Lawrence Berkeley National Laboratory



GEOMETRIC ABERRATIONS IN FINAL FOCUSSING FOR HEAVY ION FUSION



**Author**
Neuffer, D.

**Publication Date**
1978-11-01

Presented at and to be published in the
Proceedings of the Heavy Ion Fusion
Workshop, Argonne National Laboratory,
Argonne, Illinois, September 19-26, 1978

LBL-8304

# GEOMETRIC ABERRATIONS IN FINAL FOCUSSING FOR HEAVY ION FUSION

David Neuffer

November 1978

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

LAWRENCE BERKELEY LABORATORY • UNIVERSITY OF CALIFORNIA • BERKELEY

# GEOMETRIC ABERRATIONS IN FINAL FOCUSSING FOR HEAVY ION FUSION*

David Neuffer
Lawrence Berkeley Laboratory

The final focus of a heavy ion fusion (HIF) accelerator requires focussing of an intense ion beam on an extremely small target spot. This requirement has led to some concern that third-order aberrations will significantly distort focussing determined by first-order theory.[1,2] Earlier calculations have confirmed that this is true for a number of possible HIF beam/target combinations.[3] This note develops these calculations to a more general formulation which can be used to estimate third-order distortion without detailed calculations. Several candidate HIF beams are presented in some detail as examples. Some general ideas on the constraints which third-order aberrations place on HIF parameters are developed.

## I. Theoretical Formulation

The non-relativistic equations of motion to third-order can be derived from the Lorentz equation:

$$m \ddot{\vec{x}} = \frac{e}{c} \vec{V} \times \vec{B} \tag{1}$$

We change the independent variable from time t to the longitudinal position z using

$$\frac{dz}{dt} = V_o/(1 + x'^2 + y'^2)^{1/2}$$

(The notation $u' = \frac{du}{dz}$ is used throughout.)

Equation (1) becomes two equations:

$$\begin{aligned} (1 + y'^2)x'' - x'y'y'' &= \frac{e}{mV_o c} (1 + x'^2 + y'^2)^{3/2}(-B_y + y'B_z) \\ -x'y' + (1 + x'^2)y'' &= \frac{e}{mV_o c} (1 + x'^2 + y'^2)^{3/2}(B_x - x'B_z) \end{aligned} \tag{2}$$

*This work was supported by the Div. of Laser Fusion and the Div. of High Energy and Nuclear Physics of the Dept. of Energy

The magnetic field $\vec{B}$ is obtained from the gradient of a scalar potential $\Phi$ with quadrupole symmetries:

$$\Phi = -xy\phi(z) + \frac{1}{12}(x^2+y^2)\,xy\phi''(z) \qquad \text{(to third-order)} \tag{3}$$

We use the relation $\frac{e\vec{B}}{mV_oc} = -\vec{\nabla}\Phi$ to define the units of $\Phi$.

Substituting $\vec{B}$ into the equations of motion, separating x" from y", and keeping terms only up to third order, we find for x":

$$\begin{aligned} x'' = &-\phi x + \left(-\frac{3}{2}\,x'^2x - \frac{1}{2}\,y'^2x + x'y'y\right)\phi \\ &+ xyy'\phi' + \left(\frac{y^2x}{4} + \frac{x^3}{12}\right)\phi'' \end{aligned} \tag{4}$$

This equation of motion is applied to a quadrupole system, with $\phi$ constant throughout the interior of the quadrupoles and $\phi'$ and $\phi''$ nonvanishing only at the edges. To first order particle motion is given by the unperturbed equations:

$$x'' = -\phi x, \qquad y'' = \phi y$$

and a Green's function integration can be used to find the higher order corrections. The correction to x is

$$\begin{aligned} \Delta x = \int_0^{z_F} G_x(z_F,\xi)\cdot\Big\{ &\left(-\frac{3}{2}\,x'^2x - \frac{1}{2}\,y'^2x + x'y'y\right)\phi \\ &+ xyy'\phi' + \left(\frac{y^2x}{4} + \frac{x^3}{12}\right)\phi''\Big\}\,d\xi \end{aligned} \tag{5}$$

where $G_x(z_F,\xi) = x_o(z_F)x_e(\xi) - x_e(z_F)x_o(\xi)$ (6)

and $x_o$, $x_e$ are the odd and even solutions of the unperturbed equations. If we choose the points 0 and $z_F$ where $\phi = \phi' = \phi'' = 0$, we can use partial integrations to remove the derivatives of $\phi$ from the integration 5.

