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Structural gravity and fixed effects☆

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ABSTRACT

The gravity equation for trade flows is one of the most successful empirical models in economics and has long played a central role in the trade literature (Anderson, 2011). Different approaches to estimate the gravity equation, i.e. reduced-form or more structural, have been proposed. This paper examines the role of adding-up constraints as the key difference between structural gravity with "multilateral resistance" indexes and reduced-form gravity with simple fixed effects by exporter and importer. In particular, estimating gravity equations using the Poisson pseudo-maximum-likelihood estimator (Poisson PML) with fixed effects automatically satisfies these constraints and is consistent with the introduction of "multilateral resistance" indexes as in Anderson and van Wincoop (2003).

1. Introduction

The gravity equation is one of the most successful empirical models in economics and has been the focus of a very extensive literature in international trade (Anderson, 2011). The very good fit of the gravity equation for bilateral trade flows has long been recognized since Tinbergen (1962) and the many papers that followed.[1]

Various ways to specify and estimate the gravity equation have been proposed (see Feenstra, 2004; Head and Mayer, 2014). Specifications vary broadly along two dimensions. A first dimension concerns the error term. The second is the degree of model structure that is imposed on the estimation. Among the estimation approaches available, one possibility is to use the Poisson pseudo-maximum likelihood method (Poisson-PML). Santos Silva and Tenreyro (2006) show that Poisson-PML consistently estimates the gravity equation for trade and is robust to different patterns of heteroskedasticity and measurement error, which makes it preferable to alternative procedures such as ordinary least squares (using the log of trade flows) or non-linear least squares (in levels).[2]

There are also different trends in the specification of supply-side and demand-side effects in the gravity equation. Early papers have simply used total (multilateral) expenditures and total output for supply- and demand-side terms. It has been recognized, however, that adjustments are necessary to account for differences in market thickness across destinations (captured by the "inward multilateral-resistance index" in Anderson and van Wincoop, 2003) and source countries (captured by the "outward multilateral resistance index"). There are now two main ways to account for these adjustments. A set of papers introduces exporter and importer fixed effects to capture both market-size effects and multilateral-resistance indexes in a simple way (e.g. Harrigan, 1996; Redding and Venables, 2004). Another trend instead imposes more structure on the gravity equation. This approach has been put forward by Anderson and van Wincoop (2003), Anderson and Yotov (2010), and Balistreri and Hillberry (2007), with some variations in the restrictions imposed on the demand side (e.g., Fieler 2011) or supply side (e.g., Costinot, Donaldson and Komunjer, 2012).[3]

In this paper, I show that estimating gravity with Poisson PML and fixed effects is consistent with the equilibrium constraints imposed by more structural approaches such as those of Anderson and van

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[1] Note that most gravity equation estimates focus on the cross-section. Lai and Trefler (2002) is one of the few exceptions; they find that the gravity equation framework does not perform as well in time series.

[2] Poisson-PML is also consistent with the presence of zero bilateral trade flows, which are highly prevalent in disaggregated data. An alternative method by Helpman, Melitz and Rubinstein (2008) involves a 2-step estimation to structurally account for zeros.

[3] A growing literature also uses the MPEC approach, as in Balistreri et al (2011).

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Wincoop (2003) and Anderson and Yotov (2010). In particular, the estimated fixed effects in the Poisson PML specification are consistent with the definition of outward and inward multilateral resistance indexes and the equilibrium constraints that they need to satisfy. Therefore, gravity regressions with fixed effects and Poisson PML can be used as a simple tool to solve the estimation problem raised by Anderson and van Wincoop (2003).

More generally, the constraints imposed on multilateral-resistance indexes in the structural-gravity framework are equivalent to imposing adding-up constraints on the sum of trade flows for each source country and each destination. This result is valid for any estimator. However, when the Poisson-PML estimator is used, these constraints are automatically satisfied as long as we have exporter and importer fixed effects and consistent data. This adding-up property is specific to Poisson-PML regressions and could also be useful for other applications where we want to constrain the sum of fitted values to be fixed, because other estimators do not automatically satisfy adding-up constraints.[4]

In the last section, I estimate gravity equations and provide quantitative examples to illustrate these points. First, these results imply that the test of structural gravity performed by Anderson and Yotov (2010) is bound to support structural gravity when Poisson-PML is used. I verify this assertion using consistent data where outward trade flows sum up to output and inward trade flows sum up to expenditures. Secondly, I find large deviations between fitted output and observed output when gravity is estimated with importer and exporter fixed effects, especially with ordinary least squares (OLS) and Gamma-PML estimators. I also find large differences between multilateral-resistance indexes depending on whether they are constructed from importer or exporter fixed effects, unless we impose additional constraints on these indexes. Thirdly, there are systematic biases depending on market size. With OLS and Gamma-PML, the sum of fitted trade flows tends to be larger than observed output for large countries and smaller than observed output for small countries. This points to undesirable properties of OLS and Gamma-PML when no constraints on multilateral-resistance indexes are imposed.

2. The gravity model

A wide range of trade models generate relationships in bilateral trade flows that can be expressed by the following set of equations. For each exporter i and importer j, trade flows X_{ij} should satisfy:

$$X_{ij} = \frac{Y_i}{\Pi_i^{-\theta}} \cdot D_{ij}^{-\theta} \cdot \frac{E_j}{P_j^{-\theta}}. \tag{1}$$

In this equation, Y_i refers to total output in country i; E_j refers to total expenditure in country j; D_{ij} captures trade costs from i to j; and the parameter θ reflects the elasticity of trade flows to trade costs, which may have different structural interpretations depending on the model, as described below. Finally, the terms $P_j^{-\theta}$ and $\Pi_i^{-\theta}$ are called "multilateral resistance" indexes by Anderson and van Wincoop (respectively "inward" and "outward" resistance indexes). These two resistance terms should satisfy the following constraints for consistency, which define the "structural gravity" framework (Anderson, 2011).

Definition. "*Structural gravity*": *The patterns of trade flows X_{ij} are consistent with the "structural gravity" framework if they satisfy Eq. (1) with the following two constraints on multilateral-resistance terms P_j and Π_i:*

$$P_j^{-\theta} = \sum_i \frac{Y_i D_{ij}^{-\theta}}{\Pi_i^{-\theta}} \tag{2}$$

$$\Pi_i^{-\theta} = \sum_j \frac{E_j D_{ij}^{-\theta}}{P_j^{-\theta}}. \tag{3}$$

These equations define P_j and Π_i. Given output Y_i, expenditures E_j and trade costs $D_{ij}^{-\theta}$, the solution in $P_j^{-\theta}$ and $\Pi_i^{-\theta}$ to this system of two equations is unique, up to a constant (the proof of uniqueness is provided with Lemma 3 in Appendix A). As noted by Anderson and Yotov (2010), when $P_j^{-\theta}$ and $\Pi_i^{-\theta}$ satisfy Eqs. (2) and (3), $\lambda P_j^{-\theta}$ and $\Pi_i^{-\theta}/\lambda$ are also solutions, for any number $\lambda > 0$. This indeterminacy calls for a normalization; we thus impose $P_0 = 1$ for a benchmark importer $j = 0$. These equations can also be defined at the industry or product level. For convenience, I do not add industry subscripts but all results in the paper hold within each industry (as in Anderson and Yotov, 2010 and 2012).

This system of equations can be derived from various types of models. It is consistent with models based on Armington (1969) and Krugman (1980) with a constant elasticity of substitution in consumer preferences (Anderson and Van Wincoop, 2003; Redding and Venables, 2004, Fally, Paillacar and Terra, 2010, among many others). In these models, $\theta + 1$ corresponds to the elasticity of substitution. Models based on Melitz (2003), such as Chaney (2008), can also generate gravity equations, as above. In this case, the equivalent of θ would be the coefficient of the Pareto distribution of firm productivity; the coefficient is inversely related to productivity dispersion. As shown by Eaton and Kortum (2002), Ricardian models of trade are also fully consistent with gravity. In this case, the trade-cost elasticity θ corresponds to one of the coefficients of the Frechet distribution of productivity across product varieties (again, the coefficient is inversely related to productivity dispersion).[5] In all of the above-mentioned models, the inward multilateral resistance index $P_j^{-\theta}$ can be expressed as a function of the price index in the importing market. In turn, $\Pi_i^{-\theta}$ captures the degree of competition faced by exporter i.

