_s of $O(1)$ Tev is experimentally exciting [1], and theoretically intriguing. While no complete, phenomenologically acceptable model of this sort exists, all partial models require new bosons and fermions at, or sometimes considerably below, M_s . Models which are supersymmetric at M_s are very highly constrained and therefore both interesting and frustrating to model builders. Perhaps the major theoretical motivation for such models is that the gauge hierarchy problem seems more tractable in supersymmetric grand unified theories than in non-supersymmetric theories [2]. In this letter we illustrate how supersymmetry can lead to massless scalars. In a forthcoming paper we will discuss further aspects of this problem as well as other problems associated with low energy supersymmetry [3].

It will be convenient to isolate three separate aspects of the gauge hierarchy problem. The first is why some scalars, namely the Higgs scalars of the Weinberg-Salam model are effectively massless at tree level when the natural mass scale is the grand unification scale M_x ? Secondly, why do such scalars not receive masses of order αM_x from radiative corrections? These problems can be called fine tuning at the tree level and fine tuning order by order in perturbation theory. Finally an explanation is required for a negative mass squared of order M_W^2 which allows these scalars to trigger the weak interaction symmetry breakdown. Ideas from supersymmetry may be useful for each of these aspects separately; in this letter we will be concerned only with the first two. It is clear that an acceptable resolution of the first two would be that an unbroken chiral symmetry prevents some fermions from acquiring a mass. Supersymmetry then keeps their scalar partners massless down to a scale M_s . In many models this simply does not work.

Realistic supersymmetric models seem to need sets of light chiral multiplets which are real under the gauge symmetry and are therefore not protected from acquiring a large mass [3].

This is not the only way in which supersymmetric theories may be used to solve the hierarchy problem. Non-renormalization theorems [5] imply that in such theories particles do not receive radiative corrections to their masses other than from wave function renormalization. Since supersymmetry is unbroken down to M_S the second problem is automatically solved; there are no radiative corrections of order αM_X to masses which are zero at the tree level. This is a considerable advance, but now the problem of masslessness at the tree level becomes more acute and worthy of serious consideration.* In this letter we demonstrate that massless scalars, not produced by fine tuning of parameters in the Lagrangian, are quite common in supersymmetric theories. In some toy models we illustrate how several varieties of massless scalars appear.

It is not sufficient that a supersymmetric grand unified theory have massless scalars at the tree level. Equally important is that

*This problem was recently considered by the authors of ref. 6. Unfortunately, these authors never wrote a Lagrangian, so it is not clear whether or not their vacua can be realized. In non-supersymmetric gauge theories, it is almost always possible to write a Higgs potential which yields the desired vacuum. In supersymmetric theories this is rarely the case. It is therefore essential to write such a potential and not to discuss a few isolated terms.

none of them should be capable of mediating proton decay. This requires that an irreducible representation of the grand unified group G should be split: while some of its components are massless some should be superheavy. We will write down a semi-realistic model in which scalar representations are split in an interesting and useful way. Proton decay and the incorporation of matter supermultiplets are discussed, but the model is not complete because no mechanism for the breaking of supersymmetry and the weak interactions is given.

To establish notation we begin by describing the generic form of a supersymmetric grand unified theory in which the gauge symmetry G is broken but the supersymmetry is not. Such a theory is constructed from gauge supermultiplets V^a with vector, fermion and scalar components (V^a, λ^a, D^a) and chiral supermultiplets ϕ_i which have fermion and scalar components (ψ_i, ϕ_i, F_i). The non-gauge interactions of the chiral fields are described by a superpotential $W(\phi_i)$ which is at most cubic in the fields. On eliminating the D^a auxiliary fields from the gauge interactions, the tree level effective scalar potential may be written as

$$V = D^a D^a + F_i F_i, \text{ where } F_i = - \left| \frac{\partial W}{\partial \phi_i} \right|, \quad D^a = (\phi^\dagger T^a \phi), \text{ and } T^a \text{ are the}$$

generators of the group G in the representation of ϕ . The vacuum expectation values (vevs) of the scalar fields (chosen so that $\langle D^a \rangle$ and $\langle F_i \rangle$ are zero, leaving supersymmetry unbroken) break the gauge symmetry down to G' . If we write $\phi_i = A_i + iB_i$ then for the models we have investigated the vevs can be made real: $\langle B_i \rangle = 0$. There are usually several degenerate vacua corresponding to different unbroken subgroups G' , consequently $\langle A_i \rangle$ is not completely determined. Since this vacuum degeneracy is not lifted to any finite order in perturbation theory [2, 5], the choice of $\langle A_i \rangle$ is a matter of some delicacy. We

offer the following prescription. A particular G' is chosen, and the most general vev consistent with such a choice is made. Contrast this with non-supersymmetric gauge theories in which one chooses parameters in the Lagrangian to arrange for the desired G' to be the lowest energy vacuum.

