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Nonlinear Muskingum Model for Flood Routing in Irrigation Canals Using Storage Moving Average

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Abstract: The Muskingum routing method involves the calculation of the storage volume in river reaches. The accurate calculation of the storage volume is difficult to achieve with the nonlinear Muskingum model (NMM). This paper couples the storage moving average (SMA) with NMM to produce a Muskingum storage moving average model (MUSSMAM), which has improved the accuracy of storage volume and output hydrograph calculation over the NMM. The parameters of the NMM and the weighting coefficients in the SMA are treated as dimensionless parameters in the combined MUSSMAM formulation. A hybrid method that merges the generalized reduced gradient (GRG) solver with an evolutionary (EV) solver is employed to obtain the best combination of parameter values of the MUSSMAM. The sum of the squared deviation (SSQ) between observed and routed inflows is the objective function optimized with the hybrid GRG-EV solver. The proposed MUSSMAM is tested with experimental, real, and multimodal hydrograph-routing problems. These results demonstrate that MUSSMAM reduces the SSQ by 2, 10, and 29 percent compared to the NMM in experimental, real, and multimodal problems, respectively. **DOI:** **10.1061/(ASCE)IR.1943-4774.0001000.** *© 2016 American Society of Civil Engineers.*

Author keywords: Flood routing; Nonlinear Muskingum model (NMM); Storage moving average (SMA).

Introduction

Despite various recent investigations that dealt with newly developed models in several domains of water resource systems, such as reservoir operation (Ahmadi et al. 2014; Bolouri-Yazdeli et al. 2014; Ashofteh et al. 2013a, 2015c, a), groundwater resources (Bozorg-Haddad et al. 2013; Fallah-Mehdipour 2013d), conjunctive use operation (Fallah-Mehdipour 2013c), design-operation of pumped-storage and hydropower systems (Bozorg-Haddad et al. 2014), flood management (Bozorg-Haddad et al. 2015b), water project management (Orouji et al. 2014; Shokri et al. 2014), hydrology (Ashofteh et al. 2013b), qualitative management of water resources systems, (Orouji et al. 2013b; Bozorg-Haddad et al. 2015a), water distribution systems (Seifollahi-Aghmiuni et al. 2013; Soltanjalili et al. 2013; Beygi et al. 2014), agricultural crops (Ashofteh et al. 2015c), sedimentation (Shokri et al. 2013), and algorithmic developments (Ashofteh et al. 2015b), only a few of these focus on the nonlinear Muskingum model (NMM) for flood routing (Bozorg-Haddad et al. 2015d, e).

Floods are recurrent phenomena that cause considerable damage in urban, industrial, and agricultural regions (Loáiciga 2001). The prediction of flood characteristics is indispensable in preventing or alleviating flood damage. Hydraulic and hydrologic methods are used to route hydrographs in open channels. Hydraulic methods, which are based on numerical and mathematical techniques, entail complex and time-consuming processes to complete flood routing. In contrast, most of the hydrologic methods consist of simple processes (Orouji et al. 2013a). The Muskingum model is a hydrologic flood routing model based on the continuity and storage equations. Accurate estimation of its parameters is necessary to compute accurate outflow hydrographs (Chow et al. 1988; Barbetta et al. 2011). The NMM is a variant of the Muskingum model that has been used in several investigations. Various methods have been used to estimate the parameters of the NMM. Those methods include segmented least-mean-squared (LMS) method (Gill 1978), the genetic algorithm (GA) (Mohan 1997), harmony search (HS) (Kim et al. 2001), the Lagrange multiplier (LM) method (Das 2004), Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization (Geem 2006), particle swarm optimization (PSO) (Chu and Chang 2009), the immune clonal selection algorithm (ICSA) (Luo and Xie 2010), the Nelder-Mead simplex (NMS) method (Barati 2011), the hybrid GA and NMS method (Barati 2012), differential evolution (DE) (Xu et al. 2012), the shuffled frog-leaping algorithm (SFLA) (Orouji et al. 2013a), hybrid HS and BFGS optimization (Karahan et al. 2013), and the generalized reduced gradient (GRG; Barati 2013a; Hamedi et al. 2014).

The accurate calculation of storage volume greatly influences the prediction of outflow hydrographs in the Muskingum model. Koussis et al. (2012) showed that errors grow quickly after a few spatial steps of flood routing; hence, some sort of data conditioning is needed to control noise amplification. Moving averaging

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(MA) and auto regressive (AR) modeling are commonly used in the hydrologic analysis of time series. Koussis et al. (2012) applied MA to improve outflow hydrograph prediction. Several other modifications to the Muskingum method have been effected in recent years to improve its prediction accuracy (Birkhead and James 2002). Easa (2013a) proposed NMM, including four parameters to increase the degrees of freedom. Moreover, Easa (2013b) improved NMM performance by varying the exponent parameter of the storage equation.

