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Adaptive versus Eductive Learning: Theory and Evidence *

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Abstract

Adaptive and eductive learning are two widely used ways of modeling the process by which agents learn a rational expectation equilibrium (REE). In this paper we report on an experiment where we exploit differences in the conditions under which adaptive and eductive learning converge to REE so as to investigate which approach provides the better description of the learning behavior of human subjects. Our results suggest that the path by which the system converges appears to be a mixture of both adaptive and eductive learning model predictions.

JEL Classification: C91, C92, D83, D84

Keywords: Rational Expectations, Adaptive Learning, Eductive Learning, Experimental Economics.

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1 Introduction

How do agents learn a rational expectations equilibrium (REE) if they do not initially find themselves in such an equilibrium? This important, foundational question has generated a large literature in macroeconomics (see, e.g., surveys by Sargent (1993), Grandmont (1998), Evans and Honkapohja (2001)). In this paper we focus on two different but related approaches to addressing this question.

Perhaps the most widely used approach to modeling learning behavior (beginning, e.g., with Bray (1982)) is to suppose that agents are boundedly rational *adaptive* learners and to ask whether their use of a given real-time adaptive learning model that allows for a REE as a possible solution converges in the limit to that REE. An alternative, off-line approach advocated, e.g., by Guesnerie (1992, 2002), is to suppose that learning is a mental process involving (possibly collective) introspection that takes place in some notional time and that leads agents to understand and instantly coordinate upon or "educe" the REE solution.[1] Both approaches to learning place restrictions on the model under which learning agents are able to learn the REE. Our aim here is to test the validity of these restrictions for the "learnability" of REE using controlled laboratory experiments. Further, in model parameterizations where both approaches predict that the REE is stable under learning (or "learnable"), the two approaches nevertheless predict different *speeds of convergence* by which agents should be able to learn the REE. If agents are adaptive learners it should take more than a single period for their price forecasts to converge to the REE value. By contrast, if agents are eductive learners and understand the model, their price forecasts should instantaneously convergence to the REE value.

Evans (2001) highlights the different restrictions of the two different approaches to learning, and invites empirical and experimental testing of the different theoretical predictions. Specifically he writes:

> "Which is the appropriate way to model economic agents will ultimately be a matter for empirical and *experimental* research. It is likely that the answer depends on the circumstances, for example, in experiments, on the details of the setting and the types of *information* provided to

[1] These two approaches are also considered as two broad classes for belief formation in a recent survey of expectations in macroeconomics by Woodford (2013).

> the subjects. A plausible conjecture is that when a model is simple and transparent as well as *eductively stable*, agents will coordinate rapidly on the REE....If a model has no eductively stable REE, but has an REE that is adaptively stable, then a plausible conjecture is that there will still be convergence to the REE, at a rate governed by the accumulation of data....The eductive results provide a caution, however, that coordination in such cases may not be robust." (Evans 2001, p. 581 emphasis added).

In this paper we follow up on Evans's invitation to compare adaptive versus eductive learning approaches. Indeed, the manner in which agents might go about learning a rational expectations equilibrium is an important, fundamental yet unresolved issue; there are many ways to model this learning process and it would be useful to have a consensus on which approach (or combination of approaches) are more empirically valid than others.[2] Understanding the manner in which agents learn is also important for policy purposes. For instance, if agents can educe REE prior to making decisions via the mental, collective introspective process described by eductive learning, then policy ineffectiveness propositions that arise under rational expectations may have full standing. However, if agents learn REE adaptively and over time, then policy interventions are likely to be effective in the short-run in the determination of economic variables. Thus, the *manner* in which agents learn is an important empirical question.

Ideally, one would like to address the question of how agents form expectations using non-experimental field data, but unfortunately, properly incentivized field data on individual-level expectations are not generally available. Survey evidence, e.g., on inflationary expectations, consumer confidence, etc. *are* available, but these data are not properly incentivized in that constant rewards or, more typically, no reward at all for participation in such surveys, yield poor incentives to report truthful beliefs. Even setting such incentive problems aside, to use survey data on expectations one would have to know the precise structure of the economic environment, i.e., the data generating process in which agents were forming their expectations, knowledge that is typically unavailable and/or subject to some dispute. For these reasons, a laboratory experiment offers the better means of collecting data on expectations as truthful revelation can be properly incentivized (using quadratic loss scoring rules) and the

[2] Here we focus on just two approaches, but there are several other approaches including Bayesian learning, evolutionary learning and near-rational (calculation-cost) learning.

control of the laboratory allows for precise implementation of the model environment (data generating process) in which agents' expectations matter for the realizations of economic variables.[3]

We run an experiment where subjects make predictions on the market price in a simple cobweb economy. The result is in general a mixture of the predictions of both theories. The market price converges reliably to the REE when both learning approaches predict convergence. Some of these markets converge in the first period or within the first five periods, while others take more periods to converge. For the markets where the REE is learnable under adaptive learning but not eductive learning, some markets still converge to the REE within 50 periods while relatively more markets fail to converge.

The organization of the remainder of paper is as follows: section 2 discusses related literature, section 3 presents the theoretical model, section 4 discusses the experimental design and hypotheses, section 5 reports the experimental results, and section 6 concludes.

2 Related Literature

In terms of experimental design, our work is related to "learning–to–forecast" experiments (as pioneered by Marimon and Sunder (1993)), that involve versions of the cobweb market model with negative feedback (or strategic substitutes). Hommes et al. (2000) provide the first experimental test of such a cobweb economy, and this study has been followed by Sonnemans et al. (2004), Hommes et al. (2007), Heemeijer et al. (2009), Sonnemans and Tuinstra (2010), Bao et al. (2012, 2013) and Beshears et al. (2013). Hommes (2011) surveys the literature. The differences between the present study and those earlier papers are as follows. First, subjects in all of these prior studies do not precisely know the model of the economy (i.e., the data generating process) which makes it impossible for them to apply eductive learning as that type of learning (as demonstrated below) requires full knowledge of the model thereby enabling introspective reasoning about the proper forecast. By contrast, sub-

[3]See Duffy (2014) for further arguments in support of using laboratory evidence to evaluate macroeconomic models and assumptions as well as a survey of the literature on experimental macroeconomics.

jects in our experiment *are* informed about the model economy and so they *can* in principle apply eductive learning, or even directly solve for the REE using the perfect foresight condition. Second, all prior experiments using the cobweb model employ a group design where both learning and strategic uncertainty can influence the speed of the convergence to the REE. By contrast, we have both a group ("oligopoly market") treatment *and* an *individual*-decision making ("monopoly" market) treatment. The monopoly treatment rules out strategic uncertainty as a factor and serves as an important baseline for assessing the extent of rational play among subjects. Third, all prior learning–to–forecast experiments involving linear cobweb models use a data generating process for the market price equation that has a coefficient on expected prices, α, that is smaller than 1 in absolute value.[4] Finally, we explicitly test restrictions on the stability of REE under two different learning approaches. By contrast, most of the existing experimental literature on whether and how agents learn a REE in cobweb economies has been concerned with characterizing the type distribution of (adaptive) learning behaviors without regard to any stability under learning criteria, and certainly not a comparison of different learning criteria, as we present in this paper.

Since subjects in our experiment know how the price is determined as a function of price forecasts, (i.e., they know the data generating process) our experiment is also related to an experimental literature on "guessing" or "beauty contest" games (see, e.g., Nagel (1995), Duffy and Nagel (1997), Ho et al. (1998) Grosskopf and Nagel (2008) among others). In these guessing games, subjects are asked to guess a number. The winning guess, (which is similar to a market price and which yields the winner a large prize), is a known function of the average guess (or average opinion which is similar to the mean price forecast). A main finding from this literature is that the winning number is initially very far from the rational expectations equilibrium though it gets closer to that prediction with experience. In our experiment we consider forecasting by a group of three subjects (in our "oligopoly" setting) as well as an individual forecasting treatment (our "monopoly" setting) and we also examine whether our results for the monopoly treatment are closer to the REE relative to the oligopoly treatment. The winning number in beauty contest games is typically a linear function, $\rho \times$ the mean guess, where $\rho \in (0, 1)$ which is similar to a learning–to–forecast experiment

[4]Hommes et al. 2007 report on a *nonlinear* cobweb model experiment where $|\alpha| > 1$ in the REE, but the focus of that study is on adaptive learning alone, as subjects do not know the model and therefore cannot apply eductive learning.

with positive feedback (strategic complements). There are also some guessing game experiments where $\rho \in (-1, 0)$ such as Sutan and Willinger (2009). The difference between our work and their paper is that we provide a more detailed description of the model that generates the price that agents are seeking to forecast and we vary the value of ρ (equivalently, our α) so as to explore the implications of differing stability results under the adaptive and eductive approaches to learning. As in a typical macroeconomic model, we also add a shock term in the price determination equation, a setup that is not usually found in number guessing games. Our framework can also be extended easily to a real intertemporal design where shocks are autocorrelated.

Finally, since we have both monopoly (individual decision-making) and oligopoly (group decision-making) treatments, our paper is related to experimental studies on oligopoly markets, for example, Bosch-Doménech and Vriend (2003), Huck et al. (1999), and Offerman et al. (2002). These oligopoly market experiments use learning–to–optimize designs where subjects submit a quantity choice directly and price forecasts are not elicited. By contrast, we ignore quantity choices and focus on price forecasts (expectations) using a learning-to-forecast design.[5] Gaballo (2013) provides conditions under which convergence to REE happens under eductive learning in an oligopoly market setting with a small number of producers, which is slightly different from the competitive market version as in Guesnerie (1992, 2002). Our monopoly vs. oligopoly design is also helpful for investigating the important role played by common knowledge of rationality in eductive learning. In this respect our paper is related to other experimental studies exploring the role of common knowledge of rationality in market settings, for example, the "money illusion" experiments by Fehr and Tyran (2005, 2007, 2008) and the asset market experiments by Akiyama et al. (2012, 2013).

[5]In a learning-to-forecast design, subjects submit a price forecast and a computer program uses that forecast to optimally determine the subject's quantity decision. By contrast, in a learning–to–optimize design, subjects submit a quantity choice directly; their price forecast is not elicited, though it is implicit in their quantity decision. See Bao et al. (2013) for a comparison of these two approaches.

3 Theoretical Model

3.1 Cobweb economy

We consider a simple version of a cobweb model as presented in Evans and Honkapohja (2001) that is based on Bray and Savin (1986). We chose the cobweb model as it was the model originally used by Muth (1961) to illustrate the notion of a REE. It is also simple enough to explain to subjects and has the critical feature that expectations matter for outcomes, here price realizations, while outcomes can in turn matter for beliefs as subjects interact under the same model environment repeatedly. The cobweb model is one of demand and supply for a single perishable good and consists of the two equations:

$$D_t = a - bp_t,$$
$$S_t = cp_t^e + \eta_t.$$

Here, D represents demand, S supply, a, b, and c are parameters, which are usually assumed to be positive, p_t is the period t price of the good, $p_t^e = E_{t-1}[p_t]$, and η_t is a mean zero supply shock.[6]

Assuming market clearing, the reduced form equation for prices is given by:

$$p_t = \mu + \alpha p_t^e + \nu_t, \tag{1}$$

where $\mu = \frac{a}{b}, \alpha = -\frac{c}{b}, \text{and} \nu_t = \frac{\eta_t}{b}$.