Various theoretical features have been used to generate structural gravity equations, including a constant elasticity of substitution, Pareto distributions of productivity (Chaney, 2008, Costinot et al., 2011) and Frechet distributions (Eaton and Kortum, 2002). The key ingredient is that trade flows can be written as a product of an exporter term, an importer term and a term reflecting trade costs (separability condition). Another key ingredient is a consistent definition of output and expenditures.

Formally, Head and Mayer (2014) define "general gravity" when trade flows can be written as $X_{ij} = exp[e_i - \theta log D_{ij} + m_j]$ where e_i is invariant across importers and m_j is invariant across importers j. "General gravity" is in fact equivalent to "structural gravity" when output equals the sum of outward trade $Y_i = \sum_j X_{ij}$ and expenditures equal the sum of inward trade $E_j = \sum_i X_{ij}$. When trade satisfies the "general gravity" condition, we can re-express trade as in Eq. (1) with a unique set of inward and outward multilateral-resistance indexes satisfying Eqs. (2) and (3). This is shown formally in Lemma 3 in Appendix A. This equivalence has important empirical implications, which are illustrated with Lemma 1A and 1B in the next section.

3. Gravity with fixed effects

To estimate Eq. (1), there are broadly two approaches which differ in the treatment of exporter terms $\frac{Y_i}{\Pi_i^{-\theta}}$ and importer terms $\frac{E_j}{P_j^{-\theta}}$.

A first approach, the reduced-form, simply introduces exporter and importer fixed effects e_i and m_j without imposing any constraints on these terms. This approach ignores the structure proposed by Eqs. (2) and (3). The estimated equation can then be written:

$$X_{ij} = exp\left[e_i - \theta log D_{ij} + m_j\right] . \varepsilon_{ij} \tag{4}$$

[4] For instance, Poisson-PML could be useful in consumption choice models where the sum of expenditures is fixed for given subsets of observations.

[5] Gravity equations can also be motivated by Heckscher–Ohlin and specific-factor models (see Evenett and Keller, 2002).

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where ε_{ij} denotes an error term. Note that the two full sets of exporter and importer fixed effects are not of full rank.[6] In the remainder of the paper, the restriction $\hat{m}_0 = 0$ is imposed for the benchmark country $j = 0$. The trade cost variable, $logD_{ij}$, is often assumed to be a linear combination of the log of physical distance, dummies for common language, colonial links and free trade agreements, etc.[7] The use of fixed effects makes the gravity equation very easy to estimate. Various estimators have been used: ordinary least squares (in log), non-linear least-squares, Poisson-PML, Gamma-PML and negative binomial estimators have been employed to estimate Eq. (4). The results in this section apply to any of these estimators while the next section highlights particular properties of Poisson-PML.

Instead of using dummy variables, a more structural approach pioneered by Anderson and van Wincoop (2003) is to define exporter and importer terms as $e_i = log\left(\frac{Y_i}{\Pi_i^{-\theta}}\right)$ and $m_j = log\left(\frac{E_j}{P_j^{-\theta}}\right)$ and impose the following conditions on estimated multilateral resistance indexes $\widehat{\Pi_i^{-\theta}}$ and $\widehat{P_j^{-\theta}}$ (along with the normalization $P_0 = 1$)[8]:

$$\widehat{P_j^{-\theta}} = \sum_i \frac{Y_i \widehat{D_{ij}^{-\theta}}}{\widehat{\Pi_i^{-\theta}}} \tag{5}$$

$$\widehat{\Pi_i^{-\theta}} = \sum_j \frac{E_j \widehat{D_{ij}^{-\theta}}}{\widehat{P_j^{-\theta}}} \tag{6}$$

where E_j refers to *observed* expenditure by country j, Y_i refers to *observed* output in i, and $\widehat{D_{ij}^{-\theta}}$ is the estimated term for trade cost. Anderson and van Wincoop (2003) minimize the sum of squared errors in log while imposing Eqs. (5) and (6) as constraints in the minimization. Anderson and Yotov (2010) estimate Eq. (4) with fixed effects in a first step to obtain $\widehat{D_{ij}^{-\theta}}$ and then solve Eqs. (5) and (6) in a second step to obtain inward and outward resistance indexes. Head and Mayer (2014) propose estimating gravity with "structurally reiterated least squares" (SILS) by: i) estimating Eq. (4) with fixed effects to obtain $\widehat{D_{ij}^{-\theta}}$; ii) solving Eqs. (5) and (6) to obtain inward and outward resistance indexes (which depend on $\widehat{D_{ij}^{-\theta}}$); and iii) reiterating the first step using the second-step estimates of multilateral resistance indexes instead of fixed effects to obtain an updated estimate of $\widehat{D_{ij}^{-\theta}}$. Steps ii) and iii) are then reiterated until convergence is achieved.

While the structural approach exploits additional restrictions on multilateral-resistance indexes, these two approaches are not very different. In fact, the fixed-effect estimation is consistent with the structural-gravity framework if we use fitted output $\widehat{Y}_i \equiv \sum_j \hat{X}_{ij}$ and fitted expenditures $\widehat{E}_j \equiv \sum_i \hat{X}_{ij}$ instead of observed output and expenditures (where $\hat{X}_{ij}$ refers to fitted trade flows from the estimation of Eq. (4) with fixed effects). We can then redefine the system of Eqs. (5) and (6) (where $P_j^{-\theta}$ and $\Pi_i^{-\theta}$ are the two unknowns) in terms of fitted output and expenditures $\widehat{Y}_i$ and $\widehat{E}_j$ instead of observed values Y_i and E_j:

$$\widetilde{P_j^{-\theta}} = \sum_i \frac{\widehat{Y}_i \widehat{D_{ij}^{-\theta}}}{\widetilde{\Pi_i^{-\theta}}} \tag{7}$$

$$\widetilde{\Pi_i^{-\theta}} = \sum_j \frac{\widehat{E}_j \widehat{D_{ij}^{-\theta}}}{\widetilde{P_j^{-\theta}}}. \tag{8}$$

Thus, we obtain Lemma 1A:

Lemma 1. A) Substituting fitted output and expenditures

If Eq. (4) is estimated with importer and exporter fixed effects, the terms $\widetilde{P_j^{-\theta}}$ *and* $\widetilde{\Pi_i^{-\theta}}$ *defined by* $\widetilde{P_j^{-\theta}} \equiv \frac{\widehat{E}_j}{\widehat{E}_0} exp(-\hat{m}_j)$ *and* $\widetilde{\Pi_i^{-\theta}} \equiv \widehat{E}_0 \widehat{Y}_i\ exp(-\hat{e}_i)$ *are the unique solutions of Eqs. (7) and (8) (using fitted output, fitted expenditures and estimated trade costs* $\widehat{D_{ij}^{-\theta}}$*).*[9]

In other words, fitted values from the fixed effects regressions are consistent with the two general-equilibrium conditions imposed by the gravity model if we use fitted expenditures and output instead of solving for multilateral-resistance indexes with observed expenditures and output.[10] Another illustration of the role adding-up constraints and separability is the following equivalence. The estimation of structural gravity (using observed output and expenditures) is in fact equivalent to including fixed effects and imposing the sum of fitted trade to equal observed output and expenditures for each source and each destination:

Lemma 1. B) Imposing observed output and expenditures

If Eq. (4) is estimated with importer and exporter fixed effects $\hat{e}_i$ and $\hat{m}_j$, imposing $\sum_j \hat{X}_{ij} = Y_i$ and $\sum_i \hat{X}_{ij} = E_j$ is equivalent to imposing $\hat{e}_i = log\left(\frac{Y_i}{\widetilde{\Pi_i^{-\theta}}}\right)$ and $\hat{m}_j = log\left(\frac{E_j}{\widetilde{P_j^{-\theta}}}\right)$ and the restrictions (5) and (6) using observed output Y_i and expenditures E_j.

It is important to note that, in general, the sum of fitted trade does not add up to observed output and expenditures unless such a constraint is explicitly added in the estimation. Hence, it is important to either redefine output and expenditures (Lemma 1A) or impose fitted trade to sum up to observed expenditures and output (Lemma 1B). In Section 5, I illustrate the deviations between observed output and fitted output with various estimators. Section 5 shows that there are systematic deviations between fitted output and observed output depending on market size, which constitutes an argument for estimating structural gravity and imposing the sum of fitted trade. As shown in the next section, the Poisson-PML estimator is an exception: fitted output and expenditures always equal observed output and expenditures as long as exporter and importer fixed effects are included.

