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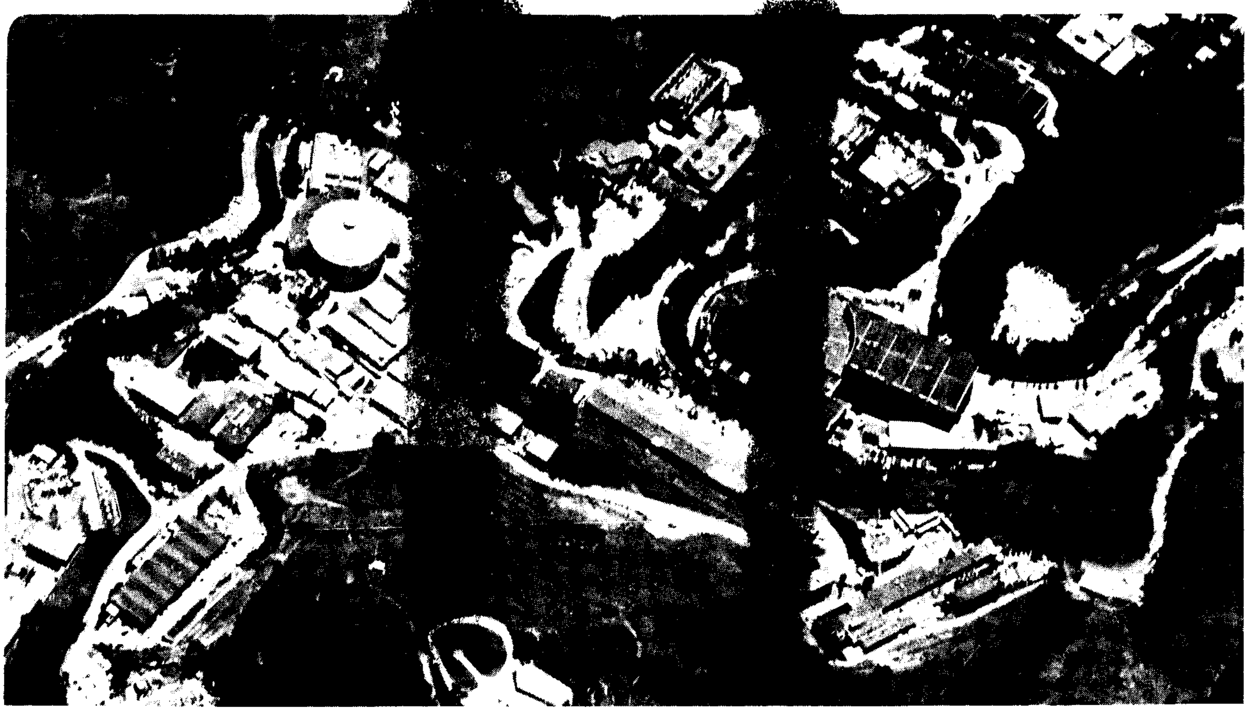

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Production of Heavy Higgs Bosons: Comparisons of Exact and Approximate Results

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Abstract

The WW and ZZ mechanisms for the production of very heavy Higgs bosons are examined in detail. The differential cross section with respect to the energies of the final state fermions is calculated analytically. The total cross section for the elementary process $q_1q_2 \to q_1'q_2'H$ is calculated using this analytic result and compared to the effective W approximation. The latter is typically 5 - 30% too large. A simple parameterization of the quark luminosity spectra is presented and used to determine the production of Higgs bosons of mass 0.2 - 1 TeV in pp collisions for $\sqrt{s}$ = 20 TeV and 40 TeV.

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I. Introduction

The production of a very heavy ($M_H > 2M_W$) Higgs boson has received increasing attention with the proposal to build a pp collider with center of mass energy $\sqrt{s}$ = 40 TeV.[1] The two primary production mechanisms for the very heavy Higgs boson are gg $\to$ H [2] and WW $\to$ H (or ZZ $\to$ H).[3] In the latter, the W's (or Z's) are virtual, the result of bremsstrahlung from the quarks in the proton. The W's tend to be collinear with the beam, as are the virtual γ's in the analogous two-photon process in e^+e^- collisions. Thus it is possible to make an equivalent W-approximation analogous to the usual Weizacker-Williams approximation.[4,5,6]

To assess the quality of the equivalent W-approximation I calculate the $q_1q_2 \to q_1'q_2'H$ subprocess exactly and in the equivalent W-approximation. The approximation is found to be equivalent to assuming a particular integral is proportional to $\delta(\theta^2)$, where θ is the angle between the exiting quarks in the q_1q_2 c.m. frame.

The full calculation of $q_1q_2 \to q_1'q_2'H$ cannot be done analytically, but for the WW $\to$ H case it is possible to calculate analytically the differential cross section $d\sigma/dE_1'dE_2'$ where E_1' and E_2' are the final quark energies.

The comparison of the approximate and exact (numerical plus analytic) calculations shows that the approximation is good, typically, to 15%. For relatively low $\sqrt{s}$ this increases to 30-50%. For very high $\sqrt{s}$, the error decreases, but does not go to zero. There appear to be corrections of order M_W^2/M_H^2.

To calculate cross sections for pp $\to$ HX, parton luminosities are needed. I have parameterized the results of Eichten *et al.*[7] (EHLQ) at $\sqrt{s}$ = 40 TeV using a simple three parameter form. These are then used to calculate the Higgs boson production cross section for M_H = 0.2 -1.0 TeV. The gg $\to$ H mechanism is calculated as well, with a t-quark mass of 40 GeV. Above M_H = 300 GeV, the sum of the WW and ZZ mechanisms dominates the Higgs production.