The system has a unique rational expectation equilibrium where

$$p_t^{e,*} = \frac{\mu}{1-\alpha} \quad \text{and } p_t^* = p_t^{e,*} + \nu_t. \tag{2}$$

3.2 Theoretical Predictions

As Evans (2001) shows, the unique REE of this model (2) is stable under adaptive learning (i.e., it is "learnable") if $\alpha < 1$. However, under the eductive learning

[6]Bray and Savin and Evans and Honkapohja use a somewhat richer model in which the supply equation, $S_t = cp_t^e + \delta w_{t-1} + \eta_t$, where w_{t-1} is an observable exogenous variable affecting supply, e.g., weather in period $t-1$, that follows a know process (i.i.d. mean 0 or possibly AR(1)). For simplicity we study the case where $\delta = 0$, but we think it would also be interesting to study cases with such exogenous forcing variables as well.

approach, the REE is learnable only if $|\alpha| < 1$ (See, e.g., Evans (2001) or Evans and Honkapohja (2001, section 15.4). [7]

To be more precise, adaptive learning consists of a general class of backward looking learning rules that make use of past information and the specific type of adaptive learning rule that we consider in this paper is "least squares learning" which is widely used. In assuming that agents learn in this adaptive fashion, we suppose that they do not know or they ignore any information about the price determination equations of the economy. Instead, they start out by choosing a random prediction for the price in period 1, p_1^e. The adaptive agents' "perceived law of motion" for the price at time t is that it is equal to some constant, a, plus noise, ϵ_t, i.e., $p_t^e = a + \epsilon_t$, which has the same functional form as the REE solution. Given this perceived law of motion and the assumption that the adaptive agents are least squares learners, it follows that, in each period $t > 1$, agents' price forecast is equal to the sample average of all past prices given the available history:

$$p_t^e = \frac{1}{t-1} \sum_{s=1}^{t-1} p_s. \tag{3}$$

Evans and Honkapohja (2001, section 2.3 and 2.4 for a simple linear case with an additional exogenous variable) provide a general proof, based on matrix operations, as to why the REE in this simple cobweb system is learnable via adaptive, least squares learning provided that $\alpha < 1$.[8] For readers without prior knowledge about adaptive learning to capture the idea of this modeling approach, we provide an alternative proof for the non-stochastic version of this model (namely, ignoring the noise term ν_t since it has mean zero and small variance) based on mathematical induction in the appendix.

For the experiment we parameterized the cobweb model as follows: $\mu = 60$ and $\nu_t \sim N(0, 1)$. Different subjects to the same history of realizations of ν_t. We consider

[7] We recognize that other learning approaches may impose different restrictions on the parameters of the cobweb economy to ensure converge to the REE. For example, Hommes and Wagener (2010) find that when agents use the evolutionary learning model of Brock and Hommes (1997), the market price may converge to a locally stable two cycle when $\alpha \in [\frac{1}{2}, 1]$.

[8] See also Evans and Honkapohja (2001), p. 149 for a more sophisticated non-stochastic non-linear negative feedback model with decreasing gain in the multivariate case based on Evans and Honkapohja, (2000).

three different values for α which comprise our three treatment values (T) for this variable: $T1 : \alpha = -0.5$, $T2 : \alpha = -0.9$, $T3 : \alpha = -2$ and $T4 : \alpha = -4$. The REE predictions associated with these three choices are $T1 : p^{e,*} = 40$, $T2 : p^{e,*} = 31.58$, $T3 : p^{e,*} = 20$ and $T4 : p^{e,*} = 12$, respectively.

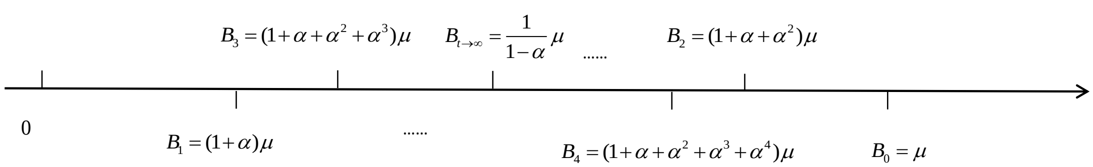
Eductive learning has two versions, the basic, single-dimensional version found in Guesnerie (1992) and a more general N-dimensional version in Guesnerie (2002). Eductive learning is based on iterated elimination of strategies that are not best responses, in our case, the elimination of unlikely price forecasts.[9] This iteration occurs in a competitive market environment where each individual producer has no market power (therefore, the rational expectation equilibrium corresponds with the *competitive equilibrium*). Each producer has perfect *individual rationality*, namely, they have no problem solving for the rational expectations equilibrium of the system, but they face possible *strategic uncertainty* about the actions of other producers.[10] The eductive learning model describes the learning process via which agents iteratively eliminate non-rationalizable strategies (price forecasts) from their strategy space. If this process leads to elimination of all other strategies aside from predicting the rational expectation equilibrium, then the rational expectation equilibrium is said to be *eductively stable*. We would like to emphasize that in this sense, eductive learning is a *social learning* process (Vriend, 2000) as the agents' learning behavior is also conditioned upon others' decisions, while adaptive learning is mostly an *individual learning* process where agents learn from the history of the realized market price and interact with others only indirectly.

In this paper, we focus on the single-dimensional version of eductive learning as in Guesnerie (1992) since our cobweb model is a simple, one product market. The eductive learning process works in the following way: in notional period 0, each agent knows that it is rational to forecast $p_t = \frac{\mu}{1-\alpha}$, but may not know whether the other agents form similar predictions. However, since all agents know that $p_t = \mu + \alpha p_t^e$, that prices should be non-negative and that $\alpha = -\frac{c}{b} < 0$, it follows that agents can logically rule out the possibility that any other agent would forecast prices greater

[9] Eductive learning is the counterpart of rationalizability (Bernheim 1984, Pearce 1984) in games.

[10] We acknowledge that the ability of agents to solve the REE from equation (1) is not explicitly included as a part of individual rationality defined by Guesnerie (1992) on page 1257. But immediately after that, on page 1258, Guesnerie makes the comment that the rational expectation equilibrium is also the unique Nash Equilibrium of the game. Therefore, the subjects should be able to find the REE/Nash Equilibrium by solving the game as if they are perfectly rational.

than μ[11], and so it can be regarded as common knowledge that no one is going to forecast $p_t > \mu$; in notional period 1, knowing that no one is going to make a price forecast that is larger than μ, and substituting this constraint into the price equation, $p_t = \mu + \alpha p_t^e$, agents should all infer that no one will forecast prices lower than $\mu + \alpha\mu = (1+\alpha)\mu$; in notional period 2, using the same reasoning, agents can rule out price forecasts greater than $\mu + \alpha(\mu + \alpha\mu) = (1 + \alpha + \alpha^2)\mu$, etc. More generally, in notional period t, the new forecast boundary created by this iterative process will be $(1 + \alpha + \alpha^2 + ... + \alpha^t)\mu$. If $|\alpha| < 1$, this process will tighten the interval range of possible price forecasts to a single point, the REE. When $|\alpha| < 1$, in the limit, the two boundaries become a single point, $\lim_{t\to\infty} \sum_{s=1}^{t} \alpha^s \mu = \frac{\mu}{1-\alpha}$. This iterative, notional time eductive learning process is illustrated in Figure 1. By contrast, when $\alpha < -1$, agents cannot rule out any price forecasts starting from notional period 1, because $\mu + \alpha\mu < 0$. Hence, in the case where $\alpha < -1$, the REE is not eductively stable, though as shown earlier, it is stable under the adaptive learning dynamics. This difference in the criteria for convergence to REE is the main hypothesis that our experiment addresses.



The Iterative Process of Eductive Learning in Notional Periods

Figure 1: An illustration of the iterative process in notional time under eductive learning. The process creates a boundary, B_t, in notional time period t, and excludes numbers that are larger/smaller than this boundary in even/odd notional periods. When $|\alpha| < 1$ the boundaries move closer to each other with each iteration so that the interval eventually tightens to a single point, i.e., $\lim_{t\to\infty} \sum_{s=1}^{t} \alpha^s \mu = \frac{\mu}{1-\alpha}$.

In our experiment we keep all parameterizations of the model constant across

[11]Since the literature on eductive learning typically assumes that $\alpha < 0$ as the starting point, when we prove that the REE is not eductively stable when $|\alpha| > 1$, we only focus on $\alpha < -1$, case because the $\alpha > 1$ case is already ruled out by the assumption that $\alpha < 0$.

treatments varying only the value of α. Specifically, we consider three different treatment values for α, $T1$: $\alpha = -0.5$, $T2$: $\alpha = -0.9$ and $T3$: $\alpha = -2$. We further differentiate between monopoly and oligopoly markets. Since eductive learning is a social learning process, only the oligopoly market design provides an environment where *both* adaptive learning and eductive learning can be properly implemented. In that case, both learning theories predict that subjects will learn the REE in treatments $T1$ and $T2$, but under $T3$, the REE is "learnable" only if agents are adaptive learners; according to the educative learning approach, the REE should not be stable under learning in $T3$ where $|\alpha| > 1$. This is our main hypothesis to be tested. In addition, as a robustness check on the oligopoly market behavior, we also explore a monopoly market treatment involving individual decision-making. Different from the oligopoly design, in the monopoly design, the REE is learnable under eductive learning in all treatments, since eductive learning assumes that agents have no difficulty solving for the REE by themselves. In other words, *the predictions of the two learning models diverge only in the oligopoly T3 treatment.* Finally, we also consider differences in speeds of convergence; when an REE is stable under eductive learning, convergence should, in principle, be instantaneous while under adaptive learning, it can take several periods for the economy to converge to the REE depending on the initial price forecasts.

Besides the competitive market version by Guesnerie (1992), Gaballo (2013) derives the eductive stability condition for oligopoly markets with small number of firms. The condition reads:

$$\frac{N}{N-2} < \alpha_N < 1, \tag{4}$$

where like α_N also stands for the expectation feedback coefficient in the price generation function, and N is the number of firms in the market. According to this criteria, the oligopoly $T3$ treatment is eductively stable since $-2 \in [-3, 1]$. To address this issue, we run an additional treatment $T4$ where $\alpha = -4$ as robustness check for our results[12]. If the markets still converge to the REE, that serves as strong evidence that favors adaptive over eductive learning.

[12]We thank a referee who recommends us to run this additional treatment.

4 Experimental Design

4.1 Treatments

We employ a 4×2 design where the treatment variables are (1) the four different values of the slope coefficient, α, and (2) the number of subjects in one experimental market: either just one subject–the "monopoly" market case or three subjects–the "oligopoly" market case. The monopoly versus oligopoly design is helpful in investigating the role of strategic uncertainty. Eductive learning assumes first that agents are perfectly rational, i.e., that they can perfectly solve for the REE when they have complete information about the model (as in our design) and second, that there is common knowledge of rationality, namely, that each player knows the other players are rational, each knows that the others know that they are rational and so on. In monopoly markets, common knowledge of rationality is not an issue since the single agent faces no uncertainty about his own level of rationality. Deviations from REE in the monopoly market are violations of the assumption of individual rationality. However, since individual rationality is a precondition for common knowledge of rationality, the extent to which deviations from individual rationality arise in the monopoly markets is useful for understanding behavior in the oligopoly markets where common knowledge of rationality plays a role. If there is some doubt as to whether other market participants can form rational expectation forecasts, as evidenced by monopoly market forecasts that are not immediately equal to REE values, then forecasting the REE price in the oligopoly setting may no longer be a best response. If it takes some time for subjects to learn the REE in the monopoly setting, then it should take at least as much time or longer for subjects to learn the REE in the oligopoly setting, as group members would first have to establish that there was common knowledge of rationality.

A noted earlier, our three treatment values for α are given by:

Treatment 1 ($T1$): weak negative feedback treatment, $\alpha = -0.5$.

Treatment 2 ($T2$): medium level negative feedback treatment, $\alpha = -0.9$.

Treatment 3 ($T3$): strong negative treatment, $\alpha = -2$.

Treatment 4 ($T4$): very strong negative treatment, $\alpha = -4$.

As shown in the prior section, the REE should be learnable under adaptive expectations for four three values of α. Generally negative feedback systems converge much faster than positive feedback systems (Heemeijer et al. 2009, Bao et al. 2012). In the oligopoly design, the REE should be learnable under the competitive market version of eductive learning only in treatments $T1$ and $T2$, but not in $T3$. The REE is learnable under the oligopoly version of edutive learning in the first three treatments, but not in $T4$. While in the monopoly design, the REE is learnable under eductive learning in all treatments.

Our experiment makes use of a learning to forecast ("LtFE") experimental design. Subjects play the role of an advisor who makes price forecasts. Subjects are paid according to the accuracy of their own price forecast and so are incentivized to provide good price forecasts. In the monopoly treatment, the time t price forecast of the one subject, i, who is associated with each monopoly market, $p^e_{i,t}$, determines the price forecast for that market, i.e., $p^e_t = p^e_{i,t}$ which is then used to determine the actual price, p_t, for that monopoly market according to equation (1). By contrast, in the oligopoly treatment, we use the mean of the three subjects' individual price forecasts for period t as the market price forecast, i.e., $p^e_t = \frac{1}{3}\sum_{i=1}^{3} p^e_{i,t}$, which is then used to determine the actual price, p_t, for each oligopoly market, again according to equation (1). One important advantage of our LtFE design is that when the subjects are paid according to their forecast accuracy instead of according to the profit from their production decision, they have *no incentive* to take their *market power* into consideration. When they are paid purely based on forecasting accuracy, predicting the REE (competitive equilibrium) will be the only Nash Equilibrium of this prediction game, where the forecasting error is minimized, and the payoff is maximized for every subject in the same market. If subjects were instead paid according to the profit their firm earned, they might have an incentive to play the Cournot-Nash equilibrium, or the collusive equilibrium, which are different from the competitive market setting used in both the adaptive and eductive learning literatures.