4. Structural fit of Poisson PML

A now widely-used strategy (following Santos Silva and Tenreyro, 2006) is to estimate Eq. (4) using Poisson pseudo-maximum-likelihood. The Poisson-PML estimator identifies the coefficients using

[6] The sum of importer dummy variables equals the sum of exporter dummy variables.

[7] Note that θ cannot be identified from the coefficients for physical distance and usual trade costs variables. What is estimated is the product of θ with the elasticity of trade costs w.r.t these variables. A special case would be to use tariffs (as in Caliendo and Parro, 2014), for which the coefficient should in principle equal θ.

[8] Anderson and van Wincoop (2003) focus on a special case with symmetric trade costs and output being equal to expenditures. The results here allow for asymmetry so that they are also valid at the industry level where output and expenditures can largely differ. See e.g., French (2014) for potential aggregation issues.

[9] The normalization $\widetilde{P_0^{-\theta}} = 1$ is satisfied given that we impose $\hat{m}_0 = 0$ in the estimation.

[10] Note that the fixed effects should not be held constant for counter-factual simulations (such as the border removal in Anderson and van Wincoop, 2003). While fixed effects may be consistent with estimated trade costs, fitted output and expenditures, multilateral-resistance indexes need to be recomputed and fixed effects adjusted accordingly if trade costs are changed in the counter-factual exercise.

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the same first-order conditions that are used by the maximum-likelihood estimator derived from the Poisson distribution. However, Poisson-PML does not require the dependent variable to be Poisson distributed. The estimation procedure is fairly easy to implement and robust to misspecifications (Gourieroux, Monfort and Trognon, 1984). As shown by Santos Silva and Tenreyro (2006), the first-order conditions associated with Poisson-PML provide a natural estimator, whether or not trade flows follow a Poisson distribution.[11]

In addition, the Poisson-PML estimator has special properties if we compare fitted output and expenditures to their observed counterparts. When there are no missing observations,[12] we obtain the following result:

Lemma 2. *If Eq. (4) is estimated using Poisson PML with exporter fixed effects, fitted production equals observed production. Similarly, when importer fixed effects are included, fitted expenditures by importer and product equal observed expenditures*:

$$\sum_j \hat{X}_{ij} = \sum_j X_{ij} = Y_i \quad and \quad \sum_i \hat{X}_{ij} = \sum_i X_{ij} = E_j.$$

This lemma is directly derived from the first-order conditions associated with the Poisson-PML approach (see Appendix for details).[13]

According to Lemma 1B, imposing consistency of the multilateral-resistance indexes with the structural gravity framework is equivalent to imposing the sum of fitted trade to equal output and expenditures for each country. Because these constraints are systematically satisfied with Poisson-PML, we obtain this very practical result:

Proposition 1. If Eq. (4) is estimated using Poisson PML with exporter and importer fixed effects, the two multilateral-resistance terms defined by $\widehat{P_j^{-\theta}} \equiv \frac{E_j}{E_0}\ exp(-\hat{m}_j)$ and $\widehat{\Pi_i^{-\theta}} \equiv E_0 Y_i\ exp(-\hat{e}_i)$ are the unique solutions of Eqs. (5) and (6), where E_j and Y_i refer to observed expenditures and output.

Anderson and Yotov (2012) suggest comparing unconstrained fixed effects and theory-consistent multilateral-resistance indexes (solving Eqs. (5) and (6)) as a "test" of structural gravity. Unfortunately, Proposition 1 shows that such a test is bound to succeed if the Poisson-PML is used as an estimator; it is therefore not a test of structural gravity. Anderson and Yotov (2012) do not actually find a perfect fit. An explanation is that trade flows do not perfectly add up to output Y_i and expenditures E_j in the data (information on output and international trade flows generally comes from different sources). I do find a perfect fit using GTAP data with harmonized information on trade, output and expenditures (see Section 5).

Proposition 1 adds to other advantages of using fixed effects and Poisson PML, and complements the arguments provided by Santos Silva and Tenreyro (2006). An important point to note is that Lemma 2 and Proposition 1 hold even if the dependent variable does not actually follow a Poisson distribution. No assumption is needed on the distribution of trade flows except that the conditional mean of trade flows is positive (Poisson-PML also allows for zero trade flows).

Moreover, these properties are specific to Poisson-PML, which is the only PML estimator that yields Lemma 2. If we estimate the gravity equation in logs with OLS and fixed effects, we obtain: $\sum_j log\hat{X}_{ij} = \sum_j logX_{ij}$ and $\sum_i log\hat{X}_{ij} = \sum_i logX_{ij}$, which do not imply equality between the sums in level. If we estimate gravity by taking trade flows in levels and minimizing the sum of squared errors (NLLS), the inclusion of exporter and importer fixed effects implies $\sum_j \hat{X}_{ij}X_{ij} = \sum_j X_{ij}^2$ and $\sum_i \hat{X}_{ij}X_{ij} = \sum_i X_{ij}^2$. For Gamma-PML, the inclusion of fixed effects implies that the ratio $\frac{X_{ij}}{\hat{X}_{ij}}$ averages to unity for each exporter and each importer. None of these equalities implies equality between $\sum_j \hat{X}_{ij}$ and $\sum_j X_{ij}$ or between $\sum_i \hat{X}_{ij}$ and $\sum_i X_{ij}$.[14]

This specificity of Poisson-PML is stated formally in Proposition 2 below. In a more general setting, let us denote by y_i the left-hand-side variable for observations indexed by i, with $y_i \in \mathbf{R}^+$, and by $\hat{y}_i \in \mathbf{R}^{+*}$ the fitted value. A pseudo-maximum-likelihood estimator maximizes the following objective function:

$$\max_{\lambda_i} \sum_i logf(y_i, \lambda_i)$$

where, for each $\lambda > 0$, $f(y, \lambda)$ is the p.d.f. of a random variable with mean λ. We further impose that λ depends log-linearly on K independent variables $x_i^{(k)}$ indexed by k where the coefficients b_k have to be estimated:

$$\lambda_i = exp\left(\sum_k b_k x_i^{(k)}\right).$$

Hence $\hat{y}_i = \lambda_i$ when λ_i is the solution of the above maximization. For any subset A of observations, we define the dummy variable D_A as having a value 1 for observations in A and zero otherwise. We can now uniquely characterize the Poisson-PML estimator as follows:

Proposition 2. The Poisson-PML estimator is the only pseudo-maximum-likelihood estimator such that, for any subset A of observations and its associated dummy variable D_A, the inclusion of D_A in the set of right-hand-side variables implies that the sum of fitted values $\sum_{i \in A} \hat{y}_i$ equals the sum of observed values $\sum_{i \in A} y_i$ over the set A.

Since maximum-likelihood estimators can be considered as a special case of PML (when the likelihood function to maximize is derived from the assumed distribution of the dependent variable), Proposition 2 also implies that no ML-estimator other than Poisson-ML satisfies this adding-up property.[15]

The proof (in the appendix) is organized in two steps. First, such an estimator is necessarily from the linear-exponential family, i.e. estimators for which $\frac{\partial logf}{\partial log\lambda}$ is a linear function of y. I show that this is the only class of estimators for which regressing a variable y_i on a constant term yields the average $\bar{y} = \frac{1}{N}\sum_i y_i$ as the fitted value (Lemma 4 in Appendix A). The second step shows that $\frac{\partial logf}{\partial log\lambda}$ must also be linear in λ to satisfy the properties of Proposition 2; this corresponds to the Poisson-PML estimator.

5. Illustrations

To what extent does the estimation of structural gravity differ from reduced-form gravity with fixed effects? In this section, I examine various specifications to illustrate the previous findings. I compare output and expenditures to the sum of fitted trade flows for each country, as well as multilateral-resistance indexes implied by either importer or exporter fixed effects.

[11] Poisson-PML does not require the dependent variable to be an integer and is consistent with over-dispersion (i.e. with a conditional variance larger than the conditional expectation). Santos Silva and Tenreyro (2011) provide additional evidence on the good performance of PPML by also allowing for a large fraction of zeros. For more details, see also: http://personal.lse.ac.uk/tenreyro/LGW.html.

[12] The case of missing observations for internal trade flows is discussed in Appendix B.

