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RESONANT NEUTRINO OSCILLATIONS AND STELLAR COLLAPSE

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ABSTRACT

The Mikheyev-Smirnov-Wolfenstein mechanism for resonant amplification of neutrino oscillations is shown to occur in collapsing presupernova stellar cores if there exist massive unstable neutrinos which mix with the electron neutrino. The relevant massive neutrino mass range is 200 eV to 25 keV, and the required vacuum mixing angle is $\theta \geq 10^{-6}$ rad. Neutrinos with these characteristics have been independently proposed to solve some galaxy formation problems and are suggested by familon models of the weak interaction. It is shown that adiabatic conversion of electron neutrinos into such massive neutrinos would occur during stellar collapse with resultant readjustment of lepton numbers and small entropy generation. These changes have implications for the supernova explosion mechanism.

Subject headings: elementary particles — neutrinos — stars: interiors — stars: supernovae

I. INTRODUCTION

Recently Mikheyev and Smirnov (1986) have pointed out that there is a resonant condition for the matter oscillations between neutrino flavors described by Wolfenstein (1978, 1979). In a seminal paper, Bethe (1986) has shown that this Mikheyev-Smirnov-Wolfenstein (hereafter MSW) effect has an adiabatic solution for the density distribution of the Sun. This implies that the high-energy ν_e produced by ^{8}B decay in the Sun would be adiabatically transformed into ν_μ, if the difference of the squares of the vacuum masses, Δ, is of order 10^{-4} (eV)^2 and the vacuum mixing angle θ corresponding to these oscillations is $\theta \geq 0.01$ rad. The solar neutrino problem is thus neatly solved. Another solution to the solar neutrino problem involving the transformation of lower energy ν_e has been found by several groups (Turner, Kolb, and Walker 1986; Rosen and Gelb 1986; Haxton 1986).

The other areas of astrophysics where neutrinos are important are cosmology and stellar collapse. Notably standard big bang nucleosynthesis puts a limit on the number of lepton generations (≤ 4; Steigman *et al.* 1986), and the estimates on the Hubble constant and deceleration parameter put an upper limit on stable light neutrino masses (Cowsik and McCelland 1972). Assuming closure density ($\Omega = 1$) is due to such neutrinos, Schramm and Freese (1984) show that the sum of stable neutrino masses is ≤ 100 eV. If globular cluster age constraints are added, this limit is reduced to ≤ 30 eV. However, some scenarios for galaxy formation would prefer a massive neutrino ($m_\nu > 100$ eV) which would allow some small-scale structure, like galaxies, to develop and survive early on (see Turner, Steigman, and Krauss 1984), but which would decay to a light neutrino within the Hubble time so that the present mass density of the universe would no longer be dominated by the more massive nonrelativistic neutrinos and closure density would not be exceeded (see the discussions by Turner, Steigman, and Krauss 1984; Gelmini, Schramm, and Valle 1984). These decaying neutrino scenarios attempt to combine the strength of rapid galaxy formation in cold dark matter models with the low galaxy scale Ω of the hot dark matter models. Such models have the advantage of reducing the mass on galaxy scales to well below $\Omega = 1$ while retaining $\Omega = 1$ on large scales. There are fairly stringent limits on the decay products and lifetimes of such massive neutrinos due to the observed isotropy of the microwave background. This point will not be discussed further here, but it suffices to point out that there exists a class of theories for the weak interaction in which massive unstable neutrinos exist with masses in the 100 eV to 1 MeV range. These are the so called "familon" (see Grinstein, Preskill, and Wise 1985) or family symmetry models, and the majoron models (Gelmini and Roncadelli 1981). Massive neutrinos in the familon model decay to light neutrinos via emission of a very weakly interacting Goldstone boson which does not thermalize, and so limits due to microwave background isotropy can be circumvented.

The thrust of this paper will be to point out that if such massive neutrinos exist with masses roughly in the range suggested by the familon or majoron models, then the MSW mechanism for enhanced neutrino oscillations leads to a possibly different picture of supernova core collapse with implications for attaining a supernova explosion.

It is instructive to remember that the gravitational binding energy released when the core of a massive presupernova star (essentially an iron white dwarf of mass $M \approx 1.4\ M_\odot$) collapses to a hot neutron star is $\sim 10^{52}$ ergs. Electron captures ensure that more than 95% of this energy will be trapped in the collapsing core in the form of a degenerate sea of electron-type neutrinos, ν_e. The short mean free path of neutrinos at nuclear density leads to release of the thermal energy over a time scale of the order of 1 s. This process eventually would leave a cold neutron star (binding energy = 10^{53} ergs). The energy of a supernova explosion is known to be $\sim 10^{51}$ ergs, only 10% of the initial gravitational binding energy release, and this is believed to result from hydrodynamic coupling of infall kinetic energy into an outgoing shock (Bethe *et al.* 1979) and/or from neutrino heating of the material behind this shock (Wilson 1985; Bethe and Wilson 1985; Arnett 1986).

Any process of lepton number nonconservation which might tap into the energy reservoir in the degenerate ν_e sea could possibly change the energetics of the supernova collapse. In fact, the aforementioned majoron model allows for just such a possibility: the ν_e could interact with thermally produced majorons to produce ν_μ, $\bar{\nu}_\mu$ or ν_τ, $\bar{\nu}_\tau$; thus, quickly running the neutrino chemical potentials to zero and unblocking electron capture. Beta equilibrium may be reestablished in these models at a much larger entropy per baryon, changing completely the standard picture of stellar collapse (Kolb, Tubbs, and Dicus 1982).

It will be shown that the MSW mechanism operating between ν_e and a massive neutrino (hereafter designated as the mystery neutrino, ν_x, which could be ν_μ, ν_τ or a fourth lepton generation neutrino. We will discuss this further in § II*b*) represents an amplification of lepton number violation, leading to a scenario in some ways similar to the majoron collapse picture, with lepton number readjustment (lower electron fraction Y_e) and some (much smaller) entropy generation.

The increase in entropy-per-baryon in units of Boltzmann's constant k is found to be $\Delta s/k \leq 1$ (compare to $s/k \leq 1.5$ for the standard model of stellar collapse) with a substantial drop in Y_e for ν_x masses of between 300 eV and 25 keV, and vacuum mixing angles $\theta \geq 10^{-6}$ rad. The change in entropy is not more than this due to the rapid reestablishment of beta equilibrium.

Section II of this paper will detail the MSW mechanism in an environment of relativistic electrons and neutrinos, while § III will discuss the adiabatic conversion of ν_e and the associated drop in neutrino chemical potential μ_{ν_e}. Section IV will outline the deviations from beta equilibrium and entropy generation via electron capture, neutrino capture, and neutrino scattering. Finally, § V will give the conclusion on what can be said about supernova core collapse if these massive neutrinos exist.

II. RESONANT NEUTRINO OSCILLATIONS

a) The High Density and Temperature MSW Mechanism

The collapsing core of a presupernova star is characterized by low temperatures and high densities with degenerate distributions of leptons. During collapse, in the standard model (see Bethe *et al.* 1979), the temperature remains close to $kT \sim$ a few MeV, while $\sim$90% of the baryons are in bound nuclei, giving an entropy-per-baryon $s/k \approx 1.5$ (in units of Boltzmann's constant k).

