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Author
Glendenning, N.K.

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Lawrence Berkeley Laboratory
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N.K. Glendenning

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Norman K. Glendenning

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ABSOLUTE LIMIT ON ROTATION OF GRAVITATIONALLY BOUND STARS

NORMAN K. GLENDENNING
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, U.S.A.

ABSTRACT

We seek an absolute limit on the rotational period for a neutron star as a function of its mass, based on the minimal constraints imposed by Einstein's theory of relativity, Le Chậtelier's principle, causality and a low-density equation of state, uncertainties in which can be evaluated as to their effect on the result. This establishes a limiting curve in the mass-period plane below which no pulsar that is a neutron star can lie. For example, the minimum possible Kepler period, which is an absolute limit on rotation below which mass-shedding would occur, is 0.33 ms for a $M = 1.442 M_\odot$ neutron star (the mass of PSR1913+16). If the limit were found to be broken by any pulsar, it would signal that the confined hadronic phase of ordinary nucleons and nuclei is only metastable, an extraordinary conclusion.

INTRODUCTION

There are many reasons why fast pulsars are interesting. We stress only one, the possibility that an altogether different state of matter exists besides the confined phase of ordinary hadrons and nuclei, – a self-bound state that it is the absolute ground state of the strong interaction. Fast pulsars can be signals of such a state, as will be explained, and they are the natural place in which the most plausible candidate for it would be created. We therefore seek a limit on the minimum possible period of pulsars assuming that they are neutron stars, – stars that are bound by the gravitational interaction. If a pulsar with period below our limit, which is obtained with minimal and impeccable principles and constraints listed below, should be found, it cannot be a neutron star, but must be of an entirely different nature (Glendenning 1992). The only alternative to gravitationally bound stars are stars that are self-bound and absolutely stable. We shall show that if the equilibrium density at which they are self-bound is sufficiently high, they can have shorter periods than neutron stars.

This is a different approach to the meaning of fast rotation than has otherwise been investigated. Other works have investigated how fast stars composed of matter obtained from particular models can rotate. Unfortunately the most that can be said if a pulsar is discovered that has a shorter period than the shortest for any of the models is that those theories of dense matter are not correct in their details. While this is interesting, and indeed probably disappointing for the theorists associated with the failed equations of state, nothing fundamental

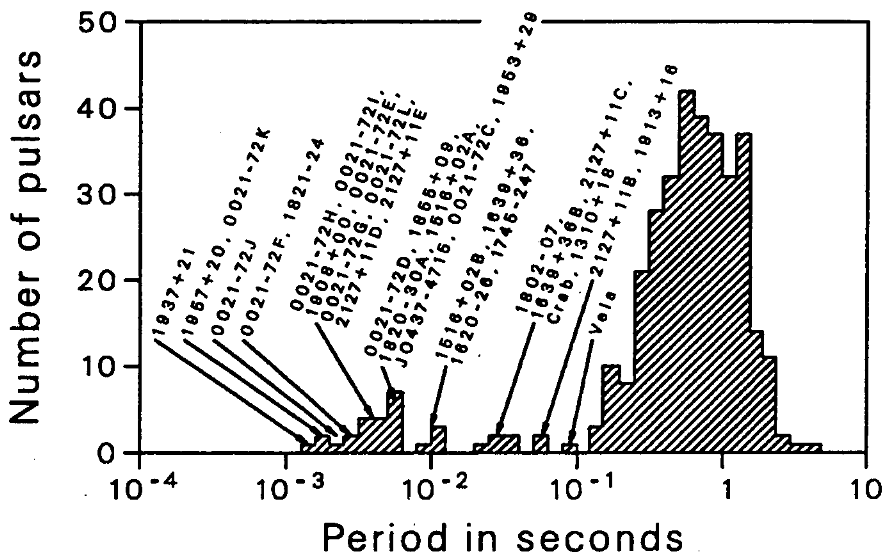

FIGURE I Number of pulsars as a function of period, as of about 1991. Many new discoveries have been made, but none lie below the lowest shown here. There is an instrumental cutoff in sensitivity at around 1 ms., as explained in the text.

has been learned.