II. Kinematics

We consider the production of a particle, H, with momentum l' and mass M_H via $q_1q_2 \to q_1'q_2'V_1V_2 \to q_1'q_2'H$, that is, each quark emits a virtual vector boson. See Fig. 1. The momenta of the incident (final) quarks are p_1, p_2 (p_1',p_2'). The momenta of the virtual V_1 and V_2 are k_1 and k_2. The quarks are taken to be massless with couplings

$$\bar{q}_1' \gamma_\mu (g_{v1} + g_{a1}\gamma_5) q_1 V_1^\mu \qquad (2.1a)$$

$$\bar{q}_2' \gamma_\mu (g_{v2} + g_{a2}\gamma_5) q_2 V_2^\mu \qquad (2.1b)$$

For $V = W$, $g_v = g/(2\sqrt{2})$, $g_a = -g/(2\sqrt{2})$, while for $V = Z$, $g_v = g(T_{3L}/2 - \sin^2\theta_W Q)/\cos\theta_W$, $g_a = -g\, T_{3L}/(2\cos\theta_W)$. As usual, T_{3L} represents the third component of weak isospin and Q is the electric charge The electroweak coupling, the weak mixing angle, and the fine structure constant are related by

$$4\pi\alpha = e^2 = g^2 \sin^2\theta_W \qquad (2.2)$$

and

$$G_F/\sqrt{2} = g^2/(8M_W^2) \qquad (2.3)$$

In the q_1q_2 center of mass, the initial energies are $E_1 = E_2 = \sqrt{\hat{s}}/2$, and we define η and ζ in terms of the final quark energies

$$E_1' = (1-\eta)\sqrt{\hat{s}}/2 \qquad (2.4)$$

$$E_2' = (1-\zeta)\sqrt{\hat{s}}/2 \qquad (2.5)$$

so that the energy of H is

$$E_H = l'_0 = (\eta + \zeta)\sqrt{\hat{s}}/2 \qquad (2.6)$$

The final state momenta lie in a plane and

$$(l')^2 = (p_1' + p_2')^2 = E_H^2 - M_H^2. \qquad (2.7)$$

We define the angle θ by

$$\cos\theta = -\hat{p}_1' \cdot \hat{p}_2' \qquad (2.8a)$$

and find

$$\cos\theta = 1 - 2(\zeta\eta - M_H^2/s)/(1-\zeta)(1-\eta) \qquad (2.8b)$$

The kinematic boundaries occur for $\cos\theta = \pm 1$, and the allowed region is

$$\zeta\eta \geq M_H^2/s \qquad (2.9)$$

$$\zeta + \eta \leq 1 + M_H^2/s \qquad (2.10)$$

See Fig. 2, the Dalitz plot for the three body final state.

The complete specification of the final state is accomplished by fixing the orientation of the final state plane relative to fixed axes in the q_1q_2 center of mass system. If the incident quarks are unpolarized, there is azimuthal symmetry about the incident momenta direction so only two rather than three Euler angles are essential. We choose a standard orientation with the final state plane perpendicular to the z-axis which is defined by $\hat{p}_1$. The standard orientation has $\hat{p}_1'$ along the x-axis. The final state orientation is defined relative to this standard by rotating about the y-axis by β, then about the new z-axis by α. In terms of the fixed axes,

$$\hat{p}_1' = \cos\alpha \cos\beta\, \hat{x} + \sin\alpha\, \hat{y} - \sin\beta \cos\alpha\, \hat{z} \qquad (2.11a)$$

$$\hat{p}_2' = -\cos(\alpha-\theta) \cos\beta\, \hat{x} - \sin(\alpha-\theta)\, \hat{y} + \sin\beta \cos(\alpha-\theta)\, \hat{z} \qquad (2.11b)$$

The Lorentz scalars are

$$p_1 \cdot p_2 = \hat{s}/2; \qquad p_1' \cdot p_2' = (1-\zeta)(1-\eta)(1+\cos\theta)\hat{s}/4$$

$$p_1' \cdot p_2 = (1-\eta)(1-\cos\alpha \sin\beta)\hat{s}/4\ ; \quad p_1 \cdot p_2' = (1-\zeta)(1-\cos(\alpha-\theta)\sin\beta)\hat{s}/4$$

$$p_1 \cdot p_1' = (1-\eta)(1+\cos\alpha \sin\beta)\hat{s}/4; \quad p_2 \cdot p_2' = (1-\zeta)(1+\cos(\alpha-\theta)\sin\beta)\hat{s}/4$$

$$k_1^2 = -(1-\eta)(1+\cos\alpha \sin\beta)\hat{s}/2; \quad k_2^2 = -(1-\zeta)(1+\cos(\alpha-\theta)\sin\beta)\hat{s}/2$$

$$(2.12)$$

The cross section is related to the invariant matrix element by

$$d\sigma = \frac{(2\pi)^{-4}}{16\,\hat{s}} \left| M \right|^2 dE_1'\, dE_2'\, d\alpha\, d\cos\beta$$

$$= 2^{-10}\,\pi^{-4} \left| M \right|^2 d\eta\, d\zeta\, d\alpha\, d\cos\beta \qquad (2.13)$$