Depending on the initial model, the collapse begins when the central density is about $\rho \approx 10^{10}$ g cm^{-3} and the central lepton fraction is about $y_e \approx 0.43$ (see Weaver, Woosley, and Fuller 1985). The collapse is initiated by a combination of photodisintegration of nuclei and electron capture. Subsequent electron captures during the collapse lead to copious production of electron neutrinos, ν_e. At densities in excess of $\rho y_e \approx 10^{11}$ g cm^{-3} these ν_e are trapped and thermalized, and the electrons, neutrinos, and nucleons rapidly approach beta equilibrium. Neutrino pairs can be produced through the plasmon and other processes. Collapse proceeds until a hydrodynamic bounce takes place at $\rho \geq 5 \times 10^{14}$ g cm^{-3}.

The collapsing core above neutrino trapping and below bounce density is characterized by a relativistically degenerate sea of electrons with Fermi energies, or chemical potentials, μ_e, in the range 25–220 MeV, a nearly degenerate Fermi-Dirac distribution of electron neutrinos, with μ_{ν_e} in the range 10–170 MeV, and relatively smaller numbers of $\nu_\mu(\bar{\nu}_\mu)$ and $\nu_\tau(\bar{\nu}_\tau)$ with $\mu_\nu = 0$. Throughout most of the collapse $(\mu_{\nu_e}/kT) \approx 15$.

The physical origin of the MSW neutrino oscillation mechanism is related to the fact that the square of the effective mass of the electron neutrino $(m_{\nu_e}^2)_{\rm eff}$ added to the square of the vacuum mass $(m_{\nu_e}^2)_{\rm vac}$ can be equal to the equivalent sum for some heavier neutrino species, ν_x. Here $(m_{\nu_e})_{\rm eff}$ is that part of the total mass contributed by interactions. Following Mikheyev and Smirnov (1985), we can express the resonant condition as (for the vacuum mixing angle $\theta \ll 1$),

$$(m_{\nu_e}^2)_{\rm vac} + (m_{\nu_e}^2)_{\rm eff} = (m_{\nu_x}^2)_{\rm vac} + (m_{\nu_x}^2)_{\rm eff} . \quad (1)$$

This equality of masses is possible because, although $(m_{\nu_x})_{\rm vac} \gg (m_{\nu_e})_{\rm vac}$, the electron neutrino can have a larger effective mass due to matter interactions which are larger than for ν_x.

Note that the effective mass discussed above is the self-energy part of the propagator mass as discussed in Wolfenstein (1978) and Fukugita *et al.* (1986). For the Sun this is just the case for ν_e and either ν_μ or ν_τ. The larger effective mass for ν_e is then due exclusively to the charged current exchange contribution to the forward scattering amplitude. The contributions to the effective masses for both ν_e and ν_μ or ν_τ would be identical for neutral current forward scattering and so these terms would cancel out on both sides of eq. (1). This was the case considered by Wolfenstein (1978).

The supernova case differs on two points: the electrons are relativistic, unlike in the Sun; and the neutral current contributions to the effective masses from the degenerate electron neutrino sea differ for ν_x and ν_e due to the ν_e exchange diagram. The two processes responsible for differences in $(m_{\nu_e}^2)_{\rm eff}$ and $(m_{\nu_x}^2)_{\rm eff}$ are shown in Figures 1*a* (charged current exchange) and 1*b* (neutral current exchange). Let us consider the first of these processes to begin with.

Consider for now only the charged current exchange of ν_e and arbitrarily relativistic electrons. The total Lagrangian density in this case is

$$L_{\rm TOT} = \bar{\psi}_{\nu_e}(i\partial\!\!\!/ - m_{\nu_e})\psi_{\nu_e} + \bar{\psi}_e(i\partial\!\!\!/ - m_e)\psi_e - \frac{G_F}{\sqrt{2}}(\bar{\psi}_{\nu_e}\gamma_\mu(1-\gamma_5)\psi_{\nu_e})(\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_e) , \quad (2a)$$

where m_{ν_e} is the vacuum electron neutrino mass, m_e is the electron mass, ψ_e and ψ_{ν_e} are the appropriate lepton fields, and G_F is the weak coupling constant. Note that the charged current exchange term has been Fierz transformed.

If we define an effective interaction potential A_μ that the electron current generates, we have

$$A_\mu \equiv \frac{G_F}{\sqrt{2}}[\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_e] = (\phi, \boldsymbol{A}) , \quad (2b)$$

where ϕ is the time component and $\boldsymbol{A}$ the spatial vector potential. The Dirac equation derived from equations (2a) and (2b) is

$$\left(i\frac{\partial}{\partial t} - \phi\right)\psi_{\nu_e} = \left[\alpha\cdot\left(\frac{1}{i}\nabla - \boldsymbol{A}\right) + \beta m_{\nu_e}\right]\psi_{\nu_e} , \quad (2c)$$

where α and β are Dirac matrices and $\hbar = c = 1$. This Dirac equation implies the following kinematic relation,

$$(E_{\nu_e} - \phi)^2 = (\boldsymbol{p}_{\nu_e} - \boldsymbol{A})^2 + m_{\nu_e}^2 , \quad (3a)$$

where E_{ν_e} and $\boldsymbol{p}_{\nu_e}$ are the electron neutrino's energy and momentum, respectively. This must be averaged over the electron distribution function in the rest frame of the stellar material to obtain the following expression for the effective mass of the neutrino,

$$m_{\rm eff}^2 = 2E_{\nu_e}\langle\phi\rangle - \langle 2\boldsymbol{p}_{\nu_e}\cdot\boldsymbol{A}\rangle + \langle A^2\rangle - \langle\phi^2\rangle , \quad (3b)$$