So that is the challenge, not merely to find the fastest pulsar, nor to rule out particular equations of state, both worthy accomplishments, but to discover, if our limit is broken, that matter of ordinary nucleons and nuclei is only a metastable phase, and that the true ground state is matter of a different nature, – matter that is self-bound in bulk by the strong interaction rather than by gravity. That it must be self-bound and the absolute ground state follows because as the pressure falls toward the outer region of the star, the matter otherwise would come into equilibrium with ordinary neutron star matter, which would be gravitationally bound to the star, and then it would have the same rotation limit that we establish for such stars! The most likely candidate is strange quark matter. It has so far eluded us to either prove or disprove that this state, postulated by Bodmer (1971) and independently by Witten (1984), is the true ground state, and that the whole universe as far as has been detected, occupies a metastable phase, however long-lived. (The present composition of the universe, hadronic matter as far as we can see in all luminous objects, *does not* shed any light on this fundamental question, because the ground state of the phase that the universe *is* in is Fe^{56} and there is very little of that, and for well understood reasons.)

Our approach is very much akin to Ruffini's mass limit for neutron stars, – so important for the detection of black hole candidates of a few solar masses.

The background against which we report this work is shown in Fig. I. The cutoff at PSR 1937+21 ($\sim$ 1.6 ms) may only reflect the well known roll off of detection efficiency between 1 and 1.5 ms, due to the interstellar dispersion of the frequencies in the radio signal, and the necessary compromises in frequency binning, sampling rate and computer time devoted to Fourier analysing the data as corrected by an unknown column height of dispersive plasma. It could happen that the coincidence of detector sensitivity at this time happens to fall at the shortest period that pulsars have. But it is tantalizing to press these limits in the hope of a fundamental discovery.

MINIMAL CONSTRAINTS

We adopt the following as the minimal principles and constraints on a gravitationally bound star:

1. Einstein's general relativistic equations for stellar structure hold.
2. The matter of the star satisfies $dp/d\rho \geq 0$ which is a necessary condition that a body is stable both as a whole, and also with respect to the spontaneous expansion or contraction of elementary regions away from equilibrium (Le Chatelier's principle).
3. Causal constraint for a *perfect fluid*; a sound signal cannot propagate faster than the speed of light, $v(\epsilon) \equiv \sqrt{dp/d\epsilon} \leq 1$, which is the appropriate expression for sound signals also in general relativity (Curtis 1950).
4. The high density equation of state, whatever it is, matches continuously in energy and pressure to the low-density one of Baym, Pethick and Sutherland (1971).

The last constraint effects only the surface of the star, and as we have shown, leads to very little uncertainty in the shortest possible period (Glendenning 1992).

METHOD

The maximum mass star, M_{max}, in the sequence of gravitationally bound compact stars belonging to a given equation of state, has the minimum Kepler period since the mass is the largest and the radius is the smallest because of the gravitational attraction. Therefore we can incorporate the minimal constraints into a variational search over a very flexible parameterization of the equation of state that even includes the possibility of a constant pressure phase transition. We minimize $f(M,P) = w_1(M - M_{max})^2 + w_2P^2$ where w_1, w_2 are weights. We use a modified Levenberg-Marquardt method for finding the minimum of this non-linear function of its arguments. The function has its minimum when $M = M_{max}$ and P is the least possible Kepler period for that mass. From numerical solutions for relativistic *rotating* stars it has been found that the Kepler period of the maximum mass star of a given equation of state can be found to better than 10 % from the mass and radius of the *non-rotating* maximum mass star, which is a solution of the Oppenheimer-Volkoff equations. The empirical relation is given by a numerical factor times the classical relation that balances gravity and centrifuge (Friedman, Ipser & Parker 1989, Haensel & Zdunik 1989, Weber & Glendenning 1990),

$$\Omega_K \approx 0.625(M/R^3)^{1/2}. \tag{1}$$

We consider only uniform rotation because it extremizes the total energy for fixed angular momentum and baryon number (Hartle and Sharp 1967, Hegyi 1977), and is the only kind of rotation available to dense stars with viscosity, a short time after their creation.