An important issue is how to allow for eductive learning. This is an off–line, notional time concept so it is not so clear how to capture or measure this kind of learning in real time. Here we focus on the stability differences as pointed out by Evans (2001) as our main test of whether agents are eductive or adaptive learners. Still, an important issue is whether subjects understand the model and have sufficient time for introspection. Under adaptive learning, agents are not assumed to know the model while under eductive learning they do know the model. What we have chosen to do is to fully inform subjects about the model, in particular about the price determination equation, (1) – see the written experimental instructions in the Appendix for the details on how this information was presented to subjects. Thus the agents in our model have more information than is typically assumed under adaptive learning specifications, but at the same time, they have all the information they need to be eductive learners. We felt that, in order to put the two learning approaches on an equal footing for comparison purposes we would have to eliminate any informational differences between the two learning approaches, which could serve as a confounding factor in our analysis. Thus we provide subjects with complete and common information about the model across all of our six treatments. Further, we did not impose any time limits on subjects' decision-making so as not to limit the type of introspective reasoning associated with the eductive approach. Indeed, we captured subjects' decision time as a variable in order to better understand whether there were any differences in decision time across treatments $T1 - T3$, or between individuals and groups in our monopoly and oligopoly treatments.

Based on the theoretical analysis of the last section, we formulate the following testable hypotheses. The underlying priory is that agents use adaptive learning, and the results favor edutive learning if the hypotheses are rejected.

Hypothesis 1. *The market price and price expectations in all treatments converge to the unique rational expectation equilibrium given in (2).*

As in section 2, both adaptive and eductive learning theories predict that market price forecasts and market prices will converge to the REE in treatments $T1$ and $T2$. In treatments $T3$ and $T4$, the REE is learnable under adaptive learning. It is learnable under eductive learning in the monopoly design but not in the oligopoly design for the competitive version of eductive learning. If Hypothesis 1 is rejected, and forecasts and prices do not converge to the REE in treatment $T3$, the experimental results favor eductive learning over adaptive learning.

Hypothesis 2. *Given that the market price and price expectations converge to the REE, convergence never takes place in the first period of the experiment.*

Since convergence under adaptive learning takes place more gradually and in real time while eductive learning happens in notional time, the convergence should take place in the first real period that is incentivized for monetary payment if agents are eductive learners, or after a few periods if agents use adaptive learning. If Hypothesis 2 is rejected, the experimental results favor eductive learning over adaptive learning.

Hypothesis 3. *Agents spend no more time in making their decisions in each period of treatment $T3$ and/or $T4$ as compared with each period of treatments $T1$ or $T2$ (in the oligopoly design).*

Since eductive learning can involve considerable introspective reasoning in notional time, which we take to be the period prior to the first incentivized market forecasting period, it may require more time for agents to reach a decision. In particular, the REE is predicted to be more difficult to learn under eductive learning in treatments $T3$ and/or $T4$ as compared with treatments $T1$ and $T2$ in the oligopoly design. Since decision time is a typical measure of the cognitive cost to agents of making decisions, if Hypothesis 3 is rejected, it suggests that making a decision in treatment $T3$ and/or $T4$ is indeed more difficult than in treatments $T1$ or $T2$, and the results favor eductive learning over adpative learning.

4.2 Number of Observations

The experimental data was collected in a number of sessions run at the CREED Lab of the University of Amsterdam. Subjects had no prior experience with our experimental design and were not allowed to participate in more than a single session of our experiment. Each session consisted of 50 periods over which the treatment parameters for that session were held constant (i.e., we used a "between subjects" experimental design). Table 1 provides a summary of the number of subjects or markets (independent observations) for each of our six treatments. Note that in the monopoly treatment, each subject acted alone in a single market, so the number of subjects equals the number of independent observations (markets) in that setting. By contrast, in the oligopoly treatment, each market consisted of three firms (subjects), so while we have more subjects in our oligopoly treatments, we nevertheless have fewer 3-firm markets (independent observations) for the oligopoly treatments. Each session averaged about 1 hour and 10 minutes in duration. The average payoff was 20.3 euros across all three monopoly treatments and 19.7 euros across all three oligopoly treatments.

Treatment Conditions	Monopoly	Oligopoly	Total No. Subjects
	No. Markets /Subjects	No. Markets / Subjects	
T1	14 / 14	10 / 30	44
T2	12 / 12	10 / 30	42
T3	13 / 13	11 / 33	46
T4	14 / 14	10 / 30	44
Totals	53 / 53	41 / 123	176

Table 1: Number of Markets (Independent Observations) and Subjects in the eight Treatments of the Experiment

4.3 Computer Screen

Figure 2 shows the computer screen we developed for the experiment in the treatment where $\alpha = -0.5$. Subjects were asked to enter a forecast number in the box and then to click "send" to submit their forecast in each period. Since the price and price expectation were restricted to be non-negative, the range of possible prices should be $[0, 60]$ according to equation (1).[13] However, restricting the price forecast range to $[0, 60]$ would be equivalent to directly imposing the first step in the eductive learning process. Therefore, we restricted the price forecast range to $[0, 100]$ in the experiment, which is less suggestive. The subjects are told in the instructions that neither the price nor their price predictions can be negative. The upper forecast bound of 100 was not indicated in the written instructions, but subjects would see a pop-up window indicating that a forecast larger than 100 is not allowed if they attempted to submit a price forecast that was greater than 100. Notice that the computer decision screen presented subjects with information and graphs of past prices, their own prior forecasts as well as realizations of shocks. The screen was refreshed with updated information once all subjects had submitted forecasts and the market price was determined. Notice further that at the top of the decision screen, the price determination equation (1) with the treatment specific value of α was always available for subjects to view, just above the input box where they were asked to submit their price prediction in each period.

[13]If $p_t^e > 0$ and given that $\alpha < 0$ it follows that $p_t = 60 + \alpha p_t^e < 60$.

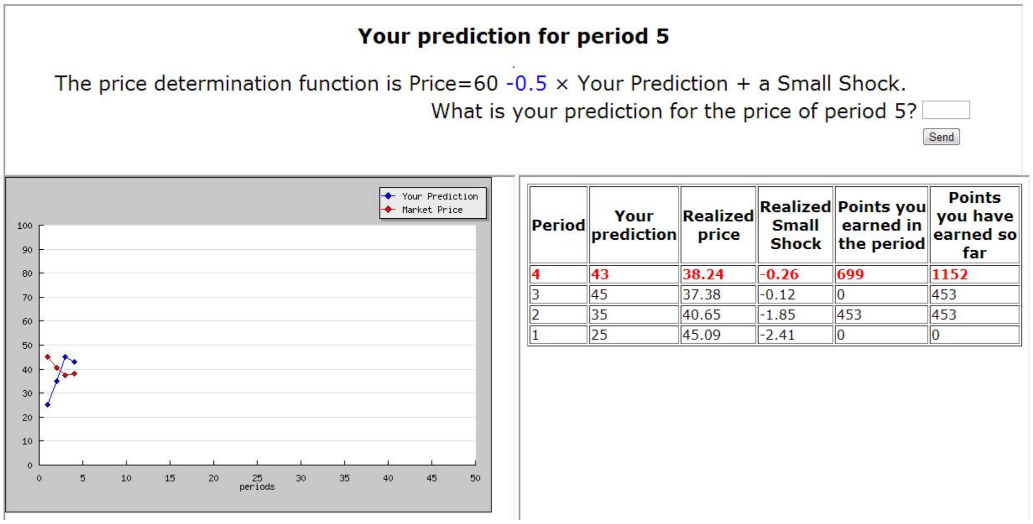


Figure 2: The computer decision screen used in the experiment for the treatment where $\alpha = -0.5$ and the subject is a monopolist in the local market. Note: the price and price expectations shown in this figure are random inputs by the authors for illustration purposes and are not taken from any experimental data.

4.4 Payoff Function

Subjects earned points during the experiment that were converted into euros at the end of the experiment according to a known and fixed rate. The period payoff function for subjects (in points) is a decreasing quadratic function of their market price prediction error, and was given by:

$$\text{Payoff for Forecasting Task for Subject (Firm) } h = \max\left\{1300 - \frac{1300}{49}(p_t - p^e_{h,t})^2, 0\right\}. \quad (5)$$

Notice that subjects earn 0 points if their own, individual price forecast error is greater than 7, and they earn a maximum of 1300 points for a perfect forecast. Subjects' point totals from all 50 periods were converted into to euros at the end of each session at a known and fixed rate of 1 euro for every 2600 points. Thus, over 50 periods, each subject's maximum earnings were (1300×50) / 2600 = 25 euros.

5 Experimental Results

5.1 Market Dynamics

5.1.1 Monopoly Markets

Figure 3 plots the *average* price expectation (forecast) against the respective REE price expectation using data from *all* markets (individual observations) of each of the four monopoly treatments. According the theoretical analysis in Section 3, convergence in terms of the market price or in terms of price expectations are equivalent. We choose to plot the *price expectations* instead of the *market prices* to rule out the influence of the noise term ν_t.[14] We observe that the price expectation in all three treatments appears to converge to the REE, although at different speeds (we will quantify this speed of convergence later in section 5.2). The adjustment towards REE is observed to be fastest in $T1$ and slowest in $T4$.

Figure 4 plots the disaggregated price expectation paths for each *individual* market for each of the three monopoly treatments against the respective REE. As this figure reveals, it may take up to 25 periods for some markets to converge, e.g., in treatment $T3$, and there are a lot of extreme outcomes, e.g., price forecasts such as 0 and 60. From these results we preliminarily conclude that adaptive learning is correct in predicting the convergence outcome across all three treatments including treatment $T3$, however the time path of convergence for some markets often resembles a real-time demonstration of the eductive, introspective learning process, in particular, the dampened cycling of price expectations over time in some markets. Further, if we look at self-reported strategies from a questionnaire solicited from subjects following the end of the experiment (as we do later in section 5.6), it seems that several subjects directly solved for the REE using $p^{e,*} = \frac{\mu}{1-\alpha}$, which indicates that those subjects were applying eductive reasoning.

Table 2 reports the mean and variance of price expectations across all markets in each of the four monopoly treatments for the entire sample of 50 periods as well as for the first 25 and last 25 periods of the sample. Confirming the impression given

[14] The results for market prices are thus very similar to the results for market price forecasts, but due to the noise term, market prices have a higher variance.

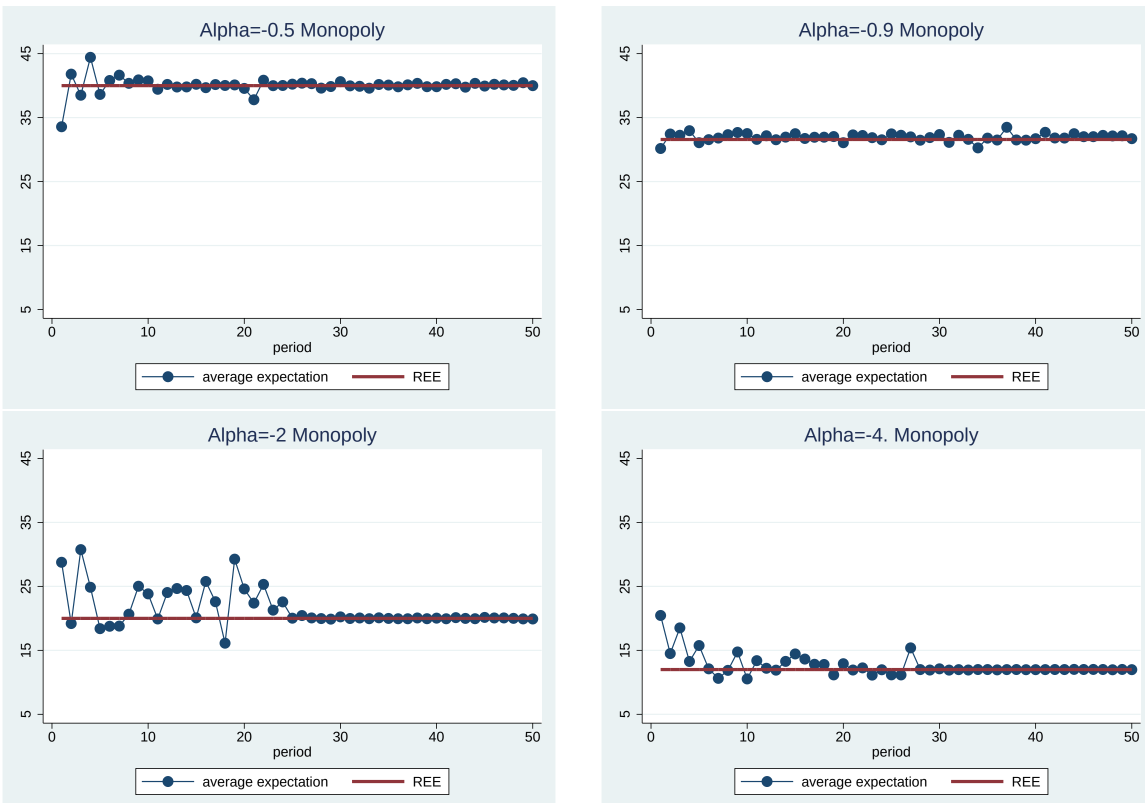


Figure 3: The average price expectation against the REE in each of the three treatments in the monopoly design.

