Weed Optimization Algorithm for Optimal Reservoir Operation

Hamid-Reza Asgari[1]; Omid Bozorg Haddad[2]; Maryam Pazoki[3]; and Hugo A. Loáiciga, F.ASCE[4]

Abstract: This study introduces the weed optimization algorithm (WOA) to optimal reservoir operation. The WOA is a metaheuristic optimization method inspired by weeds' life cycle. The effectiveness of the WOA is demonstrated with the optimization of mathematical functions and reservoir systems. The WOA is applied in continuous-time and discrete-time formulations of reservoir-operation optimization and its results are compared with global optimal solutions obtained with nonlinear programming (NLP), linear programming (LP), and the genetic algorithm (GA). The results show the WOA's fast convergence to solutions that are very near the global optimal solutions of the reservoir optimization problems. **DOI: 10.1061/(ASCE)IR.1943-4774.0000963.** *© 2015 American Society of Civil Engineers.*

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Introduction

Reservoir operation is based on a series of rules that determine the amount of water that is stored and released under different system conditions. These reservoir operation rules determine how reservoir water is allocated during periods of droughts, normal climate, or wet climate.

Methods used for the optimal operation of reservoirs can be classified into two main categories: classic algorithms, and evolutionary and metaheuristic algorithms. Although classic methods are relatively simple, they have limitations such as the possibility of not achieving global optima, convergence to local optima, and being hindered by high dimensionality (the curse of dimensionality problem). Evolutionary and metaheuristic algorithms are generally inspired by natural phenomena. One of the advantages of the latter algorithms is that they generally converge to near-global optima for any well-defined optimization problems. In addition, they can solve multiobjective problems. The main disadvantage of evolutionary and metaheuristic algorithms is the long processing time needed to converge to a solution. This has led many researchers to search for and produce newer, computationally more efficient, evolutionary and metaheuristic algorithms.

Many classic and metaheuristic optimization techniques have been recently developed and applied in various aspects of water resources systems such as reservoir (Fallah-Mehdipour et al. 2011, 2012a, 2013a), hydrology (Orouji et al. 2013), water-resources management (Bozorg Haddad et al. 2010b; Fallah-Mehdipour et al. 2012b), irrigation (Bozorg Haddad et al. 2009; Fallah-Mehdipour et al. 2013b), power plants (Bozorg Haddad et al. 2011c), structures (Bozorg Haddad et al. 2010a), distribution networks (Seifollahi-Aghmiuni et al. 2011, 2013), aquifers (Bozorg Haddad and Mariño 2011), infrastructures (Karimi-Hosseini et al. 2011), and algorithmic developments (Shokri et al. 2013). None of these works dealt with the application of the weed optimization algorithm (WOA) in water resources systems, or, in particular, to solve reservoir optimal operation.

Concerning the application of evolutionary and metaheuristic algorithms to reservoir operation, Esat and Hall (1994) resorted to the genetic algorithm (GA) to optimize reservoir operation for energy production and water for irrigation. Oliveira and Loucks (1997) employed the GA to evaluate rules concerning the operation of multi-reservoir systems. Sharif and Wardlaw (2000) implemented the GA in several multi-reservoir systems and obtained solutions very close to those calculated with dynamic programming (DP). Ahmed and Sarma (2005) compared the GA's performance with that of stochastic dynamic programming and reported that the GA was superior in calculating desired solutions for optimizing multiobjective reservoir operation. Tospornsampan et al. (2005) applied the simulated annealing (SA) algorithm to optimize the operation of a multi-reservoir system. Kumar and Reddy (2006) implemented the ant colony optimization (ACO) metaheuristic algorithm to optimize the operation of a multiobjective reservoir. Bozorg Haddad et al. (2006) introduced the honey-bee mating optimization (HBMO) metaheuristic algorithm to reservoir operation. Bozorg Haddad et al. (2008) used the HBMO and nonlinear programming (NLP) for the design and operation of a single and multiple reservoir system. Wang et al. (2011) introduced the multi-tier interactive GA (MIGA) for long-term optimization of reservoir operation. Jothiprakash et al. (2011) used the GA and stochastic dynamic programming (SDP) for the operation of a five-reservoir system in Kodaiyar, India. Ostadrahimi and et al. (2012) calculated operation rules of a multi-reservoir system using the multipopulation approach in multi-swarm particle swarm optimization (MSPSO) algorithm. Ngoc et al. (2013) applied the constrained GA to derive optimal operation principles of multiobjective reservoirs. Bozorg Haddad et al. (2014) applied the bat algorithm

[1]Dept. of Irrigation and Reclamation Engineering, Faculty of Agricultural Engineering and Technology, College of Agriculture and Natural Resources, Univ. of Tehran, Karaj, 3158777871 Tehran, Iran. E-mail: Asgari.HR@ut.ac.ir

[2]Associate Professor, Dept. of Irrigation and Reclamation Engineering, Faculty of Agricultural Engineering and Technology, College of Agriculture and Natural Resources, Univ. of Tehran, Karaj, 3158777871 Tehran, Iran (corresponding author). E-mail: OBHaddad@ut.ac.ir

[3]Assistant Professor, Faculty of Environment, College of Environmental Engineering, Univ. of Tehran, Karaj, 3158777871 Tehran, Iran. E-mail: MPazoki@ut.ac.ir

[4]Professor, Dept. of Geography, Univ. of California, Santa Barbara, CA 93106-4060. E-mail: Hugo.Loaiciga@ucsb.edu

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(BA) to determine the optimal operation of reservoir policies for Karoon 4 and four-reservoir system in continuous domain. Bozorg Haddad et al. (2015) used the water cycle algorithm (WCA) to determine the optimal operation of reservoir policies of the Karoon 4 reservoir. They also compared the results of the WCA with those obtained with the GA and NLP.