The result is, as previously derived by Meads,[4]

$$\begin{aligned} \Delta x = &\int_0^{z_F} \phi^2\, G_x\left(-xy^2 - \frac{1}{3}\,x^3\right)d\xi && \text{A} \\ &+ \frac{1}{2}\int_0^{z_F} \phi\, G_x\, x'\,(x^2 + y^2)\,d\xi && \text{B} \\ &+ \int_0^{z_F} \phi\, G_x\left(-x\,(x'^2 + y'^2) + x'yy'\right)d\xi && \text{C} \end{aligned} \tag{7}$$

In deriving 7 from 5, we have used relationships such as $x'' = -\phi x$, $y'' = +\phi y$, $G_x'' = -\phi G_x$. The expression for $\Delta y$ is obtained from 7 by the transformation $x \leftrightarrow y$, $\phi \rightarrow -\phi$, with the warning that $G_y$, also given by equation 6, is different from $G_x$.

Equation 7 can be compared with equation 5 to separate the contributions of fringe fields ($\psi'$, $\psi'' \neq 0$) from geometric contributions.

Particle acceptance may be increased in final focussing by use of octupole magnets. An octupole field adds a term

$$\Delta\Phi = -\frac{1}{3}(x^3y - y^3x)\psi(z)$$

to the magnetic potential and a term

$$\Delta x_{oct} = + \int_0^{z_F} G_x\, \psi\, (\frac{x^3}{3} - y^2x)d\xi \qquad D \qquad (7)$$

to equation 7. $\Delta y_{oct}$ is obtained from equation 7D by the exchange $x \to y$ and $\psi \to + \psi$. In the calculations of Sections II and III no octupole elements have been included, however.

The third-order corrections $\Delta x$, $\Delta y$ to the particle positions can be calculated for individual rays using equation 7 and such calculations are used to determine the particle acceptances displayed in Section III. However, in the following section approximations valid for the HIF final focus are used to derive simplified formulas which can be used to estimate the effect of third-order aberrations in an arbitrary HIF system.

## II. Approximate Formalism

We reduce equation 7 by expressing its terms using the transfer matrix from $z = 0$ to $z = z_F$

$$\begin{pmatrix} x_e(z) & x_0(z) \\ x_e'(z) & x_0'(z) \end{pmatrix} = \begin{pmatrix} M_{11}(z) & M_{12}(z) \\ M_{21}(z) & M_{22}(z) \end{pmatrix} \qquad (8)$$

We assume the beam is parallel at the entrance to the final focus ($M_{21}(0) = 0$) and the target is at a point focus ($M_{11}(z_F) = 0$). Also because of the final focus constraints, $M_{12}(z_F) = \frac{x_0 r_T}{\varepsilon} = \beta_x(0)\beta_F$ where $x_0$ is the maximum beam amplitude at $z = 0$, $r_T$ is the target spot radius, $\pi\varepsilon$ is the transverse emittance, and $\beta_x(0)$, $\beta_F$ are the initial and final beta function values.

Then

$$G_x(z_F,\xi) = M_{11}(\xi)\ \beta_x(0)\beta_F$$
$$G_x'(z_F,\xi) = M_{21}(\xi)\ \beta_x(0)\beta_F \qquad (9)$$

with similar expressions for $G_y$, $G_y'$.

We calculate ($\Delta x$, $\Delta y$) for a distribution of particle rays, each with some initial coordinates ($x,x',y,y'$) with

$$(x,x',y,y') = (X_0u, \frac{\varepsilon}{X_0} u', Y_0v, \frac{\varepsilon}{Y_0} v') \equiv (X_0u, X'_0u', Y_0v, Y'_0v') \quad (10)$$

$X_0$ and $Y_0$ are the maximum x and y amplitudes of the beam envelope at z = 0 and $(u,u',v,v')$ are numbers which lie between -1 and +1. (For a K-V distribution $u^2 + u'^2 + v^2 + v'^2 = 1$)

The first order value of x(z) is $x(z) = X_0u\, M_{11}(z) + X'_0\, u' M_{12}(z)$ (11) with similar expressions for x'(z), y(z), y'(z).

