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ABSTRACT

We construct from the S-matrix in the Dirac picture a general expression for the S-operator and show that the S-operator theory of weak interactions proposed by Pietschmann and Nilsson is only a special case of it. We critically discuss this theory.

Although the V-A theory of weak interactions described in terms of a simple phenomenological Lagrangian is not renormalizable, the answers obtained in first-order perturbation theory are all finite and show excellent agreement with experiments. All attempts to recast it into a true Lagrangian field theory yielding finite results to all orders in perturbation theory have failed so far.

Instead of using the Lagrangian formalism, one can also formulate the theory of weak interactions as an S-operator theory, so that these troublesome higher orders are excluded in a trivial way. The

S-operator, which reproduces exactly all the results of the first-order Lagrangian theory, is given by

$$S = I - i \int d^4x H(x) . \quad (1)$$

For the Hamiltonian density $H(x)$ we have the well-known current-current form

$$H(x) = \frac{G}{\sqrt{2}} J^\mu(x) J_\mu^+(x) , \quad (2)$$

where $J_\mu(x)$ is the sum of charged leptonic and charged hadronic currents (Cabibbo currents).

$$J_\mu(x) = \ell_\mu(x) + \cos \theta j_\mu(x) + \sin \theta s_\mu(x) . \quad (3)$$

The quantities $j_\mu(x)$ and $s_\mu(x)$ are the strangeness preserving and strangeness changing charged hadronic currents, belonging to a single SU(3) octet, and transforming as π^+ and K^+ respectively, and both containing vector and axial-vector parts.

The form (1) of the S-operator can only be valid at low energies, because it is not unitary. In this respect, it is plagued by the same difficulties as the first-order Lagrangian theory. Now Pietschmann and Nilsson [1] proposed a theory of weak interactions which, in an expansion to first order in G , coincides with the S-operator (1), but which also allows the calculation of higher order effects and which fulfills the requirement of unitarity. They defined the S-operator by the relation

$$S = \exp \left(-i \int d^4x H(x) \right) . \quad (4)$$

The only difference with regard to the Lagrangian formulation is that the above authors dropped the time-ordering operator in front of the exponential.

We now construct from the S-matrix in the Dirac picture a general expression for the S-operator and we give the conditions which lead to the special form (4).

In the Dirac picture the S-matrix is given by the covariant expression [2]

$$\begin{aligned}
 S &= T \exp\left(-i \int d^4x H(x)\right) \\
 &= 1 + (-i) \int d^4x H(x) + (-i)^2 \int d^4x \int d^4y \theta(x_0 - y_0) H(x) H(y) \\
 &\quad + (-i)^3 \int d^4x \int d^4y \int d^4z \theta(x_0 - y_0) \\
 &\quad \quad \quad \theta(y_0 - z_0) H(x) H(y) H(z) + \dots, \quad (5)
 \end{aligned}$$

where the interaction Hamiltonian density has the property that

$$[H(x), H(y)] = 0 \quad \text{for } (x - y)^2 < 0.$$

We define the following quantities:

$$\begin{aligned}
 G_1 &= \int d^4x H(x) \\
 G_2 &= \int d^4x \int d^4y \epsilon(x_0 - y_0) H(x) H(y) \\
 G_3 &= \int d^4x \int d^4y \int d^4z \epsilon(x_0 - y_0) \epsilon(y_0 - z_0) H(x) H(y) H(z) \\
 &\text{etc.} \quad (6)
 \end{aligned}$$

For the subsequent discussion it is convenient to cast these functions G_v in a different form. For G_2 we find after some calculation

$$G_2 = \int d^4x \int d^4y [H(x), H(y)], \quad (7)$$

where we assumed that it is permissible to interchange the order of integration. All the other G_v 's for $v > 2$ can be written in a similar fashion as integrals over $(v - 1)$ - fold commutators between the interaction Hamiltonian densities. Using these functions G_v in (5) we obtain a series expansion for the S-operator of the form

$$S = 1 + \left[(-i)G_1 + \frac{(-i)^2}{2} G_2 + \frac{(-i)^3}{4} G_3 + \dots \right] + \frac{1}{2} \left[(-i)G_1 + \frac{(-i)^2}{2} G_2 + \frac{(-i)^3}{4} G_3 + \dots \right]^2 + \dots \quad (8)$$