III. Matrix element and cross section

The WWH coupling is $-i(g\,M_W)g^{\mu\nu}$ while the ZZH coupling is $-i(gM_Z)\,g^{\mu\nu}/\cos\theta_W$. Indicating the VVH coupling generically by $-iAg^{\mu\nu}$ the Lorentz invariant matrix element for Fig. 1 is

$$M = A\, \bar{u}(p_1')\, \gamma_\mu(g_{v1} + g_{a1}\gamma_5)\, u(p_1)\ \ \bar{u}(p_2')\, \gamma^\mu\, (g_{v2} + g_{a2}\gamma_5)\, u(p_2)$$

$$\times\, (k_1^2 - M_{V_1})^{-1}\, (k_2^2 - M_{V_2}^2)^{-1} \qquad (3.1)$$

Squaring, averaging over initial spins and summing over final spins gives

$$\left| M \right|^2 = A^2\,(1/4)\ \mathrm{Tr}\, \not{p}_1'\, \gamma_\mu\, \not{p}_1\, \gamma_\lambda\, (g_{v1}^2 + g_{a1}^2 + 2\, g_{v1} g_{a1}\, \gamma_5)$$

$$\times\, \mathrm{Tr}\, \not{p}_2'\, \gamma^\mu\, \not{p}_2\, \gamma^\lambda\, (g_{v2}^2 + g_{a2}^2 + 2\, g_{v2}\, g_{a2}\, \gamma_5)$$

$$\times\, (k_1^2 - M_{V_1})^{-2}\, (k_2^2 - M_{V_2})^{-2} \qquad (3.2)$$

If we define $g_L = (g_v - g_a)/2$ and $g_R = (g_v + g_a)/2$, then

$$g_v^2 + g_a^2 = 2\,(g_R^2 + g_L^2) \qquad (3.3a)$$

$$2\, g_v\, g_a = 2\,(g_R^2 - g_L^2) \qquad (3.3b)$$

Additionally we set

$$C = 64\,(g_{L1}^2\, g_{L2}^2 + g_{R1}^2\, g_{R2}^2) \qquad (3.4a)$$

$$D = 64\,(g_{L1}^2\, g_{R2}^2 + g_{R1}^2\, g_{L2}^2) \qquad (3.4b)$$

Then after computing the indicated traces we have

$$\left| M \right|^2 = A^2\, \frac{C\, p_1 \cdot p_2\, p_1' \cdot p_2' + D\, p_1 \cdot p_2'\, p_1' \cdot p_2}{(k_1^2 - M_{V_1}^2)^2\, (k_2^2 - M_{V_2}^2)^2} \qquad (3.5)$$

For the WW → H process, $g_R = 0$ so $D=0$. We consider this case first. The cross section is

$$d\sigma = \frac{C A^2 (1 + \cos\theta)\, d\eta\, d\zeta\, d\alpha\, d\cos\beta}{2^{10} \pi^4 s^2 (1-\eta)(1-\zeta)}$$

$$\times \left[1 + \cos\alpha \sin\beta + \frac{2M_W^2}{(1-\eta)s} \right]^{-2}$$

$$\times \left[1 + \cos(\alpha-\theta) \sin\beta + \frac{2M_W^2}{(1-\zeta)s} \right]^{-2} \tag{3.6}$$

We recall that $\cos\theta$ is determined by ζ and η in Eq. (2.8).

In the Appendix we show that the integral

$$J(x,y,\theta) = \int_0^{2\pi} d\alpha \int_{-1}^{1} d\cos\beta \left[x + \cos\alpha \sin\beta\right]^{-2} \left[y + \cos(\alpha-\theta) \sin\beta\right]^{-2} \tag{3.7}$$

can be done analytically with the result

$$J(x,y,\theta) = 4\pi\left\{ \frac{3}{\Delta^2} \left(\frac{1}{\sqrt{\Delta}} \tanh^{-1}\frac{\sqrt{\Delta}}{xy-\cos\theta} - \frac{xy - \cos\theta}{(x^2-1)(y^2-1)} \right) \right.$$

$$\times (x - y\cos\theta)(y - x\cos\theta)$$

$$+ \frac{\cos\theta}{\Delta^{3/2}} \tanh^{-1} \frac{\sqrt{\Delta}}{xy - \cos\theta}$$

$$\left. + \frac{x^2 + y^2 - 3xy\cos\theta + 1}{\Delta (x^2-1)(y^2-1)} \right\} \tag{3.8}$$

where

$$\Delta = x^2 + y^2 - 2xy\cos\theta - 1 + \cos^2\theta \tag{3.9}$$

The differential cross section for the $WW \to H$ process (where $D=0$) can be written in terms of J:

$$d\sigma = \frac{C A^2}{2^{10} \pi^4 s^2} \frac{2(1 + \cos\theta)}{(1-\zeta)(1-\eta)} J\left(1+\frac{2M_W^2}{(1-\eta)s},\ 1+\frac{2M_W^2}{(1-\zeta)s},\ \theta\right) d\eta\, d\zeta \tag{3.10}$$

Since the analytical expression for J is rather opaque, we pause to consider an approximate calculation.

