

# UC Berkeley

## Working Papers

### Title

Implementing A Dynamic O-D Estimation Algorithm within the Microscopic Traffic Simulator Paramics

### Permalink

<https://escholarship.org/uc/item/0n62j6nq>

### Author

Garcia, Reinaldo C.

### Publication Date

2002-06-01

CALIFORNIA PATH PROGRAM  
INSTITUTE OF TRANSPORTATION STUDIES  
UNIVERSITY OF CALIFORNIA, BERKELEY

## **Implementing A Dynamic O-D Estimation Algorithm within the Microscopic Traffic Simulator Paramics**

**Reinaldo C. Garcia**

*University of California, Irvine*

**California PATH Working Paper  
UCB-ITS-PWP-2002-4**

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department Transportation, Federal Highway Administration.

The contents of this report reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the State of California. This report does not constitute a standard, specification, or regulation.

Report for TO 4121

June 2002

ISSN 1055-1417

**IMPLEMENTING A DYNAMIC O-D ESTIMATION ALGORITHM  
WITHIN THE MICROSCOPIC TRAFFIC SIMULATOR PARAMICS**

**Reinaldo C. Garcia  
Institute of Transportation Studies  
PATH Research Program  
University of California at Irvine  
Irvine, 94720 CA**

**January 29, 2002**

## **Executive Summary**

California has long recognized the potential for applying electronic and other transportation systems technologies to address the significant mobility and economic challenges in the state, and the rest of the nation. Through an aggressive Advanced Transportation Systems Program, Intelligent Transportation Systems (ITS) are being researched, built, and tested for deployment. These ITS will address today's transportation needs and those of the twenty-first century. An important element of this program is the California Advanced Transportation Management Systems Testbed (California ATMS Testbed) located in Orange County, California<sup>1</sup>.

The Testbed is an integrated approach to the development and deployment of advanced technologies in the management of urban transportation. Its operation is based on real-time, computer assisted traffic management and communication. Management responsibility for the development and implementation of the Testbed is vested in the University of California at Irvine (Institute of Transportation Studies), being the microscopic traffic simulator, Paramics, the primary simulator tool for the Testbed development environment.

Paramics is a shell or framework for a comprehensive and extensive transportation simulation laboratory. Paramics offers important and unprecedented features, such as high performance and scalability, to handle realistic real world traffic networks under ITS. Nevertheless, Paramics has its own limitations, particularly relating to the model's ability to interface with dynamic O-D estimation. This Work addresses the continuing effort to expand the Paramics capabilities, particularly with the dynamic O-D estimation problem, making it a more complete tool to evaluate the expected net benefits of ATMS applications.

## **Introduction**

The problem of dynamic estimation and prediction of freeway origin-and-destination (O-D) matrices has received increasing attention lately because of its applicability to on-line traffic management systems. Time-dependent O-D matrices are essential input for advanced traffic management and information systems. Applying the information contained in time-varying O-D matrices, it is possible to project traffic demands up to selected time horizon and predetermine optimal control and routing policies that achieve some desirable system objectives. Therefore, a capability for effectively estimating time-varying O-D demands can significantly improve the operational efficiency of on-line traffic management systems. This capability can be implemented and studied applying traffic simulators.

The prime objective of using traffic simulators within an ITS context is to serve as tools for dynamic transportation management. Simulators can play two distinct roles: (a) as an off-line evaluation/design tool, and (b) as an on-line control/guidance tool. Both roles cover numerous ATMS applications such as: provision of traveler information and route guidance, freeway adaptive control (adaptive signal control, adaptive ramp metering, etc.), incident detection and management, automated toll collection, and assessment and management of environmental impact of transportation design. The off-line role is fairly easier, as real-time operation is not as pressing a need as it is the case for the on-line role. However, being a simulator fast enough, it can be used for both functions (off-line and on-line). Since those functions are inherent objectives of any Traffic Management Center, and of the ATMS Testbed in our particular case, the efficiency and scalability of the simulator is regarded as a key factor to analyze the transportation network in study.

