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# Intelligent Cruise Control Systems and Traffic Flow Stability <sup>1</sup>

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## Abstract

In analogy to the flow of fluids, it is expected that the aggregate density and the velocity of vehicles in a section of a freeway adequately describe the traffic flow dynamics. The conservation of mass equation together with the aggregation of the vehicle following dynamics of controlled vehicles describes the evolution of the traffic density and the aggregate speed of a traffic flow. There are two kinds of stability associated with traffic flow problems - string stability (or car-following stability) and traffic flow stability. We make a clear distinction between traffic flow stability and string stability, and such a distinction has not been recognized in the literature, thus far. String stability is stability with respect to intervehicular spacing; intuitively, it ensures the knowledge of the position and velocity of every vehicle in the traffic, within reasonable bounds of error, from the knowledge of the position and velocity of a vehicle in the traffic. String stability is analyzed without adding vehicles to or removing vehicles from the traffic. On the other hand, traffic flow stability deals with the evolution of traffic velocity and density in response to the addition and/or removal of vehicles from the flow. Traffic flow stability can be guaranteed only if the velocity and density solutions of the coupled set of equations is stable, i.e., only if stability with respect to automatic vehicle following and stability with respect to density evolution is guaranteed. Therefore, the flow stability and critical capacity of any section of a highway is dependent not only on the vehicle following control laws and the information used in their synthesis, but also on the spacing policy employed by the control system. Such a dependence has practical consequences in the choice of a spacing policy for adaptive cruise control laws and on the stability of the traffic flow consisting of vehicles equipped with adaptive cruise control features on the existing and future highways. This critical dependence is the subject of investigation in this paper. This problem is analyzed in two steps: The first step is to understand the effect of spacing policy employed by the Intelligent Cruise Control (ICC) systems on traffic flow stability. The second step is to understand how the dynamics of ICC system affects traffic flow stability. Using such an analysis, it is shown that cruise control systems that employ a constant time headway policy lead to unacceptable characteristics for the traffic flows.

**Key Words:** Intelligent Cruise Control Systems, Traffic Flow Stability, String Stability, Advanced Vehicle Control Systems, Advanced Traffic Management Systems.

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# 1 Introduction

In this paper, we consider automated traffic flows. In an automated traffic flow, every vehicle in the traffic is equipped with an Intelligent Cruise Control (ICC) system. In this paper, we will show that any ICC system employing a constant time headway spacing policy leads to unacceptable characteristics for automated traffic flows.

A pertinent question is what are acceptable characteristics for a traffic flow and what are not. A desirable characteristic of a traffic flow is that any density or velocity disturbances must attenuate as it propagates upstream<sup>2</sup>. Otherwise, these disturbances will be felt at every point upstream from the source of the disturbance. For example, density disturbances that arise from a sudden influx of vehicles after a football game or during the beginning or end of a commute hour, will be felt at some instant of time at all points upstream.

The following logical question, then, arises regarding the propagation of density and/or velocity disturbances downstream: Is such a propagation of density disturbances acceptable? We contend that downstream propagation of such disturbances is not undesirable, whether they decay or amplify. For example, consider a uniform traffic flow where all vehicles maintain the same velocity irrespective of their intervehicular distance. If a vehicle merges into such a traffic with the prescribed velocity, the resulting density disturbance propagates forward contributing to the throughput of the highway section. It may, in fact, be desirable that disturbances propagate downstream amplified, so that addition of a vehicle upstream may increase the throughput downstream. However, the conservation of mass principle would require that the vehicle speeds increase to unacceptable and/or unattainable levels. Moreover, such a scenario is unattainable since automatic vehicle following control laws typically incorporate the information of downstream vehicle and downstream sections of the highway and not the information of upstream vehicles/sections. Such a desirable amplification of downstream disturbances, even if attainable, may not be Lyapunov stable. By Lyapunov stability, we mean that the density disturbances or perturbations, that satisfy the boundary conditions, remain bounded.

Stability in the sense of Lyapunov [23] does not suffice for achieving an acceptable characteristic for a traffic flow. We require that any disturbance must attenuate as it propagates upstream for an acceptable characteristic of a traffic flow. In this paper, we rigorously show that there are arbitrarily small density disturbances that do not decay as they propagate upstream, if all vehicles in the traffic are equipped with Intelligent Cruise Control Systems incorporating a constant time headway policy. We numerically verify the result with simulations.

ICC systems, if well designed, can lead to traffic flows with desirable characteristics and can significantly ease congestion. The design of an ICC system involves the following steps:

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<sup>2</sup>Consider two distinct points A and B on a section of a highway. If some vehicle crosses the point A at time  $t_1$  and crosses the point B at time  $t_2$ , then B is downstream from A (or equivalently, A is upstream from B), if  $t_2 > t_1$ .

1. **Design of a spacing policy:** A spacing policy is a rule that dictates how the speed of an automatically controlled vehicle must be regulated as a function of the following distance.
2. **Design of a control system to regulate the vehicle velocity according to the designed spacing policy:** String stability is an issue in the control system design. Intuitively, it implies that spacing errors in regulating the following distance according to the specified spacing policy must decay upstream from one vehicle to another.

The distinction between string stability and traffic flow stability can be explained by drawing an analogy to the decentralized market systems. In a free market, there are a large number of commodities exchanged by the economic agents, namely, the consumers and producers. The price of a commodity is dependent on its supply and demand, and the price of other commodities in the market. The law of composite commodities, formulated on the basis of the works of Lange, Hicks and Leontief [17, 40], simplifies the study of the free market. In plain words, it states that if, in a set of commodities, the price of any commodity relative to a chosen reference commodity remains constant, then, for all analytical and practical purposes, the dynamics of the price of the member commodities in the set can be examined by studying the price of a composite commodity, which is an appropriately weighted average of the prices of the member commodities. It is possible that while the relative prices of commodities remain constant, the absolute price of each commodity decreases precipitously. *However, this does not constitute an acceptable behavior of the market.*

In the vehicle following context, string stability ensures that the spacing between vehicles in the string remain nearly the same, obeys the spacing policy at all times; therefore, knowledge of the position and velocity of a chosen reference vehicle is sufficient to determine the position and velocity of any other vehicle in the string with a reasonable degree of exactness. While string stability is guaranteed, there is no guarantee that the speed of vehicles in the string do not decrease to a level where the flow rate of vehicles entering the section is greater than the flow rate of vehicles leaving the section, or increase indefinitely. Akin to the condition demanded by the law of composite commodities, string stability enables a macroscopic description of the traffic speed dynamics from the microscopic speed dynamics of the automated vehicles. *Thus, though a string may be stable, the characteristics of the flow may be totally undesirable.*

The macroscopic behavior of traffic is an aggregation of the behavior of vehicles in every section of the freeway. An understanding of the macroscopic behavior of traffic is essential for an effective traffic flow control strategy. Since the behavior of every controlled vehicle is governed by the ICC system, traffic flow stability and critical traffic flow parameters are strongly dependent on the spacing policies employed by the ICC system. In the literature, several ICC laws have been proposed without any regard to the traffic flow stability and can result in the aggravation of the traffic flow problems, instead of easing them. The focus of this paper is to illustrate how the spacing policies used in vehicle following control laws alter the macroscopic behavior of automated traffic, thus affecting the traffic flow stability. This paper, thus, provides a framework for

analyzing traffic flow stability and control problems.

## 2 Background

Freeway traffic flow control requires mathematical models of traffic flow. These models must capture the essential character of the traffic flow behavior so that effective state estimation, filtering and control algorithms can be developed. Traffic flow models of various granularities have been developed by the researchers in the past to support simulation and control design.

Microscopic or vehicle following models consider a string of vehicles following each other in a single lane. They explicitly consider the capabilities of a vehicle and reaction time of its driver in a distance regulation task. The first microscopic models known are due to Reuschel, [33] and Pipes, [32]. They hypothesize that each driver maintains a separation distance proportional to the speed of their vehicle plus a distance headway at standstill that includes the length of the lead vehicle. Currently, microscopic models of a freeway contain manual driver behavior and they are used primarily for simulation studies.