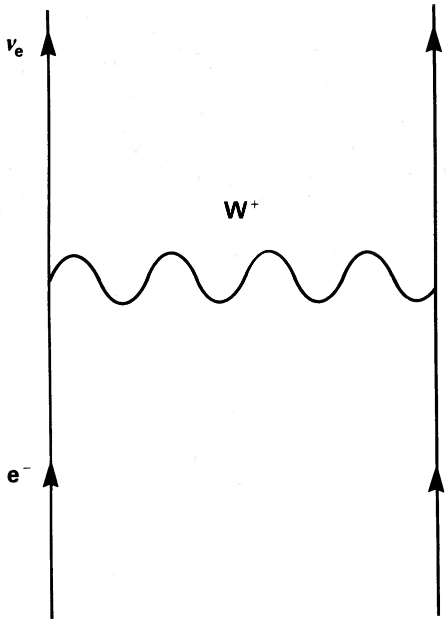

FIG. 1*a*

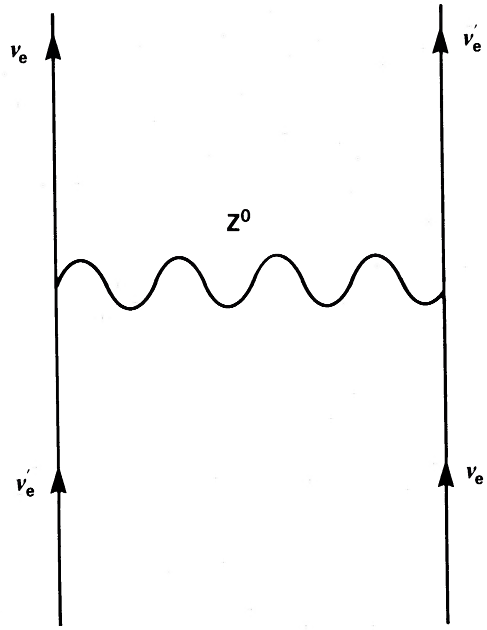

FIG. 1*b*

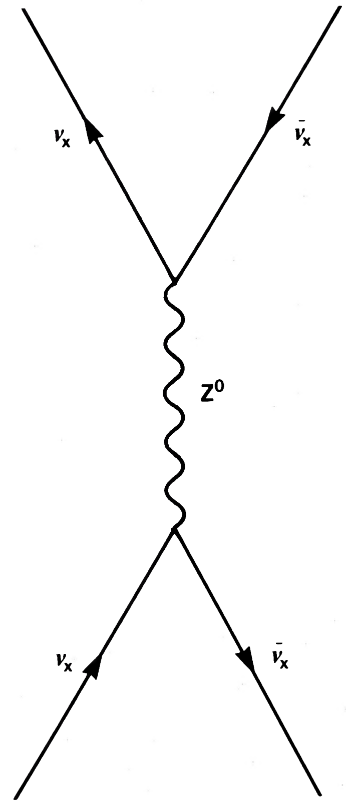

FIG. 1*c*

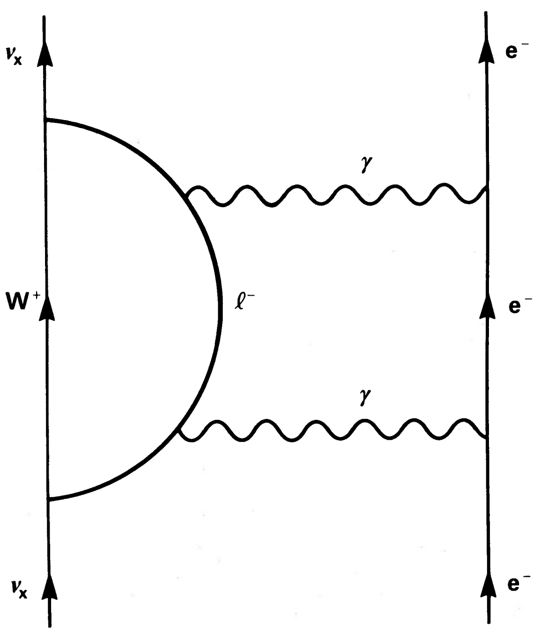

FIG. 1*d*

FIG. 1.—(*a*) The $e - \nu_e$ charged current exchange process. This process is responsible for all of the effective mass difference between neutrinos in the Sun and most of the difference in stellar core collapse. (*b*) The neutral current exchange contribution for neutrino-neutrino scattering. There is no exchange contribution for $\nu_x - \nu_e$ scattering. (*c*) The annihilation contribution to the scattering amplitude for ν_x. (*d*) This process contributes to the effective mass for ν_x. The l^- is the heavy lepton associated with ν_x. There would be a similar diagram for electron neutrinos, ν_e, in which case l^- is to be identified with the electron.

where the bracket represents an average over the electron distribution function.

Consider the second term on the right in equation (3b). The dot product term averages to zero in the isotropic distribution of electrons, $\langle 2\boldsymbol{p}_{\nu_e} \cdot \boldsymbol{A} \rangle = 0$ in the absence of large-scale fluid currents in the star. Note that, although this term vanishes, it has nothing to do with Lorentz invariance. The $2E_{\nu_e}\langle \phi \rangle$ term is just the Wolfenstein (1978) result

$$m_{\rm eff}^2({\rm eV}^2) \approx 2E_{\nu_e}\langle \phi \rangle$$
$$\approx 1.5184 \times 10^3(\rho_{10}\, y_e)\left(\frac{E_{\nu_e}}{\rm MeV}\right), \qquad (4a)$$

where ρ_{10} is the matter density in units of 10^{10} g cm^{-3} and y_e is the ratio of electrons to baryons.

The last terms on the right in equation (3b) are considerably smaller due to the extra factor of the weak coupling constant G_F and the fact that the $\langle A^2 \rangle$ and $-\langle \phi^2 \rangle$ terms are equal and opposite in the limit of completely relativistic electrons. Altogether this term makes a small negative contribution to $m_{\rm eff}^2$ (in eV^2),

$$\langle A^2 \rangle - \langle \phi^2 \rangle \approx -2G_F^2\left\{\left[\bar{e}\gamma_0\left(\frac{1-\gamma_5}{2}\right)e\right]^2 - \left[\bar{e}\gamma_i\left(\frac{1-\gamma_5}{2}\right)e\right]^2\right\}$$
$$\approx -(2.88 \times 10^{-27})(\rho y_e)^2\, \frac{m_e^2 c^4}{\langle E_e^2 \rangle}$$
$$\approx -1.83 \times 10^{-9}(\rho_{10}\, y_e)^{4/3}, \qquad (4b)$$

where e is the electron spinor and $(1 - \gamma_5)/2$ is a left-handed projection operator in the limit of completely relativistic electrons. We are interested in densities up to $\rho_{10} \leq 10^5$, so that the contribution to $m^2_{\rm eff}$ from equation (4b) is always negligible compared to that from equation (4a). We note, however, that the second order correction in G_F to the effective mass in equation (4b) is certainly not the only one: there are many second-order corrections to the neutrino effective mass in Weinberg-Salam theory, and they are all negligible for our purposes (see below for a discussion of radiative corrections, however). We refer the reader to Fukugita *et al.* (1987) for a full discussion of second-order terms.

