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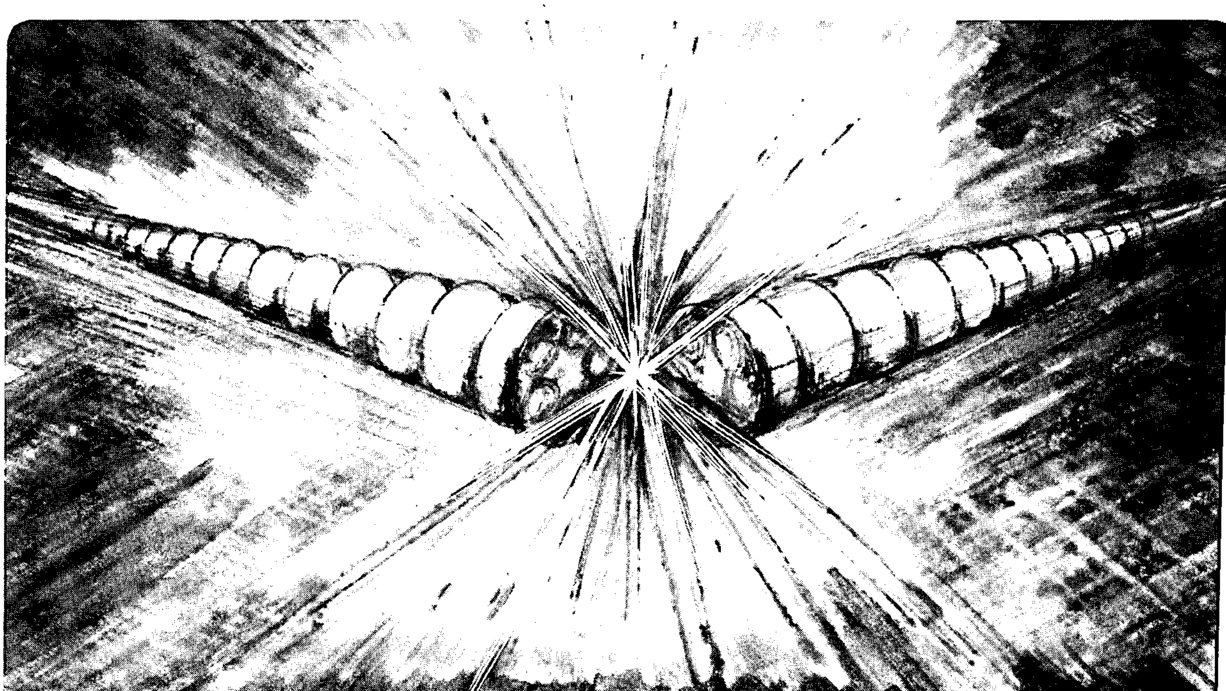

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The Use of Undulator Radiation in VUV and Soft X-Ray Radiometry

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Abstract

A new generation of synchrotron radiation light sources covering the VUV, soft x-ray and hard x-ray spectral regions is under construction in several countries, designed specifically to use periodic magnetic undulators and low-emittance electron or positron beams to produce high-brightness near-diffraction-limited synchrotron radiation beams. It should be possible to use specially designed undulators and wigglers in the new synchrotron light facilities as tunable narrow band radiometric sources in the VUV and soft x-ray regions. An introduction to the physics of undulator radiation is followed by a discussion of some of the consequences of maximizing source performance, including high beam power, harmonics, optics contamination, and the unusual spectral and angular properties of undulator radiation. The limitations of the presently planned undulators as radiometric sources and the design criteria for a possible radiometry undulator will be discussed.

Introduction

The properties of undulator radiation and undulator magnets dominate the design of the electron storage ring and the experimental apparatus planned for the new synchrotron light sources.

The radiation produced by an accelerated charge is given by[1]

$$\frac{d^2E}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{+\infty} \hat{n}\times(\hat{n}\times\beta)e^{+i\omega(t-(\hat{n}\cdot r(t))/c)}dt\right|^2. \tag{1}$$

Here E is the energy radiated by the charged particle, Ω is the solid angle, $\hat{n}$ is the unit vector from the charge to the observer, β is $\mathbf{v}/c$, and $r(t)$ is the vector describing the position of the particle.