The restriction that $W(\phi_i)$ not contain $\phi_i^\dagger$ implies that the potential at minimum ($\langle F_i \rangle = 0$) has a symmetry at least as large as $\bar{G}$, the complexification of G . $\bar{G}$ has twice as many generators as G , therefore the breaking of n gauge generators in $G \rightarrow G'$ results in $2n$ real scalars not receiving masses from $F_i F_i$. Of these, n are eaten by gauge vectors, and the rest acquire masses from $D^a D^a$. The corresponding n two-component Weyl fermions are eaten by the gauge fermions in the supersymmetric extension of the Higgs mechanism. No massless scalars have resulted so we will be interested in deviations from this generic form which we now describe in specific examples.

Consider an $O(N)$ gauge model with two fundamental chiral fields

ϕ , ϕ' and one singlet χ described by

$$W_1 = -a\chi + b\chi\phi\phi + c\chi\phi'\phi' \quad (1)$$

This and all our models can be made natural in the strong sense [2] by the use of R and discrete symmetries. The singlet field drives the gauge symmetry breaking, but the only constraint on the vevs is that $F=0$, i.e. $b\langle A_1 \rangle^2 + c\langle A_1' \rangle^2 = a$. We will choose G' to be $O(N-1)$ in which case the most general structure of the vevs is*

*We could also have chosen G' to be $O(N-2)$.

$$\langle A_1 \rangle = \begin{pmatrix} v \\ 0 \\ 0 \\ \vdots \\ \vdots \end{pmatrix} \quad \langle A_1' \rangle = \begin{pmatrix} v' \\ 0 \\ 0 \\ \vdots \\ \vdots \end{pmatrix} \quad \langle \chi \rangle = 0 \quad (2)$$

with $bv^2 + cv'^2 = a$. The vev of D is zero since ϕ and ϕ' are real representations. It is easily seen that F^2 gives masses only to A_1 , B_1 , A_1' , B_1' and χ . Of the remaining $4(N-1)$ scalars, $(N-1)$ are eaten and $(N-1)$ get masses from D^2 terms, and $2(N-1)$ are massless. These massless scalars are 'pseudos' resulting from the fact that the symmetry of the non-gauge interaction is $\overline{O(N)} \times \overline{O(N)}$ and is broken to $\overline{O(N-1)} \times \overline{O(N-1)}$. The situation is very similar to the non-supersymmetric theory, but for our purposes the most significant difference is that these masses do not receive radiative corrections.

Now imagine that we had not given the most general vev consistent with our choice of $G' = O(N-1)$. In particular suppose that we had tuned v' to be zero. Then A_1' and B_1' become massless. We call such particles 'accidentals.' They are to be avoided since it is not clear that fine tuning of parameters in vevs is any better than fine tuning of parameters in a Lagrangian.

Our second toy model has massless particles which are not associated with internal symmetries or fine tuning. The gauge symmetry G is $O(10)$ and the superpotential is

$$W_2 = -a^2\chi + b\chi\Sigma_{ij}\Sigma_{ji} + c\phi_i\Sigma_{ij}\phi_j' \quad (3)$$

where ϕ and ϕ' are two 10's, Σ is an adjoint, and χ is a singlet.

Suppose that G' is chosen to be $SU(3) \times U(1) \times O(4) \equiv SU(3) \times U(1) \times SU(2)_L \times SU(2)_R$ so that

$$\langle \phi \rangle = \langle \phi' \rangle = \langle \chi \rangle = \langle \Sigma_B \rangle = 0$$

and

$$\langle \Sigma_A \rangle = v \begin{pmatrix} \sigma & 0 & 0 & 0 & 0 \\ 0 & \sigma & 0 & 0 & 0 \\ 0 & 0 & \sigma & 0 & 0 \\ 0 & 0 & 0 & \sigma & 0 \\ 0 & 0 & 0 & 0 & \sigma \end{pmatrix} \quad (4)$$

where each element in $\langle \Sigma_A \rangle$ is a 2×2 matrix: $\sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