Equations (9) and (10) can be inserted in equation (7) obtaining an expression for $\Delta x$ of the form

$$\Delta x = U_{1111} X_0^3 u^3 + U_{1133} X_0 Y_0 uv^2 + U_{1112} X_0^2 X'_0 u^2 u' + \cdots (10 \text{ terms total}) \quad (12)$$

In (12) the $U_{ijkl}$ are third-order transport coefficients. For HIF final focussing $X_0$ and $Y_0$ are large, and we can find approximate expressions for $\Delta x$ and $\Delta y$ by ignoring all terms proportional to powers of $X_0'$ and $Y_0'$.

Then $$\Delta x \cong U_{1111} X_0^3 u^3 + U_{1133} X_0 Y_0^2 uv^2 \quad (13)$$

$$U_{1111} = -\beta_x(0)\beta_F \int_0^{z_F} \left(\frac{\phi^2 M_{11}^4}{3} + \frac{\phi}{2} M_{11}^2 M_{21}^2\right) dz \quad (14)$$

$$U_{1133} = -\beta_x(0)\beta_F \int_0^{z_F} [\phi^2 M_{11}^2 M_{33}^2 - \frac{\phi}{2} M_{21}^2 M_{33}^2 + \phi M_{11}^2 M_{43}^2 - \phi M_{11} M_{43} M_{21} M_{33}] dz \quad (15)$$

with a similar expression for $\Delta y$.

To evaluate these coefficients we consider the family of final focussing solutions derived by Garren in the 1976 HIF Summer Study.[5] These are doublet solutions with the x-coordinate the FDO coordinate and the apertures of the two quads given by $R_F = R_D = X_0$; the solutions are expressed in terms of a single parameter b where $b = \frac{LBr_T}{B\rho\varepsilon}$, and L is the final drift distance, B the FD pole tip field and $B\rho$ is the ion rigidity.

Other Garren parameters are

$$k = \frac{L^2 B}{B\rho X_0}, \quad x = \frac{X_0}{L\theta_0}, \quad y = \frac{Y_0}{L\theta_0}, \quad \text{with } \theta_0 = \frac{\varepsilon}{r_T}.$$

We define normalized third-order matrix elements $U^N_{ijkl}$ in terms of these parameters by

$$U_{1111}^N = X_o^3 U_{1111} = - L\theta_o^3 k^{3/2} x^4 \int_o^{\psi_F} M_{11}^2 \left(\frac{M_{11}^2}{3} + \frac{M_{12}^2}{2\phi}\right) d\psi \qquad (16)$$

where $d\psi = |\phi| dz$

which we write as

$$U_{1111}^N = - L\theta_o^3 g_x(b)$$

Also we write $U_{1133}^N = U_{1133} X_o Y_o^2 = - L\theta_o^3 h_x(b)$

$$U_{3333}^N = U_{3333} Y_o^3 = - L\theta_o^3 g_y(b) \qquad (17)$$

$$U_{3311}^N = U_{3311} Y_o X_o^2 = - L\theta_o^3 h_y(b)$$

which defines parameters $g_x$, $g_y$, $h_x$, $h_y$ which are functions of the Garren parameter b. It can be shown that, for parallel to point focussing, $U_{3311}^N = U_{1133}^N$ and $h_x = h_y \equiv h$. The parameters $g_x$, $g_y$ and h are tabulated in Table I, which supplements Table A8-7I in reference 5. Figure 1A displays the same information graphically.

For FDO focussing $\Delta x > \Delta y$ and $\Delta x = U_{1111}^N u^3 + U_{1133}^N uv^2$. The maximum value of $uv^2$ is .39 for a K-V elliptical distribution and the evaluation of $\Delta x$ is dominated by $U_{1111}^N$ (see Figure 1)

We define a quantity

$$\Delta x_{max} = U_{1111}^N$$

and in Figure 1B we have graphed particle acceptance at the focus as a function of $\Delta x_{max}/r_T$ assuming two possibilities of initial particle distributions at $z = 0$: 1) a K-V distribution, or 2) a "W" distribution (particles uniformly populating the interior of the K-V hyper-ellipse).

The criterion for acceptance is

$$r_F^2 = x_F^2 + y_F^2 = (x_F^0 + \Delta x)^2 + (y_F^0 + \Delta y)^2 \leq r_T^2 \qquad (18)$$

where $x_F^0$, $y_F^0$ are first-order values. Figure 1B shows particle acceptance falls off steeply with $\Delta x_{max}/r_T > 1$. As noted in Reference 3 particle acceptance is improved by moving the target from the focus to the position of least confusion, i.e., maximum particle acceptance, which is about 1% closer to the focussing doublet. The effect of this is also displayed in Figure 1B.