Transportation networks are dynamic systems that exhibit continuous changes in both the supply and the demand conceptual sides. Unexpected events inevitably occur, changing the supply side of the network. Any intervention by the Traffic Management Center can change the supply side, motivating drivers to change their behavior in several ways, including en-route and pre-trip route re-choice (within the day or from day to day). Similar dynamics take place regarding drivers choice of departure times in response to the dynamically changing supply conditions. The collective user behavior and response in this fashion give rise to dynamic

---

<sup>1</sup> The Testbed encompasses two contiguous sub-areas in Orange County, including virtually all of the major decision points for freeway travelers in the region.

demand profiles. Therefore, for any simulator to prove useful for dynamic transport management, it should be capable of:

(i) capturing the dynamics of supply, in terms of the detailed configuration of the transportation network, and its performance in response to the Traffic Management Center intervention;

(ii) capturing the dynamics of demand, in terms of dynamic user behavior in response to observed supply, either directly or via traveler information systems;

(iii) capturing the complex dynamic interaction between supply and demand;

(iv) performing an analysis faster than real-time to allow for proactive (based on predicted conditions) rather reactive (based on observed conditions) dynamic transport management.

This Work implements a dynamic O-D estimation algorithm for the Paramics microscopic simulator. The next sections give not only a complete description of the O-D estimation problem but also a review of previous research related to the dynamic O-D estimation issue. Finally, a description of the chosen algorithm is described.

## **Modeling Dynamic OD Demand Estimation**

### ***Overview of the Dynamic OD Estimation Problem***

For many network problems it is important to know not only the link flows but also the origins and the destinations of the trips flowing in the system. Examples of small networks where some of the Origin-Destination table (O-D table) could be useful are: traffic circles (or roundabouts), weaving sections of freeways which can be viewed as networks with five links, two origins and two destinations, and ordinary street intersections which can also be modeled as small networks. In order to devise effective designs and control strategies for these systems, O-D information is necessary because the systems traffic performance depends heavily on the distribution of the vehicle paths across O-D s, e.g., on the turning percentages for an intersection.

If traffic conditions were time-independent then the relevant information could be summarized by means of an O-D trip table of stationary flows. However, for time-dependent O-D tables, the information required from field data is increased manifold, and the demand estimates obtained for this case can be unlikely of being very accurate, unless a dynamic O-D profile is applied (Daganzo (1997)). Specifically, with the information contained in the time-varying O-D matrices, it is possible to project network users travel demands up to a time horizon of interest and pre-determine optimal control and routing policies that achieve some desirable system-wide objectives.

Time-dependent O-D distributions are the outcomes of motorists trip plans, given network traffic conditions and users specific characteristics. For on-line traffic management these O-D matrices can be obtained by using information contained existing O-D data and link traffic counts measured at each time interval. Those O-D data are estimates of travel demands during previous time intervals and, therefore, are associated with random errors in a dynamic system. Link traffic counts, on the other hand, suffer from measurements errors, and so forth. A dynamic O-D estimation and prediction model that predicts time-varying O-D demands from these two sources of information should recognize the uncertainty inherent in the dynamic traffic system and provide the flexibility to accommodate different degrees of error.

The problem is to estimate time-dependent O-D matrices given discrete time series of on-ramp, off-ramp, and mainline traffic counts for a network system. Two basic features characterize the problem:

- (a) the dynamic O-D estimation is in general an undetermined problem, because at each time interval the number of system equations that formulate the relationship between the time-varying O-D flows and observed link traffic counts is usually far less than the number of O-D pairs. All static models in the literature make use of some previous matrices or presumed distribution to obtain a unique solution. In contrast, most dynamic methods employ time series of traffic counts to set up more constraints so that the system equations become overdetermined. A unique O-D matrix can thus be obtained with recursive or nonrecursive system identification techniques (Nihan and Davis (1989)).
- (b) The link traffic counts measured at each time interval not only are composed of associated O-D flows but also should take the time-lag effect into consideration. In other words, the traffic count measured at exit  $j$  during time interval  $t$  is the summation of O-D flows originated from all origins (entrances) upstream of  $j$  during time intervals  $t, t-1, \dots$ , that arrived at exit  $j$  during time interval  $t$ . Therefore, a set of system equations is needed to describe the spatial and temporal relationships between a vector of time-varying O-D flows and the observed link traffic counts.