Most researchers have focused on improving the calculation of the storage volume in NMM. This paper explores other ways to predict accuracy and modifying the structure of the NMM. The primary contributions of this research are as follows:

1. A new algorithm called the storage moving average (SMA) is added to the NMM to produce a new model, herein called the Muskingum storage moving average model (MUSSMAM), which has improved storage volume calculation and parameter fitting performance.
2. The parameters of the NMM and the weighting coefficients in the SMA are modeled as dimensionless parameters whose optimal values are determined by the MUSSMAM.
3. The MUSSMAM is formulated using a spreadsheet.

Methodology

The sections entitled "Nonlinear Muskingum Model (NMM)" and "Moving Average (MA)" present the details of NMM and MA, respectively. The next section, "Proposed Muskingum Model (MUSSMAM)," presents the formulation details of the proposed model and its parameter estimation algorithm.

Nonlinear Muskingum Model (NMM)

The importance of flood routing in natural channels and rivers is well recognized in hydrologic engineering. One of the most frequently used hydrologic methods is the Muskingum model, introduced by McCarthy (1938) after studies of the Muskingum River basin in Ohio. In the original Muskingum model, the stream-flow routing procedure is based on the hydrologic continuity equation and the storage equation, which are written as follows:

$$\frac{dS}{dt} \approx \frac{\Delta S}{\Delta t} = I - O \tag{1}$$

$$S = K[XI + (1 - X)O] \tag{2}$$

where S, I, and O are simultaneous amounts of storage, inflow, and outflow, respectively; t is time; $\Delta S/\Delta t$ refers to the change of the storage volume during a time interval Δt; K is the proportionality coefficient, which has a value close to the flow travel time through the river reach; and X is the weighting factor, which depends on the channel type. In natural channel reaches, the relationship between the weighted flow $[XI + (1 - X)O]$ and storage S is not always linear, which explains why the linear Muskingum model [Eq. (2)] exhibits prediction error in many outflow hydrographs (Tung 1985). The NMM was suggested by Gill (1978), who added an exponent parameter m to the assumed linear form of Eq. (2), given as

$$S = K[XI + (1 - X)O]^m \tag{3}$$

where m is the parameter that accounts for the nonlinearity of flood-wave behavior and is greater than 1 for nonlinear models. Let the observed inflow, calculated outflow, and calculated storage at time interval j be I_j, $\hat{O}_j$, and S_j, respectively, where the expression $j = 0, 1, 2, \ldots, N$ denotes the routing procedure time intervals. The main steps of the NMM routing procedure are given as follows (Tung 1985; Geem 2006):

1. Assume values for the three hydrologic parameters (K, X, and m).
2. Calculate the initial storage S_0 using the following equation, where the initial outflow is assumed to equal the initial inflow ($\hat{O}_0 = I_0$):

$$S_0 = K[XI_0 + (1 - X)\hat{O}_0]^m \quad j = 0 \tag{4}$$

3. Calculate the rate of change of the storage volume at time interval j as follows:

$$\frac{\Delta S_j}{\Delta t} = -\left(\frac{1}{1 - X}\right)\left(\frac{S_j}{K}\right)^{1/m} + \left(\frac{1}{1 - X}\right)I_j \quad j = 0, 1, \ldots, N - 1 \tag{5}$$

4. Calculate the storage at time interval j as follows:

$$S_j = S_{j-1} + \Delta t\left(\frac{\Delta S_{j-1}}{\Delta t}\right) \quad j = 1, 2, \ldots, N \tag{6}$$

5. Calculate the outflow at time interval j using the following equation [notice that most previous studies have I_{i-1} rather than I_i in the equation (Geem 2011)]:

$$\hat{O}_j = \left(\frac{1}{1 - X}\right)\left(\frac{S_j}{K}\right)^{1/m} - \left(\frac{X}{1 - X}\right)I_{j-1} \quad j = 1, 2, \ldots, N \tag{7}$$

6. Increase the index j by 1 and repeat steps 3–5 until the routing procedure reaches time interval N.

The routing procedure is repeated with different parameter values until a reasonable fit between the calculated and observed outflows is obtained.

Moving Average (MA)

Consider a variable Y that has values at times $(t_1, t_2, t_3, \ldots, t_N)$ to form the time series $(Y_1, Y_2, Y_3, \ldots, Y_N)$. The p-order moving average of Y generates a moving-averaged time series $Z_1, Z_2, \ldots, Z_{N-p+1}$:

$$\begin{aligned} Z_1 &= w_1 Y_1 + w_2 Y_2 + \cdots + w_p Y_p \\ Z_2 &= w_1 Y_2 + w_2 Y_3 + \cdots + w_p Y_{p+1} \\ &\vdots \\ Z_{N-p+1} &= w_1 Y_{N-p} + w_2 Y_{N-p+1} + \cdots + w_p Y_N \end{aligned} \tag{8}$$

where $w_1, w_2, \ldots, w_p$ are the weighting coefficients of the p-order MA (Koussis et al. 2012).