Figures 1A and 1B can be used to determine the acceptability of a particular HIF final focussing system.

## III. Examples of Final Focussing Systems for HIF

As individual examples of final focussing for HIF we consider two possible cases in detail. The first example we consider was suggested by Keefe[6]:

19 GeV $U^{+4}$ with $r_T$ = 1.25 mm, B = 4T. The Garren parameter b can be found from

$$b = 2.7 \frac{L(m)}{\varepsilon_N(\text{cm-mR})}$$

where $\varepsilon_N = \beta\gamma\varepsilon$ is the normalized emittance. We calculate final focussing with L = 10 m and L = 5 m, which demonstrates that smaller L is favored, but L $\lesssim$ 5 m is probably impractical. The results are displayed in Table 2A.

If we require an acceptance of a "W" distribution of greater than 75% at the focus (~ 90% at the point of "least confusion"), we require $\varepsilon_N \lesssim 0.98$ (cm-mR) for L = 5 m and $\varepsilon_N \lesssim 0.6$ for L = 10 m. In both cases the maximum quadrupole aperture $X_0$ must be less than 0.18 m.

Keefe[6] has suggested that this value of $\varepsilon_N$ can be obtained from a larger value of emittance at the exit of the accelerator $\varepsilon_0$ by splitting the beam transversely into N separate beams. If this is achieved without dilution, a final emittance per beam $\varepsilon_F = \varepsilon_0/\sqrt{N}$ can be obtained at the final focus. Thus if $\varepsilon_N$ = 3 cm-mR, N $\geq$ 9 (for L = 5 m) is required.

A second example is suggested in the Hearthfire III proposal of Arnold et.al.[1]:

20 GeV $Xe^{+8}$ $r_T$ = 0.8 mm, L = 5 m, $B\rho$ = 30.3 T-m, and $b = 6.3 \frac{L\ (m)}{\varepsilon_N(\text{cm-mR})}$

at B = 4T. The results for our calculations are contained in Table 2B. 75% acceptance demands $\varepsilon_N < 0.5$ and $X_0 < 0.09$ m. This is a smaller value of $\varepsilon_N$ than the 2.8 cm-mR value desired in Reference 1.

## IV. General Comments on Geometric Aberrations

A general requirement for a successful final focus (for FDO focussing)is:

$$\frac{\Delta x_{max}}{r_T} \lesssim 1 \qquad \frac{L\theta_o^3}{r_T} g(b) \lesssim 1 \tag{19}$$

This can be re-expressed in terms of $\varepsilon$, $r_T$, b and $\rho(\rho = B\rho/B)$ as

$$\frac{\rho\varepsilon^4}{r_T^5} b\, g(b) \lesssim 1 \text{ or } \varepsilon \lesssim \rho^{-1/4} r_T^{5/4} (bg(b))^{-1/4} \tag{20}$$

We define a parameter $\Gamma(b) = (bg(b))^{-1/4}$ which is tabulated in Table I. $\Gamma(b)$ is maximum at b = 6 with a value of 0.17; it is roughly constant for b $\lesssim$ 20, and varies as $1/\sqrt{b}$ for large b. It is important to note that for b < 6 the Garren doublet solutions become somewhat unrealistic and triplet solutions are preferred, and Table I is not useable.

Equation (20) can be used to estimate maximum allowable emittance for a given HIF system. Larger target size is greatly favored. It has not yet been determined how equation (20) is modified by including octupole elements for correction and/or by higher order aberrations. These questions will be investigated by a number of researchers.[7,8]

We thank A. A. Garren, L. Smith, S. Fenster, E. Colton and D. Keefe for useful comments.

## References

1. R. C. Arnold, et.al., "Hearthfire Reference Concept 3: A Rapid Cycling Synchrotron System", ANL-ACC-6, June 16, 1978.
2. L. Smith, private communication.
3. D. Neuffer, HI-FAN-36, May 18, 1978.
4. P. F. Meads, Jr., Ph.D. Thesis, UCRL-10807, May 15, 1963.
5. A. A. Garren, Appendix 8-9 in the ERDA Summer Study of Heavy Ions for Inertial Fusion, LBL-5543, July, 1976.
6. D. Keefe, private communications.
7. S. Fenster and E. Colton, private communications.
8. D. Neuffer, private communications.