Therefore, time-dependent O-D matrices represent network users' trip demands given network traffic conditions. For a freeway system, for instance, the demand for a given O-D pair represents the number of vehicles entering the freeway segment from an upstream entrance and destined to a downstream exit. For a general network, time-varying link traffic counts (e.g., entrance, exit, and mainline section counts) are available for some links from traffic surveillance systems. The problem is to determine time-independent O-D matrices given discrete time series of entrance, exit, and mainline traffic counts for a network system.

### ***Previous Models Applied for the Dynamic O-D Estimation Problem***

In the literature, various approaches have already been applied for estimation and predictions of dynamic O-D matrices. They can be categorized into two families: statistical inference approaches and state space models. A large number of statistical inference methods in this field have been proposed. Maher (1983) presented a Bayesian statistical approach to estimate time-varying O-D flows for both intersections and small networks. The basic assumption of this



method is the multivariate normality of both existing information and observations and accurate knowledge of the assignment matrix based on a proportional assignment algorithm.

Cremer and Keller (1987) and Nihan and Davis (1987) have presented a number of least squares (LS) and weighted (generalized) LS procedures for the estimation of dynamic O-D flows on small networks or intersections. These models are based on the assumption that the time taken by vehicles to traverse an intersection or network either is small and can be ignored or is equal to some fixed number of time intervals (*constant speed assumption*), which is not a suitable assumption for a freeway or a general network, especially one in the presence of traffic congestion. Nihan and Davis (1989) presented approaches based on both *prediction error* (PE) minimization and *maximum likelihood* (ML) to estimate the turning proportion at an intersection. The research demonstrated that the ML algorithm produces biased but more efficient estimates, whereas the PE minimization approach generates unbiased but less efficient estimates. Bell (1991) considered platoon dispersion in a simple network and proposed two real-time O-D flow estimation models based on the constrained recursive least squares method. The unknown variables to be estimated are both O-D proportions and platoon dispersion fractions.

A Markovian freeway traffic model was presented by Davis (1993) estimating exit traffic that assumed the exit traffic of a given freeway section depends only on its current total traffic for a time interval. Dynamic O-D matrices are obtained by minimizing the difference between the observed and predicted exit traffic using a nonlinear least squares method. Hellinga and Van Aerde (1994) have presented an analytic approach for determining estimates of the mean O-D demands and the reliability of such estimates of the mean O-D demands and the reliability of such estimates on the basis of statistical sampling theory. The best estimate of the total number of departures between any origin and destination (on O-D pair) is the number of probe departures between them, scaled by the estimate of the level of market penetration. The sampling error of the estimator is a function of the sample size and the market penetration rate of probe vehicles. This estimator, however, may give biased and inefficient estimates because it assumes uniform expansion of market penetration across all O-D pairs (Ben-Akiva, Ashok and Yang (1994)).

Another category of methods for estimating dynamic O-D flows is *state space* models. The objective of state estimation is to track variables that characterize some dynamic behavior, by processing observations afflicted by errors. State estimators rely on a model to relate the state variables to each other, to the observations and to the forcing (Norton (1986)).

