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Publication Date

2015-11-01

DOI

10.1016/j.physletb.2015.09.064

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PII: S0370-2693(15)00739-X
DOI: http://dx.doi.org/10.1016/j.physletb.2015.09.064
Reference: PLB 31367

To appear in: *Physics Letters B*

Received date: 10 July 2015
Revised date: 25 September 2015
Accepted date: 28 September 2015

Please cite this article in press as: A. Prokudin et al., Scheme dependence and Transverse Momentum Distribution interpretation of Collins-Soper-Sterman resummation, *Phys. Lett. B* (2015), http://dx.doi.org/10.1016/j.physletb.2015.09.064

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Scheme dependence and Transverse Momentum Distribution interpretation of Collins-Soper-Sterman resummation

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Abstract

Following an earlier derivation by Catani-de Florian-Grazzini (2000) on the scheme dependence in the Collins-Soper-Sterman (CSS) resummation formalism in hard scattering processes, we investigate the scheme dependence of the Transverse Momentum Distributions (TMDs) and their applications. By adopting a universal C-coefficient function associated with the integrated parton distributions, the difference between various TMD schemes can be attributed to a perturbative calculable function depending on the hard momentum scale. We further apply several TMD schemes to the Drell-Yan process of lepton pair production in hadronic collisions, and find that the constrained non-perturbative form factors in different schemes are consistent with each other and with that of the standard CSS formalism.

Keywords: Quantum Chromo Dynamics; Resummation; CSS formalism; TMD factorization; Semi-Inclusive Deep Inelastic Scattering; Drell-Yan process

1. Introduction

The Transverse Momentum Distributions (TMDs) and the nucleon tomography in momentum space have attracted strong interest in recent years [1, 2]. TMDs provide a unique opportunity to investigate the novel correlations between the parton momentum and the nucleon spin. They unveil the strong interaction QCD dynamics in a manifest way, such as the gauge invariance leading to the sign change [3, 4] of certain TMDs in different hard scattering processes, and the QCD factorization and evolution which are crucial for predicting the scale dependence of the spin asymmetries. On the theory side, the TMDs are not straightforward extensions [5] of the conventional collinear parton distribution functions (PDFs). They hold special properties that differ them from collinear PDFs and play important roles in high energy scattering. The associated phenomena are direct consequences of perturbation gauge theory computation of the famous Sudakov form factors [6] back in 1950s.

When one applies the TMD factorization to physical processes, one has to consider the associated QCD dynamics in the definition of TMDs and in the phenomenological studies. Especially, large logarithmic corrections from high order perturbative calculations have to be taken into account and resummed [7] to all orders. In addition, the naive gauge invariant TMD definition contains the so-called light-cone singularities at higher orders and needs to be regulated [8]. Several ways to implement such a regularization have been proposed in the literature and they introduce the scheme dependence in TMDs and their applications [8, 9, 10, 11, 12, 13]. The goal of this paper is to investigate such scheme dependence, which is of crucial importance for applying the TMDs in hard scattering processes and extracting the associated nucleon structure from experiments.

In the context of the standard Collins-Soper-Sterman (CSS) resummation formalism [7], the TMDs are expressed in terms of the collinear parton distributions via an additional factorization at small $b \ll 1/\Lambda_{QCD}$, where b represents the Fourier conjugate variable associated with the transverse momentum $k_\perp$. The final expressions for the measured cross-sections differential in transverse momentum of the observed particles do not depend how we define the TMDs

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at the first place when such relations to collinear PDFs are used. In other words, in all TMD formalisms of Refs. [8, 9, 10, 11, 12, 13], one will obtain the same results as that of the standard CSS resummation. However, as discussed in an early paper by Catani, de Florian, and Grazzini [14], even in conventional CSS formalism there is freedom to separate the so-called hard factor, H, which depends on the running coupling at the hard momentum scale Q from the C-coefficient functions associated with the integrated parton distribution functions where running coupling depends on $\mu_b = c_0/b$ with $c_0 = 2e^{-\gamma_E}$. It was referred to in Ref. [14] as the scheme dependence of CSS resummation. The relation between different schemes was further demonstrated by an order by order proof [14]. The relevant derivations with explicit results up to next-to-next-leading order for Drell-Yan, Higgs boson, di-photon production processes have been extensively discussed in Ref. [15]. The same argument applies to the scheme dependence in the TMD formalism as well [7, 8, 9, 10, 11, 12, 13]. By adopting a universal C-coefficient function associated with the collinear parton distributions [14], the connections between different schemes can be attributed to the hard coefficients and can be established order by order in perturbation theory. As a result, all the TMD scheme dependence can be accounted for and the schemes can be unified and compared to the standard CSS resummation in description of the experimental data in phenomenological studies.