IV. Evaluation of $J(x,y,\theta)$ as $x,y \to 1$.

In the high energy limit, we are interested in values of $x = 1+2M_H^2/s(1-\eta)$ and $y = 1+2M_H^2/s(1-\zeta)$ near unity. As the vector boson mass becomes less and less important, the process more and more ressembles the two-photon process in e^+e^- collisions. That process is most easily analyzed in the equivalent photon approximation (Weizacker-Williams approximation). The photons are treated as real (thus necessarily transverse) and collinear with the initial beams. For the $WW \to H$ process the analogous kinematics have the quarks emitting W's by bremsstrahlung, with the quarks continuing in the forward direction. Both longitudinal and transverse W's are produced. Because of the form of the WWH coupling, the longitudinal W's make the dominant contribution to H production.[3,4,5] The forward scattering kinematics correspond to the limits

$$\cos\alpha \to -1$$
$$\sin\beta \to +1$$
$$\theta \to 0 \qquad (4.1)$$

Thus we are led to consider an expansion of the integrand in J, with $\alpha = \pi - \nu$, and $\beta = (\pi/2) - \epsilon$, with ν and ϵ small. We replace

$$\cos\alpha \to -1 + \nu^2/2$$
$$\sin\beta \to 1 - \epsilon^2/2 \qquad (4.2)$$

so that

$$J \to \int d\epsilon\, d\nu \left[x-1 + (\nu^2 + \epsilon^2)/2 \right]^{-2} \left[y-1 + ((\nu+\theta)^2 + \epsilon^2)/2 \right]^{-2} \qquad (4.3)$$

The integral is dominated by the region of small ϵ and ν. We can take the integral to extend over the full ϵ-ν plane. Writing

$$r = \epsilon\,\hat{x} + \nu\,\hat{y}$$
$$\theta = \theta\,\hat{y} \qquad (4.4)$$

we have

$$J \to \int d^2r \; \frac{16}{\left[r^2 + 2(x-1) \right]^2 \left[(r+\theta)^2 + 2(y-1) \right]^2} \qquad (4.5)$$

Since we know that J is concentrated near $\theta = 0$, we approximate it by

$$\tilde{J} \propto \delta(\theta^2) \qquad (4.6)$$

and evaluate

$$\int d^2\theta\, \tilde{J} = 16 \int d^2r \left[r^2 + 2(x-1) \right]^{-2} \int d^2\theta \left[(r+\theta)^2 + 2(y-1) \right]^{-2}$$
$$= \frac{4\pi^2}{(x-1)(y-1)} \qquad (4.7)$$

Thus within the present approximation,

$$J \to \frac{4\pi}{(x-1)(y-1)}\,\delta(\theta^2) \qquad (4.8)$$

Inserting this approximation in Eq. (3.10) we have

$$d\sigma = \frac{C A^2}{2^8 \pi^3 M_W^4}\, \delta\!\left(\frac{4(\zeta\eta - M_H^2/s)}{(1-\zeta)(1-\eta)} \right) d\zeta\, d\eta \qquad (4.9)$$

Using the boundaries indicated in Eqs. (2.9) and (2.10), we determine the total cross section

$$\sigma = \frac{C A^2}{2^{10} \pi^3 M_W^4} \int_{M_H^2/s}^{1} d\eta \int_{M_H^2/s\eta}^{1+(M_H^2/s)-\eta} d\zeta \;(1-\zeta)(1-\eta)\,\delta(\zeta\eta - M_H^2/s) \qquad (4.10)$$

For the WW→H process, we have $A = gM_W$ and $C = g^4 = (4\pi\alpha)^2/\sin^4\theta_W$ and we find

$$\sigma = \frac{1}{16 M_W^2} \left(\frac{\alpha}{\sin^2\theta_W} \right)^3 \left[\left(1 + M_H^2/s \right) \log(s/M_H^2) - 2 + 2M_H^2/s \right] \qquad (4.11)$$

which is the result of Chanowitz and Gaillard[4], and of Dawson.[5] The latter used the answer to infer the equivalent WW luminosity in pp collisions.

V. Comparison of exact results and the equivalent W-approximation

In Fig.3 is shown the cross section for H production from collisions of a u-quark and a d-quark arising from the WW mechanism (excluding contributions from ZZ virtual states) as obtained from Eq. (3.10) by numerical integration. The calculations are done with $\sin^2\theta_W = 0.22$ which gives

$$g^W_{Lu} = g^W_{Ld} = 0.23$$

$$g^W_{Ru} = g^W_{Rd} = 0$$

$$g^Z_{Lu} = 0.13 ; \quad g^Z_{Ld} = -0.16$$

$$g^Z_{Ru} = -0.05; \quad g^Z_{Rd} = 0.03$$

The Chanowitz-Gaillard-Dawson approximation is compared to the complete calculation in Fig. 4. Except for $\sqrt{s}$ not much bigger than M_H, the approximation is quite good. It appears that as $\sqrt{s} \to \infty$, the approximation is not exact, but has corrections of order M_W^2/M_H^2.