The Kalman Filtering (KF) algorithm is the most typical state space model that has been intensively investigated. Cremer and Keller (1987) have presented a KF algorithm that uses a

random walk model as a dynamic equation. They concluded that if the system is not completely observable, the KF algorithm generates biased estimates. Simultaneously, Nihan and Davis (1987) proposed a similar KF form. Their paper concluded that the KF estimator shows faster convergence than the gradient method and can be more robust with respect to the choices of initial values and other system design parameters. Okutani (1987) applied the KF algorithm to estimate dynamic O-D flows in a small network in which the state variables are those time-dependent O-D flows. The empirical results showed that the more link traffic counts are observed, the more accurate the estimator results. Ashok and Ben-Akiva (1993) provided an O-D matrix updating framework that formulates the problem as a Kalman filtering in which the state vector consists of O-D flows deviations from previous estimates based on historical data. In the case study, the authors assumed that the assignment matrices are time-invariant because of the constant speed assumption, an assumption that is not realistic in view of traffic flow variations. Chang and Wu (1994) presented an on line freeway estimation algorithm that is also based on the KF algorithm. This model considers the effect of travel time variability and uses mainline and on-ramp traffic information (counts) in the filtering process. The state variables to be estimated are the fractions of O-D flows and the assignment proportions, which might violate the normality assumption of the state variable. Furthermore, there is no guarantee that the state variable estimates are between 0 and 1.

An improved KF method for estimating dynamic freeway O-D probabilities was proposed by Zijpp and Hamerlag (1994). This model provides a way to approximate the noise variance in the KF algorithm by using a trip generation model. The simulated and empirical results indicated that the improved KF method combined with a historical data base performs better than other approaches. Wu and Chang (1995) presented a recursive-based framework to estimate dynamic O-D flows in a small network. This model considers travel-time variability and incorporates the measurements from dynamic screenline flows to enrich the information used to determine the time-varying O-D flows. The simulation results demonstrate the same findings that Okutani (1987) reported earlier.

As this literature review demonstrates, state-of-the-art methods for the estimation and prediction of dynamic O-D flows from link traffic counts lack the capability of predicting the effects of demand diversion O-D flow distributions. Moreover, some of the reviewed methods apply the O-D proportion instead of the O-D flows as state variables. Unfortunately, there is no guarantee that the natural constraints can be satisfied when this is done.

A KF based dynamic algorithm freeway O-D estimation algorithm with route switching and time-varying traffic characteristics considerations is presented in Hu et al. (2000). Their model extends the previous research in the field (Ashok and Ben-Akiva (1993)) by incorporating dynamic traffic characteristics and a behavioral component within the KF algorithm. The effects of traffic congestion or traffic information provided to motorists are explicitly captured by incorporating a route-switching model. Because of the difficulty in acquiring real-world O-D data that consist of demand diversion behavior, the proposed methodology is evaluated through simulation experiments. Preliminary test results demonstrated the capability of the proposed algorithm in predicting dynamic freeway O-D demands. Those results also illustrated the importance of using time-varying parameters and accounting for the effect of traffic information in the estimation and prediction of dynamic O-D demands.

The work developed for task order TO 4121 implements the algorithm by Hu et al. (2000) to predict and forecast the right O-D demands in a network. The next section of this report describes in detail the algorithm proposed by Hu et al. (2000).

### ***Description of the Algorithm Implemented***

The dynamic traffic system studied by Hu et al. (2000) is represented by a linear, finite dimensional stochastic system as depicted in Figure 1.

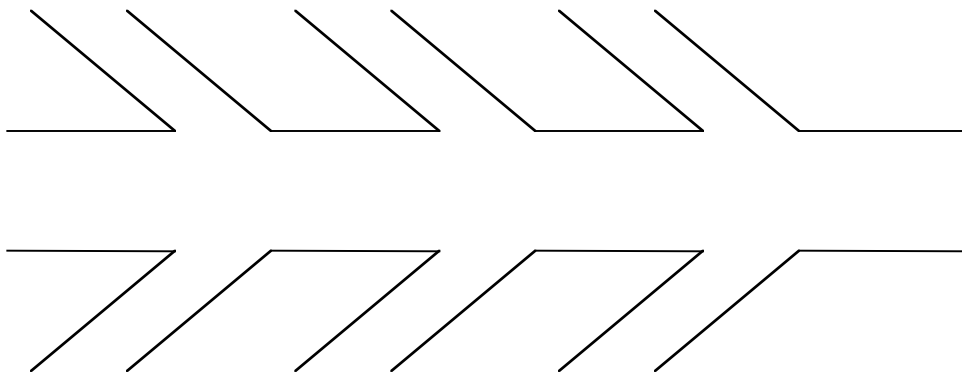


Figure 1 - Freeway section studied by Hu et al. (2000).