Proposed Muskingum Model (MUSSMAM)

In the MUSSMAM, the calculation of the storage volume has been improved by using the storage volume predictor (S^P) and the storage volume corrector (S^C). The S^P is the same as the calculated storage volume in the method given by Tung (1985), and S^C is estimated by using SMA. In other words, S_j^C is the volume storage improvement at the jth time interval, calculated by the weighted averaging of the predicted storage volumes S_{j-1}^P, S_j^P, and S_{j+1}^P, at the $j-1$th, jth, and $j+1$th time intervals, respectively. The main steps of the MUSSMAM routing procedure are as follows:

1. Assume values for the three hydrologic parameters (X, K, and m), and the weighting coefficients (w_{-1}, w_0, w_1) whose sum adds up to 1.

2. Calculate S_0^P as follows, where the initial outflow is considered to be the same as the initial inflow:

$$S_0^P = K[XI_0 + (1-X)O_0]^m \tag{9}$$

3. Calculate the temporal storage rate ΔS_j as follows:

$$\frac{\Delta S_j}{\Delta t} = -\left(\frac{1}{1-X}\right)\left(\frac{S_j^P}{K}\right)^{1/m} + \left(\frac{1}{1-X}\right)I_j \quad j = 0, 1, \ldots, N \tag{10}$$

4. Calculate S_j^P as follows:

$$S_j^P = S_{j-1}^P + \Delta t\left(\frac{\Delta S_{j-1}}{\Delta t}\right) \quad j = 1, \ldots, N+1 \tag{11}$$

5. Increase the index j and repeat steps 3–4 until the simulation S_j^P reaches time $N+1$.
6. [Storage moving average (SMA) section]: Calculate the storage volume corrector at time interval j as follows:

$$S_j^C = w_{-1}S_{j-1}^P + w_0 S_j^P + w_1 S_{j+1}^P \quad j = 1, \ldots, N \tag{12}$$

where w_{-1}, w_0, and w_1 are the weight coefficients for the predicted values S_{j-1}^P, S_j^P, and S_{j+1}^P, respectively, used for calculating S_j^C.
7. Calculate the outflow at time interval j as follows:

$$\hat{O}_j = \left(\frac{1}{1-X}\right)\left(\frac{S_j^C}{K}\right)^{1/m} - \left(\frac{X}{1-X}\right)I_{j-1} \quad j = 1, \ldots, N \tag{13}$$

8. Repeat steps 6–7 until a stopping criterion is satisfied or a maximum number of iterations is reached, whichever comes first.

The accuracy of the MUSSMAM procedure is measured by the sum of the square of the deviations between observed and computed outflows (SSQ), given by

$$\text{Min.SSQ} = \sum_{j=1}^{N}(O_j - \hat{O}_j)^2 \tag{14}$$

In this model, the storage volume corrector is obtained with a three-order MA of the storage volume predictor. Therefore, the proposed model is MUSSMAM (w_{-1}, w_0, w_1). Now, if the storage volume corrector is obtained using a two-order MA of the storage volume predictor, two models named MUSSMAM (w_{-1}, w_0, 0) and MUSSMAM (0, w_0, w_1) can be created by changing the SMA part in MUSSMAM (w_{-1}, w_0, w_1), according to Eqs. (15) and (16), respectively:

$$S_j^C = w_{-1}S_{j-1}^P + w_0 S_j^P \quad j = 1, \ldots, N \tag{15}$$

$$S_j^C = w_0 S_j^P + w_1 S_{j+1}^P \quad j = 1, \ldots, N \tag{16}$$

The proposed MUSSMAM consists of the constraints of Eqs. (9)–(13) and the objective of Eq. (14). The model, which is nonlinear and nonconvex, was solved using the Solver package available in Microsoft Excel. Solver can solve a wide range of optimization problems from linear programming to complex optimization problems. Solver includes two approaches to finding global optimal solutions: (1) the GRG solver (Lasdon et al. 1978), and (2) the evolutionary solver (EV) (*Premium Solver Platform*). The GRG relies on local search algorithms, which may converge in a few iterations but generally may lack global optimality. In contrast, the EV is a natural phenomenon–mimicking algorithm. This algorithm searches randomly for the near-global optimal solution, which may yield different solutions on different runs. The EV is computationally intensive and poor in terms of convergence performance. The GRG solver needs initial estimates of the parameters or variables being solved for, while the EV requires the determination of the algorithmic parameters such as mutation rate, population size, and random seed.