The other major source of difference in effective mass between ν_e and ν_x is due to the *exchange* diagram in neutral current $\nu_e - \nu_e$ scattering, shown in Figure 1*b*. The contribution of this diagram to the interaction matrix element is

$$\frac{-G_F}{2\sqrt{2}}\,[\bar{\nu}'_e \gamma_\mu(1-\gamma_5)\nu_e][\bar{\nu}_e \gamma_\mu(1-\gamma_5)\nu'_e]\,, \tag{5a}$$

where ν_e and ν'_e are the electron neutrino spinors. After Fierz transformation (which introduces another minus sign) and averaging over the isotropic distribution function for electron neutrinos (all vector dot products average to zero) we obtain for the contribution to the effective mass to order G_F

$$m^2_{\rm eff} \approx \frac{4G_F}{\sqrt{2}}\,\rho y_{\nu_e} E_{\nu_e}\,, \tag{5b}$$

where y_{ν_e} is the number of electron neutrinos per baryon (so that the number density of ν_e is $N_{\nu_e} = \rho y_{\nu_e}$) and E_{ν_e} is the electron neutrino energy. Putting equations (4a) and (5b) together, we obtain an approximation for the total difference in effective mass for ν_e over ν_x

$$m^2_{\rm eff}({\rm eV}^2) \approx (1.5184 \times 10^3)\left(\frac{E_{\nu_e}}{\rm MeV}\right)\rho_{10}(y_e + y_\nu)\,. \tag{6}$$

Equation (6) is a fair approximation for the effective mass of ν_e for the infall epoch of supernovae only. During this epoch the temperatures are small ($kT \approx 1$ MeV) compared to the electron neutrino Fermi energy ($\mu_\nu \approx 45$ MeV) so that the number of $\nu_e \bar{\nu}_e$ pairs is small and there are very few anti-electron-neutrinos. The number of antineutrinos $\bar{\nu}_x$ from pairs is also small compared to the number of ν_e. In fact, at $\rho = 1.2 \times 10^{13}$ g cm^{-3}, $y_e \approx 0.30$, $y_{\nu_e} \approx 0.06$, and the calculation of Mayle and Wilson (1986) gives the ratio of number densities as $(n_{\bar{\nu}_x}/n_{\nu_e}) \approx 10^{-4}$ and $(n_{\bar{\nu}_e}/n_{\nu_e}) \approx 10^{-5}$. This asymmetry in the number of $\bar{\nu}_e$ and $\bar{\nu}_x$ in principle leads to an extra effective mass term for ν_x given in Figure 1*c*, which yields

$$(m^2_{\nu_x})_{\rm eff}({\rm eV})^2 = 1.5184 \times 10^3(\rho_{10}\, y_{\bar{\nu}_x})\left(\frac{E_{\nu_x}}{\rm MeV}\right). \tag{7a}$$

There is a similar expression for $(m^2_{\nu_e})_{\rm eff}$ in terms of $y_{\bar{\nu}_e}$. All of these terms are negligible during infall as compared with equation (6) since the number of antineutrinos is small. This will not necessarily be the case after core bounce when the shock will raise the temperature to $kT \geq 10$ MeV. These and other issues relating to late time neutrino heating scenarios will be addressed in a detailed subsequent paper (Fuller *et al.* 1987).

Finally, there is another possible source of difference in effective mass between ν_e and ν_x due to radiative vertex corrections. These corrections have been discussed and pointed out by Sehgal (1985), and the prototype interaction is shown in Figure 1*d*. The extra effective mass term acquired by the electron neutrino over the ν_x via this process is of order

$$m^2_{\rm eff} \approx \frac{G_F}{\sqrt{2}}\,\rho y_e E_\nu \left(\frac{\alpha}{\pi}\right)^2 \ln\left(\frac{m_x^2}{m_e^2}\right), \tag{7b}$$

where $\alpha(\approx 1/137.04)$ is the fine structure constant, m_e is the electron mass, and m_x is the mass of the charged lepton associated with ν_x. If we identify the ν_x with a ν_τ, then $m_\tau \approx 1784.2$ MeV and equation (7b) makes a negligible contribution to the effective mass difference. Clearly this may not be the case if ν_x is to be identified with another generation of leptons where m_x might be much larger. Current limits on m_x for a fourth generation lepton are $m_x \geq 41$ GeV (Cline 1987).

We follow Pontecorvo (1958), Wolfenstein (1978, 1979), and Mikehyev and Smirnov (1985) and denote the vacuum mixing angle as θ, so that the neutrino flavor eigenstates $|\nu_e\rangle$ and $|\nu_x\rangle$ are related to the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ (with masses m_1 and m_2, respectively) as

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle\,, \\ |\nu_x\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle\,. \end{aligned} \tag{8a}$$

If we denote a general neutrino state as a linear combination of the two flavor eigenstates as

$$|\nu(\tau)\rangle = \nu_e(t)|\nu_e\rangle + \nu_x(t)|\nu_x\rangle\,, \tag{8b}$$

and note that the energy of the relativistic neutrinos is $E = (p^2 + m^2)^{1/2} \approx p + m^2/2p$, then the Schrödinger equation for the time evolution of the coefficients in equation (8b) is

$$i\frac{d}{dt}\begin{bmatrix}\nu_e(t)\\ \nu_x(t)\end{bmatrix} = H\begin{bmatrix}\nu_e(t)\\ \nu_x(t)\end{bmatrix}. \tag{8c}$$

The instantaneous Hamiltonian in matter is given in the flavor basis as

$$H \approx \left[p + \frac{1}{2p}(m_1^2 + m_2^2) + \frac{\Delta_{\rm eff}}{4p}\right]I + \frac{1}{4p}\begin{bmatrix}\Delta_{\rm eff} - \Delta\cos 2\theta & \Delta\sin 2\theta \\ \Delta\sin 2\theta & \Delta\cos 2\theta - \Delta_{\rm eff}\end{bmatrix}, \tag{8d}$$

where I is the identity matrix, the difference in the squares of the vacuum masses is $\Delta \equiv m_1^2 - m_2^2$, and where $\Delta_{\rm eff}$ is the difference in the squares of the effective masses due to matter interactions. We will assume that in vacuum a close identification can be made between m_{ν_x} and the heavy mass eigenstate $m_2(\gg m_1)$, so that $\Delta \approx m^2_{\nu_x}(m^2_{\nu_e} \approx 0)$. As discussed above, a good approximation for $\Delta_{\rm eff}$ is given by equation (6), so that

$$\Delta_{\rm eff} \approx m^2_{\rm eff}\,, \qquad \Delta \approx m^2_{\nu_x}\,. \tag{8e}$$

Equation (8c) can be solved by finding the instantaneous mass eigenstates which diagonalize the Hamiltonian in equation (8d). If the instantaneous matter mixing angle leading to this diagonalization is ϕ and if a neutrino is created as a ν_e at time $t = 0[\nu_e(0) = 1, \nu_x(0) = 0]$, then the probability that it has transformed into a ν_x is

$$|\langle\nu_x|\nu(t)\rangle|^2 = (\sin^2 2\phi)\left(\sin^2\frac{\pi c t}{L}\right), \tag{9a}$$

where L is the matter oscillation length, and c the speed of light.

