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Publication Date

2016

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UNIVERSITY OF CALIFORNIA,
IRVINE

Essays on Agent Heterogeneity in Macroeconomics

DISSERTATION

submitted in partial satisfaction of the requirements of the degree of

DOCTOR OF PHILOSOPHY

in Economics

by

Jose Luis Luna Alpizar

Dissertation Committee:

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2016

DEDICATION

“During this fight, I’ve seen a lot of changing, in the way you feel about me, and in the way I feel about you. In here, there were two guys killing each other, but I guess that’s better than twenty million. I guess what I’m trying to say, is that if I can change, and you can change, everybody can change!”

— Sylvester Stallone, *Rocky IV*

I dedicate this dissertation to my Mom.

TABLE OF CONTENTS

	Page
LIST OF FIGURES	vi
LIST OF TABLES	viii
ACKNOWLEDGMENTS	ix
CURRICULUM VITAE	x
ABSTRACT OF THE DISSERTATION	xiii
1 A Search Model of the Labor Market with Heterogeneous Workers	1
1.1 Introduction	1
1.2 Related Literature	4
1.3 Job Rationing and Moral Hazard	9
1.3.1 The Basic Model	9
1.3.2 Job Rationing	17
1.4 Worker Heterogeneity	25
1.4.1 The firm's behavior under heterogeneity.	26
1.4.2 Job Rationing among different workers	33
1.5 Simulation	35
1.5.1 Calibration	35
1.5.2 Simulation of the model with a representative worker	37
1.5.3 Simulation of the model with worker heterogeneity	40
1.5.4 The importance of the skill distribution	43

1.6	Final Remarks	45
2	Worker Heterogeneity and the Asymmetric Effects of Minimum Wages	47
2.1	Introduction	47
2.2	Related Literature	51
2.3	The model	53
2.3.1	The Model's Environment	54
2.3.2	Equilibrium	60
2.3.3	Minimum Wage	65
2.4	Evidence on Asymmetric Effects of Minimum Wages on Labor Market Outcomes	71
2.4.1	Data	72
2.4.2	Estimation Strategy	76
2.4.3	Results	78
2.5	Quantitative Exercises	86
2.5.1	Calibration	86
2.5.2	Changes in the Minimum Wage.	91
2.6	Concluding Remarks	98
3	Exchange Rate Dynamics in a Simple Model with Heterogeneous Expectations	100
3.1	Introduction	100
3.2	A model for currency exchange rate with heterogeneous beliefs	104
3.2.1	Exchange rate model	105
3.2.2	Evolutionary dynamics and heterogeneous beliefs	108
3.3	The Brown-von Neumann-Nash (BNN) Dynamic	111
3.4	Steady states and local stability analysis	115
3.5	Numerical Analysis of Global Dynamics	118

3.6 Conclusions	129
Bibliography	132
Appendices	143
Appendix A	143
Appendix B	156
Appendix C	175

LIST OF FIGURES

1.1	Worker Surplus and the No-Shirk Condition (NSC)	14
1.2	Nash Bargaining and Efficiency Wages.	19
1.3	Composition of Equilibrium Unemployment	21
1.4	The firm’s hiring strategy.	28
1.5	Equilibrium Unemployment Across the Cycle.	38
1.6	Unemployment Elasticity and Job Rationing	39
1.7	Equilibrium Outcomes with Heterogeneity.	41
1.8	Equilibrium Unemployment for different distribution variances.	44
2.1	The firm’s hiring strategy	63
2.2	Wage schedule under a minimum wage $m < w^E(\theta)$	67
2.3	Wage schedule under a minimum wage $m' > \max\{w^E(\theta), w^E(\theta')\}$	69
2.4	Suboptimal use of minimum wages	71
2.5	Wage Distributions by Educational Attainment, 1994-2013	74
2.6	Mean Low-Education Labor Market Outcomes, 1994-2013	75
2.7	The firm’s hiring strategy	87
2.8	Wage Schedule	89
2.9	Estimated Productivity Distribution	91
2.10	The Efficiency Wage and the Minimum	93
2.11	Labor Market Outcomes by Deciles	94
2.12	Total Employment and Labor Force Participation	96
2.13	Aggregate Welfare	97
3.1	Bifurcation Diagrams for θ	120
3.2	Bifurcation Diagrams for θ	121
3.3	Phase Plots	122
3.4	Largest Lyapunov Exponent for Different values of θ	123

3.5	Bifurcation Diagrams for C	124
3.6	Largest Lyapunov Exponent for Different Values of C	125
3.7	Phase Plots	126
3.8	Bifurcation Diagrams for s	127
3.9	Bifurcation Diagrams for σ^2	128

LIST OF TABLES

1.1	Baseline Parameter Values in Simulations of the Model	36
1.2	Standard Deviations Across the Cycle by Deciles	42
A1	Outcomes by deciles for different technology levels	153
A2	Outcomes by deciles	154
A3	Outcomes by deciles	155
B1	Descriptive Statistics by Age and Educational Attainment, 1994-2013.	163
B2	Minimum Wage Effects on Employment.	164
B3	Minimum Wage Effects on Labor Force Participation.	165
B4	Minimum Wage Effects on Search Intensity.	166
B5	Minimum Wage Effects on Hours.	167
B6	Minimum Wage Effects on Wages.	168
B7	Minimum Wage Effects on Low-Education Employment by Age, 1994-2013	169
B8	Minimum Wage Effects on Low-Education Labor Force Participation by Age, 1994-2013.	170
B9	Minimum Wage Effects on Low-Education Search Intensity by Age, 1994-2013	171
B10	Minimum Wage Effects on Low-Education Weekly Hours by Age, 1994-2013.	172
B11	Minimum Wage Effects on Low-Education Wages by Age, 1994-2013	173
B12	Low-Wage Mean Labor Market Outcomes	174

ACKNOWLEDGMENTS

I would like to thank my dissertation committee for all their support. I am particularly indebted to Guillaume Rocheteau for being a great influence and role model for all these years. Having the privilege of working with you change my life forever. I thank Bill Branch for his invaluable input and guidance.

I also thank all the Macro crew for their help and support. Thank you: Eric Swanson, Jose Antonio Rodriguez-Lopez, Mario Silva, Ayushi Bajaj, and Arghya Bhattacharya.

Last but not least, I thank my fellow graduate student friends: Kim Makuch, Nanneh Chehras, Ian Finn, and Brian Asquith. I would not have made it without you. Love you guys.

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ABSTRACT OF THE DISSERTATION

Essays on Agent Heterogeneity in Macroeconomics

By

Jose Luis Luna Alpizar

Doctor of Philosophy in Economics

University of California, Irvine, 2016

Professor Guillaume Rocheteau, Chair

Heterogeneous agents models have become the norm in modern macroeconomics as the limitations of the representative-agent paradigm and the importance of studying household heterogeneity grow in recognition. Agent heterogeneity may not only be important to accurately capture the description of an aggregate equilibrium. Also, the representative agent assumption may hide many distributional effects and therefore could change the answer to many normative questions usually given by representative agent models.

This dissertation contains three chapters exemplifying ways in which the consideration of heterogeneous agents in the modelling of macroeconomic phenomena has important repercussions for the predictions of the model and its normative implications. Chapters 1 and 2 show the importance of accounting for worker heterogeneity in the analysis of labor markets. Chapter 1 presents a search and matching model of unemployment with heterogeneous workers which's main features, are ex-ante worker heterogeneity and undirected search. These features enable the model to replicate the empirical correlations between labor market outcomes and proxy variables for worker productivity. The model displays job rationing, which makes it useful to understand the high levels of unemployment observed in deep recessions. It also constitutes a versatile tool for the analysis of several labor-market aspects in which worker heterogeneity could play an important role, such as the impact of employment policies that are believed to have asymmetric effects across the labor force.

Chapter 2 provides an example of such applications by analyzing the effects of increments of a minimum wage. It explores theoretically and empirically the notion that minimum wages affect low-skill workers asymmetrically due to productivity differences. Using the model presented in chapter 1, with the incorporation of endogenous search intensity to account for the effects that minimum wages could have on worker participation, I show that a rising minimum wage lowers the employment and labor force participation of low-productivity workers by pricing them out of the market, while it increases the employment, participation, and wages of more productive workers that remain hireable. Chapter 2 also contains an empirical analysis that investigates and ultimately validates the model's predictions of changes in the minimum wage. Within the labor market for low-education (high school or lower) workers, increments in the minimum wage have diametrically opposed effects: they reduce the employment and labor force participation of teenagers with less than high school education, while increasing the employment and labor force participation of mature workers with high school educational attainment. A calibrated version of the model targeting the low-education labor market shows that, despite its opposite effects across the labor force, an increase in the minimum wage negatively impacts aggregate employment, labor force participation, and social welfare.

Chapter 3 investigates the existence of complex dynamics in the behavior of exchange rates due to heterogeneity in the expectations of their future value. A simple model of exchange rate dynamics featuring traders with heterogeneous expectations is introduced. The model is based on the asset pricing model in Brock and Hommes (1998) and features the BNN dynamic presented in Brown et al. (1950), a dynamic with desirable properties absent in other dynamics used in the literature. The chapter shows that even this simple model can easily generate complex and even chaotic dynamics in the exchange rate because of the interaction of traders with different beliefs. An important implication is that long-term exchange rate prediction is, in theory, difficult.

Chapter 1

A Search Model of the Labor Market with Heterogeneous Workers

1.1 Introduction

Search-and-matching models have become the workhorse tool for the analysis of unemployment and labor dynamics due to their analytical tractability and rich comparative statics. Recent literature has pointed out that these models can be further enhanced to better replicate some empirical observations by lodging sources of unemployment other than matching frictions. Michailat (2012) points out that during deep recessions, workers queuing outside factories to get a job is an observation at odds with standard search models, which predict that if firms could hire workers right on the spot, at minimum recruitment costs, the rate of unemployment should be low. This inconsistency suggests that other factors besides matching frictions intervene in the job creation process and must be accounted for in the modeling so two sources of unemployment can be captured: job rationing, defined as the level of unemployment that would prevail in a frictionless market, and frictional unemployment, defined as the additional unemployment due to matching frictions.

This chapter presents a search-and-matching model of unemployment that incorporates

worker moral hazard to microfoundate job rationing. Building on Mortensen (1989), Mortensen and Pissarides (1999), and Rocheteau (2001), I embed the Shapiro-Stiglitz (1984) efficiency wages framework into a search-and-matching labor market and show that worker moral hazard and imperfect monitoring impose incentive constraints on wages that prevent the market from reaching full employment even in the absence of matching frictions.¹ The specific form and consequences of efficiency wages vary across the literature but they generally make wages include a premium above the market average to motivate workers. However, the average wage is set by firms themselves so in equilibrium all firms pay the same premium which results in higher wages and therefore higher unemployment. Thus, the wage incentives firms give to make the inside job offer more attractive are reinforced through a deterioration of the worker's outside options; firms open fewer vacancies so workers experience longer unemployment spells. Consequently, job rationing arises as a disciplinary device without assumptions of decreasing returns to scale or extremely low productivity.

The model predicts that job rationing is more severe during economic downturns since low productivity disables the use of high wages as motivation, so employees can be incentivized only by potentially longer unemployment spells. Unlike previous work, the model predicts some degree of job rationing even in high-productivity states. This novel result is a direct consequence of moral hazard. In the model, the worker's decision to shirk depends on his valuation of a job; workers won't risk their job by shirking if their employment surplus is large enough. The surplus of a current job depends on the wage and outside opportunities, high wages and potential long unemployment spells make workers value their job and avoid shirking. In high-productivity states firms open more vacancies, raising the exit rate of unemployment and driving down its expected length. In this new environment, workers find it easier to get a job so current matches are less valuable; a wage adjustment is necessary to ensure worker's effort. This is the role of the efficiency wage, it creates a wage floor that

¹I decided to use the term efficiency wages to relate my work with the literature but given the way wages work in the model they are better described as incentive wages. Phelps (1994) already recognizes that this title is more accurate.

adjusts with the exit rate of unemployment to compensate workers for their improved job finding conditions. In a tight market, workers would find a job almost instantaneously so the wage required to motivate them would have to be so large that they could not be hired profitably. Firms must ration jobs even in high productivity states to keep the efficiency wage affordable. Therefore, job rationing persists as a motivational device even in high productivity states.

Another novel insight of the model is the way shifts in productivity affect job rationing relative to frictional unemployment. According to previous work, the share of total unemployment due to job rationing always decreases with higher productivity.² In contrast, I show that the way this share changes with productivity depends on the elasticity of the matching function with respect to unemployment. When the matching function is relatively inelastic, an increase in vacancies due to higher productivity greatly improves the exit rate of unemployment and creates disincentivizing effects that counteract the incentives of higher wages, job rationing must remain relatively unchanged as an incentive device; higher productivity disproportionately reduces frictional unemployment. If the matching function is elastic enough, more vacancies do not generate large disincentivizing effects so the higher wages from higher productivity are enough to incentivize workers. Job rationing decreases relative to frictional unemployment because it is no longer needed.

When ex-ante worker-skill heterogeneity is introduced, the model offers a tractable and versatile framework to analyze asymmetric outcomes across the labor force such as diverse unemployment rates, unemployment volatilities, and wages. The model implies that low-skilled workers have higher unemployment rates that wildly fluctuate with productivity, whereas high-skilled workers experience much lower unemployment rates that remain relatively constant, both well documented facts in empirical studies.³ Also consistent with the

²Michaillat (2012) and Ferraro(2013).

³The fact that different kinds of workers experience different unemployment dynamics is well documented. The literature usually classifies workers by age and education and in general finds that younger workers with less education experience higher and more volatile unemployment levels. See Ferraro (2013) and Grossman (2013).

data is the prediction that the wages of high-skilled workers are relatively more volatile than those of the low-skilled. An insight of the model is that at the core of these asymmetries is the composition of the idiosyncratic unemployment rates, low-skilled unemployment has a larger share of job rationing, which is more elastic to productivity shocks than frictional unemployment. Thus, in low aggregate productivity states, the unemployment pool is characterized by a distribution of skills that is skewed to the left; low skilled workers are over represented. The skewness degree depends on aggregate productivity; when aggregate productivity is high, the distribution of skills in the unemployment pool closely replicates the skill distribution in the workforce.⁴ The diversity in labor market outcomes across the labor force generated by the model creates an ideal framework for the analysis of policies affecting workers differently according to their skills and productivity.

The remainder of the chapter is organized as follows. The next section presents an overview of the related literature. Section 1.3 introduces a search-and-matching unemployment model with moral hazard. I introduce the basic features of the model assuming a representative worker and show the existence of job rationing. Section 1.4 introduces skill heterogeneity among workers and characterizes the behavior of job rationing for workers with different skills. Section 1.5 calibrates the model and presents numerical simulations to quantitatively assess the predictions of the model. Section 1.6 presents concluding remarks. Proofs and derivations are included in appendix A.

1.2 Related Literature

There are several strands of the literature on unemployment to which the current work relates to. Firstly, the works motivating this chapter which share the purpose of proposing

⁴Clark and Summers (1981), Jaimovich and Siu (2009), Gomme et al. (2005), and Lindquist (2004) show that less skilled individuals have much more volatile hours of work than more skilled workers. Lower-skilled individuals also reset their wages less frequently and have less volatile. Daly et al. (2012) conclude that less skilled individuals have stickier wages than high-skilled individuals. Lester (2014) finds that college educated individuals have less volatile hours but more volatile wages over the business cycle compared to individuals with less than college education. Ferraro (2013) finds that low-skilled workers experience higher unemployment rates and that they account for most of the variation in aggregate unemployment.

search-and-matching models of unemployment that can explain its asymmetric behavior at different stages in the cycle, or more precisely, present mechanisms that explain why jobs are rationed in recessions. Michaillat (2012) shows that the Pissarides (2000) and Hall (2005) models predict asymptotic convergence to full employment in the absence of matching frictions, which makes them unsuitable for the analysis of recessionary unemployment which is characterized by an acute job shortage regardless of the matching environment. He proposes a model where job rationing arises during recessions as the result of combining decreasing returns to scale and exogenous wage rigidity. Ferraro (2013) evinces the lack of job rationing in models with endogenous separation rates a la Mortensen-Pissarides (1994). He presents a model that relies on worker heterogeneity and extremely low levels of productivity to generate job rationing at an aggregate level. In his model, during severe economic downturns productivity is so low that the total surplus of a match with low-skilled workers is negative, leaving them completely out of work.

An important theoretical contribution of my model is the way job rationing arises as a result of the constraints that moral hazard and imperfect monitoring impose on wages, making the assumptions of decreasing returns to scale or extremely low productivity unnecessary. In both of the models above mentioned, the presence of job rationing is contingent upon bad states of the economy, meaning that in economic expansions matching frictions are the only cause of unemployment. In contrast, in my model the existence of job rationing does not depend on the state of the economy. The inability of wages to achieve full employment persists regardless of productivity; it is rather its intensity that fades as productivity increases. The risk of moral hazard is always present so in any equilibrium there is always the need of unemployment as a disciplinary device. However, the unemployment rate necessary to motivate a worker decreases with the product of the match. This means that both rationing and frictional unemployment always coexist but their shares in total unemployment change depending on productivity. This is another contrasting feature of my model; unlike the previous works, whether the share of job rationing increases or decreases with productivity

depends on the matching function, that is, on the way the probabilities of finding a job and filling a vacancy affect the incentives of workers and firms. This characteristic highlights important mechanisms behind the incentives that both sides of the market have. This is a new insight in the literature.

To generate the wage constraints that create job rationing, I follow the canonical shirking-efficiency wages framework in Shapiro and Stiglitz (1984).⁵ Other works have embedded the shirking model in a job search environment. The model I present builds on Mortensen (1989) where potential causes for the indeterminacy and persistence of unemployment are explored under a search equilibrium framework with different assumptions about wage determination. A subtle difference between this model and mine is that instead of having a utility of shirking equal to their utility of leisure, I assume that production is costly for the worker so workers choosing to shirk save the disutility of labor. In his model, the frequency of inspections is an endogenous variable whereas in my model, for simplicity, it is exogenously given. Mortensen and Pissarides (1999) present a version of the same model with a fixed inspection rate. The model I present closely follows Rocheteau (2001), where the Pissarides (2000) and the Shapiro and Stiglitz (1984) models are combined and worker heterogeneity is introduced. Other models that consider efficiency wages in a search-and-matching environment are Sattinger (1990), Larsen and Malcomson (1999).

Since efficiency wages are an important component of my model, it is important to underscore the literature that provides evidence of the implementation of efficiency wages. Evidence of efficiency wages in practice comes in the form of a negative relationship between higher wages and alternative ways of regulating employee's effort (supervision). Zaharieva (2010) presents a statistical summary of a large European data set collected by the researchers

⁵Although their model does not include matching frictions, they introduce the idea that unemployment is a multifactorial phenomenon and that a convincing unemployment model must try to integrate all these components. About their own works they write: "The type of unemployment studied here is not the only or even the most important source of unemployment in practice. We believe it is however, a significant factor in the observed level of unemployment, especially in lower-paid, lower-skilled, blue collar occupations. It may well be more important than frictional or search unemployment in many labor markets". This remark is completely in the spirit of my research.

and shows that about 50% of firms prefer to dismiss workers rather than to reduce base wages in response to an output shock. At the same time one of the two major reasons for avoiding wage reduction is to maintain high effort and working morale. Georgiadis (2013) finds evidence for the implementation of efficiency wages in low-skilled jobs. He exploits a natural experiment provided by the 1999 introduction of the UK National Minimum Wage to test for efficiency wage considerations in a low-wage sector. He finds evidence consistent with a wage-supervision trade-off for non-managerial employees that provides support to the shirking model. Krueger and Summers (1988) find evidence supporting the presence of efficiency wages by looking at the wage differentials of equally skilled workers across industries. More recently, the literature in management control has explored the effect of efficiency wages on employee behavior and social norms finding evidence suggesting the implementation of some sort of efficiency wage. For example, Chen and Sandino (2012) find that high levels of employee compensation can deter employee theft.

Although there is evidence of the implementation of some sort of efficiency wage, the theory is not free of criticism. The most important being the so-called bonding critique succinctly described in Carmichael (1985). The critique states that the involuntary nature of unemployment in efficiency wages models can disappear under more complex wage schemes such as employment fees or bonds posted by workers when initially hired and forfeited if found cheating. Other possible solutions are performance bonds, entrance fees, upward sloping age-earnings profiles, pensions and other deferred compensation schemes. These complex hiring mechanisms can reduce wages to market-clearing levels or at least reduce unemployment to a level where the marginal worker is indifferent between being employed and the alternative thus making all remaining unemployment voluntary. There are theoretical objections to these hiring schemes, mainly regarding moral hazard on the side of the firm and capital constraints on workers. Akerlof and Katz (1986) show that in the most natural framework of the shirking model, in the absence of upfront bonding, upward sloping wage profiles generate wages above market clearing level. This means that in the absence of perfect capital

markets, bonding and deferred payment could only reduce the severity of unemployment. An even more important objection to the critique is the fact that these complex contracts are rarely observed in practice. The purpose of the present work is finding mechanisms whereby unemployment arises for reasons other than matching frictions, the voluntary or involuntary nature of the unemployment is no relevant for the analysis, even if some complex hiring mechanism eliminated the involuntary nature of unemployment, the qualitative results of the model would still hold. For this reason I leave issues related to the bonding critique for future research.

Finally, my model is related to several other models that explore worker heterogeneity in search-and-matching environments. A great deal of these works focus on the way heterogeneity affects the volatility and cyclical nature of unemployment and vacancies. Pries (2008) addresses the Shimer (2005) puzzle by introducing worker heterogeneity, a modification that makes the model exhibit considerably greater volatility. In the same spirit is Epstein (2012) introduces worker heterogeneity in production capacity to explain the strong procyclicality of the vacancy-unemployment ratio. In accordance with these results, when worker heterogeneity is incorporated in my model, it shows that the predicted aggregate unemployment is larger and more volatile the larger the variance of the skill distribution.

Another issue that models with worker heterogeneity investigate is the asymmetries of unemployment experiences across the labor force. Grossman (2013) focuses on how models with heterogeneous labor can affect trade patterns and how globalization affects the distribution of wages and unemployment across the complete spectrum of worker types via its impact on sorting and matching in the labor market. Shi (2002) analyzes an economy with workers with heterogeneous skills and skilled-based technologies. The model generates wage inequality among identical unskilled workers, as well as between-skill inequality. The fact that in my model heterogeneity is modeled as an ex-ante phenomenon, allows it to generate idiosyncratic unemployment rates with a unique composition of frictional and rationing unemployment for workers with different productivities. This fact helps the model account for

the differences in unemployment volatility and wages observed across different workers.

1.3 Job Rationing and Moral Hazard

1.3.1 The Basic Model

In this section I present a search-and-matching unemployment model with moral hazard and imperfect monitoring based on Mortensen (1989), Mortensen and Pissarides (1999), and Rocheteau (2001). I show that, unlike conventional models, the model features job rationing as the result of incentive compatibility constraints imposed on wages. The environment is similar to the basic Pissarides (2000) model with the important difference that shirking is allowed and there is imperfect monitoring. Time is continuous and it is denoted by t . Agents are risk neutral and there is a unit mass of workers and a continuum of firms which can be matched with one worker at most.

For simplicity, there are two levels of work intensity (disutility of work) e or 0 , and this is known by both firms and workers. If the worker exerts effort, the per-unit of time product of a match is ay , where $y \in \mathbb{R}^+$ is worker productivity and $a \in \mathbb{R}^+$ is an aggregate technology parameter. If the worker shirks, the product of a match is zero. The effort exerted by the worker is observable only after an inspection, which obeys a Poisson process with an exogenous arrival rate $\lambda \in \mathbb{R}^+$. If the worker is caught shirking the match is terminated. There are no reputational effects so upon meeting a worker, firms do not know whether the worker has a shirking history or not.

The number of matches made per-unit of time is given by the constant-returns matching function $h(u, v)$, differentiable and increasing in both arguments where u is the fraction of workers who are unemployed and v is the number of vacancies as a fraction of the labor force. Labor market tightness is defined as $\theta \equiv v/u$. The rate at which an unemployed worker finds a job is defined as $f(\theta) \equiv h(u, v)/u = h(1, \theta)$. Similarly, the rate at which a vacancy is filled is given by $q(\theta) \equiv h(u, v)/v = h(1/\theta, 1)$. Unemployed workers find a job more easily in a

tighter market, that is, when there are more vacancies relative to the job seekers. Similarly, firms fill vacancies faster when there are more unemployed workers relative to vacancies. The flow cost of an open vacancy is $\gamma \in \mathbb{R}^+$. Matches can be terminated by an exogenous shock following a Poisson process with parameter $s \in \mathbb{R}^+$. There is no on-the-job search, therefore only unemployed workers can search for a job.

1.3.1.1 Worker Behavior

Once matched, it is the worker's decision to exert effort or shirk. This decision is made based on the lifetime expected utility of taking these actions. A non-shirker is a worker who chooses not to shirk in all periods while his current job lasts. He gets a wage w and suffers a disutility e , per unit of time, and could have his exogenously job terminated with probability s . The lifetime expected utility of a non-shirker, E , obeys the asset-pricing equation

$$rE = w - e + s(U - E), \quad (1.1)$$

where $r \in \mathbb{R}^+$ is the time rate preference and U is the lifetime expected utility of an unemployed worker. If E represents the asset value of employment, (1.1) states that the opportunity cost of holding a job without shirking is equal to the current income flow minus the disutility of effort plus the expected capital loss from a change of state.

The expected lifetime utility of someone who chooses to shirk, S , during a length of time dt , satisfies

$$S = wdt + \exp(-r dt) \{ \Pr [\min(\tau_s, \tau_\lambda) \leq dt] U + (1 - \Pr [\min(\tau_s, \tau_\lambda) \leq dt]) E \}, \quad (1.2)$$

where τ_λ is the length of time until the next inspection and τ_s is the duration of a job.⁶ According to (1.2), during the time interval dt a shirker receives a real wage wdt and has no

⁶These two processes are characterized by an exponential distribution with parameters λ and s respectively.

disutility from work, he loses his job if he is caught shirking or if the match is terminated by an idiosyncratic shock. If neither of these two events occur during the time interval dt , the employed worker stops shirking in all subsequent periods. Notice that, all else remaining equal, if it is optimal for a worker to shirk during a length of time dt , it will be optimal to keep shirking for the next length of time dt , so if a worker decides to shirk he will do it for all subsequent periods. A worker will chose to exert effort over shirking if and only if the lifetime expected value of not shirking is greater the lifetime expected value of doing so. After some manipulation, as dt approaches zero we have that

$$E \geq S \quad \iff \quad E - U \geq \frac{e}{\lambda}, \quad (1.3)$$

this is the no-shirking condition (NSC) and its derivation is shown in appendix A. It states that in order to encourage a worker to exert effort in the production process, his surplus must be at least equal to e/λ , the expected disutility from working before the next inspection. When a workers decides to shirk he saves the disutility of effort e but has an expected capital loss of $\lambda(E - U)$. In equilibrium, workers will never have an incentive to shirk since firms will never hire a worker if they cannot guarantee their effort, so their lifetime expected utility of unemployment satisfies

$$rU = b + f(\theta)(E - U), \quad (1.4)$$

where b represents the income in unemployment. According to (1.4), when an unemployed worker finds a job he becomes a non-shirker. It is important to remark that the permanent income of unemployed workers is increasing with market tightness since the probability of coming into contact with a firm increases with more vacancies per worker, making the average duration of unemployment smaller. Combining (1.1) and (1.4) we get an expression for the worker's surplus of a match

$$E - U = \frac{w - e - b}{r + s + f(\theta)}. \quad (1.5)$$

A worker will accept a match if and only if $E - U$ is positive and, according with the NSC (1.3), will choose not to shirk if and only if it is greater than e/λ .

1.3.1.2 Firm Behavior

The present discounted value of expected profits from a vacant job (V) must satisfy the Bellman equation

$$rV = -\gamma + q(\theta)(J - V), \quad (1.6)$$

where γ is the per-unit of time cost of keeping a vacancy open, and J is the value function of a filled vacancy. This last equation states that the capital cost of an open vacancy has to be exactly equal to the rate of return of the vacancy, i.e., the flow costs of recruiting plus the expected capital gain. The asset value of an occupied vacancy satisfies a similar asset-price equation:

$$rJ = ay - w + s(V - J). \quad (1.7)$$

The capital gain of a filled vacancy is equal to the income flow, $ay - w$, plus the expected capital loss when the match is destroyed. The free-entry condition implies that there are no exploitable profits left from opening another vacancy so $V = 0$. Thus, from (1.6) we get $J = \gamma/q(\theta)$. By substituting this expression into (1.7) we get the vacancy supply condition (VSC)

$$ay - w = (r + s) \frac{\gamma}{q(\theta)}. \quad (1.8)$$

Equation (1.8) uniquely defines equilibrium market tightness and captures the essence of the model's dynamics. Vacancies are posted by employers up to the point where the expected surplus of forming a match is exactly offset by the expected recruiting costs. Expected recruiting costs rise due to a congestion effect, more firms posting vacancies rises market tightness making the instant probability of finding a job decrease. These are the negative search-externalities that firms looking for workers have on each other.

1.3.1.3 Wage Schedule

Wages will be specified to satisfy the NSC at all times. This creates an additional constraint on the bargaining process. Assuming wages are settled via Nash-bargaining with β as the worker's bargaining power, we have

$$w = \arg \max (E - U)^\beta J^{1-\beta} \text{ s.t. } E - U \geq \frac{e}{\lambda}. \quad (1.9)$$

When the NSC is not binding, the wages are determined in the same way as in conventional models. Using (1.5) and (1.7), and solving for w , the expression for the Nash-bargaining wage is

$$w_{NB} = ay \left[\frac{\beta(r + s + f(\theta))}{r + s + \beta f(\theta)} \right] + \left[\frac{(r + s)((1 - \beta))}{r + s + \beta f(\theta)} \right] (b + e). \quad (1.10)$$

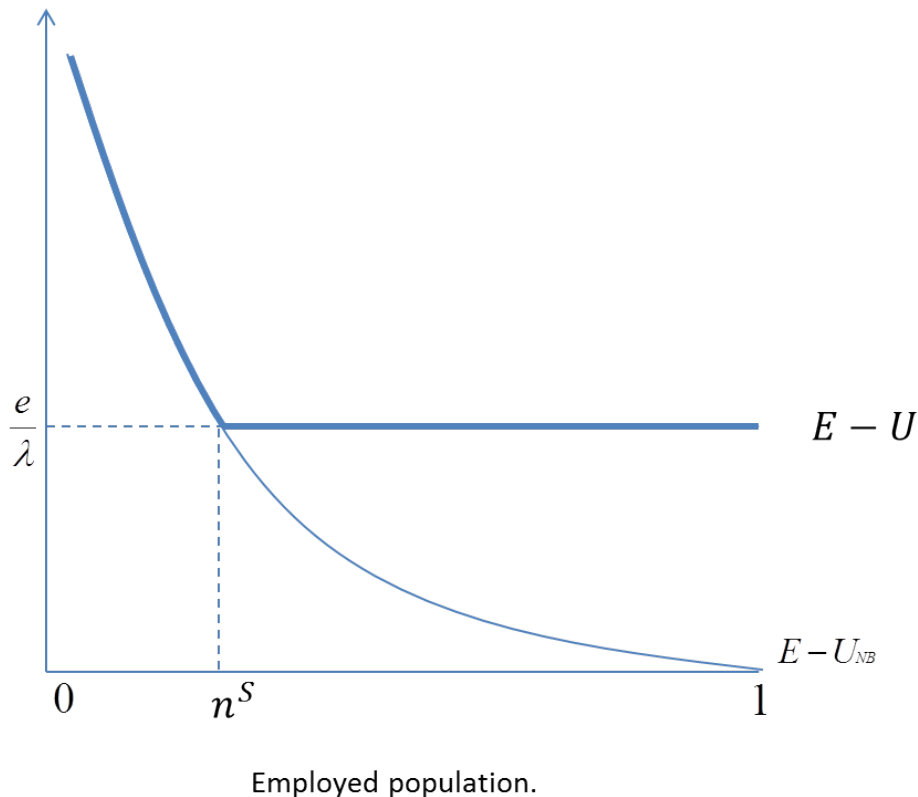
Under Nash-bargaining the worker's surplus of a match takes the form

$$E - U = \frac{\beta(ay - e - b)}{r + s + \beta f(\theta)} \equiv E(\theta) - U(\theta)_{NB}. \quad (1.11)$$

I will refer to the worker surplus generated by Nash-bargaining wages as $E(\theta) - U(\theta)_{NB}$. Notice that under Nash-bargaining, worker surplus asymptotically goes to zero as the market gets tighter, that is, $\lim_{\theta \rightarrow \infty} E(\theta) - U(\theta)_{NB} = 0$. From (1.10) we can observe that the worker's threat point in wage bargaining increases with market tightness, at the limit $w_{NB} = ay$.

This means that under Nash-bargaining wages, when the market is too tight, the NSC is

Figure 1.1: Worker Surplus and the No-Shirk Condition (NSC)



According to the NSC, a worker will participate in the production process only if the surplus he gets from a match is at least equal to e/λ . Firms know this so they will only hire a worker if they can guarantee at least that level of surplus. Under Nash bargaining worker surplus, $E(\theta) - U_{NB}$, drops to zero as the economy converges to full employment, making it necessary to implement efficiency wages, which guarantees a worker surplus of e/λ , after some level of employment n^S .

inevitably violated as worker surplus gets closer to zero. Since firms will hire workers only if the participation of the worker can be guaranteed, when NSC cannot be satisfied with Nash-bargaining wages, firms will pay higher wages to incentivize workers. The minimum wage that a firm must pay to induce effort from the worker is the wage that makes the NSC binding. Substituting (1.5) into the NSC and solving for w with an equality, we get that the efficiency wage is

$$w_E = b + e + \frac{e}{\lambda} (r + s + f(\theta)). \quad (1.12)$$

This is the minimum wage required to encourage workers to exert effort. Without the

threat of moral hazard, i.e., if perfect monitoring was possible, the unconstrained Nash-Bargaining wage would be enough to guarantee the worker's participation in the production process, however if his actions are not perfectly observable, guaranteeing his participation requires a moral hazard premium defined as the difference between the efficiency wage and the unconstrained Nash-Bargaining. It can be showed that this premium is inversely related to productivity and it increases with the relative value of the worker's outside options. If his current job is likely to end or if the expected length of unemployment is short, the outside options are relatively more valuable so the moral hazard premium must be larger. Consistent with the efficiency-wage literature, the no-shirking wage is higher when the effort to be exerted is larger or the detection probability is lower. Notice that the efficiency wage is an increasing function of market tightness just like the Nash-bargaining wage but unlike it, it is not bounded above. This is the result of the fact that the moral hazard premium goes to infinity along with market tightness. If the worker's valuation for his job must be kept above a threshold to prevent shirking, as the market tightens, the wage premium must get increasingly large to compensate the worker for his outside options.

Depending on whether the NSC is binding, equilibrium wages are determined either using Nash-bargaining or with the expression for efficiency wages so the solution to (1.9) can be specified as

$$w(\theta) = \begin{cases} w_{NB}(\theta), & E(\theta) - U(\theta)_{NB} \geq \frac{e}{\lambda}, \\ w_E(\theta), & E(\theta) - U(\theta)_{NB} < \frac{e}{\lambda}. \end{cases} \quad (1.13)$$

It can be verified that this is a continuous function of market tightness and it is strictly increasing with θ . This wage schedule guarantees that whenever Nash-bargaining wages cannot induce effort from the worker, the worker is paid the minimum to do so. Figure 1.1 shows the logic behind the wage schedule. Using (1.11) we can verify that $\lim_{\theta \rightarrow \infty} E(\theta) - U(\theta)_{NB} = 0$. For $e > 0$ and $\lambda < \infty$, this implies that $\lim_{\theta \rightarrow \infty} w(\theta) = \lim_{\theta \rightarrow \infty} b + e + (r + s + f(\theta)) \frac{e}{\lambda} = \infty$, so under the wage scheme described in (1.13), equilibrium wages increase to infinity as

the economy converges to full employment.

This is the gist of the departure from conventional models; in the presence of moral hazard and imperfect monitoring, full employment is unattainable. In the standard Pissarides Model, where equilibrium wages are determined using Nash-bargaining, as the economy converges to full employment the equilibrium wage converges to the product of the match, extracting all the surplus from the firm as a result of increased bargaining power for the workers. In the presence of moral hazard, wages must always be established considering the incentive compatibility constraint, participation of the worker in the production process must be ensured at all times. As market tightness increases, the benefits from shirking grow higher so to encourage a worker to exert effort, a higher wage must be offered. As the labor market is closer to full employment, shirking becomes so attractive that no wage can be large enough to encourage workers to exert effort. The firm's surplus will turn negative before full employment can be achieved.

There are essentially two endogenous variables in the model: θ and u . Equilibrium market tightness is determined by (1.8). To determine equilibrium unemployment we use the fact that at a steady state the inflow and outflow from unemployment must be equal. The size of the unemployed population is given by $u = 1 - n$ at all times so the equality of inflow and outflow of unemployed workers gives

$$u = \frac{s}{s + f(\theta)}, \tag{1.14}$$

this expression tell us that for a given separation rate and market tightness, there is a unique equilibrium unemployment rate.

Definition 1. *A steady-state equilibrium is a collection $(u, \theta, w) \in \mathbb{R}^{3+}$ satisfying (1.14), (1.8), and (1.13).*

I am implicitly assuming that all matches, new and old, set wages according to (1.13) at all times, as if would be the case if wages were being constantly renegotiated. In equilibrium

neither workers nor firms will have incentives to break the matches and workers will never have incentive to shirk.

1.3.2 Job Rationing

Now I show that the presence of moral hazard and imperfect monitoring creates job rationing. In neoclassical models, job rationing is the unemployment arising from the failure of equilibrium wages to clear the labor market. This definition must be adapted to a search-and-matching environment where bilateral meetings take place and wages are usually assumed to be the product of a Nash-bargain between employer and employee. In this scenario, following Michailat (2012), I define job rationing as the unemployment that persists when recruitment costs are zero. In standard search-and-matching models unemployment arises due to matching frictions which increase the expected cost of opening a vacancy as market tightens. If opening a vacancy was a costless activity, there would be convergence to full employment since Nash-bargaining guarantees that the wage remains in the bargaining set, defined as the interval between the minimum wage acceptable to the worker and the maximum wage acceptable to the firm, at all times. Without recruitment costs, the existence of job rationing requires that the settled wage is above the bargaining set so although workers would be willing to fill a vacancy, firms cannot hire them profitably. In Michailat (2012) job rationing arises from wages only partially adjusting to productivity shocks and the assumption of decreasing returns to scale. In Ferraro (2013) the bargaining set disappears in economic downturns due to extremely low productivity. In his model, if productivity is low enough, the product of a match is less than the worker's outside option so the total surplus of a match is negative for entire portions of the labor force. An important theoretical contribution of the present work is the way job rationing arises as a result of the constraints imposed on wages due to moral hazard.

To see if job rationing exists it is necessary to determine what the unemployment rate would be if job creation was entirely determined by productivity and wages just as it would

happen in a frictionless environment.

Definition 2. *Job rationing is defined as the unemployment that persists as recruitment costs disappear. That is to say, job rationing u^R is given by*

$$u^R = \frac{s}{s + f(\theta^R)}$$

where θ^R is such that

$$ay - w(\theta^R) = \lim_{\gamma \rightarrow 0} (r + s) \frac{\gamma}{q(\theta^R)}.$$

It is important to remark that although hiring costs getting infinitely small is conceptually different from a frictionless economy, their equilibrium effects on the labor market are equivalent. When opening vacancies and keeping them open is free, firms will open infinitely many vacancies to completely overcome the frictions in the matching process.

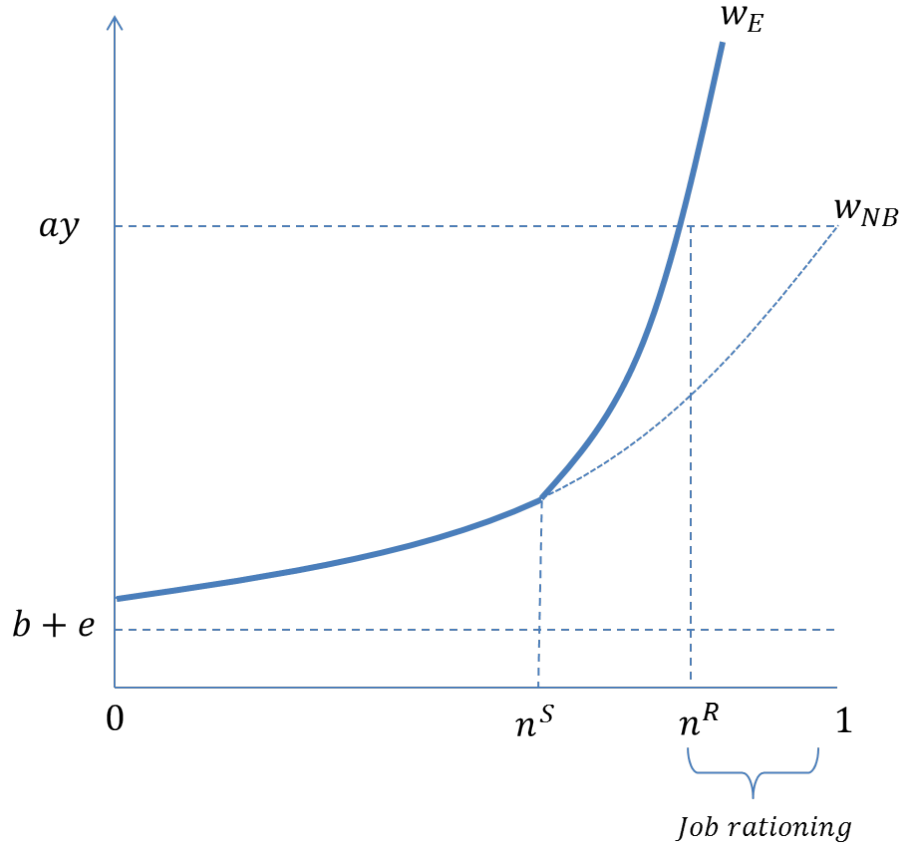
Proposition 1. *There exists job rationing in the model.*

Proof: Appendix A.