GENERAL RELATIVISTIC LIMIT ON ROTATION

It may also be of interest to note a limit that can be derived from the least number of constraints. Assuming only that Einstein's equations of stellar structure hold, then $M/R < 4/9$ for *any* static star (Weinberg 1972). Using the approximate relation for the Kepler angular velocity referred to above we obtain

$$P \approx 10.1M\left(\frac{M}{R}\right)^{3/2} > \frac{273}{8}M = 0.167\frac{M}{M_\odot} \text{ ms} \qquad \text{(any star)}, \tag{2}$$

which for PSR1913+16 is $P > 0.24$ ms. Since the limit on M/R follows from the structure of Einstein's equations the above limit on period applies both to neutron stars as well as to hypothetical stars made of self-bound matter. The GR forbidden region is marked in Fig. II.

RESULTS

The results are summarized in Fig. II. We show the minimum Kepler periods for star sequences that are subject to the above constraints as a function of the corresponding maximum (limiting) mass star. Neutron stars cannot lie below the solid curve of this figure within the very small limits discussed in detail in Ref. (Glendenning 1992). We also show Kepler periods for the sequence whose maximum mass is $1.8M_\odot$. In the preceding section we derived a region that is forbidden to all stars by the structure of general relativity, and it is also shown in the figure. For a neutron star with canonical mass of $\sim 1.44M_\odot$, apparently preferred by the creation process, the lowest possible stable period of rotation cannot be less than ~ 0.33 ms. Neutron stars cannot lie below the upper hatched region. No star can lie in the lower one. Between there is an intriguing region and we shall examine the nature of stars that can occupy it in the next section.

DISCUSSION AND ALTERNATIVES

The purpose of this paper, as stated earlier, is to provide a decisive means of distinguishing pulsars that *can be* (but are not necessarily) gravitationally bound compact stars like neutron or hybrid stars (quark-core neutron stars), from those that *cannot*. The limit that we have obtained for the rotational period as a function of star mass is necessarily lower than would be obeyed by real stars because there is no physical principle that requires the equation of state, whatever it is,

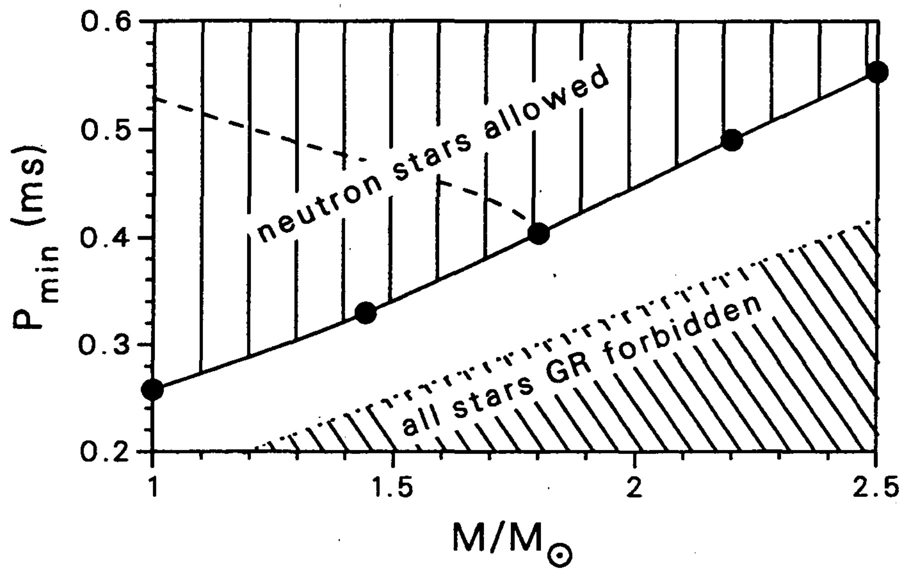

FIGURE II Minimum rotational period of neutron stars - solid curve. Calculated points are shown by dots. Periods of a sequence of stars whose maximum mass is $1.8 M_\odot$ - dashed line. Region forbidden by structure of general relativity diagonal shaded region.

to minimize the rotation period. Moreover, there are unstable pulsation modes of a rotating star associated with gravitational radiation-reaction (Lindblom & Detweiler 1977, Friedman & Schutz 1978, Friedman 1983). If a pulsar has a rotation period *smaller* than any of the periods for these critical modes, it will spin down by gravitational radiation until its period approaches that of the *largest* of the unstable modes, – all of which occur at *larger* period than the Kepler period (Lindblom 1986, Ipser & Lindblom 1989, Weber & Glendenning 1990, Weber, Glendenning and Weigel 1990, Ipser & Lindblom 1991). However, the physics that enters the estimate of these instabilities is far less certain than that which determines the Kepler period, so we use this absolute lower bound of a rotating star. Therefore if a pulsar with rotation period and mass falling below our limiting curve is found, it actually lies even further below the limit established by nature for neutron stars. So our limit is a very conservative one. The periods of real neutron stars must lie above it!