Table I - Geometric Aberration Parameters

| Garren Parameter b | $g_x(b)$ | $g_y(b)$ | $h(b)$ | $\Gamma(b)$ |
|---|---|---|---|---|
| 1.0 | 1744. | $1. \times 10^{-6}$ | 0.3 | 0.155 |
| 1.5 | 1125. | $1. \times 10^{-4}$ | 0.9 | 0.156 |
| 2.0 | 790. | $9. \times 10^{-4}$ | 1.9 | 0.159 |
| 2.5 | 592. | $4. \times 10^{-3}$ | 3.7 | 0.161 |
| 3.0 | 460. | $1.4 \times 10^{-2}$ | 5.9 | 0.164 |
| 3.5 | 371. | $4. \times 10^{-2}$ | 8.5 | 0.167 |
| 4.0 | 310. | $8. \times 10^{-2}$ | 11.4 | 0.169 |
| 4.5 | 266. | 0.2 | 14.6 | 0.170 |
| 5. | 233. | 0.3 | 17.9 | 0.171 |
| 6. | 192. | 0.7 | 25. | 0.172 |
| 7. | 167. | 1.3 | 32. | 0.171 |
| 8. | 153. | 2.2 | 39. | 0.169 |
| 9. | 144. | 3.3 | 46. | 0.167 |
| 10. | 139. | 4.5 | 54. | 0.164 |
| 11. | 136. | 6.0 | 62. | 0.161 |
| 12. | 135. | 7.5 | 70. | 0.158 |
| 14. | 135. | 11.1 | 87. | 0.152 |
| 16. | 139. | 15.2 | 104. | 0.146 |
| 18. | 143. | 19.7 | 122. | 0.140 |
| 20. | 150. | 24.3 | 141. | 0.135 |
| 22. | 156. | 30. | 161. | 0.131 |
| 24. | 163. | 35. | 180. | 0.126 |
| 26. | 171. | 40. | 201. | 0.122 |
| 28. | 179 | 46. | 222. | 0.119 |
| 30. | 187. | 53. | 245. | 0.116 |
| 35. | 209. | 68. | 300. | 0.108 |
| 40. | 232. | 85. | 360. | 0.102 |
| 45. | 256. | 102. | 420. | 0.097 |
| 50. | 280. | 122. | 480. | 0.092 |
| 55. | 306. | 138. | 540. | 0.088 |
| 60. | 331. | 163. | 610. | 0.0842 |
| 70. | 386. | 203. | 750. | 0.0780 |
| 80. | 440. | 248. | 890. | 0.0730 |
| 90. | 496. | 297. | 1040. | 0.0688 |
| 100. | 555. | 343. | 1200. | 0.0652 |

Table II: Sample Cases of FDO Focussing

A: 19 GeV $U^{+4}$, $r_T$ = 1.25 mm, $R_F = X_0$, $R_D = 0.85\ X_0$, B = 4T.

| $\epsilon_N$ (cm-mR) | $L_F$ (m) | $L_D$ (m) | $L_0$ (m) | $X_0$ (m) | $Y_0$ (m) | $\frac{\Delta x_{max}}{r_T}$ | Acceptance at Focus (K-V) | (W) |
|---|---|---|---|---|---|---|---|---|
| 2.0 | 3.36 | 3.76 | 10. | 1.00 | 0.275 | 58.5 | 6% | 21% |
| 1.5 | 2.59 | 2.73 | 10. | 0.64 | 0.203 | 26.2 | 10 | 32 |
| 1.0 | 1.84 | 1.84 | 10. | 0.37 | 0.137 | 9.5 | 19 | 47 |
| 0.75 | 1.46 | 1.42 | 10. | 0.25 | 0.104 | 4.8 | 31 | 61 |
| 0.50 | 1.07 | 1.00 | 10. | 0.15 | 0.071 | 2.0 | 56 | 83 |
| 2.0 | 3.14 | 4.22 | 5. | 0.78 | 0.15 | 37.3 | 11% | 31% |
| 1.5 | 2.39 | 2.93 | 5. | 0.46 | 0.104 | 13.2 | 18 | 46 |
| 1.0 | 1.65 | 1.84 | 5. | 0.24 | 0.068 | 3.7 | 40 | 74 |
| 0.75 | 1.29 | 1.36 | 5. | 0.16 | 0.051 | 1.6 | 66 | 92 |