Proposition 1 tells us that even if opening a vacancy was a costless activity, at some level of employment $n^R \equiv 1 - u^R < 1$, firms could not profitably keep hiring workers due to the incentive constraint on wages; not enough vacancies would be opened to make the economy converge to full employment. This is the role of equilibrium unemployment as a disciplinary device as described in Shapiro-Stiglitz(1984). There are two forces affecting the worker's decision to shirk, the wage they receive and their job opportunities in the market. Since encouraging workers only through wages becomes impossible (no finite wage by itself would be enough), unemployment arises to reduce the worker's job opportunities and make them cherish their current job enough to exert effort. Figure 1.2 provides a graphical representation of the situation.

An expression for unemployment rationing can be derived combining equations (1.8) and (1.14). As $\gamma \rightarrow 0$ equilibrium unemployment converges to

Figure 1.2: Nash Bargaining and Efficiency Wages.



According to the wage function (1.13), as the economy converges to full employment, at some point n^S , Nash-bargaining wages w_{NB} can no longer guarantee workers' participation, making the use of efficiency wages w_E necessary. Efficiency wages are designed to encourage workers by compensating them for their increasing outside opportunities as the market gets tighter. At the limit, the average spell of unemployment is close to zero, wages must be infinite to compensate the worker. This means that even without recruiting costs, the profits of the firm are equal to zero at employment level n^R .

$$u^R = 1 - n^R = \frac{(e/\lambda)s}{ay - b - e - (e/\lambda)r}. \quad (1.15)$$

Notice the importance of the separation rate in the magnitude of job rationing. According to the NSC, a higher separation rate increases the incentive to shirk since it makes a job less valuable, it will be terminated rather soon. When the separation rate is close to zero, job rationing is close to zero, firms do not ration many jobs because workers do not have strong incentives to shirk, the almost unlimited duration of jobs makes them too valuable to risk by

shirking. Comparative statics allow us to understand better the forces behind job rationing.

$$\frac{\partial u^R}{\partial ay} < 0, \frac{\partial u^R}{\partial e} > 0, \frac{\partial u^R}{\partial \lambda} < 0, \frac{\partial u^R}{\partial b} > 0, \frac{\partial u^R}{\partial s} > 0, \frac{\partial u^R}{\partial r} > 0.$$

An increase in productivity reduces unemployment rationing since it enables firms to pay higher wages instead of cutting down jobs to motivate workers. All the changes in the parameters that make unemployment more attractive than exerting effort in a job, such as a higher permanent income in unemployment, less effort necessary for production, or lower average duration of employment, have a positive impact on unemployment rationing. For a given productivity of a match, the less undesirable unemployment is, the more firms will have to restrict vacancies to encourage workers.

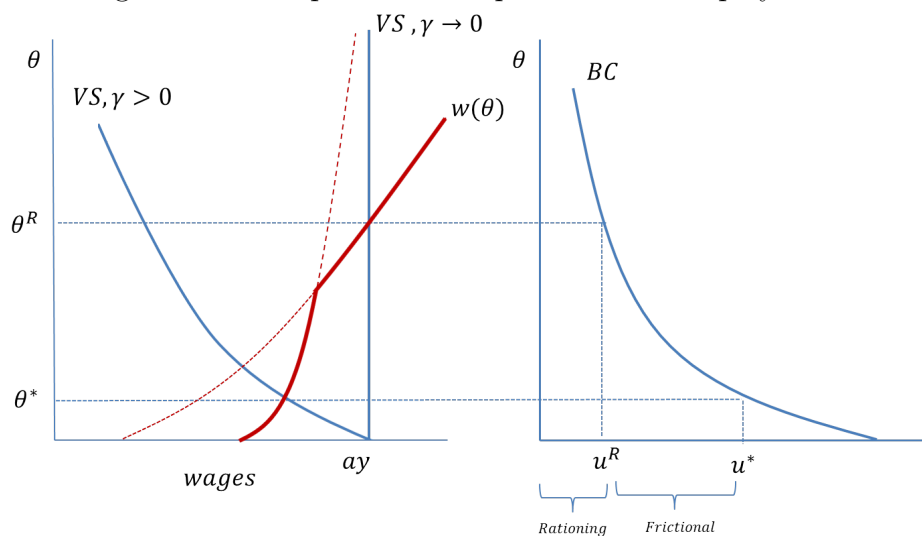
Now that I have laid out the essentials of the model I turn my attention to the source composition of unemployment. Determining the job rationing rate allows us to decompose equilibrium unemployment rate according to its causes. Equilibrium unemployment u is implicitly determined by the VSC (1.8) and the Beveridge curve (1.14). Together, these equations give a positive relationship between unemployment and the cost of opening a vacancy. For a $\gamma > 0$, we have that $u > u^R$. The extra amount of unemployment caused by the costs of opening a vacancy will be called frictional unemployment (u^f), so it can be defined as the difference between equilibrium unemployment and job rationing unemployment. That is, $u = u^f + u^r$.

To quantify the job rationing component of unemployment I focus on its share in equilibrium unemployment. The share of unemployment rationing is given by

$$R \equiv \frac{u^R}{u^R + u^F}.$$

Previous models analyzing how the composition of unemployment changes across the business cycle conclude that the share of job rationing is countercyclical, job rationing plays a more prominent role during recessions. Michailat (2012) finds that when the aggregate level of

Figure 1.3: Composition of Equilibrium Unemployment



Equilibrium unemployment level, u^* , is given by the Beveridge curve BC , and the vacancy supply condition VS . In the absence of matching frictions, the VS is vertical since for the equality to hold in (1.8), the RHS must be zero so wages must be equal to ay . The wage schedule specified in (1.13) pins down the equilibrium wage and therefore equilibrium market tightness. The unemployment level in this situation is u^R . In the presence of matching frictions, the additional unemployment due to frictions appears as VS rotates to the left. The additional unemployment is frictional unemployment.

technology is high, matching frictions account for all unemployment, but when technology is low, both job rationing and matching frictions contribute to unemployment. In his model, once technology has reached a certain level, firms always find it profitable to hire workers even in the absence of frictions. In Ferraro (2013), job rationing is a result of the skill heterogeneity in workers. In an economic downturn the productivity of the least skilled workers could be so low that is not enough to cover their reservation wages, firms could not profitably hire those workers.⁷ Both models predict that if the product of a match is high enough, all unemployment is due to frictions in the market, and that during a recession both frictional and rationing unemployment are at work. In my model this is not the case, there is always a job rationing component in unemployment regardless of the state of technology. Since efficiency wages must be infinitely large as the economy converges to full employment, no matter how high the product of a match is, there will always exist job rationing, only

⁷Notice that in the strict sense this is not job rationing, unemployment does not arise from some sort of wage rigidity but it is entirely due to low productivity.

disappearing asymptotically as the product of a match gets infinitely large. The state of the economy is here represented by the aggregate productivity parameter a .

Proposition 2. *The share of unemployment rationing decreases with productivity if and only if the elasticity of rationing unemployment is more negative than the elasticity of frictional unemployment, that is,*

$$\frac{\partial R}{\partial a} < 0 \iff \varepsilon_a^{u^R} < \varepsilon_a^{u^F}.$$

Proof: Appendix A.

As previously noted, job rationing decreases as the product of a match increases. Proposition 2 states that to have job rationing decreasing faster than total unemployment, job rationing must be more sensitive to technology shocks than frictional unemployment.⁸ Whether the increase in the product of a match has a larger effect on frictional unemployment or job rationing depends on whether it alleviates the moral hazard problem more than it helps to overcome the frictions in the labor market. The magnitude of these two effects is linked to the matching function since it determines the elasticity of the instant probability of finding job and filling a vacancy. These two elasticities are inversely related, if $\eta \in [0, 1]$ is the elasticity of $f(\theta)$, then $\eta - 1$ is the elasticity of $q(\theta)$, when $f(\theta)$ is relatively elastic, $q(\theta)$ is relatively inelastic.

With a positive productivity shock, matches become more profitable so firms decide to create more vacancies. A tighter market dampens the initial impulse to create vacancies in

⁸ Notice that the results of proposition 2 tells us that the elasticity of rationing unemployment must be less than the elasticity of frictional unemployment, we know that the elasticity of frictional unemployment is negative in the model, however, the derivative of frictional unemployment with respect to technology is indeterminate. It can be shown that the sign of this derivative goes from positive to negative as technology increases. This rather curious phenomenon is an algebraic consequence of the definition of frictional unemployment so it is also a feature of previous models, namely, Michailat (2012) and Ferraro (2013). Although this result is curious it is not relevant as long as total unemployment is decreasing with productivity, as it is the case.

two different ways. On the one hand, wages increase whether they are determined using Nash-bargaining or efficiency wages. In the first case, higher tightness increases the effective bargaining power of the worker, and in the second case, it increases his outside options making a larger remuneration necessary. This is the wage effect and it is related with job rationing. On the other hand, the expected cost of keeping a vacancy open increases due to the augmented search externalities within firms, this is the cost effect, and it is entirely related with frictional unemployment. Given the relationship between elasticities, when the wage effect is large the cost effect must be small.

When the elasticity of the instant probability of finding a job is small, the moral hazard premium remains relatively unaltered by changes in market tightness. This means that in the event of a positive productivity shock, firms can open many vacancies without having large repercussions on wages. The other side of this effect is that if the elasticity of $f(\theta)$ is small, the elasticity of $q(\theta)$ must be large. An increase in market tightness largely increases the expected cost of vacancies, so the cost effect is strong, frictional unemployment will barely be reduced. The overall outcome of these effects is a large change in the composition of unemployment; the share of rationing unemployment will be largely reduced. A small elasticity of $f(\theta)$ makes unemployment rationing relatively elastic and frictional unemployment relatively inelastic. The opposite is true when the elasticity of $f(\theta)$ is large. This result is formalized in proposition 3 where for the sake of exposition a Cobb-Douglas matching function is assumed but, as shown in Appendix A, it can be generalized to any matching function.

Proposition 3. *Let $h(u, v) = \mu u^\alpha v^{1-\alpha}$. The elasticity of the probability of filling a vacancy, α , determines how the share of job rationing changes with productivity.*

a) *If $\alpha \geq \frac{1}{2}$, then $\frac{\partial R}{\partial ay} < 0$.*

b) *If $\alpha \in (0, \frac{1}{2})$, there exist a threshold $\alpha^* \in (0, \frac{1}{2})$ determined by all the rest of the parameters such that, if $\alpha > \alpha^*$ the share of job rationing decreases with productivity, if $\alpha = \alpha^*$ it is not affected, and if it $\alpha < \alpha^*$, it increases with productivity.*

c) If $\alpha = 0$, The share of job rationing increases with productivity, that is, $\frac{\partial R}{\partial \alpha y} > 0$.

Proof: Appendix A.

Proposition 3 gives us conditions that can be empirically checked to see whether the share of unemployment rationing is countercyclical. Petrongolo and Pissarides (2001) perform a survey of the empirical literature related to the estimation of the matching function and find that when a Cobb-Douglas form is assumed, the different estimations of α place it in the range $\alpha \in (0.5, 0.7)$, which would imply that the share of job rationing decreases with productivity. As simulations in section 1.4 indicate, the share of job rationing is procyclical only at a neighborhood around $\alpha = 0$.

In this section I have laid out a basic search-and-matching model that incorporates worker moral hazard. I showed that the presence of moral hazard creates a wage constraint which, even in the absence of matching frictions, makes the equilibrium wage fail to adjust to full employment level, a situation referred to as job rationing. When matching frictions exists, equilibrium unemployment is composed of rationing unemployment and frictional unemployment. My model departs in two important ways from previous results in the literature: i) The model predicts coexistence of both rationing and frictional unemployment regardless of the aggregate productivity level, and ii) the share of job rationing is heavily determined by the matching function, more specifically, by the elasticity of the instant probability of finding a job. This is a new insight of the model that helps to understand the nature of unemployment rationing and frictional unemployment. When this probability is elastic enough, the original incentive to open vacancies as a result of higher product of a match is mostly offset by higher wages to compensate the worker for his amelioration of outside options as opposed to being mostly offset by an increase in costs of opening a vacancy. The share of job rationing remains relatively unchanged or could even increase. When the instant probability of finding a job is relatively inelastic, the share of job rationing is strongly reduced by a positive productivity shock.

According to the literature estimating the elasticity values of the matching function,

the share of job rationing is countercyclical. During a recession, a larger proportion of unemployment is due to job rationing compared to what happens during an expansion. This results suggest that the effectiveness of policies seeking to reduce unemployment are dependent on the state of the economy, during a recession policies reducing the outside options of a worker are more effective, whereas policies aimed to reduce the search frictions may be more effective during an economic expansion.

1.4 Worker Heterogeneity

Another important issue this work addresses is the employment experience of workers with ex-ante different productivities. The extension of the model incorporating worker-skill heterogeneity closely follows Rocheteau (2001). The environment is the same as in section 1.3 with the exception that there is an ex-ante distribution of workers with different skills. Firms are identical so the production flow of a match depends entirely on the worker. Search is undirected so a firm can meet workers with any skill level which is perfectly observable upon meeting.⁹

There are n different kinds of workers with productivities (ay_1, \dots, ay_n) satisfying $0 < ay_1 < \dots < ay_n$. A worker with productivity ay_i , will be referred to as a worker of type i and is hired by a firm upon meeting with an average probability of $\Pi(ay_i)$, which depends on how profitable the match is for the firm and will be discussed later. The present-discounted value of the expected income stream of an unemployed worker with idiosyncratic productivity ay satisfies

$$rU_{ay} = b + f(\theta)\Pi(ay)\{E_{ay} - U_{ay}\},$$

where U_{ay} is the value to be unemployed and $f(\theta)\Pi(ay)$ is the exit rate of unemployment.

⁹There are other search-and-matching models where search is undirected such as Acemoglu (1999), Shimer(2001), Dolado et al. (2003) and Pries (2008). Acemoglu (1999) makes a case for undirected search by pointing out that skill is imperfectly correlated with observable characteristics, such as years of education and age, making it difficult for employers to recruit workers with a particular skill level.

The value to be employed (E_{ay}) satisfies

$$rE_{ay} = w_{ay} - e + s[U_{ay} - E_{ay}],$$

where w_{ay} is the wage. By assumption, all workers face the same production effort, unemployment benefit, discount rate and separation rate. The worker's surplus of a match is

$$E_{ay} - U_{ay} = \frac{w_{ay} - e - b}{r + s + \Pi(ay)f(\theta)}. \quad (1.16)$$

It has two idiosyncratic components, wages and the exit rate of unemployment. By default, wages are established under Nash-bargaining unless the NSC binds in which case the worker gets a wage that guarantees him the minimal surplus to keep him from shirking, i.e., the efficiency wage. According to (1.3) a worker's surplus of a match in equilibrium must always satisfy

$$E_{ay} - U_{ay} = \max\left\{\frac{e}{\lambda}, E_{ay} - U_{ayNB}\right\},$$

where $E_{ay} - U_{ayNB}$ is the worker's surplus when wage is determined via Nash-bargaining.

1.4.1 The firm's behavior under heterogeneity.

The value of a vacant job for a firm satisfies the following bellman equation:

$$rV = -\gamma + q(\theta)\left[\sum_i \Pi(ay_i)\mu_i(J_{ayi} - V)\right], \quad (1.17)$$

where μ_i is the fraction of unemployed workers with productivity ay_i . The firm's asset value of an occupied job by a worker with productivity ay_i , (J_{ayi}), must satisfy

$$rJ_{ayi} = ay_i - w_{ayi} + s[V - J_{ayi}]. \quad (1.18)$$

Given the assumption that in equilibrium all profit opportunities from new jobs are exploited driving rents from vacant jobs to zero, $V = 0$, and combining equations (1.17) and (1.18) the VSC is derived:

$$\sum_i \Pi(ay_i) \mu_i (ay_i - w_{ay_i}) = (r + s) \frac{\gamma}{q(\theta)}. \quad (1.19)$$

With worker-skill heterogeneity, the profitability of a match depends on the worker so the firm's best hiring response when coming into contact with a specific worker will depend on his productivity. This worker-specific hiring response gives rise to wage dispersion and different exit rates of unemployment. The firm's hiring strategy is derived after Rocheteau (2001) and a detailed derivation is included in appendix A. Before presenting the results of the optimal hiring schedule, it is necessary to introduce some definitions.

Definition 3. For a given θ , define the segments of the real line: $C_0 \equiv (-\infty, y_L]$, $C_1 \equiv (y_L, y_M(\theta)]$, $C_2 \equiv (y_M, y_H(\theta)]$, and $C_3 \equiv (y_H, \infty)$. Where $y_L = b + e + (r + s) \frac{e}{\lambda}$, $y_M = b + e + (r + s + f(\theta)) \frac{e}{\lambda}$, and $y_H = b + e + (r + s + f(\theta)\beta) \frac{e}{\lambda\beta}$.

This definition delimits four intervals in the real line which will classify workers according to their productivity ay_i . Notice that the limits of the interval do not depend on the aggregate productivity parameter a , this characteristic allows workers to transition between intervals in response to productivity shocks. With this classification, the equilibrium hiring schedule can be specified. In the presence of heterogeneity there is the possibility of having hiring probabilities upon meeting different from zero or one.

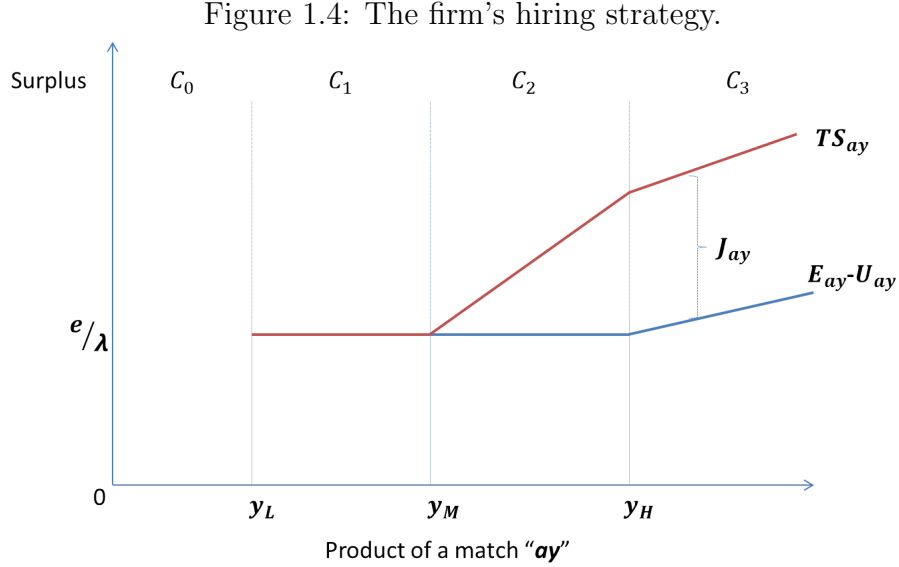
Proposition 4. For a given market tightness θ , consider a worker with productivity ay :

1. If $ay \in C_0$, then the worker is never hired, $\Pi(ay) = 0$.
2. If $ay \in C_1$, then the worker is hired with a probability $\Pi(ay) = (ay - b - e - (r + s) \frac{e}{\lambda}) / f(\theta) \frac{e}{\lambda}$, and is paid $w = ay$.
3. If $ay \in C_2$, then the worker is always hired, $\Pi(ay) = 1$, and is paid efficiency wages, $w = w_E$.

4. If $ay \in C_3$, then the worker is always hired, $\Pi(ay) = 1$, and is paid Nash-bargaining wages, $w = w_{NB}$.

Proof: Appendix A.

The rationale behind proposition 4 is summarized in Figure 4.



For a given θ , upon contact with a worker the firm's hiring response ($w_{ay}, \Pi(ay)$) will depend on the product of the match. If $ay \in C_0$, the productivity of a worker is so low that the total surplus of a match (TS) is not enough to guarantee his effort ($TS < \frac{e}{\lambda}$), no match will be made. If $ay \in C_1$, the worker is barely employable and will be discriminated with a hiring probability $\Pi(ay) \in (0, 1)$ that will reduce his outside options to the point his wage, $w_{ay} = ay$, is just enough to guarantee his effort ($TS = \frac{e}{\lambda}$). If $ay \in C_2$, the worker's productivity is high enough to generate a positive surplus for the firm ($J_{ay} = TS - \frac{e}{\lambda} > 0$) so he will always be hired ($\Pi(ay) = 1$). However, it is not large enough to guarantee his effort under Nash-bargaining so he will get the efficiency wage, $w = w_E$. If $ay \in C_3$ the match will generate positive surplus for a firm ($\Pi(ay) = 1$) and the productivity of a worker is high enough to guarantee his participation with Nash-bargaining wages, $w = w_{NB}$.

Proposition 4 describes the maximizing behavior of the firm. It specifies the firm's best response upon coming into contact with a worker with productivity ay . For a given market tightness, it describes the average hiring probability of a worker and his wage. If the product of a match is high enough ($ay \in C_3$) wage is set using Nash-bargaining since it is high enough to generate surplus for both, worker and firm, without violating the NSC. Worker surplus is above the no-shirking threshold so he will be incentivized to work, and the surplus that

a firm gets is positive so it will hire the worker with probability one. When productivity is not so high ($ay \in C_2$) then the Nash-bargaining wage is not high enough to prevent a worker from shirking, efficiency wages are necessary to ensure that the match is productive. The firm still gets a positive surplus so the worker is hired with probability one. Workers in C_2 and C_3 will be referred to as “perfectly employable” since firms can always hire them and get a strictly positive match surplus.

When $ay \in C_1$, the efficiency wage is greater than the product of a match ($w_E = b + e + (r + s + f(\theta))\frac{e}{\lambda} > ay$), the worker can still be encouraged to work with a wage equal to his productivity, $w = ay$, if his outside options are eroded by a lower probability of transition out of unemployment. If $\Pi(ay)$ were equal to one, the no-shirking wage would have to be superior to the worker’s productivity so firms would never hire them. Conversely, if $\Pi(ay)$ were equal to zero, the worker would generate positive profits for his employer so he would always be hired. The equilibrium answer to this conundrum is that employers adopt a mixed strategy, they hire the worker with a probability proportional to his productivity, that is $\Pi(ay) = (ay - b - e - (r + s)\frac{e}{\lambda}) / f(\theta)\frac{e}{\lambda}$.¹⁰ The decrease of the exit rate of unemployment can be interpreted as a disciplinary device for less productive workers. Notice that firms hiring these workers do not get any surplus from being matched so they are indifferent between hiring them or not, this is the reason I will refer to these workers as “barely employable”. If the productivity of a worker is extremely low ($ay \in C_0$), then even if the worker is fully discriminated, his no-shirking wage would have to be larger than the product of his match so he will never be hired. This differentiated treatment to workers creates wages dispersion and different unemployment rates, shares in the pool of unemployed and exit rates of unemployment.

At the steady state, the idiosyncratic unemployment rates (u_1, \dots, u_n) are constant so the equality between flows out and into unemployment give the unemployment rate of workers

¹⁰We can verify that this is indeed a probability, that is $\Pi(ay) \in [0, 1]$ by observing that it is the solution to the equation $ay_i = (1 - x)y_L + xy_M$. And by assumption $ay_i \in C_1 \equiv (y_L, y_M]$. Full derivation is included in appendix A.

of type i as

$$u_i = \frac{s}{s + \Pi(ay_i)f(\theta)},$$

moreover,

$$\frac{\partial u_i}{\partial ay_i} < 0, \frac{\partial u_i}{\partial b} > 0, \frac{\partial u_i}{\partial e} > 0, \frac{\partial u_i}{\partial s} > 0, \frac{\partial u_i}{\partial r} > 0, \frac{\partial u_i}{\partial \lambda} < 0 \forall i.$$

Total unemployment is given by

$$u = \sum_i p_i u_i,$$

where p_i is the proportion of workers type i in the labor force.¹¹ The fraction of unemployed workers with productivity ay_i is given by

$$\mu_i = \frac{p_i u_i}{u}. \quad (1.20)$$

Substituting the wages and unemployment rates into equations (1.20) and (1.19), after some algebraic manipulation the VSC can be expressed as

$$\sum_{ay_i \in C_2} p_i \left[ay_i - b - e - \frac{e}{\lambda}(r + s + f(\theta)) \right] + \sum_{ay_j \in C_3} p_j \left[\frac{(1 - \beta)(ay_j - b - e)(r + s)}{r + s + \beta f(\theta)} \right] = u \left[\frac{\gamma(r + s)(s + f(\theta))}{q(\theta)s} \right], \quad (1.21)$$

where

$$u = \sum_{ay_i \in C_0} p_i + \sum_{ay_i \in C_1} p_i \left[\frac{s(e/\lambda)}{ay_i - b - e - r(e/\lambda)} \right] + \sum_{ay_i \in C_2} p_i \left[\frac{s}{s + f(\theta)} \right] + \sum_{ay_i \in C_3} p_i \left[\frac{s}{s + f(\theta)} \right]. \quad (1.22)$$

¹¹The entire worker population has been normalized to 1 so $\sum_i p_i = 1$

Equation (1.21) uniquely determines equilibrium market tightness θ^* . It can be verified that

$$\frac{\partial \theta^*}{\partial e} < 0, \quad \frac{\partial \theta^*}{\partial \lambda} \geq 0, \quad \frac{\partial \theta^*}{\partial b} < 0, \quad \frac{\partial \theta^*}{\partial s} < 0, \quad \frac{\partial \theta^*}{\partial r} < 0,$$

and

$$\frac{\partial \theta^*}{\partial ay_i} \geq 0 \quad i = 1, \dots, n.$$

Equilibrium market tightness is strictly increasing in the productivity of all the hireable workers and it is unaffected by small changes in the productivity of workers that are not hireable. In an undirected market, the fact that increasing an idiosyncratic productivity level has a positive effect on equilibrium market tightness means that wages and unemployment rates of all workers change due to a general equilibrium effect. Given the hiring schedule, the magnitude of the derivative varies across workers depending on their productivity. Since the effect of a change in the idiosyncratic productivities on market tightness depends on the proportion of workers with that productivity, it is difficult to compare them without making assumptions about the worker-skill distribution. To overcome this problem we can compute the per-worker productivity effect on market tightness defined as

$$W_i \equiv \frac{\frac{\partial \theta^*}{\partial ay_i}}{p_i}.$$

Proposition 5. *The per-worker productivity effect on market tightness is decreasing with worker-skill, that is*

$$W_i > W_j > W_k \quad \forall ay_i \in C_1, ay_j \in C_2, ay_k \in C_3.$$

Moreover,

$$\frac{\partial W_i}{\partial ay_i} < 0 \quad \forall ay_i \in C_1.$$

Proof: Appendix A.

Proposition 5 states that the impact on equilibrium market tightness of an increase in the productivity of a single hireable worker is greater the lower his productivity. This implies that increasing the productivity of the least-skilled side of the labor force has a greater per-worker impact on market tightness than increasing the productivity of those at the top end. Increments in the productivity of a specific group of workers has positive effects on the whole labor force, higher market tightness means higher wages and lower unemployment rates, however this effect is not symmetric.

Increasing the productivity of any hireable worker in the labor force has a positive effect on market tightness, however not all workers in the labor force benefit in the same way from this general equilibrium effect. For barely employable workers, an increase in market tightness does not translate into higher wages or lower unemployment rates. These workers are being paid exactly their productivity so their wages are not affected by a tighter market. This leaves the hiring probability upon meeting as the only mechanism of adjustment to satisfy the NSC. To satisfy the NSC, since wages are fixed, their outside opportunities must be fixed too. Higher market tightness increases their instant probability of being matched with a firm but this higher arrival rate is offset by a lower probability of being hired upon meeting, their unemployment rates do not change with market tightness. This means that increments in the productivity of perfectly employable workers will not affect the wage or the unemployment rate of barely employable workers. However, when there is an increase in the productivity of barely employable workers, it improves the unemployment rate and wages of all employable workers. Moreover, the size of this effect is strictly decreasing with productivity, meaning that the highest per-worker productivity effect on market tightness is achieved by increasing the productivity of those with the lowest productivities. As it will be highlighted in the next section, behind these asymmetries is the fact that the unemployment experienced by low-skilled workers is essentially different from the unemployment experienced by high-skilled workers. The unemployment of the barely employable is entirely due to job rationing.

The model can also replicate the empirical observation that low-skilled workers are over represented in the pool of the unemployed with respect to their share in the worker population. Using (1.20) we can corroborate that $\mu_i < p_i$ if and only if $u_i < u$. The aggregate unemployment rate in the economy is given by (1.22), so it can be interpreted as the average unemployment rate. Any group of workers with an unemployment rate below the average will be over represented in the pool of unemployed. The intensity of this over and under representation in the pool of the unemployed depends on the spread of the skill distribution, a large disparity in productivities will generate strong positive skewness in the unemployment rates.

1.4.2 Job Rationing among different workers

This subsection shows that the model incorporating heterogeneity also features job rationing for all the workers and the the labor force as a whole.

Proposition 6. *The equilibrium job rationing unemployment rates for a worker with productivity ay_i is :*

$$u_i^R = \begin{cases} \frac{s(e/\lambda)}{ay_i - b - e - r(e/\lambda)}, & ay_i \notin C_0, \\ 1, & ay_i \in C_0. \end{cases} \quad (1.23)$$

Thus, $u_1^R \geq u_2^R, \dots, > u_n^R > 0$. Moreover, if the worker with the highest productivity is hireable, $ay_n \notin C_0$, firms will open vacancies to the point where the the efficiency wage is equal to the product of a match with a worker with the highest productivity. So equilibrium market tightness θ^* is such that $ay_n = b + e + (r + s + f(\theta^*))\frac{e}{\lambda}$.

Proof: Appendix A

Proposition 6 states that when there are no recruitment costs, firms will open vacancies to the point of driving the income flow of every match to zero, that is, they will open

vacancies until every worker in the economy is barely employable. Every type of worker is paid his productivity, $w_i = ay_i$, and faces an unemployment rate given by (1.23). The job rationing unemployment rate for each type of worker is inversely related to his productivity. Notice that every kind of worker will experience some degree of job rationing regardless of the aggregate level of technology, this result contrasts with Ferraro (2013) where only the least skilled workers experience job rationing. We can compute the share of job rationing for each type of worker as:

$$R_i \equiv \frac{u_i^R}{u_i}.$$

Corollary to proposition 6: *High-skilled workers experience lower shares of job rationing, i.e.,*

$$R_1 > R_2 > \dots > R_n.$$

Moreover, the idiosyncratic shares of job rationing are decreasing with productivity, i.e.,

$$\frac{\partial R_i}{\partial ay_i} < 0 \quad \forall ay_i \notin C_0.$$

The corollary tells us that the composition of the equilibrium unemployment of workers varies according to their productivity. The more skillful the worker is, the larger the role of frictions in the unemployment he experiences. Total job rationing is given by $u^R = \sum p_i u_i^R$. So the aggregate share of job rationing is given by

$$R \equiv \frac{u^R}{u}.$$

When heterogeneity is added, characterizing the behavior of the share of job rationing, either at an idiosyncratic or an aggregate level, is not as straight forward as in the case of a representative worker. The analysis depends heavily on the underlying worker-skill

distribution. Characterization of the results for an arbitrary distribution are left for further research. For the present work I rely on the numerical simulations performed in section 1.5. According to these results, obtained assuming a uniform distribution, both the idiosyncratic share of job rationing and the aggregate share of job rationing decrease with aggregate unemployment. As the aggregate technology level increases, frictions play a larger role in the generation of unemployment.

In this section I have extended the basic model described in section 1.3 to include heterogeneity among workers. In an undirected market firms can come into contact with any type of worker and their best response upon meeting a worker will depend on the worker's productivity. Workers with a high-enough productivity will always be hired and depending on the NSC they could be paid an efficiency wage or Nash-bargaining wage. In equilibrium, low-skilled workers will be discriminated by firms to reduce their outside options so the low wage corresponding to their productivity is enough to encourage them to exert effort. The model with heterogeneity also exhibits job rationing at an idiosyncratic and aggregate levels, how they are affected by productivity shocks depends on the skill distribution.

1.5 Simulation

This section presents the simulations of the model with a representative worker and the model with worker-skill heterogeneity. First, the parametrization of the model is discussed and then the results of the simulations are analyzed. This section concludes with some results that underscore the importance of considering heterogeneity in the model and the selection of a specific worker-skill distribution.

1.5.1 Calibration

Table 1.1 summarizes the baseline parameter values used in the following simulations. The objective is to illustrate the effect of changes in the parameters on the behavior of

the steady state equilibrium. When assessing these effects, only the parameter of interest changes while the rest are equal to their baseline specification. For the choice of matching function I follow the literature and assume a Cobb-Douglas $h(u, v) = \mu u^\alpha v^{1-\alpha}$, therefore

$$f(\theta) = \mu\theta^{1-\alpha} \quad \text{and} \quad q(\theta) = \mu\theta^{-\alpha}.$$

I set the elasticity of the probability of filling a vacancy to $\alpha = 0.6$ which according to Petrongolo and Pissarides (2001) is at the middle of the range of the parameter values, [0.5, 0.7], estimated across the literature. Following Shimer (2005) I consider quarters as the time unit.¹² The parameter for the efficacy of matching is set to $\mu = 1.355$ from the fact that a worker finds a job with a 0.45 probability per month so the flow arrival rate is approximately 1.35 on a quarterly basis.¹³ I set the discount rate to $r = 0.012$, equivalent to an annual discount factor of 0.953. The quarterly separation rate is set to $s = 0.10$ so the mean duration of a job is 2.5 years. The value of leisure is set to $b = 0.4$, and the bargaining power of the workers is set to satisfy the Hosios (1990) rule, that is, $\beta = 0.6$. The cost of opening vacancy is set to $\gamma = 0.213$ after Shimer (2005).

Table 1.1: Baseline Parameter Values in Simulations of the Model

	Description	Value	Source
r	Discount rate	0.012	Shimer (2005)
s	Separation rate	0.1	Shimer (2005)
μ	Efficacy of matching	1.355	Shimer (2005)
α	Unemployment-elasticity of matching	0.6	Petrongolo and Pissarides (2001)
y	Match productivity	1	Shimer (2005)
b	Unemployment benefits	0.4	Shimer (2005)
β	Worker bargaining power	0.6	Petrongolo and Pissarides (2001)
a	Technology level	1	
e	Effort intensity	0.05	Pissarides (1998)
λ	Inspection rate	0.175	Pissarides (1998)
γ	Recruiting cost	0.213	Shimer (2005)

There are two new parameters in the model, the worker effort requirement and the arrival

¹²The calibration of the base line model closely follows Shimer (2005) given the importance this paper has gained in the literature.

¹³See Shimer (2005) for further details.

rate of inspections. The lack of empirical evidence to identify either of these parameters has been a serious limitation of the efficiency wages literature. Following Pissarides (1998), I set $e = 0.05$. He sets λ so that on average a worker is detected shirking after 17 months, this implies a value of $\lambda = 1.75$.¹⁴

For the worker-skill distribution, I follow Ferraro (2013) by assuming that the distribution is uniform and I set the number of different skills in the population to $n = 200$.¹⁵ This is a discrete uniform distribution over the interval $[0.612, 1.388]$ meaning that $y_1 = 0.612$ and $y_{200} = 1.388$. The choice of these limits was made to comply with the observation made by Syverson (2011) that within four-digit SIC industries in the U.S. manufacturing sector the average difference in logged total productivity between plants in the 90th and 10th percentile is 0.651. In our setting this means that $\ln(y_{90th}/y_{10th}) = 0.651$. This would imply that the workers at the 90th percentile of the distribution are almost twice as productive as the worker at the 10th percentile. The distribution was also chosen to satisfy a mean productivity of one since the rest of the values of the parameters were taken from Shimer (2005) where the product of a match is normalized to one.

1.5.2 Simulation of the model with a representative worker

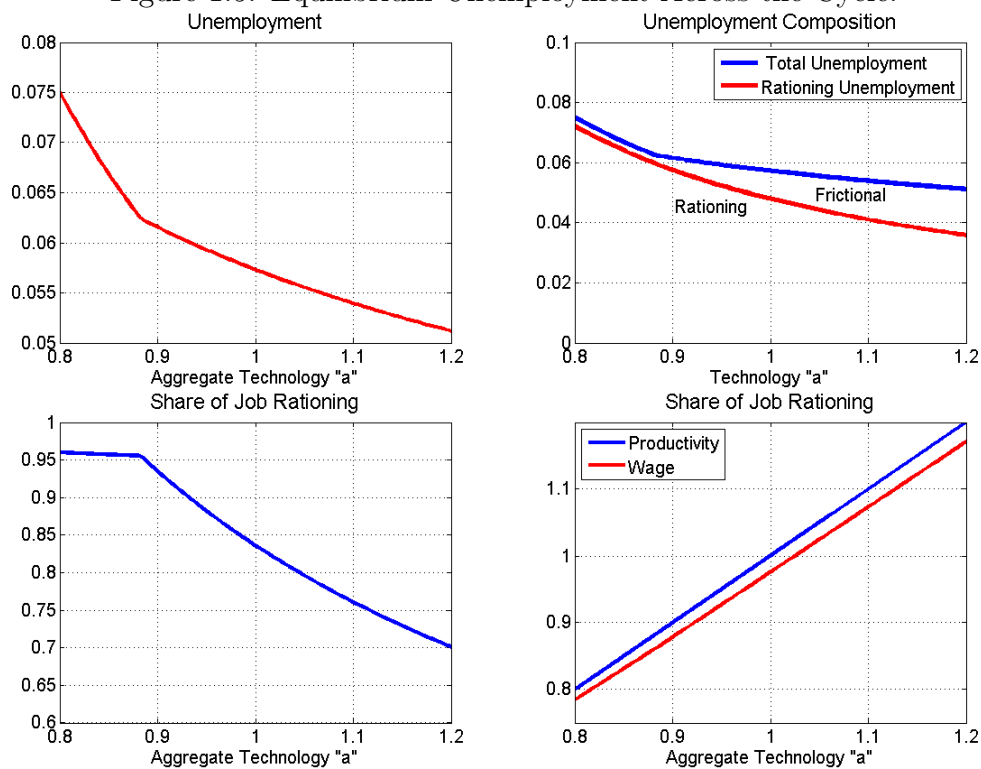
In this subsection I present the results of the model when a representative worker is considered. I am interested in how the steady-state outcomes of the model behave across the economic cycle. To simulate the economic cycle I compute and plot the steady-state

¹⁴ Pissarides (1998) considers an environment where efficiency wages are paid, however this model does not consider frictions but instead workers have to be called to a job in the market. He mentions that the role of unemployment in this setting is to discipline workers into not shirking on the job whereas in other settings, with job frictions, unemployment serves to curve wage demands. In the presence of Nash-bargaining, if there was no unemployment the worker would get all the production. In his model, wages also depend negatively on equilibrium unemployment. He recognizes the limitations of this model; the absence of empirical evidence on the effort level and the inspection rate.

¹⁵The choice of the number of different categories of worker-skills does not affect the results of the simulations as long as it is a large number. For M_{i100} the results are qualitatively the same.

outcomes of the model at different values of the aggregate technology level a , leaving the rest of the parameters constant. Michailat (2012) constructs a deseasonalized and detrended series for a using U.S. data and estimates it as a residual. He creates quarterly data for the period 1964 to 2009, during this period the parameter fluctuates roughly between 0.94 and 1.06. To clearly appreciate the contrasts, I consider a low technology level ($a = 0.08$) and a state of high technology ($a = 1.2$). The simulations are made over a grid of 100 equally spaced points.

Figure 1.5: Equilibrium Unemployment Across the Cycle.

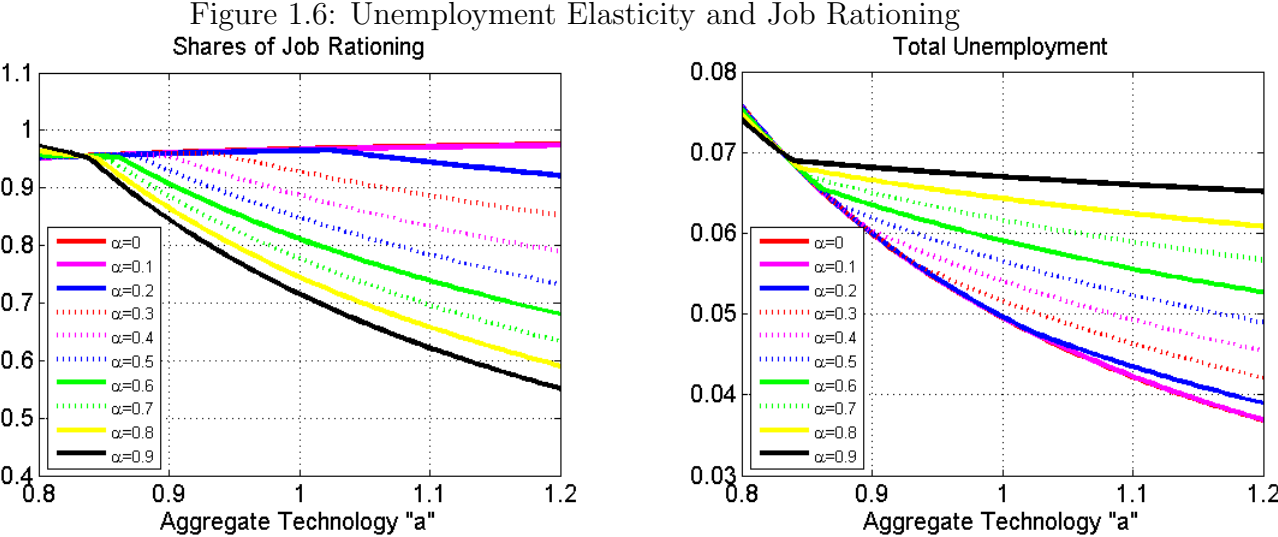


For the representative worker case, this figure shows the steady state outcome variables for different values of the aggregate technology level “a” using the baseline parameter specification. Aggregate unemployment decreases as technology improves, the kink in the slope corresponds to the change from efficiency wages to Nash-bargaining wages showing that under efficiency wages unemployment is more responsive to productivity. The share of rationing unemployment decreases with productivity and the gap between the product of a match and wages increases during an expansion.