The only general category of stars that are not subject to the bound imposed by gravitational binding are stars that are self-bound by a short-range force, like, indeed most likely, the strong interaction, – stars that would be bound even if one could switch off gravity (which neutron stars are not, since at their densities, the nucleons reside in the repulsive range of their neighbors). Such a self-bound state must be absolutely stable to beat the limit on gravitationally bound stars, as already explained in the introduction. If self-bound at a sufficiently high equilibrium energy density, such stars could have smaller rotation periods than neutron stars. The connection of the limiting rotational period of such stars and the equilibrium energy density, ϵ_e, of the hypothetical self-bound matter is trivial to establish (Glendenning 1989, 1990). Since the density (and pressure) decrease monotonically from the center to the edge of a star, and the edge occurs when $p = 0$, at which point the energy density is ϵ_e, it follows that the energy density in the interior, and the average in particular, satisfies, $\bar{\epsilon} \geq \epsilon_e$. The equality holds only when gravity is negligible, meaning that the mass is small. The average density also satisfies the identity, $M \equiv \frac{4\pi}{3} R^3 \bar{\epsilon}$. Using this relation in eq. (1) we have $P_K = 1.6(3\pi/\bar{\epsilon})^{1/2} < 1.6(3\pi/\epsilon_e)^{1/2}$. Observing also the limit imposed on M/R by general relativity, eq. (2), we can write,

$$1.21 \left(\frac{\epsilon_0}{\epsilon_e}\right)^{1/2} > \frac{P_K}{\mathrm{ms}} > 0.167 \, \frac{M}{M_\odot}, \quad (\mathrm{self-bound\ star}), \tag{3}$$

where $\epsilon_0 \approx 140$ MeV/fm^3 is the equilibrium energy density of normal nuclear matter. Obviously if ϵ_e is large enough, a few times nuclear density, the period of a self-bound star can lie in the region prohibited to neutron stars. Its energy profile is unlike those of neutron stars which fall smoothly to the outer edge of the crust. Instead a self-bound star has a sharp edge at which the energy density falls from the *high* equilibrium value of the bound matter to zero in a strong interaction length, ~ 1 fm. Such a star has a small radius for its mass as compared to a gravitationally bound star. This is what permits it to rotate rapidly without shedding matter at its equator.

SUMMARY

A neutron star cannot have a period for rotation that lies by more than a few percent below the solid curve of Fig. II. (A careful check of relaxing various of the "impeccable" assumptions enumerated above was made in Ref. (Glendenning 1992).) For a $1.442 M_\odot$ neutron star this means that the period must exceed $P \geq 0.33$ ms. This curve refers to the mass-shedding limit (Kepler period) and gravitational-wave instabilities will actually set a more stringent lower bound that lies *above* our curve. Therefore the most conservative bound for the region excluded to neutron stars is the one adopted here. If a pulsar is found with mass and period that place it below our critical curve, it must be a different kind of object and it appears that the only alternative for breaking the limit on gravitationally bound stars is a state of matter that is self-bound in bulk at an energy density larger by a factor of five to ten than normal nuclear matter and that it is absolutely stable. This would mean that the phase we *are* in of ordinary nucleons and nuclei is only metastable, but very long-lived. The most plausible candidate to date, in our opinion, is strange-quark-matter. The barrier between normal matter and this state is well understood, and careful examination reveals that the universe would have evolved along the path it did, even if strange matter were the ground state, with almost imperceptible differences, the main one being the creation of *cold* strange matter on the time-scale of evolution of stars, namely at the birth of compact stars.

What are the prospects? From the anthropocentric point of view it seems that the likelihood that *we* are in a metastable state is low. From the scientific point of view, it is very hard to make a case one way or the other. Many aspects of the problem have been examined by many authors, and no conclusive theoretical arguments are available. So finally it is an observational question; it may be a long shot, depending on which of the above points of view one takes, but it has a high payoff if the limit we have derived for gravitationally bound stars is broken!

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720