[13] Independent work by Arvis and Shepherd (2013) uncovers a similar property of Poisson-PML estimators as preserving the total sum of the dependent variable. They do not, however, examine applications to the structural gravity framework. Arvis and Shepherd (2013) also argue that this property is unique to Poisson-PML, but their argument implicitly relies on an assumption that a solution always exists, which is not true for PPML.

[14] Moreover, this property is not satisfied if we use trade shares $\frac{X_{ij}}{E_j}$ (share of imports from i for each importer j) with Poisson-PML. The sum of fitted trade for each exporter would not sum up to output in general.

[15] In turn, PML estimators can be seen as a special case of generalized-moment-method (GMM) estimators where moment conditions are exactly identified.

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5.1. Data

Data on trade flows come from the Global Trade Analysis Project (GTAP).[16] The dataset has the main advantage of providing harmonized information on production, consumption and international trade flows by country and sector. It is micro-consistent to the extent that domestic and international trade flows sum up to output for each source country and sum up to expenditures for each destination country. This is an important property since Lemma 2 and Proposition 1 would not apply otherwise. If such equalities were not satisfied in the data, the multilateral-resistance indexes implied by the fixed effects with Poisson-PML would not satisfy the structural gravity constraints based on actual output and expenditures.[17]

As usual in the gravity equation literature, I regress trade flows on various trade-cost proxies. In addition to the fixed effects or multilateral-resistance indexes, right-hand-side variables include the log of distance, a border-effect dummy (equal to one for international flows), contiguity, as well as dummies for colonial ties and common language. Data on distance and other trade costs are provided by the CEPII.

5.2. Specifications

Table 1 below describes the trade costs coefficients for various specifications using aggregate data across country pairs (excluding services). In column (1), I regress trade flows on importer and exporter fixed effects as well as on trade cost proxies using Poisson-PML. In column (2), I redo the same exercise with OLS using the log of trade flows as the dependent variable. In column (3), I minimize the sum of the squared error term, defined as the difference between observed trade flows and fitted trade flows (in log), by simultaneously imposing the structural constraints on multilateral-resistance indexes (Eqs. (5) and (6) using observed output and expenditures). In column (4), I follow the "structurally-iterated-least-squares" approach developed by Head and Mayer (2014).[18] Finally, I use non-linear least squares in column (5) and the Gamma-PML estimator in column (6), using trade flows in levels without imposing further structural constraints. I provide bootstrap standard errors for all specifications. Beyond the constraints imposed on multilateral-resistance indexes, an important source of differences across these specifications is the weight each of them places on small versus large trade flows (Santos Silva and Tenreyro, 2006; Head and Mayer, 2014). Poisson-PML and especially least squares in level (NLLS) put relatively more weight on large trade flows than do least squares in log (OLS and SILS) and Gamma-PML.

As already known in the literature, the trade cost coefficients differ across specifications. In particular, the Poisson-PML estimator yields the smallest distance coefficient while the largest coefficient is obtained with OLS and SILS (columns 2 and 4). As illustrated in Head and Mayer (2014), this difference can potentially be explained by a non-linear effect of distance, with a stronger effect on small trade flows (captured by OLS, SILS and Gamma-PML) and a weaker effect on large trade flows (captured by NLLS and Poisson-PML). Poisson-PML also yields a relatively small border effect. The estimated border effect is largest with

Table 1
Gravity equation: trade cost coefficients.

Dependent variable:	Trade flows					
	(1)	(2)	(3)	(4)	(5)	(6)
Log or level:	Level	Log	Log	Log	Level	Level
Specification:	PPML	OLS	OLS + MR	SILS	NLLS	Gamma
Distance (log)	**−0.818**	**−1.106**	**−1.225**	**−1.362**	**−1.251**	**−1.189**
	[0.072]	[0.028]	[0.029]	[0.032]	[0.193]	[0.037]
Border effect	**2.740**	**4.331**	**2.353**	**3.472**	**1.882**	**4.823**
	[0.218]	[0.271]	[0.118]	[0.109]	[0.354]	[0.295]
Contiguity	**0.404**	**1.029**	**3.534**	**0.266**	−0.009	**0.929**
	[0.120]	[0.120]	[0.080]	[0.128]	[0.221]	[0.133]
Common language	**0.502**	**0.737**	**−0.189**	**0.716**	0.288	**0.663**
	[0.146]	[0.067]	[0.078]	[0.089]	[0.252]	[0.096]
Colonial link	0.036	**0.539**	**0.376**	**1.383**	−0.122	**0.745**
	[0.146]	[0.111]	[0.108]	[0.127]	[0.280]	[0.160]
Imposing MR constraints	/	No	Yes	Yes	No	No
Countries	94	94	94	94	94	94
Observations	8836	8836	8836	8836	8836	8836

Notes: The dependent variable is bilateral trade, either in log or level; columns (1), (2), (5) and (6) include simple fixed effects by importer and exporter while columns (3) and (4) impose additional constraints on multilateral-resistance indexes (Eqs. (5) and (6) using observed output and expenditures); bootstrap standard errors in brackets; in bold: coefficients significant at 5%.

Gamma-PML and OLS when simple fixed effects are used (reduced-form gravity). As in Anderson and Van Wincoop (2003), the border effect dramatically decreases when structural constraints on multilateral-resistance indexes are added (columns 3 and 4 for OLS). Other differences between specifications include a small colonial link coefficient for Poisson-PML, a negative language coefficient for OLS when MR constraints are simultaneously imposed, and larger standard errors with non-linear least squares (NLLS).

Given these differences in trade costs estimates, it is important to gauge the relative merit of each specification. I show in the remainder of this section that traditional gravity estimates relying on OLS or Gamma-PML with fixed effects (without imposing multilateral-resistance constraints) have undesirable properties in terms of predicted output and expenditures.

5.3. Output and multilateral-resistance indexes

While Eqs. (7) and (8) are automatically satisfied across all specifications when we use fitted output and expenditures (Lemma 1A), I examine here quantitatively to what extent the traditional multilateral-resistance Eqs. (5) and (6) are violated when they are not imposed in the estimation procedure (with observed output and expenditures). Using estimates on trade costs and fixed effects, we can construct implied multilateral-resistance indexes in various ways, using either exporter or importer fixed effects. For instance, the inward multilateral-resistance index $\hat{P}_j^{-\theta}$ implied by importer fixed effects $\hat{m}_j$ can be constructed as follows:

$$\left(\hat{P}_j^{-\theta}\right)^{FM} = exp\left[-\hat{m}_j\right] \frac{E_j}{E_{USA}}$$

which satisfies the normalization imposed on the reference country ($P_{USA} = 1$). Alternatively, we can use exporter fixed effects[19] combined with estimated trade costs $\widehat{D_{ij}^{-\theta}}$:

$$\left(\hat{P}_j^{-\theta}\right)^{FX} = \sum_i \exp[\hat{e}_i] \widehat{D_{ij}^{-\theta}} E_{USA}^{-1}.$$

[16] GTAP data version 7 (Narayanan and Walmsley, 2008). Another excellent dataset with consistent information on trade flows is provided by the CEPII (Head and Mayer, 2014). The key results presented here are robust to using CEPII and Comtrade data.

[17] Because of missing observations, this requirement was not met by the data used in Anderson and Yotov (2010), which explains the discrepancy in our results.

[18] In column (3), the sum of squared errors (in log) is minimized by simultaneously imposing structural constraints. In column (4), trade costs coefficients are obtained by minimizing the sum of squared errors (in log) conditional on multilateral-resistance indexes. Multilateral-resistance indexes are then recomputed conditional on estimated trade costs. These two operations are repeated until convergence is achieved. As a result, the trade costs proxies are orthogonal to the error term with SILS (as in simple OLS) but not with the simultaneously-constrained least squares (OLS + MR). However, the R-squared is lower for the simultaneously-constrained least squares (OLS + MR) than for SILS. Hence, the second method (SILS) is more robust if we focus on the trade costs coefficients but less robust if the primary goal is to estimate MR indexes.

[19] Redding and Venables (2004) use exporter fixed effects to construct "Market Access" $\hat{P}_j^{-\theta}$.