Furthermore, this unification provides an attractive interpretation for the CSS resummation, from which we have a clear TMD interpretation of the hard scattering processes. To establish this, we apply this scheme in the global analysis of the Drell-Yan process of lepton pair production in pp collisions, and fit the associated non-perturbative form factors. In the calculations, we adopt the so-called b_*-prescription and derive the relevant perturbative coefficients following the procedure of Ref. [14]. Our results show that the non-perturbative form factors are consistent with that in the standard CSS scheme.

The rest of the paper is organized as follows. In Sec. 2, we briefly introduce the TMD schemes in hard scattering processes, and derive the relevant coefficients. In Sec. 3, we fit the experimental data of Drell-Yan type of hard processes in hadronic collisions and constrain the associated non-perturbative form factors. And finally, we conclude our paper in Sec. 4.

2. TMD Schemes

Let us start with the standard CSS resummation formalism for Drell-Yan lepton pair production processes at low transverse momentum: $A(P_A) + B(P_B) \to \gamma^*(q) + X \to \ell^+ + \ell^- + X$, where P_A and P_B represent the momenta of the incoming hadrons A and B, respectively. The differential cross section can be written as [7],

$$\frac{d^4\sigma}{dy dQ^2 d^2q_\perp} = \sigma_0^{(\mathrm{DY})} \left[\int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q;q_\perp) \right] , \tag{1}$$

where $q_\perp$ and y are transverse momentum and rapidity of the lepton pair, respectively, $\sigma_0^{(\mathrm{DY})} = 4\pi\alpha_{em}^2/3N_c sQ^2$ with $s = (P_A + P_B)^2$. In the above equation, the first term is dominant in the $q_\perp \ll Q$ region and $\widetilde{W}_{UU}$ denotes the all-order resummation result which has the following form [7, 14]:

$$\widetilde{W}_{UU}(Q;b) = H^{(DY)}(\alpha_s(Q))\, e^{-S(Q^2,b)} \sum_{q=q,\bar{q}} e_q^2\, C^{(DY)}_{q\leftarrow i} \otimes f_{i/A}(x_1,\mu_b)\, C^{(DY)}_{\bar{q}\leftarrow j} \otimes f_{j/B}(x_2,\mu_b) , \tag{2}$$

where $\mu_b = c_0/b_*$ with $c_0 = 2e^{-\gamma_E}$ and γ_E the Euler constant, $x_{1,2} = Qe^{\pm y}/\sqrt{s}$ represent the momentum fractions carried by the incoming quark and antiquark in the Drell-Yan processes, the symbol $\otimes$ for convolution in $x_1(x_2)$ and $f_{i/A}(x,\mu_b)$ and $f_{j/B}(x,\mu_b)$ stand for the collinear integrated parton distribution functions at the scale μ_b. In Eq. (2), b_*-prescription, $b \to b_* = b/\sqrt{1+b^2/b_{max}^2}$, is introduced [7]. The form factor $S(Q,b)$ contains perturbative and nonperturbative parts, such that the total form factor for quarks can be written as $S(Q,b) = S_{pert}(Q,b_*) + S_{NP}(Q,b)$,

$$S_{\mathrm{pert}}(Q,b) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[A(\alpha_s(\bar{\mu})) \ln\frac{Q^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu})) \right] , \tag{3}$$

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where A, B and C coefficients calculable order by order in perturbation theory perturbative series $A = \sum_{n=1}^{\infty} A^{(n)} (\alpha_s/\pi)^n$, $B = \sum_{n=1}^{\infty} B^{(n)} (\alpha_s/\pi)^n$, $C = \sum_{n=1}^{\infty} C^{(n)} (\alpha_s/\pi)^n$. The A, B, C coefficients can be derived [14],

$$
\begin{aligned}
&A^{(1)}_{\rm CSS} = C_F, \;\; B^{(1)}_{\rm CSS} = -\frac{3}{2}C_F, \;\; C^{(1)}_{\rm CSS} = \frac{C_F}{2}\left[(1-x) + \delta(1-x)\frac{\pi^2-8}{2}\right] , \\
&A^{(2)}_{\rm CSS} = \frac{C_F}{2}\left(C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}N_f\right) , \\
&B^{(2)}_{\rm CSS} = C_F^2\left(\frac{\pi^2}{4} - \frac{3}{16} - 3\zeta_3\right) + C_F C_A\left(\frac{11}{36}\pi^2 - \frac{193}{48} + \frac{3}{2}\zeta_3\right) + C_F N_f\left(\frac{17}{24} - \frac{\pi^2}{18}\right) ,
\end{aligned} \tag{4}
$$

in the standard CSS scheme. In the standard CSS formalism, the hard coefficient $H_{\rm CSS}(\alpha_s(Q)) \equiv 1$ for all orders.