Figure 1.5 shows the behavior of equilibrium unemployment, the composition of unemployment by its source, and wages for different values of the technology parameter. It can be observed that equilibrium unemployment decreases as the economy moves to a better state.

The abrupt change of slope in the curve corresponds to the change in wage determination described in the wage function (1.13). When the product of a match is not high enough to guarantee the participation of the worker under Nash-Bargaining, efficiency wages are implemented. As the analytical results suggested, market tightness is more elastic under efficiency wages reflected here by a steeper unemployment line. As predicted, larger realizations of technology decrease unemployment and change its composition making it more frictional, the expected cost of opening a vacancy plays a more prominent role in the decision of opening vacancies. The results are in accordance with those from previous studies with the difference that there is always some amount of rationing unemployment regardless of productivity levels. Also in accordance with the analytic results, the gap between wages and productivity widens as matches become more profitable.

To illustrate the analytic predictions about the importance of the elasticity of the matching function, I perform simulations of the model for different values of α and present it in Figure 1.6.



When the probability of finding a job is relatively inelastic (high α) the share of job rationing is decreasing with productivity and aggregate unemployment remains relatively flat. These results are interrelated, low elasticity implies that more vacancies do not translate

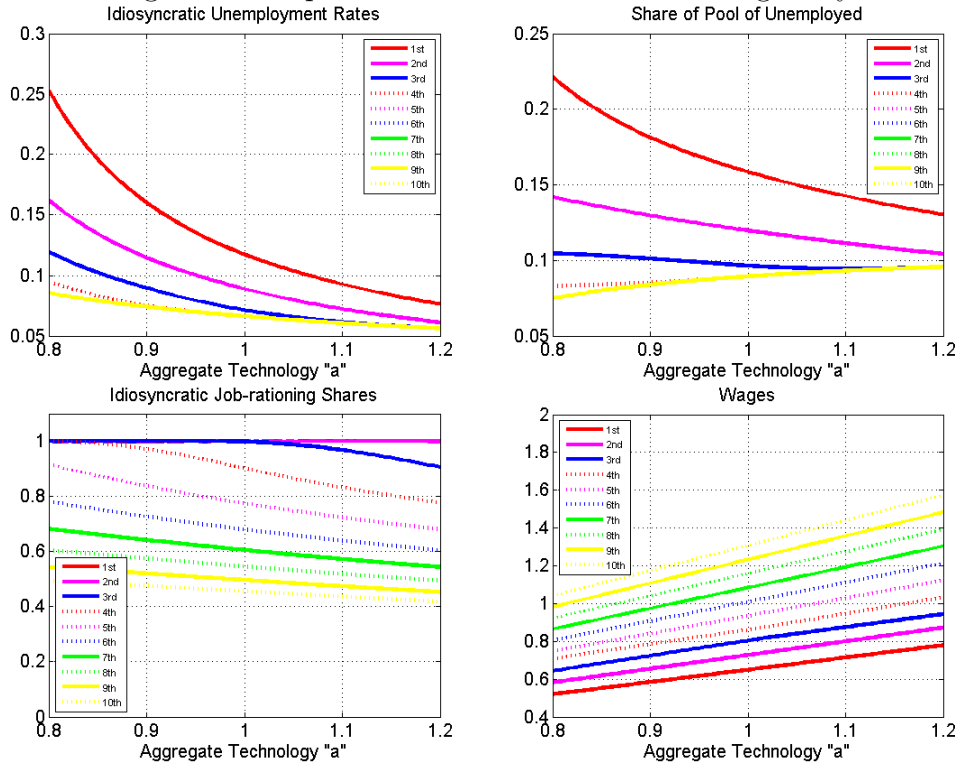
into a higher probability of finding a job, so although an increase in the productivity of a match makes firms open more vacancies, this does not translate into more employment. The outside options of an employed worker barely improve so the efficiency wage remains relatively constant. However, the inter-firm externalities are strong, so the expected costs of opening a vacancy increase so much that leave unemployment relatively unchanged. These forces make unemployment more frictional. When the probability of finding a job is relatively elastic (low α) the creation of vacancies responding to a positive productivity shock translates in to more workers finding a job, hence unemployment is more responsive to increments in productivity. The expected costs of opening a vacancy remain constant so opening a vacancy becomes relatively cheaper, more vacancies are open and the unemployment attributed to frictions is reduced as much as the unemployment attributable to moral hazard. The share of unemployment rationing stays relatively constant. In the extreme case of $\alpha = 0$, the share of rationing unemployment is procyclical, an increase in productivity reduces the frictional unemployment more than it reduces job rationing.

1.5.3 Simulation of the model with worker heterogeneity

Now I present the simulations of the model when heterogeneity is introduced. Once the skill distribution has been specified, I compute the equilibrium market tightness, idiosyncratic unemployment rates, shares of job rationing, and wages. 200 different types of workers are assumed so the results are presented in terms of averages across deciles, the first decile includes the least skilled workers and the tenth decile the most skilled.

Figure 1.7 shows the behavior of the average unemployment rates, shares in the pool of the unemployed, job rationing shares and wages by deciles for different aggregate technology levels. Table A1, included in appendix A shows the details. During a recession, the average unemployment rate of those in the bottom decile is almost 3 times as high as those in the top decile. As the economy moves to a better state, unemployment rates improve for all in the labor force and the unemployment rate gap between the bottom and the top deciles reduces

Figure 1.7: Equilibrium Outcomes with Heterogeneity.



This figure presents the idiosyncratic equilibrium outcome variables for the deciles in the skill distribution. The average unemployment rate of the lowest skilled workers is much higher and volatile than the average unemployment rate of those in the 10th decile. Also, the lowest skilled workers are over represented in the pool of the unemployed, a situation that improves with higher technology levels. The unemployment composition across deciles is different, job rationing is much more prominent in the unemployment of those with low productivity. Wages are also related to productivity, those with higher productivities have higher and more volatile wages.

to being only 1.2 times higher. Table 1.2 presents the standard deviations of these outcomes by deciles. It shows that the standard deviation of the unemployment rates of those in the bottom deciles is almost 6 times the standard deviation of the top decile. Unemployment for the low-skilled is much more volatile than unemployment for the high-skilled, decreasing much more in expansions and increasing much more in recessions. This phenomenon has its repercussions in the composition of the pool of the unemployed. In a low state of the economy those in the bottom decile represent 22% of the unemployed whereas the top decile represents only 7%. The skill distribution in the unemployment pool approximates the population skill distribution as the economy moves to a better state. In a very good state, the bottom decile represent 13% of the unemployed and the top decile 9%. One of the insights

of the model is that the asymmetries in the behavior of unemployment across deciles is a consequence of the differences in the nature of the idiosyncratic unemployment rates. These predictions are corroborated by the simulation, we can observe that job rationing is a larger component of the unemployment of those less skilled, in fact, unemployment rationing is the sole cause of unemployment for those in the bottom two deciles. In general, all deciles see a diminishing share of unemployment rationing as technology improves and, unlike other models, both kinds of unemployment persist throughout the cycle for all types of workers, having job rationing disappearing only asymptotically as the product of the match approximates infinity.

Table 1.2: Standard Deviations Across the Cycle by Deciles

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Average Unemployment Rate	0.047	0.028	0.018	0.010	0.008	0.008	0.008	0.008	0.008	0.008
Share of pool of unemployment	0.025	0.011	0.003	0.004	0.006	0.006	0.006	0.006	0.006	0.006
Job Rationing Share	0.000	0.000	0.028	0.074	0.068	0.052	0.041	0.033	0.027	0.022
Average Wage	0.076	0.085	0.088	0.095	0.110	0.119	0.129	0.138	0.147	0.156

We can observe that although the average wage of all deciles is procyclical, those of the most skilled vary more. The standard deviation of the wages in the top decile is two times the the standard deviation of those in the bottom decile. This is a direct result of the incentive constraints on the wages of least skilled workers. During a recession the wages of high-skilled workers can adjust accordingly to Nash-bargaining but the wages of low-skilled workers are constrained by the NSC.

These results can be empirically corroborated. Ferraro (2013) uses CPS micro data to analyze the behavior of groups of workers by age and education as proxies for skills, and finds that i) young and least educated workers experience average unemployment rates that are up to nine times that of primed-aged educated workers, and ii) they account for approximately 70 percent of the time series variation in the U.S. unemployment rate. Gervais et al. (2013) find a similar situation.

1.5.4 The importance of the skill distribution

In this subsection I introduce a topic for future research, the importance of worker-skill distribution in the behavior of unemployment. Recent literature emphasizes the role of heterogeneity in the labor force to understand the fluctuations in the economy. For example, Pries (2008) and Blis et al. (2011) analyze how introducing worker heterogeneity into search-and-matching models helps to emulate the observed volatility of aggregate unemployment. They find that heterogeneity does increase predicted volatility but not enough to match the volatility observed in the data.

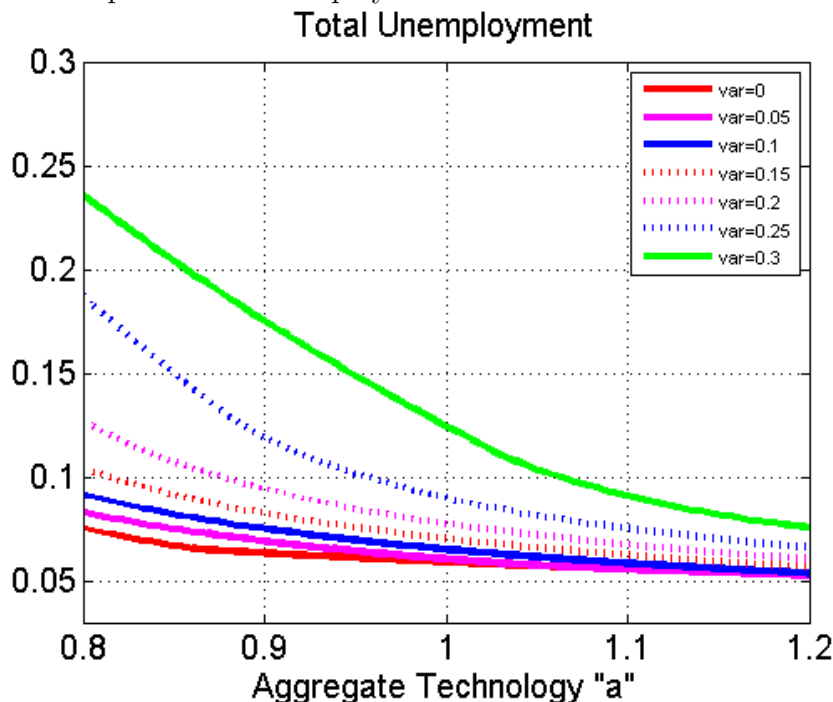
There is also an increasing literature relating demographics and skills with the observed volatility in macroeconomic variables. Jaimovich and Siu (2009) investigate the consequences of demographic change for business cycle analysis. They find that changes in age composition, which could be a proxy for skills, account for a significant fraction of the variation in business cycle volatility observed in major economies. They show that the cyclical volatility of the labor market is U-shaped as a function of age. These results suggest that the age composition of the labor force is potentially a key determinant of the responsiveness of an economy to business cycle shocks, when the labor force is mostly composed of individuals with high observed volatility in hours of work, such as the young and those close to retirement, there is more propagation in the business cycle.

Other works that emphasize how critical it is to understand cyclical movements of low-skilled workers to explain the large fluctuations in the labor market and the economy are Champagne (2014), Lagauer (2012), Champagne and Kurmann (2012), and Gorry (2013). This research underscores how important it is to incorporate the right distribution of skills into the analysis of the model with heterogeneity.

I perform the simple exercise of computing the equilibrium total unemployment across the cycle considering different worker-skill distributions. I consider uniform distributions with mean one and different variances. The results are presented in figure 1.8.

When the variance is equal to zero, the model reduces to the representative worker case

Figure 1.8: Equilibrium Unemployment for different distribution variances.



This figure shows the behavior of equilibrium unemployment for different productivity levels for skill distributions with different variances. All the distributions are uniform with mean one. When the variance is equal to zero the model reduces to the representative worker case with a productivity of one. As the variance of the distribution increases, total unemployment becomes more elastic. Aggregate unemployment increases for every realization of the technology parameter. These predictions expose the peril of the assumption of a representative agent and the importance of the skill distribution for the calibration of the model.

presented in section 1.3. As workers become more diverse aggregate unemployment increases for any technology level and also becomes more sensitive to its changes as we can observe in the steeper unemployment curve. These results suggest that i) if heterogeneity must be incorporated into the model, the calibration of the model must be changed accordingly to match the empirical moments, and ii) incorporating heterogeneity does have a great impact on the volatility predicted by the model. Future extensions of this the model here presented attempting to replicate the historic behavior of unemployment must consider how the skill distribution changes over time.

This section has presented some numerical results of the models with a representative worker and worker skill heterogeneity. The numerical results backup the analytical predictions of previous sections and unveil new insights about the importance of the matching

function in the moral hazard problem and the relevance of the skill distribution for the calibration.

1.6 Final Remarks

I have presented a framework that integrates moral hazard and worker heterogeneity into a standard search-and-matching model of unemployment. The broad contribution of this chapter is to show that these features greatly enrich the predictions of an otherwise standard search-and-matching model. Moral hazard and imperfect monitoring introduce job rationing into the model, making it more versatile and suitable to analyze the labor market situations where matching frictions have been reduced but unemployment is still persistent. When combined with worker heterogeneity, moral hazard diversifies the outcomes of the labor market across the labor force. It creates a broad classification of workers with important repercussions. There are employable workers, who generate benefits for the firm and it is the possibility of being randomly matched with one of these workers that makes firms enter the market, and there is also barely employable workers, workers with productivities so low that cannot generate positive profits for the firm but do create search externalities that make the market less efficient. Without the presence of moral hazard and imperfect monitoring, although wage dispersion could still be generated, the diversity in the unemployment rates and exit rates of unemployment would not exist. It is due to the diversity of incentive combinations necessary to align worker-firm interests at a minimum cost that all these results arise. The policy implications of the model are that the effectiveness of labor market and macroeconomic policies depends on aggregate productivity and the skill distribution in the workforce since they determine the overall and idiosyncratic share of job rationing and therefore the dynamics of unemployment. The most relevant implication has to do with policies targeting the unemployment of a particular sector in the labor force. It can also offer recommendations for programs targeting the productivity of workers such

as training programs. According to the results, the general equilibrium effect of increasing the productivity of workers on the low-side of the skill distribution is greater than effect of an increase in the productivity of the most skilled workers. This suggests that during an economic downturn, programs aimed to enhance the productivity of those with lowest skills are optimal to stimulate the economy. The model also implies that programs aimed to reduce search frictions do not affect the employment situation of low-skilled workers and their overall efficacy depends on productivity. Also, the framework of the model makes it ideal to study the effects of employment policies that are believed to have asymmetric effects across the labor force such as minimum wages laws.

Chapter 2

Worker Heterogeneity and the Asymmetric Effects of Minimum Wages

2.1 Introduction

The effects of minimum wages on labor market outcomes have been extensively investigated in economics. Most of these studies focus on low-wage industries which, although diverse in nature, share a common and heterogeneous labor supply due to low technical requirements and high substitutability among workers. This heterogeneity is often overlooked by the literature and important asymmetries in the impact of minimum wages are missed by a representative-worker assumption. This chapter explores the notion that minimum wages could affect the labor force asymmetrically due to worker heterogeneity. First theoretically, by developing a search model of unemployment with heterogeneous workers. Then empirically, by finding evidence supporting the model's main result: within the low-skill labor force, a rising minimum wage lowers the employment and labor force participation of the least productive workers as they are priced out of the market, while it increases the

employment, participation, and wages of more productive workers that remain hireable.

I develop a search-and-matching model of unemployment with ex-ante worker heterogeneity, endogenous search intensity, and worker moral hazard as in Shapiro-Stiglitz (1984). The presence of worker moral hazard creates the need to incentivize workers at all times, which is optimally done by employers through a combination of efficiency wages and the threat of long unemployment spells. Worker heterogeneity generates a diverse array of optimal incentivizing schemes that depend on the worker's productivity and lead to differences in wages and unemployment rates across the labor force.

Under these circumstances, a binding minimum wage disrupts optimal incentivizing schemes and ultimately leads to disemployment and labor-force discouragement. When wages are negotiated via Nash-bargaining, a binding minimum wage can improve the workers' bargaining position without terminating or precluding future matches. However, with efficiency wages in place, the room for wage bargaining has been exhausted and employers cannot profitably raise wages. Match termination ensues. With bleak employment prospects, the worker's best option is to stop searching for a job since it is a costly activity with no expected payoffs; a minimum wage discourages low-productivity workers from participating in the labor force.

The model predicts improvements in the labor-market conditions of workers remaining in the market after a minimum wage hike. A rising minimum wage drives the lowest-skilled workers out of the labor force, which increases average worker productivity and the expected return of filling a vacancy. Equilibrium market tightness rises in response, which increases employment and creates spillover effects on wages through higher job-finding rates and better wage-bargaining terms for workers. These improvements encourage the labor force participation and search intensity of hireable workers.

Using Current Population Survey (CPS) data I test the model's predictions. Identifying heterogeneity in the labor force is fundamental for the analysis, so I consider two-way disaggregation by educational attainment and age. Two important results emerge from the

analysis: 1) the minimum wage affects mostly low-education (high school or lower) labor markets; and 2) increments in the minimum wage have diametrically opposed effects within the low-education labor force: they reduce the employment and labor force participation of teenagers with less than high school education, while increasing the employment and labor force participation of mature workers with high school educational attainment.¹ To theoretically assess the effects of increments in the minimum wage in the low-education labor market, I calibrate the model using the empirical results. Simulations of increases in the minimum wage show that, although the effect on individual labor-market outcomes vary widely by productivity, aggregate employment, aggregate labor force participation and social welfare, defined as total output net of search and recruiting costs, decrease with a rising minimum wage. According to the simulation, increasing the binding minimum wage from \$7.25 to \$15 would cause an employment and labor force participation reduction of roughly 50%, and a 70% decrease in social welfare for the low-education labor force.

The contributions of this work are theoretical and empirical. Theoretically, it presents a tractable and versatile model for the analysis of worker heterogeneity which predicts asymmetric outcomes across the labor force such as diverse unemployment rates, labor force participation rates, and wages. This characteristic makes the model useful for the analysis of policies affecting workers differently according to their skills and productivity.

Applying this model to the analysis of minimum wages offers an additional set of advantages. The model's setting emulates an environment where a minimum wage is most likely binding and consequential; low-wage labor markets which are mostly characterized by unspecialized jobs with high substitutability between workers and no skill-signaling. The assumptions of ex-ante worker heterogeneity and random search make the model fit this description.

The model offers an intuitive and cohesive explanation of the ripple effects and the asymmetries in the impact of minimum wages. This is achieved by assuming moral hazard

¹I define mature workers as the workers aged between 25 to 59.

and imperfect monitoring in a unified low-wage labor market where all outcomes are driven by the same general equilibrium effect; changes in equilibrium market tightness. As the minimum wage binds at the low end of the worker productivity distribution, it changes the firm's incentives to open vacancies. The effects of the minimum on the outcomes of workers on the upper part of the distribution depend on whether the market tightens or loosens. The presence of moral hazard delivers stark predictions about minimum wages hikes; market tightness unambiguously increases through a "weeding-out" effect in the labor force.

The model is also capable of generating and explaining a number of other phenomena related to minimum wages in a parsimonious way. It describes the well-documented wage spillover effect as a general equilibrium result. After a minimum wage hike, market tightness increases and jobs arrive to remaining workers at higher rate than workers do to open vacancies, relative to before the hike. The worker's bargaining strength is then higher and the firm's lower, which results in higher wages.² It also sheds light on the use of suboptimal minimum wages: situations where, due to regulations, employers could actually pay workers less than the minimum and yet decide not to.³ The situation occurs even when some firms paid a starting wage below the new minimum before it became effective. In my model, a higher minimum wage increases market tightness: workers who remain hireable have better outside options, which increases the endogenous efficiency wage floor. So, it could be the case that workers earning below a new minimum before it becomes effective must be paid above the new minimum to be incentivized.

This work also contributes to the empirical literature on minimum wages by documenting that increases in the minimum wage impact low-education workers only, and that the nature and magnitude of the effects depend on education and age. My results are consistent with the bulk of literature finding negative employment and labor force participation effects for the young low-educated population, with estimated elasticities of -0.20 and -0.15 respectively. A

²For evidence on wage spillover effects see: Katz and Krueger (1992); Card and Krueger (1995); Dolado, Felgueroso, and Jimeo (1997); Teulings (2003).

³Freeman, Wayne, and Ichnioski (1981); Katz and Krueger (1991), (1992); Manning and Dickens (2002).

new finding of the present study is the positive impact on the employment and labor force participation for mature workers with high school educational attainment, with predicted elasticities of 0.05 and 0.04 respectively.

According to my results, neglecting to consider worker heterogeneity masks important intra-labor force minimum wage effects; their impact on labor market outcomes depends on the specific population under study. The implementation of a minimum wage must identify and acknowledge who the truly affected workers are, and the direction and magnitude of the impact.

The chapter is organized as follows. Section 2.2 gives a review of the related literature. Section 2.3 presents the search-and-matching model with heterogeneous workers. Section 2.4 presents empirical evidence of asymmetric minimum wage effects on labor market outcomes. Section 2.5 calibrates the model and shows the simulation's results of increases in the minimum wage. Finally, Section 2.6 concludes. Tables, derivations, and proofs are included in appendix B.

2.2 Related Literature

This work relates to several strands of the literature. First, it is related to the work studying the effect of changes in minimum wages on labor market outcomes and welfare in a Mortensen-Pissarides framework. The main difference between the present work and previous is the inclusion of worker heterogeneity in a random search environment, that is, the assumption that different workers participate in the same labor market. The best known work in the field is Flinn(2006). He considers heterogeneity in the workers' outside values to account for the labor-force participation effects that a higher minimum could create, but once workers have decided to enter the labor market, they are ex-ante identical and their productivity is determined by a random draw from a productivity distribution. This ex-post heterogeneity does not make it possible to create a link between market outcomes and

workers' individual characteristics, such as the empirical correlation between wages and age. Flinn (2010) presents an extension of that model introducing ex-ante worker heterogeneity captured by differences in the parameters of the productivity distribution determining the product of a match. With randomness in the productivity, endogenous contact rates can be derived only when directed search is considered, that is, when different workers are assumed to participate in different labor submarkets. Rocheteau and Tasci (2008) investigate the effect of minimum wages in an array of different environments within a search framework. However, they do not consider worker heterogeneity. Gorry (2013) presents a search model to explore the effects of minimum wages on experience accumulation. His model includes worker heterogeneity but search is directed.

To the best of my knowledge, this work is the first one to study minimum wages in an environment with worker heterogeneity and random search. These two characteristics absent in other models are necessary to understand how asymmetries in the way a minimum wage affects workers in the same labor market arise. With these characteristics, a minimum wage binding only for a small portion of workers has repercussions on the outcomes of all the workers in the market.

Another important difference with Flinn (2006) is that his empirical analysis focuses on estimating the workers' bargaining power to determine if the Hosios (1990) efficiency condition is satisfied and assess the welfare properties of a minimum wage. In my model, the Hosios (1990) rule of optimality does no longer hold due to the heterogeneity in the workforce and the constraint on the Nash bargaining. Whether the minimum wage has detrimental or improving welfare effects depends on the model's parameterization. Based on the results of the reduced form estimation, I assess the welfare properties of a minimum wage on the low-education labor market with a calibrated version of the model.

This work also relates to the vast empirical literature exploring the effect of minimum wages on employment, broadly reviewed in Neumark and Wascher (2007). They report that the majority of the studies give a consistent indication of negative employment effects, and

that among the papers that according to them provide the most credible evidence, almost all point to negative employment effects, both for the United States as well as for many other countries. The studies that focus on the least-skilled groups provide relatively overwhelming evidence of stronger disemployment effects for these groups. My results are consistent with the literature; teenagers and the least educated workers experience negative employment effects. My result show positive, although small, positive employment effect for 25 to 59 year-olds with high school educational attainment. To the best of my knowledge, this is the first study to find positive employment effects for this specific demographic.

This chapter also explores the effect of minimum wages on labor force participation. Previous works such as Kaitz (1970), Mincer (1976), Ragan (1977), and Wessels (1980) find that the minimum wage decreased, or left unchanged, the labor force participation rate of low-wage workers. Using more recent econometric techniques, Wessels (2005) shows that minimum wage hikes had a small but significant negative effects on the labor force participation of teenagers. My results are overall consistent with these findings; I also find significant negative elasticities for teenagers of -0.15. However, this work is the first to find significant positive effects on labor force participation on 25-59 year olds with high-school educational attainment and find a statically significant elasticity of 0.04.

2.3 The model

In this section, I present a search-and-matching model of unemployment with worker heterogeneity, moral hazard, and endogenous search intensity. The environment is the same as Pissarides (2000) chapter 5 with two important differences: workers vary in their productivity, and there is imperfect monitoring of a worker's effort as in Shapiro and Stiglitz (1984).⁴

⁴The model is based on previous models of search unemployment with moral hazard and imperfect monitoring: Mortensen (1989), Mortensen and Pissarides (1999), and Rocheteau (2001).

2.3.1 The Model's Environment

Time is continuous, endless, and is denoted by t . All agents are risk neutral and discount utility flows at rate $r \in \mathbb{R}^+$. There are n kinds of workers with ex-ante productivities y_1, \dots, y_n satisfying $0 < y_1 < \dots < y_n$. There is a continuum of identical firms which can be matched with one worker at most. Search is random or undirected; firms can be matched with any type of worker. Productivity is perfectly observable by firms and workers, so a worker with productivity y_i , hereinafter type- i worker, is hired upon meeting with an endogenous probability Π_i . As it will be shown later, Π_i is optimally chosen by firms as a motivating device.

Firms are identical, so the production flow of a match depends entirely on the worker's type.⁵ There are two levels of work intensity; e and 0. If a type- i worker exerts effort, the flow product is y_i , and the worker bears a disutility of e . If the worker shirks, the product of a match is zero. The effort exerted by the worker is observable only after an inspection, which obeys a Poisson process with an exogenous arrival rate $\lambda \in \mathbb{R}^+$. If the worker is caught shirking the match is terminated. There are no reputational effects, so upon meeting a worker, a firm does not know whether the worker has a shirking history or not.

An employed worker receives a wage $w_i \leq y_i$. When unemployed, a worker receives an income $b < y_1$, which can be interpreted as unemployment benefits or the utility a workers derives from not working. Unemployed type- i workers, must decide how actively they search for a job. This search intensity is denoted by s_i and in the model's setting, it is tantamount to labor force participation. If $s_i = 0$, the worker is not participating in the labor market and higher levels of s_i will be interpreted as higher labor force participation. Search intensity comes at a cost $c(s_i)$, where $c'(s_i) > 0$, $c''(s_i) > 0$, $c(0) = c'(0) = 0$ and $c(\infty) = \infty$. Similarly, a firm with a vacant job must incur a flow cost $\gamma \in \mathbb{R}^+$ to advertise its vacancy.

⁵Other search-and-matching models where search is undirected are Acemoglu (1999), Shimer(2001), Dolado et al. (2003) and Pries (2008). Acemoglu (1999) makes a case for undirected search by pointing out that skill is imperfectly correlated with observable characteristics, such as years of education and age, making it difficult for employers to recruit workers with a particular skill level.

Worker population is given by p_1, \dots, p_n , where p_i is the share of type- i workers. I denote the unemployment rate of each type of worker as u_i , and the number of vacancies as a fraction of the worker population as v . Labor market tightness is defined as

$$\theta \equiv \frac{v}{\sum_i p_i s_i u_i},$$

where $\sum_i p_i s_i u_i$ is the measure of active unemployed workers. The number of matches made per-unit of time is given by the constant-returns matching function $h(\sum_i p_i s_i u_i, v)$, differentiable and increasing in both arguments. The matching rate per active unemployed worker is defined as

$$f(\theta) \equiv \frac{h(\sum_i p_i s_i u_i, v)}{\sum_i p_i s_i u_i} = h(1, \theta).$$

Similarly, a firm's matching rate is given by

$$q(\theta) \equiv \frac{h(\sum_i p_i s_i u_i, v)}{v} = h(1/\theta, 1).$$

Unemployed workers are matched faster in a tighter market, that is, when there are more vacancies relative to job seekers. Similarly, firms are matched with workers faster when there are more unemployed workers relative to vacancies. Matches are terminated by an exogenous shock following a Poisson process with parameter $\delta \in \mathbb{R}^+$. There is no on-the-job search.

2.3.1.1 Worker Behavior

Once hired, it is the worker's decision to exert effort or shirk. This decision is made based on the lifetime expected utility of each action. A non-shirker is a worker who chooses not to shirk in all periods while his current job lasts. He gets a wage w_i and suffers a disutility e per unit of time, and could have his job terminated exogenously with probability δ . The lifetime expected utility of a type- i non-shirker, E_i , obeys the flow Bellman equation:

$$rE_i = w_i - e + \delta(U_i - E_i), \quad (2.1)$$

where U_i is the lifetime expected utility of a type- i unemployed worker. E_i represents the asset value of employment, so (2.1) states that the opportunity cost of holding a job without shirking is equal to the current income flow minus the disutility of effort plus the expected capital loss from a change of state.

The expected lifetime utility of a worker who chooses to shirk, S_i , during a length of time dt , satisfies

$$S_i = w_i dt + \exp(-r dt) \{ \Pr [\min(\tau_\delta, \tau_\lambda) \leq dt] U_i + (1 - \Pr [\min(\tau_\delta, \tau_\lambda) \leq dt]) E_i \}, \quad (2.2)$$

where τ_λ is the length of time until the next inspection and τ_δ is the duration of the job. These two processes are characterized by an exponential distribution with parameters λ and δ respectively. According to (2.2), during the time interval dt a shirker receives a real wage $w_i dt$ and has no disutility from work, he loses his job if he is caught shirking or if the match is terminated by an idiosyncratic shock. If neither of these two events occur during the time interval dt , the employed worker stops shirking in all subsequent periods. A worker will choose to exert effort over shirking if and only if the lifetime expected value of not shirking is greater the lifetime expected value of doing so. As dt approaches zero, it can be shown that a type- i worker will choose effort over shirking if and only if

$$E_i - U_i \geq \frac{e}{\lambda}, \quad (2.3)$$

this is the no-shirking condition (NSC) and its derivation is shown in appendix B. The condition states that in order to incentivize a worker to exert effort in the production process, his surplus from a match must be at least equal to e/λ , the expected disutility from working before the next inspection. When a workers decides to shirk he saves the disutility of effort

e but has an expected capital loss of $\lambda(E_i - U_i)$. In equilibrium, workers will never have an incentive to shirk since firms will never hire a worker if they cannot guarantee their effort, so the lifetime expected utility of unemployment of a type- i worker satisfies

$$rU_i = \max_{s_i \geq 0} \{b - c(s_i) + s_i \Pi_i f(\theta) [E_i - U_i]\}, \quad (2.4)$$

where $s_i \Pi_i f(\theta)$ is the unemployment-exit rate. According to (2.4), when an unemployed worker finds a job he becomes a non-shirker given that the NSC is always satisfied. It is important to remark that the permanent income of unemployed workers is increasing with market tightness since the probability of coming into contact with a firm increases with more vacancies per worker, which shortens the average duration of unemployment. Workers set their search intensity to maximize rU_i taking θ and the rest of the parameters as given. The optimal choice of search intensity solves:

$$c'(s_i) = \Pi_i f(\theta) [E_i - U_i]. \quad (2.5)$$

The restrictions imposed on the cost function $c(s_i)$ guarantee a unique solution to (2.5). Combining (2.1) and (2.4) the surplus of a match for a type- i worker is:

$$E_i - U_i = \frac{w_i - e - b + c(s_i)}{r + \delta + s_i \Pi_i f(\theta)}. \quad (2.6)$$

A worker will accept a match if and only if $E_i - U_i$ is positive and, according to the NSC (2.3), will choose not to shirk if and only if it is greater than e/λ .

2.3.1.2 Firm Behavior

The present discounted value of expected profits from a vacant job, V , must satisfy the Bellman equation

$$rV = -\gamma + q(\theta) \left[\sum_i \Pi_i \mu_i (J_i - V) \right], \quad (2.7)$$

where γ is the flow cost of keeping a vacancy open. The value function of a filled vacancy by a type- i worker is denoted by J_i , and μ_i is the fraction of active unemployed type- i workers in the active unemployed population. Equation (2.7) states that the capital cost of an open vacancy has to be exactly equal to the rate of return of the vacancy, i.e., the flow costs of recruiting plus the expected capital gain. The fraction of unemployed type- i workers is given by

$$\mu_i = \frac{p_i s_i u_i}{\sum_j p_j s_j u_j}. \quad (2.8)$$

The asset value of an occupied vacancy by a type- i worker satisfies a similar Bellman equation:

$$rJ_i = y_i - w_i + \delta(V - J_i). \quad (2.9)$$

Firms will hire workers only if the NSC is satisfied, so the capital gain of a filled vacancy is equal to the income flow, $y_i - w_i$, plus the expected capital loss when the match is exogenously destroyed. Each firm takes the strategy of other firms as given, i.e. they take market tightness as given, and chooses Π_i in order to maximize its expected profits. The best response function of a firm satisfies the following rule:

$$\begin{aligned} J_i - V > 0 &\Rightarrow \Pi_i = 1 \\ J_i - V < 0 &\Rightarrow \Pi_i = 0 \\ J_i - V = 0 &\Rightarrow \Pi_i \in [0, 1] \end{aligned} \quad (2.10)$$

In words, if the firm's surplus of a match is strictly positive the firm will always hire the worker. If the surplus is negative the firm will never hire the worker. If the surplus is zero, the firm is indifferent between hiring the worker and keep searching for a worker, so

the firm's best response is to hire a worker with any probability.

2.3.1.3 Wage determination

Wages are determined through Nash bargaining subject to the constraint of the NSC.⁶

Wages solve:

$$w_i = \arg \max (E_i - U_i)^\beta J_i^{1-\beta} \text{ s.t. } E_i - U_i \geq \frac{e}{\lambda}, \quad (2.11)$$

where β is the worker's bargaining power. Using (2.6) and (2.9), the expression for the unconstrained Nash-bargaining wage, w_i^N , is

$$w_i^N = y_i \left[\frac{\beta(r + \delta + s_i \Pi_i f(\theta))}{r + \delta + \beta s_i \Pi_i f(\theta)} \right] + \left[\frac{(r + \delta)(1 - \beta)}{r + \delta + \beta s_i \Pi_i f(\theta)} \right] [b + e - c(s_i)]. \quad (2.12)$$

With an unconstrained Nash-bargaining wage, the worker's surplus of employment is

$$E_i - U_i = \frac{\beta[y_i - e - b + c(s_i)]}{r + \delta + \beta s_i \Pi_i f(\theta)}. \quad (2.13)$$

When the NSC cannot be satisfied with the unconstrained Nash-bargaining wage, firms set the wage to incentivize workers. The minimum wage that a firm must pay to induce effort from the worker is the wage that makes the NSC bind. Substituting (2.6) into the NSC and solving for w with an equality, we get that the efficiency wage, w_i^E , is

$$w_i^E = b + e - c(s_i) + \frac{e}{\lambda} [r + \delta + s_i \Pi_i f(\theta)]. \quad (2.14)$$

This is the minimum wage required to incentivize workers to exert effort. Without the threat of moral hazard, i.e. if $e = 0$ or $\lambda \rightarrow \infty$, the unconstrained Nash-bargaining wage would be enough to guarantee the worker's effort into the production process. In the presence of

⁶Using formal econometric analysis, Malcomson and Mavroeidis (2011), show that this constrained Nash-bargaining mechanism fits the wage patterns in the US data better than the canonical unconstrained Nash-bargaining model in Mortensen and Pissarides (1994), or the credible bargaining model of Hall and Milgrom (2008).

moral hazard, guaranteeing effort requires a moral-hazard premium defined as the difference between the efficiency wage and the unconstrained Nash-bargaining wage, $w_i^E - w_i^N$. It can be showed that this premium is inversely related to productivity and it increases with the relative value of the worker's outside options. If his current job is likely to end or if the expected length of unemployment is short, the outside options are relatively more valuable so the moral hazard premium must be larger. Consistent with the efficiency-wage literature, the no-shirking wage is higher when the effort to be exerted is larger or the detection probability is lower. Notice that the efficiency wage is an increasing function of market tightness just like the Nash-bargaining wage but, unlike it, the efficiency wage is not bounded above. This is the result of the fact that the moral hazard premium goes to infinity along with market tightness. If the worker's valuation for his job must be kept above a threshold to prevent shirking, as the market tightens, the wage premium gets increasingly large to compensate the worker for the improvement of his outside options.

Depending on whether the NSC is binding, equilibrium wages are determined either through Nash bargaining or with the expression for efficiency wages. Using (2.3) and (2.13), the solution to (2.11) can be specified as:

$$w_i = \begin{cases} w_i^N, & y_i > b + e - c(s_i) + (r + \delta + s_i \Pi_i f(\theta) \beta) \frac{e}{\lambda \beta}, \\ w_i^E, & y_i \leq b + e - c(s_i) + (r + \delta + s_i \Pi_i f(\theta) \beta) \frac{e}{\lambda \beta}. \end{cases} \quad (2.15)$$

High-productivity workers will receive unconstrained Nash-bargaining wages while low-productivity workers will be paid an efficiency wage. This function is monotonically increasing and continuous on market tightness and productivity.

2.3.2 Equilibrium

I only consider symmetric Nash equilibria; all firms adopt the same hiring strategy. This optimal hiring strategy must satisfy the hiring rule in (2.10) and no firm must have an incentive to change its strategy given the other firms' strategies. The free-entry condition

for firms implies that the value of a vacancy is zero, $V = 0$. Using this fact, the equilibrium best response hiring function can be derived. For a given θ and s_i , the firm's best-response hiring function for a type- i worker is:

$$\Pi_i(\theta) = \begin{cases} 1, & y_i \geq y_M(\theta), \\ \frac{y_i - b - e + c(s_i) - (r + \delta)\frac{e}{\lambda}}{s_i f(\theta)\frac{e}{\lambda}}, & y_M(\theta) > y_i \geq y_L, \\ 0, & y_L > y_i. \end{cases} \quad (2.16)$$

, where $y_H(\theta) \equiv b + e - c^E + [r + \delta + s^E f(\theta)\beta]\frac{e}{\lambda\beta}$, $y_M(\theta) \equiv b + e - c^E + [r + \delta + s^E f(\theta)]\frac{e}{\lambda}$, $y_L \equiv b + e + (r + \delta)\frac{e}{\lambda}$, s^E is such that $c'(s^E) = f(\theta)\frac{e}{\lambda}$, and $c^E \equiv c(s^E)$. The derivation of this function is shown in appendix B. The function describes the firm's hiring behavior upon contact with a type- i worker. Workers with high productivities will always be hired by firms since they can be encouraged profitably; firms are strictly better off hiring them. Workers with lower productivities will be hired with a probability less than one because upon contact with these workers, firms are indifferent between hiring them and not, they generate no surplus for the firm. Workers with very low productivities cannot be encouraged without generating a negative surplus for the firms; firms will never hire these workers. Using these results, the equilibrium search intensity function for a type- i worker can be derived.

$$s_i(\theta) = \begin{cases} s_i^N, & \text{s.t. } c'(s_i^N) = \frac{\beta(y_i - e - b + c(s_i))}{r + \delta + \beta s_i^N f(\theta)} & y_i \geq y_H(\theta), \\ s^E, & \text{s.t. } c'(s^E) = f(\theta)\frac{e}{\lambda} & y_H(\theta) > y_i \geq y_M(\theta), \\ s_i^L, & \text{s.t. } c'(s_i^L) = \frac{y_i - e - b + c(s_i^L) - (r + \delta)\frac{e}{\lambda}}{s_i^L} & y_M(\theta) > y_i \geq y_L, \\ 0, & & y_L > y_i. \end{cases} \quad (2.17)$$

The results in (2.17) describe the optimal search intensity of workers according to their productivity. It can be shown that $0 \leq s_i^L \leq s_i^E \leq s_i^N$, so more productive workers will participate more intensely in the market since the gains of finding a job are larger for them.

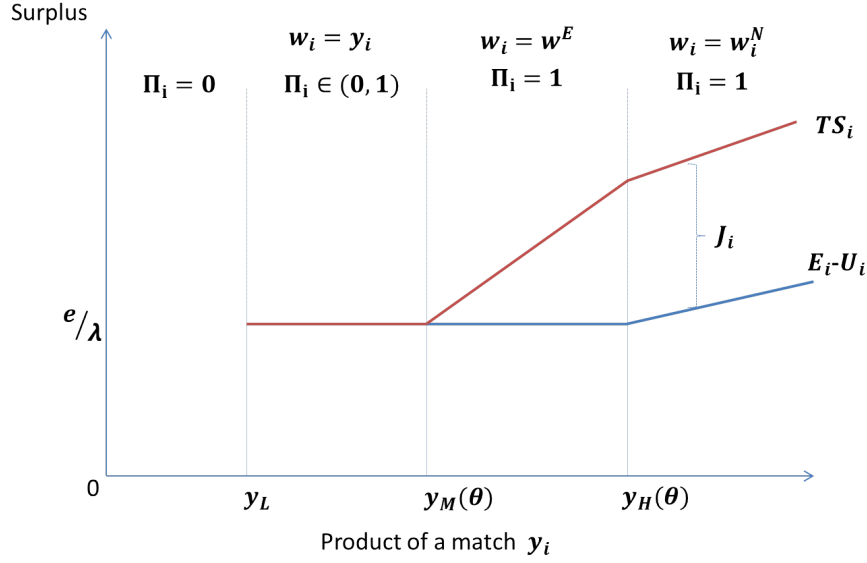
Using the information from the optimal search intensity and hiring rule functions, the wage equation (2.15) can be expressed as:

$$w_i(\theta) = \begin{cases} w_i^N(\theta), & y_i \geq y_H(\theta), \\ w^E(\theta) \equiv b + e - c^E + \frac{e}{\lambda}[r + \delta + s^E f(\theta)], & y_H(\theta) > y_i \geq y_M(\theta), \\ y_i, & y_M(\theta) > y_i \geq y_L. \end{cases} \quad (2.18)$$

Equations (2.16), and (2.18) describe the equilibrium wage-hiring incentivizing scheme for a given market tightness. Upon contact with a worker, firms hire him with a probability and a wage that ensures that the NSC is not violated. The rationale behind the incentivizing-hiring scheme is presented in Figure 2.1. If the product of a match is high enough ($y_i > y_H(\theta)$), wage is set using Nash-bargaining since it generates surplus for both, worker and firm, without violating the NSC. Worker surplus is above the no-shirking threshold so he will be incentivized to work, and the surplus that a firm gets is positive so it will hire the worker with probability one. When productivity is not so high ($y_H(\theta) \geq y_i > y_M(\theta)$), the Nash-bargaining wage is not high enough to prevent a worker from shirking, efficiency wages are necessary to ensure that the match is productive. The firm still gets a positive surplus so the worker is hired with probability one. Workers such that $y_i \geq y_M(\theta)$ will be referred to as “perfectly employable” since firms can always hire them and get a strictly positive match surplus.