Microscopic models of automated vehicle following are abundant in the literature and copious citations for the same can be found in the dissertation of Swaroop, [42].

Abstraction or aggregation of microscopic models provide critical information for macroscopic models of traffic flow. Macroscopic description of a traffic flow implies the definition of the adequate flow variables expressing the aggregate behavior of vehicles at any location and instant of time. In direct analogy to fluid flow, most macroscopic models of the traffic flow assume that traffic volume,  $q$  is equal to the product of the aggregate traffic density,  $\rho$  and the aggregate traffic velocity,  $v$ . A peculiar feature of traffic flow is that the aggregate traffic velocity decreases with increasing traffic density. Greenshields, [13] hypothesized the following steady state relationship between traffic density,  $\rho$  and speed,  $v$ :

$$v = v_f \left(1 - \frac{\rho}{\rho_{max}}\right) \Rightarrow q = v_f \rho \left(1 - \frac{\rho}{\rho_{max}}\right).$$

Here,  $v_f$  is the free speed of the traffic and  $\rho_{max}$  is the jam density. The above equation is, in essence, a constitutive relation for the traffic flow that is taking place. It can be seen that the traffic volume,  $q$  is a quadratic function of  $\rho$  in the steady state and it can be shown that the traffic volume,  $q$  increases with increasing density upto a critical density,  $\rho_{peak}$  and the corresponding traffic volume is  $q_{peak}$ . Such a characteristic ensures safety, wherein safe spacing between vehicles (and correspondingly density) is dependent on vehicle's speed. Extensive experimental data for freeway traffic consisting of manually controlled vehicles suggests instability in the traffic flow when the operating traffic density exceeds a critical value.

Macroscopic models describe the evolution of aggregate density and aggregate velocity of vehicles in every section of the freeway. Conservation of the number of vehicles relates the rate of change of density of vehicles in a section to the rate of vehicles entering and leaving it. Steady state traffic flow equation plainly states that the number of vehicles entering any section equals the

number of vehicles leaving it. Lighthill and Whitham, [20] and Richards, [34] utilized the conservation of mass equation and a volume-density characteristic,  $q = q(\rho)$  to derive some fundamental results, which have been widely used for simulation, surveillance, and control. Their results are based on the theory of kinematic waves. Lighthill and Whitham also investigated the propagation of shock waves in freeway traffic. In obtaining these results, the balance of linear momentum has been neglected, however, they employ a constitutive relation between the traffic velocity and the traffic density.

The next level of detail incorporated in the freeway traffic flow models is the dynamics of aggregate velocity of vehicles. Prominent among such models that are currently widely used, is the model due to Payne, [30]. It considers the influence of incoming traffic, and the traffic density downstream on the aggregate velocity dynamics. Speed-density characteristic is embedded into the aggregate velocity dynamics equation. This model is used in almost all recent freeway traffic control algorithms, [12] [3], [49], [28], [22], [36], [1].

Very recently, a meso-scale traffic flow model was developed by Broucke and Varaiya, [2] for Automated Highway Systems (AHS).

### 3 Traffic Flow Model and Stability

In order to model and investigate the macroscopic flow behavior of traffic consisting of vehicles equipped with Intelligent Cruise Control (ICC) systems, the following two step analysis is required:

1. The first step is to understand the effect of the spacing policy employed in automatic vehicle following on the macroscopic traffic flow dynamics,
2. The second step is to understand how the dynamics of the vehicle control system, that regulates intervehicular spacing according to the spacing policy, affects the traffic flow dynamics and its stability.

At this stage, it is important to discuss the relevant notions of stability that have been introduced in the study of solutions to integro-differential, partial differential and ordinary differential equations. For instance, in the case of partial differential equations, we can define a variety of criteria for stability. A solution is said to be “asymptotically stable”, if all arbitrary perturbations of the solutions, that yet satisfy the same boundary conditions, decay asymptotically in time. If the perturbations do not decay asymptotically in time, but remain bounded, then the solution is said to be stable. Also, if the perturbations to the basic solution remain bounded, provided the base solution satisfies some criteria (could be a measure of size, intensity etc., eg. Reynolds number), the solution is said to be conditionally stable. Energy methods and Lyapunov approaches can provide sufficient conditions for stability and they can also be used to determine instability [8, 9, 41].

Linearization is an important tool to examine the solutions of nonlinear differential equations [4, 21, 16, 25, 24]. Linearized stability analysis provides necessary conditions for local stability, i.e., if the flow is to be stable, it is necessary that the disturbances not be larger than a specified value. Similarly, it also provides sufficient conditions for local instability. It does not really

guarantee the unboundedness of solutions; while small disturbances may grow, as soon as they become sufficiently large, the linearized stability analysis is invalid, and the flow may be stable in the full nonlinear context.

The above notions of stability, while relevant, do not suffice for the investigation of the behavior of traffic flows. Here, we shall be concerned with two different criteria for stability that are central to the study of traffic flow problems: string stability and traffic flow stability.

We shall motivate the notion of string stability. Consider a string of infinite number of vehicles, each of which is equipped with an ICC system. If knowledge of the location and velocity of a vehicle in a string ensures knowledge, within reasonable bounds, of the relative placements and velocities of the other vehicles in the string, for all times, the string is said to be stable. We call this string stability. If the location and velocities of all members of the string can be determined by knowing the location and velocity of a member, in the limit as  $t \rightarrow \infty$ , we say that the string is asymptotically stable. For this reason, the string stability is typically analyzed without the any addition or removal of a vehicle in the string.

If the string of vehicles is perturbed, say by the introduction of a finite number of vehicles, and if in time, at points upstream, the string returns to the state (velocity and relative placement) before the introduction of vehicles, the traffic flow is said to be stable, i.e., we have traffic flow stability. In this sense, it is similar to what is meant by asymptotic stability of the solution to a differential equation.

The employed spacing policy must be specified prior to defining string stability. Let the employed spacing policy dictate that the desired speed,  $v_{des,j}$ , at a following distance,  $\Delta_j$ , is  $\bar{h}(\Delta_j)$ . The subscript  $j$  indicates the number of vehicles in the string preceding the vehicle under consideration. For example, the velocity of the first following vehicle in the string is  $v_1$  and so on. The function  $\bar{h}$  is one-to-one and onto at following distances of practical significance. In fact, it is a non-decreasing, continuous function of its argument. Let  $g$  denote the inverse of the function,  $\bar{h}$ .

Let each vehicle in the string be automatically controlled with its speed adjusted according to this spacing policy by the ICC system. Let  $\Delta_i, \dot{\Delta}_i$  denote the following distance and rate of change of the following distance respectively of an automatically controlled vehicle. The error in spacing,  $\epsilon_j$ , is defined in this paper as:  $\epsilon_j = \Delta_j - g(v_j)$ . The following definitions of string stability and asymptotic string stability will be used in this paper:

**Definition 3.1 (String stability):** *A string of automated vehicles is stable if given  $\epsilon > 0$ , there exists a  $\delta > 0$  such that*

$$\sup_j \max\{|\epsilon_j(0)|, |\dot{\Delta}_j(0)|\} < \delta \Rightarrow \sup_j \sup_{t \geq 0} \max\{|\epsilon_j(t)|, |\dot{\Delta}_j(t)|\} < \epsilon.$$

**Definition 3.2 (Asymptotic string stability):** *A string of automated vehicles is asymptotically stable if it is stable and if*

$$\lim_{t \rightarrow \infty} \sup_j \max\{|\epsilon_j(t)|, |\dot{\Delta}_j(t)|\} = 0.$$



Traffic flow is modeled as a continuum. While such an approximation is debatable, especially since the number of vehicles per unit length of the highway is not significantly large, we draw inspiration from rarified gas dynamics to make such an approximation [19].