We would like to emphasize that the resummation formula and the associated coefficients are uniquely determined, once the scheme is fixed. The reason is simple. In the perturbative calculations of hard processes at low transverse momentum, the large logarithms depend on two separate scales: Q and $1/b$, the hard momentum and the Fourier conjugate of the traverse momentum $q_\perp$, respectively. The resummation of these large logarithms has to take the form as in Eq. (3), as a consequence of perturbation gauge theory computation of Sudakov form factors [6, 16, 8]. Additional factors in the CSS resummation come from the fact that the collinear gluon splitting is proportional to $1/q_\perp^2$ (again a result of a gauge theory computation), for which the Fourier transformation leads to $\ln(\mu_b/\mu)$ where μ represents the PDF scale. Therefore, the integrated parton distribution is calculated at μ_b for canonical choice of the resummation. By doing so, we also resum the logarithms associated with collinear gluon radiation. The coefficients A, B, and C can be obtained from the factorization derivation, or by comparing to the fixed order perturbative calculations. For phenomenological applications, the CSS formalism has been very successful in Drell-Yan lepton pair production, $W^\pm/Z$ boson production in hadron collisions [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

As discussed in Ref. [14], there is a freedom to absorb $\alpha_s(Q)$ corrections from higher orders in the definition of hard coefficient $H^{(DY)}(\alpha_s(Q))$ of Eq. (2). Then, the associated B and C coefficients will be modified according to the renormalization group equations. This was referred to as the scheme dependence in the CSS resummation in Ref. [14]. In the following, we will apply this idea to discuss the TMD interpretation of the CSS resummation formalism, where the scheme dependence is essential in the TMD definition and factorization.

TMD factorization [9] aims at separating well defined TMD distributions in Eq. (2), such that the TMD distributions can be used in different processes in a universal manner. The b-space expression of $\widetilde{W}_{UU}$ in Eq. (2) thus can be rewritten in the TMD factorization in terms of a product of process independent TMDs and a process dependent hard factor:

$$
\widetilde{W}_{UU}(Q;b) = \mathcal{H}^{\rm TMD}_{(DY)}(Q;\mu) \sum_{q=q,\bar{q}} e_q^2\, \tilde{f}^{(sub)}_{q/A}(x_1,b;Q,\mu)\, \tilde{f}^{(sub)}_{\bar{q}/B}(x_2,b;Q,\mu) , \tag{5}
$$

where both the subtracted quark distribution $\tilde{f}^{(sub)}_q$ and hard factor $\mathcal{H}^{\rm TMD}_{(DY)}$ depend on the scheme we choose to regulate the light-cone singularity in the TMD definition. In this paper we consider three TMD schemes: (1) Ji-Ma-Yuan 2004 [10, 11]; (2) Collins 2011 [9]; (3) Lattice [13] or Collins-Soper 1981 [8]. The so-called EIS scheme was shown to be equivalent to Collins 2011 scheme [30]. Moreover, because of usage of space-like gauge link in the lattice scheme, the results in this scheme coincide with the the original Collins-Soper 81 scheme. Extensions to other formalisms can follow accordingly.

We take the example of Ji-Ma-Yuan 2004 scheme (JMY) [10, 11], where the unpolarized quark distribution is defined as

$$
f_q(x,k_\perp;\zeta,\mu_F,\rho) \;=\; \frac{1}{2}\int \frac{d\xi^- d^2b}{(2\pi)^3} e^{-ix\xi^- P^+ + i\vec{b}\cdot\vec{k}_\perp} \left\langle PS \left| \overline{\psi}(\xi^-,0,\vec{b})\mathcal{L}_v^\dagger(-\infty;\xi)\gamma^+\mathcal{L}_v(-\infty;0)\psi(0)\right| PS \right\rangle , \tag{6}
$$

with the gauge link $\mathcal{L}_v(-\infty;\xi) \equiv \exp\left(-ig\int_0^{-\infty} d\lambda\, v\cdot A(\lambda v+\xi)\right)$. The above definition contains the light-cone singularity if we take the gauge link along the light-front direction, $v^2 = 0$. The way to regulate this singularity and subtract soft gluon contribution defines the scheme for TMDs. In the JMY scheme, the gauge link is chosen to be slightly off-light-cone, such that $n = (1^-, 0^+, 0_\perp) \to v = (v^-, v^+, 0_\perp)$ with $v^- \gg v^+$. Similarly, for the TMD antiquark distribution, $\bar{v}$ was introduced, $\bar{v} = (\bar{v}^-, \bar{v}^+, 0_\perp)$ with $\bar{v}^+ \gg \bar{v}^-$. Because of the additional v and $\bar{v}$, there are additional