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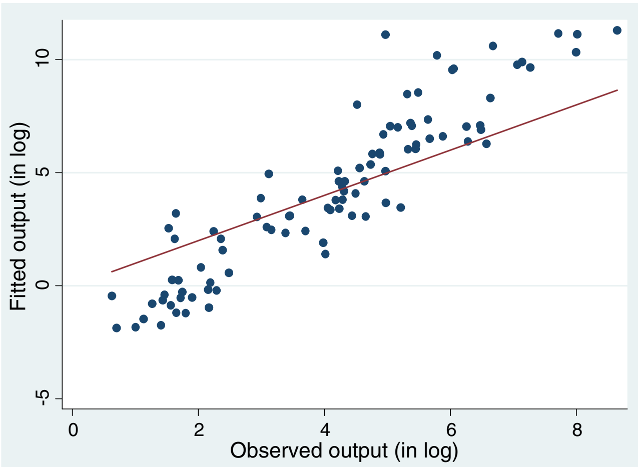

Fig. 1. Comparing fitted output (using OLS) and observed output.

The two approaches are equivalent with Poisson-PML or with additional constraints on the multilateral-resistance indexes (columns 1, 3 and 4 of Table 1 and Table 3). In other cases, there are large differences between the two definitions, comparing indexes based on importer fixed effects versus exporter fixed effects. An indicator of this misalignment is the interquartile range of $\log\left[\left(\hat{P}_i^{-\theta}\right)^{FX}/\left(\hat{P}_i^{-\theta}\right)^{FM}\right]$. It is zero for structural gravity (columns 3 and 4) and Poisson-PML (column 1), but it equals 2.769 for OLS with fixed effects (column 2), 0.369 for NLLS (column 5) and 2.290 for Gamma-PML (column 6) when only simple fixed effects are included. I find very similar results by comparing the outward multilateral-resistance index constructed with exporter fixed effects: $\left(\hat{\Pi}_i^{-\theta}\right)^{FX}$ to the one constructed with importer fixed effects: $\left(\hat{\Pi}_i^{-\theta}\right)^{FM}$.

As Lemma 1B suggests, the violations of the constraints on multilateral resistance indexes (using observed output and expenditures) imply that fitted output $\hat{Y}_i \equiv \sum_j \hat{X}_{ij}$ also largely differs from observed output $Y_i = \sum_j X_{ij}$. To be more precise, we can link the difference between fitted and actual output (in log) to the difference between the two outward multilateral-resistance indexes $\left(\hat{\Pi}_i^{-\theta}\right)^{FX}$ and $\left(\hat{\Pi}_i^{-\theta}\right)^{FM}$:

$$\Delta logY_i = log\left(\hat{\Pi}_i^{-\theta}\right)^{FM} - log\left(\hat{\Pi}_i^{-\theta}\right)^{FX} \tag{9}$$

with a similar expression for actual and fitted expenditures:

$$\Delta logE_j = log\left(\hat{P}_i^{-\theta}\right)^{FX} - log\left(\hat{P}_i^{-\theta}\right)^{FM} \tag{10}$$

where $\Delta logY_i \equiv log[\hat{Y}_i/Y_i]$ denotes the bias in fitting output and $\Delta logE_j \equiv log[\hat{E}_j/E_j]$ denotes the bias in fitting expenditures. Hence, the large differences between $\left(\hat{\Pi}_i^{-\theta}\right)^{FX}$ and $\left(\hat{\Pi}_i^{-\theta}\right)^{FM}$ translate into equally large differences between fitted and actual output. This also illustrates the point of Lemma 1B: imposing Eqs. (5) and (6) on the multilateral-resistance indexes implies that $\hat{Y}_i = Y_i$ and $\hat{E}_j = E_j$.

These differences are far from innocuous as the bias varies systematically with country size. For instance, OLS estimates inflate trade for large markets and reduce trade for small markets. This is illustrated in Fig. 1: fitted output exceeds output for the largest countries (points above the diagonal line) and tends to be smaller than observed output for the smallest markets.[20] As shown in Table 2, regressing the bias $\Delta logY_i \equiv log\hat{Y}_i - logY_i$ on observed output (in log) yields a coefficient that is large and significant for OLS and Gamma-PML (columns 1 and 3), which confirms that fitted output tends to be overinflated for larger economies. The coefficient is negative for NLLS (column 2). Similar results are obtained for $\Delta logE_i$.

Table 2
Do gravity equations inflate large countries? $\Delta logY$, $\Delta logE$ and country size.

Dependent variable:	$\Delta logY_i$			$\Delta logE_i$		
	(1)	(2)	(3)	(4)	(5)	(6)
Log output	0.738	−0.069	0.544	0.688	−0.083	0.539
	[0.065]	[0.021]	[0.066]	[0.061]	[0.014]	[0.065]
First-stage gravity:	OLS	NLLS	Gamma	OLS	NLLS	Gamma
Imposing MR constraints	No	No	No	No	No	No
Countries	94	94	94	94	94	94

Notes: OLS regressions; dependent variables: $\Delta logY_i$ (see Eq. (9)) and $\Delta logE_i$ (see Eq. (10)); robust standard errors in brackets; all coefficients are significant at the 1% level; and each column corresponds to a different specification of the gravity equation estimation in the first stage to construct $\Delta logY_i$ and $\Delta logE_i$. Note that the dependent variables $\Delta logY_i$ and $\Delta logE_i$ equal zero when we use Poisson-PML to estimate gravity or when we impose MR constraints.

Concretely, this means that importer and exporter fixed effects tend to be biased downward for large countries and upward for small countries. These biases have important implications for multilateral-resistance indexes. With OLS and Gamma-PML, the inward multilateral-resistance term $\left(\hat{P}_i^{-\theta}\right)^{FM}$ and the outward multilateral-resistance term $\left(\hat{\Pi}_i^{-\theta}\right)^{FX}$ tend to be underestimated for large markets and overestimated for small markets. Table 3 illustrates this point: $log\left(\hat{P}_i^{-\theta}\right)^{FM}$ is positively correlated with (log) output for Poisson-PML and structural gravity estimations (columns 1, 3 and 4). When OLS or Gamma-PML is used without imposing any constraint on multilateral-resistance indexes, it is slightly negatively correlated with log output (columns 2 and 6). If we instead use exporter fixed effects to construct $\hat{P}_i^{-\theta}$, these results are reversed for OLS and Gamma-PML, with much larger correlations with market size than with other specifications (columns 7 and 9).[21]

Tables 2 and 3 focus on correlations between market size and either $\Delta log\ Y_i$, $\Delta log\ E_i$ or $P_i^{-\theta}$ to illustrate the differences between specifications, but similar results are obtained if per capita income or other country characteristics are substituted for market size. In light of these results, one should be wary of trade costs coefficients and should be cautious in interpreting multilateral-resistance indexes with an estimator other than Poisson-PML if structural gravity constraints are not imposed. Imposing these constraints or using Poisson-PML appear to be good practices especially when the multilateral-resistance indexes are used in a second step for other empirical purposes (e.g., to explain wages, as in Redding and Venables, 2004, or final demand, as in Caron et al., 2014). There are still large differences in coefficients among Poisson-PML and other specifications that do impose the full structure. It is beyond the scope of the paper to argue for a specific estimator, but Poisson-PML seems particularly appealing because structural gravity constraints are automatically satisfied and the method is easy to implement.

6. Concluding remarks

This paper shows that Poisson-PML regressions exhibit interesting properties that can be particularly useful for the estimation of gravity equations for trade flows. Specifically, the estimation of gravity with Poisson-PML and exporter and importer fixed effects is consistent with a more structural approach (as in Anderson and van Wincoop, 2003) that imposes further restrictions on exporter and importer

[20] I find the same results when I examine trade within each sector: these deviations are driven by market size rather than by per capita income or other country characteristics.

[21] The results presented in Table 2 focus on the inward multilateral-resistance index $\hat{P}_i^{-\theta}$ but the same results hold for the outward multilateral-resistance index $\hat{\Pi}_i^{-\theta}$.

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Table 3
Regressing constructed inward MR indexes on observed output.