Let  $x$  denote the position of a vehicle at time  $t$ , i.e.,  $x = x(X, t)$ , where  $X$  is the state of the vehicle at some initial time,  $t_0$ . Here, we choose  $t_0 = 0$ . Since the trajectories are uniquely associated with vehicles, a vehicle under consideration can be implicitly determined from the given trajectory. Let the velocity of the vehicle be given by  $v(x(t), t)$  or simply,  $v(x, t)$ . Here, we have suppressed the dependence of the position of a vehicle on the initial state,  $X$ . The acceleration of the vehicle is, therefore, given by:

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x}.$$

The time derivative on the left hand side of the equation is the Lagrangian time derivative and is obtained by keeping the vehicle fixed. The partial time derivative on the right hand side denotes the Eulerian time derivative and is obtained by keeping  $x$  fixed. If the flow is steady, then  $\frac{\partial v}{\partial t} = 0$ .

Let the following distance of a vehicle be denoted by  $\Delta(x(t), t)$  or simply,  $\Delta(x, t)$ . Let the length of every vehicle in the traffic be denoted by  $L_c$ . Then, we define density,  $\rho(x, t)$ , as:

$$\rho(x, t) = \frac{1}{\Delta(x, t) + L_c}.$$

If string stability is guaranteed and if the errors in spacing and velocity are sufficiently small, then density is a well-behaved quantity.

Also,

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial x}.$$

We adopt the following definition of traffic flow stability in our investigation:

**Definition 3.3 (Traffic Flow Stability):** *Let  $v_0(x, t), \rho_0(x, t)$  denote the nominal state of traffic. Let  $v_p(x, t), \rho_p(x, t)$  be the velocity and density perturbations to the traffic, consistent with the boundary conditions and are such that  $v_p(x, 0) \equiv 0, \rho_p(x, 0) \equiv 0, \forall x \geq x_u$ . The traffic flow is stable, if*

1. *given  $\epsilon > 0$ , there exists a  $\delta > 0$  such that*

$$\sup_{x \leq x_u} \{|v_p(x, 0)|, |\rho_p(x, 0)|\} < \delta \Rightarrow \sup_{t \geq 0} \sup_{x \leq x_u} \{|v_p(x, t)|, |\rho_p(x, t)|\} < \epsilon,$$

*and*

2.  $\lim_{t \rightarrow \infty} \sup_{x \leq x_u} \{|v_p(x, t)|, |\rho_p(x, t)|\} = 0$ .

### 3.1 Effect of the Spacing Policy employed by ICC system on Traffic Flow Stability: (PDE approach)

In order to analyze the effect of the spacing policy on the traffic flow dynamics, consider the limiting performance of the spacing policy with an ideal ICC on each vehicle, which regulates the following distance (spacing) instantaneously with respect to the speed of the vehicle. This consideration implies that the intervehicular spacing, or equivalently, the inverse of density,  $\rho$ , is given by:

$$\frac{1}{\rho} = g(v) \Rightarrow v = h(\rho) := g^{-1}\left(\frac{1}{\rho}\right). \quad (1)$$

This condition is similar to the approximation used by Lighthill and Whitham [20] in their theory and it represents the fundamental characteristic of the traffic consisting of automatically and ideally controlled vehicles. A spacing policy specification is, therefore, a specification of the Fundamental Traffic Characteristic of the automated traffic flow.

Analogous to the flow of a fluid, traffic volume at any point is defined as  $q = \rho v$ . The evolution of traffic density is determined by the conservation of mass equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0. \quad (2)$$

As a result of the ideal cruise control system assumption, the conservation of mass equation becomes:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho h(\rho))}{\partial x} = 0. \quad (3)$$

Let  $\rho_0$  be a base solution for the density. In order to study the stability of the base solution, consider small density perturbations,  $\epsilon \rho_p$  to the base solution. Neglecting second-order terms in  $\epsilon$  and defining the characteristic wave velocity,  $c$  at a density  $\rho_0$  as:

$$c := h(\rho_0) + \rho_0 \frac{\partial h}{\partial \rho}(\rho_0), \quad (4)$$

$$\text{we get } \frac{\partial \rho_p}{\partial t} + c \frac{\partial \rho_p}{\partial x} = 0. \quad (5)$$

Clearly, the sign of  $c$  is dependent on the density of base flow,  $\rho_0$ . The behavior of the solution to the above linear partial differential equation that describes the evolution of a density disturbance depends on the sign of  $c$ . The solution is a traveling wave, i.e.,  $\rho_p = F(x - ct)$ . If  $c > 0$ , the solution is a forward traveling wave and if  $c < 0$ , the solution is a backward traveling wave. With an ideal cruise control system dynamics, when  $c < 0$ , arbitrarily small density disturbances are propagated upstream without any attenuation. While ideal cruise control system dynamics guarantees that there are no spacing errors at any time, yet density disturbance propagate upstream and unattenuated when  $c < 0$ . This is definitely an undesirable feature of automated vehicle traffic on a highway. In a constant time headway policy, the desired following distance is linearly proportional to the vehicle velocity,  $v$ . The constant of proportionality

is  $h_w$  and it is assumed that all vehicles have a length of  $L_0$ . In other words, at equilibrium,

$$\frac{1}{\rho} = L_0 + h_w v, \quad (6)$$

$$c = -\frac{L_0}{h_w}. \quad (7)$$

With a constant time headway policy,  $c$  is always negative for positive values of  $h_w$ . Since  $c$  is independent of  $\rho_0$ , the governing differential equation is a linear partial differential equation and the result holds at all operating traffic densities.

With the inclusion of lags in the cruise control system dynamics, one can show the linearized instability of traffic equilibrium if  $c < 0$ .

### 3.2 Modeling the effect of cruise control system dynamics on traffic flow behavior:

Ideal cruise control systems are a far cry from reality. Therefore, one must consider the dynamics of the control system that regulates intervehicular spacing. Variable spacing schemes to be employed in cruise control algorithms have been designed without any regard to traffic flow stability. In some instances, string stability is mistakenly assumed to guarantee traffic flow stability [7]. String stability analyses for automated vehicle following algorithms do not consider the conservation of mass equation (density evolution equation). Therefore, guaranteeing string stability does not necessarily ensure traffic flow stability. Readers interested in the details of string stability analyses are referred to some of the recent papers such as Sheikholeslam and Desoer [38], Swaroop, Hedrick, Chien and Ioannou [44] and to the doctoral dissertation of Swaroop [42]. Traffic flow stability is guaranteed only if the coupled differential equations that describe aggregate density and speed evolution exhibit stable behavior.

The following traffic flow model, which considers the reaction time,  $\tau$ , of the vehicle and the driver, was hypothesized in [30, 27, 18]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0, \quad (8)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{1}{\tau}[h(\rho) - v] - \mu(\rho) \frac{\partial \rho}{\partial x}. \quad (9)$$

The first equation expresses the conservation of the number of vehicles in the traffic flow. The second equation represents the traffic speed dynamics and is an analog of the balance of linear momentum for a fluid flow which has a specific constitutive relation for its stress. The underlying assumptions in deriving this constitutive equation have not been explicitly stated in these references. For example, if the vehicle following behavior of the vehicles constituting the traffic were unstable, can the above set of equations still represent resulting behavior of traffic flow?

A spatial discretization of the above equations has been used in [15, 5, 27]. Various researchers have used different functions for the pressure gradient term,  $\frac{\partial p}{\partial x} = \mu(\rho) \frac{\partial \rho}{\partial x}$ , but the underlying assumption is that  $\mu > 0$  in [45, 27, 5].

Our focus is to derive the constitutive equation that describes the dynamics of traffic speed from the vehicle following control laws. In the ensuing derivation, it is assumed that the acceleration of a vehicle can be controlled, i.e., it can be assigned any value. Now,

$$\begin{aligned} v_{des} &= \bar{h}(\Delta) = h(\rho), \\ e_v &:= v - \bar{h}(\Delta) = v - h(\rho), \end{aligned}$$

and thus

$$\dot{e}_v = \dot{v} - \dot{\bar{h}}(\Delta) = \dot{v} - h'(\rho)\dot{\rho}.$$

A choice of  $\dot{v}$  that ensures that the error in velocity,  $e_v$ , goes to zero asymptotically, with an exponential decaying rate,  $\frac{1}{\tau}$ , is given by:

$$\dot{v} = h'(\rho)\dot{\rho} - \frac{1}{\tau}(v - h(\rho)).$$

Note that

$$\begin{aligned} \dot{v} &= \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x}, \\ \dot{\rho} &= \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial x}. \end{aligned}$$

From the conservation of mass equation, it follows that

$$\dot{\rho} = -\rho \frac{\partial v}{\partial x}.$$

As a result, the analog of the equation describing the speed dynamics of the traffic flow is:

$$\frac{\partial v}{\partial t} + (v + \rho h'(\rho)) \frac{\partial v}{\partial x} = \frac{1}{\tau} [h(\rho) - v]. \quad (10)$$

Notice that the pressure gradient term is missing in the equation for the speed dynamics of automated traffic. In deriving this equation, it is assumed that the control actions of an automatically controlled vehicle is dependent solely on the behavior of the vehicle immediately preceding it. Therefore, such a term is missing in our derivation.