Expressions for the instantaneous matter mixing angle, ϕ, and oscillation length, L, are

$$\sin^2 2\phi = \frac{\Delta^2 \sin^2 2\theta}{(\Delta_{\text{eff}} - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}, \tag{9b}$$

and

$$L = \frac{4\pi E_\nu}{[(\Delta_{\text{eff}} - \Delta \cos 2\theta)^2 + \Delta \sin^2 2\theta]^{1/2}}. \tag{9c}$$

The instantaneous mass eigenvalues are

$$m^2 = \tfrac{1}{2}(m_1^2 + m_2^2 + \Delta_{\text{eff}}) \pm \tfrac{1}{2}[(\Delta_{\text{eff}} - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta]^{1/2}. \tag{9d}$$

We follow Wolfenstein (1978) and observe that in the limit of no matter effects ($\Delta_{\text{eff}} = 0$), we recover the vacuum oscillations of equation (8a): where $|\nu_e\rangle \approx |\nu_1\rangle$, the light eigenstate; and $|\nu_x\rangle \approx |\nu_2\rangle$, the heavy eigenstate. At extreme density, equation (6) would give $\Delta_{\text{eff}} \gg \Delta$, so that $\phi \approx \pi/2$ and $|\nu_e\rangle$ would be identified with the heavy mass eigenstate. Finally, as Mikheyev and Smirnov (1985) have pointed out there is a resonant condition,

$$\Delta_{\text{eff}} = \Delta \cos 2\theta \tag{10a}$$

where the oscillation amplitude is maximized, $\phi = \pi/4$, and the oscillation length is

$$L_{\text{res}} \approx \frac{4\pi E_\nu}{\Delta \sin 2\theta} \approx \frac{126(E_\nu/\text{MeV})\ \text{cm}}{(\Delta/\text{eV}^2)\theta}, \tag{10b}$$

where the latter approximation assumes $\theta \ll 1$.

The instantaneous mass eigenstates are completely mixed at resonance $|\nu_{1,2}\rangle = (|\nu_e\rangle \pm |\nu_x\rangle)/2^{1/2}$. Figure 2 shows the instantaneous values of the mass eigenstates as functions of density ρ (a diagram which Bethe 1986 invented to show adiabatic transformation of neutrino states in the Sun).

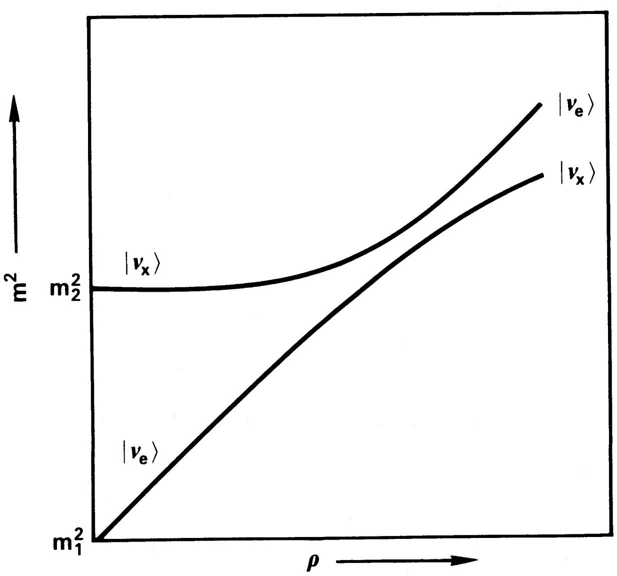

FIG. 2.—Adiabatic transformation diagram adopted from Bethe (1986). The square of the neutrino mass is plotted against the density. At zero density the mass eigenstates m_1 and m_2 correspond closely with the flavor eigenstates ν_e and ν_x. At very high density ν_e corresponds to the heavy-mass eigenstate, ν_x to the lighter mass eigenstate. Gravitational collapse can induce neutrino flavor transformation: starting at low ρ, where the neutrino state is ν_e, the collapse carries the neutrino to higher density where it will transform to ν_x. Note that at high density there are large, and different, effective mass contributions for ν_e and ν_x.

The width of the resonant region is Γ,

$$\Gamma \equiv \Delta \sin 2\theta, \tag{10c}$$

and since $\Delta_{\text{eff}} \propto \rho$ by equation (6) we see that the width of the resonance in terms of density is

$$\frac{\delta\rho}{\rho} = \frac{\Gamma}{\Delta \cos 2\theta} = \tan 2\theta. \tag{10d}$$

The change in radius, r, in the star corresponding to this width is then

$$\delta r = \left(\frac{1 d\rho}{\rho\, dr}\right)^{-1} \tan 2\theta, \tag{10e}$$

which for the Wilson and Mayle (1986) model at a central density of $\rho_c \approx 10^{13}$ g cm^{-3} and a vacuum mixing angle of $\theta = 10^{-5}$ rad gives $\delta r \approx 50$ cm over much of core.

Bethe (1986) has shown that if the density is changed slowly enough then complete adiabatic conversion from ν_e to ν_x is possible. The condition for this to occur is that $\delta r \gg L_{\text{res}}$. We will see in the next section that this condition can hold for a volume element in the star undergoing collapse.

b) Massive Neutrinos

The range of densities during stellar collapse in which the neutrinos are trapped and thermalized ($10^{11} \le \rho \le 10^{15}$) implies a rough range of ν_x masses (assuming again $m_{\nu_e} \approx 0$) by equation (6) and the resonance condition equation (10a), of $0.1 \le m_{\nu_x}(\text{keV}) \le 24$, where we have assumed $E_\nu \approx 40$ MeV, $y_e \approx 0.3$, $y_\nu \approx 0.06$, and $\theta \ll 1$. We note immediately that this is outside the range of masses of stable neutrinos implied by the cosmological considerations discussed in the introduction.

This range for an unstable neutrino mass has, however, been discussed by cosmologists in the context of galaxy formation as discussed in the first section, and it is not in conflict with cosmological limits if the mass of the ν_x is understood to arise via a family symmetry. The familon models of the weak interactions would predict that if $m_{\nu_e} \approx 0$ and $m_{\nu_x} \approx 1$ keV then the vacuum mixing angles would be very small, $\theta < 10^{-2}$ rad (see Grinstein, Preskill, and Wise 1985).

The experimental limits for the mass of the electron neutrinos are $m_{\nu_e} \le 20$ eV (Kundig *et al.* 1986) from tritium positron decay experiments, or $m_{\nu_e} \le 3.2$ eV from limits on neutrinoless double beta decay if the neutrino is a Majorana particle (see Haxton and Stephenson 1984; Bergkvist 1985). The limits on the ν_μ mass is $m_{\nu_\mu} < 250$ keV from π^+ decay, and the limits on the ν_τ mass is $m_{\nu_\tau} < 70$ MeV from the decay of the τ lepton (Vannucci 1985). The vacuum mixing angle from terrestrial oscillation experiments is not constrained in the range of Δ and θ being discussed here (Boehm and Vogel 1984).

The use of the MSW mechanism to explain the low count rate for the Davis ^{37}Cl solar neutrino experiment implies vacuum neutrino mass differences of $\Delta \approx 10^{-4}(\text{eV})^2$ for $\theta \ge 0.01$ rad (Bethe 1986). This result combined with a standard "seesaw" model of neutrino masses (Gell-Mann, Ramond, and Slansky 1979) would give ν_τ masses below the $m_{\nu_\tau} \approx 1$ keV range discussed here. However, the low-energy neutrino adiabatic conversion of low-energy solar neutrinos discussed by Kolb, Turner, and Walker (1986), Parke (1986), Parke and Walker (1986), and Haxton (1986) could give an acceptable ^{37}Cl SNU rate but a low ^{71}Ga SNU rate for the

Sun with an even smaller Δ. Distinguishing between this scenario and the Bethe picture will depend partially on the result of the ^{71}Ga experiment. It must be remembered that there are other proposed solutions to the solar neutrino puzzle (see Bahcall *et al.* 1982) which do not involve neutrino oscillations. This, combined with the cosmological interest in massive neutrinos, motivates the further discussion of effects of massive neutrinos on stellar collapse here. We will continue to refer to the massive neutrino as ν_x, but the most likely identification would be with the ν_τ, or with an as yet undiscovered fourth lepton generation neutrino.