VI. Quark luminosities and the pp→HX cross section

To calculate the production of Higgs bosons in pp collisions, we need the luminosity spectrum for the quark-quark collisions. The ratio of the quark-quark center of mass energy squared to the overall center of mass energy squared is indicated by $\tau = \hat{s}/s$. For pp collisions we have

$$(d\mathcal{L}/d\tau)_{uu} = \int_\tau^1 (dx/x)\ u(x)\ u(\tau/x) \qquad (6.1)$$

$$(d\mathcal{L}/d\tau)_{ud} = 2\int_\tau^1 (dx/x)\ u(x)\ d(\tau/x) \qquad (6.2)$$

and similarly for the various other combinations of quarks, antiquarks, and gluons. Here $u(x)dx$ is the probability that a u-quark in a proton carries a fraction of the proton's momentum between x and $x+dx$. The distributions are also function of a scale denoted by Q^2 and which we take to be M_W^2.[7]

For the parton distributions we have used Set 2 (Λ = 0.29 GeV) of Eichten *et al.*[8] (EHLQ). The resulting luminosity spectra can be parameterized as

$$(d\mathcal{L}/d\tau) = (A/\tau^\gamma)\ \exp\ (-\beta \sqrt{\tau}\,) \qquad (6.3)$$

There is no special significance to the form of the parameterization. In Table 1 we give the values of A, β, and γ for pp collisions at $\sqrt{s}$ = 40 TeV. The fits cover $0.001 < \sqrt{\tau} < 0.2$. The luminosities fall by a factor of about 10^6 over this range. The fits are good to a few percent around $\sqrt{\tau} = 0.01$ and to $10 - 20\%$ in the extreme ends of the range.

Table 1

Parameterization of the luminosity spectra at $\sqrt{s}$ = 40 TeV..
See Eqs. (6.1)- (6.3).

	A	β	γ
uu	0.323	4.57	2.57
dd	0.103	8.99	2.69
ud	0.392	6.96	2.62
$u\bar{u} = u\bar{d}$	0.251	12.81	2.67
$d\bar{u} = d\bar{d}$	0.107	14.63	2.76
$u\bar{s} = us$	0.176	13.16	2.71
$d\bar{s} = ds$	0.073	14.86	2.80
$\bar{u}\bar{u}=\bar{d}\bar{d}=\bar{u}\bar{d}/2$	0.021	20.86	2.87

Using these fits we have calculated the cross section for $pp \to HX$ through the WW and ZZ mechanisms at $\sqrt{s}$ = 20 TeV and 40 TeV (using the same $(d\mathcal{L}/d\tau)$ at the two energies). The results are shown in Table 2. We have assumed that the integral with coefficient D in Eq. (3.5) is equal to that with coefficient C, since $p_1 \cdot p_2 \; p_1' \cdot p_2'$ is nearly equal to $p_1 \cdot p_2' \; p_1' \cdot p_2$ where the integrand is most important.

Also shown in the Table are the cross sections for producing the Higgs through the gluon fusion mechanism. That cross section is

$$\sigma = (\sqrt{2}\, G_F/64)\, \pi\, (\alpha_s/\pi)^2\, (|N|^2/9)\, (\tau\, d\mathcal{L}/d\tau)_{gg} \qquad (6.4)$$

where N is a complex function of $\lambda = M_t^2/M_H^2$ for $\lambda > 1/2$:

$$N = 3\, [\, 2\lambda + \lambda(4\lambda - 1)\, f(\lambda)],$$

$$f(\lambda) = (1/2)\, \log^2(\, \eta^+/\eta^-\,) - \pi^2/2 + i\pi\, \log\, (\, \eta^+/\eta^-) ,$$

$$\eta^{\pm} = (1/2) \pm \sqrt{(1/4) - \lambda} \qquad (6.5)$$

An adequate parameterization of the gg luminosity obtained from the EHLQ structure functions is

$$(d\mathcal{L}/d\tau)_{gg} = (20325/\tau)\, \exp(\, -24.80\, \tau^{0.2}\,) \qquad (6.6)$$

This fit is accurate to within a few percent.

In Table 2 are displayed the results of these calculations. Cross sections are given both for $\sqrt{s}$ = 20 TeV and 40 TeV. The contributions from the WW, ZZ and gg processes are displayed separately. The results are in agreement with earlier calculations.[9] Above M_H = 300 GeV, the WW plus ZZ processes dominate the gg. The calculation of the gg process assumes a t-quark mass of 40 GeV.

Table 2

Cross section for $pp \rightarrow HX$ in pb via $WW \rightarrow H$, $ZZ \rightarrow H$ and $gg \rightarrow H$. For comparison, the values obtained from Eq. (4.11) for the $WW \rightarrow H$ process are shown in parentheses.

$\sqrt{s}$ =	20 TeV	40 TeV
M_H = 0.2 TeV		
WW	2.6 (3.7)	6.8 (9.2)
ZZ	0.9	2.5
gg	8.8	22.8
M_H = 0.3 TeV		
WW	1.6 (2.0)	4.5 (5.5)
ZZ	0.6	1.7
gg	1.9	5.8
M_H = 0.5 TeV		
WW	0.7 (0.8)	2.4 (2.6)
ZZ	0.3	0.9
gg	0.2	0.8
M_H = 0.7 TeV		
WW	0.4 (0.4)	1.5 (1.6)
ZZ	0.15	0.6
gg	0.03	0.2
M_H = 1.0 Tev		
WW	0.19 (0.20)	0.80 (0.83)
ZZ	0.07	0.31
gg	0.005	0.03

VI. Summary

The analytical determination of a fundamental integral in the calculation of the WW mechanism for Higgs production has facilitated our comparison of the exact result with the approximation of Chanowitz and Gaillard, and of Dawson. The 5 - 30 % accuracy of the approximation is adequate for many applications. Similar results have been obtained by Dawson.[9] Using simple parameterizations of the luminosity spectra of EHLQ, the total Higgs production cross section is found to be dominated by the WW and ZZ mechanisms for Higgs masses greater than 300 GeV.