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invariants: $\zeta_1^2 = (2v \cdot P_A)^2/v^2$, $\zeta_2^2 = (2\bar{v} \cdot P_B)^2/\bar{v}^2$, and $\rho^2 = (2v \cdot \bar{v})^2/v^2\bar{v}^2$. Accordingly, the soft factor is defined as,

$$S^{v,\bar{v}}(b) = \langle 0|\mathcal{L}_{\bar{v}}^{\dagger}(b_\perp)\mathcal{L}_{v}^{\dagger}(b_\perp)\mathcal{L}_{v}(0)\mathcal{L}_{\bar{v}}(0)|0\rangle \,. \tag{7}$$

Following the subtraction procedure of Ref. [9], we can define the subtracted TMDs in b-space in the JMY scheme as,

$$\tilde{f}^{(sub)}_{q(JMY)}(x, b; \zeta, \mu_F, \rho) \quad = \quad \frac{\tilde{f}_q(x, b; \zeta, \mu_F, \rho)}{\sqrt{S(b; \rho, \mu_F)}} \,, \tag{8}$$

where $\tilde{f}_q(x, b; \zeta, \mu_F, \rho)$ is the b-space expression for the un-subtracted TMD of Eq. (6). The evolution equations are derived for the TMDs: one is the energy evolution equation respect to ζ, the so-called Collins-Soper evolution equation [8] and the renormalization group equation associated with the factorization scale μ_F and related to anomalous dimensions of the distribution $\tilde{f}$. After solving the evolution equations and expressing the TMDs in terms of the integrated parton distributions to have a complete resummation results, we can write,

$$\begin{aligned}\tilde{f}^{(sub)}_{q(JMY)}(x, b; \zeta^2 = \rho Q^2, \mu_F = Q, \rho) \quad &= \quad e^{-S^q_{pert}(Q,b_*)-S^q_{\rm NP}(Q,b)}\, \widetilde{\mathcal{F}}_q^{\rm JMY}(\alpha_s(Q); \rho) \\ &\quad \times \sum_i C_{q\leftarrow i} \otimes f_i(x, \mu_b) \,,\end{aligned} \tag{9}$$

where we have chosen the energy parameter $\zeta^2 = \rho Q^2$ and the factorization scale $\mu_F = Q$ to resum large logarithms [10, 11]. The perturbative form factor S^q_{pert} contains contributions from the Collins-Soper evolution kernel and the renormalization equation respect to the factorization scale μ_F. Similar to the CSS resummation, b_*-prescription was applied. In the above equation, we have also followed the derivations of Ref. [14] to include the ρ-dependence in the hard factor $\widetilde{\mathcal{F}}_q$ by applying the renormalization group equation of running coupling α_s. By doing that, the C-coefficients are much simplified and have the following universal TMD form [1],

$$C^{(TMD)}_{q\leftarrow q'}(x, \mu_b) = \delta_{q'q}\left[\delta(1-x) + \frac{\alpha_s}{\pi}\left(\frac{C_F}{2}(1-x)\right)\right] \,, \tag{10}$$

$$C^{(TMD)}_{q\leftarrow g}(x, \mu_b) = \frac{\alpha_s}{\pi} T_R\, x(1-x) \,, \tag{11}$$

for the quark-quark and quark-gluon splitting case. A universal C-function in the CSS resummation formalism has also been emphasized in Ref. [15], where it was referred as the "hard" scheme. From the results in Ref. [10], see, for example, Eq. (36) of [10], we obtain

$$\widetilde{\mathcal{F}}_q^{\rm JMY}(\alpha_s(Q); \rho) = 1 + \frac{\alpha_s}{2\pi} C_F \left(\ln\rho - \frac{\ln^2\rho}{2} - \frac{\pi^2}{2} - 2\right) \,. \tag{12}$$

The above equations are derived based on the perturbative calculation and the associated QCD factorization for the TMDs. They apply to all TMD schemes [8, 9, 10, 11, 12, 13] mentioned above. The collinear divergence in the TMDs can be factorized into the integrated parton distributions as shown in Eq. (9) at small $b \ll 1/\Lambda_{QCD}$. For large b, a non-perturbative function has to be included. The universal C-coefficient function is adopted to simplify the final expression for the TMDs and minimize the higher order corrections associated with the integrated parton distributions.