However, if one were to model the effectiveness of feeding back information from the roadside devices into the cruise control law (e.g., Variable Message Signs (VMS)), such a term will be present in the equation for the dynamics of traffic speed, as we shall see later in this section.

A general form of the balance of linear momentum in continuum mechanics is given by:

$$\text{div} \mathbf{T} + \rho \mathbf{b} = \rho \frac{d\mathbf{v}}{dt}. \quad (11)$$

In the above equation,  $\mathbf{T}$  denotes the Cauchy stress tensor, and  $\mathbf{b}$  denotes the body force term.  $\mathbf{v}$  denotes the velocity field. In the case of a one-dimensional flow, the kinematical quantity,  $\mathbf{D}$  reduces to  $\frac{\partial v}{\partial x}$ , and  $\mathbf{T}$  to a scalar,  $T$ .

If

$$T = - \int_{x_0}^x \rho^2 h'(\rho) \frac{\partial v}{\partial \psi} d\psi,$$

then equation (11) reduces to equation (10). Thus, we can think of  $T$  as the stress generated in an automated traffic. Different vehicle following control laws lead to different representations for  $T$ , i.e., to different constitutive relations for the stress. In other words, we have different governing equations depending on the constitutive characterization of the traffic. In the case of traffic consisting of manually controlled vehicles only,

$$T = - \int_{\rho_0}^{\rho} \mu(p) dp.$$

In both cases, the body force,  $\mathbf{b}$ , is  $\frac{1}{\tau}[h(\rho) - v]$ .

It is worth recognizing from the continuum perspective that equation (9) corresponds to an Euler fluid, [46, 37], i.e.,

$$\mathbf{T} = -p(\rho)\mathbf{I},$$

while equation (10) corresponds to a non-local model, wherein

$$\mathbf{T} = \mathcal{F}(\rho, \mathbf{D}) = \hat{\mathbf{T}}(\mathbf{x}, t).$$

Here, the stress tensor,  $\mathbf{T}$ , is a function of the density and the symmetric part of the gradient of the velocity field,  $\mathbf{D}$ , i.e.,

$$\mathbf{D} = \frac{1}{2} \left[ \frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \left( \frac{\partial \mathbf{v}}{\partial \mathbf{x}} \right)^T \right].$$

In fact,  $\mathbf{T}$  has the special form,

$$\hat{\mathbf{T}} = - \int_{x_0}^x \rho^2 h'(\rho) \frac{\partial v}{\partial x} dx := - \int_{x_0}^x \eta(\rho) \frac{\partial v}{\partial x} dx.$$

If the stress,  $T$ , is of the form:

$$T = -p(\rho) + h(\rho) \frac{\partial v}{\partial x},$$

where  $p(\rho)$  is a given pressure function, then

$$\frac{\partial T}{\partial x} = - \frac{\partial p}{\partial x} + h(\rho) \frac{\partial^2 v}{\partial x^2} + h'(\rho) \frac{\partial \rho}{\partial x} \frac{\partial v}{\partial x}.$$

Let the body force term be equal to  $\frac{1}{\tau}[h(\rho) - v] - \frac{h'(\rho)}{\rho} \frac{\partial \rho}{\partial x} \frac{\partial v}{\partial x}$ . Then, the balance of linear momentum corresponding to this stress is

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} h(\rho) \frac{\partial^2 v}{\partial x^2} - \frac{1}{\tau} [v - h(\rho)].$$

The above equation represents the balance of linear momentum for a Navier-Stokes fluid and is known as the Navier-Stokes equation. Of course, the body force in this case is not the gravitational force as is usually encountered in fluid dynamics. A cruise control law that results in the Navier-Stokes equation for the traffic dynamics is given as follows:

$$a_{des} = \frac{1}{\tau} [h(\rho) - v] + \frac{1}{\rho} h(\rho) \frac{\partial^2 v}{\partial x^2} - \frac{1}{\rho} \frac{\partial p}{\partial x}.$$

Here,  $a_{des}$  is the control input and is the commanded acceleration of the vehicle.  $\rho$  is implemented as  $\frac{1}{\Delta+L_c}$ , where  $L_c$  is the vehicle length and  $\Delta$  is the range measurement.  $\dot{\rho}$  is known once  $\Delta$  and  $\dot{\Delta}$  are known. Note that  $\dot{\Delta}$  is the range rate measurement and can be measured. The terms  $\frac{\partial \rho}{\partial x}$  and  $\frac{\partial^2 v}{\partial x^2}$  are obtained from the roadside sensors in the following manner: Consider two roadside sensors such as loop detectors placed a distance,  $L_{section}$ , apart, and measuring the occupancy of vehicles. From these two measurements, one can approximate the gradient of density and broadcast the information to every vehicle between the two roadside sensors.

Similarly one can compute the term,  $\frac{\partial^2 v}{\partial x^2}$  in two different ways. The first method involves three roadside sensors, located some distance apart. Each sensor measures the average velocity of the traffic at the location of the sensor, and  $\frac{\partial^2 v}{\partial x^2}$  can be obtained by processing the three velocity measurements using a central difference scheme and communicating the processed information to every vehicle between the extreme sensors. The second method involves utilizing the fact that

$$\frac{\partial v}{\partial x} = -\frac{\dot{\rho}}{\rho} = \frac{\dot{\Delta}}{\Delta}.$$

The second method involves two roadside sensors separated by a distance, each measuring the relative velocity and distance between vehicles. Such a measurement can be obtained using a device such as a camera located on the roadside. Once the two measurements are obtained, one can compute the average velocity gradient at each location, using the formula above. Given the values of the velocity gradient at two locations, one can compute  $\frac{\partial^2 v}{\partial x^2}$  from the above two values and can be communicated to every vehicle on the highway between the two sensors.

### 3.3 Linearized stability analysis of the traffic flow taking ICC system dynamics into consideration

We will use the following equations for describing the traffic flow:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} &= 0, \\ \frac{\partial v}{\partial t} + (v + \rho h'(\rho)) \frac{\partial v}{\partial x} &= \frac{1}{\tau} [h(\rho) - v]. \end{aligned}$$

Note that the balance of linear momentum equation is the same as equation (10). A stationary solution admitted by the above set of equations is  $\rho = \rho_0$ ,  $v_0 = h(\rho_0)$ . Consider the following possible, perturbed solution:

$$\begin{aligned} \rho &= \rho_0 + \tilde{\rho} e^{ik(x+ct)} e^{\lambda t}, \\ v &= h(\rho_0) + \tilde{v} e^{ik(x+ct)} e^{\lambda t}. \end{aligned}$$

Substituting the above expressions into the above set of differential equations, and ignoring higher order terms:

$$\begin{aligned} (\lambda + ik(c + h(\rho_0))) \tilde{\rho} + ik \rho_0 \tilde{v} &= 0, \\ (\lambda + \frac{1}{\tau} + ik(c + h'(\rho_0))) \tilde{v} - \frac{1}{\tau} h'(\rho_0) \tilde{\rho} &= 0. \end{aligned}$$

In the above equation,  $q'_c(\rho) = \rho h'(\rho) + h(\rho)$ . For a nontrivial solution, the following conditions must hold:

$$k^2(c + h(\rho_0))(c + q'_c(\rho_0)) = \lambda\left(\lambda + \frac{1}{\tau}\right),$$

$$\lambda(c + q'_c(\rho_0)) + \left(\lambda + \frac{1}{\tau}\right)(c + h(\rho_0)) + \frac{q'_c(\rho_0) - h(\rho_0)}{\tau} = 0.$$

As  $\tau \rightarrow 0$ , we recover the traveling wave solution, with the speed of the traveling wave,  $c$ , equal to  $-q'_c(\rho_0)$ .