The following works have implemented the WOA in a variety of engineering optimization problems, but not to reservoir operation as of yet. Mehrabian and Lucas (2006) introduced the WOA. They solved two engineering problems and compared the results with the GA, memetic algorithm (MA), particle swarm optimization (PSO) algorithm, shuffled frog leading algorithm (SFLA), and the simulated annealing (SA) algorithm. Their results showed a relatively superior performance by the WOA. Mehrabian and Yousefi-Koma (2007) applied the WOA to optimize the location of piezoelectric actuators on a smart fin. Mallahzadeh et al. (2008) tested the flexibility, effectiveness, and efficiency of the WOA in optimizing a linear array of antenna and compared the computed results with those of the PSO algorithm. Sahraei-Ardakani et al. (2008) used WOA to optimize the generation of electricity. Roshanaei et al. (2009) applied the WOA to optimize uniform linear array (ULA) used in wireless networks, such as commercial cellular systems, and compared their results with those from the GA and least mean square (LMS). Mallahzadeh et al. (2009) used the WOA to design vertical antenna elements with maximal efficiency. Krishnanand and Nayak (2009) compared the effectiveness of the WOA, GA, PSO algorithm, artificial bee colony (RBC), and artificial immune (AI) by solving five basic standard mathematical problems with multivariate functions. Zhang et al. (2010) used heuristic algorithm concepts for developing the WOA. They introduced the WOA with crossover function and tested the new algorithm on standard mathematical problems and compared the results of the developed WOA with those of the standard WOA and PSO. Sharma et al. (2011) used the WOA to solve dynamic economic dispatch (DED). Their results showed that the WOA algorithms reduced production costs relative to those obtained with the PSO and AI algorithms and differential evolution (DE). Jayabarathi et al. (2012) implemented the WOA for solving economic dispatch (DE) problems. Kostrzewa and Josiński (2012) introduced a new version of the WOA and tested their algorithm on several standard mathematical problems. Abu-Al-Nadi et al. (2013) applied the WOA for model order reduction (MOR) in linear multiple-input-multiple-output systems (MIMO). Sang and Pan (2013) introduced the effective discrete WOA (DIWO) to solve the problem of flow shop scheduling with average stored buffers, and compared their results with the hybrid GA (HGA), hybrid PSO algorithm (HPSO), and the hybrid discrete differential evolution algorithm (HDDE). Saravanan et al. (2014) applied the WOA to solve the unit commitment (UC) problem for minimizing the total costs of generating electricity. They compared their results with those calculated with the GA, SFLA, PSO algorithms, Lagrangian relaxation (LR), and the bacterial foraging (BF) algorithm. Barisal and Prusty (2015) used the WOA to solve economic problems on a large scale with the aim of minimizing the costs of production and transfer of goods subject to restrictions on production, market demand, the damage caused to goods during transportation, and to alleviate other calamities.

The reviewed literature established that the WOA has not been applied to optimize reservoir operation. This study introduces the WOA to the field of reservoir operation and compares its results with those obtained with the GA, linear programming (LP), and NLP. Several comparative examples are solved to measure the performance of the WOA against those of well-established optimization methods.

Weed Optimization Algorithm

A common phenomenon in agriculture inspired the WOA. The WOA was developed on the basis of weeds' growth characteristics. Weeds are plants that grow spontaneously and may be harmful to pastures, farms, and gardens. They can easily adapt to almost any environment and new conditions. Despite its simplicity, the WOA emulates numerically many characteristics of plants such as seed production, growth, and competition. The following characteristics describe the growth of weed colonies:

- A limited number of seeds are spread in a search area.
- Each seed turns into a weed that produces seeds based on its quality in the colony.
- Produced seeds spread randomly in the environment and make new seeds.
- This process is repeated until the maximum number of plants in a colony is reached. Then, competition for survival starts between weeds so that in each stage weeds of lower quality are removed. This process continues to produce weeds of the highest quality.

The detailed steps of the WOA are as follows:

1. Start with an initial population: the initial population (P_{initial}) is produced and spread randomly in a d-dimensional search area. In fact, each plant is a solution whose location in any dimension of d-dimensional area is a decision making parameter. A bunch of several plants constitutes a colony.
2. Reproduction: In this stage plants are allowed to produce seeds according to the quality of the colony, their own quality, and the maximum and minimum number of produced seeds ($NoS_{\max}$), ($NoS_{\min}$), which the user can choose. Seed production is illustrated in Fig. 1 approximately as a linear function.

   The reproduction stage adds an important advantage to the algorithm. In evolutionary algorithms population agents constitute a range from appropriate solutions to inappropriate ones. Appropriate samples have a higher probability of reproduction than inappropriate ones, but there is always the possibility that population elements that seem inappropriate at each stage contain important information that even suitable plants lack. It is therefore probable that, with a suitable reproduction, inappropriate plants survive an unsuitable environment and find a hidden suitable environment. This process is observed in nature.
3. Spread of seeds: At this stage adoption and randomness are introduced in algorithm. The produced seeds are spread randomly with normal distribution and zero mean in a d-dimensional area. Therefore new plants spread randomly around the parent plants, but their standard deviation is

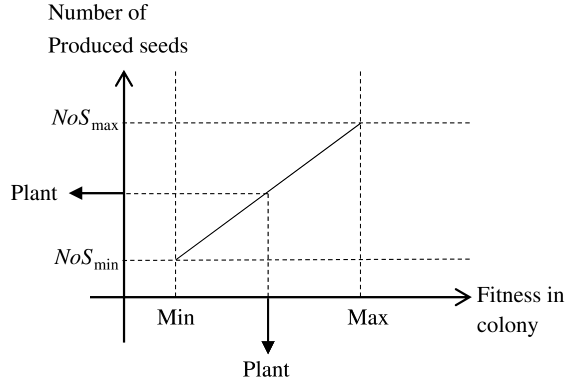

Fig. 1. Level of reproduction for each plant with respect to fitness

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variable. The standard deviation is reduced from the initial predetermined value (maximum) to a final predetermined value (minimum) according to Eq. (1)

$$\sigma_{\text{iter}} = \frac{(\text{iter}_{\max} - \text{iter})^n}{(\text{iter}_{\max})^n}(\sigma_{\text{initial}} - \sigma_{\text{final}}) + \sigma_{\text{final}} \tag{1}$$

where σ_{iter} = standard deviation of the current iteration; $\text{iter}_{\max}$ = maximum iteration number (reproduction stages); iter = current iteration number; σ_{initial} = initial standard deviation; σ_{final} = final standard deviation; and n = nonlinear modulus (Nonlinear Modulation Index) selected by the user. The probability of placing a seed far from its parent plant in the beginning of the algorithm is high, and it decreases during later stages of the algorithm when the number of appropriate plants increases.

4. Competitive exclusion: if a plant does not produce seeds it will become extinct. If all the plants produce seeds and the seeds grow, the number of plants increases exponentially. Therefore a competitive process is necessary to limit and remove some of the existing plants. After several reproductions, the number of plants in the colony reaches its maximum ($P_{\max}$). This is when the process of omitting unsuitable plants starts, and is repeated until the end of the algorithm. Fig. 2 depicts a flowchart of the WOA.

Testing the WOA with Mathematical Functions

Two mathematical functions were used in this study to test the optimizing capacity of the WOA. The Ackley unbound n-dimensional mathematical function, and a constrained mathematical problem with seven parameters was used in this test. The Ackley function is given by Eq. (2)

$$\text{Min. } f(x) = -20\exp\left(-\frac{1}{5}\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right) - \exp\left[\frac{1}{n}\sum_{i=1}^{n}\text{Cos}(2\pi x_i)\right] + 20 + e \qquad -5 \le x_i \le 5 \tag{2}$$