The system can be described by the following state space equations:

$$X_{k+1} = \sum_{r=k-p+1}^k f_k^r X_r + w_k \quad (1)$$

$$Z_k = \sum_{r=k-u}^k a_k^r X_r + v_k \quad (2)$$

where: -  $x_{k+1}$  is an  $n$ -vector of O-D flows at time  $k+1$ ;

-  $z_k$  is an  $m$ -vector of link traffic counts measured at time  $k$  ( $m \neq n$ );

-  $f_k^r$  is the ( $n \times n$ ) transition matrix which describes the effects of previous O-D flows  $x_r$  on current O-D flows  $x_{k+1}$ ;

-  $a_k^r$  is the ( $m \times n$ ) assignment matrix whose entries specify the contributions of O-D flows  $x_r$  and  $z_k$ ;

-  $w_k$  and  $v_k$  are the  $n$ -vector input (random) and  $m$ -vector output (measurement) noise processes, respectively.

The noise processes ( $w_k$  and  $v_k$ ) are assumed to be individually white noise and Gaussian processes with zero means and known covariance matrices, i.e.  $Q_k$  and  $R_k$ .

In their work, Hu et al. (2000) termed Eq. (1) as the transition equation and it was formulated as an autoregressive model of order  $p$ . et al. (2000) also termed Eq. (2) as the measurement equation, describing the temporal relationships between the dynamic O-D flows and the observed link traffic counts.

Given the above described system, the goal of the Kalman filtering algorithm is to obtain an estimate of  $x_k$  (the  $n$ -vector of O-D flows) using measurements  $z_0, z_1, \dots, z_k$  (the  $m$ -vector of link traffic counts). Mathematically, Hu et al. (2000) sought to recursively estimate the following system of state variables:

$$X_{k|k-1} = E[x_k | z_0, z_1, \dots, z_{k-1}] = E[x_k | Z_{k-1}] \quad (3)$$

$$X_{k|k} = E[x_k | z_0, z_1, \dots, z_k] = E[x_k | Z_k] \quad (4)$$

and their corresponding error covariance matrices can be described as:

$$\bullet_{k|k-1} = E[(X_k - X_{k|k-1})(X_k - X_{k|k-1})^T] \quad (5)$$

$$\bullet_{k|k} = E[(X_k - X_{k|k})(X_k - X_{k|k})^T] \quad (6)$$

where: -  $x_{k|k-1}$  is the state estimate given observations up to time  $k-1$  being termed the one-step ahead prediction and,

-  $x_{k|k}$  is the state estimate given observations up to time  $k$  being termed the filtered estimate (see Anderson and Moore (1979)).

Hu et al. (2000) assumed that the initial conditions,  $X_0$  (the mean of the initial state) and  $P_0$  (the covariance of the initial state  $X_0$ ) are given and then the Kalman filter can recursively estimate the state variables by applying the following equations:

$$\bullet_{k|k-1} = F_k \bullet_{k|k-1} F_k^T + Q_k \quad (7)$$

$$K_k = \bullet_{k|k-1} A_k^T [A_k \bullet_{k|k-1} A_k^T + R_k]^{-1} \quad (8)$$

$$\bullet_{k|k} = \bullet_{k|k-1} - K_k A_k \bullet_{k|k-1} \quad (9)$$

$$X_{k|k-1} = F_k X_{k|k-1} \quad (10)$$

$$X_{k|k} = X_{k|k-1} + K_k [z_k - A_k X_{k|k-1}] \quad (11)$$

As the transition and measurement equations are given by Eqs. (1) and (2), the dependence of the current state and measurement vectors on previous state vectors of all prior intervals up to  $s = \max(p, u)$  is introduced. Hu et al. (2000) defined  $p$  as the order of the transition equation, which is an autoregressive time series model and,  $u$  as the maximum number of time intervals required to travel between any O-D pair of the entire freeway segment. Besides,  $F_k$  ( $n_s \times n_s$ ) and  $A_k$  ( $m \times n_s$ ) represent the augmented transition and assignment matrices, respectively.

By the description of the Kalman filtering algorithm described above one can see that it requires the input and the knowledge of input data (i.e., input matrices). For instance, the matrices  $z_k$   $A_k$  are the link traffic counts and the assignment matrices, respectively, that must be given as inputs to the algorithm. The following section describes how Hu et al. (2000) obtained their input

data and the approach applied in this research to obtain from Paramics the necessary data as input.

### ***Implementation of the Kalman Filtering Algorithm***

The implementation of the Kalman filtering algorithm in Hu et al. (2000) was made in connection with DYNASMART, a mesoscopic traffic simulator (Mahmassani et al. (1993)). In some sense, that made Hu et al. (2000) implementation easier than the one in this research, as it is possible, for instance, to obtain the assignment matrix directly from DYNASMART.

At the start of the implementation of this project (task Order TO 4121), it was also considered the possible application of DYNASMART in connection with Paramics. It should also be added that such implementation had already been made in the past (in a small scale) and was called Paradyn (*Paramics* and *Dynasmart*). But, after having discussed the problem with some colleagues, it was concluded that such implementation would not have been so direct and, besides that, there would be the need to provide further training to Caltrans (the main sponsor of this project). Therefore, a new approach was developed being decided to develop API s (Advanced Programming Interfaces) to obtain the necessary data from Paramics as input to the algorithm and, then return the results (the new O-D table) back to Paramics (in Paramics format).

In this project (Task Order TO 4121), the algorithm described in Hu et al. (2000) was developed in C language. The developments of the API s to interface the implemented KF algorithm with Paramics were also started. The full implementation of the API s and the analysis of the results obtained for a network were developed in Task Order TO 4131. A discussion of the development of a dynamic O-D estimation approach by Quadstone (the developer of Paramics) is also made in Task Order 4131.

## CONCLUSIONS

An important element of California's strategy in the development of a real world transportation testbed is the California ATMS Testbed. It evaluates the potential of new technologies and strategies in the management of advanced transportation systems. The analysis of the California ATMS Testbed is made by Paramics, the microscopic traffic simulator.

The application of the Paramics simulator tool shows its flexibility and sensitivity to a variety of issues regarding real-time information, and provides a useful environment for testing and applying information supply strategies. This work extends the capabilities of Paramics, making it a more robust protocol to analyze the day to day of the urban transportation challenges. Implementing the dynamic O-D estimation and prediction of this project will make the Paramics software a more realistic protocol.

This project studied a variety of previous research related to the dynamic O-D estimation problem. This research also chose to implement a particular Kalman filtering algorithm that uses time-varying assignment matrices generated by applying Paramics. To apply the described algorithm with Paramics, the development of APIs (Advanced Programming Interfaces) have also to be done. The continuation of this project is described in Task Order 4131, where a description of Quadstone (the developer of Paramics) own efforts to develop a O-D estimation procedure is also discussed.

## REFERENCES

Ashok, K. and Ben-Akiva, M.E. (1993) *Dynamic Origin-Destination matrix estimation for real-time traffic management systems*. In *Transportation and Traffic Theory* (C.F. Daganzo, ed.), Elsevier Science Publishers B.V., 465-484.

Bell, M.G.H. (1991) *The real time estimation of origin-destination flows in the presence of platoon dispersion*, *Transportation Research, Part B*, Vol. 25, No. 2, 1157-125.