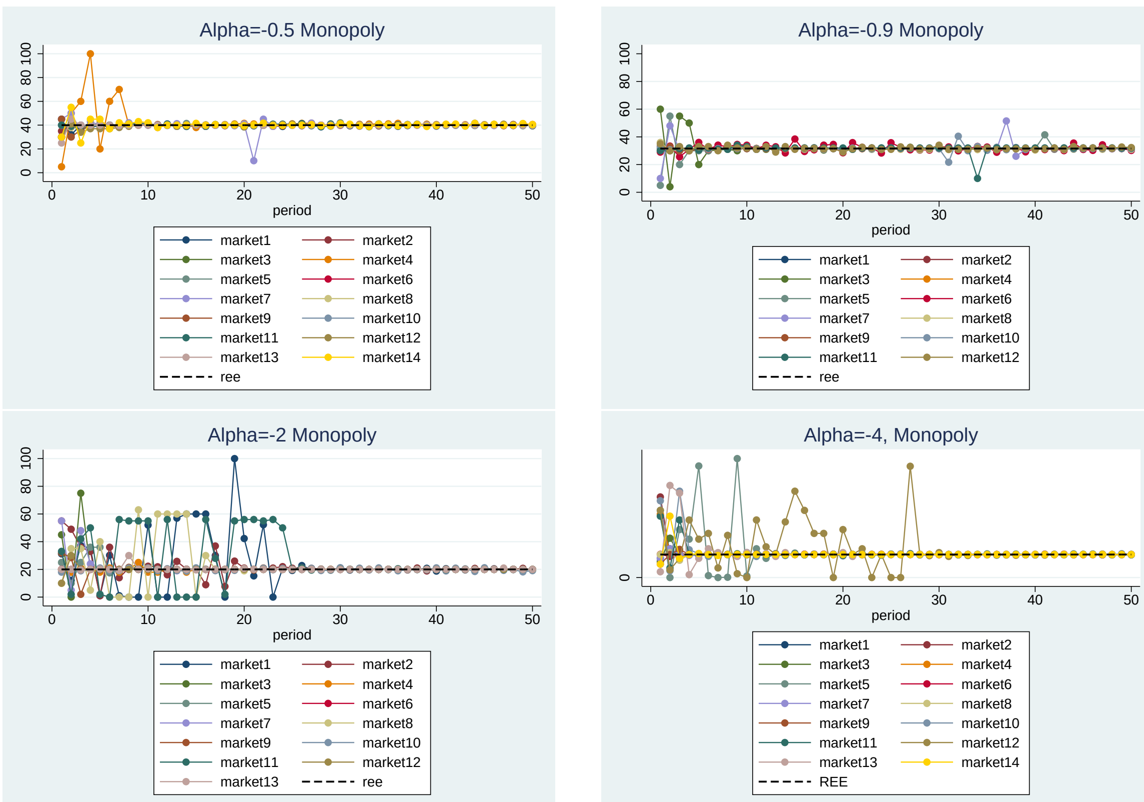


Figure 4: Disaggregated price expectations against the REE when $\alpha = -0.5, -0.9$, -2 and -4(from top to bottom) in the monopoly design.

in Figures 3-4, we observe that, on average, price forecasts converged to the REE for each treatment and that the variance in these forecasts over all 50 periods was greatest in treatment $T3$.

		Periods 1-50		Periods 1-25		Periods 26-50	
Treatment	REE	Mean	Variance	Mean	Variance	Mean	Variance
$\alpha = -0.5$	$p^* = 40$	40.02	14.18	39.97	28.30	40.07	0.23
$\alpha = -0.9$	$p^* = 31.58$	31.62	10.79	31.65	17.51	31.60	4.51
$\alpha = -2.0$	$p^* = 20$	21.46	90.76	22.88	172.70	20.03	0.13
$\alpha = -4.0$	$p^* = 12$	12.63	28.40	13.17	49.82	12.09	6.48

Table 2: Mean and variance of price expectations in each treatment ($\alpha = -0.5, -0.9, -2, -4$) of the monopoly setting.

5.1.2 Oligopoly Markets

Figure 5 plots the average price expectations (forecasts) against the respective REE price expectation using data from *all* markets of each of the four oligopoly treatments. We see that the average price expectation in all three treatments converges to the REE, although, again, at different speeds. The adjustment towards the REE is again observed to be fastest in $T1$ and slowest in $T4$.

Figure 6 plots the disaggregated average price expectations for each of the three-firm markets (independent observations) against the respective REE price for all three oligopoly treatments. Compared with the monopoly treatment, the convergence to REE in the oligopoly setting appears to be faster and more reliable in the eductively stable treatments, namely markets with $\alpha = -0.5$ and $\alpha = -0.9$. By contrast, in the eductively unstable oligopoly market treatments $T3$ and $T4$ (where $\alpha = -2$ and $\alpha = -4$), the volatility of price expectations appears to be greater and more persistent as compared with the monopoly $T3$ treatment. Indeed, one oligopoly market (Market 4) in the $T3$ treatment fails to converge to the REE within the 50 periods allowed according to our convergence criterion (more on our convergence criterion below). This finding suggests that the oligopoly market setting may facilitate learning when the REE is eductively stable as this environment aggregates the already near-rational expectations of other agents and may thus speed up the achievement of common knowledge of rationality. However, when the REE is not eductively stable, faced with the greater uncertainty that other agents may not be able to learn the REE, common knowledge of rationality is much harder to achieve, and therefore the oligopoly market setting takes longer to converge to the REE.

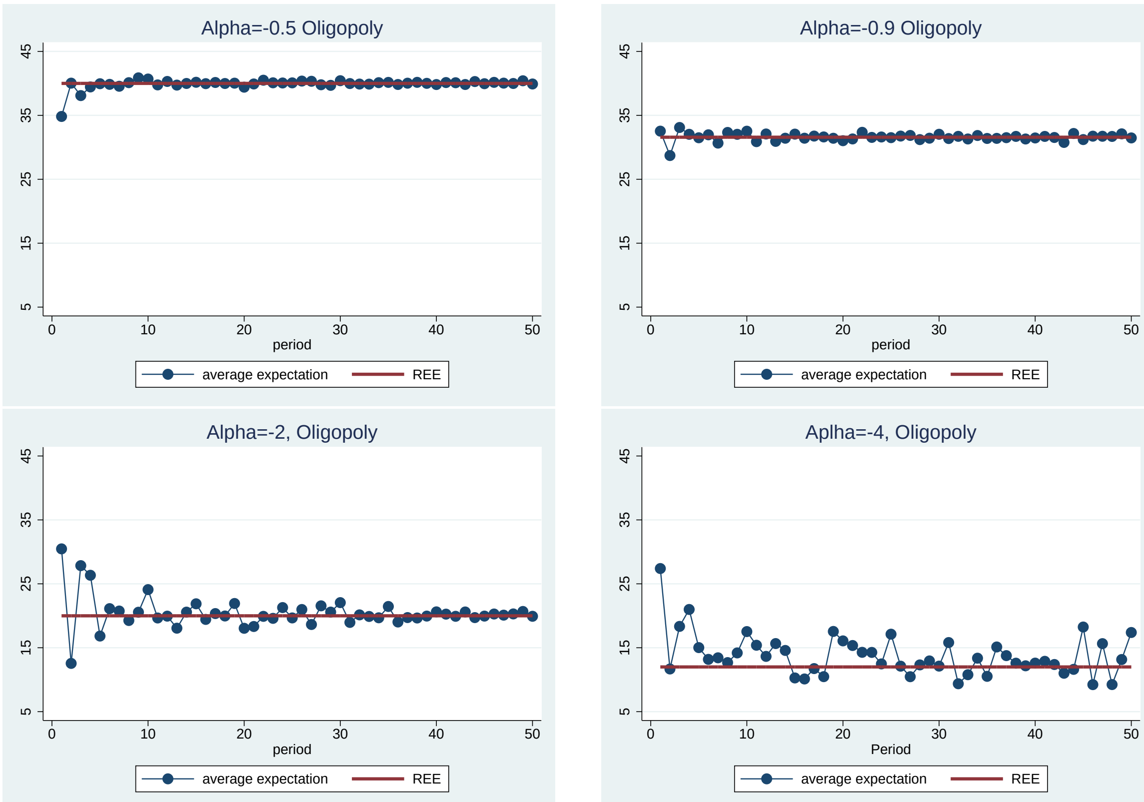


Figure 5: The average oligopoly market price expectation against the REE price when $\alpha = -0.5, -0.9, -2$ and -4 (from top to bottom) in the oligopoly design.

Table 3 reports the mean and variance of price expectations across all markets of each of the three oligopoly treatments for the entire 50 period sample and for the first 25 and last 25 periods of the sample. Consistent with Figures 5-6, we observe that, on average, price expectations converged to the REE prediction for each treatment and that the variance in price expectations in treatment 1 is the lowest at 1.14 over all 50 periods while the variance in price expectations in treatment 4 is the greatest at 77.91 over all 50 periods.

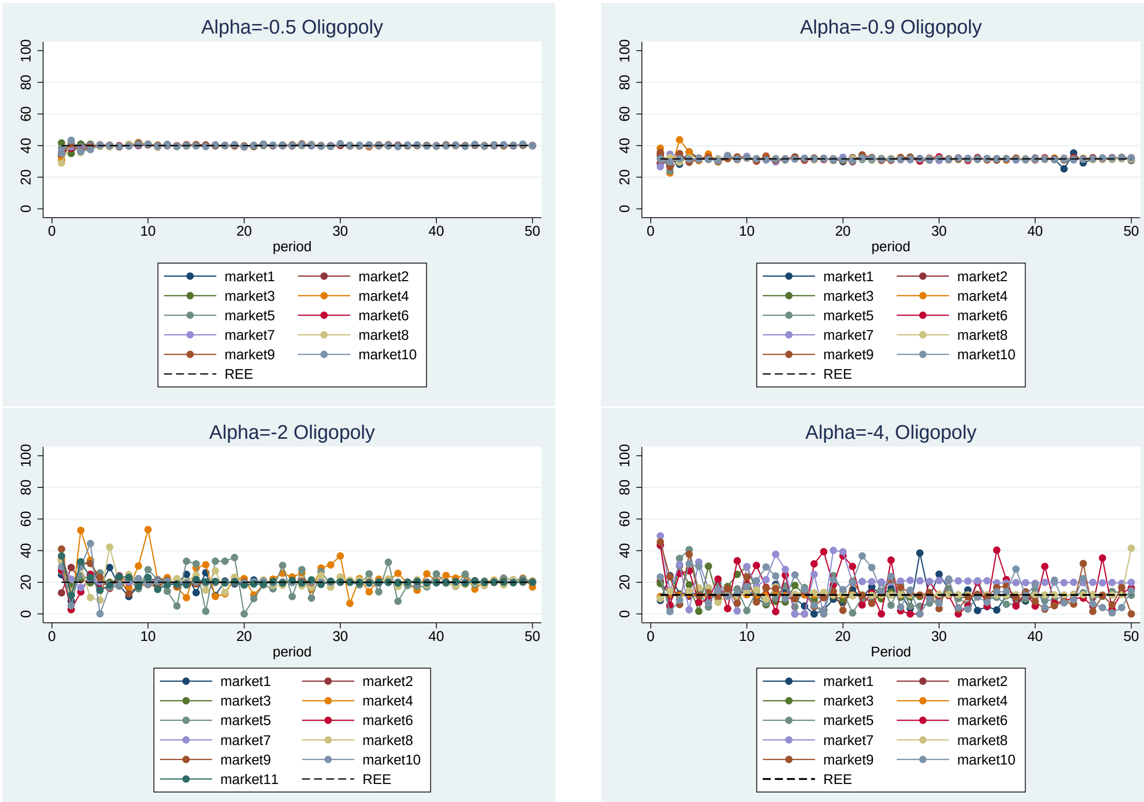


Figure 6: Disaggregated oligopoly market average price expectations against the REE price when $\alpha = -0.5, -0.9, -2$ and -4 (from top to bottom) in the oligopoly design.