A hybrid optimization technique merges the EV with the GRG. The hybrid EV-GRG method has two phases: (1) obtaining a vector

Table 1. Results of Estimating the Proposed Model MUSSMAM (w_{-1}, w_0, w_1) (Data from Wilson 1974)

J	Time (h)	I_j m^3/s	O_j m^3/s	S_j^P m^3	$\Delta S_i/\Delta t$ m^3/s	S_j^C m^3	$\hat{O}_j$ m^3/s	$(O_j - \hat{O}_j)^2$ m^3/s
0	0	22	22	175.65	0.00	—	22.00	0.00
1	6	23	21	175.65	1.42	175.94	22.03	1.06
2	12	35	21	184.16	17.64	187.82	22.74	3.03
3	18	71	26	289.99	59.76	302.39	27.28	1.64
4	24	103	34	648.57	82.65	665.72	34.68	0.46
5	30	111	44	1114.50	71.01	1159.30	43.95	0.00
6	36	109	55	1570.54	51.91	1581.30	56.63	2.67
7	42	100	66	1882.00	28.50	1887.91	67.92	3.67
8	48	86	75	2053.02	3.13	2053.67	77.01	4.05
9	54	71	82	2071.82	−18.75	2067.93	83.33	1.78
10	60	59	85	1959.33	−32.12	1952.65	85.96	0.92
11	66	47	84	1766.08	−42.87	1757.19	84.54	0.29
12	72	39	80	1508.84	−45.24	1499.46	80.54	0.29
13	78	32	73	1237.42	−44.89	1288.11	73.58	0.34
14	84	28	64	986.11	−39.28	959.96	65.24	1.53
15	90	24	54	732.44	−33.83	725.42	55.80	3.23
16	96	22	44	529.46	−25.68	524.13	46.53	6.41
17	102	21	36	375.36	−17.39	371.76	37.72	2.97
18	108	20	30	271.02	−11.15	268.70	30.54	0.30
19	114	19	25	204.13	−6.92	202.70	25.37	0.14
20	120	19	22	162.62	−2.98	162.00	21.91	0.01
21	126	18	19	144.76	−2.56	144.23	20.08	1.18
—	—	—	—	129.40	—	—	—	—
Sum	—	—	—	—	—	—	—	35.96

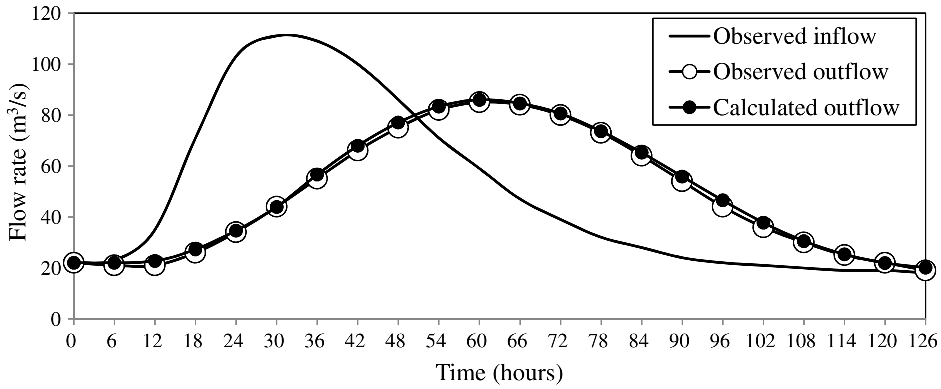

Fig. 1. Comparison of estimated hydrograph of the proposed model MUSSMAM (w_{-1}, w_0, w_1) and the observed hydrograph for the first case study [data from Wilson (1974)]

Table 2. Comparison of the Best SSQs Obtained with the Various MUSSMAM and the NMM for Case Study 1 (Data from Wilson 1974)

Model	K	X	m	w_{-1}	w_0	w_1	SSQ
NMM	0.5175	0.2869	1.8186	—	—	—	36.77
MUSSMAM (w_{-1}, w_0, 0)	0.5175	0.2869	1.8186	0	1	—	36.77
MUSSMAM (0, w_0, w_1)	0.5979	0.2955	1.8385	—	0.9654	0.0346	35.96
MUSSMAM (w_{-1}, w_0, w_1)	0.5979	0.2955	1.8385	0	0.9654	0.0346	35.96

of parameters by the EV that is used as the initial solution for the GRG method, and (2) estimation of final parameter values by GRG using the initial solution obtained in the previous step. In other words, first the EV is run, and after a few seconds of solution searching, the search algorithm is stopped. The current values in the stopped search algorithm are used as the initial estimate of the hydrologic parameters for the GRG. In this manner, the coefficient of variation (equal to standard deviation over the average) of the objective function produced by the EV-GRG method is very small.