The Ackley function has global optimum $f(x^*) = 0$ at $x^* = (0, \ldots, 0)$. Fig. 3 displays this function in a two dimensional space. It is shown from Fig. 3 that this function has many local optima. The Ackley function was solved for 5, 10, 20, 50, and 100 dimensions using the GA and the WOA with 10 runs (one run for a different initial population). The GA was implemented with the following characteristics: number of decision-making parameters = 5–100, population size = 50–100, the number of iterations ranged between 500 and 1500, the selection method was the Roulette Wheel, the Crossover Function was single point, the Constraint Dependent was uniform, the Crossover Possibility ranged between 0.7 and 0.8, and the mutation probability ranged between 0.015 and 0.05. The best values for each parameter were chosen after analyzing the parameters' sensitivities. The WOA's parameters were as follows: number of decision-making parameters = 5–100; $P_{\text{inital}} = 5$–10; $P_{\max} = 15$–30; $\sigma_{\text{initial}} = 1$–$2$; $\sigma_{\text{final}} = 0.00001$–$0.01$; $n = 3$; $NoS_{\max} = 3$–5; $NoS_{\min} = 0$–1; and $\text{iter}_{\max} = 500$–$1{,}000$. The best parameter values were chosen after analyzing the parameters' sensitivities. The results of the 10 runs corresponding to maximum, mean, and minimum standard deviation and coefficient of variation are presented in Table 1. It is shown in Table 1 that the WOA problem is more effective than

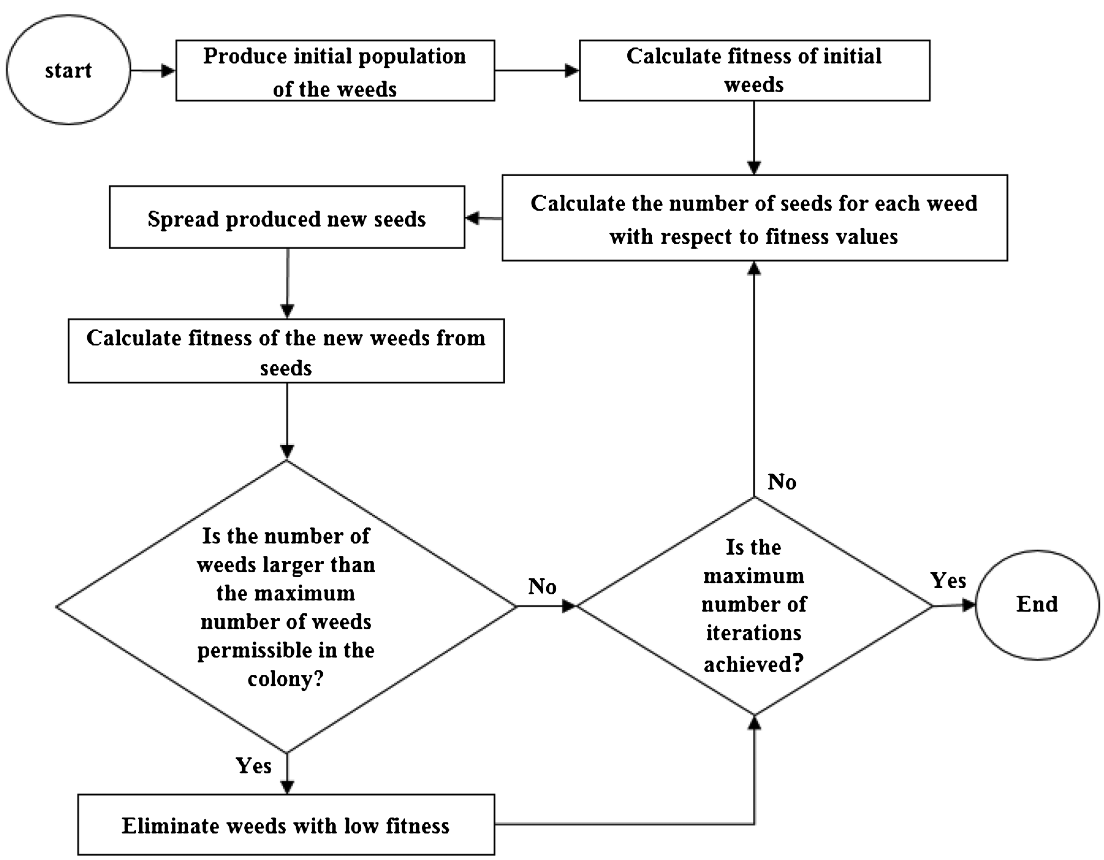

Fig. 2. Flowchart of the WOA algorithm

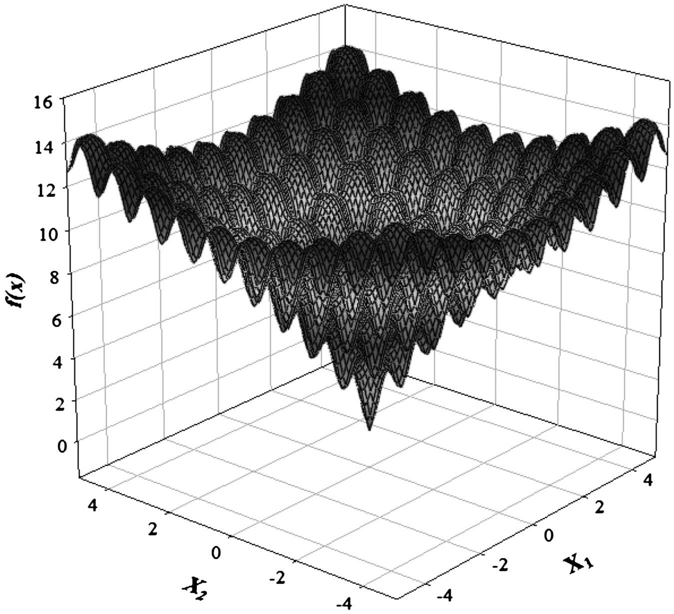

Fig. 3. Graph of the Ackley function in two dimensions

the GA in approximating the global optimum of the Ackley function.

The second test function is a constrained mathematical problem whose objective function is given in Eq. (3) and its constraints are written in Eqs. (4)–(8)

$$\text{Min. } f(X) = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7 \quad (3)$$

Subject to:

$$-127 + 2x_1^2 + 3x_2^4 + x_3 + 4x_4^2 + 5x_5 \leq 0 \quad (4)$$

$$-282 + 7x_1 + 3x_2 + 10x_3^2 + x_4 - x_5 \leq 0 \quad (5)$$

$$-196 + 23x_1 + x_2^2 + 6x_6^2 - 8x_7 \leq 0 \quad (6)$$

$$4x_1^2 + x_2^2 - 3x_1x_2 + 2x_3^2 + 5x_6 - 11x_7 \leq 0 \quad (7)$$

$$-10 \leq x_i \leq 10 \qquad i = 1, \ldots, 7 \quad (8)$$

The global minimum of Eqs. (3)–(8) is $f(X^*) = 680.630057$ at point $X^* = (2.331, 1.951, -0.489, 4.365, -0.624, 1.038, 1.597)$ (Parsopoulos and Vrahatis 2002).