		Periods 1-50		Periods 1-25		Periods 26-50	
Treatment	REE	Mean	Variance	Mean	Variance	Mean	Variance
$\alpha = -0.5$	$p^* = 40$	39.90	1.14	39.74	2.18	40.05	0.08
$\alpha = -0.9$	$p^* = 31.58$	31.60	1.61	31.61	2.82	31.58	0.45
$\alpha = -2.0$	$p^* = 20$	20.46	25.01	20.75	44.37	20.19	6.31
$\alpha = -4.0$	$p^* = 12$	13.98	77.91	15.29	97.66	12.67	55.00

Table 3: Mean price and variance of price expectations in each treatment ($\alpha = -0.5, -0.9, -2, -4$) of the oligopoly setting.

5.2 Convergence to REE

We shall declare that convergence to the REE occurs in the first period for which the absolute difference between the (average) price expectation and the REE price prediction is less than 3 and stays below 3 forever after that period. We choose a threshold of 3 for two reasons: (1) ideally, one would like to declare convergence only if the price expectation was exactly equal to the REE, but if such a criteria were used, almost no market would satisfy that criterion so that it would not be possible to make any distinctions among our treatments; (2) the threshold should not be so large that it allows for substantial deviations from the REE. We choose the two sided range $[-3, +3]$ because it is 10% of the rationalizable price range, $[0, 60]$, and one-sided deviations from REE larger than 3 (5%) of this range may be regarded as substantial. We further categorize the markets according to whether convergence happens in the first period, between periods 2 and 5, between periods 6 and 10, between periods 11 and 20, and between period 21 and 50. The results from applying our convergence criterion to each market of each treatment are reported in Table 4. In the final rows of the same table we also report the mean number of periods required for convergence (according to our criterion) in each treatment as well as the variance.

On average, it takes fewer periods for price forecasts to converge to the REE in treatment $T1$ as compared with treatments $T2$ and $T3$ in both the monopoly and oligopoly settings. There is less of a difference in the mean time to convergence between the $T2$ and $T3$ treatments of both the monopoly and oligopoly settings, though convergence is slightly faster on average in $T2$ than in $T3$ for both market settings. Surprisingly, the number of periods before convergence in $T4$ is larger than in $T1$ but smaller than in $T2$ and $T3$ in the monopoly design, while it is much larger than $T1 - T3$ in the other three treatments. A Wilcoxon Mann-Whitney test on market-level data suggests that the differences in the mean time to convergence between treatments $T1$ and each of $T2$ and $T3$, is significant at the 5% level for both the monopoly and oligopoly markets, while the differences in the mean time to convergence between treatments $T2$ and $T3$ is significant at the 5% level for the monopoly markets, but not significant for the oligopoly markets. The mean time to convergence in $T4$ is not significantly different at 5% level from all other treatments in the monopoly design, and significantly different from all other treatments in the oligopoly design. The mean time to convergence is smaller in the oligopoly deisng than in the monopoly design for the eductively stable treatments ($T1$ and $T2$, the difference is significant at 5% for $T1$ but not for $T2$), while it is larger in the oligopoly design than in the monopoly design in the eductively unstable treatments ($T3$ and $T4$, the differences are significant at 5% level for both treatments).

For both the monopoly and oligopoly markets, the variance in the number of periods before convergence is smallest in treatment $T1$. In the monopoly market treatment, the variance in the number of periods required for convergence is larger in treatment $T2$ than in treatment $T3$, but this is due to the random behavior of just a few subjects in T2 who inexplicably began experimenting with high/low price predictions after they had converged to the REE for more than 10 periods. For the oligopoly treatment, the variance in the number of periods required for convergence is also smallest in treatment $T1$ and is higher and similar in treatments $T2$ and $T3$.[15] These results generally support the notion that convergence is more difficult as the absolute value of the coefficient α becomes larger, as larger values of α make the market more unstable.

Table 4 also reveals that for treatments $T1$ and $T2$ of both the monopoly and oligopoly settings, there is at least 1 market (and often more) that converges to the REE beginning with the very first period. The fact that a market converges to the REE in the very first period may be regarded as support for the eductive learning approach. If this eductive learning criteria is relaxed to allow for convergence within the first 5 periods then about 70% of the markets in treatments $T1$ and $T2$ and about 60% in treatment $T3$ of our experiment can be said to be consistent with eductive learning. In treatment $T3$ of the oligopoly treatment, there are *no* instances of convergence to the REE in the very first period of a session and one market in this treatment failed to satisfy our convergence criterion within the entire 50 periods allowed by our experiment. These differences in outcomes between the eductively stable treatments $T1$ and $T2$ and the eductively unstable treatment $T3$ suggest that the eductive stability criterion is useful in understanding differences in the behavior of subjects in our experiment.

Figure 7 shows the fraction of markets that have converged by each period. For the oligopoly markets, it is clear that treatment $T3$ takes the greater number of

[15]We perform a Siegel-Tukey test and the result shows that none of the differences across treatments is significant at 10% level.

Convergence in period(s)	monopoly $\alpha = -0.5$	monopoly $\alpha = -0.9$	monopoly $\alpha = -2$	monopoly $\alpha = -4$	oligopoly $\alpha = -0.5$	oligopoly $\alpha = -0.9$	oligopoly $\alpha = -2$	oligopoly $\alpha = -4$
1	21.4% (3)	41.7% (5)	30.8% (4)	7.1% (1)	20.0% (2)	30.0%(3)	0.0% (0)	0.0% (0)
$[2, 5]$	57.1% (8)	25.0% (3)	30.8% (4)	78.6% (11)	80.0% (8)	60.0%(6)	27.3% (3)	0.0% (0)
$[6, 10]$	14.3% (2)	16.7% (2)	7.7% (1)	7.1% (1)	0.0% (0)	0.0%(0)	18.2% (2)	0.0% (0)
$[11, 20]$	0.0% (0)	0.0% (0)	0.0% (0)	7.1% (1)	0.0% (0)	0.0%(0)	18.2% (2)	0.0% (0)
$[21, 50]$	7.1% (1)	16.7% (2)	30.8% (4)	0.0% (0)	0.0% (0)	10.0%(1)	36.4% (4)	100.0% (10)
Average	4.4	8.2	8.8	5.5	2.1	7.1	17.3	40.3
Variance	29.5	179.8	87.0	47.3	1.1	188.8	289.2	171.6
# Obs	14	12	13	14	10	10	11	10

Table 4: Frequency distribution of the number of periods it takes for convergence to REE in each treatment. Convergence period ranges are given in the left-most column as bins [ ]. The number of observations per bin are indicated in ( ).

periods while treatment $T1$ requires the fewest periods. For the monopoly markets, although the CDF of treatment $T3$ starts out below the other two treatments, it crosses treatment $T2$ due to the very high/low numbers submitted by those few subjects in treatment $T2$ after prices had already converged to the REE for some time.

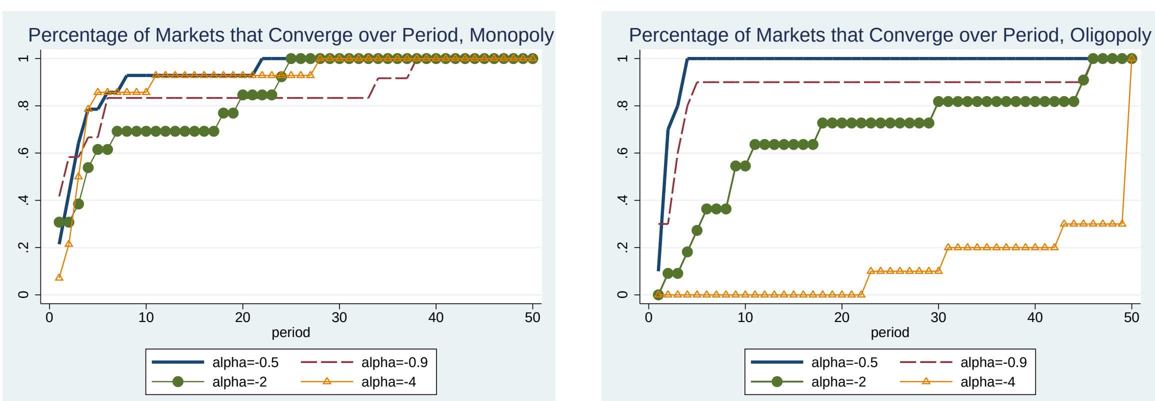


Figure 7: The fraction of markets that have converged by each period under the different treatments. The vertical axis measures the percentage of markets that converge while the horizontal axis reports the period number.

We summarize the findings in the above sections as Results 1-2 addressing Hypotheses 1-2:

Result 1. *We do not reject Hypothesis 1. Convergence to the REE obtains in all three treatments, though it may take many periods in treatment $T3$ where $\alpha = -2$.*

This finding suggests, as Evans (2001) observes, that the learning process may be a mixture of adaptive and eductive approaches when the REE is not learnable under eductive learning.

Result 2. *We reject Hypothesis 2, when the REE is learnable under eductive learning, convergence can occur immediately but it most frequently requires more than a single period. This finding again suggests that the learning path across all agents may be a mixture of real–time adaptive and notional–time eductive learning.*

5.3 Fit of the Two Learning Approaches to the Experimental Data

We next consider the fit of the two different approaches to learning to our experimental data. Table 5 reports on the mean squared error between the experimental and the simulated price expectations according to the two different learning models in all six treatments.

For the adaptive learning model, we assume that the model's predictions coincide with the actual (average) price prediction in the experimental data. To initialize a simulation of the adaptive learning model we set the initial price prediction p_1^e equal to the individual (monopoly) or average (oligopoly) forecasts made by subjects in period 1 (and period 1 only). Thereafter, the adaptive learning model specifies how all subsequent simulated prices and price predictions are determined. That is, given p_1^e, the price for period 1, p_1, is determined by equation (1). Given p_1 the adaptive learning model predicts the price for period 2 according to equation (3), and thus generates a simulated actual price for period 2 again via equation (1). In period 3, the adaptive model take the average of the simulated prices for periods 1 and 2 and makes a price prediction for period 3, which is then used to generate the simulated price for period 3 via equation (1), and so on. Thus, to sum up, the adaptive learning model uses its own simulated prices as input to generate simulated market price predictions in each period. Importantly, this simulation only loads experimental data from period 1, and makes simulated prices and predictions for the remaining 49 periods, so there are no degrees of freedom in the predictions of the adaptive learning model for each market observation.

For the eductive learning model, we assume that for each period, the simulated price prediction is $p^e_{edc,t} = \frac{\mu}{1-\alpha}$ and thus the actual market price equals the REE price, $\frac{\mu}{1-\alpha} + \nu_t$ in the treatments where eductive learning predicts that the REE is learnable, namely, all treatments of the monopoly setting, and treatments $T1$ and $T2$ of the oligopoly setting. Note that for treatment $T3$ in the monopoly setting, the eductive learning model predicts that the market price equals the REE price because eductive learning assumes that individuals are perfectly capable of solving for the REE. Since in the monopoly case, there are no other firms (agents) forming expectations so there is no strategic uncertainty regarding the decisions of other participants, it follows that the market price should equal the REE price even in the $T3$ treatment of the monopoly setting treatment. By contrast, in treatment $T3$ under the oligopoly setting, the eductive learning model does not exclude any combination of price predictions and market prices. Therefore, the MSE from eductive learning for the $T3$ version of the oligopoly setting is undefined, or 0 if we consider that the model predicts that "anything can happen." For these reasons we did not construct MSEs between the data and the eductive learning model prediction for treatment $T3$ of the oligopoly setting. However, for the other five treatments, we can calculate the MSE between the data and the eductive learning model predictions. We note that, as was the case for the adaptive learning model predictions, there are again no degrees of freedom in the predictions of the eductive learning model.