The thirty Goldstone bosons are $\Sigma_A(3 + \bar{3}, 2, 2)$, $\Sigma_A(3, 1, 1)$ and $\Sigma_A(\bar{3}, 1, 1)$;

corresponding Σ_B 's get masses from D^2 . Bracket labels are $SU(3) \times SU(2)_L \times$

$SU(2)_R$, and subscripts A and B refer to the real and imaginary scalar

components of the superfield. The only masses from F^2 are for

χ_1 , $\Sigma_{12} + \Sigma_{34} + \Sigma_{56}$, $\phi(3 + \bar{3}, 1, 1)$ and $\phi'(3 + \bar{3}, 1, 1)$. The massless

scalars are $\phi(1, 2, 2)$, $\phi'(1, 2, 2)$, $\Sigma(8, 1, 1)$, $\Sigma(1, 3, 1)$ and $\Sigma(1, 1, 3)$.

These massless particles are not related to a symmetry of the model.

Such particles can provide useful scalar representations; we call them 'serendipities'. It is clear that the number of serendipities depends on the choice made for G' .

The presence of χ in the model is inessential. It drives the gauge symmetry breakdown at a scale a . If a and b are set to zero in Eq. (3), the gauge symmetry may still break, but the scale is now arbitrary.

Having illustrated the types of massless scalars which occur in supersymmetric gauge theories, we can now specify the properties which a model must have for us to call it semi-realistic. As before, it is a model in which supersymmetry is unbroken, and has the most general vevs consistent with a chosen gauge symmetry breakdown $G \rightarrow G'$. Parameter tuning in the Lagrangian and in the vevs is forbidden. An effective theory at scale M_S may be obtained by integrating out

superheavy fields. The effective semi-realistic theory should have conventional matter generations of quarks and leptons. Two further requirements are placed on the effective theory. Firstly, it must be possible to break G' to $SU(3)_C \times U(1)_{e.m.}$ by giving vevs to scalar fields. The quarks and leptons should acquire mass at this stage. Secondly, all the renormalizable couplings of quarks and leptons must conserve baryon number. In general such semi-realistic models are neither complete or phenomenologically acceptable, however they do make progress in solving the hierarchy problem.

We now promote model 2 to a semi-realistic model. Although we still require left-right symmetry down to M_S , we now arrange for the massless scalars to be sufficient to break $SU(3) \times U(1) \times O(4)$ down to $SU(3) \times U(1)$. This is done most economically by adding 16 and $\bar{16}$ superfields E and G. The superpotential is

$$W_3 = -a^2\chi + b\chi\Sigma_{ij}^2 + c\phi\Sigma_{ij}\phi' + d\chi G^T E \quad (5)$$

It can be made natural by the R symmetry $\chi \Sigma \phi \phi' G E$ together with discrete symmetries forbidding $\phi\phi'$. The most general vevs consistent with $SU(3) \times U(1) \times O(4)$ and supersymmetry are

$$\langle \chi \rangle = \langle \phi \rangle = \langle \phi' \rangle = \langle E \rangle = \langle G \rangle = \langle \Sigma_B \rangle = 0 \text{ and}$$

$$\langle \Sigma_A \rangle = \begin{pmatrix} \sigma & 0 & 0 & 0 & 0 \\ 0 & \sigma & 0 & 0 & 0 \\ 0 & 0 & \sigma & 0 & 0 \\ 0 & 0 & 0 & \sigma & 0 \\ 0 & 0 & 0 & 0 & \sigma \end{pmatrix} \quad (6)$$

which leads to the following massless serendipities: E, G, $\Sigma(8, 1, 1)$, $\Sigma(1, 3, 1)$, $\Sigma(1, 1, 3)$, $\phi(1, 2, 2)$ and $\phi'(1, 2, 2)$. ϕ and ϕ' are split representations, with all the $SU(3)$ components superheavy. It is tempting to change the R quantum number of E and G to $1/4$ and $-1/4$, allowing

extra terms in W : $E^c \gamma^i E_{\phi 1}$ and $E^c \gamma^i E_{\phi 1}'$. The fermions of E are then assigned to be quarks and leptons which receive mass from these Yukawa couplings. This is unsatisfactory because the $SU(5)$ singlet component of E , which eventually gets a vev to break $O(4)$ to $SU(2)_L \times U(1)_Y$, gives a Dirac mass term coupling the $SU(5) \bar{5}$ in E to the 5 in ϕ .