Dependent var:	$log\left(\hat{P}_i^{-\theta}\right)^{FM}$						$log\left(\hat{P}_i^{-\theta}\right)^{FX}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output	0.263	−0.096	0.228	0.369	0.345	0.039	0.591	0.262	0.578
	[0.018]	[0.028]	[0.045]	[0.049]	[0.043]	[0.023]	[0.048]	[0.034]	[0.056]
First-stage	PPML	OLS	OLS + MR	SILS	NLLS	GPML	OLS	NLLS	GPML
MR constraints	/	No	Yes	Yes	No	No	No	No	No
Countries	94	94	94	94	94	94	94	94	94

Notes: OLS regressions; dependent variable: log inward MR based on importer fixed effects (columns 1 to 6) or exporter fixed effects (columns 7 to 9); robust standard errors in brackets; all coefficients are different from zero at a 1% significance level except in column (6); and each column corresponds to a different specification of the gravity equation estimation in the first stage to construct the inward multilateral resistance index.

terms ("multilateral resistance" indexes). Furthermore, the inclusion of exporter and importer fixed effects in the Poisson-PML estimation of gravity implies that fitted output and expenditures (defined as the sum of fitted outward and inward trade flows for each country) perfectly match observed output and expenditures, respectively. This property is unique to the Poisson-PML estimator.

When other estimators are used, estimating gravity with simple fixed effects is no longer consistent with the structural gravity framework defined by Anderson and Van Wincoop (2003), unless multilateral-resistance indexes are redefined using fitted output and expenditures instead of observed output and expenditures. In practice, however, there are large differences between observed output and fitted output implied by gravity equations with simple fixed effects, especially with OLS and Gamma-PML: total output and expenditures are biased upward for large economies and downward for smaller economies. Similarly, inward and outward multilateral-resistance indexes appear to be biased with OLS and Gamma-PML, with the sign of the bias depending on market size and on whether these indexes are constructed using importer or exporter fixed effects. Given these results, one should put more trust in specifications of gravity equations where either Poisson-PML is used or the full gravity structure is imposed.

Appendix A. Proofs of propositions

Before proving the two lemmas and propositions from the main text, Lemma 3 (below) formally states the equivalence between separability with adding-up constraints and "structural gravity" (see Section 2 on the theory background):

Lemma 3. *Suppose that trade flows satisfy*:

$$log\, X_{ij} = e_i - \theta\, log\, D_{ij} + m_j \quad (11)$$

such as θ is a constant parameter, e_i is invariant across importers and m_j is invariant across importers j. Suppose also that output and expenditures are consistent with the sum of outward and inward trade flows: $Y_i = \sum_j X_{ij}$ *and* $E_j = \sum_i X_{ij}$. *There exists a unique pair of variables P_j and Π_i (with $P_0 = 1$) such that X_{ij} is consistent with the "structural gravity" framework.*

Proof of Lemma 3. Suppose that X_{ij} can be written as a function of exporter and importer effects as well as bilateral trade costs:

$$X_{ij} = exp\left[e_i + logD_{ij}^{-\theta} + m_j\right].$$

Suppose also that output and expenditures are defined by $Y_i \equiv \sum_j X_{ij}$ and $E_j \equiv \sum_i X_{ij}$. These two equalities can be rewritten as:

$$\begin{cases} \sum_j exp\left[e_i + logD_{ij}^{-\theta} + m_j\right] = Y_i \\ \sum_i exp\left[e_i + logD_{ij}^{-\theta} + m_j\right] = E_j \end{cases}$$

or equivalently:

$$\begin{cases} \sum_j D_{ij}^{-\theta} E_0 exp(m_j) = E_0 Y_i\, exp(-e_i) \\ \sum_i D_{ij}^{-\theta} E_0^{-1} exp(e_i) = E_0^{-1} E_j\, exp(-m_j) \end{cases}.$$

After defining $P_j^{-\theta} \equiv E_0^{-1} E_j\, exp(-m_j)$ and $\Pi_i^{-\theta} \equiv E_0 Y_i\, exp(-e_i)$, and incorporating into the previous two equations, we obtain Eqs. (2) and (3):

$$\sum_j D_{ij}^{-\theta} \frac{E_j}{P_j^{-\theta}} = \Pi_i^{-\theta} \quad \text{and} \quad \sum_i D_{ij}^{-\theta} \frac{Y_i}{\Pi_i^{-\theta}} = P_j^{-\theta}. \quad (12)$$

This proves that $\Pi_i^{-\theta}$ and $P_j^{-\theta}$ are solutions of Eqs. (2) and (3). Moreover, we can check that $P_0^{-\theta} = 1$ for $j = 0$.

We still need to prove *uniqueness*: for a given set of trade costs $D_{ij}^{-\theta}$, output Y_i and expenditures E_j, the solution in $P_j^{-\theta}$ and $\Pi_i^{-\theta}$ to Eqs. (2) and (3) is unique, up to the normalization $P_0^{-\theta} = 1$.

Suppose that $\Pi_i^{-\theta}$ and $P_j^{-\theta}$, as well as $(\Pi_i')^{-\theta}$ and $(P_j')^{-\theta}$ are two solutions to Eqs. (2) and ((3)). Let us define x_i as the ratio of $(\Pi_i')^{-\theta}$ to $\Pi_i^{-\theta}$ and y_j as the ratio of $(P_j')^{-\theta}$ to $P_j^{-\theta}$. To prove that the solution is unique, we need to show that $x_i = 1$ and $y_j = 1$ for all i and j.

Using Eq. (3), we can re-write y_j as:

$$y_j = \frac{\sum_i Y_i D_{ij}^{-\theta} (\Pi_i')^{\theta}}{\sum_i Y_i D_{ij}^{-\theta} \Pi_i^{-\theta}}.$$

Given that x_i is defined as the ratio of $(\Pi_i')^{-\theta}$ to $\Pi_i^{-\theta}$, we can rewrite y_j as an average of $1/x_i$ with weights $Y_i D_{ij}^{-\theta} \Pi_i^{\theta}$:

$$y_j = \frac{\sum_i Y_i D_{ij}^{-\theta} \Pi_i^{\theta} \left(\frac{1}{x_i}\right)}{\sum_i Y_i D_{ij}^{-\theta} \Pi_i^{\theta}}.$$

Similarly, we can express x_i as a weighted average of $1/y_j$:

$$x_j = \frac{\sum_i E_j D_{ij}^{-\theta} P_j^{\theta} \left(\frac{1}{y_j}\right)}{\sum_j E_j D_{ij}^{-\theta} P_j^{\theta}}.$$

Let us now proceed by contradiction and suppose that y_j differs from unity for at least one country. Since $y_0 = 1$ for $j = 0$ with our normalization, it means that the y's are strictly different between at least two countries j. Let us denote the minimum value by $y^* = min_j\{y_j\}$. If there are at least two y_j with strictly different values, the same holds for $1/y_j$ and there is at least one country j for which $1/y_j < 1/y^*$. Since x_i is a weighted average of all the $1/y_j$'s, it implies that all x_i's are strictly smaller than $1/y^*$. This inequality is strict as long as the weights $E_j D_{ij}^{-\theta} P_j^{\theta}$ are all strictly positive, which is implicitly assumed here (zero weights for country j would imply zero inward trade for country j).

Since all x_i's are strictly smaller than $1/y^*$, we obtain that $min_i \frac{1}{x_i}$ is strictly larger than y^*. In turn, since all y's correspond to a weighted

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average of $1/x_i$'s, we obtain that y_i is strictly larger than y^* for all i. The strict inequality contradicts the assumption that the lower bound y^* is reached for at least one country and that at least two values of y differ. It proves that $y_j = 1$ for all j, and we can also conclude that $x_i = 1$.

Proof of Lemma 1A. Let us denote:

$$\hat{X}_{ij} = exp\left[\hat{e}_i + log\widehat{D_{ij}^{-\theta}} + \hat{m}_j\right]$$

where hats refer to the fitted variable in the gravity Eq. (4) estimated with fixed effects. We define fitted output and fitted expenditures by $\hat{Y}_i \equiv \sum_j \hat{X}_{ij}$ and $\hat{E}_j \equiv \sum_i \hat{X}_{ij}$.

The proof of Lemma 1A follows exactly the same steps as the proof of Lemma 3 by using fitted trade, fitted output and fitted expenditures, where the solution of Eqs. (7) and (8) would be the same as for Lemma 3 (above) using fitted expenditures and fixed effects:

$$\widetilde{P_j^{-\theta}} \equiv \hat{E}_0^{-1}\hat{E}_j exp(-\hat{m}_j) \text{ and } \widetilde{\Pi_i^{-\theta}} \equiv \hat{E}_0 \hat{Y}_i exp(-\hat{e}_i)$$

instead of $P_j^{-\theta} \equiv E_0^{-1} E_j exp(-m_j)$ and $\Pi_i^{-\theta} \equiv E_0 Y_i exp(-e_i)$.