When $y_M(\theta) \geq y_i > y_L$, the worker’s efficiency wage is greater than the product of a match ($w_i^E = b + e - c(s_i) + \frac{e}{\lambda}[r + \delta + s_i \Pi_i f(\theta)] > y_i$), the worker can still be encouraged to work with a wage equal to his productivity, $w_i = y_i$, if his outside options are eroded by a lower probability of transition out of unemployment. If Π_i were equal to one, the no-shirking wage would have to be superior to the worker’s productivity so firms would never hire them. Conversely, if Π_i were equal to zero, the worker would generate positive profits

Figure 2.1: The firm's hiring strategy



For a given θ , upon contact with a worker the firm's hiring response (w_i, Π_i) will depend on the product of the match. If $y_L \geq y_i$, the productivity of a worker is so low that the total surplus of a match (TS_i) is not enough to guarantee his effort ($TS_i < \frac{e}{\lambda}$), no match will be made. If $y_M(\theta) > y_i > y_L$, the worker is barely employable and will be discriminated with a hiring probability $\Pi_i \in (0, 1)$ that will reduce his outside options to the point his wage, $w_i = y_i$, is just enough to guarantee his effort ($TS_i = \frac{e}{\lambda}$). If $y_H(\theta) \geq y_i > y_M(\theta)$, the worker's productivity is high enough to generate a positive surplus for the firm ($J_i = TS_i - \frac{e}{\lambda} > 0$) so he will always be hired ($\Pi_i = 1$). However, it is not large enough to guarantee his effort under Nash-bargaining so he will get the efficiency wage, $w = w^E$. If $y_i > y_H(\theta)$, the match will generate positive surplus for a firm ($\Pi_i = 1$) and the productivity of a worker is high enough to guarantee his participation with Nash-bargaining wages, $w = w_i^N$.

for his employer so he would always be hired. The equilibrium answer to this conundrum is that employers adopt a mixed strategy, they hire the worker with a probability proportional to his productivity, that is $\Pi_i = [y_i - b - e + c(s_i) - (r + \delta)\frac{e}{\lambda}] / s_i f(\theta)\frac{e}{\lambda}$.⁷ The decrease in the exit rate of unemployment can be interpreted as a disciplinary device for less productive workers. Notice that firms hiring these workers do not get any surplus from being matched, so they are indifferent between hiring them and not. For this reason, I will refer to these workers as "barely employable". If the productivity of a worker is extremely low ($y_L \geq y_i$), then even if the worker is fully discriminated, his no-shirking wage would have to be larger

⁷We can verify that this is indeed a probability, that is $\Pi_i \in [0, 1]$ by observing that it is the solution to the equation $y_i = (1 - x)y_L + xy_M(\theta)$. And by assumption $y_M(\theta) \geq y_i > y_L$.

than the product of his match so he will never be hired. This differentiated treatment to workers creates wages dispersion and different unemployment rates, shares in the pool of unemployed and exit rates of unemployment.

The worker can receive either an unconstrained Nash-bargaining wage w_i^N , or an efficiency wage w_i^E . According to (2.14) and (2.18), the highest an efficiency wage can be is $w^E \equiv b + e - c^E + \frac{e}{\lambda}[r + \delta + s^E f(\theta)]$ and it corresponds to the only efficiency wage that can motivate workers without a complementary hiring discrimination. This is the wage that all perfectly employable who get an efficiency wage receive and from now on I will refer to it as “the” efficiency wage.

To determine equilibrium unemployment we use the fact that at a steady state the inflow and outflow from unemployment must be equal, that is $p_i[1 - u_i]\delta = s_i(\theta)\Pi_i(\theta)f(\theta)p_iu_i$. Solving for u_i :

$$u_i = \frac{\delta}{\delta + s_i(\theta)\Pi_i(\theta)f(\theta)}. \quad (2.19)$$

This expression states that for a given separation rate there is a unique equilibrium unemployment rate determined by equilibrium market tightness. It can be shown that $u_1 \geq u_2 \geq \dots, \geq u_n$, workers with higher productivities have lower unemployment rates. Given the assumption that in equilibrium all profit opportunities from new jobs are exploited driving rents from vacant jobs to zero, $V = 0$, and combining equations (2.7) and (2.9), the vacancy supply condition (VSC) is derived:

$$\sum_i \Pi_i(\theta)\mu_i[y_i - w_i(\theta)] = (r + \delta)\frac{\gamma}{q(\theta)}. \quad (2.20)$$

Equation (2.20) uniquely determines equilibrium market tightness θ^* . It can be verified that

$$\frac{\partial \theta^*}{\partial e} < 0, \quad \frac{\partial \theta^*}{\partial \lambda} \geq 0, \quad \frac{\partial \theta^*}{\partial b} < 0, \quad \frac{\partial \theta^*}{\partial \delta} < 0, \quad \frac{\partial \theta^*}{\partial r} < 0,$$

and

$$\frac{\partial \theta^*}{\partial y_i} \geq 0 \quad i = 1, \dots, n.$$

To complete the notation for the model before I introduce the minimum wage, a steady-state equilibrium of the model is defined as follows:

Definition 4. *A steady-state equilibrium consists of a collection of values $\{w_i, \Pi_i, s_i, u_i\}_{i=1}^n$, and θ , satisfying (2.18) (2.16) (2.17) (2.19), and (2.20).*

2.3.3 Minimum Wage

Now I introduce a minimum wage m with full compliance, that is, no wage below m will ever be paid. Hitherto, market tightness alone characterized every outcome of the market: wages, unemployment rates, etc. Although equilibrium market tightness is a function of m itself, it is convenient for the analysis to specify all outcomes as functions of a market tightness θ , and the minimum wage m . The minimum wage adds a restriction to the functions that describe the equilibrium.

Functions (2.18), (2.16), and (2.17) that describe the equilibrium can be expressed as follows:

Equilibrium wage,

$$w_i(m, \theta) = \begin{cases} y_i, & y_M(\theta) > y_i \geq \max\{m, y_L\}, \\ \max\{m, w^E(\theta)\}, & y_H(\theta) > y_i \geq \max\{m, y_M(\theta)\}, \\ \max\{m, w_i^N(\theta)\}, & y_i \geq \max\{m, y_H(\theta)\}. \end{cases} \quad (2.21)$$

Equilibrium hiring probability,

$$\Pi_i(m, \theta) = \begin{cases} 0, & \max\{m, y_L\} > y_i, \\ \frac{y_i - b - e + c(s_i) - (r + \delta)\frac{e}{\lambda}}{s_i^L f(\theta)\frac{e}{\lambda}}, & y_M(\theta) > y_i \geq \max\{m, y_L\}, \\ 1, & y_i \geq \max\{m, y_M(\theta)\}. \end{cases} \quad (2.22)$$

Equilibrium search intensity,

$$s_i(m, \theta) = \begin{cases} 0, & \max\{m, y_L\} > y_i, \\ s_i^L, & y_M(\theta) > y_i \geq \max\{m, y_L\}, \\ \max\{s^m(\theta), s^E(\theta)\}, & y_H(\theta) > y_i \geq \max\{m, y_M(\theta)\}, \\ \max\{s^m(\theta), s_i^N(\theta)\}, & y_i \geq \max\{m, y_H(\theta)\}, \end{cases} \quad (2.23)$$

where $s^m(\theta)$ solves $c'(s_i) = f(\theta)[m - e - b + c(s_i)]/[r + \delta + s_i f(\theta)]$, $s_i^N(\theta)$ solves $c'(s_i^N) = \beta[y - e - b + c(s_i^N)]/[r + \delta + \beta s_i^N f(\theta)]$, $s^E(\theta)$ solves $c'(s^E) = f(\theta)\frac{e}{\lambda}$, and s_i^L solves $c'(s_i^L) = [y_i - e - b + c(s_i^L) - (r + \delta)\frac{e}{\lambda}]/s_i^L$. When $m = 0$, (2.21), (2.22), and (2.23) reduce to (2.18), (2.10), and (2.17) respectively.

In representative worker world, analyzing the relation between efficiency wages and minimum wages would be trivial; a minimum wage below the efficiency wage has no effects on the labor market. However, introducing heterogeneity allows the possibility to have only a fraction of workers under efficiency wages being affected by the minimum wage and, through a general equilibrium effect, change the outcomes of all the participants. Figure 2.3.3 presents the wage schedule in (2.21) when $m < w^E(\theta)$.

The minimum wage is binding only for barely employable workers. Since these workers are being paid their productivity, a binding minimum can only price them out of the market by making it impossible to find a firm willing to hire them. Perfectly hirable workers will not be directly affected by the minimum wage, but will still experience ripple effects through a general equilibrium effect.

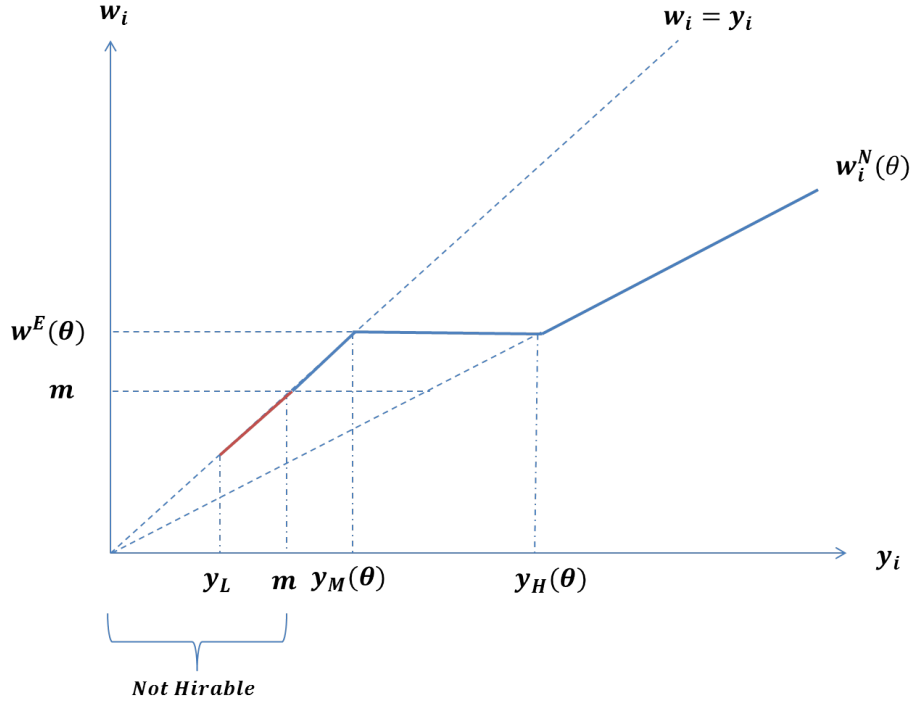


Figure 2.2: Wage schedule under a minimum wage $m < w^E(\theta)$.

Since all outcomes are defined by equilibrium market tightness, it is fundamental to determine how changes in the minimum wage affect it. The presence of moral hazard allows some stark predictions.

Proposition 7. *Let θ and θ' be the equilibrium market tightness under m and m' respectively. If $m < m' \leq w^E(\theta)$, then $\theta \leq \theta'$.*

Proof: Appendix B.

The result in proposition 7 summarizes the most important finding of this study. When an increase in the minimum wage is not large enough to make it binding for perfectly employable workers, equilibrium market tightness increases. This is a very intuitive result and a direct consequence of the presence of moral hazard. The need of worker motivation makes the workers at the low end of the productivity distribution get a wage equal to their productivity and as the minimum wage raises, it simply prices them out; the efficiency wage they receive has exhausted the possibility of a raise in the salary. Assuming full compliance with the law,

the firm has no option but to terminate the match.

According to (2.23), it is the worker's best response to stop looking for a job since it is costly activity with no expected payoffs. Workers with productivities below the minimum stop participating in the labor market, which means that the average productivity of workers participating in the labor force increases. This "weeding-out" effect in the labor force generates a more attractive environment for firms to open vacancies. The probability of being matched with a high-productivity worker increases along with the expected return of an open vacancy, which results in a higher equilibrium market tightness.

This is a sharp result of the model that contrasts with the ambiguous predictions of models without moral hazard. Without moral hazard, wages would be determined via Nash-bargaining, which leaves room for a raise in wages without representing a negative surplus for the firm. Under these conditions, a higher minimum wage increases the expected productivity of a worker, but it also increases their wages. The effect on the profitability of an open vacancy is ambiguous.

What a higher minimum wage represents for the market outcomes of different workers directly follows from Proposition 7.

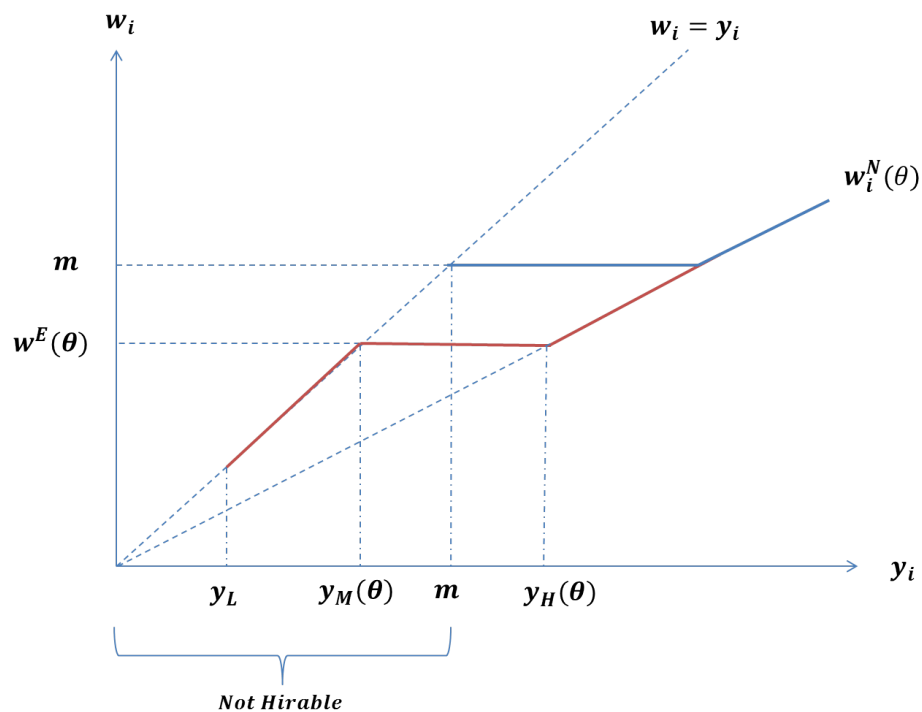
Corollary to Proposition 7 : *Let θ and θ' be the equilibrium market tightness under m and m' respectively. If $m < m' \leq w^E(\theta)$, then $u_i(m, \theta) \geq u_i(m', \theta')$, $s_i(m, \theta) \leq s_i(m', \theta')$, and $w_i(m, \theta) \leq w_i(m', \theta') \forall i \in \Omega(m')$. Also, $u_i(m', \theta') = 1$ and $s_i(m', \theta') = 0 \forall i \in \Omega^c(m')$.*

The corollary highlights the asymmetry in the effects of the minimum wage. For those workers who remain hireable after a minimum wage hike, their unemployment rates fall and the wages increase, also their participation in the labor market increases as result of these improvements. These results contrast sharply with the consequences a higher minimum wage brings for the workers that have been priced out. These workers are no longer hireable so, their unemployment rate is one. With this outlook, it is their best response to stop looking for a job, so their search intensity and participation in the labor force drop to zero.

These results apply as long as the efficiency wage remains above the minimum. When

$m' > w^E(\theta)$, two scenarios can arise. If $m' > \max\{w^E(\theta), w^E(\theta')\}$, all barely hireable workers are priced out of the market and the minimum wage could also price out workers otherwise perfectly hireable. Perfectly hireable workers remaining in the market see their wages increased to the minimum. Figure 2.3 describes the situation. In these conditions, the effects of a minimum wage are ambiguous since, on the one hand, the average productivity of the labor force increases, and on the other hand, wages increase as well. The effect that a higher minimum wage has on the expected profit of a match will heavily depend on the specific productivity distribution and the rest of the parameter values. The model's calibration for the Low-education labor market in Section 2.5 shows that this situation does not arise for realistic values of increments in the minimum. Even when the minimum increases by 100%, the efficiency wage increases and remains above the imposed wage floor. The other possibility is that $w^E(\theta) < m' < w^E(\theta)$, this particular situation generates suboptimal use of minimum wages.

Figure 2.3: Wage schedule under a minimum wage $m' > \max\{w^E(\theta), w^E(\theta')\}$.



2.3.3.1 Suboptimal use of Minimum Wages

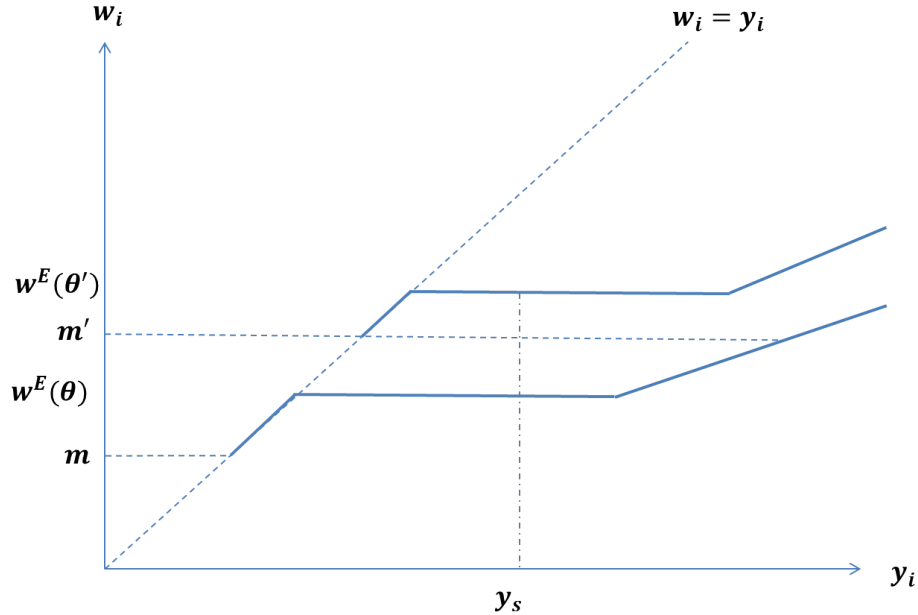
Falk, Fehr and Zenhder (2006) raise the following question: Why do profit-maximizing employers not take advantage of the possibility of reducing wages below the legal minimum, and why do they pay more than the minimum for those workers who earned less than the new minimum wage before it was introduced? This question follows from the evidence reporting low utilization of minimum wages in situations where in principle, employers could pay the minimum or less.⁸ Using data from a laboratory experiment, they argue that the introduction of a minimum wage increases workers' reservation wages due to a constant perception of what a fair wage is. Workers perceive a wage as a fair if it, to a certain degree, is above the minimum regardless of what the minimum wage is. As a result, firms end up paying wages above a new minimum even when workers were earning less than the new minimum before its introduction.

The model offers an explanation of this phenomenon that is also related to changes in the reservation wage. The efficiency wage is the minimum wage required to induce worker participation in the production process, in this sense it constitutes an effective reservation wage. Which workers get an efficiency wage and what this efficiency wage is, depends on market tightness. So a new minimum, if it changes market tightness enough, could drastically alter the wage schedule. Figure 2.4 presents a situation where the minimum wage is binding for perfectly employable workers so in principle, their new wage should be equal to the minimum, however this is not the case.

Let θ be the equilibrium market tightness under the old minimum m . Before the increase to a minimum m' , the salary for worker s was $w^E(\theta) < m'$. When the minimum wage changed to m' , worker s was still hireable and should have receive a salary equal to m' . However, a higher minimum created a tighter market with improved outside options for workers. In equilibrium, the efficiency wage has to adjust to compensate workers for this improvement. The wage recived by the worker is $w^E(\theta') > m'$. Firms are not paying the worker the

⁸Freeman, Wayne, and Ichniowski 1981; Katz and Krueger 1992; Manning and Dickens 2002

Figure 2.4: Suboptimal use of minimum wages



minimum although in principle, they could. Studies show that this practice is common. For example, Katz and Krueger (1992) report that some fast-food restaurant managers were not using the subminimum wage option because they believed that it would not attract qualified teenage workers at that wage. Notice that as long as $w^E(\theta) > m$, even if for some exception the employer could pay wages below the minimum, he will choose not to do so due to moral hazard.

Unfortunately, since the new minimum m' is above the level $w^E(\theta)$, characterizing when these situations can arise is difficult since it depends on the parameter values and the work-productivity distribution.

2.4 Evidence on Asymmetric Effects of Minimum Wages on Labor Market Outcomes

In this section, I use individual data on labor market outcomes to investigate the existence of asymmetries in the way minimum wages affect the employment, labor force par-

ticipation, search intensity, wages, and labor hours of workers with different productivities. According to the results of the model described in Section 2.3, a minimum wage lowers the employment and labor force participation of low-productivity workers while it increases the employment, encourages labor force participation, and augments wages of more productive workers. To identify heterogeneity in productivity I consider two-way disaggregation by educational attainment and age. The data suggest that older and more educated workers are more productive.

The empirical results provide support for the model's predictions and can be summarized as follows: 1) the minimum wage affects only low-education labor markets; and 2) the low-education workforce is asymmetrically affected by minimum wages depending on individual productivities. In fact, increments in the minimum wage have diametrically opposed effects; they reduce the employment and labor force participation of the younger and less educated workers (teenagers with less than high school education) while increasing the employment and labor force participation of older more educated workers (25-59 year olds with high school educational attainment). Despite the dichotomy, the disemployment and discouraging effects are much stronger than the employment and encouraging effects.

2.4.1 Data

I compile a repeated cross-sectional sample at individual level from the CEPR uniform data extracts, which are based on the Outgoing Rotation Group (ORG) of the CPS, for the years 1994-2013.⁹ The CEPR ORG extracts contain detailed information on individuals' demographic characteristics such as education, age, employment status, and hourly earnings. Using the CPS basic monthly files, I augment the data to include individual information about unemployed workers' job-searching efforts. As a proxy for job-search intensity, I use the number of different job-finding methods used by unemployed workers in the four weeks

⁹<http://ceprdata.org/cps-uniform-data-extracts/>

preceding the CPS interview.¹⁰ Each observation is merged with a monthly minimum wage variable; the federal or the state minimum, whichever is higher.¹¹ Additionally, observations are merged with data that capture overall labor market conditions and labor supply variation; monthly state-wide unemployment rates and population shares for the relevant demographic groups.¹²

Table B1 provides descriptive statistics for the different demographic groups analyzed: teens (16 to 19 year olds), young workers (16 to 24 year olds), mature workers (25 to 59 year olds), and elderly workers (60 to 64 year olds). Observations are also classified by educational attainment: Less than high school (LTHS), high school, some college, college, and advanced education.¹³ Not surprisingly, individuals with higher educational attainment are older on average. Average worked weekly hours and average hourly wage increase with educational attainment and age. Older and more educated individuals use on average more different methods to find a job. Unemployment rates drop with age and education; teenagers have the highest unemployment rate, 16.6%, while individuals with advanced education have the lowest, 2.2%. Employment and labor force participation are larger in older and more educated groups giving the contrasting employment and participation rates of 38.5% and 46.1% for teenagers, against 86.4% and 88.3% for the advanced education group.

Young workers and teenagers have been the most widely analyzed demographics in the minimum wage literature, so I report their share on each educational group. Teenagers are mostly concentrated in the LTHS group constituting almost 40% of that population. Young

¹⁰This variable is constructed using the variables PELKM1, PULKM2, PULKM3, PULKM4, PULKM5, and PULKM6 from the CPS basic monthly data. Each one of these variables allows the interviewed to choose one of the following responses: contacted employer directly/interview, contacted public employment agency, contacted private employment agency, contacted friends or relatives, contacted school/university employment center, sent out resumes/filled out application, checked union/professional registers, placed or answered ads, other active, looked at ads, attended job-training programs/courses, nothing, and other passive.

¹¹I constructed the minimum wage variable using data from the United States Department of Labor and each state's department of labor, when available, to accurately record effective dates.

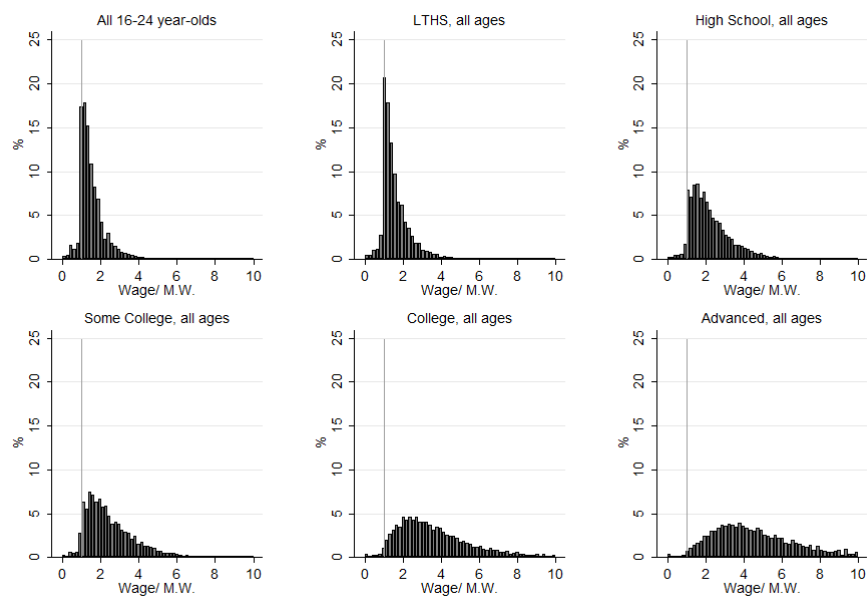
¹²Population shares are exogenous (aside from migration). Although the unemployment rate is potentially endogenous, by using state-wide unemployment rates rather than unemployment rates of the specific demographic groups, I hope to capture an aggregate demand indicator.

¹³Classifications follow Jaeger (1997) who defines high school attainment as completing the 12th grade regardless of high school diploma receipt. Advanced schooling is defined as having a master's degree, a professional school degree, or a doctorate degree.

workers constitute 44% of the LTHS group, 16% of High School group, and 19% of those with some college.

To begin the analysis of the effects of the minimum wage, I compute the share of the population in each education group that could be considered as directly affected by it; those earning a wage within a 10% range of the minimum wage. Figure 2.5 displays the wage distribution in terms of the effective minimum wage for each of the categories.

Figure 2.5: Wage Distributions by Educational Attainment, 1994-2013



Those directly affected by the minimum wage are concentrated in the youngest population, they constitute 32 % of teenagers and 19% of young workers. In terms of education groups; 20% of workers with LHTS education are impacted directly by the minimum wage and this proportion decreases with educational attainment; the share reduces to 6% for workers with high school education, 5% for workers with some college, and 1% for workers with college or advanced education. The wage distributions of younger and low educated workers concentrate closer to the minimum wage and as education increases the distributions spread out. Not surprisingly, and as next section will show, when disaggregating by educational attainment only LHTS and high school groups are affected by changes in the minimum wage. For this reason, I divide the education groups into high-education (some college, college and

advanced), and low-education (LTHS and high school). The analysis concentrates on the latter group.

According to the BLS, 26% of total jobs in 2012 had no educational requirements. On the same year, only 8% of the labor force had LTHS education. This suggests a unified labor market of significant size for workers with different educational attainment. The fact that only low-education groups are affected by the minimum wage suggests that they constitute a labor market of their own.

It is the thesis of this study that heterogeneity plays an important role in the way the minimum wage affects individuals within the same labor market. For this reason, I further disaggregate and analyze low-education groups by age, another variable commonly used as a proxy for skill.

Figure 2.6: Mean Low-Education Labor Market Outcomes, 1994-2013

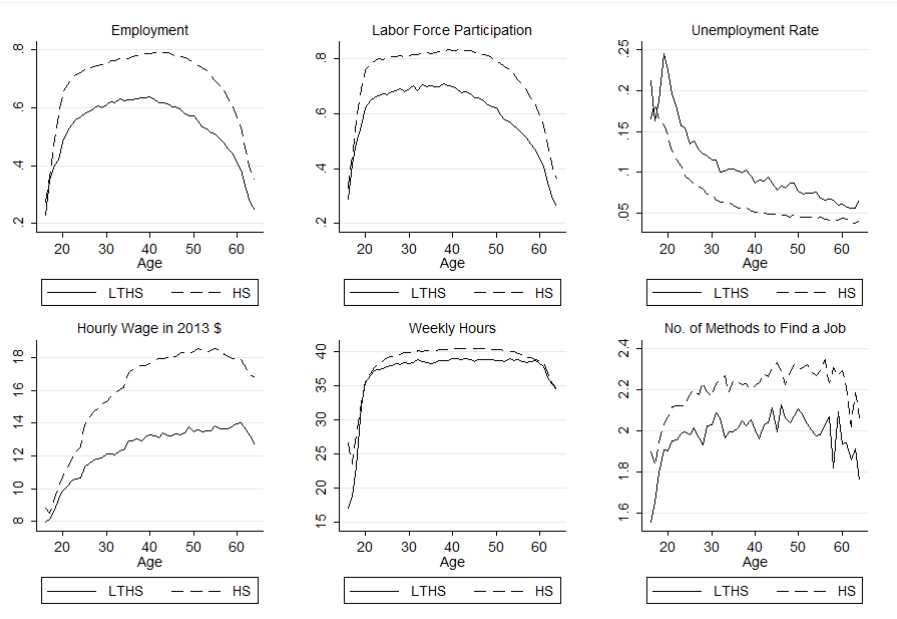


Figure 2.6 shows the average market outcomes of low-education groups by age. With the exception of unemployment rate, there is non-monotonic relation between these variables and age. The gap in outcomes between groups is relatively small for younger workers but it widens as they reach the prime of life only to start closing again as they enter the late years. Employment, wages, and weekly hours reach a maximum around 40 years of age in

both groups. Labor force participation and search intensity are a measure of labor market activity and they increase with age and are in general greater for those with high school education.

These results suggest that within the low-education labor force, mature workers with high school education are the most productive while teenagers with LTHS education are at the bottom of the productivity distribution. For the remainder of the analysis I will underscore the importance of the two-dimensional proxy for productivity, education and age, to account for worker heterogeneity. Whether completing the 12th grade actually increases human capital or merely signals aptitudes, those with high school education are on average more productive than those less educated. The differences across ages could mirror differences in experience and the natural cycle of ability decay.

2.4.2 Estimation Strategy

My objective is to estimate the effect of minimum wage increments on employment, labor force participation, search intensity, hours, and wages. I use four different specifications popular in the literature as robustness checks. All of them are estimated at individual level and with standard errors clustered at the state level to account for dependence among observations within the same state. The baseline specification is the panel difference-in-difference canonical model:

$$y_{ist} = \alpha + \beta MW_{st} + \delta Z_{st} + \lambda X_{ist} + \gamma_s + \tau_t + \varepsilon_{ist}, \quad (1)$$

where i , s , and t denote, respectively, individual, state, and time indexes. The dependent variables y_{ist} , are: a dichotomous employment variable, a dichotomous labor force participation variable, search intensity as previously defined, the natural log of weekly hours, and the natural log of hourly earnings. MW is the log of the effective minimum wage; Z is a vector of state characteristics that includes the aggregate unemployment rate, the population share

of the demographic of interest, and aggregate average wage. X is a vector of individual characteristics: race, age, education, marital status and gender. γ_s denotes the state-fixed effect and τ_t represents time dummies in months.

According to Dube, Lester, and Reich (2010), failing to control for spatial heterogeneity in trends generates biases toward negative elasticities of the dependent variable. To address this issue, I follow Allegretto, Dube, and Reich (2011) and I add two sets of controls. First, I include census division-specific time effects, which removes the variation across census divisions by controlling for spatial heterogeneity in regional economic shocks:

$$y_{ist} = \alpha + \beta MW_{st} + \delta Z_{st} + \lambda X_{ist} + \gamma_s + \tau_{dt} + \varepsilon_{ist}, \quad (2)$$

where τ_{dt} is the census division-specific time effect.¹⁴ The third specification adds state-specific linear trends that capture long-run growth differences across states:

$$y_{ist} = \alpha + \beta MW_{st} + \delta Z_{st} + \lambda X_{ist} + \gamma_s + \tau_{dt} + \pi_s \cdot t + \varepsilon_{ist}, \quad (3)$$

where $\pi_s \cdot t$ represents the time trend for state s .¹⁵ Earlier findings indicate that the minimum wage effects can take some time to fully become apparent.¹⁶ To account for possible lagged effects I estimate the distributed lag model that includes the contemporary, the six-month lag, and the one-year lag of the log of the minimum wage:

¹⁴Census divisions are: New England: ME, NH, VT, MA, RI, and CT. Middle Atlantic: NY, NJ, and PA. East North Central: OH, IN, IL, MI, and WI. West North Central: MN, IA, MO, ND, SD, NE, and KS. South Atlantic: DE, MD, DC, VA, WV, NC, SC, GA, and FL. East South Central: KY, TN, AL, and MS. West South Central: AR, LA, OK, and TX. Mountain: MT, ID, WY, CO, NM, AZ, UT, and NV. Pacific: WA, OR, CA, AK, and HI.

¹⁵According to Meer and West (2015), if changes in minimum wages affect a variable over time, through changes in growth rather than through an immediate shift, specifications including state-specific time trends will fail to capture these effects. They attenuate the estimates of the impact of the minimum wage on the growth of a variable so even real causal effects on the level of the variable can be attenuated to be statistically indistinguishable from zero. It is for this reason that a specification including only linear state-specific time trends is omitted and specification 4 including division-specific fixed effects and linear state-specific time trends should be taken with considerable skepticism.

¹⁶Baker, Dwayne, and Suchita (1999); Neumark and Wascher (1992); Neumark, Schweitzer, and Wascher (2004).

$$y_{ist} = \alpha + \beta_0 MW_{st} + \beta_1 MW_{st-6} + \beta_2 MW_{st-12} + \delta Z_{st} + \lambda X_{ist} + \gamma_s + \tau_t + \varepsilon_{ist}. \quad (4)$$

2.4.3 Results

The presentation of the results goes as follows. First, I analyze the impact of the minimum wage on high-education groups and show that the minimum wage has no statistically significant impact on any of their labor market outcomes. Then, I discuss the results for low-education workers at an aggregate level and its disaggregation by age to document differences within low-education groups. For comparison to previous work and validation of the estimation strategy, I also present and discuss the results for all teenagers.

The relevant resulting estimates of the four specifications are presented in tables B2 through B11. All tables report the coefficient of the log of the minimum wage on each of the five dependent variables and the associated elasticity. For specification 4, I report summed contemporaneous and lagged effects. For the wage and hours estimates, the dependent variable is already in logs, so the estimated coefficients are directly interpretable as elasticities. It is not my intention to enter the debate of “the right model” to identify the impact of the minimum wage on labor outcomes, but to provide evidence supporting the notion that minimum wages have asymmetric effects on the labor force. For this reason, there is no preferred specification, and I consider an effect to be significant only if there is a consistent pattern across the four specifications.

2.4.3.1 Minimum Wage Effects on High-Education Workers

Table B2 reports the estimates of the impact of changes in the minimum wage on employment. The results across the three high-educations groups vary in sign, but overall are not significant with the exception of specification 1 showing a significant employment coefficient of -0.018 with a corresponding employment elasticity of -0.022 for workers with college

education.¹⁷ Table B3 shows the estimates on labor force participation; they are statistically indistinguishable from zero and varying in sign from specification to specification. These results do not support any employment or labor force participation effects associated with minimum wage increments.

The estimated impact of minimum wages on my proxy variable for search intensity is presented in Table B4. For workers with some college, only specifications 2 and 3 show a statistically significant negative elasticity of -0.142 and -0.175. For workers with a college degree, the situation is the opposite; only specifications 1 and 4 show significant positive elasticities of 0.3 and 0.34 respectively. The coefficients on advanced education workers are statistically indistinguishable from zero. If this variable reflects indeed job-search efforts, negative coefficients would suggest that the minimum wage decreases the surplus of a match for workers with some college and it increases the surplus of a match for workers with college education despite the fact that, according to results on employment and participation, the minimum wage is not binding in this market. This situation could be due to the fact that this proxy is too imprecise and responds to some general equilibrium effect of the whole economy.

Table B6 reports the results for the log of weekly hours, which show no discernible effects on the worked hours of high-education groups. Only specifications 3 and 4 give a relatively small elasticity of -0.023 and -0.019, respectively, for workers with some college. Finally, the effects on the log of wages are displayed in Table 3.6. The estimates are consistently non-significant through high-education groups and their signs vary from specification to specification.

In summary, the results do not provide evidence of significant effects of changes in the minimum wage on labor market outcomes of high-education workers.

¹⁷The elasticity is obtained by dividing the coefficient by the fraction of employed individuals in the demographic of interest.

2.4.3.2 Minimum Wage Effects on Low-Education Workers

Now I turn to the analysis of low-education groups and teenagers. It is one of the goals of this work to stress that one way disaggregation, either by age or education, could mask worker heterogeneity, a fundamental aspect to understand the workings of the labor market. Two-way disaggregation captures heterogeneity better and enables more precise identification of the effects of minimum wages. For the analysis of low-education groups, I additionally estimate the effects on age subgroups; teenagers, young workers, mature workers, and elderly workers.

First, I discuss the estimated employment effects reported in Table B2. Consistent with previous findings, the canonical model of specification 1 produces a significant negative estimate for teenage employment elasticity of -0.084. Controlling for division-specific economic shocks and heterogeneity in the underlying employment trends, specifications 2 and 3, render estimates that, unlike previous work (Allegretto, Dube, and Reich (2011)), are significant and stronger than the estimate of the canonical model; -0.14 and -0.12 respectively. Specification 4, which includes lag terms to capture changes in growth rate, gives a negative but insignificant effect of -0.06. When disaggregated by educational attainment, LTHS teenagers show strong and significant elasticities (-0.2, -0.19, -0.19 and -0.19) while teenagers with high school education do not show effects in employment.

The estimates of the LTHS groups as a whole are insignificant across specifications although it has a teenage composition of 40%. Table 3.6 shows that behind the insignificant results is the fact that the magnitude of the effect is much related to age, only young workers display statistically significant disemployment effects.

The study's most relevant finding is the effect that hikes in the minimum wage have on 25-59 year-old workers with high school educational attainment. Table B2 shows that specifications 1, 2, and 4 produce statistically significant point estimates with elasticities of 0.02, 0.03 and 0.03 respectively. It is important to stress that the results do not contradict the bulk of studies finding negative employment effects since most of those studies focus

on teenage employment. Teenagers constitute only 6% of the workforce with high school education. Further disaggregation by age makes the sign, magnitude, and significance of the estimates vary widely across age subgroups. Table 3.6 shows that the positive employment effect is restricted to 25-59 year-olds. Elasticities range from 0.025 to 0.042 and are significant in all specifications. These results are consistent with Neumark (2007) who also reports insignificant employment effects for workers under 25 with high school education.

Now I turn to labor force participation effects reported in Table B4. Consistent with previous findings, the results show significant participation-discouraging effects among teenagers.¹⁸ Specifications 1, 2, and 3 produce significant elasticities ranging from -0.10 to -0.06. Specification 4 also predicts a negative elasticity but it is non-significant. Disaggregating by educational attainment, Table B8 shows that not all teenagers are affected equally. The minimum wage has a strong discouraging effect only on teenagers with LTHS education; the estimated elasticity is consistently significant across all specifications with values around -0.15. The results for teenagers with high school education are not statistically significant with the exception of specification 4 that gives a significant elasticity value of 0.1. The results in table B4 for the LTHS group as a whole are not significant since, according to Table table8, the minimum wage influences only the participation decisions teenagers with LTHS education. The participation decision of older workers with LTHS education is not affected.

Another key finding of this study is the participation-encouraging effects of minimum wages on mature workers with high school education. Tables B3 and B8 show that, although the elasticity estimates are statistically significant for the high school demographic as a whole, the effects of minimum wages are concentrated on workers aged between 25 and 59. All the specifications give very significant elasticity estimates ranging from 0.029 to 0.043.

Tables B4 and 3.6 contain the results for the proxy variable for search intensity. Only

¹⁸Kaitz (1970), Mincer (1976), Ragan (1977), and Wessels (1980). They estimated the effects of the minimum wage on labor force participation and found that minimum wage decreased (or did not affect) the labor force participation rate of low-wage workers. More recently Wessels (2001), and Wessels (2005) investigate the effect on teenage participation and conclude that minimum wages decreases teenage labor force participation and their proportion of new entrants into the labor force.

workers with LTHS education show a significant effect. Specifications 1 and 4 produce a significant elasticity of -0.15 and -0.19 respectively. Surprisingly, age desegregation shows no significant effects on teenagers. Only specification 4 for young workers (-0.15), specifications 1 and 4 for mature workers (-0.19 and -0.22), and specification 2 for the elderly (-1.5) show a negative and significant elasticity.