III. ADIABATIC CONVERSION OF NEUTRINO FLAVORS DURING STELLAR COLLAPSE

The collapse of the iron core of a presupernova star is very nearly homologous (that is with velocity proportional to radius) for the inner $\sim 0.6\,M_\odot$ (Brown, Bethe, and Baym 1982). This implies that for the inner homologous core the density distribution remains self-similar, and a given volume element experiences roughly the same density-time history as another. A collapse rate which fits the numerical result of Mayle and Wilson (1986) well is

$$\frac{d\ln\rho}{dt} = (100\ \mathrm{s}^{-1})\rho_{10}^{1/2}\ . \tag{11a}$$

Picture then a small volume element, for illustrative purposes say the center of the core, which is having its density increased at the rate given by equation (11). The electron neutrinos are trapped and thermalized into a roughly Fermi-Dirac distribution function with an electron neutrino chemical potential given by

$$\mu_{\nu_e} \approx (11.1\ \mathrm{MeV})(2\rho_{10}\,y_{\nu_e})^{1/3}\ , \tag{12a}$$

while the electron chemical potential is

$$\mu_e \approx (11.1\ \mathrm{MeV})(\rho_{10}\,y_e)^{1/3}\ . \tag{12b}$$

Throughout most of the density range we are interested in $[10^{11} \le \rho(\mathrm{g\ cm^{-3}}) \le 10^{15}]$ the lepton fractions are $y_e \approx 0.3$ and $y_{\nu_e} \approx 0.06$.

Examining the mass eigenstate curves in Figure 2, for a state which begins as $|\nu_e\rangle$, we see that for increasing density this state can be rotated into $|\nu_x\rangle$. Using the resonance condition (eq. [10a]) along with equation (6) for $\Delta_{\mathrm{eff}} = m_{\mathrm{eff}}^2$ and again assuming $m_{\nu_e} \approx 0(\Delta \approx m_{\nu_x}^2)$ and $\theta \ll 1$, we obtain an expression relating the density and lepton fractions at which a ν_e of energy E_{ν_e} transforms into a ν_x with mass m_{ν_x},

$$m_{\nu_x}^2(\mathrm{eV}^2) \approx (1.5184\times 10^3)(E_{\nu_e}/\mathrm{MeV})\rho_{10}(y_e + y_{\nu_e})\ . \tag{13}$$

So as the volume element has its density increased, the ν_e at the top of the Fermi-Dirac energy distribution goes through the resonance first and is transformed into ν_x. As the density increases, the transformation sweeps down the ν_e distribution. It is shown below that this transformation is adiabatic if θ is large enough.

First, it must be pointed out that it does make sense to discuss coherent processes like neutrino oscillations in the temperature and density conditions which exist in the collapsing stellar core. Note that the principal source of opacity for neutrinos in the core is due to coherent scattering on heavy nuclei. The mean free paths of neutrinos, λ_{mfp}, in the density region considered is $20\ \mathrm{cm} \le \lambda_{\mathrm{mfp}} \le 10^5$ cm. On the other hand, the oscillation length, which is a maximum at resonance, is from equation (10b) $L_{\mathrm{res}} \approx 2$ cm for $m_{\nu_x} \approx 5$ keV, $E_{\nu_e} = 40$ MeV, and $\theta \approx 10^{-4}$ rad. Therefore, under the ranges of ρ, E_ν, m_{ν_x}, and θ we consider there are very many oscillation lengths between neutrino scatterings. Considering that the phases of the neutrino states might be reset with each scattering, we see that the development in the last section would make little sense were not $L_{\mathrm{res}} \ll \lambda_{\mathrm{mfp}}$.

Given that the width of the resonance region implies the fractional change in density given in equation (10d), the collapse rate expression (eq. [11a]) yields the time to fall through the resonance

$$\Delta t_{\mathrm{res}} \approx (10^{-2}\ \mathrm{s})\frac{\tan 2\theta}{\rho_{10}^{1/2}}\ . \tag{14a}$$

The typical weak interaction time is of order 10^{-8} s, but the time scale to achieve and maintain beta equilibrium is of order $\Delta t_{\mathrm{equil}} \approx 10^{-5}$ s via neutrino-electron scattering and electron/neutrino captures (coherent scattering on nuclei does not change neutrino energy to first order). As will be discussed in the next section, maximum electron capture requires $\Delta t_{\mathrm{res}} \ll \Delta t_{\mathrm{equil}}$, implying that, roughly, $\theta < 10^{-2}$ rad.

On the other hand, the condition for adiabatic transformation of ν_e to ν_x is that $L_{\mathrm{res}} \ll \delta r$, with δr from equation (10e), so that adiabaticity requires that

$$\theta > \frac{2\times 10^{-3}(E_{\nu_e}/\mathrm{MeV})^{1/2}}{(m_{\nu_x}/\mathrm{eV})}\ , \tag{14b}$$

Combining the constraints on θ from equations (14a) and (14b) we have $10^{-7} < \theta\,(\mathrm{rad}) < 10^{-2}$ for the maximal effect in stellar collapse. Curiously the requirement of a rapid transformation implies an *upper* limit to the range of interesting vacuum mixing angles. The lower limit on interesting θ comes from either equation (14b) or the $L_{\mathrm{res}} \ll \lambda_{\mathrm{mfp}}$ conditions.

For a collapsing volume element the picture is that for a given m_{ν_x} and θ, the ν_e at the top of the Fermi-Dirac distribution are transformed first, so that the effective ν_e chemical potential is lowered from the top down, with equations (12a) and (12b) and (13) giving

$$\mu_{\nu_e}^{\mathrm{res}}(\mathrm{MeV}) \approx \frac{660(m_{\nu_x}^2/\mathrm{keV}^2)}{\rho_{10}(y_e + y_{\nu_e})}\ . \tag{15}$$

Of course, as the effective ν_e chemical potential, $\mu_{\nu_e}^{\mathrm{res}}$, is lowered, electron capture phase space is opened, and electron capture produced ν_e, along with ν_e-electron scattering, will rapidly replace the transformed ν_e. Beta equilibrium obtains before transformations begin, so that obtaining a new equilibrium state after the transformation process implies a possible generation of entropy.

We have assumed that the massive neutrinos are unstable, as in the family symmetry models. The unstable ν_x could, in principle, decay back to a light neutrino on time scales of interest in stellar collapse (~ 1 s). However, $\nu_x \to \nu_e + G$, where G is a Goldstone boson, will be blocked due to the degenerate sea of low-energy ν_e. It is possible that the neutrino transformation $\nu_e \to \nu_x$ is followed by the decay $\nu_x \to \nu_\mu + G$, where ν_μ is indicative of a neutrino with a mass intermediate between ν_e and ν_x. This process would not be strongly blocked in the collapsing stellar core. Depending on the interaction coupling constant of G with other neutrinos and the local neutrino number density this process could represent a source of nonadiabaticity in the collapse (if the Goldstone bosons escape). We are investigating this process in a subsequent paper. For now, we have assumed that the decay back of ν_x to ν_e is blocked.