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Appendix A

We wish to evaluate

$$J(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y}\int_0^{2\pi} d\alpha \int_{-1}^{1} d\cos\beta \; \frac{1}{(\cos\alpha \sin\beta + x)\,(\cos(\alpha-\theta)\sin\beta + y)} \qquad (A.1)$$

so we introduce a Feynman parameter, t:

$$J(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y}\int_0^{1} dt \int_0^{2\pi} d\alpha \int_{-1}^{1} d\cos\beta$$

$$\times \Big\{ (1-t)x + ty + [\cos\alpha\,(1-t+t\cos\theta) + t\sin\alpha\sin\theta]\sin\beta \Big\}^{-2} \qquad (A.2)$$

Redefining the variable α,

$$J(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y}\int_0^{1} dt \int_0^{2\pi} d\alpha \int_{-1}^{1} d\cos\beta$$

$$\times \Big\{ (1-t)x + ty + \sin\beta\cos\alpha\,[(1-t)^2 + 2(1-t)t\cos\theta + t^2]^{1/2} \Big\}^{-2} \qquad (A.3)$$

we can do the α integral,

$$J(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y}\int_0^{1} dt \int_{-1}^{1} d\cos\beta\; 2\pi\,[(1-t)x + ty]$$

$$\times \Big\{ [(1-t)x+ty]^2 - [(1-t)^2 + 2(1-t)t\cos\theta + t^2]\sin^2\beta \Big\}^{-3/2} \qquad (A.4)$$

and the $\cos\beta$ integral

$$J(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y}\int_0^{1} dt\; \frac{4\pi}{\Big[(1-t)x+ty\Big]^2 - \Big[(1-t)^2 + 2(1-t)\,t\cos\theta + t^2\Big]} \qquad (A.5)$$

The t integral is elementary. The result is

$$J(x,y) = 2\pi\,\frac{\partial}{\partial x}\frac{\partial}{\partial y}\,\frac{2}{\sqrt{\Delta}}\tanh^{-1}\frac{\sqrt{\Delta}}{\left(\frac{x+y}{2}\right)^2 - \left(\frac{x-y}{2}\right)^2 - \cos\theta} \qquad (A.6)$$

where

$$\Delta = x^2 + y^2 - 2xy\cos\theta - 1 + \cos^2\theta \qquad (A.7)$$

Some tedious algebra yields the final result:

$$J(x,y) = 4\pi\Bigg\{ \frac{3}{\Delta^2}\left(\frac{1}{\sqrt{\Delta}}\tanh^{-1}\frac{\sqrt{\Delta}}{xy-\cos\theta} - \frac{xy-\cos\theta}{(x^2-1)(y^2-1)}\right) \times (x - y\cos\theta)\,(y - x\cos\theta)$$

$$+ \frac{\cos\theta}{\Delta^{3/2}}\tanh^{-1}\frac{\sqrt{\Delta}}{xy-\cos\theta}$$

$$+ \frac{x^2 + y^2 - 3xy\cos\theta + 1}{\Delta\,(x^2-1)\,(y^2-1)}\Bigg\} \qquad (A.8)$$

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FIGURE CAPTIONS

1. The Feynman diagram for Higgs production through virtual W's or Z's.

2. The Dalitz plot for the final state phase space in $q_1q_2 \rightarrow q_1'q_2'H$. The variables η and ζ represent the fractions of center of mass energy lost by q_1 and q_2. See Eqs. (2.4), (2.5), (2.9), and (2.10). The plot is for $M_H^2/s = 0.2$.

3. The cross section in pb for ud→ udH from the WW process as a function of $\sqrt{s}$ in TeV for four values of the Higgs mass: $M_H = 0.3$ TeV (solid), 0.5 TeV (dotted), 0.7 TeV (dashed). 1.0 TeV (dot-dash).

4. The ratio of the cross section for ud → udH via the WW process calculated from the approximation of Eq. (4.11) to the exact calculation, as a function of $\sqrt{s}$. The values of the Higgs mass are 0.3 TeV (solid), 0.5 TeV (dashed), 0.7 TeV (dot-dash), 1.0 TeV (dotted).