Table 2. Results of the GA and the WOA for 10 Runs for Mathematical Problem with Constraints

Algorithm	Minimum	Average	Maximum	SD	Coefficient of variation
GA	683.1057	687.5666	701.9914	5.500057	0.001990
WOA	680.6351	680.6499	680.6972	0.016994	0.000020

Eqs. (3)–(8) was solved with 10 different runs (each corresponding to a different initial population) with the GA and the WOA. The solution results are listed in Table 2. The parameters in the GA were chosen after sensitivity analysis as follows: number of decision-making parameters = 7; population size = 100; number of iterations = 1,000; selection method was roulette wheel; crossover function was single point; mutation function was uniform; crossover probability = 0.8; and the mutation probability = 0.012. The WOA's parameters after sensitivity analysis were as follows: number of decision-making parameters = 7; $P_{\text{initial}} = 10$; $P_{\max} = 50$; $\sigma_{\text{initial}} = 2$; $\sigma_{\text{final}} = 0.001$; $n = 3$; $NoS_{\max} = 5$; $NoS_{\min} = 1$; and $\text{iter}_{\max} = 500$. The results of Table 2 establish that the WOA converged to 99.99% of the global optimum. Moreover, even the worst solution obtained with the WOA was better than the best solution from the GA. These results show that WOA is more effective than GA even in the type of constrained problem herein considered.

The convergence (correlation) graph for the mean value of the GA and the WOA for the 10 runs used to solve the bound problem is presented in Fig. 4. Fig. 4 shows that the WOA converged to the global optimum of the problem whereas the GA did so to a local optimum.

Reservoir Operation Model

The WOA was tested with a single-reservoir operation problem with irrigation function and with a four-reservoir problem in discrete and continuous domains. The objective function for the single-reservoir problem that meets downstream water demand is

$$\text{Min. OF} = \sum_{t=1}^{T} \left(\frac{\text{De}_t - R_t}{\text{De}_{\max}} \right)^2 \quad (9)$$

where OF = objective function of relative shortage during operation periods; t = period index; $t = 1, 2, \ldots, T$; T = number of operation periods; De_t = volume of required downstream water during operation period t; $\text{De}_{\max}$ = maximum volume of downstream water requirement during operation periods; and R_t = release volume during operation period t.

Table 1. Results of 10 Runs of the GA and the WOA for the Ackley Function with 5, 10, 20, 50, and 100 Dimensions

Dimension number	Optimization method	Minimum	Average	Maximum	SD	Coefficient of variation
5	GA	0.000153	0.000468	0.000964	0.622515	0.000292
	WOA	0.000053	0.000092	0.000123	0.243669	0.000023
10	GA	0.001519	0.000773	0.002124	0.780005	0.000603
	WOA	0.000016	0.000020	0.000023	0.103054	0.000002
20	GA	0.001519	0.003284	0.004741	0.309534	0.001016
	WOA	0.000030	0.000037	0.000044	0.134394	0.000005
50	GA	0.023687	0.107699	0.497494	1.481381	0.159543
	WOA	0.000083	0.000097	0.000106	0.071705	0.000007
100	GA	1.003671	1.140694	1.366686	0.094148	0.107394
	WOA	0.001410	0.442132	1.323137	1.350323	0.597021

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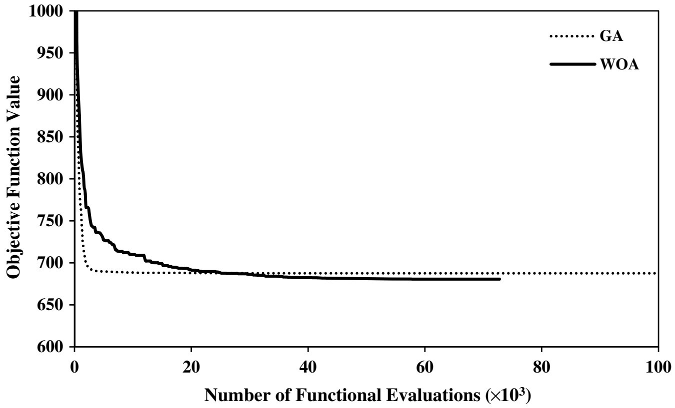

Fig. 4. Convergence of the GA and the WOA for 10 runs of the constrained mathematical problem

In the four-reservoir problem where the goal is to maximize the benefit gained from reservoir-system operation according to Eq. (10)

$$\text{Max. Be} = \sum_{i=1}^{n}\sum_{t=1}^{T} b_{i,t} \times R_{i,t} \tag{10}$$

where Be = income from allocated water; i = reservoir number; n = total number of reservoirs; $b_{i,t}$ = income function in period t for reservoir i; and $R_{i,t}$ = required volume in period t for reservoir i.

The key constraint in reservoir-operation modeling is the conservation of water volume as stated by Eq. (11)

$$S_{i,t+1} = S_{i,t} + Q_{i,t} + M_{n\times n}R_{i,t} - L_{i,t} - Sp_{i,t} \tag{11}$$

where $S_{i,t+1}$ = stored volume at the beginning of operation period $t+1$ in reservoir i; $S_{i,t}$ = stored volume at the beginning of operation period t in reservoir i; $Q_{i,t}$ = volume of monthly inflow operation period t in reservoir i; M = an $n \times n$ matrix expressing the hydraulic connectivity among reservoirs; $L_{i,t}$ = volume of losses in operation period t in reservoir i; and $Sp_{i,t}$ = volume of overflow from reservoir i in period t.

The reservoir storage falls between its maximum and minimum values

$$S\text{min}_i \leq S_{i,t} \leq S\text{max}_i \tag{12}$$

where Smin and Smax denote respectively the minimum and maximum storages of reservoir i.

The maximum and minimum values for release are defined by Eq. (13)

$$R\text{min}_i \leq R_{i,t} \leq R\text{max}_i \tag{13}$$

where $R\text{min}_i$ and $R\text{max}_i$ express the minimum and maximum allowable release volume from reservoir i, respectively.

In most reservoir operation models the losses equal the algebraic difference between precipitation and evaporation on the surface of reservoirs. Other losses, such as leakage from the bottom of the reservoir, are considered relatively small compared to other factors. Storage loss is calculated according to Eq. (14)

$$L_{i,t} = A_{i,t} \times E_{i,t} \tag{14}$$

where $A_{i,t}$ = lake surface in operation period t in reservoir i; and $E_{i,t}$ = average water level during operation period t in reservoir i. The lake surface is expressible as a function of the reservoir volume.

Reservoir overflow (spillage) occurs when the storage volume exceeds storage capacity according to the following equation:

$$Sp_{i,t} = \begin{cases} S_{i,t} - S\text{max}_i & \text{if } S_{i,t+1} > S\text{max}_i \\ 0 & \text{else} \end{cases} \tag{15}$$

The initial and final volumes are frequently required to equal each other, the so-called carryover condition, which is expressed by Eqs. (16) and (17)

$$S_{i,1} = S\text{initial}_i \tag{16}$$

$$S_{i,T+1} = S_{i,1} \tag{17}$$

where $S\text{initial}_i$ = initial reservoir volume before operation begins. A penalty function is applied when the carryover condition is not satisfied

$$P1_i = K1[S_{i,T+1} - S\text{initial}_i]^2 + c \tag{18}$$

where $P1_i$ = penalty function for violating carryover limitation; $K1$ = penalty coefficient; and c = constant value.