Note that  $\lambda = 0$ ,  $c = -q'_c(\rho_0)$  and  $k$  is a real number, satisfies the above equation. In other words, the perturbed solutions agree quite closely with the traveling wave solutions as long as the perturbed solutions are sufficiently small. Such an observation is in agreement with the simulation results and with the results of Lighthill-Whitham [20].

With a constant time headway spacing policy, the slope,  $c = -\frac{L_0}{h_w}$  and is always negative. This implies that small density disturbances propagate upstream without any attenuation.

### 3.4 Effect of Spacing Policy employed by ICC on traffic flow stability (Spatially Discretized Approach):

For the sake of illustration, consider the  $i$ th section of a  $N$ -section single lane highway as shown in the figure 0.

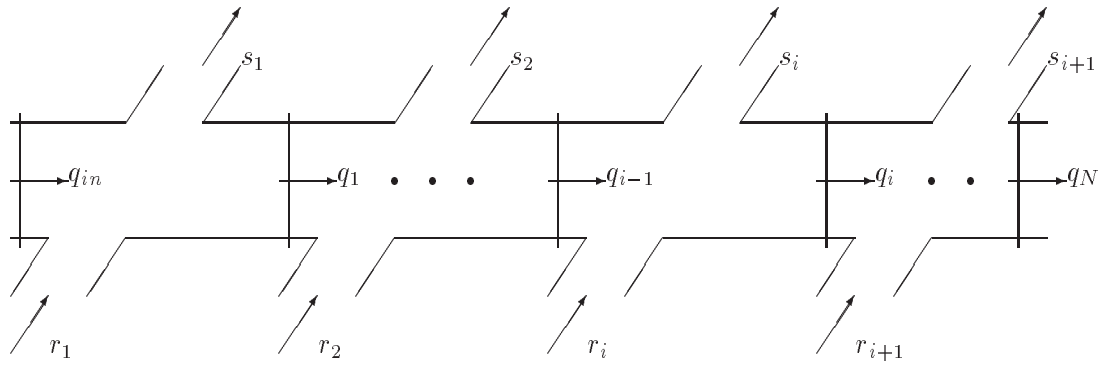


Figure 0: A schematic of a highway

Let the input traffic volume into the section be  $q_{i-1}$ , let the volume of traffic from the on-ramp into the section be  $r_i$ . Let  $s_i$  be the traffic volume exiting this section via the off-ramp and let  $q_i$  be the traffic volume from this section into the adjoining downstream section of the highway. Let  $\delta_i$  be the length of the section, and let  $N_i(t)$  denote the number of vehicles in the section at time  $t \geq 0$ . Furthermore, suppose  $v_i$  is the aggregate velocity of all vehicles in this section and  $\rho_i$  is the aggregate density of vehicles in this section. Then,  $\rho_i = \frac{N_i}{\delta_i}$ .

On balancing the number of vehicles entering and leaving the section,

$$\begin{aligned}\dot{N}_i &= q_{i-1} + r_i - s_i - q_i, \\ \Rightarrow \dot{\rho}_i &= \frac{q_{i-1} + r_i - s_i - q_i}{\delta_i}.\end{aligned}$$

We will assume the following discretization for traffic volume:

$$q_i = \alpha_i \rho_i v_i + (1 - \alpha_i) \rho_{i+1} v_{i+1}.$$

The above discretization of the traffic volume is widely used in the literature, for example, see Karaslaan and Varaiya, [15], and Stotsky, Chien and Ioannou [5], Papageorgiou [27]. The flow through a section of the highway,  $q_i$ , is assumed to be a convex combination of the theoretical traffic volume,  $\rho_i v_i$ , of the section and the theoretical traffic volume,  $\rho_{i+1} v_{i+1}$ , of its downstream neighbor. Here,  $\alpha_i$  is the factor associated with such a combination. Such a combination models the realistic dependence of the traffic flow dynamics of a section on the state of traffic downstream from it.

Equilibrium conditions are specified by the conservation of mass equation and the constitutive relationship between the traffic speed and density,  $v = h(\rho)$ .

$$\begin{aligned}q_{i-1} + r_i - s_i - q_i &= 0, i = 1, \dots, N \\ \text{or equivalently,} \\ \alpha_{i-1} \rho_{i-1} v_{i-1} + (1 - \alpha_{i-1} - \alpha_i) \rho_i v_i - (1 - \alpha_i) \rho_{i+1} v_{i+1} &= s_i - r_i, i = 2, \dots, N - 1, \\ \alpha_1 \rho_1 v_1 + (1 - \alpha_1) \rho_2 v_2 &= q_0 + r_1 - s_1, \\ \alpha_{N-1} \rho_{N-1} v_{N-1} + (1 - \alpha_{N-1}) \rho_N v_N - \rho_N v_N &= s_N - r_N, \\ \text{and } \rho_i &= h(v_i), i = 1, \dots, N.\end{aligned}$$

Let  $\rho_i^* = \eta_i(q_0, r_1, \dots, r_N, s_1, \dots, s_N)$  be the equilibrium density and  $v_i^*$  be the corresponding equilibrium traffic speed.

Define  $\tilde{\rho}_i = \rho_i - \rho_i^*$ . Stability of the traffic flow with the spacing policy can be now be analyzed via linearization of the traffic flow dynamics about the steady state operating density. Define

$$c(\rho) := \frac{d(\rho v)}{d\rho} = \left(\rho \frac{\partial h}{\partial \rho} + h\right)(\rho).$$

Then, linearization of the density evolution equations results in :

$$\begin{aligned}\dot{\tilde{\rho}}_1 &= \frac{-\alpha_1 c(\rho_1^*) \tilde{\rho}_1 - (1 - \alpha_1) c(\rho_2^*) \tilde{\rho}_2}{\delta_1}, \\ \dot{\tilde{\rho}}_i &= \frac{\alpha_{i-1} c(\rho_{i-1}^*) \tilde{\rho}_{i-1} + (1 - \alpha_{i-1} - \alpha_i) c(\rho_i^*) \tilde{\rho}_i - (1 - \alpha_i) c(\rho_{i+1}^*) \tilde{\rho}_{i+1}}{\delta_i}, \quad i = 2, \dots, N - 1, \\ \dot{\tilde{\rho}}_N &= \frac{\alpha_{N-1} c(\rho_{N-1}^*) \tilde{\rho}_{N-1} - \alpha_{N-1} c(\rho_N^*) \tilde{\rho}_N}{\delta_N}.\end{aligned}$$

Compactly, the above set of equations can be written as:

$$\dot{X} = A_{lin} X,$$



where  $X = [\tilde{\rho}_1, \dots, \tilde{\rho}_N]^T$  and  $A_{lin}$  is given by

$$A_{lin} = D_1 A D_2,$$

$$D_1 = \begin{bmatrix} \frac{1}{\delta_1} & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{\delta_2} & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & \frac{1}{\delta_{N-1}} & 0 \\ 0 & 0 & \dots & 0 & \frac{1}{\delta_N} \end{bmatrix},$$

$$A := \begin{bmatrix} \alpha_1 & (1 - \alpha_1) & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ -\alpha_1 & -(1 - \alpha_1 - \alpha_2) & (1 - \alpha_2) & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & -\alpha_2 & -(1 - \alpha_2 - \alpha_3) & (1 - \alpha_3) & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & -\alpha_{N-2} & -(1 - \alpha_{N-2} - \alpha_{N-1}) & (1 - \alpha_{N-1}) \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & -\alpha_{N-1} & \alpha_{N-1} \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -c(\rho_1^*) & 0 & \dots & 0 & 0 \\ 0 & -c(\rho_2^*) & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & -c(\rho_{N-1}^*) & 0 \\ 0 & 0 & \dots & 0 & -c(\rho_N^*) \end{bmatrix}.$$

**Stability of a constant time headway spacing policy:** In this policy, section density,  $\rho_i$  and traffic speed,  $v_i$  are related by:

$$\frac{1}{\rho_i} = L_0 + h_w v_i \Rightarrow v_i = \frac{1}{h_w} \left[ \frac{1}{\rho_i} - L_0 \right].$$

Here,  $L_0$  is the sum of intervehicular spacing at standstill and vehicle length, and  $h_w$  is the constant time headway. As a result,  $c(\rho) = -\frac{L_0}{h_w} < 0$ .

