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Development of a Dynamic Cathode Ejector Model for Solid Oxide Fuel Cell-Gas Turbine Hybrid Systems

Solid oxide fuel cell-gas turbine (SOFC-GT) hybrid systems are attractive for future power generation with ultra-low criteria pollutant and greenhouse gas emissions. One of the challenges for SOFC-GT systems is to sufficiently pre-heat incoming air before it enters the fuel cell cathode. An ejector for cathode exhaust recirculation has the benefits of reliability, low maintenance, and cost compared to either recuperators or cathode recirculation blowers, which may be also be used for air pre-heating. In this study, a dynamic Simulink model of an ejector for cathode exhaust recirculation to pre-heat incoming fuel cell air has been developed. The ejector is to be utilized within a 100 MW SOFC-GT dynamic model operating on coal syngas. A thorough theoretical development is presented. Results for the ejector were found to be in good agreement with those reported in literature.

Keywords: SOFC-GT hybrid system, IGFC, fuel cell, gas turbine, cathode ejector, cathode recirculation, dynamic modeling

1 Introduction

The need for advanced coal based power generation has led to the concept of an integrated gasification fuel cell system (IGFC). The main power block in this system is comprised of a solid oxide fuel cell (SOFC) integrated with a gas turbine (GT) operating on coal syngas. Detailed descriptions of such an integrated hybrid cycle or of a simple-cycle solid oxide fuel cell (SOFC) operating on coal syngas have been outlined in the literature [1–4]. The main benefits of IGFC systems are that they allow operation on an abundant and cheap fuel and by using syngas in place of solid coal many of the toxic emissions associated with coal based power generation are removed. Additional benefits include very high electrical efficiencies that are possible by using a SOFC-GT hybrid system [5–14], and the separated anode and cathode flows of a fuel cell that more readily enable carbon concentration for sequestration. Finally, synergies between the SOFC and various gasifiers can be exploited to further increase overall system efficiency. One such potential synergy involves creating a higher methane content syngas, which improves the cold-gas efficiency of the gasifier. This is accomplished via internal reformation within the fuel cell, which provides a means of fuel cell cooling: reducing the excess air required for cooling and averting the associated parasitic losses.

In this study, a dynamic Simulink model of an ejector for cathode exhaust recirculation is developed. The ejector is utilized within a 100 MW SOFC-GT Simulink model operating on coal syngas shown in Fig. 1.

An ejector for cathode exhaust recirculation has the benefits of reliability and low maintenance, since it possesses no moving parts, and relatively low cost compared to a cathode recirculation blower, which may also be used in this application. However, the ejector is expected to be a larger parasitic load compared to a blower in this application. Both the ejector and blower are alternatives to costly recuperators that may also be used to pre-heat incoming compressor air before it enters the fuel cell cathode.

Some groups have developed ejector models for use in fuel cell applications [15–18]. However, these groups do not outline the theoretical development in sufficient detail, with the exception of perhaps [15]. Also, none of these groups describe the detailed theoretical development of a cathode ejector model and are instead focused upon anode ejector applications. The ejector model that is the subject of this study is used in conjunction with a SOFC-GT model developed by the National Fuel Cell Research Center (NFCRC), which has been extensively peer-reviewed and validated using dynamic experimental data from a 220 kW Siemens-Westinghouse SOFC-GT system tested at the NFCRC [19–28]. The original 220 kW model has been modified to simulate a planar SOFC and scaled up to 100 MW for IGFC applications.

2 Model Development

The cathode recirculation ejector modeled herein is grounded in the theoretical and experimental work conducted by Keenan et al. [29] and extends the theoretical work of Sun et al. [30]. The ejector model solves for pressure ($P$), temperature ($T$) and Mach number ($M$) at each section of the ejector geometry $(A)$ diagrammed in Fig. 2. The ejector geometry is fixed but can be defined by the user to achieve different ejector performance characteristics.