IV. LEPTON NUMBER REDISTRIBUTION AND BETA EQUILIBRIUM

A result of all numerical calculations of stellar collapse is that beta equilibrium is rapidly established after neutrinos are trapped and thermalized (Arnett 1977). Beta equilibrium is maintained through electron and ν_e capture reactions,

$$e^- + p \rightleftarrows n + \nu_e \,, \tag{16a}$$

and when equilibrium obtains the forward (electron capture) rate and the reverse (neutrino capture) rate of equation (16a) are equal. In this case the lepton chemical potentials are related through

$$\mu_e - \mu_{\nu_e} = \hat{\mu} + \delta m \,, \tag{16b}$$

where $\hat{\mu} = \mu_n - \mu_p$ is the difference in the neutron and proton chemical potentials and $\delta m \approx 1.293$ MeV is the neutron-proton mass difference. We define the deviation from beta equilibrium to be δ

$$\delta = \frac{1}{kT}(\mu_e - \mu_\nu - \hat{\mu} - \delta m) \,. \tag{16c}$$

As discussed in the last section, electron neutrino phase space is opened as the density increases. The Fermi level of ν_e decreases as in equation (15), but electron scattering and electron capture reactions rapidly fill in the empty ν_e phase space. In what follows we will make the approximation that empty ν_e phase space is filled by electron capture reactions. In actuality $\nu_e - e$ scattering can also help refill ν_e phase space, and we will take this into account by using an estimate for the actual time it takes to establish beta equilibrium, Δt_{eq}, when both electron (or neutrino) capture and $\nu_e - e$ scattering take place. A fair estimate for the time scale to establish beta equilibrium is

$$\Delta t_{\text{eq}} = (10^{-5}\ \text{s})/\rho_{12}^{3/2} \,, \tag{17a}$$

where ρ_{12} is the density in units of 10^{12} g cm^{-3}.

If we assume that the lepton distribution functions can be approximated by zero-temperature Fermi-Dirac distributions, then after the onset of neutrino oscillations the ν_e distribution function will have a small strip missing. The lower energy edge of this strip is bounded by the lowest energy neutrino which has been adiabatically transformed and that energy is given by $\mu_{\nu_e}^{\text{res}}$ in equation (15). The top of the ν_e Fermi-Dirac distribution is roughly where it would have been had neutrino oscillations never occurred: the ν_e which have been adiabatically transformed into ν_x have all been replaced via electron capture as in equation (16a), save in a narrow empty strip where there has not been sufficient time (Δt_{eq}) to reestablish equilibrium. The energy width of this strip, for the collapse rate given in equation (11a) can be shown to be

$$\Delta\mu_{\nu_e} \approx (6.6 \times 10^3)\left(\frac{m_{\nu_x}}{\text{keV}}\right)^2 \frac{\Delta t_{\text{eq}}}{\rho_{12}^{1/2}} \,, \tag{17b}$$

where $\Delta\mu_{\nu_e}$ is in MeV. The deviation of the system from beta equilibrium is of this order, so that $\delta \approx (\Delta\mu_{\nu_e}/kT)$. We note that both $\Delta\mu_{\nu_e}$ and δ are very small compared to μ_{ν_e} and (μ_{ν_e}/kT), respectively.

Nevertheless, there can be a substantial amount of electron capture. We follow Fuller, Fowler, and Newman (1985) and give the time rate of the change of the electron fraction, $\dot{y}_E$, as

$$\dot{y}_e \approx \frac{x_h}{A}(e^{-\delta} - 1)\lambda_{e^-}^h \,, \tag{18a}$$

and the entropy generation rate as

$$\frac{\dot{s}}{k} = -\delta(\dot{y}_e) \,, \tag{18b}$$

where x_h is heavy nucleus mass fraction, A is the mean nuclear mass, and $\lambda_{e^-}^h$ is the electron capture rate for heavy nuclei. There are similar expressions for free protons. The electron capture rate is

$$\lambda_{e^-}^h = \frac{\ln 2}{\langle ft \rangle} I_e \,, \tag{18c}$$

where $\langle ft \rangle$ is the effective ft-value, taken from Fuller, Fowler, and Newman (1985), and I_e is the electron capture phase space integral. There is a similar expression for free protons. We take $\log \langle ft \rangle \approx 2.500$ for heavy nuclei and $\log \langle ft \rangle \approx 3.035$ for free protons. The thermal unblocking result of Fuller, Fowler, and Newman (1985) has been used: heavy nuclei tend to be unblocked at densities much greater than the neutrino trapping density due to the high temperatures and large nuclear masses. It can be shown that the electron capture phase space integral is

$$I_e = \int_x^y \omega_e^2(\omega_e - |q|)^2 d\omega_e \approx \frac{kT\delta}{m_e^2 c^4}(\mu_{\nu_e}^{\text{res}})^2(\mu_{\nu_e}^{\text{res}} + Q)^2 \,, \tag{18d}$$

where ω_e is the electron energy in units of $m_e c^2$, $q = Q/m_e c^2 = (\hat{\mu} + \delta m)/m_e c^2$ is the electron capture threshold Q-value, and the lower and upper limits of integration are $x = (\mu_{\nu_e}^{\text{res}} + Q)/m_e c^2$ and $y = (\mu_{\nu_e}^{\text{res}} + Q + \Delta\mu_{\nu_e})$. Ffrom the above it can be shown that the electron capture rate for heavy nuclei is

$$\dot{y}_e \approx (-1.895 \times 10^5)\frac{x_h}{A}\frac{\ln 2}{\langle ft \rangle}(\mu_{\nu_e}^{\text{res}})^2 \times (\mu_{\nu_e}^{\text{res}} + \hat{\mu} + \delta m)^2 \frac{(m_{\nu_x}/\text{keV})^2}{\rho_{12}^{1/2}} \Delta t_{\text{eq}} \,, \tag{18e}$$

where the chemical potentials are in MeV and the notation is as above. There is an analogous expression for free protons.