If all the eigenvalues of  $A_{lin}$  have negative real parts, then the equilibrium is locally stable. If atleast one eigenvalue of  $A_{lin}$  has a positive real part, the equilibrium is unstable. In case of constant time headway policy, it is easy to see that  $D_2 > 0$ . Since  $D_1 > 0$  and  $D_1, D_2$  area diagonal, it follows from Sylvester's theorem of inertia and the properties of a similarity transformation that the signature of the matrix  $A_{lin}$  is the same as the signature of the matrix,  $A$ .

With elementary matrix operations, it can be shown that the determinant of the matrix,  $A_{lin}$ , is  $(\frac{L_0}{h_w})^n \prod_1^{N-1} \frac{\alpha_j}{\delta_j} \frac{1}{\delta_N} > 0$ . Furthermore, consider the last row of  $A$ . One of the Gershgorin's disks [11] is given by  $|\lambda - \alpha_{N-1}| \leq |\alpha_{N-1}|$ , where  $\lambda$  is an eigenvalue of  $A$ . From the above facts, it is clear that  $A_{lin}$  has an eigenvalue with a positive real part, since the determinant is non-zero. Although all vehicles are assumed to have the same length, by a similar argument, it can be shown that  $A_{lin}$  has an eigenvalue with a positive real part, even when the vehicles do not have the same length. In other words, constant time headway policy always leads to an unstable traffic flow behavior.

It should be remarked that this spacing policy leads to traffic flow instability because the slope of the equilibrium volume-density curve is always negative.

Physically, this implies that the steady state traffic throughput with this spacing policy decreases with increasing traffic density. Recall that the unstable regime of the fundamental characteristic of traffic (equilibrium volume-density curve) consisting of manually controlled vehicles has a negative slope.

It should be noted that the spatial discretization is not producing any spurious results here. The solution of the non-discretized conservation of mass equation with the LW (ideal cruise control) approximation is a backward traveling wave that does not amplify or attenuate in time. *While such a solution is stable in the sense of Lyapunov, it is not desirable.* The solution of the spatially discretized approximation indicates instability. The reason is clear from the inherent lack of stability robustness for the constant time headway spacing policy as was evinced in the earlier section.

While the assumption of an ideal vehicle cruise control system is not realistic, it serves to differentiate traffic flow stability from string stability. String stability is concerned with the issue of adequacy of information to regulate the intervehicular spacing in a vehicle string according to a pre-specified spacing policy. String stability, therefore, implies the stability of aggregate traffic speed dynamics consistent with the employed spacing policy and is only a *necessary condition* for traffic flow stability. Due to the assumption of an ideal vehicle cruise control system on all vehicles constituting the traffic, string stability is vacuously guaranteed since the spacing error of every vehicle in following a vehicle ahead is always zero. Traffic flow stability, however, is also dependent on the spacing policy employed in the vehicle cruise control systems.

Since the limiting performance of constant time headway policy obtained by ideal Intelligent Cruise Control systems always leads to an unstable traffic flow behavior, all realistic Cruise Control Systems that employ a constant time headway policy, lead to an unstable traffic flow behavior, as was shown in the earlier section.

## 4 Simulation of a traffic flow:

For simulation purposes, we have considered a single section of length  $1000m$  of a single lane freeway. All the vehicles constituting the traffic on this section are automatically controlled and the control algorithm used is given in [44]. Specific values of time headway of  $1sec$  and a control gain,  $\lambda = 1$  are used in this simulation. We have assumed that the spacing between vehicles, including the car length, at standstill is  $10m$ . Entry ramp to the section is located at  $350m$  from the downstream end of the section. A vehicle merging into the automated traffic from the on-ramp is placed halfway between the vehicles closest to the section with their average velocity. Any merging vehicle induces braking in its following vehicles. Simulations indicate that  $350m$  is sufficiently long for any vehicle to pick up the speed while following a vehicle ahead. This simulation is set up such that if a vehicle does not have a predecessor in the section, it will maintain its velocity until it leaves the section. The mainline traffic volume is  $2700vehicles/hour$ . In other words, three vehicles enter every four seconds. As long as time modulo four is one, two or three, a vehicle is placed at the upstream end of the section with a velocity equal to that of its immediate

predecessor. In other words, the vehicle entering will have no velocity error initially; however, it may have a spacing error. Corresponding to this mainline traffic volume, a time headway of  $1sec$  and the standstill distance between the center of mass of the vehicles,  $L_0 = 10m$ , the uniform traffic speed is  $30m/s$ . Therefore, initially all vehicles move at  $30m/s$ .

We have considered two scenarios of merging vehicles. In the first scenario, we simulate a traffic burst, i.e a total of eight vehicles merge into the automated traffic in  $160sec$ . In the second scenario, we simulate a continuous stream of merging vehicles, with a traffic volume of  $180vehicles/hour$ . All vehicles merge from the ramp only after  $t = 50sec$ .

The following statistics are calculated every  $0.5sec$ : the mean velocity of all vehicles in the section, the number of vehicles in the section, the cumulative number of vehicles that have entered (either from the mainline or from the on-ramp) and exited the section and the position of all vehicles in the section at that time. Four plots corresponding to each of the scenario clearly paint the picture of the behavior of automated traffic flow.

The first plot depicts the evolution of aggregate traffic density. The discrete nature of counting the number of vehicles in a section and the fact that the section length can not identically be an integral multiple of intervehicular distance, the resulting density plot contains a lot of spikes. Looking at figure 2, the process of densification can be understood. For example, consider the time interval from 280 seconds to 440 seconds. In the time interval from 280 seconds to approximately 310 seconds, the frequency of a count of 34 vehicles is higher than the frequency of a count of 33 vehicles or that of count of 35 vehicles in the section. Hence, you see pointed spikes in the upward and downward direction from the count of 34 vehicles in the section in that time interval. In the time interval from 310 seconds to 380 seconds, one could clearly see a progressive increase in the frequency of the count of 35 vehicles. After time  $t = 380$  seconds, frequency of the count of 35 vehicles in the section is much higher than the frequency of the count of 34 vehicles or 36 vehicles. Hence, one can see upward and downward spikes from the count of 35 vehicles. In this figure, densification slows after  $t = 200$  seconds, by which time the burst of vehicles have already merged into the traffic. Notice that the density always increases, however, slowly. With a constant stream of merging vehicles, densification progresses at a faster pace as can be seen from figure 5.

Figures 2 and 6 show the speed evolution with this spacing policy under the two scenarios described above. The chattering nature of this plot is due to the discrete nature of counting and averaging the velocity of all vehicles in the section.

In the simulations, there are two counters - an inflow counter and an outflow counter. Whenever a vehicle enters from the mainline or the on-ramp, the inflow counter is incremented by one. Similarly, whenever a vehicle leaves the section from the downstream end, the outflow counter is incremented by one. Figures 3 and 7 show how the value of these two counters vary with time when AICC with a constant spacing policy is used. Clearly, the difference between the two counters is diverging, indicating an accumulation of vehicles in the section.

