Title
Using Astigmatic Laser Mode Converters to Produce “Donut-Shaped” Intensity Beams

Author
Vig, Rohit Kumar

Publication Date
2023-12-10

Using Astigmatic Laser Mode Converters to Produce “Donut-Shaped” Intensity Beams

Rohit Vig[1*]

[1] *Electrical and Computer Engineering Department, Henry Samueli School of Engineering and Applied Sciences, University of California, Los Angeles, 7400 Boelter Hall, Los Angeles, CA, 90024*
**rohitvig9@g.ucla.edu*

Abstract: This study demonstrates the theoretical conversion of Hermite-Gaussian modes to Laguerre-Gaussian modes using astigmatic mode converters, showcasing the creation of donut-shaped beam profiles via MATLAB simulations, with implications for optical manipulation and beam shaping applications.

INTRODUCTION
An essential problem in free-electron lasers (FELs) is microbunching instability, which describes unwanted fluctuations in the energy distribution and electron beam density. This instability is harmful because it reduces the quality and coherence of the generated laser beam, hence affecting the performance of FELs utilized in scientific research, such as material science and bioimaging. Addressing this insecurity is critical for optimizing FEL operations.[1]

A promising solution to this problem is to use a laser heater with an LG01 transverse laser mode. LG modes are distinguished by their orbital angular momentum, which can exert mechanical torque that can be used to manipulate microparticles and influence the dynamics of a beam, reducing the impacts of microbunching instability[2]. The generation of LG01 mode lasers often involves the use of a spiral wave plate in order to convert the HG mode to an LG mode. The operation of the spiral wave plate is based on changing the phase of the incoming light wave by 90 degrees, resulting in the helical wavefront that defines LG01 beams. However, spiral phase plates may have some drawbacks such as conversion efficiency and beam quality.[3]

Astigmatic laser mode converters are a possible substitute for spiral phase plates in terms of generating a “donut-shaped” intensity profile from an HG mode input beam. These converters, which are normally made up of two cylindrical lenses, provide an efficient method of producing "donut-shaped" intensity beams by using the cylindrical lenses' different focal characteristics to modify the phase and shape of the input beam, converting an HG mode to an LG mode, and vice-versa[4] with nearly 100% efficiency in an ideal scenario. Although, in a practical situation, manufacturing tolerances in focal length can reduce this efficiency, it is still worth exploring as an alternative to a spiral beam phase plate for producing donut-shaped beams.[3]

METHODS
This study performs a theoretical MATLAB simulation to demonstrate the transformation of Hermite-Gaussian (HG) modes into Laguerre-Gaussian (LG) modes utilizing an astigmatic mode conversion approach. The simulation leverages the Fourier optics framework to model the effect of cylindrical lenses on the HG beam profile. Initially, the spatial distribution of the HG modes, characterized by their respective Hermite polynomials, is computed.

$$\mathrm{HG}_{nm}(x,y,z) = H_n\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \frac{1}{w(z)} e^{-\frac{x^2+y^2}{w^2(z)}} e^{-i\frac{k(x^2+y^2)}{2R(z)}} e^{i(n+m+1)\psi(z)}$$

Subsequent Fourier transformation translates this distribution into the frequency domain, where the phase alterations induced by cylindrical lenses—represented by a quadratic phase factor—are applied selectively along one dimension. An inverse Fourier transform readjusts the beam profile back into the spatial domain, now embodying the phase characteristics of an LG mode. The superposition of the orthogonal HG modes, incorporating a $\pi/2$ phase shift, culminates in the distinctive donut-shaped intensity profile of the LG01 mode.

$$\mathrm{LG}_{pl}(r,\phi,z) = L_p^l\left(\frac{2r^2}{w^2(z)}\right) \frac{1}{w(z)} e^{-\frac{r^2}{w^2(z)}} e^{-il\phi} e^{-i\frac{kr^2}{2R(z)}} e^{i(2p+l+1)\psi(z)}$$
$$\mathrm{LG}_{01} = HG_{10}x + iHG_{01}x$$

The simulation is executed in MATLAB, utilizing the built-in Fourier transformation functions to enact the cylindrical lens' phase modulation effect, thereby emulating the optical conversion process shown in Fig. 1.