Similarly, for the Collins 2011 (JCC) scheme, we have [9, 31],

$$\begin{aligned}\tilde{f}^{(sub)}_{q(JCC)}(x, b; \zeta_c^2 = Q^2, \mu_F = Q) \quad &= \quad e^{-S^q_{pert}(Q,b_*)-S^q_{\rm NP}(Q,b)}\, \widetilde{\mathcal{F}}_q^{\rm JCC}(\alpha_s(Q)) \\ &\quad \times \sum_i C^{(TMD)}_{q\leftarrow i} \otimes f_i(x, \mu_b) \,,\end{aligned} \tag{13}$$

$$\widetilde{\mathcal{F}}_q^{\rm JCC}(\alpha_s(Q)) \quad = \quad 1 + O(\alpha_s^2) \,, \tag{14}$$

[1] In principle, we can also choose $C_{\rm CSS}$ of Eq. (4) for the C-coefficients, which will go back to the standard CSS resummation for phenomenological applications. We chose these coefficients for simplicity.

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where ζ_c is the regulation parameter in JCC scheme and the α_s correction in $\widetilde{\mathcal{F}}_q^{\rm JCC}$ vanishes [2]. Again, we emphasize that C-coefficient takes the same form as that in Eq. (10). Therefore, the scheme dependence in the TMDs only comes from the hard function $\widetilde{\mathcal{F}}_q$ as we have shown in the above equation.

Recently, there has been a motivated study to formulate the TMDs on lattice, where a different subtraction scheme was adopted, for which we have [13]

$$\begin{aligned} \tilde{f}^{(sub)}_{q(Lat.)}(x,b;\zeta^2=Q^2,\mu_F=Q) &= e^{-S^q_{pert}(Q,b_*)-S^q_{\rm NP}(Q,b)}\,\widetilde{\mathcal{F}}_q^{\rm Lat.}(\alpha_s(Q)) \\ &\quad\times\sum_i C^{(TMD)}_{q\leftarrow i}\otimes f_i(x,\mu_b)\,, \end{aligned} \tag{15}$$

$$\widetilde{\mathcal{F}}_q^{\rm Lat.}(\alpha_s(Q)) = 1+\frac{\alpha_s}{2\pi}C_F(-2)\,, \tag{16}$$

where the regulator ζ is defined as $\zeta^2=(2n_z\cdot P)^2/(-n_z^2)$ with space-like n_z: $n_z^2=-1$, $n_z\cdot P=-P_z$. As we mentioned above, lattice scheme uses the same space-like gauge link as the original Collins-Soper 1981 scheme, that is why the final expression for TMD are the same in both schemes.

Applying the above TMDs into the factorization formula of Eq. (5), and comparing to that in Eq. (2), we will find that the TMDs actually provide a special scheme for the CSS resummation in the context of Ref. [14]. We can immediately derive the relevant coefficients,

$$H^{(DY)}_{\rm TMD}(\alpha_s(Q)) = \widetilde{\mathcal{F}}_q(\alpha_s(Q))\times\widetilde{\mathcal{F}}_{\bar q}(\alpha_s(Q))\times\mathcal{H}^{\rm TMD}_{(DY)}(Q;Q)\,, \tag{17}$$

which will enter into Eq. (2) for phenomenological applications. Because the C-coefficients are universal among different TMD schemes, we conclude that $H^{(DY)}_{\rm TMD}$ will be the same in all of the three schemes discussed above. In particular, in the JMY scheme, all three factors in Eq. (17) depend on ρ, however the final result for $H^{(DY)}_{\rm TMD}$ does not depend on ρ. This demonstrates that all the TMD factorization schemes are equivalent in the context of the CSS resummation formalism, which will be used in the phenomenological applications. This can be verified from the above explicit results and from the associated hard factors calculated for different schemes at the one-loop order, and order by order proof can be done accordingly.

Further comparison also indicates that the perturbative and non-perturbative form factors for the quark and anti-quark can be related to that in the CSS formalism Eq. (2),

$$S^q_{pert}(Q,b_*) = S^{\bar q}_{pert}(Q,b_*) = S_{perp}(Q,b_*)/2\,, \tag{18}$$

$$S^q_{\rm NP}(Q,b)+S^{\bar q}_{\rm NP}(Q,b) = S_{\rm NP}(Q,b)\,, \tag{19}$$

where the perturbative form factor $S_{pert}(Q,b_*)$ takes the form of Eq. (3) with A and B coefficients for a particular TMD scheme. The above equation for the perturbative form factors can be verified explicitly from one-loop results in the TMD factorization of Refs. [9, 10, 11, 13]. Higher orders can be calculated in perturbative expansion.