The parameters of the MUSSMAM are the three hydrologic parameters (X, K, and m) and the weighting coefficients (w_{-1}, w_0, and w_1). The input data are Δt, I_j and O_j. The optimal solution also requires lower and upper bounds on the parameters. These differences between the lower and upper bounds of the three hydrologic parameters do not have to be small. The lower and upper bounds for weighting coefficients are 0 and 1, respectively.

It should be noted that, if needed, MUSSMAM (w_{-1}, w_0, 0), MUSSMAM (0, w_0, w_1), and NMM are obtained from MUSSMAM (w_{-1}, w_0, w_1) by setting $w_1 = 0$, $w_{-1} = 0$, and $w_{-1} = w_1 = 0$, respectively. Furthermore, all forms of MUSSMAM can be evaluated using the same developed spreadsheet MUSSMAM (w_{-1}, w_0, w_1) by setting the values of the respective parameters. This provides the user with the ability to evaluate all model forms easily and flexibly.

Applications and Results of the Proposed MUSSMAM

The proposed model was applied by using three different case studies involving experimental, real, and multimodal examples, whose results are presented in the following sections.

Case Study 1: Experimental Example

The example in Case Study 1 uses the inflow and outflow hydrograph discussed by Wilson (1974). There are three reasons for selecting this example:

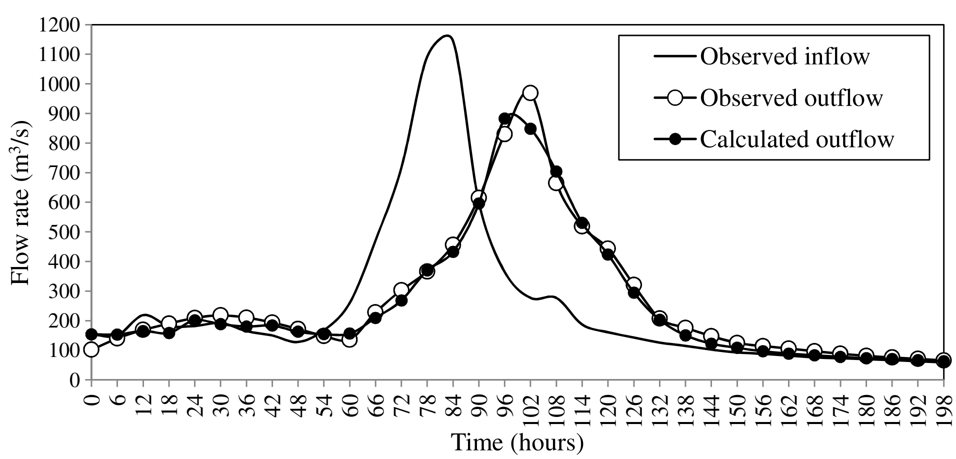

Fig. 2. Comparison of estimated hydrograph of the proposed model MUSSMAM (w_{-1}, w_0, w_1) and the observed hydrograph for the second case study

Table 3. Comparison of the Best SSQs Obtained with the Various MUSSMAM and NMM Models for Case Study 2

Model	K	X	m	w_{-1}	w_0	w_1	SSQ
NMM	0.4754	0.4092	1.5815	—	—	—	34,789
MUSSMAM (w_{-1}, w_0, 0)	0.1753	0.3503	1.7071	0.2438	0.7562	—	33,626
MUSSMAM (0, w_0, w_1)	0.6656	0.4319	1.5336	—	0.9483	0.0517	34,310
MUSSMAM (w_{-1}, w_0, w_1)	0.2295	0.3294	1.6567	0.4592	0.4048	0.1360	31,421

1. These data are shown to have a nonlinear relationship between weighted discharge and storage (Yoon and Padmanabhan 1993; Mohan 1997).
2. These data are used extensively in the literature as a benchmark problem (Gill 1978; Tung 1985; Mohan 1997; Kim et al. 2001; Geem 2006; Chu and Chang 2009; Luo and Xie 2010; Barati 2011; Xu et al. 2012; Karahan et al. 2013; Easa 2013a).
3. The performance of the proposed MUSSMAM can be compared with previous reported results for this example. The number of time steps and the duration of the time step in Wilson (1974) are $\Delta t = 6$ h and $N = 21$, respectively.

The optimal outflows and intermediate results of the proposed model [MUSSMAM (w_{-1}, w_0, w_1)] are presented in Table 1. Also, the observed and calculated hydrographs of the MUSSMAM (w_{-1}, w_0, w_1) are shown in Fig. 1. It is evident from Fig. 1 that the calculated hydrograph fits the observed hydrograph well.