The solution steps are calculated under the following assumptions:

1. One-dimensional, steady state flow of an isentropic ideal gas.
2. Primary and secondary fluids have the same molecular weight and ratio of specific heats.
3. Primary and secondary fluids are supplied at zero velocities, i.e., stagnation conditions in states (1) and (2).
At (i) the two streams meet and mixing occurs at constant pressure between (i) and (j).

Transverse shock occurs at a plane between (j) and (k).

Velocity at (3) is zero, i.e., stagnation conditions.

Air, with a ratio of specific heats of $\gamma = 1.4$, is used for both primary and secondary fluids.

The primary fluid (denoted by the subscript 1) in this study is compressor air and the secondary fluid (denoted by the subscript 2) is cathode exhaust. A table of ejector areas $A_t$, $A_1$, and $A_j = A_k$ is calculated for various pressure ratios $P_i = P_2$ using specified primary and secondary pressure and temperature stream values $P_1$, $P_2$, $T_1$, $T_2$ and $\omega$, where $\omega$ is the entrainment ratio defined by

$$\text{Entrainment Ratio} = \omega = \frac{\dot{m}_2}{\dot{m}_1} \quad (1)$$

where $\dot{m}_1$ is the mass flow rate of the primary fluid (compressor air) and $\dot{m}_2$ is the mass flow rate of the secondary fluid (cathode exhaust).

Geometry is best characterized outside of the Simulink model using methods defined by Keenan et al. [29]. The geometry necessary to create the appropriate pressure rise across the ejector $P_3 = P_2$, accounting for pressure loss in the fuel cell and meeting the fuel cell inlet pressure design point is chosen. This is accomplished by guessing $P_1$ (again this is outside of the Simulink model). $P_1$ is the stagnation pressure value at the gas turbine compressor outlet. In the model, $P_1$ and gas turbine RPM will specify a mass flow rate from the compressor map that defines $\dot{m}_1$ and also the stagnation temperature $T_1$.

For analysis of the primary nozzle (Figure 3) Table 1 is used with knowledge of ejector geometry ($A/A^*$, from Table 1, which is the area to critical area ratio at the throat needed for mach flow) to solve for $M_{1ia}$. Throughout this work the subscript (s) denotes the isotropic value and (a) the actual value.

From Figure 4, it is apparent that there are two solutions for $M_{1ia}$ for a specified area ratio $A/A^*$. In the case that $A/A^* = 1.099$, $M_{1ia}$ is either 1.37 or 0.69. If we assume a supersonic nozzle (which is used in this model) then we chose $M_{1ia} = 1.37$. Again, the value of $M_{1ia}$ comes from a look up table of supersonic Mach numbers for a given area ratio.

The actual temperature $T_{1ia}$ out of the primary nozzle will be greater than the isotropic value $T_{1is}$ due to inefficiencies accounted for by a nozzle efficiency term $\eta_n$ in the model, a value of 90% is claimed to be typical of supersonic converging-diverging nozzles [30]

$$T_{1ia} = T_1 \left[ 1 + \frac{\eta_n (\gamma - 1)}{2} M_{2ia}^2 \right]^{-1} \quad (2)$$

$T_{2is}$ is known from the cathode exit conditions.

The actual Mach number $M_{1ia}$, accounting for efficiency loss, is calculated using

$$M_{1ia} = \sqrt{\frac{2}{\gamma - 1} \left( \frac{T_1}{T_{1ia}} \right) - 1} \quad (3)$$

where $M_{1ia} < M_{1is}$.

By definition nozzle efficiency does not affect $P_i$. 