From the above one-loop results for $\widetilde{\mathcal{F}}_{q,\bar q}$ and the relevant hard factors in the TMD factorization calculated in Refs. [10, 11, 9, 13],

$$\mathcal{H}^{\rm TMD\,JCC}_{(DY)}(Q;\mu) = 1+\frac{\alpha_s(\mu)}{2\pi}C_F\left(3\ln\frac{Q^2}{\mu^2}-\ln^2\frac{Q^2}{\mu^2}+\pi^2-8\right), \tag{20}$$

$$\mathcal{H}^{\rm TMD\,JMY}_{(DY)}(Q;\mu) = 1+\frac{\alpha_s(\mu)}{2\pi}C_F\left((1+\ln\rho^2)\ln\frac{Q^2}{\mu^2}-\ln\rho^2+\ln^2\rho+2\pi^2-4\right), \tag{21}$$

$$\mathcal{H}^{\rm TMD\,Lat.}_{(DY)}(Q;\mu) = 1+\frac{\alpha_s(\mu)}{2\pi}C_F\left(\ln\frac{Q^2}{\mu^2}+\pi^2-4\right), \tag{22}$$

we obtain the one-loop expression for $H_{\rm TMD}$ as,

$$H^{(1)(DY)}_{\rm TMD} = \frac{1}{2}C_F\left(\pi^2-8\right)\,, \tag{23}$$

[2]There is an ambiguity for the ultra-violet (UV) subtraction: an additional term of $\pi^2/12$ should be added in α_s correction if we follow the standard $\overline{\rm MS}$ subtraction used in the standard CSS. Here we adopt Collins-11 prescription for the UV subtraction.

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so that

$$H^{(DY)}_{\rm TMD}(Q) \;=\; 1+\frac{\alpha_s(Q)}{2\pi}C_F\left(\pi^2-8\right)\,, \tag{24}$$

For B and C coefficients, following the derivation of Ref. [14], we will obtain

$$\begin{aligned} C^{(1)}_{\rm TMD} &= C^{(1)}_{\rm CSS}-\delta(1-x)\,H^{(1)(DY)}_{\rm TMD}/2\,,\\ B^{(2)}_{\rm TMD} &= B^{(2)}_{\rm CSS}-\beta_0\,H^{(1)(DY)}_{\rm TMD}\,, \end{aligned} \tag{25}$$

where $\beta_0=\frac{11}{12}C_A-\frac{N_f}{6}$, and $A^{(1,2)}$ and $B^{(1)}$ remain the same as the standard CSS scheme. We will apply these coefficients in the next section to analyze the Drell-Yan type of lepton pair production in hadronic processes to constrain the associated non-perturbative form factors. Note that the TMD scheme with the one-loop coefficients defined in Eqs. (10,11,24,25) coincide the "hard" scheme defined in Ref. [15].

Following the arguments of Ref. [14], the process-dependence is included in H in Eq. (2), so that C-coefficients will be universal. We can apply the same C-functions to the quark distributions in other processes, such as the Semi-Inclusive Deep Inelastic Scattering (SIDIS), for which we have the standard CSS resummation coefficient

$$B^{(2)(SIDIS)}_{CSS}=B^{(2)}_{CSS}-\beta_0\,C_F\,\pi^2/2\,, \tag{26}$$

with $B^{(2)}_{CSS}$ from Eq. (4) and all other coefficients that have been listed in Refs. [32, 33, 28]. The hard function for SIDIS is

$$H^{(SIDIS)}_{\rm TMD}(Q) \;=\; 1+\frac{\alpha_s(Q)}{2\pi}C_F\,(-8)\,, \tag{27}$$

if we choose the TMD scheme for this process.