Given fitted output, fitted expenditures and fitted expenditures, we can show that $\widetilde{P_j^{-\theta}}$ and $\widetilde{\Pi_i^{-\theta}}$ are the unique solutions. The proof is the same as above in Lemma 3 using fitted values.

Proof of Lemma 1B. The proof is again similar to Lemma 3 (above in Appendix) and Lemma 1A. Let us denote:

$$\hat{X}_{ij} = exp\left[\hat{e}_i + log\, \widehat{D_{ij}^{-\theta}} + \hat{m}_j\right]$$

where the hats refer to estimated coefficients. If we use observed output Y_i and observed expenditures E_j to define $\widehat{P_j^{-\theta}} \equiv E_0^{-1} E_j exp(-\hat{m}_j)$ and $\widehat{\Pi_i^{-\theta}} \equiv E_0 Y_i exp(-\hat{e}_i)$, the above equation becomes:

$$\hat{X}_{ij} = \frac{Y_i}{\widehat{\Pi_i^{-\theta}}} \widehat{D_{ij}^{-\theta}} \frac{E_j}{\widehat{P_j^{-\theta}}}$$

where $\widehat{\Pi_i^{-\theta}}$ and $\widehat{P_j^{-\theta}}$ replace fixed effects.

In the estimation, imposing $\sum_j \hat{X}_{ij} = Y_i$ and $\sum_i \hat{X}_{ij} = E_j$ is equivalent to imposing:

$$\begin{cases} \sum_j \frac{Y_i}{\widehat{\Pi_i^{-\theta}}} \widehat{D_{ij}^{-\theta}} \frac{E_j}{\widehat{P_j^{-\theta}}} = Y_i \\ \sum_i \frac{Y_i}{\widehat{\Pi_i^{-\theta}}} \widehat{D_{ij}^{-\theta}} \frac{E_j}{\widehat{P_j^{-\theta}}} = E_j \end{cases}$$

which, in turn, is equivalent to the constraints (5) and (6):

$$\begin{cases} \sum_j \widehat{D_{ij}^{-\theta}} \frac{E_j}{\widehat{P_j^{-\theta}}} = \widehat{\Pi_i^{-\theta}} \\ \sum_i \widehat{D_{ij}^{-\theta}} \frac{Y_i}{\widehat{\Pi_i^{-\theta}}} = \widehat{P_j^{-\theta}} \end{cases}.$$

Again, the proof of uniqueness is the same as for Lemma 3 above.

Proof of Lemma 2. As shown in Gourieroux, Monfort and Trognon (1984), the maximization of the log-likelihood associated with Poisson distributions yields simple first-order conditions and the solution is unique. They show that, if a variable y_i is regressed on a set of K variables $x_i^{(k)}$ with $k = 1,..., K$, the first-order conditions are:

$$\sum_i x_i^{(k)} (y_i - \hat{y}_i) = 0$$

for each variable k, where $\hat{y}_i$ denotes the fitted value and takes the functional form: $\hat{y}_i = exp\left[\sum_k \hat{b}_k x_i^{(k)}\right]$.

When one of the independent variables $x_i^{(k)}$ is a dummy variable D_i^A equal to one for a subset of observations $i \in A$, the first-order condition associated with this variable can be written:

$$\sum_i D_i^A (y_i - \hat{y}_i) = \sum_{i \in A} (y_i - \hat{y}_i) = 0$$

which also implies that the sum of fitted values equals the sum of observed values on this subset: $\sum_{i \in A} y_i = \sum_{i \in A} \hat{y}_i$. Using this result for the gravity equation, Lemma 2 is obtained by simply writing this first-order condition for exporter and importer fixed effects. When one of the independent variables is a dummy variable that takes the value 1 for a given exporter i and zero otherwise the first-order condition related to this dummy variable can be written:

$$\sum_j \left(X_{ij} - \hat{X}_{ij}\right) = 0$$

which proves the first part of Lemma 2. The second part of Lemma 2 is obtained by looking at the first-order condition related to importer fixed effects when we include a dummy variable that takes the value 1 only for a given importer j.

Proof of Proposition 1. Proposition 1 follows from Lemma 1B using the additional result that $\hat{Y}_i = Y_i$ and $\hat{E}_j = E_j$ when Poisson-PML is used (Lemma 2).

Proof of Proposition 2. To prove Proposition 2, I use of the following lemma which provides a simple characterization of PML estimators from the linear-exponential family:

Lemma 4. With a PML estimator from the linear-exponential family, the average $\bar{y} \equiv \frac{1}{N}\sum_i y_i$ is the fitted value when regressing the dependent variable y_i on a constant term. Conversely, if a PML estimator always yields the average as the fitted value of a regression on a constant term, then this estimator is from the linear-exponential family.

Proof of Lemma 4. Let us denote by $log\, f(y, \lambda)$ the log-likelihood function and by $\varphi(y, \lambda) = \frac{\partial log f}{\partial log \lambda}$ its first derivative w.r.t λ. The linear-exponential family of PML estimators corresponds to the special case where:

$$\varphi(y, \lambda) = g(\lambda) \cdot (y - \lambda)$$

(see Gourieroux et al., 1984).[22] With this family of estimators, it is simple to verify that the average $\bar{y} \equiv \frac{1}{N}\sum_i y_i$ satisfies the first-order condition associated with the constant term since we would have:

$$\sum_{i=1}^{N} \varphi(y_i, \bar{y}) = \sum_{i=1}^{N} g(\bar{y})(y_i - \bar{y}) = g(\bar{y}) \sum_{i=1}^{N} (y_i - \bar{y}) = 0.$$

The reciprocal part of Lemma 4 is also useful to prove Proposition 2. It mirrors Theorem 2 in Gourieroux et al. (1984) stating that strongly-consistent PML estimators are necessarily from the linear-exponential family.

Suppose that, for a PML-estimator, the average $\bar{y}$ is always the fitted value when regressing y_i on a constant term. The primary goal is to prove that $\varphi(\lambda, y)$ is linear in y.

For any given pair (y, λ) with $y > 0$ and $\lambda > 0$, and for n sufficiently large, $y' \equiv \frac{n\lambda}{n-2} - \frac{2y}{n-2}$ is also positive. I apply the property for $y_1 = y -$

[22] Gourieroux et al. (1984) define the exponential family by $f(y, \lambda) = exp[A(\lambda) + B(y) + C(\lambda)y]$ where $A(\lambda)$ has to satisfy: $A'(\lambda) = -C'(\lambda)y$ (Property 1 in Gourieroux et al., 1984). These two definitions are equivalent.

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ε, $y_2 = y + \varepsilon$ and $y_i = \frac{n\lambda}{n-2} - \frac{2y}{n-2}$ for all $i = 3,..., n$. One can check that λ is the arithmetic average of the y_i's and therefore we should have:

$$\varphi(y-\varepsilon,\lambda) + \varphi(y+\varepsilon,\lambda) + (n-2)\varphi\left(\frac{n\lambda}{n-2} - \frac{2y}{n-2}, \lambda\right) = 0.$$

I apply again the above property to the same set of y's and λ's with $\varepsilon = 0$ instead. I obtain:

$$2\varphi(y,\lambda) + (n-2)\varphi\left(\frac{n\lambda}{n-2} - \frac{2y}{n-2}, \lambda\right) = 0.$$

Combining with the previous equation, we obtain:

$$\varphi(y,\lambda) = \frac{\varphi(y-\varepsilon,\lambda) + \varphi(y+\varepsilon,\lambda)}{2}$$

which is true for any λ and $y > 0$ and any small enough $\varepsilon > 0$. Further assuming that g is twice differentiable in y with a continuous second derivative, the above equality implies that g is linear in y. Hence there exist two real functions $g(\lambda)$ and $h(\lambda)$ such that:

$$\varphi(y,\lambda) = g(\lambda)y - h(\lambda).$$

Since $\varphi(y, \lambda) = 0$ for $\lambda = y$, we also obtain that $h(\lambda) = g(\lambda)\lambda$ and $\varphi(y, \lambda) = g(\lambda)(y - \lambda)$.

Proof of Proposition 2 (continued). Since the constant term is a dummy for the full set of observations, the assumptions in Proposition 2 implies that a PML estimator satisfying the adding-up properties also yields the arithmetic average as the fitted variable of a regression on a constant term. Hence, using Lemma 4, such an estimator is from the exponential family. The exponential family is however quite large (Gaussian, Poisson, Gamma, Binomial, etc.). Now, we need to show that only the Poisson-PML estimator satisfies the adding-up properties of Proposition 2.