The mean squared errors (MSE) between the simulated and experimental data as presented in Table 5, suggest that the fit of the adaptive learning model to the experimental data generally results in a smaller MSE than does the eductive learning model; the mean MSE for the adaptive learning model is lower than for eductive learning model in all the treatments where the MSE can be calculated. A Wilcoxon signed rank test suggests that the difference between the MSEs for the adaptive and eductive learning models is significant at the 5% level for $\alpha = -0.5$ for both monopoly and oligopoly settings, for $\alpha = -0.9$ for the oligopoly setting (in all these cases the adaptive learning model generates smaller MSE on average) and not significant in the other treatments. However, there is also some heterogeneity across the different markets/observations. For example, for the oligopoly market with $\alpha = -0.9$, the adaptive learning model generates a higher MSE relative to the eductive learning model in markets 1, 2 and 5, but a lower MSE relative to the eductive learning model in markets 3, 6 and 7. This finding suggests that it is very likely that some oligopoly markets are dominated by subjects using adaptive learning, while others are dominated by subjects using eductive learning. We will provide evidence for such heterogeneity of types later in section 5.6.

5.4 Payoff Efficiency

Table 5.4 reports average payoffs and payoff efficiency (payoffs divided by 25 euros, which was the maximum amount each subject could earn when they made no forecasting errors) for each treatment. Payoff efficiency is about 90% when $\alpha = -0.5$, $\alpha = -0.9$, and a little lower, between 50%-80% when $\alpha = -2$ and $\alpha = -4$. Efficiency is higher in the oligopoly treatment than in the monopoly treatment with $T1$, and lower in the oligopoly treatment than in the monopoly treatment with $T3$ and $T4$. We performed a Wilcoxon Mann-Whitney Test using individual earnings data in the monopoly design and average earnings in each market in the oligopoly design. The results indicate that for the monopoly treatment, there is no difference in payoff efficiency between the $T1, T2$ and $T4$ treatments at the 5% level, but that payoff efficiency in the these treatments is significantly greater than payoff efficiency in the $T3$ treatment at the 5% level. In the oligopoly treatment, the difference the difference between each pair of treatments is significant at the 5% level. The low payoff efficiency in $T3$ and $T4$ in the oligopoly design suggests the task is more cognitively demanding when the REE is unstable under eductive learning.

5.5 Decision Time

We collected data on the time it took subjects to make their decisions. Specifically, in each period we measured the time, in seconds, from the start of each new period to the time at which each subject clicked "send" to submit their price forecast for that same period. Such data can be useful in understanding possible variation in the cognitive difficulty of decision-making tasks. In particular, Rubinstein (2007) provides evidence that choices requiring greater cognitive activity are positively correlated with longer decision response time. In our experiment, subjects face a more difficult task in $T3$ and $T4$ as compared with either $T1$ and $T2$ and so they may be expected to take more time to make their decisions in treatment $T3$ and $T4$ than in treatments $T1$ and $T2$. According to a referee's suggestion, we compare the average decision time in the first period only, because given the program only goes to the next period when

Learning		Monopoly				Oligopoly			
Model	Market	$\alpha = -0.5$	$\alpha = -0.9$	$\alpha = -2$	$\alpha = -4$	$\alpha = -0.5$	$\alpha = -0.9$	$\alpha = -2$	$\alpha = -4$
Adaptive	1	0.94	0.15	377.75	0.40	0.10	2.21	11.88	18.14
	2	0.33	0.38	78.71	9.60	0.18	0.50	1.26	7.15
	3	0.17	23.46	42.25	0.31	0.35	0.30	6.98	3.71
	4	109.73	0.06	0.93	9.94	0.26	4.63	69.94	182.94
	5	0.97	5.15	10.57	98.77	0.12	0.26	62.46	37.64
	6	0.16	5.40	0.13	0.45	0.27	0.73	5.40	147.64
	7	23.76	8.73	8.03	0.49	0.63	0.57	12.87	129.85
	8	0.08	0.70	223.79	0.98	0.57	0.33	12.87	20.32
	9	0.79	0.17	31.01	6.27	0.26	0.97	25.50	65.50
	10	0.34	3.94	0.67	6.00	0.63	0.28	10.49	88.17
	11	0.49	9.67	371.13	1.72			21.88	
	12	1.49	1.43	1.19	127.85				
	13	0.03		2.19	13.24				
	14	9.46			0.66				
	average	10.62	4.94	88.34	19.76	0.34	1.08	21.96	70.11
Edutive	1	2.52	0.00	384.14	0.27	0.89	2.41	-	-
	2	0.91	0.09	76.26	18.01	1.04	0.63	-	-
	3	2.17	52.14	81.08	1.72	0.68	0.42	-	-
	4	141.14	0.06	1.02	8.82	2.02	6.34	-	-
	5	3.29	30.14	13.41	121.62	0.19	1.62	-	-
	6	2.02	5.75	0.00	0.04	0.67	1.30	-	-
	7	22.05	23.43	45.04	0.28	0.46	1.14	-	-
	8	0.19	0.69	230.52	0.21	3.12	0.19	-	-
	9	3.02	0.21	10.81	0.64	0.90	1.60	-	-
	10	0.15	3.79	0.71	39.17	1.34	0.33	-	-
	11	0.49	9.76	387.83	15.49			-	
	12	3.62	1.33	4.00	137.22				
	13	5.01		2.05	50.63				
	14	13.01			8.59				
	average	14.26	10.62	95.14	28.77	1.13	1.60	-	-

Table 5: MSE between the experimental data and the two learning model predictions. The last column for eductive model is filled with “-”s because the model does not make a clear prediction when the REE is unstable under eductive learning when the subjects face strategic uncertainty from the other subjects in the same market.

Market Structure	α	Payoff	Efficiency
Monopoly	-0.5	22.9	91.6%
	-0.9	22.7	90.8%
	-2	20.1	80.4%
	-4	21.8	87.0%
Oligopoly	-0.5	23.7	94.6%
	-0.9	22.6	90.6%
	-2	16.7	65.6%
	-4	12.6	50.2%

Table 6: Payoffs and payoff efficiency across the six treatments.

every subject finishes this period, the decision time in later periods is influenced by the subjects' experience in earlier periods (e.g. if they finish early, they have to wait for the subjects who take longer time), and are thus not independent.

Figure 8 shows the empirical cumulative distribution function of decision time in the first period for treatments $T1-T4$. We find that for the monopoly treatment, the average decision time is 53.1 seconds in $T1$, 49.6 seconds in $T2$, 103.8 seconds in $T3$ and 148.5 seconds in $T4$. The difference between each of $T1$ and each of $T3-T4$ and $T2$ and $T4$ is significant at the 5% level according to the Wilcoxon Mann-Whitney test, while other differences across treatments are not significant.

In the oligopoly treatment, the results are more in line with our expectations. We find that for the monopoly treatment, the average decision time is 55.2 seconds in $T1$, 62.3 seconds in $T2$, 98.1 seconds in $T3$ and 89.2 seconds in $T4$. The difference between each of $T-T2$ and each of $T3-T4$ is significant at the 5% level according to the Wilcoxon Mann-Whitney test, while the difference between $T1$ and $T2$ or $T3$ and $T4$ is not significant. This finding supports the notion that subjects face a more difficult task in $T3$ and $T4$, and therefore require more time to make a decision.

The findings in this section are summarized by Result 3 which addresses Hypothesis 3:

Result 3. *We reject Hypothesis 3 for the oligopoly treatment, but not for the monopoly treatment. The cognitive cost of making decisions in treatment $T3$ is not significantly larger than in the other two treatments in the monopoly setting where subjects do*

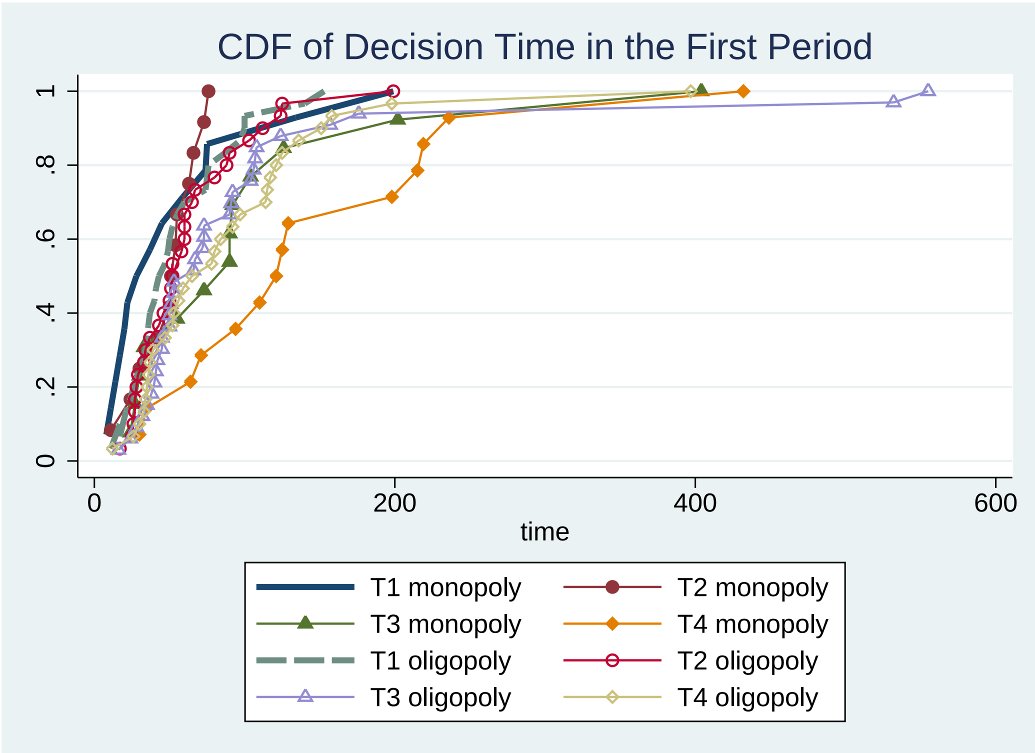


Figure 8: The empirical cdf of the time taken to complete decision tasks in treatments $T1-T4$ of the monopoly (solid line or markers) and oligopoly (long dash line or hollow markers) settings. The unit of time is seconds, as measured on the horizontal axis. The vertical axis measures the cumulative frequency.

not have to consider problems of coordination and common knowledge of rationality. By contrast, when common knowledge of rationality is an issue as in our oligopoly setting, decision time is significantly greater in treatment $T3$ relative to the other two treatments.

5.6 Categorization of Subjects into Adaptive or Eductive Learners

Finally, we try to categorize each subject in our experiment into one of three types: adaptive learner, eductive learner or neither. We do this using two different approaches and we compare the results from using each approach.

The first approach is to make categorizations based on the definition of the two types of learning. This categorization is performed as follows:

1. We consider all subjects who predict the REE in the very first period to be eductive learners. Since the REE in treatment $T2$ where $\alpha = -0.9$ is 31.58, and not an integer, taking into account that some subjects may use $\alpha = -1$ as a proxy, we consider all subjects making predictions in the range $[30, 32]$ to be eductive learners in $T2$. For the other two treatments, the REE is an integer value so to be categorized as an eductive learner, subjects must correctly predict a price of 40 in $T1$ and a price of 20 in $T3$.

2. For each subject we use their first period price forecast to initialize the adaptive learning model as given in equation (3) and we then calculate the mean squared error between the simulated predictions of that model and each individual subject's actual price predictions. If the mean squared error between actual and predicted price forecasts is smaller than 1, then the subject is classified as an adaptive learner. We choose a threshold of 1 as we wanted the threshold to be as low as possible, but at the same time to allow for subjects to engage in some rounding of numbers to integer values. Since adaptive learning does not make assumptions on the initial price prediction, the probability that one happens to come up with the REE is infinitely close to 0 under adaptive learning. If a subject meets our criteria for being categorized as both an adaptive and an eductive learner, then we classify him/her as an eductive learner. If a subject

meets neither criteria, then he/she is classified as "neither".

The second approach to type classification makes use of answers that subjects gave to a post–experimental questionnaire (see the Appendix for details). The questionnaire asked subjects a number of restricted-form questions about the type of prediction strategy they used during the experiment. We provided them with four options and we asked them to choose the option that best described how they made their predictions in the experiment. Specifically, the four options were:

1. I refer to information about past prices.
2. I make calculations based on the value of α.
3. I eliminate unlikely numbers iteratively.
4. None of the above.