The trajectory of the particle is computed by integrating the Lorentz force equation of motion

$$\frac{d\beta}{dt} = \frac{e}{\gamma mc}\beta\times\mathbf{B} \tag{2}$$

In an ideal undulator magnet, the magnetic field on-axis can be written as $\mathbf{B} = B_y\hat{y}$, with $B_y(z) = B_0\cos kz$, where $k = 2\pi/\lambda_u$. For permanent magnet systems, B_0 is proportional to the magnetization strength of the polarized magnetic material, and can be as high as 2 Tesla. An electron of energy $E = \gamma m_0c^2$ has a sinusoidal trajectory given by $x(z) = a\cos kz$, where $a = K\lambda_u/2\pi\gamma$. The maximum deflection angle is K/γ. The undulator parameter K is $\lambda_u eB_0/2\pi m_0c^2$, having the value $0.934\,B_0(T)\lambda_u(cm)$, usually of the order of unity. For a 1.5 GeV electron beam, $\gamma \simeq 3000$, so for $\lambda_u = 5$ cm a is of the order of 10μ, which is much smaller than the typical electron beam dimensions σ_x and σ_y.

The radiation produced by a single electron is essentially that of a moving dipole. The Doppler effect for such a moving source results in an angle dependent observed wavelength, given by $\lambda = \lambda_u(1-\beta_z\cos\theta)$. Here β_z is v_z/c, with v_z being the electron's average velocity in the z direction. The speed of the electron is constant travelling through the undulator magnetic field, so the transverse deflection of the trajectory results in a slowing down of the average z motion. This can be expressed as a reduced γ, given by $\gamma^* = \gamma/(1+K^2/2)^{\frac{1}{2}}$. The radiation pattern is strongly peaked in the forward direction, with an opening angle of the order of $1/\gamma$, a fraction of a milliradian. The small angle approximation may therefore be used, resulting in

$\lambda = \lambda_u(1 + K^2/2 + \gamma^2\theta^2)/2\gamma^2$. Hence, the shortest wavelengths are radiated in the forward direction, with longer wavelengths radiated off-axis.

For an undulator of length $L = N\lambda_u$, the electron radiates a Doppler-compressed wave train of length $N\lambda$, resulting in a transform limited sinc^2 spectrum, with fractional linewidth $1/N$. For undulators at the new light sources, $N \simeq 100$, resulting in a one percent spectral bandwidth for each observation angle. Near the forward direction, energy within this bandwidth is radiated into a central cone with an opening angle of $\theta = 1/\gamma^*\sqrt{N} = (1 + K^2/2)^{\frac{1}{2}}/\gamma\sqrt{N}$, a small fraction of $1/\gamma$, approximately 0.1 milliradian or less for the new light sources. This is the same as the diffraction limited angular width of the radiation pattern from a line source of length L, given by $\theta \simeq (\lambda/L)^{\frac{1}{2}}$. This is because a classical charged particle radiates a completely deterministic coherent wave field obeying the rules of diffraction. The peak intensity, or energy/unit solid angle, in the forward direction scales as N^2. The total energy radiated into the central cone therefore goes as N, resulting in roughly N times more spectral flux than from a bend magnet source.

If an optical system is used to form an image of the source of the central cone radiation, the diffraction limited 1-σ source size and angular divergence are $\sigma_r = \sqrt{\lambda L}/4\pi$ and $\sigma_{r'} = \sqrt{\lambda/L}$, resulting in a phase space (emittance) of $\lambda^2/4$. This is the same expression as for a single Gaussian laser mode[2]. These properties are in marked contrast to the broad spectral and angular features of bend magnet radiation.

The angular and energy spreads and the non-zero size of the electron beam further reduces the brightness of the source.

The total power radiated by the electron beam is proportional to $B^2\gamma^2IL$. B is dependent on magnet technology, and is about the same for all synchrotron radiation sources. Since storage ring beam emittance grows with increasing γ, the electron beam energy must be kept relatively low (1–2 GeV) to achieve the small emittance and energy spread necessary for high brightness undulator sources. Beam current is limited by the low beam energy and the small beam size, while the undulator length L is limited to a few meters for a variety of reasons. This means that the new light sources will produce about the same spectral flux as present synchrotron radiation sources, with much higher spectral brightness.

Consequences of High Performance

High Power Loading

To maximize light source performance, the longest possible straight sections and the largest N are used, since both flux and brightness increase with N. Usually the size and number of straights in the machine are fixed, and hence the length L of the undulator. This means λ_u must decrease for N to increase. For a fixed machine energy (γ) and operating wavelength λ, this means K must increase, requiring an increase in B. A large value of K_{max} means that a single undulator can cover a large tuning range by varying K, a desirable feature. Unfortunately, this also drives up the total power that must be handled by the first optical element in the beam line. For the new light sources this can be several kW.