![Fig. 2 Fuel cell cathode recirculation ejector](image-url)
Actual Kinetic Energy at Nozzle Exit

\[ \eta_a = \frac{\text{Kinetic Energy at Nozzle Exit for Isentropic Flow from the Same Inlet State to the Same Exit Pressure}}{\text{Kinetic Energy at Nozzle Exit for Isentropic Flow from the Same Inlet State to the Same Exit Pressure}} \]

\[ = \frac{V_f^2}{V_i^2} = \frac{M_i^2 T_i}{M_f^2 T_f} \]

\[ P_j \text{ is calculated using } \]

\[ P_j = P_i \left( 1 + \frac{\gamma - 1}{2} M_i^2 \right)^{-\frac{\gamma}{\gamma - 1}} \]

Mixing occurs from i to j at constant pressure \( P_{ij} = P_j \). With knowledge of \( P_j \) from the cathode exit conditions, \( M_{2i} \) can be solved using

\[ M_{2i} = \sqrt{\frac{2}{\gamma - 1} \left[ \left( \frac{P_i}{P_j} \right)^{-\frac{1}{\gamma - 1}} - 1 \right]} \]

The entrainment ratio \( \omega \) can be solved for using

\[ \frac{A_{2i}}{A_{1i}} = \frac{P_j}{P_i} \left[ \left( \frac{P_i}{P_j} \right)^{\frac{1}{\gamma - 1}} \right] \frac{1}{\left[ \left( \frac{P_i}{P_j} \right)^{\frac{1}{\gamma - 1}} - 1 \right]} \omega \sqrt{\frac{T_j}{T_i}} \]

Rearranging we get

\[ \omega = \frac{A_2 P_2}{A_1 P_1} \left( \frac{P_i}{P_j} \right)^{\frac{1}{\gamma - 1}} \left[ 1 - \left( \frac{P_i}{P_j} \right)^{\frac{1}{\gamma - 1}} \right] \sqrt{T_j/T_i} \]

The temperature at j, \( T_j \), accounts for the mixing of streams 1 and 2 at the outlet of the constant pressure mixing section and is solved for using

\[ T_j = \left\{ \left[ (\sum N \int C_p dT) + \frac{\dot{m} V^2}{2} \right]_j \right\} \left[ - (\sum N \int C_p dT) + \frac{\dot{m} V^2}{2} \right]_{out} \frac{V^2 C_p \text{ Conc.}}{\sqrt{C}} dT \]

To correct for the local speed of sound \( C^l \) which varies with temperature, the following is used

\[ M^* = \frac{V}{C^l} = \frac{M C P}{\sqrt{\gamma R T}} = M \sqrt{\frac{T}{T^*}} \]

where \( T^* \) is the local temperature

\[ T^* = \frac{2 T_o}{\gamma + 1} \]

and \( T_o \) is the stagnation condition. Substitution gives

\[ M^* = M \sqrt{\frac{T(\gamma + 1)}{2 T_o}} \]

\( M^*_j \) is solved by substitution

\[ M^*_j = \frac{M^*_{1a} + \omega M^*_{2a} \sqrt{T_2/T_1}}{1 + \omega T_2/T_1 (1 + \omega)} \]

\[ = \frac{M^*_{1a} T_{1a}(\gamma + 1)}{2 T_1} + \omega M^*_{2a} \sqrt{T_2(\gamma + 1) 2T_2 T_1} \sqrt{T_2/T_1} \]

\[ (1 + \omega T_2/T_1) (1 + \omega) \]

\[ M_j = \sqrt{\frac{2 M^*_j^2}{\gamma + 1 - M^*_j^2(\gamma - 1)}} \]

Shock is assumed to occur in the constant area section of the ejector from j to k. The equation describing a one-dimensional normal shock for an ideal gas with constant specific heats and molar mass is used to solve for \( M_k \)

\[ M_k = \sqrt{\frac{\gamma M^*_j^2}{\gamma - 1} + M_j^2 \frac{2}{\gamma - 1} \gamma M^*_j^2 - 1} \]

The pressure rise across the shock is used to find \( P_k \)
\[ P_j = P_i \]

Accounting for the temperature change associated with shock in the constant area section

\[ T_k = T_j \left( 1 + \frac{M_j^2 (\gamma - 1)}{2} \right) \left( 1 + \frac{M_k^2 (\gamma - 1)}{2} \right) \]

Finally, \( P_3 \) is determined using

\[ P_3 = P_k \left[ 1 + \frac{\eta_d (\gamma - 1)}{2} M_j^2 \right]^{\gamma/(\gamma - 1)} \]

where \( P_j = P_i \).