More specifically, we need to show that the function $g(\lambda)$ is constant and does not depend on λ. If $g(\lambda)$ is constant, the estimator would then be equivalent to the Poisson-PML estimator.

We want to prove by contradiction that $g'(\lambda) = 0$ for all $\lambda > 0$. To do so, suppose that $g'(a) > 0$ for a given a (the proof works the same way if we assume instead that $g'(a) < 0$). There exists b strictly greater than a but sufficiently close to a such as $g'(\lambda) > 0$ and $g(\lambda)$ is strictly increasing on $\lambda \in [a, b]$. Without loss of generality, we can also assume that $g(\lambda)$ never equals zero on the segment $[a, b]$.[23] We then construct a dependent and an independent variable based on these two values a and b, and show that the first-order conditions imply a contradiction.

Given these two distinct values a and b, we define a dependent and an independent variable for four observations:

- Dependent variable: $y_1 = y_2 = \frac{a+b}{2}$, $y_3 = a$ and $y_4 = b$
- Independent variable: $x_1 = x_3 = \log a$ and $x_2 = x_4 = \log b$.

Let us then regress y on x with two dummy variables: a dummy equal to one for the first two observations and a dummy equal to one for the last two observations (note that a constant term would be redundant). Let us denote by λ_i the predicted value for y_i, by a the coefficient for x_i, by γ_{12} the coefficient for the dummy variable for the first two observations and by γ_{34} the coefficient for the dummy variable for the last two observations. The fitted values are then:

$$\lambda_1 = \exp[\alpha \log a + \gamma_{12}] \qquad \lambda_2 = \exp[\alpha \log b + \gamma_{12}]$$

$$\lambda_3 = \exp[\alpha \log a + \gamma_{34}] \qquad \lambda_4 = \exp[\alpha \log b + \gamma_{34}].$$

Given the assumptions made in Proposition 2, having a dummy for the first two observations implies that the sum of the fitted values equals the sum of the dependent variables for the first two observations:

$$\exp[\alpha \log a + \gamma_{12}] + \exp[\alpha \log b + \gamma_{12}] = a + b.$$

Similarly, for the last two observations:

$$\exp[\alpha \log a + \gamma_{34}] + \exp[\alpha \log b + \gamma_{34}] = a + b.$$

These two conditions imply that the coefficient for the dummy variable is the same for both subsets of observations: $\gamma_{12} = \gamma_{34} \equiv \gamma$ and imply also that $\lambda_1 = \lambda_3$ and $\lambda_2 = \lambda_4$.

The first-order condition for the dummy for the first two observations gives:

$$g(\lambda_1)\left(\frac{a+b}{2} - \lambda_1\right) + g(\lambda_2)\left(\frac{a+b}{2} - \lambda_2\right) = 0. \tag{13}$$

In turn, the first-order condition for the dummy for the last two observations gives:

$$g(\lambda_3)(a-\lambda_3) + g(\lambda_4)(b-\lambda_4) = 0. \tag{14}$$

Taking the difference between the two conditions, and using the fact that $\lambda_1 = \lambda_3$ and $\lambda_2 = \lambda_4$, we obtain:

$$g(\lambda_1)\left(a - \frac{a+b}{2}\right) + g(\lambda_2)\left(b - \frac{a+b}{2}\right) = 0$$

which also implies that $g(\lambda_1) = g(\lambda_2)$. To obtain a contradiction, the next step is to show that the two fitted values λ_1 and λ_2 are distinct and lie on the $[a, b]$ segment.

The first-order condition in a (with $\lambda_1 = \lambda_3$ and $\lambda_2 = \lambda_4$) gives:

$$(\log a)g(\lambda_1)\left(\frac{3a+b}{4} - \lambda_1\right) + (\log b)g(\lambda_2)\left(\frac{a+3b}{4} - \lambda_2\right) = 0$$

while the sum of Eqs. (13) and (14) gives:

$$g(\lambda_1)\left(\frac{3a+b}{4} - \lambda_1\right) + g(\lambda_2)\left(\frac{a+3b}{4} - \lambda_2\right) = 0.$$

Given that $g(\lambda_1)$ and $g(\lambda_2)$ are non-zero, these two equations imply the following fitted values:

$$\lambda_1 = \frac{3a+b}{4} \qquad \text{and} \qquad \lambda_2 = \frac{a+3b}{4}.$$

Hence, combining with the results above, we obtain that: $g\left(\frac{3a+b}{4}\right) = g\left(\frac{a+3b}{4}\right)$ which contradicts the strict monotonicity of g on the $[a, b]$ segment.

Appendix B. Estimation of gravity with missing values

What happens when internal trade flows are missing? Or, equivalently, when output data have missing observations? Internal trade flows are often imputed as the difference between output and total exports. Output data are largely available at the aggregate level but industry-level data are more scarce at the industry level for developing countries.

If internal trade flows are missing for country i, then total fitted trade flows (i.e. total fitted exports) perfectly match total observed exports when exporter fixed effects are included in a Poisson-PML estimation

[23] Otherwise, we can restrict our attention on an interior segment $[a', b']$ that satisfies this property.

of gravity. The same result holds for imports when importer fixed effects are included. For each exporter i for which internal flows X_{ii} are missing, the Poisson-PML estimator imposes:

$$\sum_{j,j\neq i}\hat{X}_{ij}=\sum_{j,j\neq i}X_{ij}=X_i^{tot}\qquad\text{and}\qquad\sum_{j,\ \ j\neq i}\hat{X}_{ji}=\sum_{j,\ \ j\neq i}X_{ji}=M_i^{tot}.$$

We could then use fixed effects estimates $\hat{e}_i$ and $\hat{m}_i$ and trade costs estimates $\widehat{D_{ii}^{-\theta}}$ to infer missing internal trade flows $\hat{X}_{ii}$ and then reconstruct output and expenditures as: $\hat{Y}_i \equiv \hat{X}_{ii}+X_i^{tot}=\sum_j \hat{X}_{ij}$ and $\hat{E}_i \equiv \hat{X}_{ii}+M_i^{tot}=\sum_j \hat{X}_{ji}$. Using Lemma 1A, inferred trade flows, output and expenditures would then be consistent with the multilateral resistance indexes implied by the fixed-effects estimates. Moreover, fitted output would still equal observed output in all the cases where output data are not missing.

Appendix C. Inclusion of border dummies

In general, the estimation of Eq. (4) involves a dummy for international trade flows as one of the variables to proxy for trade costs (dummy variable B_{ij} being equal to one if $i \neq j$). Such a dummy can be identified when data on internal trade flows (X_{ii}) are available. Estimates typically exhibit large border effects ("home-bias puzzle" raised by McCallum 1995).

With Poisson-PML, the inclusion of a border effect in the gravity equation also has important implications for the sum of fitted exports. In particular, the Poisson-PML first-order condition associated with the border effect implies that the sum of fitted exports across all countries equals the sum of observed exports:

$$\sum_{i,j,i\neq j}\hat{X}_{ij}=\sum_{i,j,i\neq j}X_{ij}.$$

The proof is similar to Lemma 2. Given Lemma 2, it also means that the ratio of total fitted cross-border trade over total fitted output equals the ratio of total observed cross-border trade over total observed output in the data: $\left(\sum_{i,j,i\neq j}\hat{X}_{ij}\right)/\left(\sum_{i,j}\hat{X}_{ij}\right)=\left(\sum_{i,j,i\neq j}X_{ij}\right)/\left(\sum_{i,j}X_{ij}\right)$.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log or level:	Level	Log	Log	Log	Level	Level	
Specification:	PPML	OLS	OLS + MR	SILS	NLLS	Gamma	DATA
Ratio	0.728	0.961	0.531	0.889	0.703	0.982	0.728

The table below shows this ratio for the same estimators as in Table 1. The first column is the ratio for Poisson-PML, which is the same as in the data. With other estimators, this fitted ratio can widely differ from the data even if further constraints on multilateral-resistance indexes are imposed. OLS, SILS and Gamma-PML do not put a large weight on large trade flows, which could explain why it does not do a good job at matching international trade and output sums. It is interesting to see that simultaneously imposing the constraints on multilateral-resistance indexes (OLS + MR) has a very different outcome compared to iterating OLS and adjustments of multilateral-resistance indexes (SILS).

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