By using EV-GRG as the optimization algorithm, the MUSSMAM (w_{-1}, w_0, w_1) parameters were determined as $K = 0.598$, $X = 0.296$, $m = 1.838$, $w_{-1} = 0$, $w_0 = 0.965$, and $w_1 = 0.035$. The corresponding SSQ value was calculated as 35.96. The value of w_{-1} equals zero in the solution, which means that there is no interaction between S^P_{j-1} and S^C_j.

The best optimal parameters of NMM were found by Xu et al. (2012) to be $K = 0.5157$, $X = 0.2869$, and $m = 1.8681$, and the corresponding SSQ value as 36.77. To verify the proposed MUSSMAM and developed spreadsheet, the optimal parameters of Xu et al. (2012) are substituted in the Excel worksheet of the proposed method for $w_{-1} = w_1 = 0$. These parameters yield SSQ = 36.77, which is identical to the value obtained by Xu et al. (2012).

Table 2 shows the comparison of the best objective function for Wilson's data and parameters obtained from NMM and various MUSSMAM models, such as MUSSMAM (w_{-1}, w_0, w_1), MUSSMAM (w_{-1}, w_0, 0), and MUSSMAM (0, w_0, w_1). Given that the value of w_{-1} was determined as zero with the EV-GRG method for the Wilson data, results obtained with MUSSMAM (w_{-1}, w_0, w_1) and NMM equal the results obtained from MUSSMAM (0, w_0, w_1) and MUSSMAM (w_{-1}, w_0, 0), respectively. Also, notice that MUSSMAM (w_{-1}, w_0, w_1) decreased (i.e., improved) the SSQ by 2 percent compared to the SSQ results obtained from NMM.

Table 4. Comparison of the Computed Outflows Obtained with the Various MUSSMAM and NMM Models for the Case Study 2

	Time	Observed data (m^3/s)		Computed outflow (m^3/s)			
j	(h)	I_j	O_j	NMM	MUSSMAM (w_{-1}, w_0, 0)	MUSSMAM (0, w_0, w_1)	MUSSMAM (w_{-1}, w_0, w_1)
0	0	154	102	154	154	154	154
1	6	150	140	154	154	154	153
2	12	219	169	152	152	156	165
3	18	182	190	183	178	178	159
4	24	182	209	192	198	191	201
5	30	192	218	185	186	186	189
6	36	165	210	187	186	185	181
7	42	150	194	178	181	179	184
8	48	128	172	161	161	161	163
9	54	168	149	139	140	145	155
10	60	260	136	154	150	159	156
11	66	471	228	201	196	207	209
12	72	717	303	267	263	271	269
13	78	1092	366	347	351	355	370
14	84	1145	456	419	429	417	433
15	90	600	615	602	613	592	596
16	96	365	830	879	879	881	883
17	102	277	969	839	845	839	849
18	108	277	665	689	701	686	704
19	114	187	519	531	541	523	530
20	120	161	444	414	424	411	424
21	126	143	321	290	292	291	295
22	132	126	208	203	198	206	203
23	138	115	176	150	144	155	151
24	144	102	148	123	120	125	123
25	150	93	125	102	105	1107	108
26	156	88	114	94	94	95	97
27	162	82	106	88	88	88	89
28	168	76	97	81	82	82	83
29	174	73	89	75	76	76	77
30	180	70	81	72	73	73	73
31	186	67	76	69	69	69	70
32	192	63	71	66	67	66	67
33	198	59	66	62	62	62	63

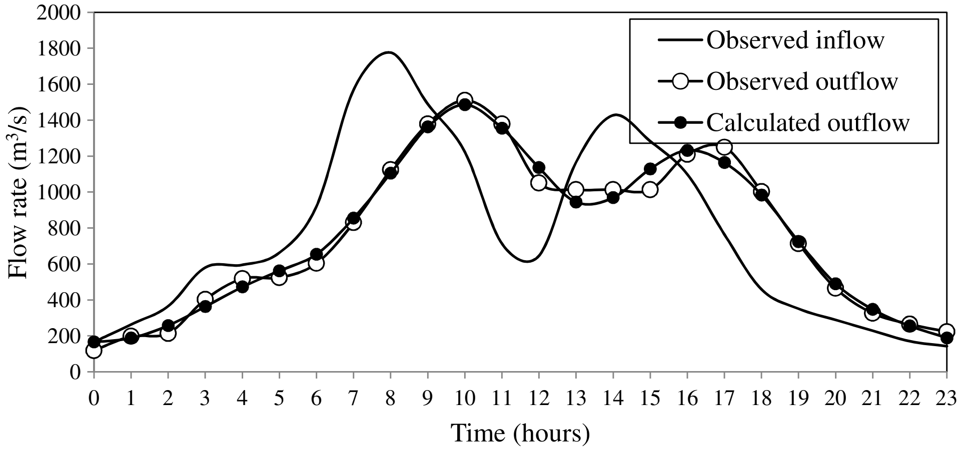

Fig. 3. Comparison of estimated hydrograph of the proposed model MUSSMAM (w_{-1}, w_0, w_1) and the observed hydrograph for the third case study