The thermodynamic state and mass flow entering or leaving the ejector system (plumbing and ejector inlet components, followed by ejector, followed by plumbing and ejector outlet components) are calculated from the flow dynamics of a plenum volume. Figure 5 schematically depicts the plenum volume considered in the current model.

Using the isentropic relation \( T_e/T = (P_o/P)^(\gamma-1)/\gamma \) which is equivalent to \( T_3/T_i = (P_3/P_i)^(\gamma-1)/\gamma \) and by substitution into Eq. (18) we get Eq. (20), which is used to solve for the exit temperature of the ejector; an important value that will determine the effectiveness of the ejector to replace recuperation in SOFC-GT cycles.

\[ T_3 = T_k \left[ 1 + \frac{\eta_d (\gamma - 1)}{2} M_j^2 \right] \]

In the model, \( P_s \) and the cathode inlet pressure must be equal. In order to approach matching ejector outlet and cathode inlet pressures, ejector geometry design is iterated externally from the model. An algebraic constraint that manipulates compressor outlet pressure \( P_3 \), ultimately ensures that these pressures match exactly.

Thus far, the model development has focused upon the steady-state performance characteristics of the cathode ejector. Since a typical ejector does not have a significant volume that would allow mass storage to occur during transients, the current formulation accounts for ejector system dynamics through use of dynamic expressions for mass storage that can occur in the volumes immediately upstream and/or downstream from the ejector itself as follows. The thermodynamic state and mass flow entering or leaving the ejector system (plumbing and ejector inlet components, followed by ejector, followed by plumbing and ejector outlet components) are calculated from the flow dynamics of a plenum volume.
where $P$ is pressure, $T$ is temperature, $V$ is the plenum volume before or after the ejector (m$^3$), and $m_{in}$ and $m_{out}$ are the mass flows (kg/s) in and out of the plenum volume (before or after the ejector). Note that the plenum volumes may include the volume associated with plumbing or other upstream or downstream components with sufficiently large volume to allow mass storage. Note also that a plenum volume with dynamic mass storage as described above may be inserted before and/or after the ejector as appropriate for the particular fuel cell system design one considers.

### 3 Results and Discussion

#### 3.1 Ejector Model Verification

The general approach employed for the ejector model is to utilize the steady state agreement with the literature [32–34]. Figure 9 indicates that for a fixed primary stream temperature of 1000 K and pressure of 500 kPa (from the cathode exit) that the entrainment ratio and ejector exit temperature decrease with increasing primary stream inlet pressure; in agreement with dynamic input/output equations.

Figure 9 indicates that for a fixed primary stream temperature of 400K (from the gas turbine compressor) and a fixed secondary stream temperature of 1000 K and pressure of 500 kPa (from the cathode exit) that the entrainment ratio and ejector exit temperature decrease with increasing primary stream inlet pressure; in agreement with the literature [35].

### 4 Conclusions

Use of an ejector for cathode exhaust recirculation in SOFC/GT systems has the benefits of reliability, low maintenance, and cost compared to either recuperators or cathode recirculation blowers. However, the dynamics of ejector operation in such applications cannot be assessed without a sufficient dynamic model. The development of a dynamic Simulink model of an ejector for cathode exhaust recirculation is presented. A thorough theoretical development is presented that may be used by other groups striving to develop and analyze the use of ejectors in fuel cell systems. Results were found to be in good agreement with those reported in literature.

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