A subject is classified as an adaptive learner if he chooses option 1, and is classified as an eductive learner if he chooses option 3. If the subject chooses option 2, it is likely that he solves the REE directly, and we also classify this type as a an eductive learner, because as discussed in section 3.2, the eductive learning model typically starts with the assumption that agents solve the REE directly from equation (1)[16]. Subjects choosing option 4 are classified as "neither". Due to a technical problem, we lost some data on self-reported strategies in the first, and relatively larger session of our monopoly market treatments, (9 markets for treatment 1, 8 markets for each of treatments 2 and 3). Nevertheless, we do have data on self-reported strategies for many of our subjects and for all six treatments.

Table 5.6 shows the number of participants who can be categorized as adaptive or eductive learners in each treatment using Approach 1 or Approach 2, as well as the overall frequency of each type classification for each treatment. Tables 9 and 10 in the Appendix report more disaggregated information on each individual subject's type using both approaches (where possible). In general, it seems that more subjects can be categorized as adaptive and/or eductive learners when $\alpha = -0.5$ than when $\alpha = -0.9$

[16]Here we do not consider the case where subjects use a mixture of observation and calculation as in Evans and Ramey (1992), though that would be an interesting extension

or -2. There are more subjects who can be categorized as adaptive learners than as eductive learners (there are in total 30 adaptive learners and 22 eductive learners using approach 1, and 38 adaptive learners and 25 eductive learners using approach 2). In particular, there is a good level of consistency between the categorizations based on our two different approaches. For 31 subjects for which both approaches yield a classification of either adaptive or eductive learners, the two approaches agree on the type assignment in 21 cases, which means that the two approaches assign the same category with a probability of $21/31 = 67.7\%$.

Approach 1								
Treatment	$\alpha = -0.5$		$\alpha = -0.9$		$\alpha = -2$		All	
Monopoly								
Adaptive	8	57.14%	2	16.67%	2	15.38%	12	30.77%
Eductive	3	21.43%	5	41.67%	3	23.08%	11	28.21%
Neither	3	21.43%	5	41.67%	8	61.54%	16	41.03%
Total	14	100.00%	12	100.00%	13	100.00%	39	100.00%
Oligopoly								
Adaptive	12	66.67%	6	33.33%	0	0.00%	18	31.58%
Eductive	4	22.22%	3	16.67%	4	19.05%	11	19.30%
Neither	2	11.11%	9	50.00%	17	80.95%	28	49.12%
Total	18	100.00%	18	100.00%	21	100.00%	57	100.00%
Approach 2								
Treatment	$\alpha = -0.5$		$\alpha = -0.9$		$\alpha = -2$		All	
Monopoly								
Adaptive	1	7.14%	1	8.33%	2	15.38%	4	10.26%
Eductive	3	21.43%	2	16.67%	3	23.08%	8	20.51%
Neither	10	71.43%	9	75.00%	8	61.54%	27	69.23%
Total	14	100.00%	12	100.00%	13	100.00%	39	100.00%
Oligopoly								
Adaptive	11	61.11%	15	83.33%	8	38.10%	34	59.65%
Eductive	7	38.89%	2	11.11%	8	38.10%	17	29.82%
Neither	0	0.00%	1	5.56%	5	23.81%	6	10.53%
Total	18	100.00%	18	100.00%	21	100.00%	57	100.00%

Table 7: Number and percentage of subjects who can be categorized as adaptive or eductive learners or neither in each treatment. Approach 1 is the approach based on first period price predictions and the mean squared error of individual price predictions from the adaptive learning model. Approach 2 is the approach based on self-reported strategies.

6 Conclusion

The process by which agents might learn a REE has been the subject of a large amount of theorizing but surprisingly there has been little empirical assessment of the leading theories of this learning process. To address this gap, we have designed and implemented a learning–to–forecast experiment in the context of a simple cobweb economy with negative feedback where expectations matter and where subjects are informed about the law of motion for prices. We are particularly interested in knowing which of two leading approaches to modeling learning – adaptive learning or eductive learning – provides the better characterization of human learning behavior in this setting. In particular, we vary the slope parameter of the price determination equation, α, in such a way that in one of our treatments the REE should not be learnable (stable under learning) if agents are eductive learners but should always be learnable if agents are adaptive learners. We further investigate different predictions between the two learning theories with regard to the speed of convergence. Finally, our experimental design includes both monopoly and oligopoly settings in order to better understand the role played by common knowledge of rationality.

In all of our treatments, even the eductively unstable cases, we observe convergence of prices to the REE, which provides evidence in support of adaptive learning and against the eductive learning approach. However, the variance in market prices is much greater in the eductively unstable treatments where $\alpha = -2$ relative to the other two eductively stable treatments where $|\alpha| < 1$. Convergence to REE is also slower in the eductively unstable case, especially in the oligopoly treatment where prices often continue to deviate from the REE until the very end of the 50 period horizon. Further, there are many instances of markets that satisfy our criteria for convergence to the REE in the very first period, which is more in line with eductive rather than adaptive learning. Indeed, our efforts to classify subjects as adaptive or eductive learners reveals a mix of both learning types (as well as many subjects who are unclassifiable). Perhaps, as Evans (2001) suggests, individuals or populations of individuals use a mixture of both adaptive and eductive learning approaches.

The cobweb economy that we study is a very simple economic model. Our experimental examination of forecasting behavior in this model is the first study in which subjects were given complete information about the economic model. In this sense, our experiment provides the most favorable conditions for the rational expectation hypothesis and for the eductive learning approach to work. Our findings confirm that the rational expectation hypothesis and rational expectation equilibrium provide a good characterization of the market outcome in this setting. Further experimental studies might be conducted where subjects are exposed to a more complicated, forward-looking dynamic economic model where forecasts matter for realizations of future state variables, as for example in a modern dynamic, stochastic general equilibrium model. We leave that extension to future research.

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A Experimental Instructions

A.1 Experimental Instructions (Monopoly)

Experimental Instructions

Welcome to this experiment in economic decision-making. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's experiment. There is no talking for the duration of this session. If you have a question at any time, please raise your hand and your question will be answered in private.

General information

Imagine you are an advisor to a farm that is the only supplier of a product in a local market. In each time period the owner of the farm needs to decide how many units of the product he will produce. To make an optimal decision each period, the owner requires a good prediction of the market price of the product in each period. As the advisor to the farm owner, you will be asked to predict the market price, p_t of the product during 50 successive time periods, t=1,2,...,50. Your earnings from this experiment will depend on the accuracy of your price predictions alone. The smaller are your prediction errors, the greater will be your earnings.

About the determination of the market price p_t

The actual market price for the product in each time period, t, is determined by a market clearing condition, meaning that it will be the price such that demand equals supply for that period.

The amount demanded for the product depends on the market price for the product. When the market price goes up (down) the demand will go down (up). The supply of the product on the market is determined by the production decision of the farm owner. Usually, a higher (lower) price prediction by you causes the farm owner to produce a larger (smaller) quantity of the product which increases (decreases) the supply of the product on the market. Therefore, the actual market price p_t in each period depends upon your prediction, p_t^e, of the product's market price. More precisely, equating demand and supply, we have that the market price of the product is determined according to:

$$p_t = \max(60 - \alpha p_t^e + \eta_t, 0)$$

This means that the price cannot be below 0. The parameter α is different for different local markets. You will see the α value for your own local market on your decision page during the experiment. This α parameter will remain the same for your local market for all 50 periods of the experiment. η_t is a small random shock to the supply caused by non-market (demand/supply) factors, for example, weather conditions. This small shock is randomly drawn each period and is sometimes positive, sometimes negative and sometimes zero. It is not correlated across periods. This small shock is normally distributed. The long term mean value of this small shock is 0, and the standard deviation is 1.

Here is an example: Suppose the parameter α is 0.8 in your local market. Suppose further that you price prediction for the period is 35, and the realization of the shock η_t is -0.2. Using the equation given above, the market price is then determined as:

$$p_t = 60 - 0.8 * 35 - 0.2 = 31.8$$

Note that in this case your forecast error, $|p_t^e - p_t|$, is 35-31.8=3.2. This forecast error of 3.2 would determine your points for the period as discussed below.

Please also note that this example is for illustration purposes only. The value of the parameter α in your local market may be different from 0.8. The precise value of alpha and the equation for the determination of the market price in your local market is given on your decision page.

About your task

Your only task in this experiment is to correctly predict the market price in each time period as accurately as possible. The only constraint on your predicted price is that it cannot be less than zero (negative), since the actual price itself can never be less than zero. At the beginning of the experiment you are asked to give a prediction for the price of your farm's product in period 1. Note that, while there are several farms being advised by a forecaster like you in each period, these different local markets are totally separate from your own so what happens in other markets does not have any influence on your market. After all forecasters have submitted their predictions for the first period, the local market price for period 1 will be determined and will be revealed to you. Based the accuracy of your prediction in period 1, your earnings will be calculated. Subsequently, you are asked to enter your prediction for period 2. When all forecasters have submitted their predictions for the second period, the market price for that period in your local market will be revealed to you and your earnings will be calculated, and so on, for all 50 consecutive periods.

Information

Following the first period, you will see information on your computer screen that consists of 1) a plot of all past prices together with your market predictions and 2) a table containing the history of your past forecasts and payoffs, as well as realized market price and the shock term η_t.

About your payoff

Your payoff depends on the accuracy of your price forecast. The earnings shown on the computer screen will be in terms of points. When your prediction is p_t^e and the market price is p_t your payoff is a decreasing function in your prediction error, namely the distance between your forecast and the realized price.

$$Payoff_t = \max[1300 - \frac{1300}{49}(p_t^e - p_t)^2, 0]$$

Recalling the example above, if your forecast error for the period t, $|p_t^e - p_t|$, was 3.2, then according to the payoff function you would earn 1028.33 points for the period.

Notice that the maximum possible payoff in points you can earn from the forecasting task is 1300 for each period, and the larger is your prediction error, $|p_t^e - p_t|$, the fewer points you earn. You will earn 0 points if your prediction error is larger than 7. There is a Payoff Table on your desk, which shows the points you can earn for various different prediction errors.

At the end of the experiment your *total points* earned from *all 50 periods* will be converted into Euros at the rate of 1 Euro for every 2600 points that you earned. Thus, the more points you earn, the greater are your Euro earnings.

Questions?

If you have questions about any part of these instructions at any time, please raise your hand and an experimenter will come to you and answer your question in private.

A.2 Experimental Instructions (Oligopoly)

Welcome to this experiment in economic decision-making. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's experiment. There is no talking for the duration of this session. If you have a question at any time, please raise your hand and your question will be answered in private.

General information

Imagine you are an advisor to a farm that is **one of the three** main suppliers of a product in a local market. In each time period the owner of the farm needs to decide how many units of the product he will produce. To make an optimal decision, the owner requires a good prediction of the market price of the product in each period. As the advisor to the farm owner, you will be asked to predict the local market price, p_t of the product during 50 successive time periods, $t = 1, 2, 3, ...50$. Your earnings from this experiment will depend on the accuracy of your price predictions alone. The smaller are your prediction errors, the greater will be your earnings.

About the determination of the market price p_t

The actual market price for the product in each time period,t, is determined by a market clearing condition, meaning that it will be the price such that demand equals supply for that period.

The amount demanded for the product depends on the market price for the product. When the market price goes up (down) the demand will go down (up). The supply of the product on the market is determined by the production decision of the farm owners. Usually, a higher (lower) price prediction by the advisors causes the farm owners to produce a larger (smaller) quantity of the product which increases (decreases) the supply of the product on the market. Therefore the actual market price p_t in each period depends upon the average prediction, $\overline{p}_t^e$ of the product's market price. For example, if the predictions made by the advisors are $p_{1,t}^e$, $p_{2,t}^e$ and $p_{3,t}^e$ respectively, $\overline{p}_t^e = \frac{1}{3}(p_{1,t}^e + p_{2,t}^e + p_{3,t}^e)$. Equating demand and supply, we have that the market price of the product is determined according to:

$$P(t) = 60 - \alpha \overline{p}_t^e + \eta_t$$

This means that the price cannot be below 0. The parameter α will be shown on your decision page during the experiment. This α parameter will be the same for all three farms in your local market and for all 50 periods. Note also that η_t is a small random shock to the supply caused by non-market (demand/supply) factors, for example, weather conditions. This small shock is randomly drawn each period and is sometimes positive, sometimes negative and sometimes zero. It is not correlated across periods. This small shock is normally distributed. The long term mean value of this small shock is 0, and the standard deviation is 1.