A penalty function is applied when reservoir storage is less than Smin

$$P2_{i,t} = K2[S\text{min}_i - S_{i,t}]^2 + d \tag{19}$$

where $P2_{i,t}$ = penalty function for violating minimum reservoir storage; $K2$ = penalty coefficient; and d = constant value. In general, the sum of the penalty functions is defined by Eq. (20). Depending on whether it is a maximization or minimization problem, it is added to or subtracted from the objective function, respectively

$$P = \sum_{i=1}^{n} P1_i + \sum_{i=1}^{n}\sum_{t=1}^{T} P2_{i,t} \tag{20}$$

in which P = sum of penalty functions.

Case Studies of Reservoir Operation

Single-Reservoir Operation Problem

The Bazoft reservoir's data for 5 years (1955–1960) was considered as a case study test the WOA. Bazoft dam is located in Chaharmahal and Bakhtiari, Iran, and it was built on the Bazoft River. It is a concrete arch dam whose height equals 160 m. Average annual flow into the reservoir is estimated at 2012 million cubic meters. Maximum and minimum of reservoir storage are 450 and 142 million cubic meters, respectively (Fallah-Mehdipour et al. 2011). Fig. 5 displays reservoir inflow and downstream water demand for the Bazoft reservoir. It is shown in Fig. 5 that in most periods the amount of monthly Bazoft River flow is less than the downstream water demand.

The Bazoft reservoir's surface-storage equation is

$$A_t = -0.0000002S_t^2 + 0.0128S_t + 0.7605 \tag{21}$$

where A_t is in $10^3 \times \text{km}^2$ and S_t is in $10^6 \times \text{m}^3$.

In this study the carryover condition (17) was not applied because of the long operation period. Only the penalty function (19) was used with coefficients $K2 = 2$, $d = 10$.

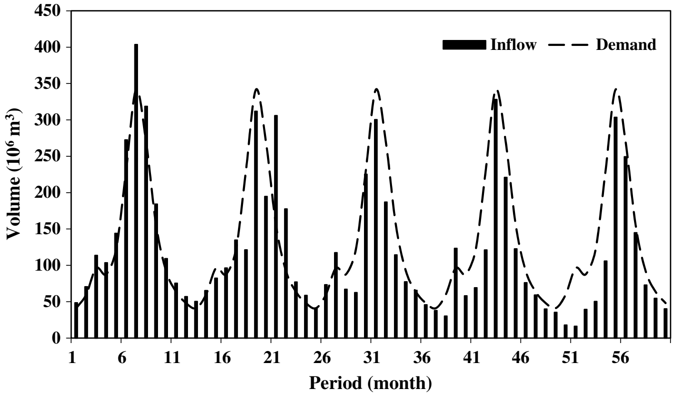

Fig. 5. Monthly discharge against water demand for the Bazoft reservoir in the five-year operation period

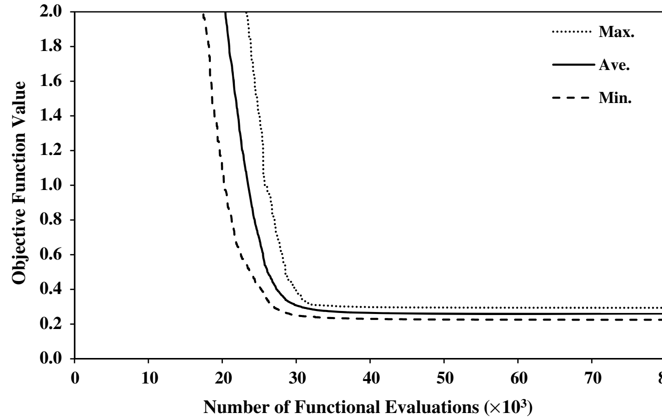

Fig. 6. Convergence of GA for 10 runs of the single-reservoir problem

The operation model of Bazoft reservoir was solved employing the NLP Solver from *Lingo 13.0*, and with the GA and the WOA. Sensitivity analyses were employed to determine the GA and WOA parameters. Next, the Bazoft reservoir's operation was optimized using 10 runs (each corresponding to a different initial population). The parameters of the GA were as follows: number of decision making parameters = 60; population size = 60; number of iterations = 2,000; selection method was the roulette wheel; the crossover function was single point; mutation function was uniform; crossover probability = 0.8; and the mutation probability = 0.015. The WOA parameters were as follows: number of decision-making parameters = 60; $P_{\text{initial}} = 10$; $P_{\max} = 30$; $\sigma_{\text{initial}} = 10$; $\sigma_{\text{final}} = 0.1$; $n = 3$; $S_{\max} = 5$; $S_{\min} = 0$; and $\text{iter}_{\max} = 1{,}000$. The values of the minimum, maximum, and mean standard deviation and the coefficient of variation for the 10 runs are listed in Table 3. According to Table 3 the best objective function value was 0.2250 for the GA, and 0.1624 for the WOA. The WOA optimal solution equaled 99.38% of the global optimum calculated with NLP. The WOA achieved lower standard deviation and coefficient of variation than the GA. Also, the worst value obtained employing WOA was better than the best solution from the GA.

The GA convergence graph is portrayed in Fig. 6. This graph shows the minimum and maximum value of the objective function of 10 runs for 2,000 iterations. Fig. 6 shows that the GA converged after approximately 30,000 evaluations of the objective function.

Table 3. Results of 10 Runs of the GA and the WOA for Single-Reservoir System

Run	GA	WOA	NLP
1	0.2731	0.1711	—
2	0.2755	0.1627	—
3	0.2614	0.1655	—
4	0.2779	0.1624	—
5	0.2346	0.1624	—
6	0.2661	0.1625	—
7	0.2250	0.1626	—
8	0.2400	0.1725	—
9	0.2348	0.1624	—
10	0.2937	0.1724	—
Minimum	0.2250	0.1624	0.1614
Average	0.2583	0.1656	—
Maximum	0.2937	0.1656	—
SD	0.0218	0.0038	—
Coefficient of variation	0.0843	0.0229	—

The WOA convergence graph is shown in Fig. 7. This diagram shows the minimum and maximum value of the objective function in 10 runs for 1,000 iterations. Fig. 7 shows that the WOA converged approximately after 5,000 evaluations of the objective function.

The graph of mean values of the objective function for 10 runs of the GA and the WOA is depicted in Fig. 8. It is evident in Fig. 8 that the WOA converged quickly to a solution, while the GA exhibited slower convergence.

The results of release values obtained from NLP, GA, and WOA for downstream requirements are shown in Fig. 9. Fig. 9 establishes that the results from NLP meet downstream demand fully until period 29. From period 29 onward, the results from NLP method exhibited the smallest number of failed periods, while GA had the maximum number of failed periods. Comparing release results from the NLP method with the WOA's release results in Fig. 9 indicates that release values obtained with the WOA are very close to those calculated with the NLP method in all periods. The largest difference between the results from NLP and the WOA occurred in period 47 and equaled 3 million cubic meters, whereas in other periods the deficit (difference) approximates the value of zero.

Fig. 10 graphs storage volumes calculated with NLP, GA, and WOA. The three methods calculated reservoir storage similarly up to period 30. After this period, there are larger differences between the stored volumes calculated with NLP and the GA. Conversely, the stored volumes obtained with the WOA correspond closely to those calculated with NLP.