The effect on hours by education level and its disaggregation by age are shown in tables 5 and 10. Minimum wages do display a significant impact in worked hours for workers with high school education as a whole. However, for teenagers and workers with LTHS education, the estimates are negative and significant; the four specifications indicate very significant elasticities; -0.12, -0.22, -0.21, and -0.12 for teenagers; and -0.07, -0.11, -0.1, and -0.06 for LTHS workers. Further disaggregation shows that reduction in hours concentrates on teenagers with LTHS education with negative elasticities ranging from -0.24 to -0.16 and significant in all specifications. Teenagers with high school education report negative and smaller effects, significant only for specifications 2, 3, and 4. Within LTHS workers, the effects vary non-monotonically with age and specification. For teenagers, all specifications are significant, for young workers only specifications 2 and 3 are significant with coefficients of -0.15 and -0.16 respectively. For 25-59 year-olds, only specifications 1, 2 and 4 report significant results of -0.05, -0.07, and -0.05. Elderly workers report significant results in specifications 1 and 4 with elasticities of -0.16 and -0.19. Taken together, the estimates suggest that the size of the effect is inversely related to age and education.

Finally, I discuss the wage effect of the minimum wage. Consistent with previous findings (Neumark (2007), Allegretto, Dube, and Reich (2011)) the results in Tables B6 and B11 give a positive and statistically significant wage effect for teenagers regardless of the specifications. The estimated elasticities range from 0.14 to 0.16. For LTHS and high school groups, only specifications 2 and 3 show statistically significant wage effects. Age disaggregation shows that the wage effects are concentrated in the youngest populations of both education groups. All the specifications report a significant positive effect that is strongest in teenagers with

LTHS education, ranging from 0.17 to 0.22, and is weakest in the group of 16 to 24 year olds, ranging from 0.08 to 0.14. No significant effects on wages can be found in older groups regardless of their education.

2.4.3.3 Theoretical Implications of the Results

Now I analyze the empirical results in the light of the model's framework. The evidence indicates that changes in the minimum wage affect labor market outcomes of low-education groups only. According to the model, this situation is explained by the fact that low-education groups and high-education groups belong to different labor markets and the minimum wage is binding only in the low-education labor market. If the minimum wage binds in the high-education groups, the share of workers affected by the changes must be negligible. The ripple effects observed in the low-education labor market indicate that the proportion of workers in that market who are affected directly by hikes in the minimum wage must be large enough to have considerable changes in equilibrium market tightness. Figure 2.5 shows that this is the case, the proportion of workers with LTHS and high school education with wages barely above the minimum is much larger than those in high-education groups.

Age disaggregation shows that the effects are concentrated mostly in two demographics. Teenagers with LTHS education are adversely affected with disemployment and lower participation, and mature workers with high school education who are encouraged with positive employment effects. According to the model, although these two groups participate in the same market, the contrasting effects indicate significant productivity differences. LTHS teenagers must be concentrated at the bottom of the productivity distribution, while mature workers with high school education concentrate at the top. The lack of significance in the impact on other demographics in the same labor market is explained by the fact that those groups are scattered around the center of the productivity distribution. Consequently, there are some individuals being adversely affected and others are benefited, rendering an

average change difficult to identify by the regressions. This finding points out that there are two fundamental components to average productivity: educational attainment and age. Figure 2.6 and the results in table 12 reinforce the notion of the double dimensionality in the determination of productivity.

The model predicts that more productive individuals will receive higher wages, will have lower unemployment rates and will participate more actively in job search. We can observe that average labor market outcomes indicate that the most productive individuals are on average mature workers with high school education, followed by other mature workers, the elderly, and young workers in no particular order.¹⁹ An innovative feature of the empirical approach is the use of the number of different methods to find a job as a proxy for workers' search efforts. The results show no discernable significant effects of the minimum wage on this variable. This contradicts the theoretical prediction that labor force participation and search intensity should move in the same direction and are, in fact, the same decision. The inconsistency casts doubt on the validity of this proxy variable and the results should serve as reference for future studies attempting to find a valid proxy for search intensity in the context of search models. The regressions indicate that labor force participation is a better proxy for search intensity.

Although many previous studies distinguish between older and younger teens to look for labor substitution effects, the results show that this approach is limited since the substitution does not occur within teenagers but is directed towards older and more educated workers. A word of caution about substitution; the employment effects predicted by the model have a broader interpretation than worker-for-worker labor substitution, they could also be interpreted as destruction of lower productivity matches and creation of new more productive matches. For this reason, the model could also be consistent with empirical work not finding labor substitution effects in a specific industry. For example, Dube, Lester and

¹⁹In some market outcomes elderly workers outperform mature workers, however the labor market conditions of elderly workers are understandably determined by conditions other than productivity, making it difficult to fully be consistent with the model

Reich (2011) do not find evidence of labor substitution within the restaurant workforce. Theoretically this result could be explained by the fact that the minimum wage does not really change the profitability of an employee-employer match, either because, in this industry, the minimum wage is too low to be binding or does not bind due to special considerations for tipped workers.

The model does not distinguish between hours worked and employment levels, a reduction in the unemployment rate could be interpreted either as more hours worked by individuals or as more workers being employed, so theoretically hours and employment levels in the data should be closely linked. The numbers of hours worked by teenagers does move in the same direction as employment, it decreases with an increase of the minimum wage. However, the hours worked by mature workers are not affected by changes in the minimum wage but their employment levels increase slightly. This could be attributed to legal restrictions on the maximum number of hours and does not contradict the model's predictions.

The model predicts spillover effects on wages for all the workers remaining in the workforce and their size depends mostly on the incentivizing scheme the workers is on; workers close to the minimum that do not need to be incentivized through the threat of longer spells of unemployment have the greatest effect. Workers at the top of the productivity distribution whose wages are not subject to the NSC constraint have weaker gains in wages. The fact that spillover effects can be detected only in young workers in both education groups suggests that a large proportion of this demographic could be in the class of workers that are paid an efficiency wage. The effects found are contingent upon employment so they do not reflect the employment losses that come along with the wage increases for the group as a whole. Theoretically all workers that remain employed must see their wages being increased, however for those workers at the top of the distribution the effect could be so small that is not identifiable in the regressions.

2.5 Quantitative Exercises

In this section, I assess the quantitative properties of the model to evaluate the effects of an increase in the minimum wage on employment, labor force participation, wages and social welfare.

2.5.1 Calibration

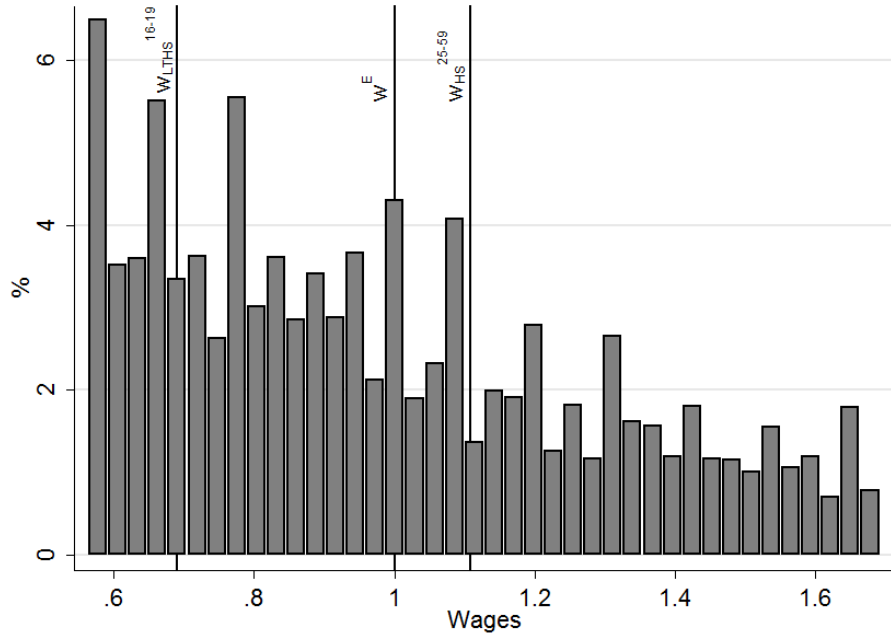
Following the results in Section 2.4, the models' calibration simulates the low-education labor market, where minimum wage changes are consequential for the market outcomes. A productivity distribution must be specified based on the wages obtained from the CPS micro data. Observed wages are expressed in terms of the minimum wage by dividing them by the effective minimum. I restrict my attention to LTHS and high school observations that are less than three times the minimum wage but no less than the minimum since the model assumes compliance with the law. The resulting average wage is 1.78 times the minimum. After normalizing the wage distribution so the average wage is equal to one, the minimum wage is equal to $m_0 = 0.57$ and it corresponds with the lowest wage in the distribution, that is $w_1 = 0.58$. Wage observations are grouped into 40 intervals to create a wage distribution with 40 different values giving the lowest wage, $w_1 = 0.58$, and the highest wage $w_{40} = 1.7$. Figure 2.7 shows the resulting wage distribution.

Section 2.4 shows that when a minimum wage increases, it is teenagers with LTHS education who are the most perversely affected workers while mature workers with high-school education benefit from the increase. This observation motivates the key identifying assumption of the calibration,

$$w_{LTHS}^{16-19} < w^E \leq w_{HS}^{25-59}, \quad (2.24)$$

where w_{LTHS}^{16-19} is the average wage of teenagers with LTHS education and w_{HS}^{25-59} is the average wage of 25 to 59 year olds with high school education. Using this assumption I pin

Figure 2.7: The firm's hiring strategy



text here

down the model's parameters and ultimately the underlying productivity distribution. The unemployment rates for these subpopulations can be computed from the data which generates four values ($w_{LTHS}^{16-19} = .7$, $w_{HS}^{25-59} = 1.1$, $u_{LTHS}^{16-19} = .19$, $u_{HS}^{25-59} = .055$) that along with a choice of a value w^E , and equations (2.21), (2.22), and (2.23) and (2.19), form the following system of equations:

$$c'(s^{HS}) = f^* \left[\frac{w_{HS}^{25-59} - e - b + c(s^{HS})}{r + \delta + s^{HS} f^*} \right],$$

$$u_{HS}^{25-59} = \frac{\delta}{\delta + s^{HS} f^*},$$

$$c'(s^E) = f^* \frac{e}{\lambda},$$

$$w^E = b + e - c(s^E) + \frac{e}{\lambda} (r + \delta + s^E f^*), \quad (2.25)$$

$$u_{LTHS}^{16-19} = \frac{\delta(e/\lambda)}{w_{LTHS}^{16-19} - b - e + c(s^L) - r(e/\lambda)},$$

$$c'(s^L) = \frac{w_{LTHS}^{16-19} - e - b + c(s^L) - (r + \delta)\frac{e}{\lambda}}{s^L},$$

subject to

$$w^E - w_1 \leq c(s_1) - c(s^E) + \frac{e}{\lambda} s^E f^*. \quad (2.26)$$

The restriction (2.26) ensures the labor force participation of all workers under the current minimum. For given values of b , δ , and r , the system defines e , λ , f^* , the theoretical search intensity of LTHS teenagers s^L , and the theoretical search intensity of mature high school workers s^H .

The time period is set to a quarter. I set $r = .012$ corresponding to an annual discount factor of 0.953. Also, b is set to 0.2, which corresponds to an income replacement ratio of 40% of the lowest productivity worker. For the choice of δ , I use the fact that the average unemployment duration for low-education workers is 1.8 quarters and the average unemployment rate is 8.1%. Using (2.19), I get a value of $\delta = 0.05$.

Some assumptions about the functional form of the matching function and the cost of search are necessary. For the choice of matching function I assume a Cobb-Douglas $h(\sum_i p_i s_i u_i, v) = \tau (\sum_i p_i s_i u_i)^\eta v^{1-\eta}$, therefore

$$f(\theta) = \tau \theta^{1-\eta}, \quad \text{and} \quad q(\theta) = \tau \theta^{-\eta}.$$

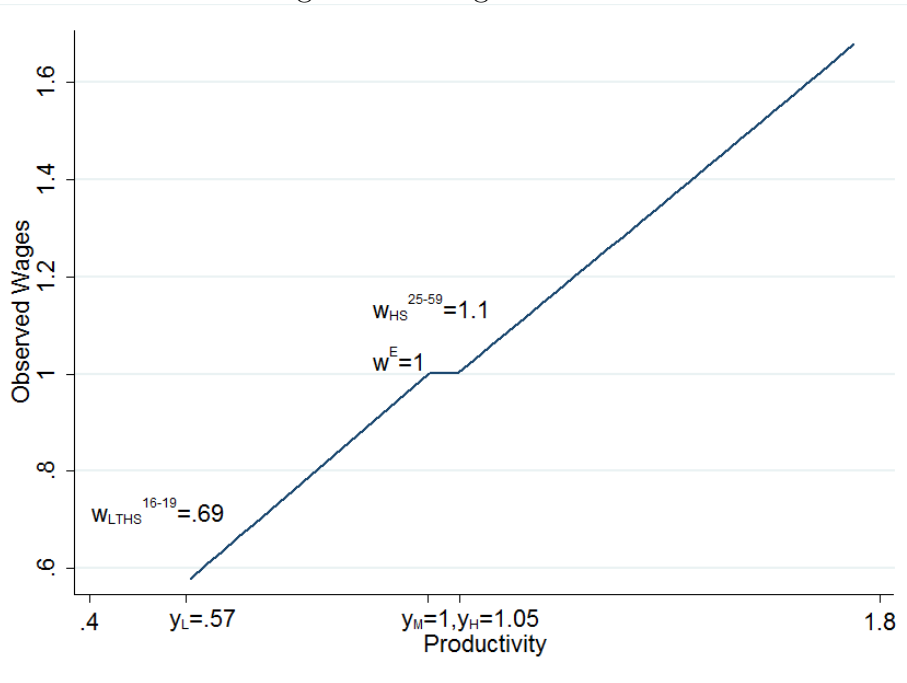
I set the elasticity of the probability of filling a vacancy to $\eta = 0.6$, which according to Petrongolo and Pissarides (2001) is at the middle of the range of the parameter values, [0.5, 0.7], estimated across the literature. Following Christensen et al. (2005), the cost of search function is

$$c(s) = c_0 \frac{s^{\alpha+1}}{\alpha + 1},$$

with $\alpha = 1.18$ and $c_0 = 1$ as the normalization the calibration allows. The bargaining power of the workers is set to satisfy the Hosios (1990) rule, that is $\beta = 0.6$.

The calibration requires a value for w^E , and according to the wage function (2.21), there should be a relatively large concentration of workers with this wage. Inspecting the wage distribution in Figure 7, $w^E = 0.78$ seems a good candidate. However, with this choice the resulting values from the system of equations do not satisfy (2.26). The next candidate is $w^E = 1$, which returns values of $e = 0.295$, $\lambda = 0.24$, $f^* = 1.27$, and satisfies (2.26). With these values and using (2.21), the implied wage schedule is derived and presented in Figure 2.8.

Figure 2.8: Wage Schedule



The wage function is not injective, to recover the domain some assumption must be made. The calibration offers a value for $y_M = 1$ and a value for $y_H = 1.05$, I will assume that the weight on the efficiency wage is distributed uniformly between these two values. This creates

2 extra bins, so number of different productivities is now 42, with the lowest productivity being $y_1 = 0.58$, and the highest, $y_{42} = 1.75$.

Now, it is necessary to derive the productivity distribution p_1, \dots, p_{42} . According to the model, workers with higher productivities have lower unemployment rates. This means that the empirical wage distribution over represents high-productivity workers and under represents low-productivity ones since wages are observed conditional on employment. Let d_i be the share of observed wages w_i . Wages are observed contingent upon employment, so according to the model

$$d_i = \frac{(1 - u_i)p_i}{\sum_j (1 - u_j)p_j}.$$

Values u_i and s_i cannot be observed directly from the data for every worker type- i . However, under the assumption that LTHS teenagers are at the barely employable side of the wage distribution and mature workers are perfectly employable, an approximation for the share of type- i workers is

$$p_i \approx d_i \frac{Emp}{Emp_{LTHS}^{16-19}} \text{ if } w_i < w^E,$$

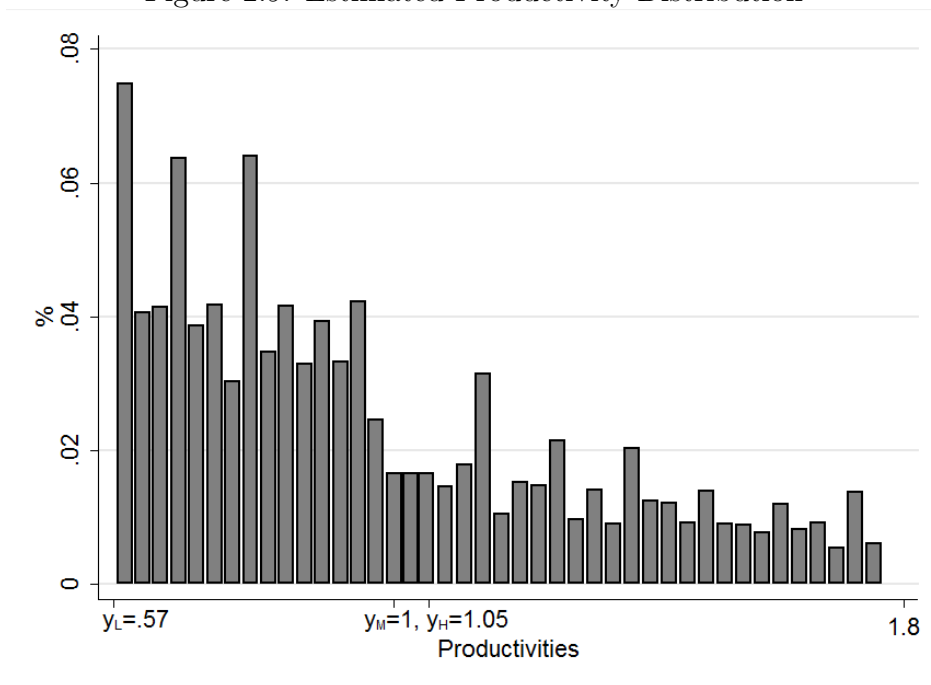
and,

$$p_i \approx d_i \frac{Emp}{Emp_{HS}^{25-59}} \text{ if } w_i \geq w^E,$$

where Emp is the employment rate of all the LTHS and high school population, Emp_{LTHS}^{16-19} is the employment rate of teenagers with LTHS education, and Emp_{HS}^{25-59} is the employment rate of mature workers with high school education. The resulting productivity distribution is presented in Figure 2.9.

Using the estimated productivity distribution, a new system of equations can be created to pin down the remaining parameter values γ , and τ :

Figure 2.9: Estimated Productivity Distribution



$$\tau\theta^{*1-\eta} = f^* = 1.27,$$

$$\sum_i \Pi_i \mu_i(\tau, \theta^*) (y_i - w_i(\tau, \theta^*)) = (r + \delta) \frac{\gamma}{\tau\theta^{*-\eta}},$$

$$\sum_j p_j s_j(\tau, \theta^*) u_j(\tau, \theta^*) = 0.08.$$

The first equation comes from the definition of $f(\theta)$, the second equation is the VSC, and the third equation is the aggregate unemployment rate of 8%. The resulting parameter values are $\gamma = 0.95$, $\tau = 1.4$, and $\theta^* = 0.8$. Table 13 summarizes the parameter values.

2.5.2 Changes in the Minimum Wage.

In this section, I use the calibrated model to investigate the effects of increments in the minimum wage on employment, labor force participation, and total welfare. The model was

Table 13: Parameter Values in Simulations of the Model.

	Parameter	Value	Source
r	Discount rate	0.012	0.953 annual discount factor
b	Unemployment benefits.	0.2	40% income replacement ratio for the lowest productivity
η	Unemployment-elasticity of matching	0.6	Petrongolo & Pissarides (2001)
β	Worker bargaining power	0.6	Hosios (1990)
δ	Separation rate.	0.05	Average unemployment duration: 1.8 quarters, average unemployment rate 8.1%
α	Search cost parameter	1.18	Christensen et. al (2005)
e	Effort intensity	0.295	
λ	Inspection rate	0.24	
τ	Efficacy of matching	1.4	
c_0	Search cost parameter	1	
γ	Recruiting cost parameter	0.95	

Notes: Most of the displayed values have been rounded since they are derived as the solution to a set of equations. Simulations use the real values.

calibrated so that the minimum wage is not binding for any type of workers; the experiment will be to see how the steady-state variables change with a higher minimum all else remaining equal. The fact the calibration targets exclusively the low-education workers should be borne in mind when interpreting the simulation's results and particularly when the welfare exercise results are reported.

The first important result of the simulations is that, under this calibration, the efficiency wage remains above the minimum. As the minimum wage increases from m to m' , the equilibrium efficiency wage increases from $w^E(\theta)$ to $w^E(\theta')$, such that $m' < w^E(\theta')$. Figure 2.10 shows how these two wages behave. We can observe that, at least for increments of less than 100%, the efficiency wage adjusts so that it is always above the minimum although the difference between the two wages closes.

This means that the minimum wage is aggravating the moral hazard risk in the market; as it increases, it improves the hireable workers' working conditions so much that instead of receiving the unconstrained wage that once was enough to motivate them, now they must receive an efficiency wage. The situation is severe enough to generate the sub-optimal

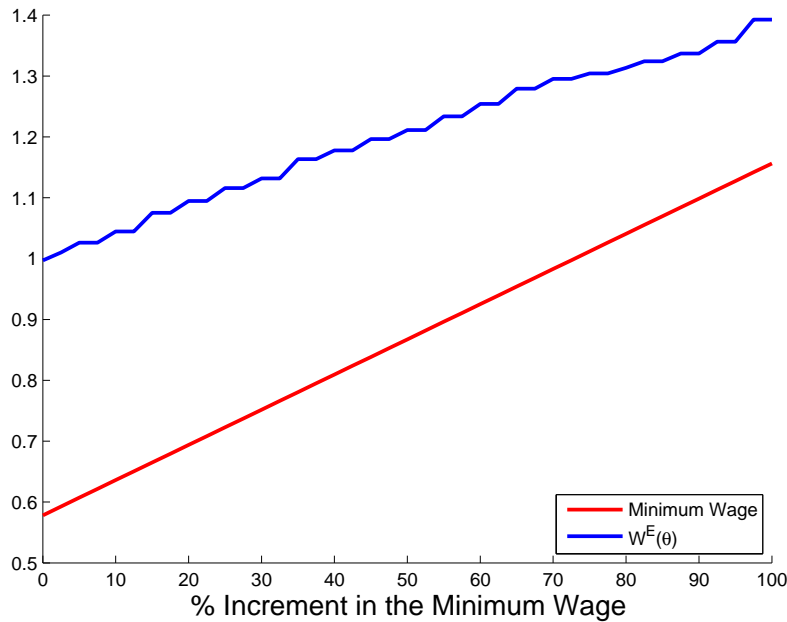


Figure 2.10: The Efficiency Wage and the Minimum

minimum wage utilization described in Section 2.3.

For example, with a 90% increment the minimum goes from approximately 0.57 to 1.1. A worker earning a wage of 1 before the increment, instead of receiving a wage of 1.1, after the increment receives a wage above 1.3. The minimum wage is being under used. The premium above the minimum arises because of the increment in the risk of moral hazard. This means that the minimum wage increases market tightness, so there will be asymmetries in the outcomes of different workers as described in Section 2.3.

First I analyze individual market outcomes. Figure 2.11 shows the impact by deciles in the worker population. Panel a) shows the effects that of an increase in the minimum wage on labor force participation. Workers with the lowest productivity, the first decile, have productivities very close to the minimum wage so a 10% increase in the minimum would reduce their participation in the labor force by 60%. An increase of 18% would completely price them out and drive them out of the labor force. Increasing the minimum further, would affect more productive deciles the same way. A 100% increase would drive half of the worker population out of the market. The asymmetry in the participation effect is visible in

the graph; as less productive workers are discouraged from participating, remaining workers increase their participation and the effect is stronger for workers at the high end of the distribution. Despite the asymmetry, the discouraging effects for low-productivity workers are much stronger. This is consistent with the results in Section 2.4 which show that the labor force discouraging effects on LTHS teenagers is much stronger than the encouraging effects for mature workers with high school educational attainment. Panel b) showing the effect on employment tells a similar story. As the employment falls for the least productive workers, those that remain hireable have small increases in employment. The situation is also in agreement with the results in Section 2.4.

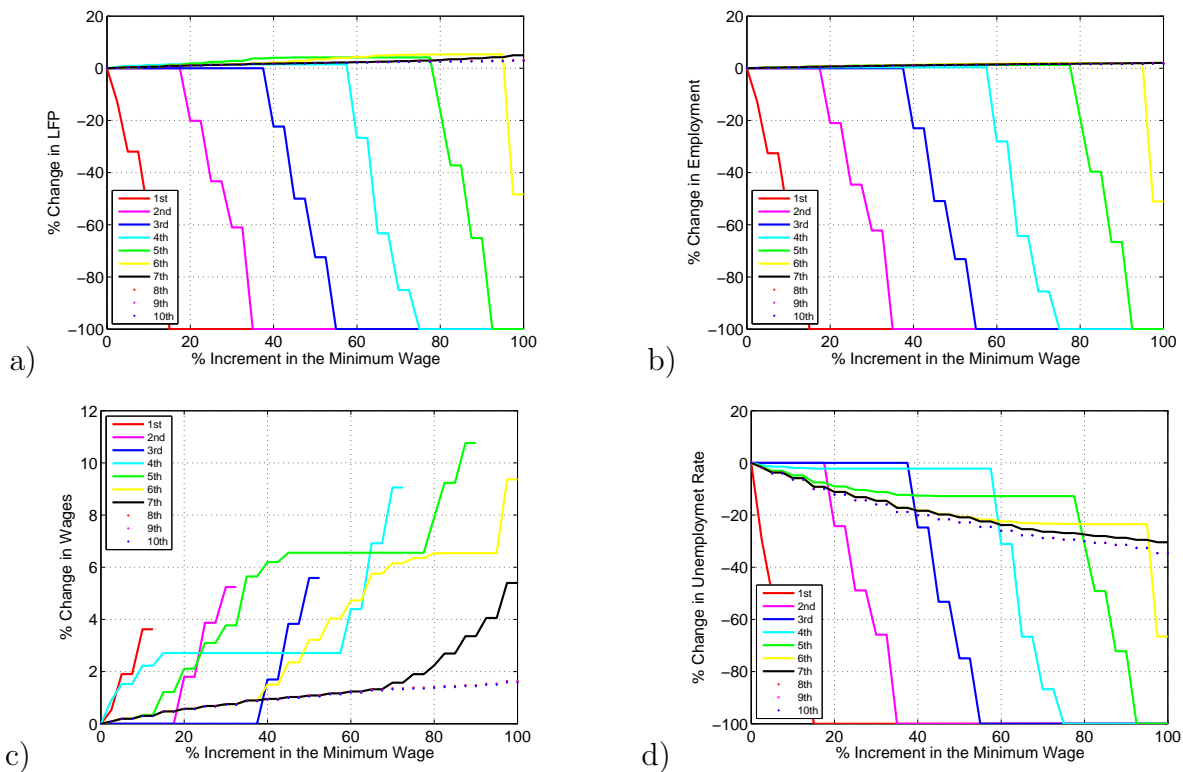


Figure 2.11: Labor Market Outcomes by Deciles

Panel c) shows the wage effects across deciles. Consistent with the theoretical predictions and the empirical results in Section 2.4, the simulations show wage spillover effects. A 10 % increase in the minimum wage increases the average wage of the lowest productivity decile above 3%. The effect on wages is reflected on the wages of perfectly employable workers

and it decreases in intensity as productivity increases. The average wage of the fourth decile augments by 1%, and the average wage of top deciles by around 0.5%.

The situation is misleading. Wages are contingent on employment so the minimum wage is not really increasing wages in the first decile since these are barely employable workers who are being paid as much as they can. The increase in the average wage is due to the fact that a higher minimum drives some of the workers in the lowest decile out of the market, so the average wage in that decile increases because only the most productive workers have wages to report. This is not the case for the top 6 deciles who truly see their wages increase due to the general equilibrium effect, and not because workers are being driven out of the market.

This could explain why the wage spillover effects in Section 2.4 are significant only for teenagers and not for mature workers with HS education. Workers at the middle of the productivity distribution show strong increases on average wages since the minimum is trimming off low wages. After a hike in the minimum wage, teenagers reporting their wages are those productive enough to remain hireable.

Panel d) shows the effects on unemployment rates. The situation is misleading since the unemployment rate takes into account the search intensity that unemployed workers put into finding a job. For example, a 10% increase reduces the unemployment rates of the least productive and the most productive workers for very different reasons. Just as in the wage effects, the unemployment rates of workers at the low end of the distribution decreases because participation decreases, many workers stop reaching for a job, so after a minimum hike their unemployment rate is lower. On the other hand, perfectly employable workers see their unemployment rates decreases as their participation increases because they are actually more employable in a tighter market.

Now I analyze the effects on the market at an aggregate level. I start with aggregate employment and aggregate labor force participation. I define aggregate employment as the total amount of total workers employed, that is,

$$\text{Total Employment} = \sum_i p_i(1 - u_i).$$

Total labor force participation is defined as the total amount of employed workers plus the measure of unemployed workers searching for a job,

$$\text{Labor Force Participation} = \sum_i p_i(1 - u_i) + \sum_i p_i s_i u_i.$$

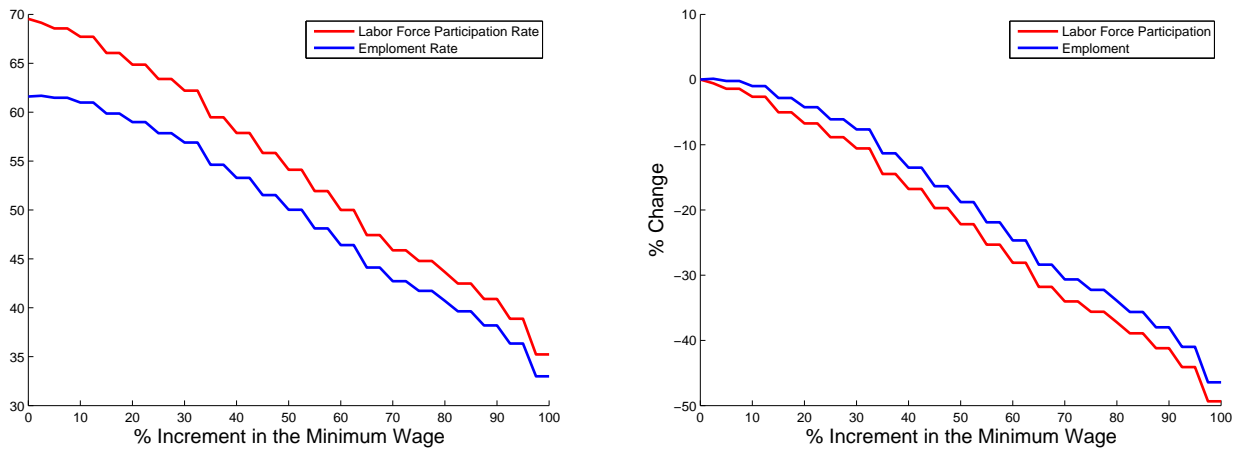


Figure 2.12: Total Employment and Labor Force Participation

Figure 2.12 presents the simulation's results. They confirm what is inferred from the desegregated results, increments in the minimum wage have a much stronger negative employment effect on low-productivity workers, than the positive employment effects they have on more productive workers. Overall, total employment falls drastically after a minimum wage increment. The effect on total labor force participation is similar; a hike in the minimum has an overall negative impact.

The fact that labor force participation and employment fall as the minimum wage increases does not entail a reduction of aggregate welfare in the market. To assess the welfare properties of an increase of the minimum wage, I compute aggregate welfare as the sum of all agents' utilities, that is

$$\text{Aggregate Welfare} = \sum_i p_i u_i r U_i + \sum_i p_i (1 - u_i) r E_i + \sum_i p_i (1 - u_i) r J_i + v r V.$$

At the steady state it can be expressed as

$$\text{Aggregate Welfare} = \sum_i p_i u_i [b - c(s_i)] + \sum_i p_i (1 - u_i) [y_i - e] - \theta \left[\sum_i p_i s_i u_i \right] \gamma.$$

Figure 2.13 presents the simulation's results. Increments in the minimum wage generate a reduction in aggregate welfare. The strong negative welfare effects of the minimum wage are due to the fact that most of low-education workers concentrate right above the minimum so even small increments drive a considerable percentage of workers out of the labor force. Figure 2.11 shows that an increase of 100% in the minimum drives half of the labor supply out of the market, which makes social welfare drop significantly.



Figure 2.13: Aggregate Welfare

2.6 Concluding Remarks

This chapter has explored the notion that minimum wages affect the labor force asymmetrically due to worker heterogeneity. I developed a search model of unemployment predicting that due to the presence of moral hazard, a rising minimum wage will price out of the labor force low-productivity workers and will increase the employment, encourage the labor force participation, and raise the wages of workers that remain hireable. These predictions hold in CPS micro data once I focus on the low-education labor market and I disaggregate workers by age and education.

The study's results have important implications. First, they emphasize the inherent characteristics of the labor market that makes its analysis particular. Not only is it riddled with trading frictions, it also displays heterogeneity among participating agents and relatively strong government intervention. I have shown that including heterogeneity in the modeling of the market is consequential for the insights obtained from the model. Models of unemployment must incorporate all these elements to better understand the implications of labor market policies.

On the pragmatic side, the results emphasize that the aggregation level matters for the assessment of the disrupting effects of an imposed wage floor. They enrich the debate of “who” is truly affected by the minimum, by adding the notion of “how” they are affected. Even within the same labor market, not all workers are equally impacted, so a clear understanding of these differences is necessary if the goal of a minimum wage is to change the labor conditions of a specific group. If, on the other hand, the performance of a minimum is to be judged by its broad impact on low-wage labor markets, the results show that despite the asymmetries, the consequences of a higher minimum are detrimental for employment, labor force participation and welfare.

According to my results, the initiative to increase the federal minimum wage to \$15 an hour in the United States would generate large-scale employment and welfare losses. For those states where the binding minimum is equal to the federal minimum, fixed at \$7.25 per

hour since 2009, the proposed new minimum would represent a 107% increase. According to the simulation for the low-education labor market, this increment would bring a decrement of around 50% in employment and labor force participation, and a reduction of 70% in social welfare. States with a relatively high minimum wage would experience smaller, although still considerable, losses. For example, California with a minimum wage of \$9.00 would see a reduction of roughly 30% in employment and labor force participation, and a decrease of 55% in social welfare in the low-education labor market.

Chapter 3

Exchange Rate Dynamics in a Simple Model with Heterogeneous Expectations

3.1 Introduction

Traditional models assume that movements in the exchange rate are caused by unexpected changes in the macroeconomic fundamentals, or “news”, and that this causal relationship remains stable across time. Some celebrated examples of these models are the seminal Dornbusch (1976) model and the portfolio-balance model. See Frankel (1987) for an extensive review of these models. Although popular for their tractability and logical appeal, these workhorse models are not capable of accurately predicting fluctuations in exchange rates.

Several empirical studies have documented how the fundamentals in the economy cannot be linked to the erratic nature of exchange rates. For example, Meese and Rogoff (1983) show that a random walk model performs as well as various structural and time series models when trying to make out-of-sample predictions. Baxter and Stockman (1989), and Flood and Rose (1995) found that while the movements from fixed to flexible exchange rates lead to

dramatic increases in the exchange rate volatility, no such increase could be detected in the volatility of the underlying economic variables. All these puzzles have led some economists to consider the evolution of exchange rates as a stochastic phenomenon leaving its forecast to technical analysis and time series procedures with little room for economic theory.

Another feature of traditional models challenged by research is their assumption of a representative agent. There is now abundant evidence that foreign exchange market participants have heterogeneous expectations about future exchange rates. See Frankel and Froot (1987), Ito (1990), MacDonald and Marsh (1996), Elliott and Ito (1999) and Menkhoff et al. (2008). This characteristic provides a chance to reintroduce economic theory into models by considering heterogeneity and bounded rationality among traders. The diversity of traders' expectations introduces nonlinear features capable of generating complex dynamics and even chaos. Although the presence of chaos in the exchange dynamics is controversial. See Torkamani et al. (2007) where they analyze the behavior of the exchange rate of many foreign currencies against the Irani Rial and find complex chaotic dynamics. Resende and Zeidan(2008) who obtain Lyapunov exponents for different currencies' expectations upon weekly data within the 1984 – 88 period and do not find evidence of deterministic chaos in the expectations of exchange rates. it is worth exploring further.

This situation has motivated research analyzing heterogeneous beliefs in the foreign exchange market. For my purpose, two studies stand out for their similarities with the present work. They develop models where the interaction of agents with heterogeneous beliefs generate complex dynamics in the foreign exchange market. The first one is De Grauwe and Grimaldi (2005). They analyze the workings of a nonlinear currency exchange model in which agents hold different beliefs about the future exchange rate. In their model, there are two types of agents in the foreign exchange market, fundamentalists and chartists, a common way to model heterogeneity in the literature. See seminal work by Frankel and Froot (1990) or more recently De Grauwe and Grimaldi (2006) and Wieland and Westerof(2005). Menkhoff (2009) presents a literature review.

Whereas fundamentalists anticipate that exchange rates move towards their long-run equilibria, modeled via balanced current accounts, chartist take positions in line with recent exchange rate changes (i.e. they extrapolate exchange rate trends). The nonlinear structure of their model is capable of generating very complex exchange rate dynamics that could explain some of the empirical puzzles. A caveat of their model is that its nonlinear structure is not simple enough to derive analytic solutions, so their analysis relies entirely on simulation techniques using plausible values for the parameters. To determine the evolution of the population of traders with different beliefs they don't rely on evolutionary game theory but instead, they define their own dynamic that is similar in nature to the well known logit dynamic.

The other work closely related to this work is Chiarella et al. (2007). They present a continuous time model that extends the classical Dornbusch (1976) exchange rate model with homogeneous expectations to incorporate heterogeneous beliefs and bounded rationality into a dynamic model of foreign exchange. The exchange rate is determined by the interaction of portfolio managers who base their predictions on a weighted average of the different expectations among traders. In their paper, traders can be either fundamentalists or chartists. A major drawback of their model is that the proportion of traders using a particular forecasting strategy is exogenously determined leaving partially aside the traders' maximizing behavior. Their model generates very complex dynamics in the market, including the existence of multiple steady state equilibria, the persistence of deviations of the market exchange rate from its fundamental value, and wild market fluctuations.

The present work shares its purpose with the two studies above mentioned, it presents a very simple model with heterogeneous expectations capable of generating very complex dynamics for the exchange rate. This model takes the structure of the asset pricing model in Brock and Hommes (1998) which is a financial market application of the heterogeneous evolutionary framework of Brock and Hommes (1997). Traders make profits by currency arbitrage and their current actions depend on their expectations about the future exchange

rate. Traders can be of two types, sophisticated or naive, with both types having different payoffs for their choice of forecasting tool. My model deviates from the chartist versus fundamentalist approach and instead it allows both kinds of traders to choose between rational expectations and chartist prediction as forecasting strategy.

Since every trader's profit depends on his choice of forecasting tool and the actions of all other traders through the determination of the future exchange rate, the situation can be analyzed as a game. I make use of the Brown-von Neumann-Nash (BNN) dynamic presented in Brown et al.(1950) to determine the evolution of the proportions of traders using each strategy. One of the appealing features of the BNN dynamic is that the steady states of the dynamical system will correspond to the Nash equilibria due to the property of Nash stationarity. Since one of the main features of this work is the use of the BNN dynamic, a section is devoted to introduce it.

Unlike previous works in the literature, the model presented here is simple enough to derive analytical solutions. I find that the existence of steady states depends on the interest rate differential and on the cost of using rational expectations. When using rational expectations is a costly activity and there is an interest rate differential, the unique steady state has all naive traders choosing chartist forecasting and the exchange rate being equal to its fundamental value. The chapter focuses on the behavior of naive traders since sophisticated traders are assumed to be a minority that selects rational expectations as forecasting tool. In this steady state, the exchange rate is continually determined only by the fundamentals of the economy so naive traders have no incentive to acquire a costly forecasting tool to predict it. Its local stability depends on the proportion of sophisticated traders and the kind of beliefs traders using chartists prediction have. When there are few sophisticated traders, who always use rational expectations, the relative loss of using the chartist forecast is higher due to the powerful negative feedback effect, more traders believing that the exchange rate will increase cause a deeper drop in the exchange rate so the miscalculation of the future exchange rate is more severe.

I present simulations of the model to analyze the long run behavior of the system for different parameter values. Phase plots and bifurcation diagrams demonstrate the complexity of the system and, under certain parameter values, show the existence of strange attractors where the possible paths of the dynamical system form irregular dense sets, strong evidence of the presence of chaos. Chaos in the system is asserted with the computation of the largest Lyapunov exponent. I show that chaos can arise under a wide range of parameter values.

The implications of complex dynamics of this sort driving the exchange rate are not small. Chaos would imply that it is very difficult, perhaps even impossible, to find a pattern in the movement of exchange rates linked to the fundamentals. Even in the absence of random shocks, the slightest miscalculation in the parameters could still result in dramatic imprecisions in long-term forecasts.

This chapter is organized as follows, Section 3.2 presents a model for exchange rate determination with different forecasting rules. Section 3.3 introduces the reader to the BNN dynamic and presents the population dynamics. Section 3.4 presents the analysis of the steady states and their stability. Section 3.5 presents arguments for complex dynamics using numerical simulations. Section 3.6 concludes.

3.2 A model for currency exchange rate with heterogeneous beliefs

In this section I develop a simple exchange rate model with heterogeneous expectations taking the structure from the model presented in Brock and Hommes (1998), a financial application of the heterogeneous expectations evolutionary switching framework of Brock and Hommes (1997).