Table 1. Supplementary Materials-MATLAB Code

Code	https://docs.google.com/document/d/1SKh3pKdfMtsxrhygawCBUcqYIrdYqOX4v2Qn2TECjI8/edit?usp=sharing

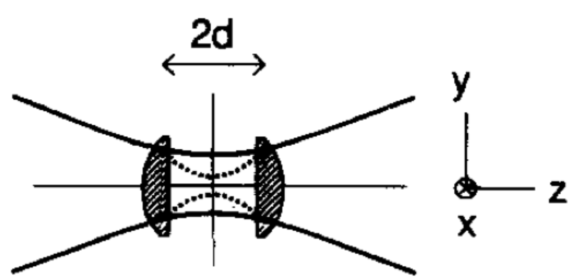

Fig. 1. Two cylindrical lenses as a mode converter for a beam, 2d is the confocal parameter [4]

RESULTS AND INTERPRETATION

The main result of this paper is the successful theoretical demonstration of converting Hermite-Gaussian (HG) modes, as seen in Figure 2, into Laguerre-Gaussian (LG) modes using an astigmatic mode converter in MATLAB simulations. This conversion is simulated through the Fourier transformation of HG beam profiles, application of a cylindrical lens phase effect, and inverse Fourier transformation to obtain the LG mode. The distinctive donut-shaped intensity profile characteristic of the LG01 mode, as seen in Figure 3, is a key outcome, validating the theoretical framework.

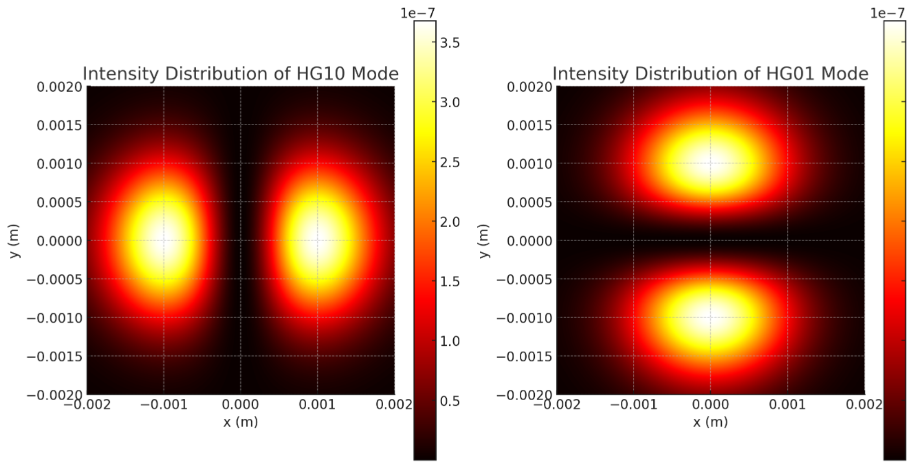

Fig. 1. HG Mode intensity profiles.

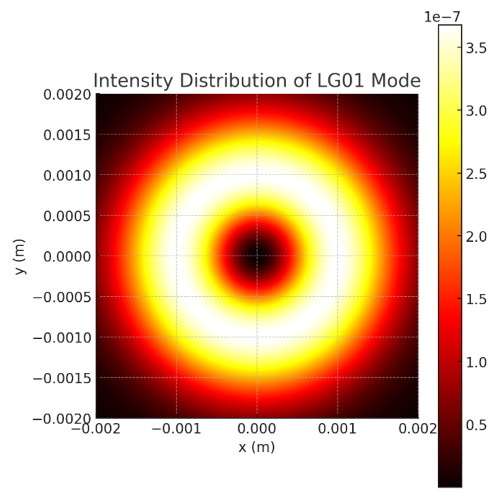

Fig. 2. Donut-Shaped LG01 Intensity profile.

CONCLUSIONS

Overall, this paper demonstrates that cylindrical lenses can be employed to efficiently transform HG modes into LG modes, with the potential for practical applications in various fields like optical trapping, communications, and beam shaping in free-electron lasers. This knowledge contributes to the understanding of beam mode transformations using simple optical components, offering an alternative to more complex methods like spiral phase plates.

REFERENCES

1. Tang, Jingyi, et al. "Laguerre-gaussian mode laser heater for microbunching instability suppression in free-electron lasers." Physical review letters 124.13 (2020): 134801.
2. Saleh, Bahaa E. A., and Malvin Carl Teich. "Fundamentals of Photonics." 3rd ed. Wiley Series in Pure and Applied Optics, 2019, p. 315.
3. Courtial, J., Padgett, M.J. "Performance of a cylindrical lens mode converter for producing Laguerre–Gaussian laser modes." Optics Communications, vol. 159, no. 1–3, 1999, pp. 13-18, ISSN 0030-4018, https://doi.org/10.1016/S0030-4018(98)00599-9.
4. Beijersbergen, M.W., Allen, L., van der Veen, H.E.L.O., Woerdman, J.P. "Astigmatic laser mode converters and transfer of orbital angular momentum." Optics Communications, vol. 96, no. 1–3, 1993, pp. 123-132, ISSN 0030-4018, https://doi.org/10.1016/0030-4018(93)90535-D.