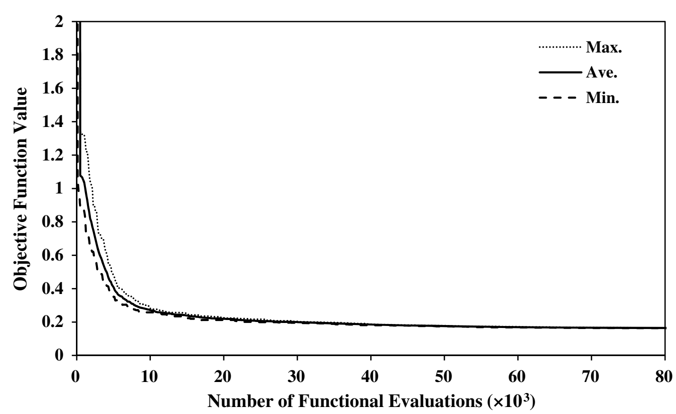

Fig. 7. Convergence of the WOA for 10 runs of the single-reservoir problem

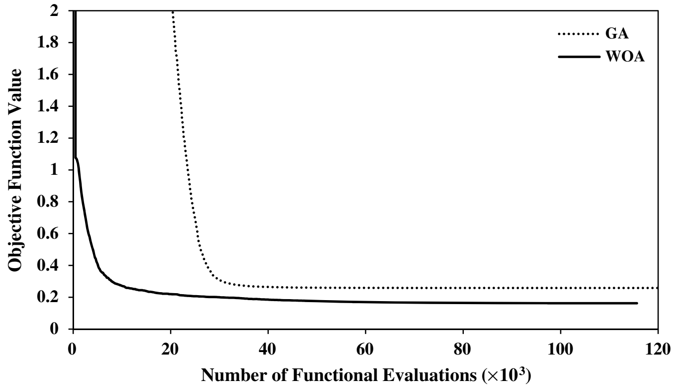

Fig. 8. Average of the objective function for 10 runs with the GA and the WOA for single-reservoir problem

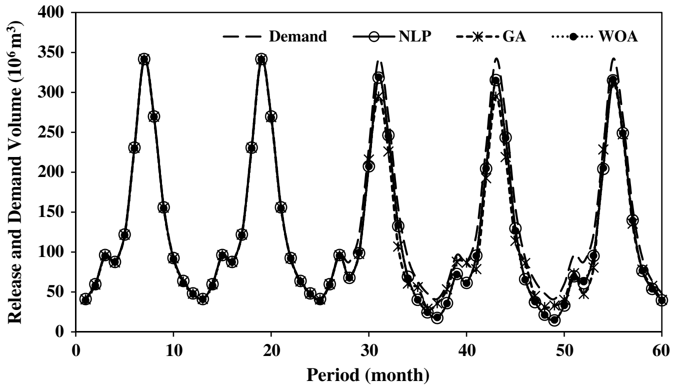

Fig. 9. Results of release volumes with NLP, GA, and WOA against demand, for single-reservoir problem to meet agricultural demand

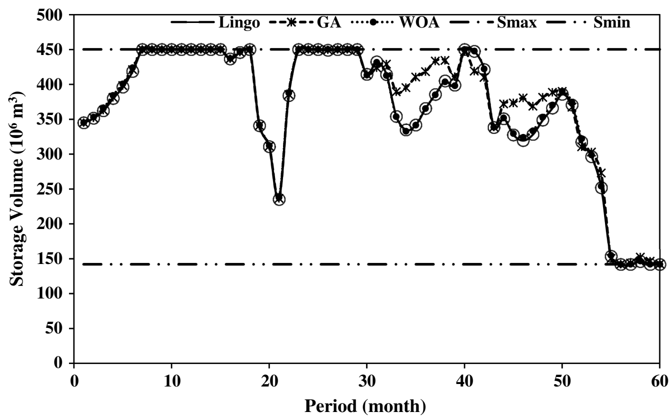

Fig. 10. Results of reservoir storage volume from NLP, GA, and WOA for single-reservoir to meet agricultural demand

Four-Reservoir System Problem

A schematic of the four-reservoir system is depicted in Fig. 11. The connectivity matrix for the four-reservoir system is

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & -1 \end{bmatrix} \tag{22}$$

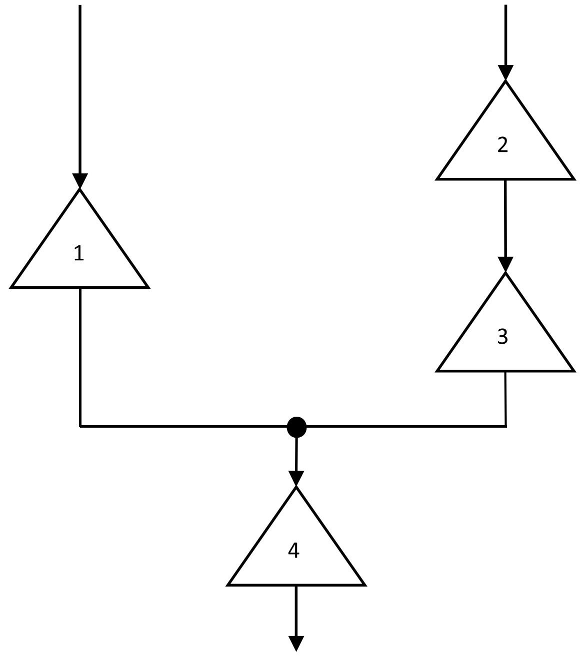

Fig. 11. Schematic of the four-reservoir system

The water released from reservoirs is used for irrigation and hydroelectricity. Reservoir losses were not considered in the operation model of this system. The following penalty function was employed to prevent exceeding the maximum storage:

$$P3_{i,t} = K3[S_{i,t} - S\max_{i,t}]^2 \tag{23}$$

This four-reservoir system was operated over a 12-period interval with each period lasting 2 h. The goal of the operation was maximization of the benefit of released water.

Continuous-Time, Four-Reservoir, Operation Problem

Te Chow and Cortes-Rivera (1974) introduced this problem, which used a continuous-time formulation of reservoir operation optimization. Murray and Yakowitz (1979) solved it. The minimum value of release for all reservoirs is 0.005 units and the maximum value of release for reservoirs 1– 4 equals 4, 4.5, 4.5, and 8 units respectively. The minimum reservoir storage is one unit and the maximum reservoir storage differs among periods.

The water storage volume at the beginning of operation equaled the final storage volume at the end of operation and is equal to 6, 6, 6 and 8 units for reservoirs 1–4, respectively. The penalty coefficients in Eqs. (18) and (19) equal $K1 = K2 = 40$, $c = d = 0$ (Heidari et al. 1971; Bozorg Haddad et al. 2011a).