3.2.1 Exchange rate model

Agents are traders trying to profit from currency arbitrage. They can invest in two different countries, a domestic country with interest rate r_d , and a foreign country with interest rate r_f .¹ Interest rates are assumed constant through time and exogenous. Let p_t be the spot exchange rate defined as the domestic price of a unit of foreign currency, and z_t be the total amount of foreign currency that a trader holds at time t . A trader's wealth dynamic is given by

$$W_{t+1} = (1 + r_d)(W_t - p_t z_t) + (1 + r_f)(p_{t+1} z_t), \quad (3.1)$$

where W_t denotes the total wealth of a trader at time t . Since both interest rates remain fixed throughout the analysis, domestic currency can be considered as a risk-free asset whereas foreign currency could be considered as a risky asset since its profitability depends on the unknown future currency exchange rate. Defining $R_d \equiv (1 + r_d)$ and $R_f \equiv (1 + r_f)$ we get

$$W_{t+1} = R_d W_t + (R_f p_{t+1} - R_d p_t) z_t. \quad (3.2)$$

All agents are assumed to be myopic mean-variance maximizers so at each period they choose the optimal amount of foreign currency z_t^* that solves

$$\max_{z_t} \left\{ E_t [W_{t+1}] - \frac{a}{2} V_t [W_{t+1}] \right\}, \quad (3.3)$$

where a is the risk aversion parameter, and $E[\cdot]$ and $V[\cdot]$ are the expectation and the variance operator respectively. Solving the maximization problem we get the optimal demand for foreign currency

¹There is an implicit assumption that all traders face the same dilemma in terms of currency rates and interest rate differentials, they are all domestic traders deciding whether or not they will invest in a foreign currency with a higher and riskier interest rate. I treat all traders as belonging to the same country, a standard assumption in these kinds of models. See Marey (2004) for a model that differentiates between domestic and foreign traders. He investigates the plausibility of standard exchange rate expectation mechanisms, which are favored over rational expectations in survey data long horizons, in an artificial economy with heterogeneous traders.

$$z_t^* = \frac{E_t [R_f p_{t+1} - R_d p_t]}{a V_t [R_f p_{t+1} - R_d p_t]}. \quad (3.4)$$

A key simplifying assumption in the model presented in Brock and Hommes (1998) is that the variance in equation (4), the variance of the excess return of foreign investment over domestic investment, is known and constant in time. Imposing this assumption we get

$$z_t^* = \frac{R_f E_t [p_{t+1}] - R_d p_t}{a \sigma^2}, \quad (3.5)$$

where $\sigma^2 = V_t [R_f p_{t+1} - R_d p_t] \forall t$. Notice that if the no arbitrage condition of the uncovered interest rate parity $E_t [p_{t+1}] = \frac{R_d}{R_f} p_t$ holds then the optimal amount of foreign currency for each trader would be zero due to their risk aversion. Following De Grauwe and Grimaldi (2005) and De Grauwe and Markiewicz (2013), the exchange rate can be written as follows

$$p_t = p_t^* + x_t,$$

where $p_t^* = f(m_t)$ is the fundamental exchange rate determined by m_t , a vector of fundamental variables in the economy, so x_t is the deviation of the observed exchanged rate from its fundamental value. For convenience, I assume that the fundamental exchange rate evolution satisfies $p_{t+1}^*(m_{t+1}) = p_t^*(m_t) \frac{R_d}{R_f}$ and that the fundamental value and its evolution are known by every trader.² Under these assumptions, the expectation of the future exchange rate becomes $E_t [p_{t+1}] = p_{t+1}^* + E_t [x_{t+1}]$. Substituting into equation (5) we can write the optimal demand of foreign currency in terms of the expected deviations from its benchmark fundamental value

$$z_t^* = \frac{R_f E_t [x_{t+1}] - R_d x_t}{a \sigma^2}. \quad (3.6)$$

²Notice that this assumption in our model is not very restrictive. The fundamental exchange rate is a theoretical concept that will only affect the dynamics of the model through the way traders form expectations about future exchange rates so it would suffice to assume that all the traders believe that the fundamental price evolves according to $p_{t+1}^*(m_{t+1}) = p_t^*(m_t) \frac{R_d}{R_f}$.

Heterogeneity among traders is introduced in the form of different beliefs about future deviations from the fundamental exchange rate, so traders can be classified according to the forecasting strategy they use. With K different forecasting strategies, the total demand for foreign currency is given by sum of the individual demands

$$Z^D = \sum_{k=1}^K q_{kt} \frac{R_f E_{kt}[x_{t+1}] - R_d x_t}{a\sigma^2},$$

where q_{kt} is the proportion of the trader population using forecasting strategy k at time t , and $E_{kt}[x_{t+1}]$ is the predicted deviation from the fundamental exchange rate given by forecasting strategy k . The general model for financial applications presented in Brock and Hommes (1998) focuses on the special case of zero supply of outside shares in the market. According to Evans (2002) most of the transactions in the foreign exchange market is inter-dealer trading so, *ceteris paribus*, having a zero supply of outside currency, i.e. $Z^S = 0$, seems a plausible assumption for this model. From the market clearing condition $Z^S = Z^D$ we derive the market equilibrium pricing equation in terms of deviations from the fundamental exchange rate

$$\frac{R_d}{R_f} x_t = \sum_{k=1}^K q_{kt} E_{kt}[x_{t+1}]. \quad (3.7)$$

In a model with a representative trader with rational expectations, all traders would have the same expectations about future deviations from the fundamental exchange rate. In this case the market equilibrium equation (7) simplifies to

$$\frac{R_d}{R_f} x_t = E_t[x_{t+1}]. \quad (3.8)$$

This equality states that today's deviation from the fundamental value must equal tomorrow's expected value of the deviation discounted by the ratio of the foreign interest rate over domestic interest rate. Iterating this equation to infinity we get the rational expectations solution

$$x_t^* = \lim_{n \rightarrow \infty} (R_f/R_d)^n E_t[x_{t+n}]. \quad (3.9)$$

Under the assumption that every trader expects that in the long term the exchange rate will go back to its fundamental value, i.e.

$$\lim_{n \rightarrow \infty} E_t[x_{t+n}] = 0. \quad (3.10)$$

The rational expectations exchange rate is determined and it is equal to its fundamental value, it is the price that would prevail in an efficient market with rational traders only.

3.2.2 Evolutionary dynamics and heterogeneous beliefs

This section describes how the proportion of the trader population using forecasting strategy k , q_{kt} , evolves over time. Traders will choose their forecasting strategies depending on their past performance according to the BNN dynamic described in section 2. Payoffs to different strategies are associated with a cost of using them. Differences in cost reflect the fact that some forecasting tools are more computationally intensive or require hiring an external consultant. Brock and Hommes (1998) use realized profits as their measure of evolutionary fitness. Following them, accumulated realized profits take the form

$$U_{kt} = (R_f p_t - R_d p_{t-1}) z_{kt-1} - C_k + w U_{k,t-1}. \quad (3.11)$$

The first term in (11) represents the realized profit in t for a trader using strategy k in $t - 1$ and it is composed of the excess return of foreign investment over domestic investment times the demand for foreign currency in $t - 1$. C_k is the average per period cost of using strategy k and it will be equal to zero for simple forecasting strategies and may be positive for more sophisticated ones. $w \in [0, 1]$ is a memory parameter indicating the importance that past realized profits have in strategy selection. I will focus in the case with no memory, i.e.

$w = 0$, so the fitness measure equals the net realized profit in the last period. Substituting $p_t = p_t^* + x_t$ and given that $p_{t+1}^* = p_t^* \frac{R_d}{R_f}$, we can express realized profits in terms of exchange rate deviations from its fundamental value

$$U_{kt} = (R_f x_t - R_d x_{t-1}) z_{kt-1} - C_k. \quad (3.12)$$

Now I specify a particular version of the model by defining the two forecasting strategies available to traders; rational expectations and chartist prediction. My model deviates from the chartist versus fundamentalist approach, a common way of modeling expectation heterogeneity in the literature of foreign exchange markets. I follow the setting in Brock and Hommes (1998) where agents have rational expectations and simple linear forecasting to choose from. Those traders selecting rational expectations form their forecast according to the mathematical conditional expectation given all available information, they do not make systematic mistakes and their expectations are, on average, correct. The rational expectations forecasts takes the form $E_{Rt}[x_{t+1}] = x_{t+1}$. Traders choosing chartists forecasting make predictions in line with past exchange rate changes (i.e they extrapolate exchange rate trends). For simplicity, I assume that chartist prediction is a linear function of the latest two observations available, namely, $E_{Ct}[x_{t+1}] = x_t + \theta(x_t - x_{t-1})$ where $\theta \in \mathbb{R}$ is the trend parameter. Notice how different values of θ represent different beliefs about the future exchange rate. For example, $\theta = 1$ is equivalent to extrapolating the trend perfectly, believing that the next increase (or decrease) in the exchange rate will be the same as the last one observed. $\theta = 0$ is equivalent to thinking that there will be no further changes in the exchange rate. When $\theta = -1$ traders believe that the exchange rate will go back to its previous level.

An important assumption of the model is the existence of two different types of traders in the market, those that are better adapted to the market and those that are less adapted. For those that are better adapted, having rational expectations has no cost. We can think of this adaptation as greater experience with the workings of the market, superior skills or superior information required to form rational expectations. I refer to this kind of traders

as “sophisticated” and they will be assumed to be a small fixed minority representing a proportion $s \in (0, 1)$ of the total trader population.³ For the other type of traders that is not as well adapted to the market, having rational expectations is a costly activity. I will refer to this kind of trader as “naive” and they represent a fraction $1 - s$ of the entire trader population. If naive traders choose to use rational expectation as forecasting tool they must pay C at each period. Chartist prediction is assumed to be costless for both types of players.

A key aspect of the model is that the proportion of sophisticated traders is so small that it hardly has an impact on the determination of the exchange rate, so for the sake of simplicity, I will assume that sophisticated traders will always opt for using rational expectations. It is sensible to think that if using rational expectations is costless then there is no reason to use chartist forecasting. Under this assumption the market clearing condition (7) takes the form

$$\frac{R_d}{R_f} x_t = [(1 - s)q_t + s]x_{t+1} + (1 - q_t)(1 - s)[x_t + \theta(x_t - x_{t-1})], \quad (3.13)$$

where q_t is the proportion of naive traders using rational expectations at time t , therefore $1 - q_t$ is the proportion of naive traders using chartist forecast at time t . From (13) we can derive the dynamics for the exchange rate deviations from its fundamental value

$$x_t = \frac{x_{t-1}(\frac{R_d}{R_f}) - (1 - s)(1 - q_{t-1})(1 + \theta) + x_{t-2}(1 - s)(1 - q_{t-1})\theta}{(1 - s)q_{t-1} + s} \quad \forall t. \quad (3.14)$$

Given the assumption that sophisticated traders always choose to use rational expectations and that they constitute a small fixed minority of the population, their actions and their size are fixed so from now on they will be considered as another parameter in the system “ s ”, the autonomous proportion of traders using rational expectations. The rest of

³As long as this proportion of players remains a minority, how small it really is does not affect the qualitative findings of the study. The implications for the results of different sizes of this proportion are analyzed in a later section of the chapter.

the analysis will focus on the choice made by the naive traders. With these two specific forecasting strategies, the realized profits for naive traders in terms of exchange rate deviations from its fundamental in equation (12) take the form

$$U_{Rt} = [R_f x_t - R_d x_{t-1}]^2 \frac{1}{a\sigma^2} - C \quad (3.15)$$

$$U_{Ct} = [R_f x_t - R_d x_{t-1}] [x_{t-1}(R_f(1 + \theta) - R_d) - x_{t-2}R_f\theta] \frac{1}{a\sigma^2}. \quad (3.16)$$

Equation (15) is the realized profits from using rational expectations and (16) is the realized profits from using chartist forecasting.

3.3 The Brown-von Neumann-Nash (BNN) Dynamic

There is a broad literature dealing with macroeconomic phenomena that makes use of evolutionary game theory dynamics to model switching proportions of a population choosing different strategies. For example, Brock and Hommes (1997) study chaos in the Cobweb model with heterogeneous beliefs under the logit dynamic. Branch and McGough (2008) analyze the Cobweb model using a modification of the replicator dynamic. Waters (2009) analyzes the Cobweb model using the BNN dynamic. Brock and Hommes (1998) study chaos in financial markets with the use of a multinomial logit model.

The BNN dynamic has certain properties absent from all of the alternative dynamics used in the literature, namely positive correlation, Nash stationary, and existence, uniqueness, and continuity of solutions. These properties could be considered as minimal conditions for an intuitively appealing dynamic capable to model the evolution of heterogeneous strategies. These conditions relate aggregate behavior under evolutionary dynamics to incentives in the underlying games. The first such condition, positive correlation (PC), requires that whenever a population is not at rest, its strategies' growth rates be positively correlated with their

payoffs. This condition demands a weak but fundamental connection between individual incentives and disequilibrium aggregate dynamics. The second condition, Nash stationary (NS), ask that the rest points of the mean dynamic be precisely the Nash equilibria of the game being played. Dynamics satisfying NS display a basic agreement between the evolutionary dynamic and the traditional game-theoretic notion of equilibrium play. Existence, uniqueness, and continuity of solutions (EUC) requires a dynamic to admit exactly one solution from each initial state. It also requires solutions to change continuously as the initial state varies.

Imitative dynamics, exemplified by the replicator dynamic, satisfy the disequilibrium condition PC and EUC however, because pure imitation precludes the introduction of unused strategies, imitative dynamics admit rest points that are not Nash equilibria, and so fail Nash stationarity.⁴ Best response dynamics, such as the logit dynamic, satisfy modified versions of PC and NS but fail EUC (Sandholm 2005). Since this dynamic's law of motion is discontinuous, its behavior, even over short time spans, is quite sensitive to initial states. Thus, while solutions to the best response dynamic exist and are upper hemi-continuous in their initial conditions, multiple solution trajectories can emanate from a single initial state.

The BNN dynamic is the most prominent example of an excess payoff dynamic. In fact, it is from this family of dynamics that it inherits all its convenient properties.⁵ Under excess payoff dynamics, agents receive opportunities to choose new strategies according to Poisson processes, choosing stochastically from the available strategies when such opportunities arise. Both revision rates and choice probabilities are functions of the strategies excess payoffs, i.e., the difference between the strategies payoffs and the population average payoff. Sandholm (2005) shows that every excess payoff dynamic is well-behaved in the sense of satisfying properties EUC, NS and PC. I use the discrete time version of the BNN dynamic presented in Sandholm (2005). Let S be the finite set of strategies available to a population and $U_{k,t}$

⁴This aspect is closely related to the Inventiveness criterion in Waters (2009) who also presents a justification for the use of the BNN dynamic.

⁵See Sandholm (2005) for an analysis of the subject.

be the payoff or fitness measure to strategy $k \in S$ at time t . Then, the proportion of the population using strategy k at time t , $q_{k,t}$, evolves according to

$$q_{k,t+1} = \frac{q_{k,t} + \sigma(\pi_{k,t})}{1 + \sum_{j \in S} \sigma(\pi_{j,t})} \quad \forall k \in S, \quad (3.17)$$

where $\sigma(\pi_{k,t}) = \max\{\pi_{k,t}, 0\}$ and $\pi_{k,t}$ is the excess payoff to strategy k and it is defined as $\pi_{k,t} = U_{k,t} - \bar{U}_t$, the difference between the payoff to strategy k and the population average payoff given by $\bar{U}_t \equiv \sum_{j \in S} q_{j,t} U_{j,t}$.

The BNN dynamic is not free from shortcomings. For example, the information requirements it imposes could be unrealistic. It requires players to know the mean payoff of the population and unless this information is provided by a central planner, it is not readily available to agents in typical large population settings. The BNN dynamic could also present problems with its analysis. Although continuous in the payoffs, it might not be globally differentiable since its construction eliminates negative excess payoffs. However, as will be seen later, this is not an issue in this study.

The model for exchange rate determination with heterogeneous beliefs presented in De Grauwe and Grimaldi (2005) is, to the best of my knowledge, the only study previous to this one that endogenizes the proportions of traders using a particular forecasting strategy. To model the proportion of traders using each forecasting strategy they define their own dynamic, very similar in nature to the logit dynamic. As mentioned above, this dynamic could present some problems,⁶ my model attempts to improve upon this situation by using the BNN dynamic.

For the two strategies described in section 2 the excess payoffs take the form

$$\pi_{R,t} = U_{R,t} - \bar{U}_t = (1 - q_t)(U_{R,t} - U_{C,t}) = (1 - q_t)\delta(x_{t-1}, q_{t-1}) \quad (3.18)$$

⁶See Waters (2009) or Sandholm (2010) for the inconveniences of using the logit dynamic.

$$\pi_{C,t} = U_{C,t} - \bar{U}_t = -q_t(U_{R,t} - U_{C,t}) = -q_t\delta(x_{t-1}, q_{t-1}), \quad (3.19)$$

where $\delta \equiv U_{Rt} - U_{Tt}$ is the payoff difference function. Equation (18) expresses the difference between the profit from using rational expectations and the mean profit among naive traders,⁷ equation (19) does the same for chartist forecasts. Substituting the dynamics for x_t (equation (14)), into (15) and (16), the payoff difference function takes the form

$$\delta(x_{t-1}, q_{t-1}) = \frac{[x_{t-1}^2(R_d - R_f(1 + \theta))^2 + x_{t-1}x_{t-2}R_f(R_d - R_f(1 + \theta))(1 + \theta) + x_{t-2}^2\theta R_f^2]}{a\sigma^2} * \frac{(1-s)(1-q_{t-1})}{[(1-s)q_{t-1} + s]^2} - C. \quad (3.20)$$

The evolution of (x_t, q_t) is determined by the payoff difference function, the price dynamics and the BNN evolutionary dynamic. The analysis of the system is complicated by the non-negativity restriction within the BNN dynamic, $\sigma(\pi_{k,t}) = \max\{\pi_{k,t}, 0\}$, so to clarify the analysis I describe the motion of q_t in the form below using the payoff difference function

$$q_t = \begin{cases} 1 - \frac{1-q_{t-1}}{1+(1-q_{t-1})\delta(\cdot)} & \text{if } \delta(x_{t-1}, q_{t-1}) > 0 \\ \frac{q_{t-1}}{1-q_{t-1}\delta(\cdot)} & \text{if } \delta(x_{t-1}, q_{t-1}) \leq 0 \end{cases}. \quad (3.21)$$

The proportion of naive traders using rational expectations will increase or decrease depending on the sign of $\delta(x_{t-1}, q_{t-1})$.

⁷There is the implicit assumption that naive traders make their decision based only on the performance of other naive traders.

3.4 Steady states and local stability analysis

The previous sections describe a model where foreign exchange rate is determined by the interaction of traders in the market. Trader's actions depend on the forecasting strategy they choose according to their past performance, determined on past realizations of the exchange rate. This recursive mechanism constitutes a dynamical system for the exchange rate and the strategies traders use. The suitability of the model depends on how well the dynamic can mimic the exchange rate's fluctuations. To assess this, it is necessary to study the system's long-term behavior so this section deals with the question: what can we know about the orbits of the dynamical system for different parameter sets and for different initial states? This simple system could converge to a periodic or an aperiodic cycle depending on the parameter values. First I analyze the existence and local stability of steady states. Fortunately, the dynamics of this system of equations is tractable enough to be analyzed with standard theoretical tools. Let F be the evolution function where $F(x_t, q_t) = (x_{t+1}, q_{t+1})$ and it is composed by (14), (20) and (21). A steady state for this dynamical system is a point $Z \in \mathbb{R}^2$ such that $F(Z) = Z$. The existence of a steady state will depend on the interest rate differential, the cost naive traders have to pay to have rational expectations and the trend parameter. Proposition 8 summarizes all possible steady states.

Proposition 8. *The following conditions characterize the steady state:*

- i) If $R_d \neq R_f$ and $C > 0$, the unique steady state is $(0, 0)$.
- ii) If $R_d \neq R_f$ and $C = 0$, the unique steady states are all the points of the form $(0, q)$ with $q \in [0, 1]$.
- iii) If $R_d = R_f$ and $C > 0$, then the unique steady states are all the points of the form $(x, 0)$ with $x \in R$ provided that $\theta = 1$. If $\theta \neq 1$, then all the points of the form (x^*, q) with $q \in [0, 1)$ can be a steady state where

$$x^* \equiv \sqrt{\frac{C[(1-s)q + s]^2 a \sigma^2}{(1-s)(1-q)(Rd - \theta R_f)^2}}.$$

iv) If $R_d = R_f$ and $C = 0$, then any point (x, q) with $q \in [0, 1]$ and $x \in R$ is a steady state provided that $\theta = 1$. If $\theta \neq 1$ then any point $(0, q)$ with $q \in [0, 1]$ can be a steady state.

Proof: See Appendix C.

Given the assumption of two different types of traders in the market, sophisticated traders who can use rational expectations at no cost, and naive traders who must pay a price to use rational expectations, the analysis will be restricted to the case where it is costly for naive investors to use rational expectations, i.e. $C > 0$. As stated in proposition 8, the steady states will depend on the existence of an interest rate differential. When the interest rates are not the same domestically as abroad, i.e. $R_d \neq R_f$, then the only possible steady state has all naive traders using chartist forecasting and the exchange rate being equal to its fundamental price. In the special case of no interest rate differential other steady state equilibria are possible. However, the local stability of all the possible steady states when $C > 0$, regardless of the interest rates, reduces to the same expression. The values of the trend parameter θ that can induce a steady state depend on the proportion of sophisticated traders, there can be a locally stable steady state under any θ if the proportion of sophisticated traders is large enough. Proposition 9 formalizes these results.

Proposition 9. *When $C > 0$ a steady state is locally stable if and only if*

$$\frac{R_d/R_f - 1}{1 - s} \leq \theta \leq \frac{1 - R_d/R_f - 2s}{s - 1}. \quad (3.22)$$

Furthermore, the width of this interval is increasing in s , the autonomous proportion of traders using rational expectations.

Proof: See Appendix C.

An assumption of the model is that s is very small, only a minority of the traders can be considered “sophisticated” enough to have rational expectations at no cost. The interval of values of θ that induce a locally stable steady state is larger as the proportion of sophisticated

traders increases, and as the proportion of sophisticated traders goes to zero the possibility of having a stable steady state disappears. As the bifurcation diagrams for parameter θ in the next section will show, when the proportion of sophisticated traders is a minority, this interval becomes very small. The interest rate ratio does not have an impact on the width of the interval but it does affect the location of the interval. As the domestic interest rate increases over the foreign interest rate the interval moves to the right, all the values of θ in the interval increase, meaning that the beliefs that the last observed change is the start of a trend, should be stronger to induce local stability in the steady state.

The local stability condition in the steady state with an interest rate differential has an intuitive explanation. In this steady state the exchange rate is equal to its fundamental value and all naive traders are using chartist forecasting. If some exogenous shock moved the exchange rate slightly above its fundamental value, depending on parameter θ , traders using chartist forecast will expect the exchange rate to keep rising ($\theta > 0$), to remain the same ($\theta = 0$), or to partially move back to its fundamental value ($\theta < 0$). Notice that due to the constant foreign currency supply, the model presents negative expectational feedback. A higher expected exchange rate increases the demand for foreign currency so to keep the demand constant, next period's observed exchange rate has to actually decrease. This negative feedback effect that traders using chartist forecasts have on the exchange rate is partially offset by those traders using rational expectations. So the fewer the traders using rational expectations the more powerful the negative feedback will be. When there are few sophisticated traders, who always use rational expectations, the relative loss of using the chartist forecast is higher due to the powerful negative feedback effect, more traders believing that the exchange rate will increase cause a deeper drop in the exchange rate so the miscalculation of the future exchange rate is more severe. In this situation naive traders have a stronger incentive to switch and forecast using rational expectations to avoid the miscalculation. If the proportion of sophisticated traders is relatively high then the negative feedback effect on the realized exchange rate is weaker and the relative losses of keep using

chartist forecast are less severe, there is less incentive for the naive traders to switch to rational expectations. This is why extreme beliefs, higher absolute values of θ , require larger proportions of sophisticated traders to achieve local stability. From equation (22) we see that the interest rate differential also plays a part in stability. A larger interest rate differential exacerbates the incentive of naive traders to change to rational expectations since it makes the relative losses from using chartist forecasting higher. For a given θ , a higher interest rate differential requires a larger proportion of smart traders to reduce the incentives and make naive traders keep using chartist forecasting.

A steady state is not the only kind of converging set of the dynamical system, it is possible to have convergence to a k -period cycle or even to strange attractors. Analytically we can only investigate steady states so the next section presents numerical simulations of the system to give us a better idea of its global behavior.

3.5 Numerical Analysis of Global Dynamics

This section presents simulations for the system defined by equations (15), (20), and (21) under different parameter values. Analysis with bifurcation diagrams and phase plots is standard in the literature featuring non-linear dynamics and aims to give a fast exposition of the global dynamics of the model and to corroborate analytical results. Bifurcation diagrams are a simple and systematic way to find stable cycles numerically and they are obtained by plotting a large number of points in an orbit (the time path of the system) after a transient phase for a large number of equally spaced parameter values in the parameter interval under consideration.⁸ The bifurcations will help us identify values of the parameter for which the converging orbit changes radically. The change in the attracting set is better observed in phase diagrams where we can also detect strange attractors with a complicated fractal structure, a clear sign of chaos.⁹ To determine the presence of chaos in the system I compute

⁸Brock and Hommes (1998).

⁹Strange attractors are unique from other phase-space attractors in that one does not know exactly where on the attractor the system will be after a certain number of periods. Two points on the attractor

the largest Lyapunov exponent for different parameters. Chaotic systems are characterized by extreme sensitivity on initial conditions, nearby initial states diverge exponentially fast creating completely different time paths. The Lyapunov exponent measures the average exponential rate of divergence of nearby states, so for an initial state converging to a locally stable steady state or a stable k-cycle, the corresponding Lyapunov exponent is negative, whereas in a chaotic time path the Lyapunov exponent is positive. The numerical definition of chaos tells us that a dynamical system is called chaotic if there exists a set of initial states of positive Lebesgue measure, such that the Lyapunov exponent is strictly positive. Hommes (2013) offers a review of all these methods.

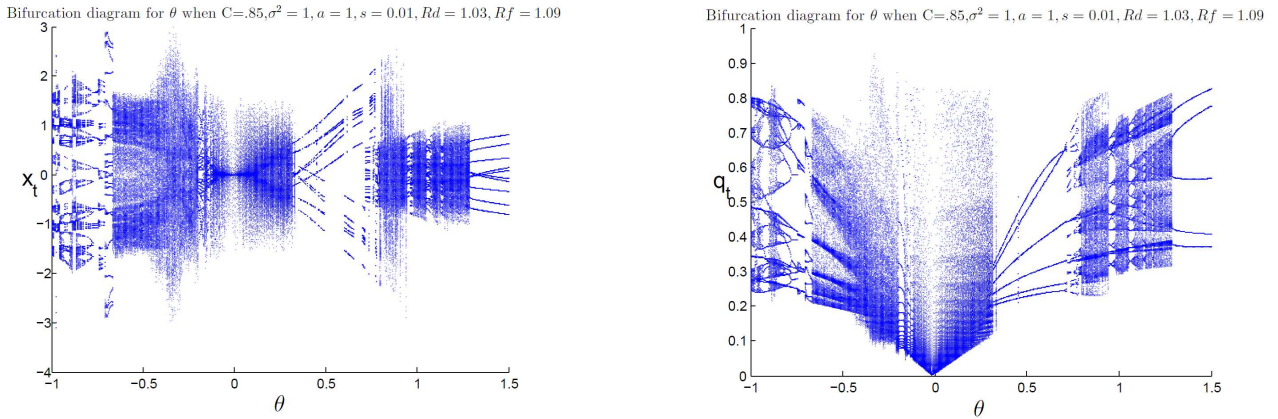
All the bifurcation plots presented are performed with the following parameters values only varying one parameter at the time. Trend parameter θ is set to one. The cost of using rational expectations for the regular traders is set to $C = .085$. The variance of the excess return is $\sigma^2 = 1$ just like the risk aversion parameter set to $a = 1$. The proportion of sophisticated traders is $s = 0.01$, the domestic interest rate is 3% and the foreign interest rate is 9%. Each Bifurcation diagram was created plotting 500 points of the orbit after a transient period of 10,000. The interval considered for each parameter was divided into 1000 equally spaced point grid. In all cases, the behavior of the dynamics in the long run does not depend on the initial conditions.

First I analyze how the trend parameter θ affects the dynamics of the system. As a reminder, θ determines how traders using chartist forecasting form their beliefs. $\theta = 1$ is equivalent to extrapolating the trend perfectly, believing that the increase (or decrease) in the exchange rate for the next period will be the same as the last one observed. $\theta = 0$ is equivalent to believing that there will be no further changes in the exchange rate. When $\theta = -1$ chartist traders believe that the exchange rate will go back to its previous level. Figure 3.1 shows the bifurcation diagrams for the exchange rate deviations from its fundamental and the

that are near each other at a point in time will be arbitrarily far apart in later periods. The only restriction is that the state of the system remain on the attractor. Strange attractors are also unique in that they never close on themselves, the motion of the system never repeats (non-periodic). The motion we are describing on these strange attractors is what we mean by chaotic behavior.

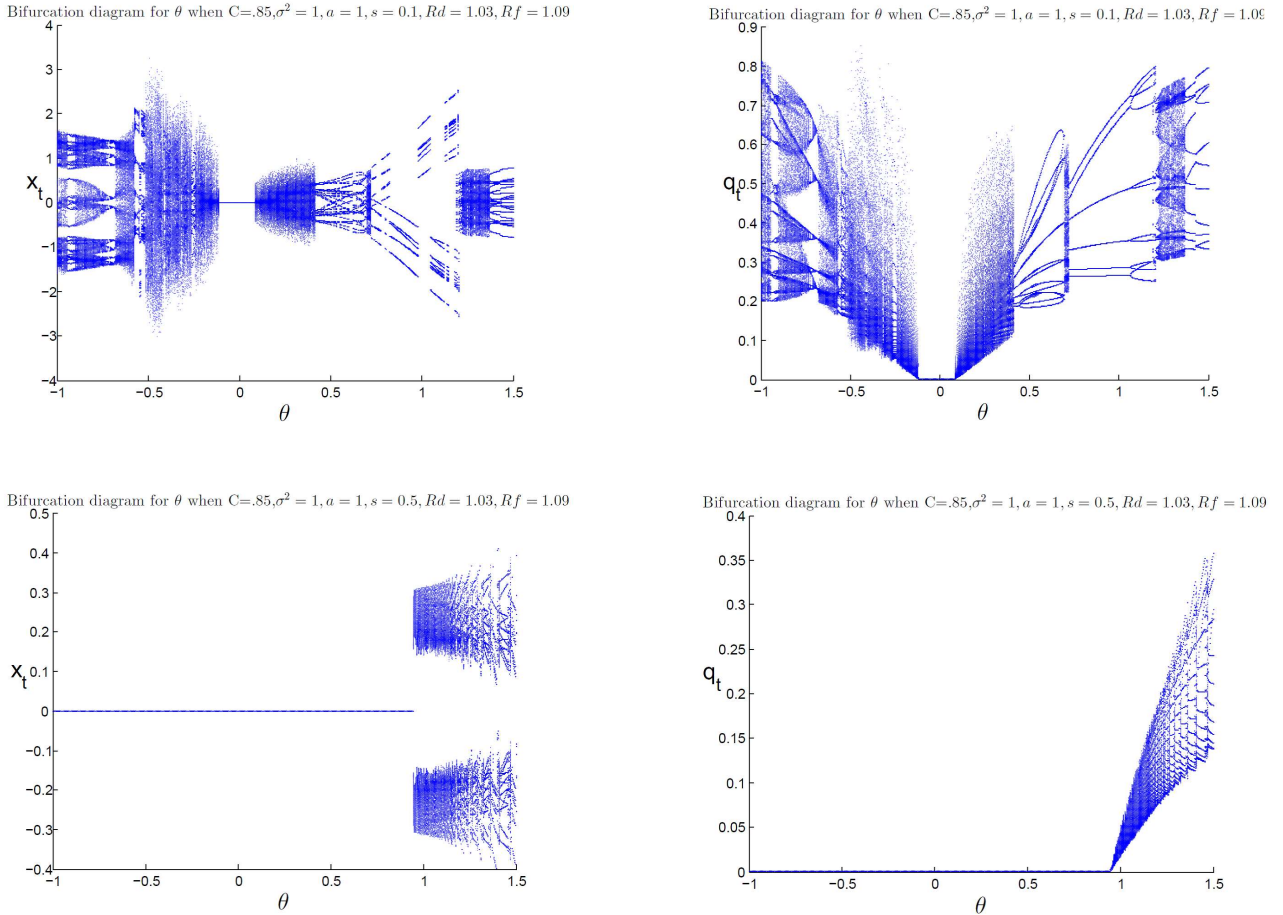
proportion of naive traders using rational expectations.

Figure 3.1: Bifurcation Diagrams for θ



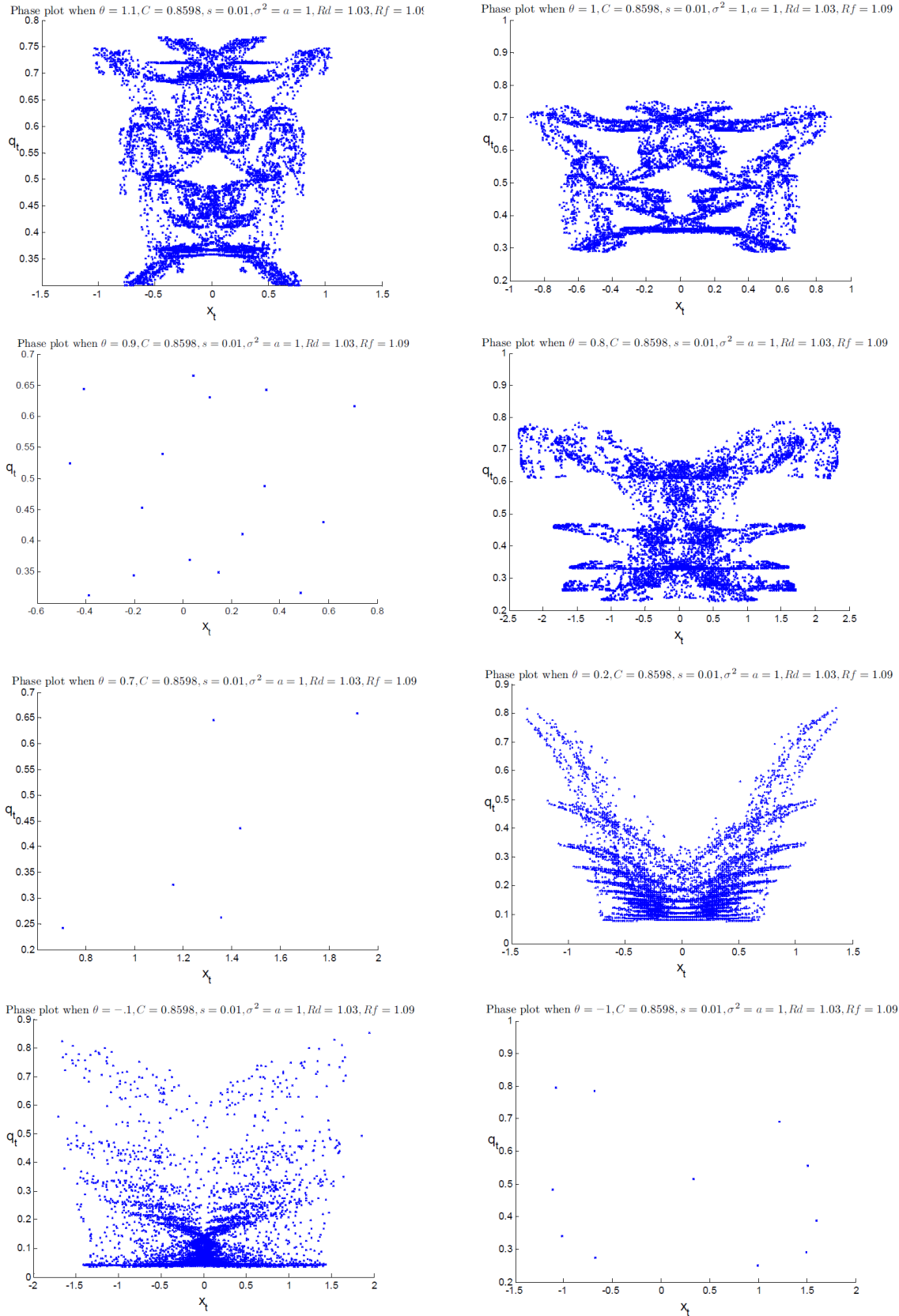
We can observe that depending on the value of θ , the convergence set switches from periodic orbits to dense sets and convergence to the steady state $(0,0)$ is achieved only in a very small interval of θ . As shown in the previous section, the range of values for θ that induce local stability in the steady state depends on the parameter s , a larger size of sophisticated traders can support a wider range of values of θ that create local stability in the system. Figure 3.2 shows the same bifurcation diagrams for $s = 0.1$ on the top and $s = 0.5$ on the bottom.

Figure 3.2: Bifurcation Diagrams for θ



We can observe how as the proportion of sophisticated trades increases, the range of values of θ for which the dynamical system converges to the steady state expands. When the proportion of sophisticated traders is half of the total trader population the range of values for θ that induce local stability is quite large, it would require very extreme beliefs to not have a locally stable steady state. These bifurcation diagrams suggests the occurrence of an infinite succession of period-doubling bifurcations with periods of stable and unstable cycles. To appreciate how drastically the parameter θ changes the long run behavior of the system, next I present the phase plots corresponding to different values of the parameter. Figure 3.3 shows the phase plots for $\theta = 1.1, 1, 0.9, 0.8, 0.7, 0.2, -0.1, -1$. Each panel shows trajectories

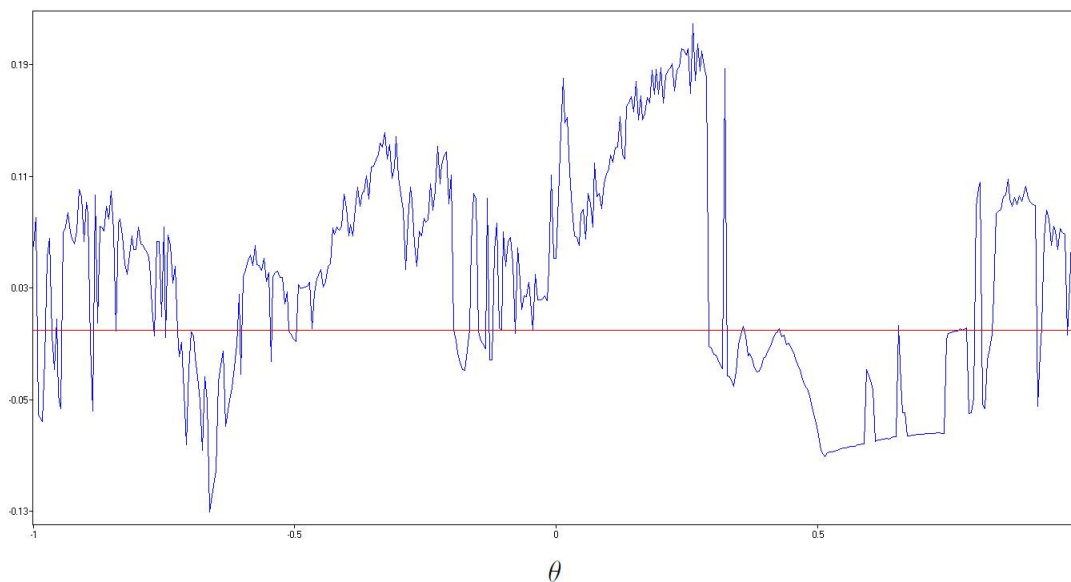
Figure 3.3: Phase Plots



of length 15000 for the dynamical system after a period of 30000 simulations.

We can see how different beliefs about the trend of the exchange rate change the converging set of the dynamical system. The parameter θ generates transitions from periodic to aperiodic cycles, small changes in something as subjective as the beliefs have drastic changes in the converging set. The presence of strange attractors makes prediction even harder, even if the trend parameter was not changing, in the presence of a strange attractor, predictions about the future are invalid if the current position in the system is not known with infinite precision. Although the phase plots present dense sets with an apparently fractal structure, it is necessary to test chaos in a formal way by computing the largest Lyapunov exponents for different values of θ . The numerical definition of chaos tells us that a dynamical system is called chaotic if there exists a set of initial states of positive Lebesgue measure, such that the Lyapunov exponent is strictly positive. Figure 3.4 presents the Largest Lyapunov exponents¹⁰ for different values of θ .

Figure 3.4: Largest Lyapunov Exponent for Different values of θ



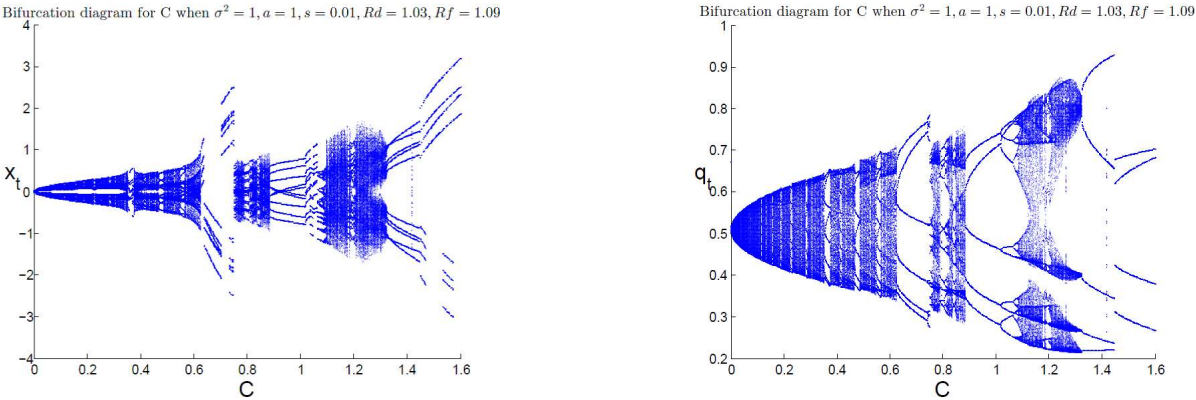
We can see that for many values of θ the Lyapunov exponent is positive, indicating

¹⁰All the computations of the largest Lyapunov exponent were performed using the E&F Chaos Software package available at <http://www1.fee.uva.nl/cendef/>.

the presence of chaos. This graph shows infinitely many spikes, indicating changes in the converging set due to any variation in the parameter. At bifurcations of cycles, the Lyapunov exponent touches zero, for example, when period-doubling bifurcations occur from a stable 2-period cycle to a stable 4-period cycle, a stable 8-period cycle, etc. The switching from negative to positive indicates the transition from a periodic cycle to a non-periodic cycle. This is perhaps the most significant result in the model, the presence of chaos and how small changes in something as subjective, volatile and immeasurable as the beliefs about the trend the exchange rate will follow could change entirely the nature of the long-run behavior of the system. Even in θ remained fixed, making predictions about future exchange rates would be extremely hard in the presence of chaos, if we add the fact that θ is immeasurable and could be constantly changing, the task of prediction is impossible.