The superpotential W_3 is therefore interpreted as being the Higgs sector of the theory. The three generation matter sector is

$$W_4 = \sum_{x, y=1}^3 \left(f_{xy} \psi^c x \gamma^i \psi \phi_1 + g_{xy} \psi^c x \gamma^i \psi y^i \right) \quad (7)$$

where ψ is a 16 dimensional spinor with R charge $1/4$ and $W_3 + W_4$ is invariant under $\psi \rightarrow -\psi$, and γ_i are the $O(10)$ gamma matrices in the Weyl basis.

The arrangement of quarks and leptons in ψ is conventional [7]. In non-supersymmetric $O(10)$ gauge models, a real 10 leads to the same mass for up and down quarks. This relation is destroyed by making the 10 complex. Our supersymmetric model has the advantage that the 10's, like all other chiral superfields, are necessarily complex.

Notice that any coupling involving three matter multiplets is potentially disastrous as the scalar partners of the quarks are now able to mediate proton decay at an unacceptable rate. In supersymmetric $SU(5)$ they can only be removed by a non gauge symmetry, but in $O(10)$ the gauge symmetry forbids such terms.

This model satisfies our requirements for a semi-realistic model. The effective theory at scale M_g does have scalars which could break $SU(3) \times U(1) \times O(4)$ to $SU(3) \times U(1)_{e.m.}$. Matter fermions would then acquire masses, but would have no renormalizable baryon number violating interactions. The proton decays through higher dimension operators which result from the exchange of four types of particles: superheavy

gauge bosons, superheavy gauge fermions, superheavy Higgs bosons and their fermionic partners. The first three give proton decay at a rate proportional to M_x^{-4} as in conventional grand unified theories. The last one gives a proton decay matrix element $\propto \frac{f_{xy}^2}{M_x}$ and this is suppressed by the small Higgs coupling f_{xy} .*

The actual value of the proton lifetime is of course sensitive to the value of M_x . Above M_g , the $SU(3)$ coupling constant in our model is not asymptotically free; we will not therefore have a perturbative unification and are consequently unable to calculate M_x . It is clear that this model has much structure at M_g . The symmetry breaking at M_g must be such that E , G , the non-superheavy pieces of ψ , Higgs fermions, matter scalars, and gauge fermions have sufficient mass to avoid detection.

It is tempting to consider an alternative version of the model with superpotential $W_3 + W_4$ in which G' is chosen to be $SU(3) \times SU(2) \times U(1)_Y$. The most general vevs for this breaking are

$$\langle \Sigma_A \rangle = v \begin{pmatrix} \sigma & 0 & 0 & 0 & 0 \\ 0 & \sigma & 0 & 0 & 0 \\ 0 & 0 & \sigma & 0 & 0 \\ 0 & 0 & 0 & \epsilon \sigma & 0 \\ 0 & 0 & 0 & 0 & \epsilon \sigma \end{pmatrix} \quad (8)$$

*We disagree with the estimate made by Weinberg [8] of the rate of proton decay produced by this matrix element in the model of Georgi and Dimopoulos [4]. We obtain a contribution to the decay width from this process comparable to the decay width from superheavy gauge boson exchange in conventional $SU(5)$.

$$\langle E_A \rangle = \begin{pmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot \\ v_E & v_G & v_X & v_X & v_X & v_X \end{pmatrix} \quad \langle G_A \rangle = \begin{pmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot \\ v_G & v_X & v_X & v_X & v_X & v_X \end{pmatrix} \quad \langle \psi_{x_A} \rangle = \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \quad (8)$$

with $bv^2(6 + 4\epsilon^2) + dv_E^2 v_G^2 = a^2$ and $v_G^2 = v_E^2 + 2v_X^2$ so that the supersymmetry is unbroken. The model is not semi-realistic for two reasons. There are no massless weak scalar doublets coupled to quarks and leptons unless ϵ is tuned to be zero. The doublets $\phi(1, 2, 2)$ and $\phi'(1, 2, 2)$, which were previously massless serendipities, have become accidentals. Secondly, unless v_X is tuned to be zero, the effective theory at M_g does not even contain conventional quarks and leptons. It is precisely to avoid such tunings that we preferred the choice of $G' = SU(3) \times U(1) \times O(4)$.

We have illustrated how gauge models with unbroken supersymmetry can lead to several varieties of massless particles when the gauge symmetry is broken. Having proposed a procedure for choosing vevs naturally, we wrote down a semi-realistic model in which quarks and leptons could acquire masses without giving proton decay at an unacceptable rate. Although no parameters are tuned, work remains to be done on the low energy symmetry breakdown before the model can be made realistic.

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