3. Non-perturbative Form Factors and TMD Interpretation

As we mentioned above, we will apply the b_*-prescription for the non-perturbative form factors. We will follow the SIYY parameterization [29]. This parameterization is motivated by a phenomenological study [28] and is inspired by matching to perturbative calculations of the Sudakov form factors [34, 35]. It has the following form,

$$S_{\rm NP}(Q,b) \;=\; g_2\ln(b/b_*)\ln(Q/Q_0)+g_1b^2\,, \tag{28}$$

with the initial scale $Q_0^2=2.4$ GeV^2 and cut-off parameter $b_{max}=1.5$ GeV^{-1}. The parameters $g_{1,2}$ have been fitted to the experimental data of Drell-Yan type processes in Ref. [29] using the standard CSS formalism ($H\equiv 1$). In the study of Ref. [29] it was found that the experimental data are consistent with x-independent non perturbative factors. In the following studies, we will take the above simple form of Eq. (28). Since we will compare the TMD schemes to the standard CSS scheme, we will keep all relevant parameters in non-perturbative factors fixed except for the changes in the coefficients $H^{(1)}$, $C^{(1)}$ and $B^{(2)}$.

We compare our results to the same set of the experimental data sets as those used in Ref. [29]. The data sets include the Drell-Yan lepton pair production from fixed target hadronic collisions from R209, E288 and E605 [36, 37, 38], and Z boson production in hadronic collisions from Tevatron Run I and Run II [39, 40, 41, 42]. We proceed with the fit of the experimental data using standard CSS, done in Ref. [29] and the TMD-schemes described in this paper. Notice that all TMD-schemes have exactly the same hard factor $H^{(DY)}_{\rm TMD}$, so by doing a single fit we effectively obtain underlying TMDs in either Collins 2011 [9], Lattice [13], or Ji-Ma-Yuan 2004 [10, 11] schemes. The fitted parameters are found to be,

$$\text{SIYY [29]}:\quad g_1=0.212,\quad g_2=0.84\,,\quad \text{total }\chi^2=168, \tag{29}$$

$$\text{SIYY}_{\rm TMD}:\quad g_1=0.212,\quad g_2=0.84\,,\quad \text{total }\chi^2=168\,, \tag{30}$$

where the first line is for the standard CSS scheme fit [29], the second for the TMD-scheme JMY and JCC with coefficients in Eqs. (10,11,23,25). There is hardly any difference between the two fits. There is no difference in the

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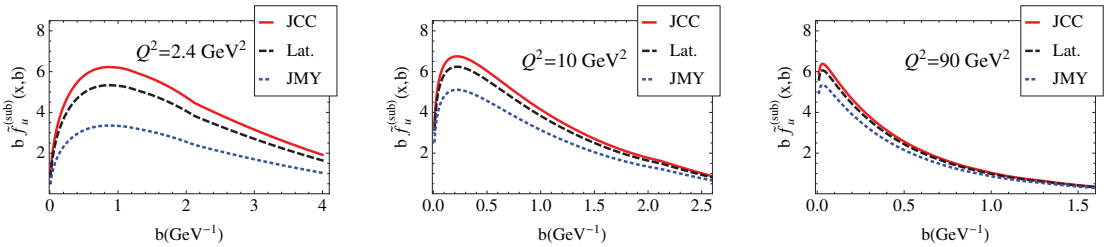

Figure 1: TMD up-quark distributions $\tilde{f}_u^{(sub.)}(x = 0.1, b)$ as functions of b at different scale $Q^2 = 2.4$, 10, 90 (GeV^2) for three different schemes, from the top to the bottom: JCC [9], Lattice [13], JMY [10, 11] ($\ln\rho = 1$).

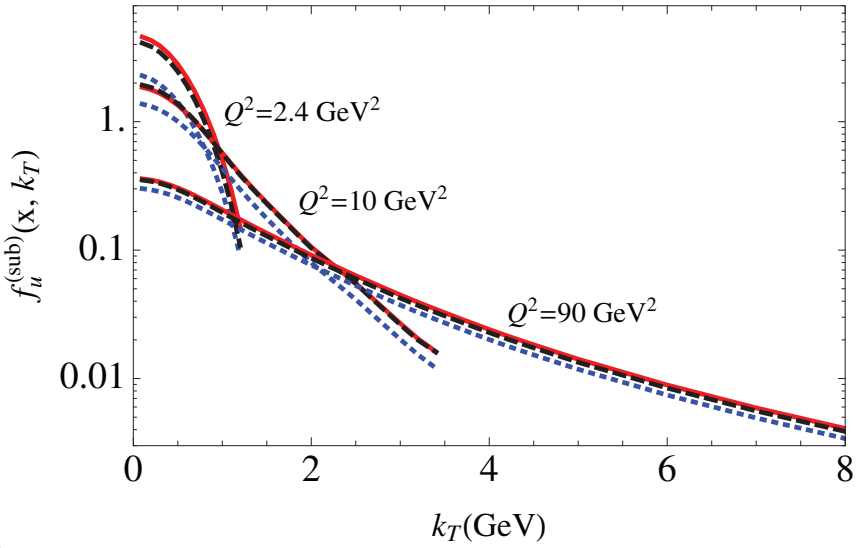

Figure 2: TMD up-quark distributions $f_u^{(sub.)}(x = 0.1, k_\perp)$ as functions of the transverse momentum $k_\perp$ (GeV) at three different scales $Q^2 = 2.4$, 10, 90 (GeV^2) for three different schemes, in each group for a particular value of Q^2 from the top to the bottom: JCC [9], Lattice [13], JMY [10, 11] ($\ln\rho = 1$).