Every dot in figures 4 and 8 has two coordinates. The horizontal coordinate

is time and the vertical coordinate is the position from the upstream end. A dot represents the position of a vehicle (vertical coordinate) at a given time (horizontal coordinate). A lower value of the vertical coordinate implies that the vehicle is close to the upstream end of the section. The number of vehicles in the section at any given time can be found by counting the number of dots which have the given time as the horizontal coordinate. The intervehicular spacing distribution can be similarly inferred. If there are more dots in a part of the section at a given time, that part of the section is congested and appears dense in the plot. One can clearly see the upstream propagation of density

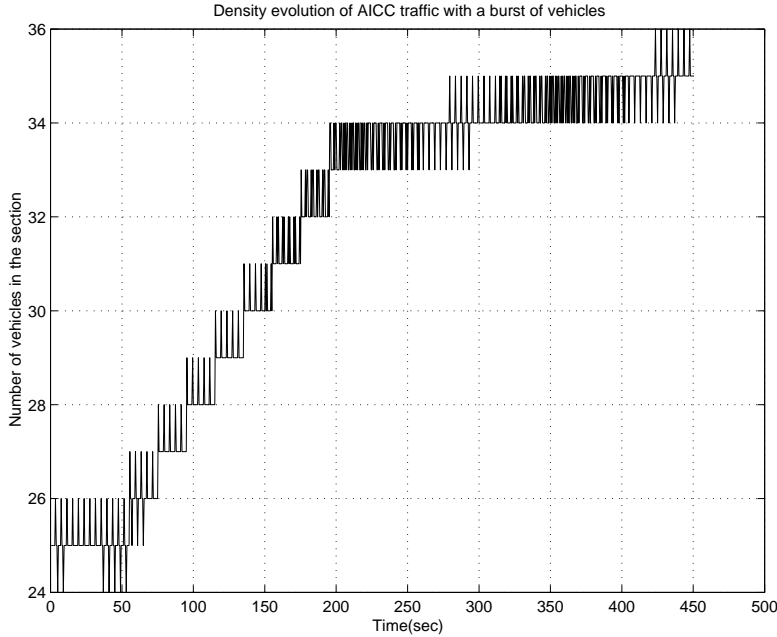


Figure 1: Aggregate density evolution of traffic with AICC with a burst of on-ramp merging vehicles

disturbances resulting from vehicles merging into the automated traffic. In both scenarios, the upstream end of the section is congested. In fact, vehicles are stopped at the upstream end of the section.

## 5 The problem of specifying a spacing policy

Since a constant time headway spacing policy leads to traffic flows with undesirable stability characteristics, it is natural to ask how to synthesize a spacing policy with desirable stability characteristics. For example, let the following be the specifications of a desirable traffic characteristic for autonomous vehicles:

1. A maximum capacity of  $A_{max}$  at a fraction,  $\beta$ , of the jam density,  $\rho_{max}$ . Here,  $\rho_{max}$  is the density that corresponds to a situation where the front and rear bumpers of any two adjoining vehicles touch each other.

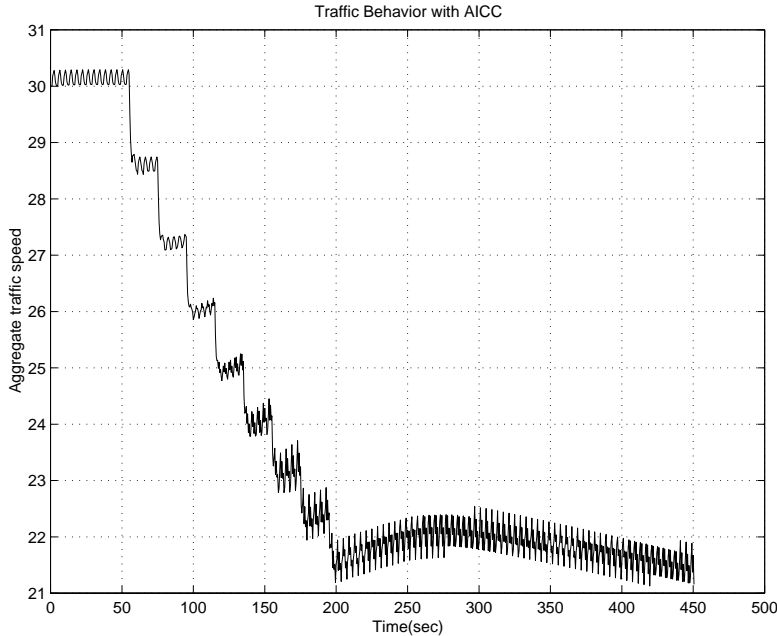


Figure 2: Aggregate traffic speed with AICC with a burst of on-ramp merging vehicles

2. It is desired that the traffic flow be stable over the entire possible density regime,  $[0, \rho_{max}]$ . By this, we mean that the density disturbances must attenuate as they propagate upstream.
3. The capacity is always a positive number.

The impossibility of the second specification with an autonomous vehicle following law is shown in [43] using Rolle's theorem. Traffic flow stability can only be guaranteed up to the peak density value if autonomous vehicles have negligible actuation and sensing delays.

One can synthesize many different characteristic curves that satisfy the first and last specifications. One such characteristic curve is synthesized from the formulae given in [6, 43]:

$$v^* = v_f \left(1 - \left(\frac{\rho}{\rho_{max}}\right)^l\right)^m$$

For example, with  $l = 1$ ,  $m = \frac{\rho_{max} - \rho^*}{\rho^*}$ , where  $\rho^*$  is the specified critical traffic density, and

$$v_f = \frac{C_{max}(1 + lm)^{\frac{1+lm}{m}}}{\rho_{max}(lm)^m}$$

the specifications on critical density and maximum capacity are met.

The sensitivity of  $v^*$  on  $\rho^*$  is a specification on the actuators, sensors and the vehicle following control system. The feasibility of such a spacing policy is then determined by the capabilities of the control system.

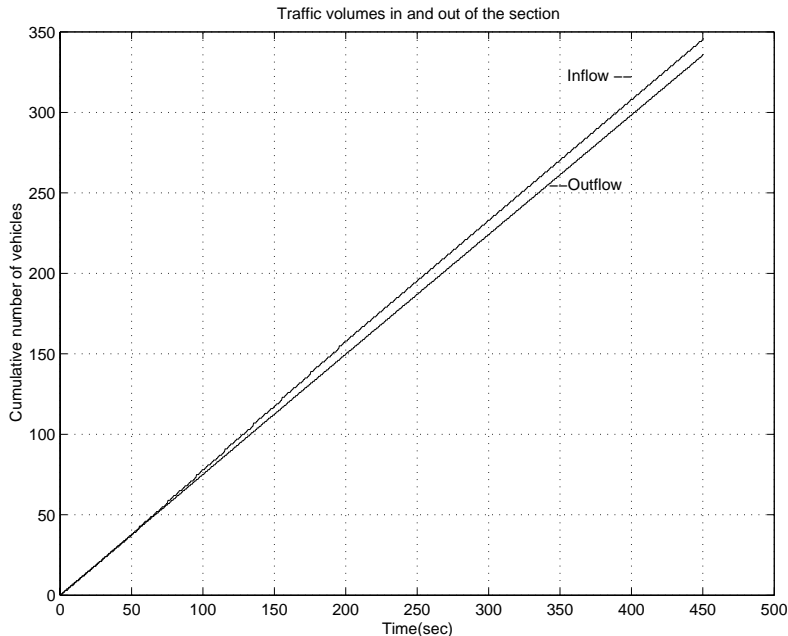


Figure 3: Traffic Inflow and Outflow from the section with AICC with a burst of on-ramp merging vehicles

## 6 Summary and Conclusions

In this paper, we have addressed the issue of the effect of vehicle following control laws on the behavior of traffic flow. To aid the investigation, we have motivated physically and then defined mathematically, what is meant by string stability, what is meant by traffic flow stability and what the difference between the two is. We have explicitly stated the assumptions in deriving the constitutive equation that describes the speed dynamics of the traffic flow. Then, we have shown that traffic flow is unstable if every vehicle is equipped with an automatic vehicle following system that employs a constant time headway policy. We have numerically verified the upstream, unattenuated propagation of disturbances, if every vehicle in the traffic were equipped with an ICC system employing a constant time headway policy. At the end of this paper, we have provided a framework for designing cruise control laws that take their effect on traffic flow into consideration.