This series can easily be summed up and the result is

$$S = \frac{1 - iK}{1 + iK}, \quad (9)$$

where the K-operator is given by

$$K = \sum_{v=1}^{\infty} \frac{(-i)^{v-1}}{2^v} G_v. \quad (10)$$

Since K is an hermitian operator, the S-operator is unitary and can therefore be written in terms of an hermitian operator Q as

$$S = \exp(2iQ). \quad (11)$$

A comparison of (9) and (11) yields for the operator Q :

$$Q = -\text{arctg } K . \quad (12)$$

Thus the most general form of the S-operator fulfilling the requirement of unitarity is given by

$$S = \exp(-2i \text{ arctg } K) . \quad (13)$$

From (13) we can immediately read off the conditions under which we obtain the special form (4) of Ref. [1]: (1) Take only the first term in the expansion of $\text{arctg } K$, i.e. K , and (2) neglect in the expression (10) for K all terms G_ν with $\nu \geq 2$ [3].

Since for pure leptonic reactions the commutator between the interaction Hamiltonian densities is zero, we see from (7) and the subsequent remark there, that the second condition is in this case exactly fulfilled.

But this is no longer true for semileptonic and nonleptonic decays. Even in a first-order expansion of $\text{arctg } K$ the other terms G_ν with $\nu \geq 2$ have to be taken into account in the expression (10) for K and influence appreciably the renormalization of the coupling constants.

Apart from the fact that it is extremely complicated to calculate higher order contributions from the general expression (13) - it is somewhat more convenient to work with (9) - we want to emphasize that this S-operator theory of weak interactions has considerable drawbacks.

Since we have no longer a time-ordering operator, we have no causal propagators and therefore the theory is not crossing symmetric (no Wick-theorem). In an expansion of a matrix element of (13) with respect to G , each term can be represented by a number of graphs. But these graphs are not Feynman graphs, because their lines do not correspond to particle propagators, and therefore they do not represent any virtual process developing in time. Because all lines correspond to particles on the mass shell, no vacuum graphs will occur. For these reasons the physical interpretation of the contributions of these graphs is obscure, e.g. graphs which look like self-energy corrections do not lead to mass renormalizations.

It turns out, that all higher order terms are finite apart from a single infinite parameter which, in principle, can be absorbed in a coupling-constant renormalization. This infinity has its analogy in standard quantum electrodynamics and is there absorbed in a wave function renormalization. This difficulty is related to the ambiguity of letting the initial and final times go to $\pm\infty$ and can there formally be overcome by an adiabatic hypothesis. We thus see that the S-operator theory does not overcome all renormalization difficulties.

We want to make a final comment about analyticity. From the derivation of the S-operator (13) one could conjecture that we obtain from diagrams of a certain order the same singularities as from the analogous ones in Lagrangian field theory. A check on the lower order diagrams proves this. From this check it can also be seen that under

the conditions of taking only the first term in the expansion of $\arctg K$ and by neglecting the terms G_ν with $\nu \geq 2$ [4] one gets fewer singularities as supposed in Ref. [1].

We conclude by saying that although this theory fulfills the requirement of unitarity, even though in a more or less trivial way, we believe that in view of the above mentioned drawbacks and difficulties this kind of approach is not very fruitful and to speak of a consistent theory is at least somewhat exaggerated.

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FOOTNOTES AND REFERENCES

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1. H. V. R. Pietschmann and J. S. Nilsson, Phys. Rev. 142, 1173 (1966).
 2. See for example, S. S. Schweber, An Introduction to Relativistic Quantum Field Theory (Row, Peterson and Company, Evanston, 1961).
 3. In view of (7) and the subsequent remark there the second condition can also be formulated by saying that one assumes that the commutator between the interaction Hamiltonian densities vanishes everywhere.
 4. For pure leptonic processes this second condition is trivially fulfilled as already remarked above. We emphasize once more that both conditions together are equivalent to dropping the time-ordering operator in (5).

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