The trend parameter of chartist forecast is not the only parameter that has great implications for the long-run behavior of the system. The cost naive traders must pay to use rational expectations is also very important for the long run behavior of the system. Figure 3.5 shows the bifurcation diagrams for C .

Figure 3.5: Bifurcation Diagrams for C



This figure shows the long-run behavior of the system for different values of C , the cost of using rational expectations for naive traders. We can appreciate that for this particular

parameter setting there are no stable steady states for positive values of C , and the system switches between cycles of different periods and aperiodic sets as well. In the aperiodic cases, the orbits create irregular dense sets that are typical in strange attractors. Figure 3.7 illustrates this situation and presents evidence of the existence of strange attractors in the system. Each of the panels in figure 3.7 shows trajectories of length 15,000 for the dynamical system after a period of 30,000 simulations with different values of C . The parameter configuration of each diagram is the same except for the value of C .

Panel a) Shows a strange attractor, b) presents the convergence of the system to a 42 period cycle, c) presents a 84 period cycle, d) presents a 336 period cycle, e) shows convergence to a 7,645 period cycle and f) shows convergence to a strange attractor. It is remarkable how the converging set is transformed so abruptly by changes of the magnitude of a few hundred thousandths in the parameter. Very small variations in C induce the system to converge to different periodic sets and strange attractors, the slightest variation in C could change entirely the long-run behavior of the system making it very hard to make any inference. As shown in figure 3.6, even for a fixed value of C , the system could present chaotic dynamics. Figure 3.6 show the largest Lyapunov exponent for different values of C .

Figure 3.6: Largest Lyapunov Exponent for Different Values of C

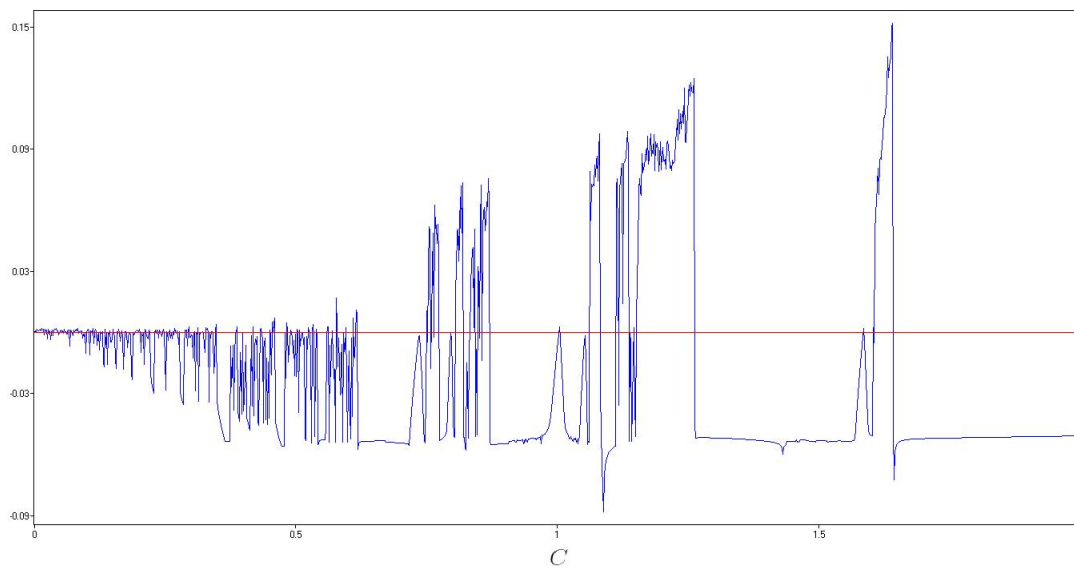
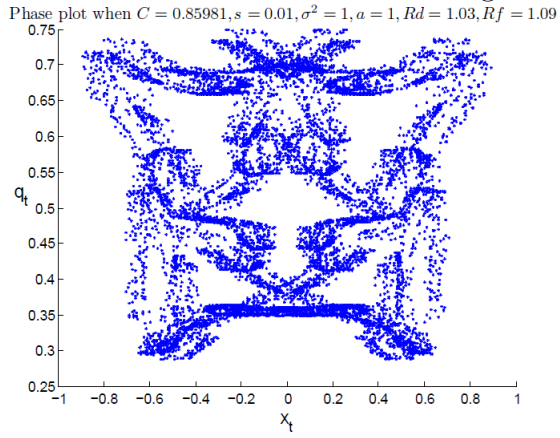
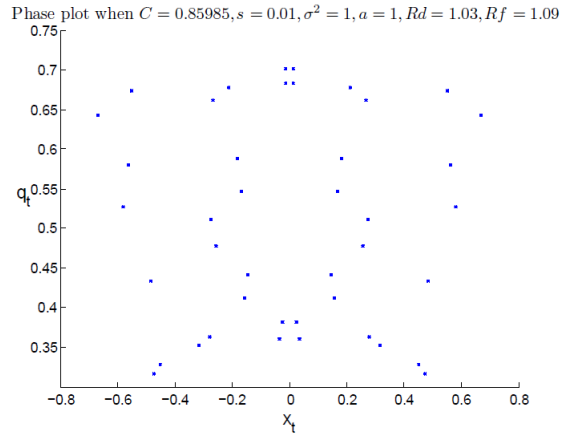


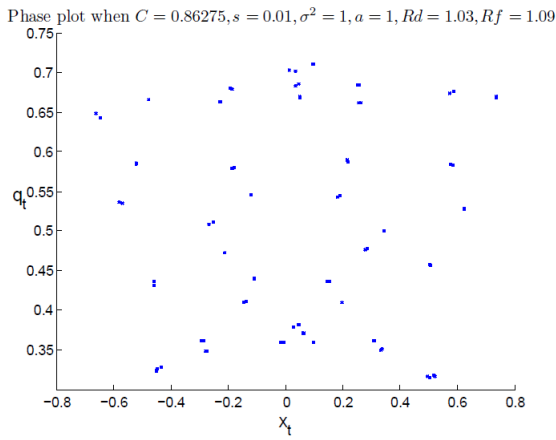
Figure 3.7: Phase Plots



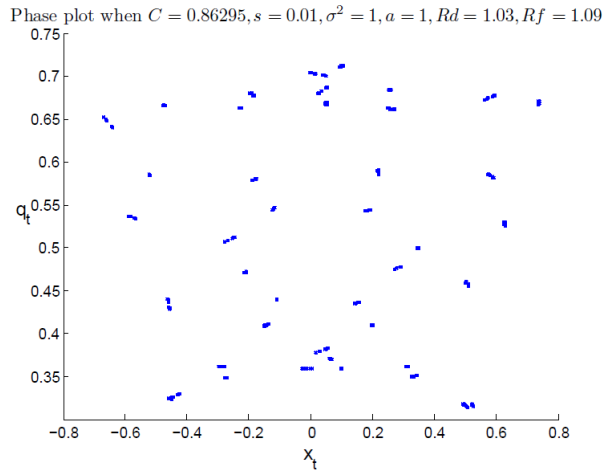
a)



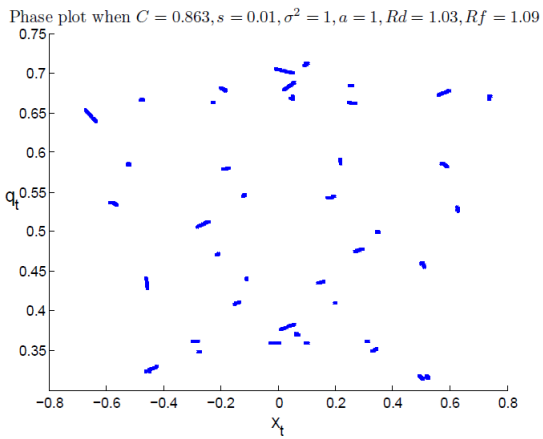
b)



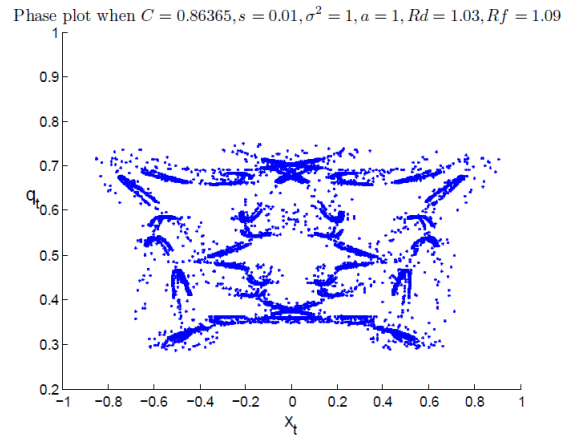
c)



d)



e)



f)

The Lyapunov exponent is positive for some values of C , indicating the presence of chaos. I have shown that a configuration of parameters that makes the system chaotic can easily arise. Switching between periodic cycles and strange attractors can be caused by a very slight variation in the parameters even if everything else remains constant. Without the exact knowledge of the value of each one of the parameters of the system, any accurate prediction about the future is impossible. For completeness, I show the bifurcation diagrams for the proportion of sophisticated traders and the variance of the expected return of foreign investment over domestic investment.

Figure 3.8: Bifurcation Diagrams for s

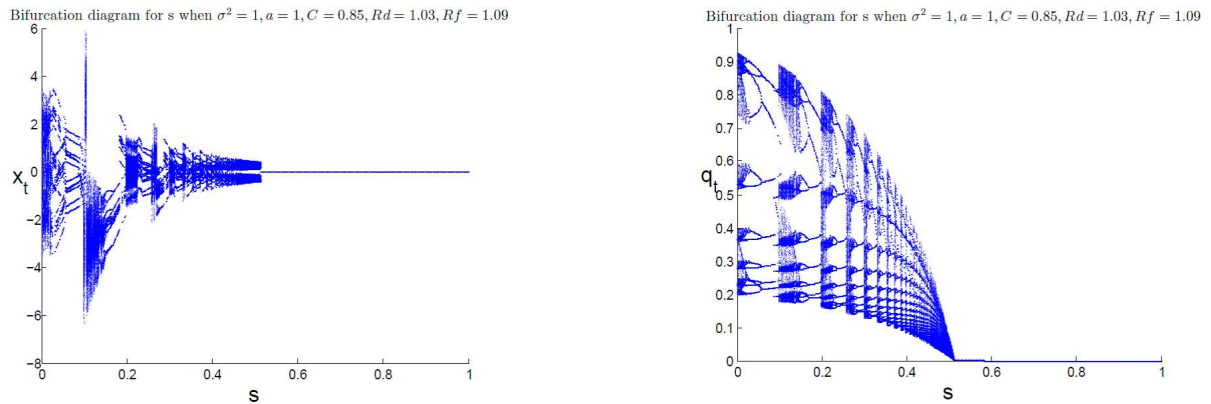
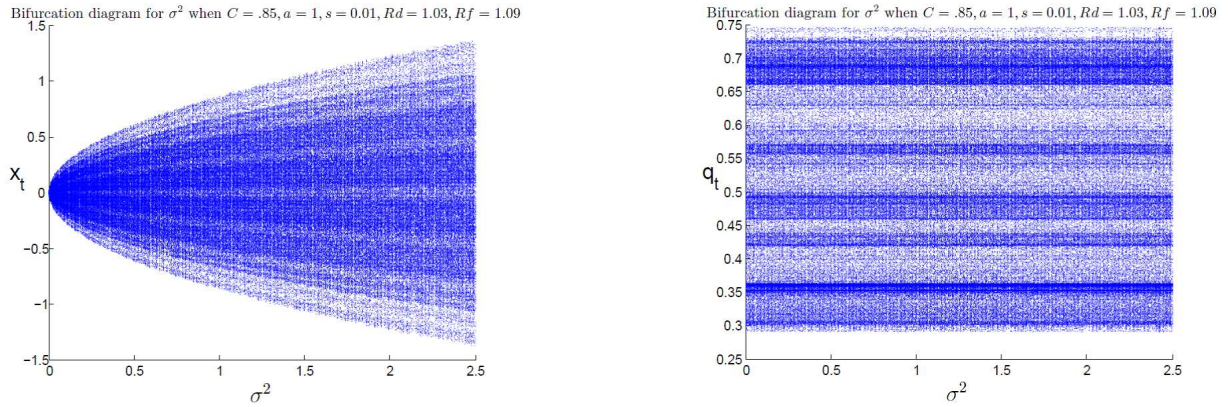


Figure 3.8 shows the bifurcation diagram for parameter s , the proportion of sophisticated traders. As previously observed, this parameter plays an important role for the stability of a steady state and as expected, the steady state $(x, q) = (0, 0)$ is locally stable only if the proportion of sophisticated traders is unrealistically large, for this particular configuration of parameters more than half of the traders would need to be sophisticated to induce stability in the system, a contradiction to the initial assumption of the model that they constitute a minority. Figure 3.9 shows the bifurcation diagram for the variance of the excess return of foreign investment over domestic investment.

We can observe that larger values of these parameters have an amplifying effect on the

Figure 3.9: Bifurcation Diagrams for σ^2



range of the values of the converging orbit however, it does not change its nature. The previous analysis proves that a very simple model for foreign currency can easily generate very complicated dynamics. Complex dynamics of this sort could be behind some of the empirical puzzles in exchange rate behavior. For example, the disconnect puzzle that refers to the fact that the market deviates substantially and for relatively long periods of time from its fundamental value. Results show that such disconnections are a natural outcome of the nonlinear dynamics introduced by heterogeneous beliefs. It is the inherent dynamics of the market what creates these fluctuations, a feature that makes it unnecessary to invoke exogenous events and random components to explain why exchange rates deviate from their fundamental values. Another empirical puzzle that could easily be caused by these dynamics are the structural breaks observed in the data, situations usually explained in the literature by policy changes. In our model these changes can be explained by very small variations in the parameter values. Figure 3.7 shows how slight movements in the cost of using rational expectations can change the converging orbit of the system entirely, if a mechanism similar in nature was at work in the market then we could expect to see this structural breaks whenever the cost of using a forecasting strategy or some other parameter changed. Thus, in a nonlinear world, structural breaks in the link between the exchange rate and its fundamentals could

occur frequently even without changes in the policy regime. In summary, the movements of the exchange rate are likely to be driven by a nonlinear speculative dynamics that makes it difficult or near to impossible to explain their behavior.

3.6 Conclusions

Motivated by the challenges traditional foreign exchange models face when explaining the anomalies and puzzles observed in reality, this chapter presented a model featuring nonlinear dynamics caused by heterogeneity in the traders' beliefs. It follows a series of studies (De Grauwe and Grimaldi (2005) and Chiarella et al. (2007)) that investigate the idea of heterogeneous beliefs among traders as the source of the complexity in the behavior of the exchange rate.

The model emphasizes the traders' utility maximizing behavior as the source of the complexity in the market. Namely, it makes use of the BNN dynamic to model the proportion of traders using a particular strategy to predict future exchange rates, a feature absent in previous models. The BNN dynamic has appealing properties that make it desirable to model macroeconomic phenomena. Besides tractability, the dynamic is continuous in the payoffs and has the properties of positive correlation, existence, uniqueness, and continuity of solutions, and Nash stationary.

The model departs from the fundamentalist versus chartist approach and instead it considers rational expectations versus chartist forecasting with different costs. A key aspect of the model is the existence of two different kinds of traders, sophisticated traders, who don't face a cost for rational expectations, and naive traders, who face a cost for using rational expectations. The model is capable of generating very complex and even chaotic dynamics that resemble the data while remaining tractable and parsimonious.

I have showed that the existence of steady states in the system depends on the interest rate differential and on the cost of using rational expectations. For a realistic configuration of

the model, i.e. when there exist an interest rate differential and using rational expectations is more costly than using a simple trend forecasting tool, there is only one steady state. This unique steady state has the currency exchange rate being equal to its fundamental value and all naive traders using chartist forecasting. The analytical results show that the local stability of this steady state depends on the proportion of sophisticated traders, it requires an unrealistically large proportion to make it locally stable. These results suggest that instability is related to the cost of gathering information to make accurate predictions. If information about the state of the market was more accessible to traders, the unpredictability of the dynamics could be mitigated. The structure of the foreign exchange market makes it hard to consider that such access to information could have place in reality.

Simulations of the model are presented to analyze the long run behavior of the system for different parameter values. Phase plots and bifurcation diagrams demonstrate the complexity of the system and, under certain parameter values, the existence of strange attractors where the paths of the dynamical system form irregular dense sets, strong evidence of the presence of chaos. The presence of chaos is confirmed by the computation of the largest Lyapunov exponent for different parameters. This study shows that in a simple setting and under fairly general conditions, chaotic dynamics in the exchange rate can arise as a result of the interaction of traders with different beliefs. The fact that chaos could be an intrinsic feature of the market has important implications for the way we think about models predicting the exchange rate behavior.

Models presenting complex behavior are good candidates for explaining exchange rate fluctuations. The analysis of the interactions and changes between heterogeneous strategies is a field that offers rich possibilities for future work although the implications of complex dynamics of this sort driving the exchange rate are not small. Chaos would imply that it is very difficult, perhaps even impossible, to find a pattern in the movement of exchange rates linked to the fundamentals. Even in the absence of random shocks, the slightest miscalculation in the parameters could still result in dramatic imprecisions in long-term

forecasts.

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Appendix A

Proof of Proposition 1

First, it is necessary to prove that when recruiting costs go to zero, efficiency wages are paid. From the VSC (1.8) we get an inverse relationship between equilibrium market tightness θ^* and recruitment costs γ . As recruitment costs go to zero, market tightness goes to infinity. Accordingly to (1.13), for very large values of θ , efficiency wages must be paid.

With efficiency wages, the job creation condition (1.8) becomes

$$ay - b - e - \frac{e}{\lambda}(r + s + f(\theta)) = (r + s) \frac{\gamma}{q(\theta)}.$$

From this expression it can be verified that as $\gamma \rightarrow 0$, $\theta \rightarrow \theta^R$, where $\theta^R = f^{-1}([ay - b - e - \frac{e}{\lambda}(r + s)]/)$. In the absence of recruiting costs, θ^R is the equilibrium market tightness. Using the Beveridge curve (2.19), we have that the equilibrium employment level in the absence of recruitment costs is $n^R = \frac{f(\theta^R)}{s + f(\theta^R)} < 1$, the economy does not converge to full employment. \square

Proof of Proposition 2

$$\frac{\partial S^R}{\partial ay} = \frac{\frac{\partial u^R}{\partial ay} u^F - \frac{\partial u^F}{\partial ay} u^R}{(u^R + u^F)^2} < 0, \quad \Leftrightarrow \quad \frac{\partial u^R}{\partial ay} u^F < \frac{\partial u^F}{\partial ay} u^R \Leftrightarrow \frac{\partial u^R}{\partial ay} \frac{ay}{u^R} < \frac{\partial u^F}{\partial ay} \frac{ay}{u^F}$$

. \square

Proof of Proposition 3

Proposition 3 follows from the next theorem:

Theorem 1. *Let $\eta(\theta) \in [0, 1]$ be the elasticity of $f(\theta)$. A sufficient condition to have a countercyclical share of job rationing, i.e. $\frac{\partial S^R}{\partial ay} < 0$, is*

$$\eta(\theta) < \frac{s + f(\theta)}{s + 2f(\theta)}. \quad (23)$$

Proof. Taking all the parameters as fixed, the equilibrium market tightness is given by the piecewise implicitly defined function:

$$\theta^*(ay) = \begin{cases} ay - b - e - \frac{e}{\lambda}(r + s + f(\theta)) - \frac{\gamma(s+r)}{q(\theta)} = 0, & E - U = \frac{e}{\lambda}, \\ \frac{(1-\beta)(ay-b-e)(r+s)}{r+s+\beta f(\theta)} - \frac{\gamma(s+r)}{q(\theta)} = 0, & E - U < \frac{e}{\lambda}. \end{cases}$$

The reader can verify that this is a continuous function. This is a piecewise differentiable function and the derivative is given by

$$\frac{\partial \theta^*}{\partial ay} = \begin{cases} \frac{1}{\frac{e}{\lambda} f'(\theta) - \frac{\gamma(r+s)}{q(\theta)^2} q'(\theta)}, & E - U = \frac{e}{\lambda}, \\ \frac{(1-\beta)}{\frac{(1-\beta)(ay-b-e)\beta f'(\theta)}{(r+s+\beta f(\theta))} - \frac{\gamma}{q(\theta)^2} q'(\theta)}, & E - U > \frac{e}{\lambda}. \end{cases}$$

Rationing share of unemployment can be expressed as:

$$S^R = \frac{1 - n^R}{1 - n(\theta^*)}. \quad (24)$$

Let $n^* \equiv n(\theta^*)$. Taking the derivative of (32):

$$\frac{\partial S^R}{\partial ay} = \frac{(1 - n^*)(-\frac{\partial n^R}{\partial a}) - (1 - n^R)(-\frac{\partial n^*}{\partial ay})}{(1 - n^*)^2}. \quad (25)$$

Using $n(\theta^*) = f(\theta^*)/(s + f(\theta^*))$, and n^R we get the expressions:

$$\frac{\partial n(\theta^*)}{\partial ay} = \frac{f'(\theta)s}{(s + f(\theta))^2} \frac{\partial \theta^*}{\partial ay} = (1 - n^*(\theta)) \frac{f'(\theta)}{s + f(\theta)} \frac{\partial \theta}{\partial ay},$$

$$\frac{\partial n^R}{\partial ay} = \frac{(e/\lambda)s}{(ay - b - e - (e/\lambda)r)^2} = \frac{1 - n^R}{(ay - b - e - (e/\lambda)r)}.$$

Substituting these derivatives into (25), we get

$$\frac{\partial S^R}{\partial ay} = \frac{(1 - n^R)}{(1 - n^*)} \left[\frac{f'(\theta)}{(s + f(\theta))} \frac{\partial \theta}{\partial ay} - \frac{1}{(ay - b - e - (e/\lambda)r)} \right].$$

This expression is negative if and only if

$$f'(\theta) \frac{\partial \theta^*}{\partial ay} \left[ay - b - e - \frac{e}{\lambda} r \right] - (s + f(\theta)) < 0. \quad (26)$$

Depending on whether the NSC is binding or not, θ^* is given by different equations. First consider the case where the NSC is binding, $E - U = \frac{e}{\lambda}$. In this case we have

$$f'(\theta) \frac{\partial \theta^*}{\partial ay} = \frac{e}{\lambda} - \frac{\gamma(r + s)q'(\theta)}{q(\theta)^2 f'(\theta)}. \quad (27)$$

At equilibrium $\frac{\gamma(r+s)q'(\theta)}{q(\theta)^2} = ay - b - e - \frac{e}{\lambda}(r + s + f(\theta^*))$, we also have $\frac{q'(\theta)}{q(\theta)f'(\theta)} = \frac{1-\eta(\theta)}{\eta(\theta)f(\theta)}$, substituting into (27) we have that (26) is satisfied if and only if

$$\frac{(ay - b - e - \frac{e}{\lambda}r)\eta(\theta)f(\theta)}{\frac{e}{\lambda}\eta(\theta)f(\theta) + (ay - b - e - \frac{e}{\lambda}(r + s + f(\theta^*))(1 - \eta(\theta)))} - (s + f(\theta)) < 0.$$

After some manipulation this expression becomes

$$\frac{(ay - b - e - \frac{e}{\lambda}(r + s + f(\theta^*)))(\eta(\theta)f(\theta) - (1 - \eta(\theta))(s + f(\theta)))}{\frac{e}{\lambda}\eta(\theta)f(\theta) + (ay - b - e - \frac{e}{\lambda}(r + s + f(\theta^*))(1 - \eta(\theta)))} < 0,$$

further manipulation gives

$$\eta(\theta) < \frac{s + f(\theta)}{s + 2f(\theta)}.$$

Now consider the case when the NSC is not binding, $E - U > \frac{e}{\lambda}$. Making all the substitutions and using the fact that at an equilibrium $\frac{(1-\beta)(ay-b-e)(r+s)}{r+s+\beta f(\theta)} = \frac{\gamma(s+r)}{q(\theta)}$ we can express (26) as

$$\frac{(r + s)(ay - b - e - \frac{e}{\lambda}r)(r + s + \beta f(\theta))\eta f(\theta)}{(ay - b - e) [\beta \eta f(\theta) + (r + s)(1 - \eta)(r + s + \beta f(\theta))]} - (s + f(\theta)) < 0,$$

after some manipulation the expression becomes

$$(ay-b-e) [(r+s)(r+s+\beta f(\theta))(\eta f(\theta) - (s+f(\theta))(1-\eta)) - (s+f(\theta))\beta\eta] - \frac{e}{\lambda} r\eta f(\theta)(r+s+\beta f(\theta)) < 0. \quad (28)$$

The only term with undefined sign is $(r+s)(r+s+\beta f(\theta))(\eta f(\theta) - (s+f(\theta))(1-\eta))$, the rest are negative, so a sufficient condition to have the whole expression being negative is

$$\eta f(\theta) - (s+f(\theta))(1-\eta) < 0,$$

or differently expressed:

$$\eta(\theta) < \frac{s+f(\theta)}{s+2f(\theta)}.$$

This is a sufficient condition to have the derivative being negative regardless of whether the NSC is binding or not.

□

Theorem 1 gives sufficient conditions for the share of job rationing to be countercyclical without any assumption about the specific form of the matching function or any specific parameter values. The proof of Proposition 3 follows from theorem 1. By assumption the matching function takes the form $h(v, u) = \mu v^{1-\alpha} u^\alpha$. With this specification the elasticity of the instant probability of finding a job is constant and equal to $\eta(\theta) = 1 - \alpha$. Condition (23) Takes the form:

$$1 - \alpha < \frac{s + \mu\theta^{1-\alpha}}{s + 2\mu\theta^{1-\alpha}}. \quad (29)$$

- a) If $\alpha > \frac{1}{2}$, condition (29) always holds.
- b) If $\alpha \in (0, \frac{1}{2})$ condition (29) depends on the equilibrium market tightness which depends on the rest of the parameters.
- c) If $\alpha = 0$, condition (29) never holds. If the NSC is binding, this is a sufficient condition

to have $\frac{\partial S^R}{\partial ay} > 0$. If the NSC does not bind then eq.(28) is always positive so $\frac{\partial S^R}{\partial ay} > 0$. \square

Proof of Proposition 4

Consider the firms' hiring decision in an undirected market with workers with heterogeneous skills. I only consider symmetric Nash equilibria (NE). Each employer takes the strategy of others employers as given and chooses a probability $\Pi(y)$ of recruiting the worker with idiosyncratic productivity y in order to maximize his expected profits. Because in equilibrium the value of a vacancy is zero ($V = 0$), the best response function of an employer satisfies the following rule:

$$J_y > 0 \implies \Pi(y) = 1$$

$$J_y < 0 \implies \Pi(y) = 0$$

$$J_y = 0 \implies \Pi(y) \in [0, 1].$$

The employer accepts to recruit a worker with probability one if this worker generates positive profits for the firm, if the profits are negative the firm never hires the worker and if it makes no profits the the firm is indifferent between hiring or not.

Lemma 2. *A match will never form if the NSC is not satisfied.*

Proof: Assume the NSC is not satisfied. Then, by assumption $ay = 0$ which implies that the value of a match is $J = -\frac{w}{(r+s)}$. A firm will only accept the match if $w = 0$. If $w < b$ the worker will not accept the match. By assumption $b > 0$ so at $w = 0$ workers will not accept the match. Since both worker and firm must accept the match, if the NSC cannot be satisfied the match will never form. \square

Lemma 3. *If $ay < y_L \equiv b + e + (r + s)\frac{e}{\lambda}$ then the NSC can never be satisfied.*

Proof: Assume that $ay < y_L$ and that the NCS is satisfied, that is:

$$E_y - U_y = \frac{w_y - e - b}{r + s + \Pi(y)f(\theta)} \geq \frac{e}{\lambda}.$$

For parameters e, b, λ, r, s , and taking θ as given. Considering the restrictions $w_y \leq ay$ and $\Pi(y) \in [0, 1]$. The largest the RHS of the inequality can be is

$$\frac{ay - e - b}{r + s} \geq \frac{e}{\lambda} \iff ay \geq e + b + \frac{e}{\lambda}(r + s).$$

By assumption $ay < e + b + \frac{e}{\lambda}(r + s)$ which contradicts the statement above. \square

With these lemmas, the proof of proposition 4 follows:

1. If $ay \in C_0$, then the worker is never hired, $\Pi(y) = 0$.

Proof: If $ay \in C_0$ then $ay < y_L \equiv b + e + (r + s)\frac{e}{\lambda}$ so by lemma 5.2 the NSC cannot be satisfied. By lemma 5.1 a match will never form, $\Pi(ay) = 0$ \square

2. If $ay \in C_1$, then the worker is hired with a probability $\Pi(y) = (ay - b - e - (r + s)\frac{e}{\lambda}) / f(\theta)\frac{e}{\lambda}$, and is paid $w = ay$.

Proof: We must prove that to have a match forming it must be the case that $w = ay$ and $\Pi(y) = (ay - b - e - (r + s)\frac{e}{\lambda}) / f(\theta)\frac{e}{\lambda}$.

Assume that $w < ay$, this would imply $J_y > 0$, so $\Pi(y) = 1$. Then

$$E_y - U_y = \frac{w - e - b}{r + s + f(\theta)} < \frac{ay - e - b}{r + s + f(\theta)} \leq \frac{y_M - e - b}{r + s + f(\theta)} = \frac{b + e + (r + s + f(\theta))\frac{e}{\lambda} - e - b}{r + s + f(\theta)} = \frac{e}{\lambda}$$

The NSC is violated.

If $w > ay$ then $J_y < 0$, matches will never form. This proves that if a match must form with $ay \in C_1$ then it must be that $w = ay$.

The probability of hiring a worker with this productivity is the result of a symmetric Nash Equilibrium. Firms know what the average probability of hiring this worker is and each firm takes it as given. Notice that with $w = ay$ and $\Pi(y) = (ay - b - e - (r + s)\frac{e}{\lambda}) / f(\theta)\frac{e}{\lambda}$ we have $E_y - U_y = \frac{e}{\lambda}$. If $\Pi(y) > (ay - b - e - (r + s)\frac{e}{\lambda}) / f(\theta)\frac{e}{\lambda}$ then a firm coming into contact with the worker would not hire him because the NSC is violated and cannot encourage him by paying a higher wage. If $\Pi(y) < (ay - b - e - (r + s)\frac{e}{\lambda}) / f(\theta)\frac{e}{\lambda}$ then the firm could pay

a worker $w = ay - \epsilon$ to make a profit and still not violate the NSC, but since every firm would do the same then $\Pi(y) = 1$, so the NSC could not be satisfied.

We can verify that this is indeed a probability, that is $\Pi(y) = (ay - b - e - (r + s)\frac{e}{\lambda}) / f(\theta)\frac{e}{\lambda} \in [0, 1]$ by observing that it is the solution to the equation $ay_i = (1 - x)y_L + xy_M$. And by assumption $ay_i \in C_1 \equiv (y_L, y_M]$. \square

3. If $ay \in C_2$, then the worker is always hired, $\Pi(y) = 1$, and is paid efficiency wages, $w = w_E$.

Proof: If $ay \in C_2$, then under Nash-Bargaining wages the NSC is violated since $w_{NB} > w_E$. The minimum wage that firms will pay to encourage workers is the efficiency wage. Since $ay > w_E$ we have $J_y > 0$ so $\Pi(y) = 1$. \square

4. If $ay \in C_3$, then the worker is always hired, $\Pi(y) = 1$, and is paid Nash-Bargaining wages, $w = w_{NB}$.

Proof: If $ay \in C_3$, then $w_{NB} > w_E$, the NSC is always satisfied, since under Nash Bargaining the profit for a firm is always positive then $\Pi(y) = 1$. \square

Proof of Proposition 5

Making use of (1.19) and implicit differentiation we can compute :

$$W_i \equiv \frac{\partial\theta/\partial ay_i}{p_i} = \frac{\frac{\gamma(r+s)}{q(\theta)} \frac{(s+f(\theta))}{s} \left[\frac{s(e/\lambda)}{(ay_i - b - e - r(e/\lambda))^2} \right]}{d\theta} \quad \forall ay_i \in C_1$$

$$W_j \equiv \frac{\partial\theta/\partial ay_j}{p_j} = \frac{1}{d\theta} \quad \forall ay_j \in C_2$$

$$W_k \equiv \frac{\partial\theta/\partial ay_k}{p_k} = \frac{\frac{(1-\beta)(r+s)}{r+s+\beta f(\theta)}}{d\theta} \quad \forall ay_k \in C_3$$

where,

$$d\theta = \sum_{C_2} p_k \frac{e}{\lambda} f' + \sum_{C_3} p_j \frac{(1-\beta)(ay_j - b - e)(r+s)\beta f'}{(r+s+\beta f(\theta))^2} + G(\theta)$$

and,

$$G(\theta) \equiv \frac{\gamma(r+s)}{s} \left[\frac{f'(\theta)}{q(\theta)} \left(\sum_{C_0} p_i + \sum_{C_1} \frac{p_i s (e/\lambda)}{ay_j - b - e - r(e/\lambda)} \right) - \sum_i p_i u_i (s + f(\theta)) \left(\frac{q'(\theta)}{q(\theta)^2} \right) \right]$$

We have that $W_j > W_k \forall ay_j \in C_2, ay_k \in C_3$ if and only if

$$1 > \frac{(1-\beta)(r+s)}{r+s+\beta f(\theta)} \iff f(\theta) > -(r+s)$$

, which is always the case. Also $W_i > W_j \forall ay_1 \in C_1, ay_j \in C_2$ if and only if

$$\frac{\gamma(r+s)}{q(\theta)} > \frac{e}{\lambda} (s + f(\theta)) \tag{30}$$

From equation (1.19) we get that at an equilibrium

$$\frac{\gamma(r+s)}{q(\theta)} > \frac{e}{\lambda} \frac{(s + f(\theta))}{su}$$

since $su < 1$, this condition implies (30). \square

Proof of Proposition 6

Assume that $ay_m \notin C_0$. When $\gamma \rightarrow 0$, by (1.21) and (1.22), the equilibrium market tightness increases to the point of the RHS of the equation is close to zero. This happens at the point where $ay_m = w_e = b + e + (s + r + f(\theta)) \frac{e}{\lambda}$. By definition 3, this implies that any hireable worker with productivity ay_i must be in Class 1. \square

The No-Shirking Condition (NSC)

The expected lifetime utility of someone who chooses to shirk (S) during a length of time dt , satisfies

$$S = wdt + \exp(-rdt) \{ \Pr [\min(\tau_s, \tau_\lambda) \leq dt] U + (1 - \Pr [\min(\tau_s, \tau_\lambda) \leq dt]) E \}.$$

Where $\min(\tau_s, \tau_\lambda)$ is a Poisson process with parameter $\lambda + s$ which yields

$$S = wdt + \exp(-rdt) \{ (1 - \exp(-(s + \lambda)dt)) U + (\exp(-(s + \lambda)dt)) E \}.$$

Using power series

$$S = wdt + (1 - rdt + o(dt)) \{ [(s + \lambda)dt + o(dt)] U + (1 - (s + \lambda)dt + o(dt)) E \}$$

with $\lim_{dt \rightarrow 0} o(dt)/dt = 0$. Rearranging

$$S = wdt + (1 - rdt) \{ (s + \lambda)dtU + [1 - (s + \lambda)dt] E \} + o(dt)$$

Substituting $wdt = sEdt + edt - sdt(U - E)$.

$$S = E + edt - \lambda dt(E - U) - rdt^2(s + \lambda)(E - U) + o(dt),$$

. When dt approaches zero, the worker's optimal strategy is not to shirk if and only if

$$S - E \simeq [e - \lambda(E - U)] dt \leq 0$$

\Leftrightarrow

$$E - U \geq \frac{e}{\lambda}.$$

Table A1: Outcomes by deciles for different technology levels

a=.8	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Average Unemployment Rate	0.253	0.162	0.119	0.095	0.086	0.086	0.086	0.086	0.086	0.086
Share of pool of unemployment	0.221	0.142	0.105	0.083	0.075	0.075	0.075	0.075	0.075	0.075
Job Rationing Share	1.000	1.000	1.000	1.000	0.915	0.781	0.682	0.604	0.543	0.493
Average Productivity	0.519	0.582	0.644	0.706	0.769	0.831	0.894	0.956	1.018	1.081
Average Wage	0.519	0.582	0.644	0.706	0.750	0.806	0.864	0.923	0.982	1.040
% in C_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
% in C_1	1.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
% in C_2	0.000	0.000	0.000	0.000	0.350	0.000	0.000	0.000	0.000	0.000
% in C_3	0.000	0.000	0.000	0.000	0.650	1.000	1.000	1.000	1.000	1.000

Table A2: Outcomes by deciles

a=1	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Average Unemployment Rate	0.117	0.089	0.071	0.066	0.066	0.066	0.066	0.066	0.066	0.066
Share of pool of unemployment	0.158	0.120	0.096	0.089	0.089	0.089	0.089	0.089	0.089	0.089
Job Rationing Share	1.000	1.000	0.999	0.902	0.775	0.680	0.605	0.545	0.496	0.456
Average Productivity	0.649	0.727	0.805	0.883	0.961	1.039	1.117	1.195	1.273	1.351
Average Wage	0.649	0.727	0.804	0.861	0.935	1.009	1.083	1.158	1.232	1.306
% in C_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
% in C_1	1.000	1.000	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000
% in C_2	0.000	0.000	0.100	0.150	0.000	0.000	0.000	0.000	0.000	0.000
% in C_3	0.000	0.000	0.000	0.850	1.000	1.000	1.000	1.000	1.000	1.000

Table A3: Outcomes by deciles

a=1.2	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Average Unemployment Rate	0.077	0.061	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.066
Share of pool of unemployment	0.130	0.104	0.096	0.096	0.096	0.096	0.096	0.096	0.096	0.089
Job Rationing Share	1.000	1.000	0.907	0.777	0.680	0.604	0.544	0.494	0.453	0.456
Average Productivity	0.779	0.872	0.966	1.060	1.153	1.247	1.340	1.434	1.528	1.351
Average Wage	0.779	0.872	0.944	1.033	1.123	1.213	1.303	1.393	1.482	1.306
% in C_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
% in C_1	1.000	0.950	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
% in C_2	0.000	0.050	0.150	0.000	0.000	0.000	0.000	0.000	0.000	0.000
% in C_3	0.000	0.000	0.850	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Appendix B

Derivation of the No-Shirking Condition (NSC)

The expected lifetime utility of a worker who chooses to shirk during a length of time dt , satisfies

$$S_i = w_i dt + \exp(-r dt) \{ \Pr [\min(\tau_\delta, \tau_\lambda) \leq dt] U_i + (1 - \Pr [\min(\tau_\delta, \tau_\lambda) \leq dt]) E_i \},$$

where $\min(\tau_\delta, \tau_\lambda)$ is a Poisson process with parameter $\lambda + \delta$. This yields:

$$S_i = w_i dt + \exp(-r dt) \{ (1 - \exp(-(\delta + \lambda) dt)) U_i + (\exp(-(\delta + \lambda) dt) E_i) \}.$$

Using power series:

$$S_i = w_i dt + (1 - r dt + o(dt)) \{ [(\delta + \lambda) dt + o(dt)] U_i + (1 - (\delta + \lambda) dt + o(dt)) E_i \},$$

with $\lim_{dt \rightarrow 0} o(dt)/dt = 0$. Rearranging terms:

$$S_i = w dt + (1 - r dt) \{ (\delta + \lambda) dt U_i + [1 - (\delta + \lambda) dt] E_i \} + o(dt).$$

From (2.1), substituting :

$$S_i = E_i + e dt - \lambda dt (E_i - U_i) - r dt^2 (\delta + \lambda) (E_i - U_i) + o(dt),$$

As $dt \rightarrow 0$, the worker's optimal decision is not to shirk if and only if

$$E_i - S_i \simeq [\lambda (E_i - U_i) - e] dt \geq 0$$

\iff

$$E_i - U_i \geq \frac{e}{\lambda}.$$

Derivation of the Equilibrium Best Response Hiring Function

First is necessary to derive two lemmas.

Lemma 4. *A match will never form if the NSC is not satisfied.*

Proof: Assume the NSC is not satisfied. Then, the worker's optimal behavior is to shirk, so nothing is produced. This implies that value of a match is $J = -w_i/(r + \delta)$, a firm will only accept the match if $w_i = 0$. If $w_i < b$ the worker will not accept the match. By assumption $b > 0$ so with $w_i = 0$ workers will not accept the match. Since both worker and firm must accept the match, if the NSC cannot be satisfied, the match will never form.