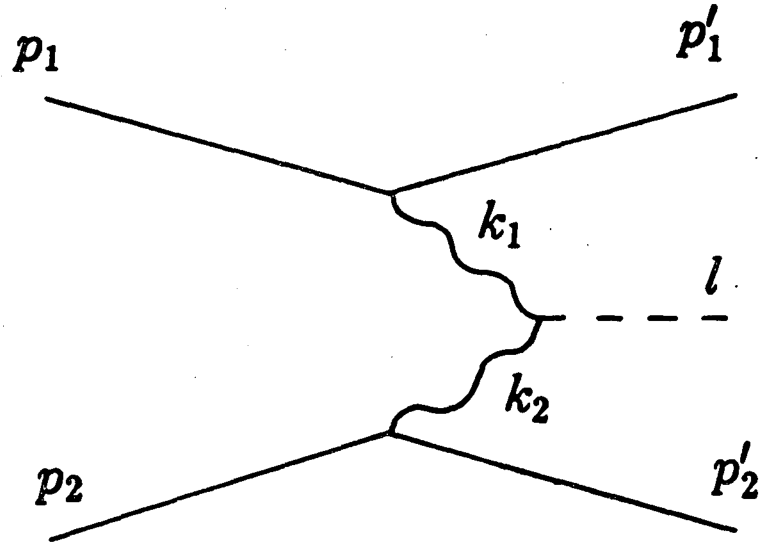

Fig. 1

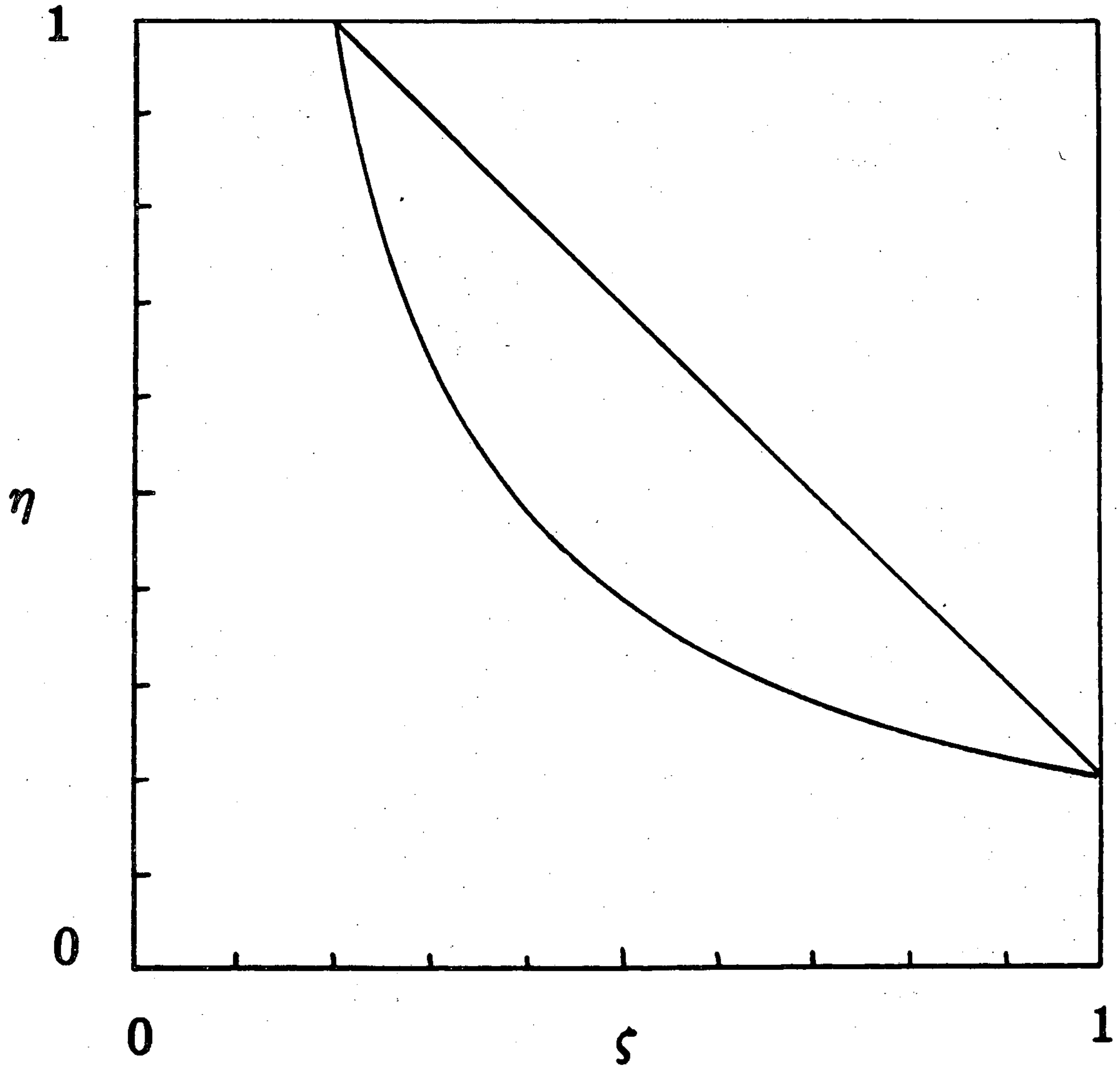

Fig. 2

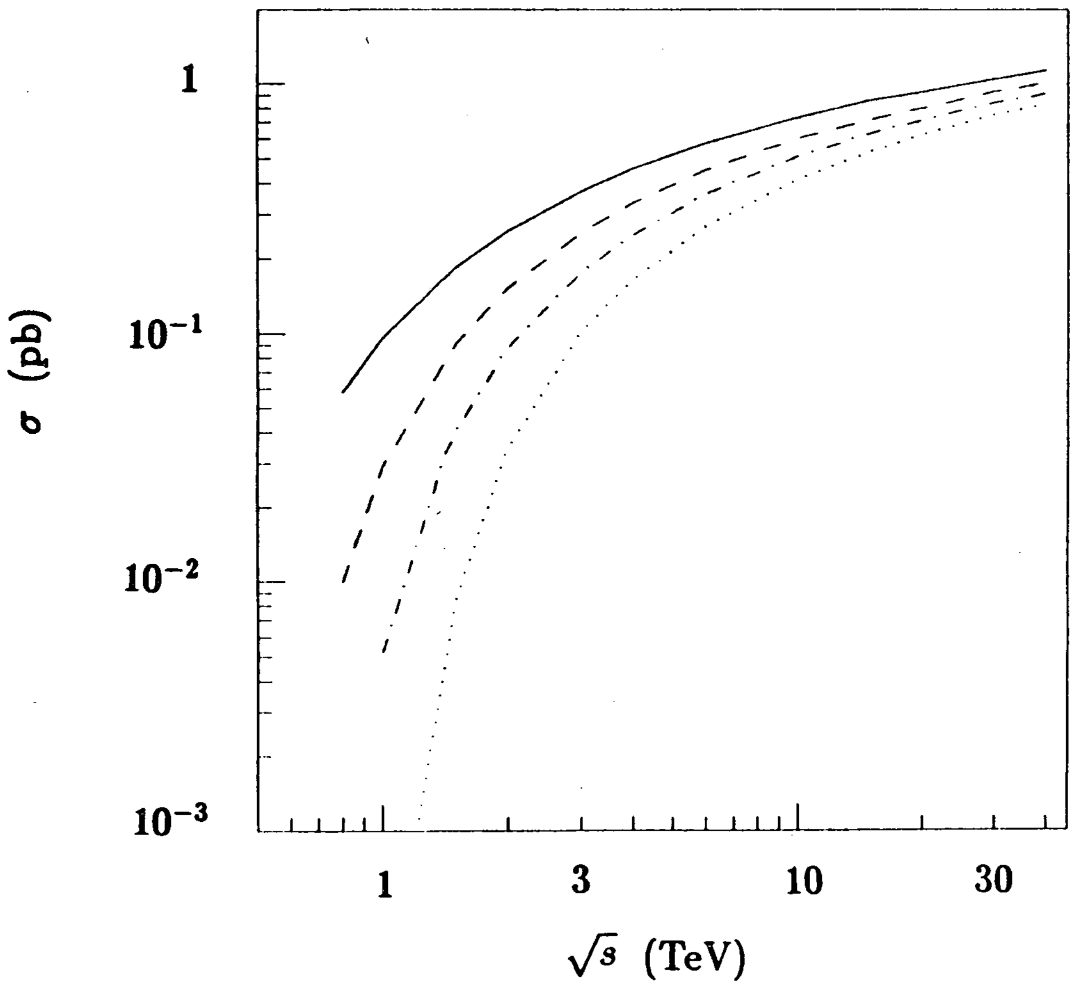