B: 20 GeV $Xe^{+8}$, $r_T$ = 0.8 mm, B = 4T

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2.80 | 2.03 | 2.39 | 5. | 0.903 | 0.222 | 175. | 6% | 11% |
| 1.50 | 1.14 | 1.17 | 5. | 0.334 | 0.113 | 31. | 10 | 28 |
| 1.00 | 0.800 | 0.817 | 5. | 0.194 | 0.076 | 11.2 | 16 | 41 |
| 0.75 | 0.655 | 0.620 | 5. | 0.134 | 0.058 | 6.0 | 25 | 55 |
| 0.50 | 0.476 | 0.440 | 5. | 0.082 | 0.040 | 2.5 | 49 | 76 |
| 0.40 | 0.402 | 0.367 | 5. | 0.063 | 0.032 | 1.6 | 62 | 88 |
| 0.25 | 0.283 | 0.253 | 5. | 0.036 | 0.021 | 0.6 | 100 | 100 |

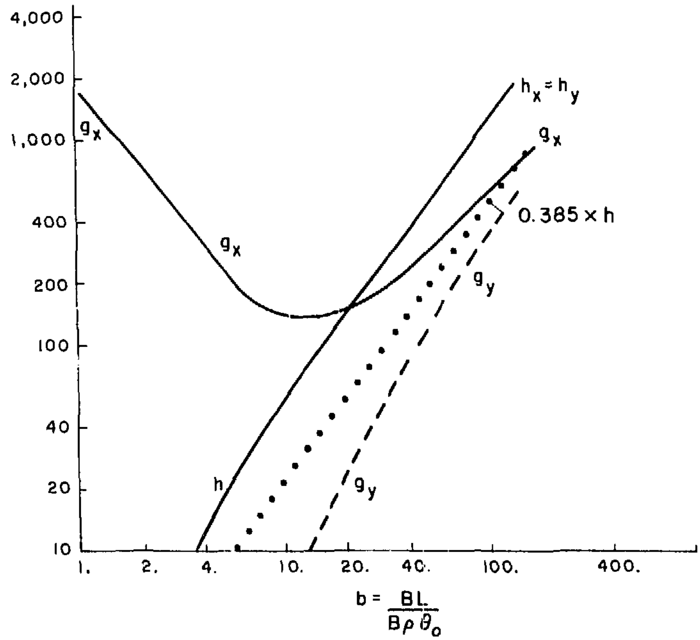


Figure 1A - GEOMETRIC ABERRATION PARAMETERS: $\begin{matrix} g_x & h_x \\ g_y & h_y \end{matrix}$