The value of the global optimum and optimal operation rule for the four-reservoir system problem in continuous domain was calculated employing the LP solver from *Lingo 13.0*. Next, the GA and the WOA were used to solve this problem for 10 runs each corresponding to a different initial population. The GA was implemented with the number of decision making parameters = 48; population size = 200; number of iterations = 8,000; the selection method was the roulette wheel; crossover function was single point; mutation function was uniform; crossover probability = 0.7; and mutation probability = 0.06. The parameters of the WOA, after sensitivity analyses, were as follows: the number of decision making parameters = 48; $P_{\mathrm{initial}} = 10$; $P_{\max} = 50$; $\sigma_{\mathrm{initial}} = 4$; $\sigma_{\mathrm{final}} = 0.01$; $n = 4$; $S_{\max} = 5$; $S_{\min} = 0$; and $\mathrm{iter}_{\max} = 20{,}000$. The results of the 10 runs are listed in Table 4. It is shown in Table 4 that the WOA converged to 99.95% of the global optimum while

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Table 4. Results of the GA and the WOA for 10 Runs of the Four-Reservoir Problem in Continuous Time

Run	GA	WOA	LP
1	298.89	307.90	—
2	300.47	307.54	—
3	298.36	308.10	—
4	299.25	308.15	—
5	300.35	307.25	—
6	300.08	307.91	—
7	299.87	308.00	—
8	300.45	307.70	—
9	300.01	308.01	—
10	299.20	306.99	—
Minimum	298.36	306.99	—
Average	299.69	307.75	—
Maximum	300.47	308.15	308.29
SD	0.6890	0.3640	—
Coefficient of variation	0.0023	0.0011	—

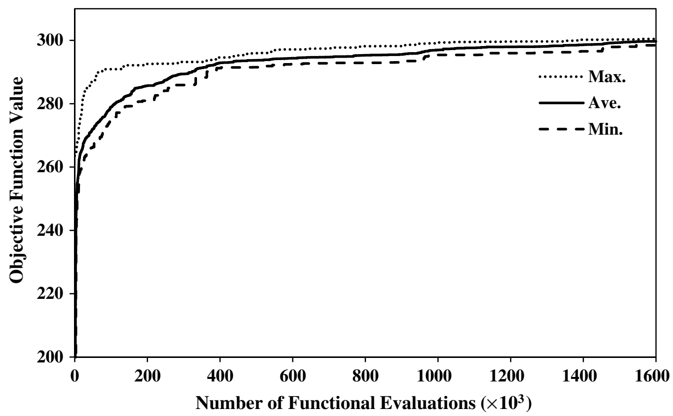

Fig. 12. Convergence of the GA for 10 runs of the four-reservoir problem in continuous time

the GA converged to 97.46% of the global optimum. The WOA exhibited lower values of standard deviation and change coefficient that those calculated with the GA. In addition, the worst solution obtained with the WOA was better than the best solution reached by GA. The value of the global optimum obtained with *Lingo* equaled 308.29, whereas the best values reached by GA and WOA in 10 runs were 300.47 and 308.15, respectively.

The convergence graphs of GA and WOA for the 10 runs are shown in Figs. 12–14. These graphs show the smallest and largest values of the metaheuristic function in 10 runs. It is shown in Fig. 14 that the GA converges to the solution faster than the WOA up to about 700,000 functional evaluations because of its larger population. Thereafter, the WOA exhibits better convergence to the optimal solution than the GA. Moreover, the results of release values obtained with LP, GA, and the WOA are graphed in Fig. 15. Fig. 16 represents storage volumes calculated with the LP, the GA and the WOA.

Discrete-Time, Four-Reservoir, Operation Problem

This problem was introduced and solved by Larson (1968) using a discrete-time formulation of the reservoir-operation optimization problem. It was also solved by Esat and Hall (1994) and Wardlaw and Sharif (1999). The water released from the reservoirs is used for irrigation and hydroelectricity. The minimum value of release for all reservoirs equals zero and the maximum value of release for reservoirs 1– 4 equals 10, 10, 10, and 15 units, respectively. The value of the initial water storage for all reservoirs is 5 units, and the final value of reservoir storage at the end of operation period for reservoirs 1– 4 equals 5, 5, 5, and 7 units respectively.

The operation model of four-reservoir system was implemented with LP in *Lingo* software. Then, the optimal operation rule was calculated with 10 runs of the WOA and the results of

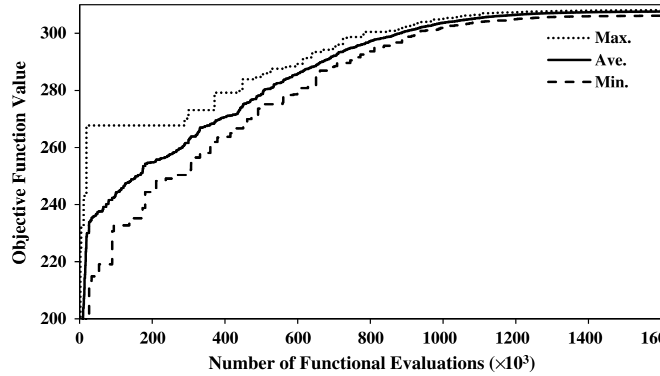

Fig. 13. Convergence of the WOA for 10 runs of four-reservoir problem in continuous time

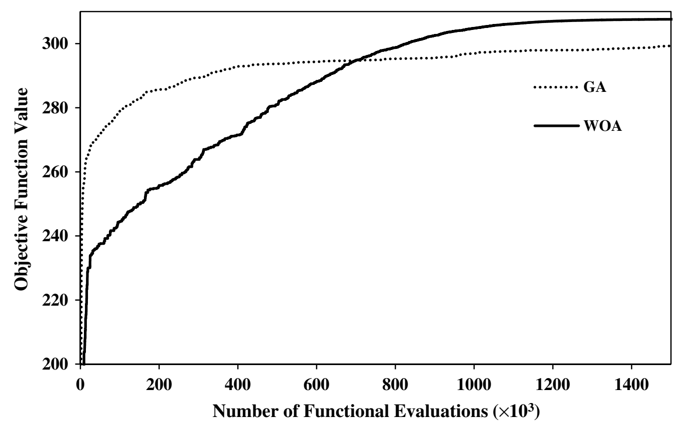

Fig. 14. Average of the objective function of 10 runs from the GA and the WOA for four-reservoir problem in continuous time

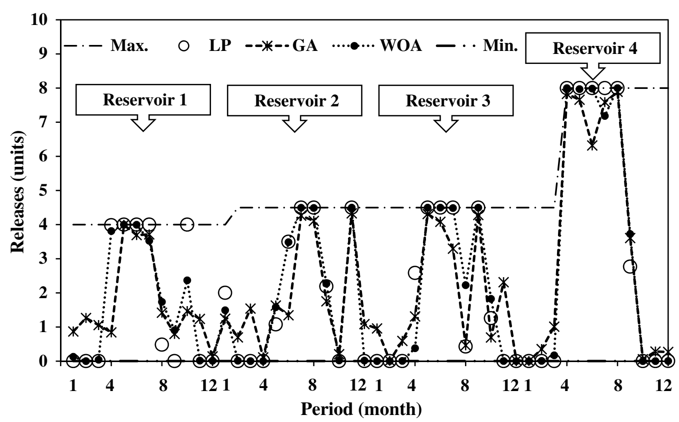

Fig. 15. Results of release volumes with LP, GA, and WOA against water demand for four-reservoir problem in continuous time