Lemma 5. *If $y_i < y_L \equiv b + e - c(s_i) + (r + \delta)\frac{e}{\lambda}$, then the NSC can never be satisfied.*

Proof. Assume that $y_i < b + e - c(s_i) + (r + \delta)\frac{e}{\lambda}$ and that the NCS is satisfied, that is:

$$E_i - U_i = \frac{w_i - b - e + c(s_i)}{r + \delta + s_i \Pi_i f(\theta)} \geq \frac{e}{\lambda}.$$

Taking θ and s_i as given and considering the restrictions $w_i \leq y_i$ and $\Pi_i \in [0, 1]$, the largest $E_i - U_i$ can be is

$$\frac{y_i - e - b + c(s_i)}{r + \delta} \geq \frac{e}{\lambda} \iff y_i \geq b + e - c(s_i) + (r + \delta)\frac{e}{\lambda}.$$

By assumption $y_i < b + e - c(s_i) + (r + \delta)\frac{e}{\lambda}$, which contradicts the statement above. □

Now we must show that for a given θ and $\{s_j\}_{j=1}^n$, the firm's best-response hiring function for a type- i worker is:

$$\Pi_i = \begin{cases} 1, & y_i \geq b + e - c(s_i) + (r + \delta + s_i f(\theta)) \frac{e}{\lambda}, \\ \frac{y_i - b - e + c(s_i) - (r + \delta) \frac{e}{\lambda}}{s_i f(\theta) \frac{e}{\lambda}}, & b + e - c(s_i) + (r + \delta + s_i f(\theta)) \frac{e}{\lambda} > y_i \geq b + e - c(s_i) + (r + \delta) \frac{e}{\lambda}, \\ 0, & b + e - c(s_i) + (r + \delta) \frac{e}{\lambda} > y_i. \end{cases}$$

1. If $b + e - c(s_i) + (r + \delta) \frac{e}{\lambda} > y_i$, then $\Pi_i = 0$.

Proof: If $b + e - c(s_i) + (r + \delta) \frac{e}{\lambda} > y_i$, by lemma 2 the NSC cannot be satisfied. By lemma 1 a match will never form, so $\Pi_i = 0$. This is also a Nash equilibrium since no firm has incentive to deviate from this probability. If they hired a worker with some $\Pi_i > 0$, they would be strictly worse off since the NSC cannot be satisfied. \square

2. If $y_i \geq b + e - c(s_i) + (r + \delta + s_i f(\theta)) \frac{e}{\lambda}$ then $\Pi_i = 1$.

Proof: Given the assumption $V = 0$, according to (2.10), if $J_i > 0$ then $\Pi_i = 1$. By (2.9) $J_i > 0$ if and only if $y_i - w_i \geq 0$. From (2.15), there are two scenarios where this can happen:

a) If $w_i = w_i^N$, from the Nash-bargaining solution : $y_i - w_i^N \geq 0$. So from (2.15) if $y_i > b + e - c(s_i) + (r + \delta + s_i f(\theta) \beta) \frac{e}{\lambda \beta}$, then $\Pi_i = 1$.

b) If $w_i = w_i^E$, $y_i - w_i^E > 0$ if and only if $y_i > b + e - c(s_i) + \frac{e}{\lambda} (r + \delta + s_i \Pi_i f(\theta))$. So if $y_i > b + e - c(s_i) + \frac{e}{\lambda} (r + \delta + s_i f(\theta))$, then $\Pi_i = 1$.

Since $b + e - c(s_i) + (r + \delta + s_i f(\theta) \beta) \frac{e}{\lambda \beta} > b + e - c(s_i) + \frac{e}{\lambda} (r + \delta + s_i f(\theta))$, if $y_i \geq b + e - c(s_i) + (r + \delta + s_i f(\theta)) \frac{e}{\lambda}$ then $\Pi_i = 1$. This is a Nash equilibrium since, upon being matched with a worker with a productivity above this threshold, any firm will be strictly better off hiring the worker, no firm has incentives to deviate. \square

3. If $b + e - c(s_i) + (r + \delta + s_i f(\theta)) \frac{e}{\lambda} > y_i \geq b + e - c(s_i) + (r + \delta)$ then $\Pi_i = [y_i - b - e + c(s_i) - (r + \delta) \frac{e}{\lambda}] / s_i f(\theta) \frac{e}{\lambda}$.

Proof: According to (2.10) if $J_i = 0$, any $\Pi_i \in [0, 1]$ is a best response. If $J_i = 0$ then $y_i = w_i$ and according to (2.15) this is only possible if $w_i = w_i^E$, so $y_i = b + e - c(s_i) + \frac{e}{\lambda} (r + \delta + s_i \Pi_i f(\theta))$. With $\Pi_i \in [0, 1]$ there is a range of productivities that make

$J_i = 0$. This range is $y_i \in [b + e - c(s_i) + \frac{e}{\lambda}(r + \delta), b + e - c(s_i) + \frac{e}{\lambda}(r + \delta + s_i\Pi_i f(\theta))]$. Upon contact with a worker in this range, any $\Pi_i \in [0, 1]$ is a best response but only $\Pi_i = [y_i - b - e + c(s_i) - (r + \delta)\frac{e}{\lambda}]/s_i f(\theta)\frac{e}{\lambda}$ is a symmetric Nash equilibrium. To see this, assume that upon contract with a type- i worker, all firms adopted a strategy $\Pi_i > [y_i - b - e + c(s_i) - (r + \delta)\frac{e}{\lambda}]/s_i f(\theta)\frac{e}{\lambda}$. If this was the case then $y_i < w_i^E$ so $J_i < 0$ and firms would have the incentive to deviate to $\Pi_i = 0$.

If, on the other hand, the strategy was $\Pi_i < [y_i - b - e + c(s_i) - (r + \delta)\frac{e}{\lambda}]/s_i f(\theta)\frac{e}{\lambda}$, then $y_i > w_i^E$ so $J_i > 0$ and firms would have the incentive to deviate to $\Pi_i = 1$. \square

Derivation of the Equilibrium Optimal Search Intensity

Optimal search intensity is given by function (2.17) for a given θ and Π_i . According to (2.5) and the wage schedule in (2.15), optimal participation intensity is determined by

$$c'(s_i) = \max \left\{ \Pi_i f(\theta) \left[\frac{\beta[y_i - e - b + c(s_i)]}{r + \delta + \beta s_i \Pi_i f(\theta)} \right], \Pi_i f(\theta) \frac{e}{\lambda} \right\}. \quad (31)$$

Consider a worker with productivity such that $y_i = b + e - c(s_i) + (r + \delta + s_i \Pi_i f(\theta) \beta) \frac{e}{\lambda \beta}$, according to (2.15), $w_i = w_i^E$; and according to (2.16), $\Pi_i = 1$. His optimal participation effort is given by $c'(s_i) = f(\theta) \frac{e}{\lambda}$, which I will denote as s^E . This means that any worker with productivity $y_i > b + e - c(s^E) + (r + \delta + s^E f(\theta) \beta) \frac{e}{\lambda \beta}$ will set his wage according to $w_i = w_i^N$, and $\Pi_i = 1$. He will determine his participation intensity according to

$$c'(s_i) = \frac{\beta(y_i - e - b + c(s_i))}{r + \delta + \beta s_i f(\theta)},$$

where the resulting s_i is such that $s_i > s^E$. According to (2.16) any worker with $y_i \geq b + e - c(s_i) + (r + \delta + s_i f(\theta)) \frac{e}{\lambda}$, will be hired with a probability $\Pi_i = 1$, so workers $b + e - c(s^E) + (r + \delta + s^E f(\theta) \beta) \frac{e}{\lambda \beta} > y_i \geq b + e - c(s_i) + (r + \delta + s_i f(\theta)) \frac{e}{\lambda}$, will choose optimal effort $c'(s_i) = f(\theta) \frac{e}{\lambda}$, so their participation intensity is s^E . Substituting we get the boundaries of productivity with participation intensity s^E is $y_H = b + e - c^E + (r + \delta + s^E f(\theta) \beta) \frac{e}{\lambda \beta}$, and

$$y_M = b + e - c^E + (r + \delta + s^E f(\theta)) \frac{e}{\lambda}.$$

For workers with $y_i < b + e - c(s^E) + (r + \delta + s^E f(\theta)) \frac{e}{\lambda}$, according to (2.16), $\Pi_i = [y_i - b - e + c(s_i) - (r + \delta) \frac{e}{\lambda}] / s_i f(\theta) \frac{e}{\lambda}$. Substituting this last expression in (2.16), I get that the optimal participation intensity is given by

$$c'(s_i) = \frac{y_i - e - b + c(s_i) - (r + \delta) \frac{e}{\lambda}}{s_i}.$$

The LHS of this expression is negative if $y_i < e - b + c(s_i) - (r + \delta) \frac{e}{\lambda}$, in which case optimal participation is $s_i = 0$. Substituting this value creates a lower bound for positive participation intensities at $y_i - e - b - (r + \delta) \frac{e}{\lambda}$. Any worker with a value below this threshold will not participate. \square

Proof of Proposition 7

For the proof of this proposition, some set notation is convenient. Let $\{1, \dots, n\}$ be the set of all workers with productivities $\{y_1, \dots, y_n\}$, for a given equilibrium market tightness θ , and minimum wage m , define:

- The set of barely employable workers: $L(\theta) = \{i \in \{1, \dots, n\} \mid y_M(\theta) > y_i \geq y_L\}$.
- The set of perfectly employable workers with efficiency wages: $M(\theta) = \{i \in \{1, \dots, n\} \mid y_H(\theta) > y_i \geq y_L\}$.
- The set of perfectly employable workers with Nash-bargaining wages: $H(\theta) = \{i \in \{1, \dots, n\} \mid y_i \geq y_H(\theta)\}$.
- The set of employable workers under the minimum wage m : $\Omega(m) = \{i \in \{1, \dots, n\} \mid y_i \geq m\}$.
- The set of non-employable workers under the minimum wage m : $\Omega^c(m) = \{i \in \{1, \dots, n\} \mid i \notin \Omega(m)\}$.

Equilibrium market tightness is given by the VSC (2.20), which can be expressed as:

$$K(m, \theta) \equiv \sum_i \Pi_i(m, \theta) \mu_i(m, \theta) [y_i - w_i(m, \theta)] = (r + \delta) \frac{\gamma}{q(\theta)}.$$

First, I must prove that $K(m, \theta)$ is decreasing in θ . So, I show that for any θ and θ' s.t. $\theta < \theta'$, $K(m, \theta) \geq K(m, \theta')$.

To do so, I can rewrite

$$K(m, \theta) = \sum_{i \in \Omega(m) \cap L(\theta)} \Pi_i(m, \theta) \mu_i(m, \theta) [y_i - w_i(m, \theta)] + \sum_{i \in M(\theta) \cup H(\theta)} \Pi_i(m, \theta) \mu_i(m, \theta) [y_i - w_i(m, \theta)]$$

Since $M(\theta) \cup H(\theta) = \Omega(m) \cap [M(\theta) \cup H(\theta)]$ given the assumption $m < w^E(\theta)$. Notice that according to ((2.21)), $y_i = w_i(m, \theta) \forall i \in \Omega(m) \cap L(\theta)$. Also, from (2.22) $\Pi_i(m, \theta) = 1 \forall i \in M(\theta) \cup H(\theta)$. So we can write

$$K(m, \theta) = \sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta) [y_i - w_i(m, \theta)].$$

From (2.19), (2.22), and (2.23) we have that $\sum_i p_i s_i(m, \theta) u_i(m, \theta) \geq \sum_i p_i s_i(m, \theta') u_i(m, \theta')$, and $s_i(m, \theta) u_i(m, \theta) = s_i(m, \theta') u_i(m, \theta') \forall i \in L(\theta)$, so from (2.8), $\mu_i(m, \theta) \leq \mu_i(m, \theta') \forall i \in L(\theta)$. Since $\sum_i \mu_i(m, \theta) = 1$, we have that

$$\sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta) = 1 - \sum_{i \in L(\theta)} \mu_i(m, \theta),$$

so

$$\sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta) \geq \sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta').$$

Using that $[M(\theta) \cup H(\theta)] \supseteq [M(\theta') \cup H(\theta')]$ and that according to (2.21), $y_i - w_i(m, \theta) \geq y_i - w_i(m, \theta') \forall i$, we have that

$$\sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta) \geq \sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta') \implies \sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta) \geq \sum_{i \in M(\theta') \cup H(\theta')} \mu_i(m, \theta').$$

From this last week inequality it follows that

$$\sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta)[y_i - w_i(m, \theta)] \geq \sum_{i \in M(\theta') \cup H(\theta')} \mu_i(m, \theta')[y_i - w_i(m, \theta')].$$

So $K(m, \theta) \geq K(m, \theta')$ for any θ and θ' s.t. $\theta < \theta'$.

Now I must show that for any m and m' s.t. $m < m' < w^E(\theta)$, where θ is the equilibrium market tightness under m , $K(m, \theta) \leq K(m', \theta)$.

To see this notice that according to (2.23) and (2.20) and (2.19), for any m and m' s.t. $m < m' < w^E(\theta)$, $s_i(m, \theta)u_i(m, \theta) \geq s_i(m, \theta')u_i(m, \theta') \forall i \in \Omega^c(m')$, and $s_i(m, \theta)u_i(m, \theta) = s_i(m, \theta')u_i(m, \theta') \forall i \in \Omega(m')$. So $\mu_i(m, \theta) \leq \mu_i(m', \theta) \forall i \in \Omega(m')$. Using that $w_i(m, \theta) = w_i(m', \theta), \forall i \in M(\theta) \cup H(\theta)$ and $M(\theta) \cup H(\theta) = \Omega(m) \cap [M(\theta) \cup H(\theta)] = \Omega(m') \cap [M(\theta) \cup H(\theta)]$, we have

$$\sum_{i \in M(\theta) \cup H(\theta)} \mu_i(m, \theta)[y_i - w_i(m, \theta)] \leq \sum_{i \in M(\theta') \cup H(\theta')} \mu_i(m', \theta)[y_i - w_i(m', \theta)].$$

Now that I have established that the LHS of the VSC is non-increasing in θ and non-decreasing in m for small changes in m . The proof is straight forward.

For a given m^* , let θ^* be the equilibrium market tightness that solves the VSC (2.20), that is

$$K(m^*, \theta^*) = (r + s) \frac{\gamma}{q(\theta^*)}.$$

Let m' be such that $m^* < m' \leq w_E(\theta^*)$. Since $K(m, \theta)$ is non-decreasing in m for $\Delta m < w_E(\theta^*) - m^*$, we have

$$K(m', \theta^*) \geq (r + s) \frac{\gamma}{q(\theta^*)}. \quad (32)$$

So the equilibrium market tightness θ' that give the equality in (32) must be such that $\theta' \geq \theta^*$, given that the LHS of (32) is decreasing in θ and the RHS is increasing in θ . \square

Table B1: Descriptive Statistics by Age and Educational Attainment, 1994-2013.

	By Age					By Educational Attainment			
	16-19	16-24	25-59	60-64	LTHS	H. School	S. College	College	Advanced
Age	17.4 (1.1)	19.8 (2.6)	41.8 (9.7)	61.9 (1.4)	32.7 (15.9)	40.4 (13.2)	38.9 (13.1)	41 (11.5)	45 (10.6)
No. of Hours Worked	24.12 (13.3)	31.55 (14)	40.6 (12.8)	37.8 (14.6)	33.7 (14.5)	39.21 (12.6)	38.61 (13.5)	40.89 (12.9)	42.59 (13)
Wage	8.94 (4.9)	11.12 (6.8)	21.9 (15.2)	22.6 (20.2)	11.47 (6.6)	16.32 (9.6)	18.36 (11.9)	27.27 (18.23)	34.51 (21.2)
No. of Methods to Find a Job.	1.78 (1)	1.94 (1.1)	2.32 (1.3)	2.23 (1.2)	1.86 (1.1)	2.18 (1.2)	2.3 (1.3)	2.52 (1.3)	2.56 (1.4)
Unemployment Rate	16.58%	11.96%	4.55%	4.14%	12.63%	6.59%	4.90%	3.08%	2.20%
Employment	38.5%	54.1%	78.3%	50.1%	46.1%	70.9%	75.7%	82.9%	86.4%
Labor Force Participation	46.1%	61.5%	82%	52.3%	52.7%	75.9%	79.6%	85.6%	88.3%
16-19 year-olds %	100.0%	47.6%	0%	0%	37.4%	6.0%	4.1%	0.0%	0.0%
16-24 year olds %	100%	100%	0%	0%	44.2%	16.2%	19.2%	6.4%	0.0%
Share within 10% of MW	32.4%	18.8%	3.2%	3.6%	19.8%	5.9%	4.94%	1.35%	0.6%
Observations	438,772	920,844	3,821,826	377,615	757,518	1,620,648	1,429,204	885,237	427,678

Total number of observations is 5,120,285. Standard deviations are reported in parenthesis for continuous variables. Hourly wages expressed in 2013 dollars. Variable used for hourly wages is wage3 from the CEPR ORG extracts; the NBER recommended wage variable. For the hours variable I used hourslw from CEPR ORG extracts; it measures actual hours worked on all jobs the week before the interview. The share of the population within a 10% of de minimum wages is computed by dividing wages by their corresponding effective minimum wage, if this ratio is between .9 and 1.1 the observation counts for this share.

Table B2: Minimum Wage Effects on Employment.

Specification		(1)	(2)	(3)	(4)
High-Education, All Ages					
Some College	coefficient	0.003	-0.001	0.000	0.005
	s.e.	(0.009)	(0.012)	(0.011)	(0.010)
	elasticity	0.003	-0.002	0.001	0.007
College	coefficient	-0.018*	0.006	0.009	-0.025
	s.e.	(0.010)	(0.010)	(0.012)	(0.017)
	elasticity	-0.022*	0.007	0.011	-0.031
Advanced	coefficient	-0.022	-0.003	0.001	-0.024
	s.e.	(0.014)	(0.015)	(0.016)	(0.014)
	elasticity	-0.026	-0.004	0.001	-0.027
Low-Education, All Ages					
L.T. High School	coefficient	-0.008	0.010	-0.032	0.013
	s.e.	(0.018)	(0.024)	(0.022)	(0.021)
	elasticity	-0.010	0.044	-0.049	0.024
High School	coefficient	0.020**	0.028**	0.015	0.027**
	s.e.	(0.009)	(0.016)	(0.010)	(0.014)
	elasticity	0.028**	0.040**	0.021	0.038**
Teenagers					
	coefficient	-0.032**	-0.052**	-0.046**	-0.024
	s.e.	(0.016)	(0.027)	(0.022)	(0.019)
	elasticity	-0.084**	-0.136**	-0.120**	-0.061

The dependent variable is a dummy variable for employment. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Elasticities are calculated by dividing the coefficient by the relevant employment to population ratio. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. Observations reporting to be out of the labor force due to retirement or disability were excluded. Observations: Teenagers 438,402; LTHS 755,968; HS 1,618,616; SC 1,428,006; College 884,821; Advanced 427,505.

Table B3: Minimum Wage Effects on Labor Force Participation.

Specification		(1)	(2)	(3)	(4)
High-Education, All Ages					
Some College	coefficient	-0.002	0.000	-0.002	0.002
	s.e.	(0.008)	(0.012)	(0.011)	(0.009)
	elasticity	-0.002	0.000	-0.003	0.002
College	coefficient	-0.014	0.010	0.010	-0.020
	s.e.	(0.009)	(0.011)	(0.011)	(0.013)
	elasticity	-0.017	0.012	0.012	-0.024
Advanced	coefficient	-0.015	0.000	0.001	-0.016
	s.e.	(0.011)	(0.012)	(0.014)	(0.013)
	elasticity	-0.017	0.000	0.002	-0.018
Low-Education, All Ages					
L.T. High School	coefficient	-0.005	0.023	-0.026	0.012
	s.e.	(0.021)	(0.025)	(0.021)	(0.025)
	elasticity	-0.010	0.044	-0.049	0.024
High School	coefficient	0.022**	0.029**	0.025*	0.032***
	s.e.	(0.009)	(0.015)	(0.013)	(0.010)
	elasticity	0.029**	0.038**	0.033*	0.042***
Teenagers					
	coefficient	-0.029*	-0.048*	-0.042**	-0.015
	s.e.	(0.015)	(0.026)	(0.021)	(0.018)
	elasticity	-0.062*	-0.103*	-0.092**	-0.033

The dependent variable is a dummy variable for labor force participation. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Elasticities are calculated by dividing the coefficient by the relevant labor force participation to population ratio. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. Observations reporting to be out of the labor force due to retirement or disability were excluded. Observations: Teenagers 438,402; LTHS 755,968; HS: 1,618,616; SC 1,428,006; College 884,821; Advanced 427,505.

Table B4: Minimum Wage Effects on Search Intensity.

Specification	(1)	(2)	(3)	(4)	
High-Education, All Ages					
Some College	coefficient	-0.088	-0.328*	-0.403**	-0.134
	s.e.	(0.157)	(0.194)	(0.194)	(0.166)
	elasticity	-0.038	-0.142*	-0.175**	-0.058
College	coefficient	0.299*	0.076	0.099	0.341**
	s.e.	(0.169)	(0.270)	(0.261)	(0.169)
	elasticity	0.119*	0.030	0.039	0.135**
Advanced	coefficient	0.046	0.089	-0.301	0.125
	s.e.	(0.272)	(0.424)	(0.466)	(0.317)
	elasticity	0.018	0.035	-0.117	0.049
Low-Education, All Ages					
L.T. High School	coefficient	-0.274**	-0.159	-0.067	-0.357**
	s.e.	(0.137)	(0.151)	(0.215)	(0.149)
	elasticity	-0.147**	-0.086	-0.036	-0.192**
High School	coefficient	0.002	-0.233	-0.326	0.008
	s.e.	(0.113)	(0.144)	(0.230)	(0.198)
	elasticity	0.001	-0.107	-0.150	-0.006
Teenagers					
	coefficient	-0.015	-0.089	-0.032	-0.126
	s.e.	(0.104)	(0.171)	(0.209)	(0.114)
	elasticity	-0.009	-0.050	-0.018	-0.071

The dependent variable is the number of methods used to find a job. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Elasticities are calculated by dividing the coefficient by the mean number of methods to find a job used in each demographic. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. Additionally, each specification includes a control for unemployment duration, a possible cause of endogeneity; the variable used is *undur*. Observations: Teenagers 31,499; LTHS 43,998; HS 68,341; SC 48,469; College 20,902; Advanced 7,466.

Table B5: Minimum Wage Effects on Hours.

Specification		(1)	(2)	(3)	(4)
High-Education, All Ages					
Some College	coefficient	-0.015	-0.022	-0.023*	-0.019*
	s.e.	(0.011)	(0.015)	(0.013)	(0.011)
College	coefficient	-0.016	0.018	0.019	-0.014
	s.e.	(0.016)	(0.013)	(0.013)	(0.017)
Advanced	coefficient	-0.023	0.008	0.018	-0.025
	s.e.	(0.015)	(0.021)	(0.021)	(0.016)
Low-Education, All Ages					
L.T. High School	coefficient	-0.065**	-0.105***	-0.096***	-0.063**
	s.e.	(0.027)	(0.034)	(0.035)	(0.031)
High School	coefficient	-0.015	-0.018	-0.021*	-0.015
	s.e.	(0.010)	(0.015)	(0.012)	(0.011)
Teenagers	coefficient	-0.120***	-0.218***	-0.210***	-0.123***
	s.e.	(0.034)	(0.051)	(0.048)	(0.036)

The dependent variable is the log of weekly hours worked. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. Observations: Teenagers 162,009; LTHS 335,014; HS 1,105,666; SC 1,039,471; College 702,036; Advanced 348,697.

Table B6: Minimum Wage Effects on Wages.

Specification		(1)	(2)	(3)	(4)
High-Education, All Ages					
Some College	coefficient	0.003	-0.001	0.000	0.005
	s.e.	(0.009)	(0.012)	(0.011)	(0.010)
College	coefficient	-0.005	0.026	0.016	-0.003
	s.e.	(0.035)	(0.025)	(0.024)	(0.035)
Advanced	coefficient	-0.037	-0.004	-0.013	-0.052
	s.e.	(0.040)	(0.031)	(0.035)	(0.040)
Low-Education, All Ages					
L.T. High School	coefficient	0.053	0.102**	0.066*	0.054
	s.e.	(0.035)	(0.041)	(0.034)	(0.042)
High School	coefficient	0.009	0.050*	0.045*	0.004
	s.e.	(0.029)	(0.030)	(0.025)	(0.031)
Teenagers					
	coefficient	0.152***	0.163***	0.136***	0.163***
	s.e.	(0.026)	(0.037)	(0.032)	(0.028)

The dependent variable is the log of hourly wage. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. Observations: Teenagers 163,027; LTHS 312,185; HS 1,004,419; SC 946,401; College 623,667; Advanced 302,711.

Table B7: Minimum Wage Effects on Low-Education Employment by Age, 1994-2013

Specification	LTHS				High School			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
16-19								
coefficient	-0.067***	-0.060**	-0.058***	-0.058***	0.042	-0.037	0.002	0.046
s.e.	(0.021)	(0.026)	(0.022)	(0.023)	(0.033)	(0.055)	(0.051)	(0.038)
elasticity	-0.215***	-0.192**	-0.187***	-0.186***	0.082	-0.073	0.004	0.038
16-24								
coefficient	-0.049**	-0.038	-0.043*	-0.037*	0.021	-0.002	0.040	0.022
s.e.	(0.020)	(0.026)	(0.023)	(0.021)	(0.020)	(0.030)	(0.027)	(0.022)
elasticity	-0.141**	-0.109	-0.123*	-0.107*	0.034	-0.003	0.064	0.036
25-59								
coefficient	0.018	0.026	-0.027	0.042	0.019**	0.032**	0.025*	0.030***
s.e.	(0.026)	(0.026)	(0.032)	(0.029)	(0.009)	(0.014)	(0.014)	(0.010)
elasticity	0.030	0.044	-0.046	0.073	0.025**	0.042**	0.033*	0.040***
60-64								
coefficient	0.030	-0.007	-0.034	0.067	0.024	0.044	0.020	0.035
s.e.	(0.034)	(0.057)	(0.077)	(0.040)	(0.031)	(0.030)	(0.033)	(0.036)
elasticity	0.093	-0.020	-0.104	0.206	0.053	0.095	0.043	0.077

The dependent variable is a dummy variable for employment. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Elasticities are calculated by dividing the coefficient by the relevant employment to population ratio. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. Observations reporting to be out of the labor force due to retirement or disability were excluded. LTHS Observations; 16-19: 283,310; 16-24: 334,182; 25-59: 368,059; 60-64: 53,727.HS Observations; 16-19: 96,467; 16-24: 251,226; 25-59: 1,234,738; 60-64: 132,652.

Table B8: Minimum Wage Effects on Low-Education Labor Force Participation by Age, 1994-2013.

Specification	LTHS				High School				
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	
16-19	coefficient	-0.066***	-0.053**	-0.052**	-0.055**	0.041	-0.046	0.003	0.063*
	s.e.	(0.021)	(0.025)	(0.023)	(0.023)	(0.030)	(0.052)	(0.041)	(0.034)
	elasticity	-0.170***	-0.137**	-0.134**	-0.144**	0.068	-0.076	0.005	0.103*
16-24	coefficient	-0.037*	-0.017	-0.024	-0.026	0.022	-0.014	0.020	0.032*
	s.e.	(0.021)	(0.025)	(0.022)	(0.023)	(0.018)	(0.029)	(0.025)	(0.019)
	elasticity	-0.088*	-0.040	-0.056	-0.061	0.031	-0.019	0.028	0.045*
25-59	coefficient	0.009	0.031	-0.026	0.028	0.023***	0.034***	0.026**	0.034***
	s.e.	(0.028)	(0.027)	(0.033)	(0.032)	(0.009)	(0.013)	(0.013)	(0.010)
	elasticity	0.014	0.048	-0.040	0.044	0.029***	0.043***	0.033**	0.042***
60-64	coefficient	0.044	0.005	-0.050	0.081	0.018	0.051*	0.019	0.031
	s.e.	(0.037)	(0.057)	(0.078)	(0.043*)	(0.033)	(0.031)	(0.033)	(0.039)
	elasticity	0.125	0.016	-0.145	0.235	0.037	0.107*	0.039	0.064

The dependent variable is a dummy variable for labor force participation. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Elasticities are calculated by dividing the coefficient by the relevant labor force participation to population ratio. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. Observations reporting to be out of the labor force due to retirement or disability were excluded. LTHS Observations; 16-19: 283,310; 16-24: 334,182; 25-59: 368,059; 60-64: 53,727.HS Observations; 16-19: 96,467; 16-24: 251,226; 25-59: 1,234,738; 60-64: 132,652.

Table B9: Minimum Wage Effects on Low-Education Search Intensity by Age, 1994-2013

Specification	LTHS				High School			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
16-19								
coefficient	-0.095	-0.053	0.132	-0.222	0.304	-0.014	-0.191	0.177
s.e.	(0.147)	(0.207)	(0.244)	(0.156)	(0.209)	(0.318)	(0.360)	(0.225)
elasticity	-0.057	-0.032	0.079	-0.132	0.154	-0.007	-0.097	0.090
16-24								
coefficient	-0.150	-0.245	-0.082	-0.267*	0.073	-0.257	-0.270	0.039
s.e.	(0.151)	(0.186)	(0.239)	(0.155)	(0.173)	(0.215)	(0.263)	(0.210)
elasticity	-0.087	-0.141	-0.047	-0.154*	0.036	-0.125	-0.131	0.019
25-59								
coefficient	-0.384*	-0.005	-0.005	-0.448*	-0.011	-0.244	-0.363	-0.013
s.e.	(0.216)	(0.214)	(0.270)	(0.251)	(0.135)	(0.186)	(0.231)	(0.143)
elasticity	-0.190*	-0.002	0.010	-0.222*	-0.005	-0.109	-0.162	-0.006
60-64								
coefficient	-1.406	-2.836*	-1.484	-0.753	0.058	-0.505	-1.102	-0.067
s.e.	(0.905)	(1.631)	(2.260)	(0.985)	(0.522)	(0.856)	(1.045)	(0.524)
elasticity	-0.744	-1.500*	-0.785	-0.398	0.027	-0.232	-0.506	-0.031

The dependent variable is the number of methods used to find a job. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Elasticities are calculated by dividing the coefficient by the mean number of methods to find a job used in each demographic. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. Additionally, each specification includes a control for unemployment duration, a possible cause of endogeneity; the variable used is undur. LTHS Observations; 16-19: 19,522; 6-24: 24,992; 25-59: 18,212; 60-64: 794.HS Observations: 16-19: 9,034; 16-24: 21,988; 25-59: 44,340; 60-64: 2,013.

Table B10: Minimum Wage Effects on Low-Education Weekly Hours by Age, 1994-2013.

Specification	LTHS				High School			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
16-19								
coefficient	-0.159***	-0.238***	-0.230***	-0.162***	-0.077	-0.213***	-0.214***	-0.099*
s.e.	(0.049)	(0.067)	(0.069)	(0.054)	(0.055)	(0.070)	(0.073)	(0.057)
16-24								
coefficient	-0.072	-0.150***	-0.157***	-0.066	-0.020	-0.058*	-0.049	-0.028
s.e.	(0.045)	(0.053)	(0.057)	(0.054)	(0.023)	(0.035)	(0.035)	(0.025)
25-59								
coefficient	-0.051**	-0.070**	-0.035	-0.053**	-0.013	-0.012	-0.017	-0.010
s.e.	(0.023)	(0.030)	(0.030)	(0.024)	(0.011)	(0.015)	(0.012)	(0.014)
60-64								
coefficient	-0.161**	-0.183	-0.175	-0.185**	-0.044	-0.024	-0.028	-0.069
s.e.	(0.084)	(0.128)	(0.125)	(0.088)	(0.041)	(0.045)	(0.043)	(0.045)

The dependent variable is the log of weekly hours worked. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. LTHS Observations: 16-19: 84,525; 16-24: 110,979; 25-59: 207,541; 60-64: 16,494. HS Observations: 16-19: 47,621; 16-24: 152,067; 25-59: 895,720; 60-64: 57,879.

Table B11: Minimum Wage Effects on Low-Education Wages by Age, 1994-2013

Specification	LTHS				High School			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
16-19								
coefficient	0.194*** (0.029)	0.186*** (0.041)	0.174*** (0.040)	0.218*** (0.031)	0.094*** (0.037)	0.139*** (0.053)	0.102* (0.057)	0.087** (0.041)
s.e.								
16-24								
coefficient	0.169*** (0.030)	0.160*** (0.044)	0.140*** (0.041)	0.184*** (0.033)	0.083*** (0.031)	0.141*** (0.043)	0.110*** (0.037)	0.083** (0.035)
s.e.								
25-59								
coefficient	-0.015 (0.046)	0.062 (0.047)	-0.001 (0.040)	-0.020 (0.057)	-0.003 (0.030)	0.033 (0.032)	0.029 (0.026)	-0.008 (0.032)
s.e.								
60-64								
coefficient	0.079 (0.099)	0.173 (0.092)	0.231 (0.101)	0.090 (0.112)	-0.017 (0.050)	0.035 (0.066)	0.070 (0.076)	-0.034 (0.061)
s.e.								

The dependent variable is the log of hourly wage. Results are reported for the coefficients of the natural logarithm of the minimum wage for specifications (1) to (3). For specification (4) the coefficient is the sum of the contemporary and lagged effects. Standard errors clustered at state level are reported in parentheses. Significance levels are as follows: *** 1 percent, ** 5 percent, * 10 percent. Each specification includes individual controls for age, gender, race, marital status, education, as well as controls for seasonally adjusted unemployment rate, and the relevant population share for each demographic group. State-fixed effects are also included. LTHS Observations: 16-19: 85,030; 16-24: 110,744; 25-59: 187,331; 60-64: 14,110. HS Observation: 16-19: 47,771; 16-24: 150,447; 25-59: 804,586; 60-64: 49,386.

Table B12: Low-Wage Mean Labor Market Outcomes

		16-19	20-24	25-59	60-64
Wages	LTHS	8.3	10.3	12.9	13.5
	High School	9.6	11.6	17.2	17.3
Unemployment Rate	LTHS	19%	18.3%	9.4%	5.9%
	High School	16.3%	11.8%	5.5%	4.1%
Labor Force Participation	LTHS	38.5%	65.2%	64.3%	34.5%
	High School	60.1%	78.5%	79.6%	48%
Search Intensity	LTHS	1.68	1.95	2	1.9
	High School	1.97	2.1	2.24	2.17

Appendix C

Proof of Proposition 8

To prove proposition 8, two lemmas are necessary.

Lemma 6. *If there is an interest rate differential, i.e., if $R_d \neq R_f$, then any candidate for a steady state must have $x = 0$, if $R_d = R_f$ then any $x \in \mathbb{R}$ could be a component of a steady state.*

Proof: Given the definition of a steady state, it must be the case that at any steady state $x_t = x_{t-1} = x_{t-2}$, so the evolution for the exchange rate deviation from the fundamental exchange rate (14) takes the form

$$x = x \left[\frac{R_d}{R_f} - (1-s)(1-q) \right] \frac{1}{(1-s)q + s}. \quad (33)$$

Any steady state must satisfy this equation. If $R_d \neq R_f$ the equation can be satisfied if and only if $x = 0$. If $R_d = R_f$ then (23) turns into an identity so any $x \in \mathbb{R}$ could be a component of a steady state.

□

Lemma 7. *Let*

$$x^s \equiv \sqrt{\frac{C[(1-s)q + s]^2 a \sigma^2}{(1-s)(1-q)(R_d - \theta R_f)^2}}.$$

If $C = 0$, then $q = 1$ can always be part of a steady state. If $C = 0$ and $\theta = \frac{R_d}{R_f}$ then any $q \in [0, 1]$ can be part of a steady state. If $C = 0$ and $\theta \neq \frac{R_d}{R_f}$ and $x = 0$, then any $q \in [0, 1]$ can be a steady state. If $C > 0$ and $\theta = \frac{R_d}{R_f}$, then only $q = 0$ could be part of a steady state. If $C > 0$ and $\theta \neq \frac{R_d}{R_f}$ and $x = \pm x^$ then any $q \in [0, 1)$ can be steady state. If $C > 0$ and $\theta \neq \frac{R_d}{R_f}$ and $x = \pm x^s$ then only $q = 0$ can be a steady state.*

Proof: The dynamic of the proportion of naive traders using rational expectations is determined by equations (20) and (21). In a steady state these equations take the form

$$q = \begin{cases} 1 - \frac{1-q}{1+(1-q)\delta(\cdot)} & \text{if } \delta(x, q) > 0 \\ \frac{q}{1-q\delta(\cdot)} & \text{if } \delta(x, q) \leq 0 \end{cases} \quad (34)$$

$$\delta(x, q) = \frac{x^2(R_d - \theta R_f)^2 (1-s)(1-q)}{a\sigma^2 [(1-s)q + s]^2} - C \quad (35)$$

First I will show that if $\delta(x, q) > 0$ then a steady state will exist only if $C = 0$, and at this steady state $q = 1$. Assume that at a steady state $\delta(x, q) > 0$ so by (24) any steady state must satisfy

$$q = 1 - \frac{1-q}{1+(1-q)\delta(x, q)} \quad (36)$$

This equation always holds for $q = 1$, but if $q = 1$ and $C > 0$ equation (24) is negative so there would be a contradiction to the initial assumption that it is positive. So if $C > 0$, $q = 1$ cannot be a steady state. For $q \neq 1$, equation (26) can only be satisfied if $(1-q)\delta(x, q) = 0$, since we are assuming $\delta(x, q) > 0$ then this equation is true if and only if $q = 1$ which can be a steady state only if $C = 0$.

Now, I focus on the case where $\delta(x, q) \leq 0$. Assume that at a steady state $\delta(x, q) \leq 0$, then by (24) any q candidate to form a steady state must meet

$$q = \frac{q}{1 - q\delta(x, q)} \quad (37)$$

This equation holds if and only if $q\delta(x, q) = 0$. If $q \neq 0$ then it must be that $\delta(x, q) = 0$ which by (25) implies

$$\frac{x^2(R_d - \theta R_f)^2 (1-s)(1-q)}{a\sigma^2 [(1-s)q + s]^2} = C \quad (38)$$

If $C = 0$ and $\theta = \frac{R_d}{R_f}$ then both sides of the equation are equal to zero so any $q \in [0, 1]$ can be part of a steady state. Similarly if $C = 0$ and $x = 0$ then any $q \in [0, 1]$ can be part of a steady state. If $C > 0$ and $\theta = \frac{R_d}{R_f}$ then (28) cannot hold and there can only be a steady state if $q = 0$. If $C > 0$ and $\theta \neq \frac{R_d}{R_f}$ then at a steady state it must be that $x = \pm x^*$, since $\delta(x^*, q) = 0 \forall q \in [0, 1)$, so if these conditions hold then any $q \in [0, 1)$ can be a steady state. If $C > 0$ and $\theta \neq \frac{R_d}{R_f}$ and $x \neq \pm x^*$, then $q\delta(x, q) = 0$ can only be true if $q = 0$, so any steady state must have $q = 0$.

□

Proof of Proposition 8

i) By Lemma 6 if $R_d \neq R_f$ then any steady state must have $x = 0$. By Lemma 7 if $C > 0$ and $x = 0$ then only $q = 0$ can be part of a steady state. Hence, the only possible steady state is $(0, 0)$.

ii) By Lemma 6 if $R_d \neq R_f$ then any steady state must have $x = 0$. By Lemma 7 if $C = 0$ and $x = 0$ then any $q \in [0, 1]$ can be part of a steady state, so all the possible steady states have the form $(0, q)$ with $q \in [0, 1]$.

iii) By Lemma 6 if $R_d = R_f$ then any $x \in \mathbb{R}$ can be part of a steady state. By Lemma 7 if $C > 0$ and $\theta = \frac{R_d}{R_f}$ then $q = 0$ can always be a steady state. Similarly if $C > 0$ and $\theta \neq \frac{R_d}{R_f}$, there can be a steady state only if $x = \pm x^s$ in which case any $q \in [0, 1)$ can be part of a steady state.

iv) By Lemma 6 if $R_d = R_f$ then any $x \in \mathbb{R}$ can be part of a steady state. By Lemma 7 if $C = 0$ and $\theta = 1$ then any $q \in [0, 1]$ can be part of a steady state. Similarly if $C = 0$ and $\theta \neq 1$ then to have a steady state it is necessary that $x = \pm x^*$, if this is true then any $q \in [0, 1)$ can be a steady state.

□

Proof of Proposition 9

To prove Proposition 9, I will show that for every possible steady state that could arise when $C > 0$, the same condition for local stability always must hold. First consider the case when $R_d \neq R_f$. Under these conditions the only possible steady state is $(0, 0)$. The Jacobian around the steady state is

$$J_{(0,0)} = \begin{pmatrix} \left[\frac{R_d}{R_f} - (1 + \theta)(1 - s) \right] \frac{1}{s} & 0 \\ 0 & 1 \end{pmatrix} \quad (39)$$

Hence the steady state is non-hyperbolic with eigenvalues 1 and $\left[\frac{R_d}{R_f} - (1 + \theta)(1 - s) \right] \frac{1}{s}$. So $(0, 0)$ is a locally stable steady state if and only if $-1 \leq \left[\frac{R_d}{R_f} - (1 + \theta)(1 - s) \right] \frac{1}{s} \leq 1$. This condition holds if and only if

$$\frac{R_d/R_f - 1}{1 - s} \leq \theta \leq \frac{R_d/R_f - 1 + 2s}{1 - s} \quad (40)$$

Now consider the case when $R_d = R_f$. If $\theta = 1$, then the steady states are in the form $(x, 0)$ with $x \in \mathbb{R}$. Notice that with these values, we have $\delta(x, q) < 0$, since the first element of equation (20) is equal to zero and $C > 0$. The Jacobian around these steady states is

$$J_{(x,0)} = \begin{pmatrix} [2s - 1] \frac{1}{s} & 0 \\ 0 & 1 \end{pmatrix}$$

The eigenvalues of the Jacobian are 1 and $[2s - 1] \frac{1}{s}$. So the steady state will be locally stable if and only if $s > 1/3$. Notice that the Jacobian in (29) and the stability condition in (30) are a generalization of this situation.

Now consider the case when $R_d = R_f$ and $\theta \neq 1$. By lemma 7 we have that the possible steady states under these conditions are those (x^*, q) with $q \in [0, 1)$ and x^* as defined in

Lemma 27 We have that $\delta(x^*, q) = 0 \forall q \in [0, 1]$. So the Jacobian around these steady states takes the form

$$J_{(x^*, q)} = \begin{pmatrix} [s + \theta(1 - s)]^{\frac{1}{s}} & 0 \\ 0 & 1 \end{pmatrix}$$

The local stability condition for this Jacobian is $0 \leq \theta \leq 2s/(1-s)$. Notice again that this is a specific case of the Jacobian in (29) and the stability condition in (30). I have shown that the local stability condition of every steady state possible when $C > 0$ can be represented by condition (30). To appreciate how the interval of values of parameter θ that create local stability changes with s , let the width of this interval be $W = \frac{R_d/R_f - 1 + 2s}{1-s} - \frac{R_d/R_f - 1}{1-s} = \frac{2s}{1-s}$, the derivative of the width of the interval with respect to s is $\frac{\partial W}{\partial s} = 2/(1-s)^2 > 0$, so the interval always expands with s .

□