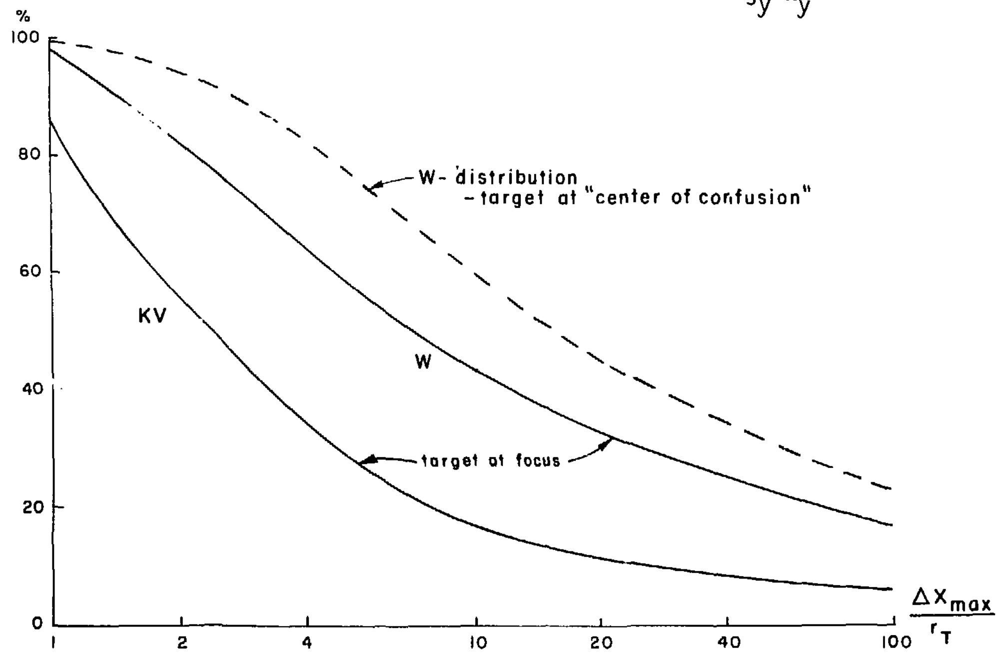


Figure 1B - PARTICLE ACCEPTANCE AS FUNCTION OF $\frac{\Delta X_{max}}{r_T}$

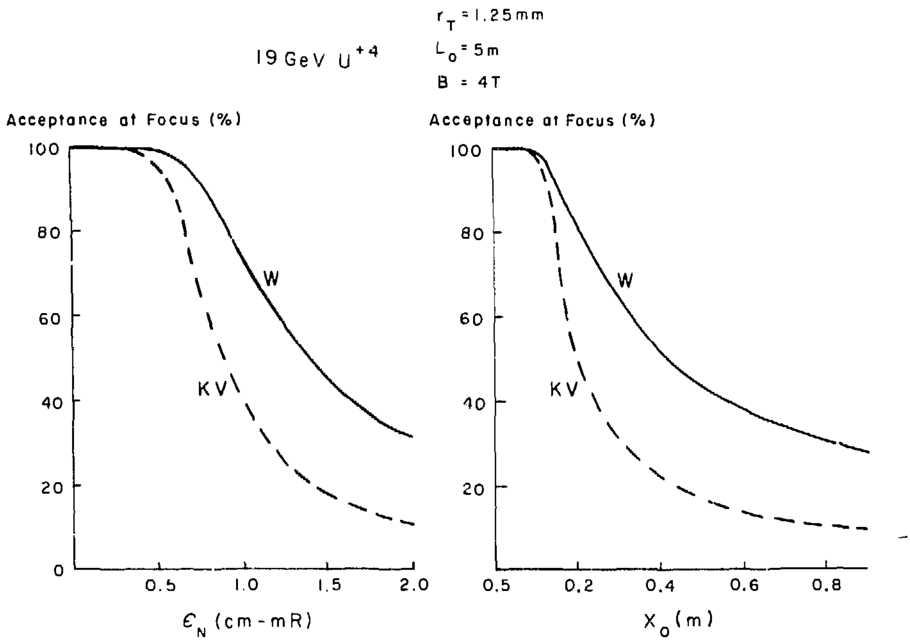


Figure 2A - PARTICLE ACCEPTANCE AS A FUNCTION OF NORMALIZED EMITTANCE ($\varepsilon_N$) AND QUADRUPOLE APERTURE ($X_0$) FOR 19 GeV $U^{+4}$

20 GeV $Xe^{+8}$ $r_T$ = 0.8 mm, $L_0$ = 5 m, B = 4 T

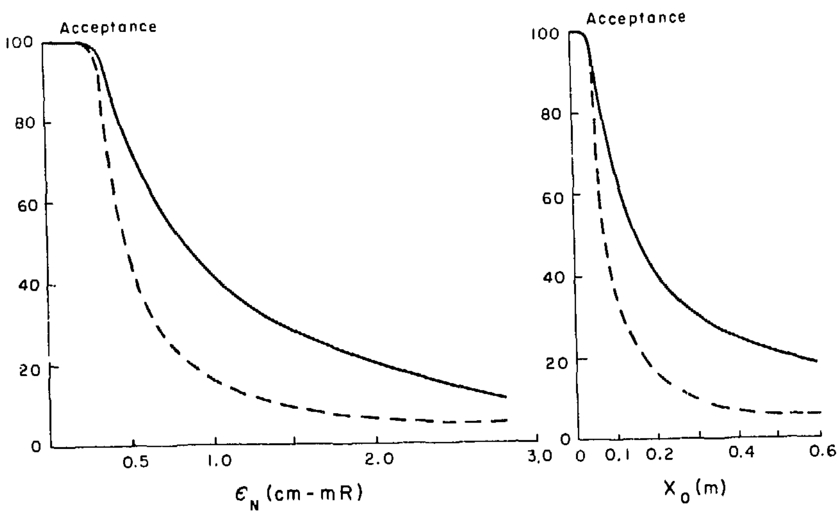


Figure 2B - PARTICLE ACCEPTANCE (%) FOR 20 GeV $Xe^{+8}$