Here is an example: Suppose the parameter α is 0.8 for all three farms in your market. Suppose further that you prediction for the price is 30 and the predictions by the other two advisors in your market are 35 and 40 respectively. Finally, suppose that the realization of the shock, η, is -0.2. The market price is in your three farm local market is then determined as follows:

$$p_t = 60 - 0.8 \times \frac{1}{3}(30 + 35 + 40) - 0.2 = 31.8$$

Note that in this case your forecast error (the distance between your forecast and the market price), $|p_t^e - p_t|$, is $|30 - 31.8| = 1.8$. This forecast error would be used to determine your points for the period as discussed below.

Please also note that this example is for illustration purposes only. The value of the parameter may be different from 0.8. The precise value of α and the equation for the determination of the market price in your local market are given on your decision page.

About your task

Your only task in this experiment is to correctly predict the market price in each time period as accurately as possible. The only constraint on your predicted price is that it cannot be less than zero (negative), since the actual price itself can never be less than zero. At the beginning of the experiment you are asked to give a prediction for the price in period 1. There are several markets of various products and each of them consists of 3 farms, and each of the farms is advised by a forecaster like you. These different local markets are totally separate from your own market so what happens in other markets does not have any influence on your market. After all forecasters have submitted their predictions for the first period, the local market price for period 1 will be determined and will be revealed to you. Based on the accuracy of your prediction in period 1, your earnings will be calculated. Subsequently, you are asked to enter your prediction for period 2. When all forecasters have submitted their predictions for the second period, the market price for that period in your local market will be revealed to you and your earnings will be calculated, and so on, for all 50 consecutive periods.

Information

Following the first period, you will see information on your computer screen that consists of 1) a plot of all past market prices together with your market price forecasts and 2) a table containing the history of your past forecasts and payoffs, as well as realized market prices and the shock term, η_t.

About your payoff

Your payoff depends on the accuracy of your price forecast. The earnings shown on the computer screen will be in terms of points. When your prediction is and the market price is your payoff is a decreasing function of your prediction error, namely the distance between your forecast and the realized price. Specifically:

$$payoff = \max[1300(1 - \frac{(p_t^e - p_t)^2}{49}), 0]$$

Notice that the maximum possible payoff in points you can earn from the forecasting task is 1300 for each period, and the larger is your prediction error, the fewer points you earn. You will earn 0 points if your prediction error is larger than 7. There is a Payoff Table on your desk, which shows the points you can earn for various different prediction errors.

At the end of the experiment your total points earned from all 50 periods will be converted into Euros at the rate of 1 Euro for every 2600 points that you earned. Thus, the more points you earn, the greater are your Euro earnings.

Questions?

If you have questions about any part of these instructions at any time, please raise your hand and an experimenter will come to you and answer your question in private.

B Payoff Table

Table 8 is the payoff table used in this experiment.

Payoff Table for Forecasting Task Your Payoff=max$[1300 - \frac{1300}{49}(\text{Your Prediction Error})^2, 0]$ 2600 points equal 1 euro							
error	points	error	points	error	points	error	points
0	1300	1.85	1209	3.7	937	5.55	483
0.05	1300	1.9	1204	3.75	927	5.6	468
0.1	1300	1.95	1199	3.8	917	5.65	453
0.15	1299	2	1194	3.85	907	5.7	438
0.2	1299	2.05	1189	3.9	896	5.75	423
0.25	1298	2.1	1183	3.95	886	5.8	408
0.3	1298	2.15	1177	4	876	5.85	392
0.35	1297	2.2	1172	4.05	865	5.9	376
0.4	1296	2.25	1166	4.1	854	5.95	361
0.45	1295	2.3	1160	4.15	843	6	345
0.5	1293	2.35	1153	4.2	832	6.05	329
0.55	1292	2.4	1147	4.25	821	6.1	313
0.6	1290	2.45	1141	4.3	809	6.15	297
0.65	1289	2.5	1134	4.35	798	6.2	280
0.7	1287	2.55	1127	4.4	786	6.25	264
0.75	1285	2.6	1121	4.45	775	6.3	247
0.8	1283	2.65	1114	4.5	763	6.35	230
0.85	1281	2.7	1107	4.55	751	6.4	213
0.9	1279	2.75	1099	4.6	739	6.45	196
0.95	1276	2.8	1092	4.65	726	6.5	179
1	1273	2.85	1085	4.7	714	6.55	162
1.05	1271	2.9	1077	4.75	701	6.6	144
1.1	1268	2.95	1069	4.8	689	6.65	127
1.15	1265	3	1061	4.85	676	6.7	109
1.2	1262	3.05	1053	4.9	663	6.75	91
1.25	1259	3.1	1045	4.95	650	6.8	73
1.3	1255	3.15	1037	5	637	6.85	55
1.35	1252	3.2	1028	5.05	623	6.9	37
1.4	1248	3.25	1020	5.1	610	6.95	19
1.45	1244	3.3	1011	5.15	596	$error \geq 0$	
1.5	1240	3.35	1002	5.2	583		
1.55	1236	3.4	993	5.25	569		
1.6	1232	3.45	984	5.3	555		
1.65	1228	3.5	975	5.35	541		
1.7	1223	3.55	966	5.4	526		
1.75	1219	3.6	956	5.45	512		
1.8	1214	3.65	947	5.5	497		

Table 8: Payoff Table for Forecasters

C Alternative Proof of the Stability Condition of Adaptive Learning based on Mathematical Induction

Without loss of generality, let $p_1^e = p^* + \Delta$, where Δ is the difference between the period 1 prediction and the REE. Substituting this forecast into equation (1), we obtain $p_1 = \mu + \alpha(p^* + \Delta)$. Since $p^* = \mu + \alpha p^*$, this expression simplifies to $p_1 = p^* + \alpha\Delta$. In period 2, the prediction is the price in period 1, $p_2^e = p_1 = p^* + \alpha\Delta$. Substituting this prediction into equation (1) and simplifying, yields $p_2 = \mu + \alpha p_2^e = p^* + \alpha^2\Delta$. In period 3, the prediction should be the average price in periods 1 and 2, $p_3^e = \frac{p_1+p_2}{2} = p^* + \frac{1}{2}\alpha(\alpha+1)\Delta$. Substituting this prediction into equation (1) and simplifying yields $p_3 = \mu + \alpha p_3^e = p^* + \frac{1}{2}\alpha^2(\alpha+1)\Delta$. By iterating in this fashion it is not difficult to find that in general, for period t, $p_t^e = \frac{1}{t-1}\sum_{s=1}^{t-1} p_s = p^* + \frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+t-2)}{1\times 2\times 3...(t-1)}\Delta$ and so $p_t = \mu + \alpha p_t^e = p^* + \alpha\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+t-2)}{1\times 2\times 3...(t-1)}\Delta$.

Clearly this system converges to the REE whenever the ratio $\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+t-2)}{1\times 2\times 3...(t-1)}$ goes to 0. This ratio consists of $t-1$ components in both the numerator and the denominator. We can pair the components in the numerator and the denominator according to the sequence, namely, let α be paired to 1, $\alpha+1$ be paired to 2, ..., $\alpha+t-2$ be paired to $t-1$. When $\alpha > 1$, each component of the numerator is larger than its paired number in the denominator. Therefore $\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+t-2)}{1\times 2\times 3...(t-1)}$ will increase over time with t, diverging away from 0. When $\alpha = 1$, the ratio is exactly equal to 1. When $-1 < \alpha < 1$, each component in the numerator has a smaller absolute value than its paired number, so the ratio will decrease with t, and goes to 0 as $t \to \infty$.

When $\alpha < -1$, we make a slightly different re-matching of the components in the numerator and the denominator. First, let m be an integer such that $\alpha + m - 1 < 0$ and $\alpha + m > 0$. We re-state the ratio as $\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+m-1)(\alpha+m)(\alpha+m+1)...(\alpha+t-2)}{1\times 2\times 3...(t-m-1)(t-m)(t-m+1)...(t-1)}$. We then "cut" the numerator into two parts, $N_1 = \alpha(\alpha+1)(\alpha+2)...(\alpha+m-1)$ and $N_2 = (\alpha+m)(\alpha+m+1)...(\alpha+t-2)$, and we also cut the denominator into two parts, $D_1 = 1\times 2\times 3...(t-m-1)$ and $D_2 = (t-m)(t-m+1)...(t-1)$. We pair N_2 to D_1, namely, $\alpha+m$ to 1, $\alpha+m+1$ to 2, ... $\alpha+t-2$ to $t-m-1$. It is not difficult to see that each item in N_2 is smaller than the paired item in D_1 ($\alpha+m < 1$, $\alpha+m+1 < 2$, ... $\alpha+t-2 < t-m-1$), and therefore that $\frac{(\alpha+m)(\alpha+m+1)(\alpha+m+2)...(\alpha+t-2)}{1\times 2\times 3...(t-m-1)}$ decreases with t, and goes to 0 as $t \to \infty$. There remain m extra components in both the numerator and the denominator. In the numerator, $|N_1| = |\alpha(\alpha+1)(\alpha+2)...(\alpha+m-1)| < |\alpha^m|$ is a finite number, while in the denominator, $D_2 = (t-m)(t+m+1)...(t-1)$ goes to infinity as $t \to \infty$. Therefore, the remaining fraction $\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+m-1)}{(t-m)(t-m+1)...(t-1)}$ also goes to 0 as $t \to \infty$. It follows that, under adaptive (least squares) learning, the system converges to the REE provided that $\alpha < 1$ and diverges from the REE only if $\alpha > 1$.

D Categorization of Subjects

	$\alpha = -0.5$		$\alpha = -0.9$		$\alpha = -2$		$\alpha = -4$	
	Categorized	Reported	Categorized	Reported	Categorized	Reported	Categorized	Reported
exp1	A		E				A	E
exp2	A		A					E
exp3	A				A			E
exp4			E		E		A	E
exp5	A							
exp6	A				E		A	A
exp7							A	E
exp8	E		A				E	E
exp9	A		E	E		A		E
exp10	E	E		E	E	A		E
exp11	E		E			E		E
exp12	A	A		A		E		
exp13	A	E			E	E		A
exp14		E					A	E

Table 9: Categorization of subjects into adaptive and eductive learners in the monopoly setting. “A” means adaptive learner. “E” means eductive learner. We leave the cell blank for subjects we can not categorize into either of the two types. “Categorized” means categorization according to the first approach where we use the definition of the learning rules. “Reported” means categorization is done according to the second approach based on self-reported strategies.

	$\alpha = -0.5$		$\alpha = -0.9$		$\alpha = -2$		$\alpha = -4$	
	Categorized	Reported	Categorized	Reported	Categorized	Reported	Categorized	Reported
exp11	E	E	E					E
exp12	A	A		A		E		
exp13	A	A		A		E		E
exp21	A	E	A	A	E			A
exp22	E	E	A	A		E		A
exp23	A	A	A	A		A	E	E
exp31	A	E	A	A		A		E
exp32	A	A	E	A	E	E	E	E
exp33		A		A		A	E	
exp41	A	A		A	E	E		E
exp42		A		A		A	E	E
exp43	A	E		A		E		
exp51	E	E	A	A		A		E
exp52	A	A		A	E	A	E	A
exp53	A	A	E	E				A
exp61	E	A		A		E		E
exp62	A	A		A		E		A
exp63	A	E	A	E				A
exp71	A	A		A		A	E	E
exp72		A	E	E		A		A
exp73	A	A		A		E	E	E
exp81		A	A	E		E		E
exp82	E	E	E	A		A	E	E
exp83	A	E	E	E			E	E
exp91	A	A		A		E		E
exp92	E	A	E	E		A		
exp93		A	E	A		A	E	E
exp101		A	E	A		A		A
exp102	A	A	E	A				A
exp103	E	E	A	A		A		A
						A		
						A		
						E		

Table 10: Categorization of subjects into adaptive and eductive learners in the oligopoly setting. "A" means adaptive learner. "E" means eductive learner. We leave the cell blank for subjects we can not categorize into either of the two types. "Categorized" means categorization according to the first approach where we use the definition of the learning rules. "Reported" means categorization is done according to the second approach based on self-reported strategies.