We have used this expression for the electron capture rate in a one-zone collapse code to estimate the change in y_e and entropy generation due to adiabatic transformation of ν_e into ν_x. The one-zone collapse code uses the collapse rate in equation (11a) and treats the thermodynamics as described in Fuller (1982). We have assumed that the trapped lepton fraction is $y_e \approx 0.38$. Figure 3 shows the subsequent change in y_e due to adiabatic neutrino oscillations for ν_x masses between 300 eV and 20 keV. When the mass of the ν_x is low, neutrino oscillations set in at a relatively low density. At low densities the beta equilibrium time scale is long, so that a relatively large amount of ν_e phase space is opened. The opened ν_e-phase space is filled by subsequent electron capture reactions yielding a large drop in y_e. For the highest ν_x masses shown ($m_{\nu_x} > 15$ keV), Figure 3 shows that $-\Delta y_e \leq 0.04$. In fact, this is an artifact of the electron capture approximations shown above. If beta equilibrium were achieved on an infinitesimally short time scale $-\Delta y_e$ would be equal to the original y_{ν_e}, as each transformed ν_e would be replaced by one electron capture reaction. Likewise, the lower m_{ν_x} end of Figure 3 shows $-\Delta y_e \approx 0.25$

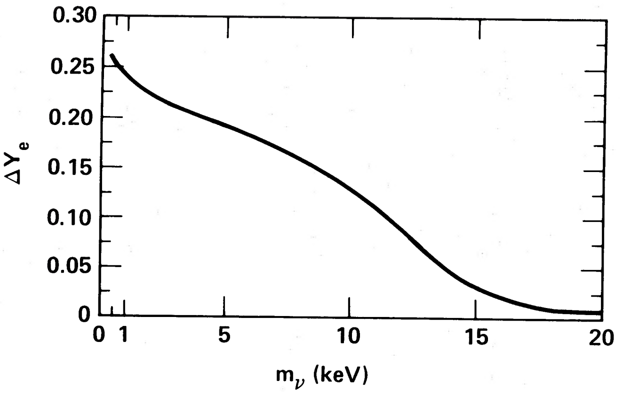

FIG. 3.—Change in electron fraction $-\Delta y_e$ as a function of the ν_x mass in keV (assuming $m_{\nu_e} \approx 0$). Here Δy_e is estimated by the approximation derived in the text which tends to overestimate the effect for small m_ν and underestimate it for large m_ν. For large m_ν the $-\Delta y_e$ should asymptotically approach the original y_{ν_e} at the onset of neutrino transformation, $-\Delta y_e \approx 0.06$. At small $m_\nu(\leq 1$ keV) phase space considerations limit $-\Delta y_e$ to ~ 0.15. On average $-\Delta y_e \approx 0.07$ is a fair approximation to the actual effect for the range of ν_x masses considered,

which is too large since the simple result in equation (18e) will tend to overestimate the electron capture phase space. Figure 3, therefore, only indicates the trend, with the average being $-\Delta y_e \approx 0.07$, which is roughly the initial y_{ν_e}.

The entropy generation corresponding to Δy_e in Figure 2 is always small since the entropy generation rate (eq. [18b]) is quadratic in the small deviation from beta equilibrium. In all cases considered $\Delta s/k \leq 0.1$.

V. CONCLUSIONS FOR THE SUPERNOVA MECHANISM

The drop in the electron fraction, y_e, discussed above could be quite significant for the mechanism of the supernova explosion. In the standard core bounce model (Bethe *et al.* 1979) the energy of the newly formed shock is related to the size of the homologous core. The mass of this core is essentially the instantaneous Chandrasekhar mass, which is determined by the total pressure. The pressure is dominated by the contribution from the relativistic electrons because of their very high Fermi energy. The mass of the homologous core is related to the electron fraction by

$$M_{\rm HC} \approx 5.8 \langle y_e^2 \rangle \, M_\odot \,, \tag{19}$$

so that a drop in y_e implies a drop in $M_{\rm HC}$ and, hence, in the energy of the shock. Furthermore, a smaller homologous core mass means there will be more low-entropy "Fe" rich material for the shock to photodisintegrate, which will entail further degradation of the shock energy. This trend argues against the viability of the standard core-bounce/prompt explosion model if neutrinos with the assumed properties exist (see Burrows and Lattimer 1983). On the other hand, if the core-bounce/prompt explosion scenario were the only process for producing Type II supernovae then one could argue that neutrinos with the assumed properties cannot exist because supernovae occur. In our case a $\Delta y_e \approx -0.07$ implies a reduction in the initial shock energy of 30%, ensuring the nonviability of the standard model in the presence of neutrinos with the assumed properties.

Of course, there is an alternative to the standard core-bounce/prompt explosion model; namely, the late time neutrino heating mechanism (Wilson 1985), which provides a means of reenergizing a stalled shock using thermal neutrino energy. If neutrinos with the assumed properties exist then the reduction in the homologous core mass discussed above ensures that the late-time neutrino heating mechanism would be involved in any subsequent supernova explosion. We are beginning to investigate whether the lepton and entropy distributions, homologous core mass, and initial shock energy resulting from a stellar collapse with massive neutrinos will result in a supernova explosion via the late-time neutrino heating mechanism.

Interestingly, there is another independent process whereby the MSW mechanism can help the late-time neutrino heating process (which has anyway suffered from producing a supernova explosion of relatively low energy, $\sim 10^{50}$ ergs; Wilson 1985). The late-time neutrino-heating scenario for reenergizing a stalled shock can be very roughly pictured as resulting from the emission of thermal, blackbody, distributions of ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$, ν_τ, $\bar{\nu}_\tau$ from a neutrino "photosphere" at a density of 10^{11} g cm^{-3}. These neutrinos freely stream above the photosphere out to the region just inward of the stalled shocks at a density of 10^8–10^9 g cm^{-3}. Here the ν_e and $\bar{\nu}_e$ can be captured via the charged current processes

$$\nu_e + n \rightarrow p + e^- \,, \tag{20a}$$

$$\bar{\nu}_e + p \rightarrow n + e^+ \,, \tag{20b}$$

which results in local energy deposition and, hence, increased pressure which reenergizes the shock (Bethe and Wilson 1985). The ν_μ, ν_τ can be adiabatically transformed to ν_e as they move down the density gradient of the star in accord with Figure 2. This is the high-density to low-density MSW transformation which would operate in the Sun, not the low-density to high-density transformation characteristic of stellar collapse. To undergo the MSW transformation in the region between the neutrino photosphere and the base of the shock the ν_μ, ν_τ neutrinos must have masses in the range $9 \leq m_\nu(\text{eV}) \leq 240$. This range in mass is from equation (6) assuming $y_{\nu_e} \approx 0$, $y_e \approx 0.37$, E_{ν_e} is characteristic of thermal energies (3–10 MeV), and $0.05 \leq \rho_{10} \leq 10$. We see that if the transformation is adiabatic, the initial energy deposition rate behind the shock will be very roughly a factor of 1.5 larger because we have essentially increased the ν_e luminosity by a factor of 3. Adiabaticity will be maintained if the vacuum mixing angles involved are $\theta > 10^{-4}$ rad. Note that this range in neutrino masses only overlaps the range of interest for stellar collapse at the upper end. In fact, for the lower end of ν_μ and ν_τ masses considered these neutrinos could be stable and be within cosmological limits.

We have shown that massive neutrinos and the MSW mechanism for resonant enhancement of neutrino oscillations can have interesting effects for current models of the supernova explosion mechanism. In particular, massive unstable neutrinos, suggested in some galaxy formation scenarios, would change the current stellar collapse model by lowering the electron fraction and thereby reducing the homologous core mass and initial shock energy. We show how this could happen by a reverse adiabatic neutrino transformation induced by gravitational collapse. Furthermore, we show that the late-time neutrino-heating mechanism can be aided by adiabatic transformation of ν_μ and ν_τ neutrinos into ν_e.

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