Case Study 2: Real Example

The second case study is an example of the 1960 flood in the Wye River in the United Kingdom [Natural Environment Research Council (NERC) 1975]. This flood event demonstrates the use of flood-routing techniques very well because (1) the 69.75-km stretch of the Wye River from Erwood to Belmont has no tributaries and very small lateral inflow (Bajracharya and Barry 1997); and (2) this event presents a pronounced nonlinear relationship between weighted-flow and storage volume (Barati 2013b), similar to Wilson's data. This case study includes $\Delta t = 6$ h and $N = 33$. This flood was first studied by O'Donnell (1985) with a linear Muskingum model. Also, these data were used by Karahan et al. (2013), Barati (2013b), and Hamedi et al. (2014) to estimate the parameters of NMM using the HS-BFGS, NMS, and GRG methods, respectively.

A comparison of the observed hydrograph and calculated hydrograph using the proposed model [MUSSMAM (w_{-1}, w_0, w_1)] of this case study is presented in Fig. 2, where the estimated hydrograph fits the observed hydrograph well.

The optimal parameters of the MUSSMAM (w_{-1}, w_0, w_1) are $K = 0.2295$, $X = 0.3294$, $m = 1.6567$, $w_{-1} = 0.4592$, $w_0 = 0.4048$, and $w_1 = 0.1360$, and the corresponding SSQ value is 31,421. In addition, the best SSQ of the NMM for the parameter vector ($K = 0.4754$, $X = 0.4092$, $m = 1.5815$) is 34,789, values obtained by Hamedi et al. (2014). It is evident from these results that MUSSMAM (w_{-1}, w_0, w_1) improves the fitting of the outflow data by 10 percent compared to the best result with the NMM for this example.

Tables 3 and 4 show the comparison of the best objective functions and the corresponding outflows obtained from NMM and various MUSSMAM models in the second case study. It is evident from Tables 3 and 4 that MUSSMAM (w_{-1}, w_0, w_1) has the best (i.e., lowest) values of the objective functions. In addition, these results show that the computed value of SSQ by various MUSSMAM models is smaller than the NMM. Also, notice that the SSQ of the MUSSMAM (w_{-1}, w_0, 0) showed a 2 percent improvement over the SSQ of the MUSSMAM (0, w_0, w_1).

Case Study 3: Multimodal Example

The third case study uses the data of Viessman and Lewis (2003). The input data of inflow-outflow hydrographs are shown in Fig. 3. This example involves a flood multiple peak hydrograph, with $\Delta t = 1$ day and $N = 23$. The estimated outflow hydrograph using the proposed model [MUSSMAM (w_{-1}, w_0, w_1)] is also shown in Fig. 3.

The optimal parameters of MUSSMAM (w_{-1}, w_0, w_1) are obtained as $K = 0.5463$, $X = 0.4099$, $m = 1.2141$, $w_{-1} = 0.00$, $w_0 = 0.8453$, and $w_1 = 0.1547$ by using the EV-GRG algorithm, and the SSQ value of the corresponding output hydrograph was calculated as 52,057. In this example, similar to Wilson's data, the EV-GRG method neglects the interaction between S^P_{j-1} and S^C_j in achieving the best value for SSQ.

The best SSQ of the NMM is 76,911, as shown by Easa (2013a). In addition, applying the proposed MUSSMAM ($w_{-1} = w_1 = 0$, set equal to zero in the spreadsheet) to obtain the optimal values of the parameters of NMM yields $K = 0.4754$, $X = 0.4092$, $m = 1.5815$, and SSQ $= 73{,}399$. This shows that the SSQ value of the NMM calculated in this paper is 3,512 less than the SSQ obtained with the NMM by Easa (2013a). Also, notice that the computed value of SSQ of MUSSMAM (w_{-1}, w_0, w_1) is much smaller than the NMM (SSQ $= 73{,}399$). These results demonstrate that the application of MUSSMAM (w_{-1}, w_0, w_1) substantially improves the fit to observed outflows (up to 29 percent for SSQ).

A comparison of the best objective functions obtained from NMM and various MUSSMAM models in the third case study is presented in Table 5. Considering the fact that the parameter value of w_{-1} was determined as zero, the results obtained from MUSSMAM (w_{-1}, w_0, w_1) and NMM equal the results obtained from MUSSMAM (0, w_0, w_1) and MUSSMAM (w_{-1}, w_0, 0), respectively. Also, the outflows estimated by NMM and various MUSSMAM models are listed in Table 6. It is clear that the computed outflows of MUSSMAM (w_{-1}, w_0, w_1) are better than those calculated with the NMM.