Harmonics

The undulator spectrum is rich in harmonics, with even harmonics forbidden by symmetry on-axis. The number of significant harmonics is $n_c = (3/4)K(1 + K^2/2)$, increasing as K^3 for large K, and, the fraction of total radiated power in the first harmonic is $P_1/P_{tot} = 1/(1 + K^2/2)^2$. The maximum useful harmonic limited by undulator quality and the electron beam emittance and energy spread, so the higher harmonics are just extra heat to deal with. In addition, harmonics can pass through grating monochromator systems and show up in the output beam, and may have to be filtered out. The optimization of undulator performance can lead to difficult optical engineering problems: how to remove extra heat with thermal distortions kept to a minimum to preserve brightness.

Optics Contamination

The extra power produced when central brightness and flux are maximized can pose a serious optics contamination problem. Synchrotron radiation hitting optical surfaces generates an intense photoelectron flux, which can lead to deposition of carbonaceous contamination, reducing reflectivity.

Effects on the Storage Ring

In an undulator magnet the peak mid-plane field depends on the gap between pole tips or magnet blocks, $B_y \simeq \exp(-\pi g/\lambda_u)$[3]. If the λ_u is decreased, as desired, g must decreased also to increase B and K. This means that the electron beam must pass through an increasingly long and narrow vacuum chamber. Such a small gap may make it difficult or impossible to inject beam into the storage ring or have a long stored beam lifetime.

Strong undulator fields, combined with the relatively low energy of the electron beam required, result in significant electron beam focussing effects, making it necessary to re-tune the machine when the undulator gap is varied. This may have an adverse effect on electron beam lifetime, position, or size around the ring.

Off-axis Radiation

As L and N are increased, the fact that the observer of the undulator output is not infinitely far away becomes increasingly important. Typical undulator to experiment distances are of the order of 10 meters, while L can be as large as 5 meters. This places the experiment in the Fresnel region[4], where for an off-axis detection point the variation of the observation angle θ from one end of the undulator to the other results in a chirped radiation pulse. Furthermore, since the upstream end of the undulator is farther away from the observer than the downstream end, the inverse square law results in significant amplitude modulation of the chirped wave, even for on-axis measurements. An analysis leads to the result that if the off-axis observation angle θ is greater than $\sigma_{r'}\sqrt{D/L}$, then the chirp effect broadens the narrow undulator spectral peaks[5]. For larger off-axis angles, the broadening is large enough to overlap different harmonics, and all the spectral advantages of using undulators are lost. The apparent source size $\Delta x = L \sin\theta$ increases off-axis, decreasing source brightness.

Undulators for Radiometry

The narrow spectral bandwidth of undulator radiation is a useful feature for radiometry. The presence of many higher harmonics is not, however. This and the desire to maintain tuneability by changing the magnetic gap means the device should have a low K, on the order of 0.5. As N increases, the electron beam properties become more important. Since it is hard to measure beam size, angular divergence, and energy spread with great accuracy, the number of undulator periods N should not be too large, with $N \approx 20$ probably being adequate. This gives a fundamental bandwidth of 5%, which is still quite monochromatic. The use of low K and small N has the further advantage of reducing the effect of random magnet errors of the output performance[6], making the undulator performance closer to ideal and reducing the difficulty of meeting the required mechanical and magnetic design tolerances.

The magnetic field of a real undulator must satisfy Laplace's equation in free space. This means that the ideal $\cos kz$ dependence described above neglects variation in the x and y directions, the presence of higher spatial harmonics in the (non-sinusoidal) field, and the effects of the finite length and width of a real device. A field varying as $\cos kz$ must also depend on y, in the form $B_y(y,z) = B_0 \cos kz \cosh ky$. This expression still neglects end effects and finite pole width, however. The resulting variation of B_y and hence K with y position of the electron beam means that electron beam alignment is a critical adjustment. Using a relatively large value of λ_u makes this less serious, however. The undesirable Fresnel region effects described above also make a relatively modest length L a better choice. In addition, a short, low K, longer period undulator can operate with a larger gap, and with less effect on the operation of the storage ring. Interestingly, these design choices are opposite to the usual ones made to maximize undulator performance for use as a light source.