Fig. 3

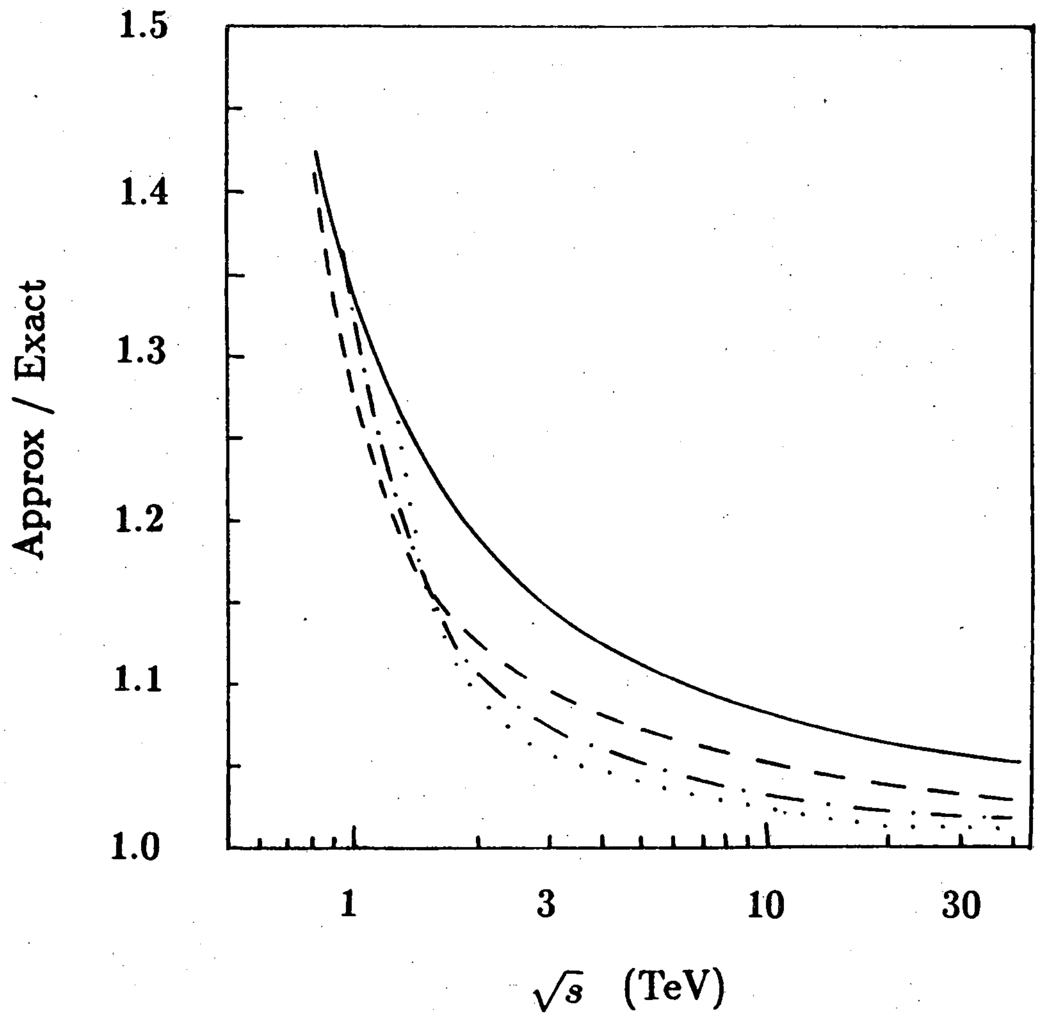

Fig. 4

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