Table 5. Comparison of the Best SSQs Obtained by the Various MUSSMAM and NMM Models for Case Study 3

Model	K	X	M	w_{-1}	w_0	w_1	SSQ
NMM	0.0764	0.1673	1.4454	—	—	—	73,399
MUSSMAM (w_{-1}, w_0, 0)	0.0764	0.1673	1.4454	0	1	—	73,399
MUSSMAM (0, w_0, w_1)	0.5463	0.4099	1.2141	—	0.8453	0.1547	52,057
MUSSMAM (w_{-1}, w_0, w_1)	0.5463	0.4099	1.2141	0	0.8453	0.1547	52,057

Table 6. Comparison of the Computed Outflows Obtained with the Various MUSSMAM and NMM Models for Case Study 3

j	Time (days)	Observed data (m^3/s) I_j	O_j	Computed outflow (m^3/s) NMM	MUSSMAM (w_{-1}, w_0, 0)	MUSSMAM (0, w_0, w_1)	MUSSMAM (w_{-1}, w_0, w_1)
0	0	166.2	118.4	166.2	166.2	166.2	166.2
1	1	263.6	197.4	166.2	166.2	187.9	187.9
2	2	365.3	214.1	263.2	263.2	257.9	257.9
3	3	580.5	402.1	346.8	346.8	363.1	363.1
4	4	594.7	518.2	505.2	505.2	472.3	472.3
5	5	662.6	523.9	563.1	563.1	562.0	562.0
6	6	920.3	603.1	620.8	620.8	654.0	654.0
7	7	1,568.8	829.7	773.8	773.8	854.8	854.8
8	8	1,775.5	1,124.2	1109.5	1,109.5	1,104.7	1,104.7
9	9	1,489.5	1379	1381.7	1,381.7	1,361.8	1,361.8
10	10	1,223.3	1,509.3	1,460.5	1,460.5	1,485.4	1,485.4
11	11	713.6	1379	1,389.1	1,389.1	1,355.6	1,355.6
12	12	645.6	1,050.6	1,133.5	1,133.5	1,135.8	1,135.8
13	13	1,166.7	1013.7	890.7	890.7	944.5	944.5
14	14	1,427.2	1013.7	983.0	983.0	968.9	968.9
15	15	1,282.8	1,013.7	1,168.0	1,168.0	1,128.5	1,128.5
16	16	1,098.7	1,209.1	1,236.2	1,236.2	1,232.0	1,232.0
17	17	764.6	1,248.8	1,192.9	1,192.9	1,164.5	1,164.5
18	18	458.7	1,002.4	1,019.8	1,019.8	983.8	983.8
19	19	351.1	713.6	743.0	743.0	723.7	723.7
20	20	288.8	464.4	501.3	501.3	489.2	489.2
21	21	228.8	325.6	345.1	345.1	346.9	346.9
22	22	170.2	265.6	245.2	245.2	255.3	255.3
23	23	143	222.6	168.9	168.9	188.5	188.5

Concluding Remarks

Several researchers have focused on modifying the structure of the NMM to improve the fitting goodness of this model. This fact motivated the research presented in this paper, which aimed at modifying the structure of the NMM to achieve greater capabilities in routing outflow hydrographs. For this, the SMA was coupled with NMM and called MUSSMAM.

The following conclusions can be drawn:

- The proposed model, MUSSMAM, is a third-order MA of the storage volume [MUSSMAM (w_{-1}, w_0, w_1)], or a second-order MA of the storage volume [MUSSMAM (w_{-1}, w_0, 0) and MUSSMAM (0, w_0, w_1)]. MUSSMAM (w_{-1}, w_0, 0), MUSSMAM (0, w_0, w_1), and NMM are special cases of MUSSMAM (w_{-1}, w_0, w_1).
- MUSSMAM was formulated on a spreadsheet that is directly solved with the optimization software Solver in Excel. This software can calibrate the Muskingum parameters using (1) the GRG solver, and (2) the EV solver. To overcome the disadvantages of the GRG and EV, the optimization model was efficiently solved using EV and GRG jointly. The *Solver* software is ubiquitous and widely accessible for rapid implementation.
- The amount of parameter-fitting improvement of the proposed model varies with the type of case study. Results illustrated that MUSSMAM (w_{-1}, w_0, w_1) yields optimal SSQ that are 2, 10, and 29 percent smaller (i.e., better) than the corresponding values calculated with NMM in the experimental, real, and multimodal problems, respectively. In addition, these results showed that the computed values of the SSQ calculated with MUSSMAM (w_{-1}, w_0, 0) and MUSSMAM (0, w_0, w_1) are equal or smaller (i.e., better) than those obtained with the NMM. The proposed model is intended as a practical and helpful tool for practitioners concerned with flood routing.

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