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comparisons to the experimental data either. It demonstrates the effective equivalence between all schemes in the phenomenological studies. Theoretically, the difference could come from higher orders, such as α_s^2 in $H_{\rm TMD}$ and the coefficients at N^{3}LL for the resummation which are beyond what we have considered in this paper and Ref. [29].

With the non-perturbative form factors determined from the experimental data, we can compare the TMD quark distributions in different schemes by evaluating them using Eqs. (14, 16, 9). Ji-Ma-Yuan 2004 scheme has a residual dependence on the value of ρ, we fix it by choosing $\ln\rho = 1$.The transverse momentum dependence in three schemes is calculated by Fourier transformation respect to b using Eqs. (9,13,15). In Fig. 1, as an example, we plot the up-quark distributions $b\tilde{f}_u^{(sub)}(x = 0.1, b)$ at $x = 0.1$ for different schemes at different scale $Q^2 = 2.4$, 10, 90 (GeV^2) as functions of b (GeV^{-1}) and in Fig. 2 we plot $f_u^{(sub)}(x = 0.1, k_\perp)$ as function of the transverse momentum $k_\perp$. One can see from Fig. 1 that at low values of Q^2 the non-perturbative part of the distribution becomes very important and the values of $b > b_{max}$ dominate the result in $k_\perp$ space. At higher values of Q^2 the large b tail of the distribution is suppressed and the whole distribution can be computed using mainly perturbative regime $b < b_{max}$. In this regime the results will have a relatively low sensitivity to the non-perturbative input of TMD evolution. Again, the difference between different schemes is due to the coefficient $\widetilde{\mathcal{F}}_q$ in Eqs. (12,14,16). Because the difference is proportional to $\alpha_s(Q)$, it will become smaller at higher scale Q as shown in Figs. 1, 2. Similar plots have been shown in Ref. [31] for the quark distributions in the JCC scheme, however, using the previous BLNY parameterization [17] for the non-perturbative form factors obtained in CSS resummation. In our calculation we have consistently used relation between different schemes and the fitted non-perturbative form factors.

4. Conclusion

In this paper, we have investigated the scheme dependence in the TMD parton distributions and factorizations to describe the experimental data of hard scattering processes in hadron collisions. The equivalence between different schemes can be proven in perturbation theory order by order following the procedure of a similar study of Catani-de Florian-Grazzini 2000 [14]. We have studied three such schemes, Collins 2011 [9], Lattice [13], or Ji-Ma-Yuan 2004 [10, 11], and have demonstrated the equivalence between them and equivalence to the standard CSS method. The associated coefficients are illustrated at one-loop order.

With TMD scheme dependence embedded in the coefficients, $\widetilde{\mathcal{F}}$ and $\mathcal{H}^{\rm TMD}$, the resummation formulas have been applied to the Drell-Yan type of lepton pair production in pp collisions, and the associated non-perturbative form factors are determined from the global fit. We found that the TMD-schemes produce the same phenomenological results as compared to the standard CSS scheme for the resummation. More importantly, the parameters of the associated non-perturbative form factors are also found to be the same in all schemes.

Using the fitted parameters, we can calculate the TMD quark distributions as functions of the transverse momentum. We have compared the results from three different schemes. This comparison becomes useful in the TMD interpretation of the experimental results.

In this paper we explored the spin-average quark distributions. Similar studies can be carried out for all other TMDs. In particular, the quark Sivers function, which describes the correlation of the transverse momentum of the quark and the nucleon spin, can be formulated in the CSS resummation formalism. The non-perturbative form factors for Sivers function, however, will be different from the unpolarized quark distributions discussed in this paper. In order to determine these non-perturbative factors one needs to perform a global fit to the existing SIDIS data. We leave that for a future publication.

5. Acknowledgments

We thank C. P. Yuan for discussions and comments. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under contracts No. DE-AC02-05CH11231 (PS, FY), No. DE-AC05-06OR23177 (AP).

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