Until recently, designs for undulator radiation sources have used estimated spectral output based on the ideal field discussed above using the assumption that the observer is infinitely far away from the source [2, 7, 8, 9, 10, 11]. The resulting expressions for the radiation of a single electron, involving sums of Bessel functions, are convolved with electron beam angular and spatial Gaussian distributions to account for emittance effects. Electron energy spread is usually added as a reduction factor at the end of the calculation. This approach neglects Fresnel region effects, and further assumes that the radiation from electrons with different angles and positions in the beam may be added via a convolution. This approximation neglects the variation of K with y discussed above, as well as any focussing effects the undulator field may have on the electron beam. In addition, it is assumed that electron energy spread is a small correction. The ends of the undulator are treated in an idealized, nonphysical way, with the $\cos kz$ dependence terminating abruptly, unlike the complicated end-effect behavior observed in actual undulators. For undulators with relatively small values of N, this approach may incorrectly estimate the spectral output by a large factor, as can be seen in Figs 1. and 2.

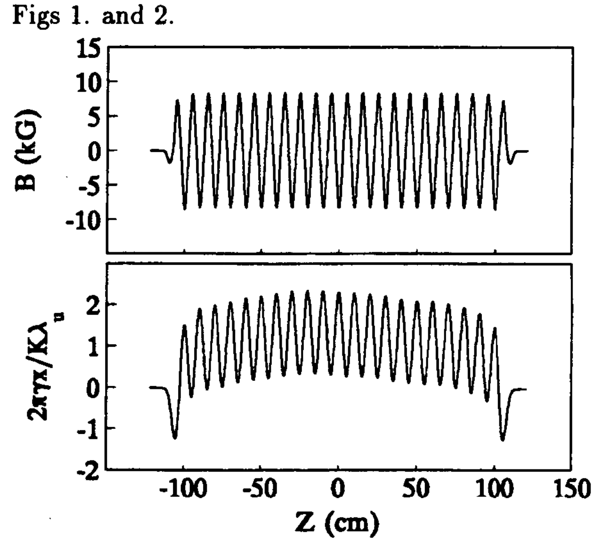

Figure 1. The measured field and the calculated trajectory for the TOK undulator[12].

The electron trajectory has been scaled to unit peak oscillation amplitude. The magnetic field is plotted in kilogauss versus z position in cm.

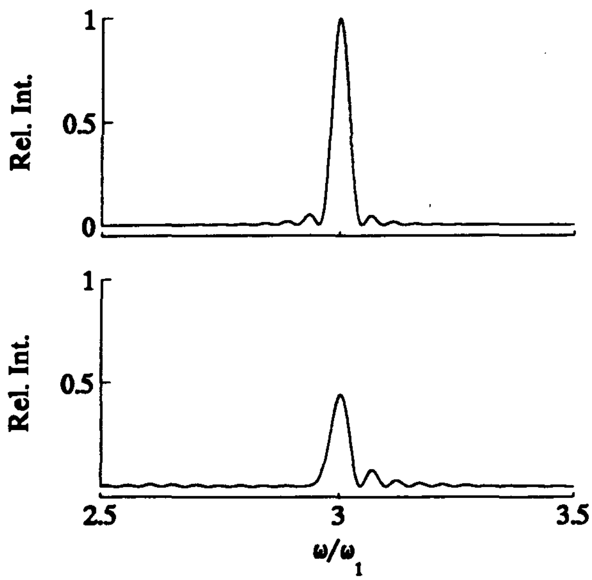

Figure 2. A comparison between the calculated spectral output for the ideal $\cos kz$ field versus the actual undulator.

In this case the spectral output was computed from the measured field by integrating the trajectory and evaluating the integral in Eq. 1. The calculations were done for the third harmonic, in the single electron (zero-emittance) case. The large intensity difference is caused by the difference in the ends of the trajectory. Measurements made using visible undulator light, where the zero-emittance approximation is a good one, reproduced the asymmetric spectral shape seen in Fig. 2., although absolute flux measurements were not performed[12]. All this points out the necessity of precision magnetic measurements and correct calculation of the expected spectral output.

In addition, recent work at BESSY[13] based on measured fields and including emittance via the convolution method has shown good absolute agreement between calculated and measured spectra.

Conclusions

If the critical characteristics of the electron beam (beam energy, energy spread, horizontal and vertical beam size and angular divergence) are known accurately and reproducibly and the parameters of the undulator are chosen conservatively, then precision magnetic measurements of the undulator field will make it possible to calculate the absolute undulator spectral output from first principles. Undulator radiation can then provide a useful radiometric standard, with relative spectral purity, tuneability, well defined and calculable spectral and angular properties, with enough intensity to do absolute measurements.

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