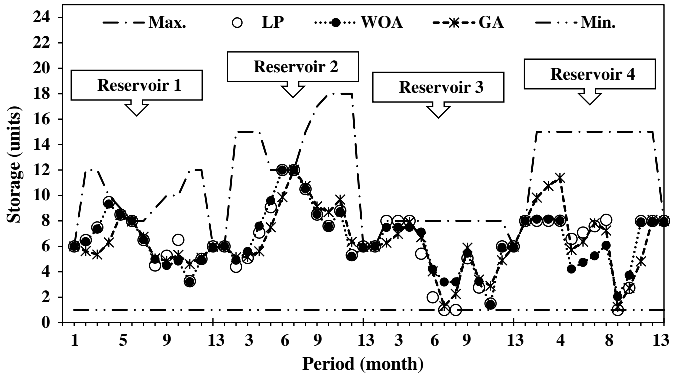

Fig. 16. Results of reservoir storage volume from LP, GA, and WOA for four-reservoir problem in continuous time

Table 5. Results of the GA and the WOA for 10 Runs of the Four-Reservoir Problem in Discrete Domain

Run	GA, Wardlaw and Sharif (1999)	WOA	LP
1	Not reported	401.1	—
2	Not reported	401.1	—
3	Not reported	401.2	—
4	Not reported	401.2	—
5	Not reported	401.3	—
6	Not reported	401.2	—
7	Not reported	401.3	—
8	Not reported	401.3	—
9	Not reported	401.1	—
10	Not reported	401.3	—
Minimum	Not reported	401.1	—
Average	Not reported	401.21	—
Maximum	401.3	401.3	401.3
SD	Not reported	0.03801	—
Coefficient of variation	Not reported	0.00009	—

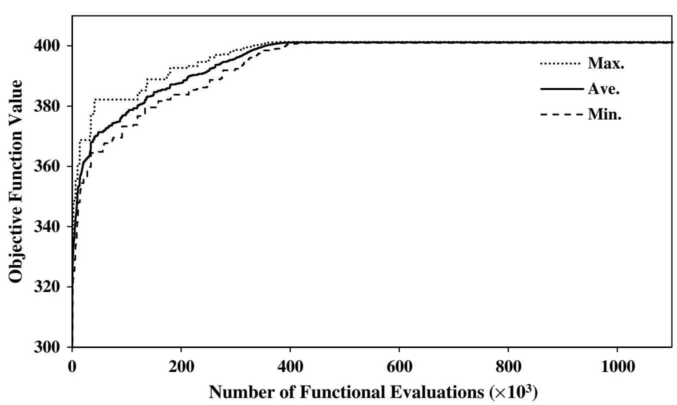

Fig. 17. Convergence of the WOA for 10 runs for four-reservoir problem in discrete domain

Wardlaw and Sharif (1999) were reported as those from the GA. Also, the values of the minimum, mean, maximum, standard deviation and the coefficient of variation for the 10 runs are listed in Table 5. In the WOA the number of decision-making parameters = 48; $P_{\text{initial}} = 10$; $P_{\max} = 40$; $\sigma_{\text{initial}} = 3$; $\sigma_{\text{final}} = 1$; $n = 3$; $S_{\max} = 5$; $S_{\min} = 0$; and $\text{iter}_{\max} = 5{,}000$. The best value

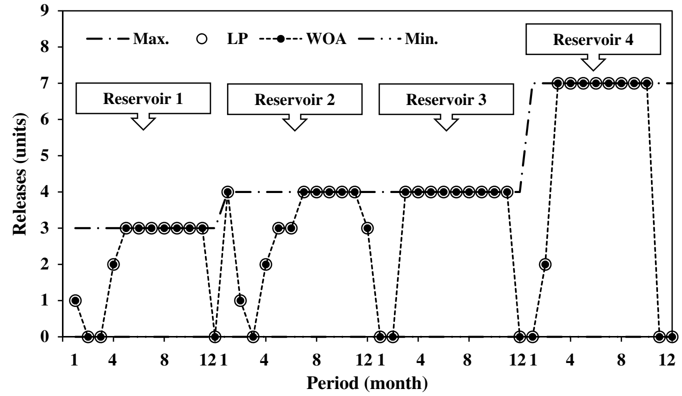

Fig. 18. Results of releases volumes with LP and WOA for four-reservoir problem in discrete domain

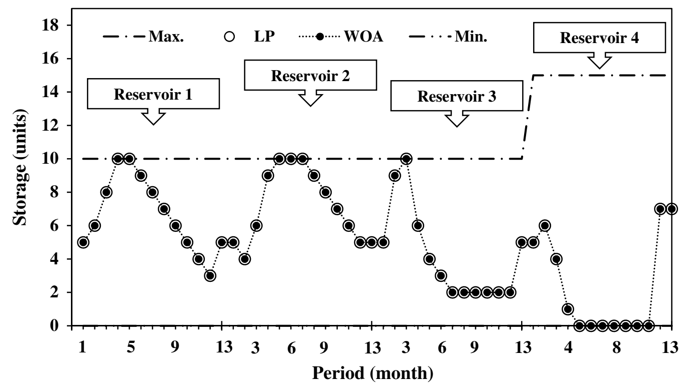

Fig. 19. Results of reservoir storage volume from LP and WOA for four-reservoir problem in discrete domain

of the objective function of the WOA in 10 runs was 401.3. The low standard deviation and coefficient of variation for the WOA's 10 runs shows that this algorithm performs well in solving reservoir-operation problems defined in discrete space. The convergence graph for the WOA for the 10 runs is shown in Fig. 17. This graph shows the minimum and maximum value of the objective function in 10 runs for 5,000 iterations. It is clear from Fig. 17 that the WOA exhibited acceptable convergence since the initial iterations.

The release values obtained with LP method and the WOA are plotted in Fig. 18. It can be seen in Fig. 18 that for all reservoirs in all periods the calculated release volumes with WOA are fully consistent with the calculated release volumes from LP.

Fig. 19 graphs calculated storage volumes with LP and the WOA. It is shown in Fig. 19 that the storage volumes calculated with the WOA in all periods are consistent with the volumes calculated with LP.

Conclusion

This study introduced and implemented the WOA to reservoir operation optimization, and compared the WOA results with those from NLP, LP and the GA. This study's results have demonstrated a superior performance of the WOA compared with that of the GA in solving several optimization problems, constrained or unconstrained. The value of the metaheuristic function obtained with WOA was 38.79% better than that from GA for the single-reservoir problem, and 2.53% percent better for the continuous four-reservoir problem. This indicates a better capacity of the WOA to achieve near-optimal solutions than the GA. Analysis of convergence graphs showed that the GA converges to a near solution faster than the WOA in the initial iterations, but the WOA converges faster to very near the global optimal solution in the latter iterations. The WOA exhibited lower standard deviation and coefficient of variation than the GA, proving the former algorithm superior accuracy and precision over the GA. Moreover, the WOA's convergence to the optimum in the discrete-time, four-reservoir, operation problem also demonstrates its capacity to solve problems of this type. It is recommended that further development and testing of the WOA with different problems be done to fully assess its capabilities to solve a variety of water-resources optimization algorithms.

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