Ben-Akiva, M.E., Ashok, K. and Yang Q. (1994), *Commentaries on a statistical analysis of the reliability of using RGS vehicle probes as estimators of dynamic O-D departure rates*, *IVHS Journal*, Vol. 2, No. 1, 21-44.

Chang, G.L. and Wu, J. (1994) *Recursive estimation of time-varying Origin-Destination flows from traffic counts in freeway corridors*, *Transportation Research, Part B*, Vol. 28, No. 2, 141-160.

Cremer, M. and Keller, H. (1987) *A new class of dynamic methods for the identification of Origin-Destination flows*, *Transportation Research, Part B*, Vol. 21, No. 2, 117-132.

Daganzo, C. (1997), *Fundamentals of Transportation and Traffic Operations*, Elsevier Science Ltd., Great Britain.

Davis, G. A. (1993) *Estimating freeway demand patterns under impact of uncertainty on ramp controls*, *ASCE Journal of Transportation Engineering*, Vol. 119, No. 4, 489-503.

Gartner, N.H. (1982) *OPAC: A demand-responsive strategy for traffic signal control*, *Transportation Research Record*, **906**, 75-81.

Gartner, N.H. (1983) *Simulation study of OPAC: A demand-responsive strategy for traffic signal control*. In Gartner N.H. and Wilson N.H. (eds.), *Transportation and Traffic Theory* (pp. 233-250). New York: Elsevier Science Publishing Company.

Hellinga, B. and Van Aerde, M. (1994) *A statistical analysis of the reliability of using RGS vehicle probes as estimators of dynamic O-D departure rates*, *IVHS Journal*, Vol. 2, No. 1, 21-44.



Hu, S.R., Madanat, S.M., Krogmeier, J.V. and Peeta, S. (2000), *Estimation of dynamic assignment matrices and OD demands using adaptive Kalman Filtering*, Working paper, 2000 (to appear in the ITS Journal).

Maher, M. J. *Inferences on trip matrices from observations on link volumes: a Bayesian statistical approach*, Transportation Research, Part B, Vol. 17, No. 6, 435-447.

Mahmassani, H.S. and Liu, Yu-Hsiu (1999), *Dynamics of commuting decision behavior under advanced traveler information systems*, Transportation. Research, **7C**, 91-107

Norton, J.P. (1986) *An introduction to identification*, Academic Press, Inc. London, U.K..

Okutani, I. (1987) *The Kalman filtering approach in some transportation and traffic problems*. In International Symposium on Transportation and Traffic Theory (N.H. Gartner and N.H.M. Wilson, eds.), Elsevier Science Publishing Company, Inc., 397-416.

Peeta, S. (1994), *System optimal dynamic traffic assignment in congested networks with advanced information systems*, Doctoral Dissertation, The University of Texas at Austin.

Peeta, S. and Mahmassani, H.S. (1995), *Multiple user-class real-time traffic assignment for online operations: a rolling horizon solution framework*, Transportation. Research, **3C**, 83-98.

Nihan, N.L. and Davis, G.A. (1987) *Recursive estimation of Origin-Destination matrices from Input/Output counts*, Transportation Research, Part B, Vol. 21, No. 2, 149-163.

Nihan, N.L. and Davis, G.A. (1989) *Application of prediction-error minimization and maximum likelihood to estimate intersection O-D matrices from traffic counts*, Transportation Science, Vol. 23, No. 2, 77-90.

Zijpp, N.J. and Hamerslag, R. (1994) *An improved Kalman filtering approach to estimate Origin-Destination matrices for freeway corridors*. Presented at 73<sup>rd</sup>. Annual Meeting of the Transportation Research Board, Washington, D.C.

Wu, J. and Chang, G.L. (1995) *Estimation of time-varying O-D matrices with dynamic screenline flows*. Presented at 74<sup>th</sup>. Annual Meeting of the Transportation Research Board, Washington, D.C.