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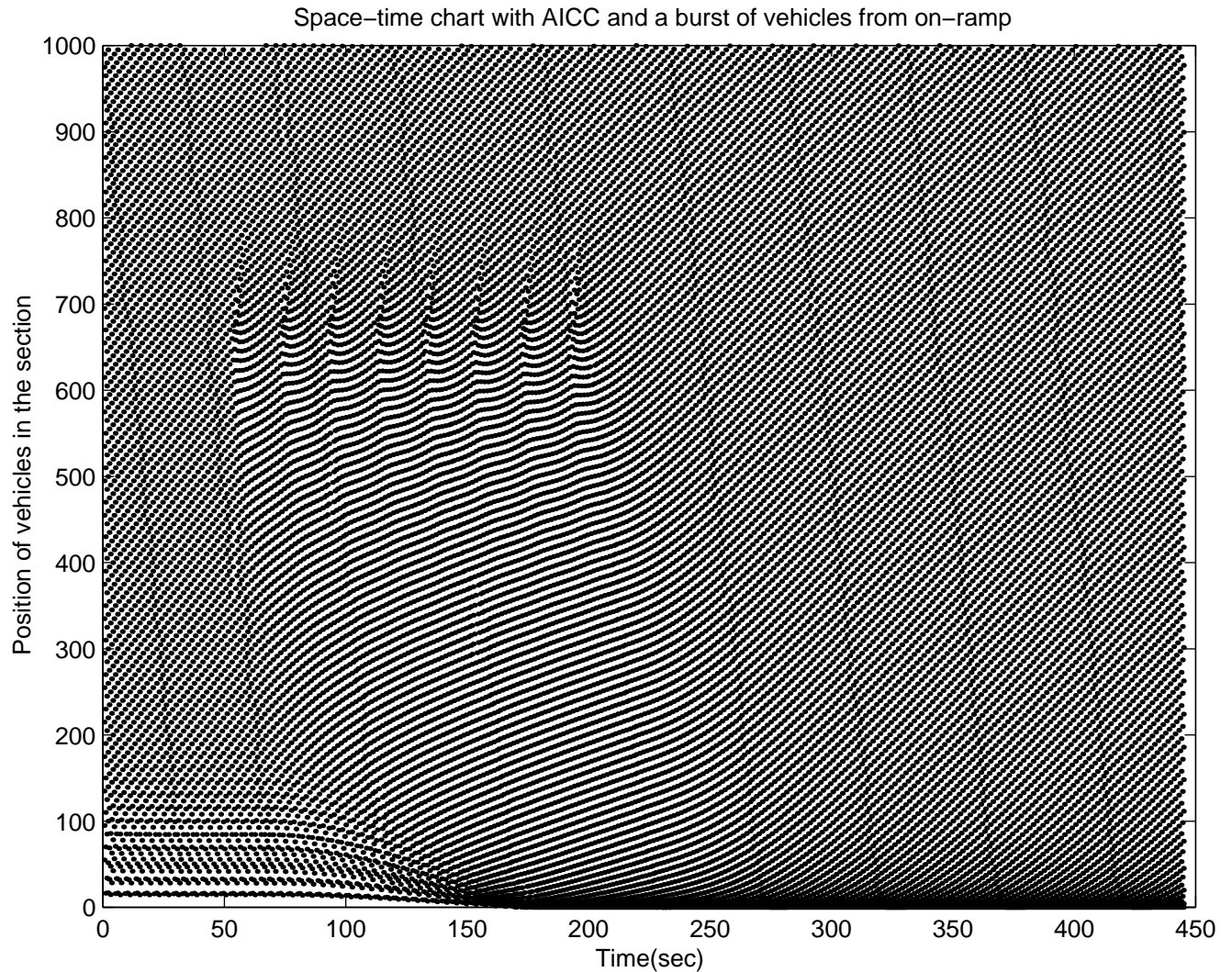


Figure 4: Space-time chart of AICC traffic with a burst of on-ramp merging vehicles

## References

- [1] Blinkin, M. Ya, "Problem of Optimal control of traffic flow on highways," *Automation and Remote control*, vol.37, pp. 662-667, 1976.
- [2] Broucke, Mireille., Varaiya, P., "A theory of traffic flow for Automated Highway Systems," *Transportation Research*, Vol. C, August, 1996.
- [3] Cremer, M., Papageorgiou, M., "Parameter identification for a traffic flow model," *Automatica*, vol. 17, pp. 837-843, 1981.
- [4] Chandrasekhar, S., "Hydrodynamic and Hydromagnetic Stability," Oxford Press, 1961.
- [5] Chien, C. C., Zhang, Y., Stotsky, A., Ioannou, P., "Roadway Traffic Controller design for automated highway systems," pp. 2425-30, Vol. 3, 1994.
- [6] Drew, D.R., "Traffic Flow Theory and Control," McGraw-Hill, New York, 1968.

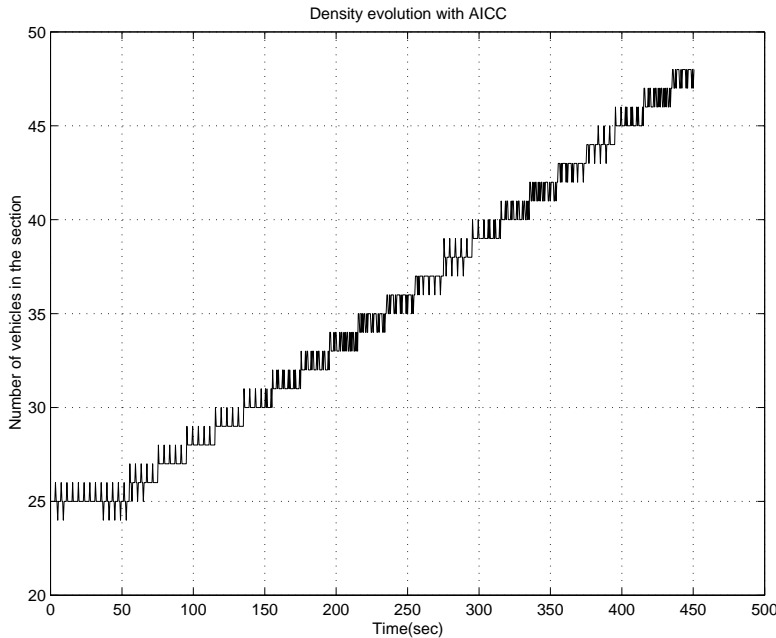


Figure 5: Aggregate density evolution of traffic with AICC with a stream of on-ramp merging vehicles

- [7] Fancher, P., Bareket, Z., “Influence of AICC on traffic flow,” In Proceedings of the Advanced Vehicular Control, pp. 402-407, 1994.
- [8] Galdi, G. P., Padula, M., “A new approach to energy theory in the stability of fluid motion,” Archive for rational mechanics and analysis, Vol. 110, no. 3, Springer-Verlag, 1990.
- [9] Galdi, G. P., Rionero, S., “Weighted Energy Methods in Fluid Dynamics and Elasticity,” Springer Lecture Notes in Mathematics, No. 1134, 1985.
- [10] Gazis, D. C., Herman, R., Potts, R., “Car-following theory of steady state traffic flow,” Operations Research, vol. 7, pp. 499-595, 1959.
- [11] Golub, Gene, H., Van Loan, Charles, F., “Matrix Computations,” The Johns Hopkins University Press, 1989.
- [12] Greenlee, T. L., Payne, H.J., “Freeway Ramp metering Strategies for Responding to Incidents,” Proceedings of the 1977 IEEE Conference on Decision and Control, pp. 987-992, 1997.
- [13] Greenshields, B. D., “A study in Highway capacity”, Highway Research Record, vol. 14, p. 468, 1934.
- [14] Ioannou, P., Chien, C, “Autonomous Intelligent Cruise Control,” IEEE Transactions on Vehicular Technology, Vol.42, pp. 657-72, 1993.
- [15] Karaslaan, U., Varaiya, P., Walrand, J., “Two proposals to improve freeway traffic flow,” In Proceedings of the American Control Conference, pp. 2539-44, 1991.
- [16] KampedeFeret, J., “Sur la décroissance de lenergie cinetique dun fluide viqueux incompressible occupant un domaine borne ayant pour frontiere



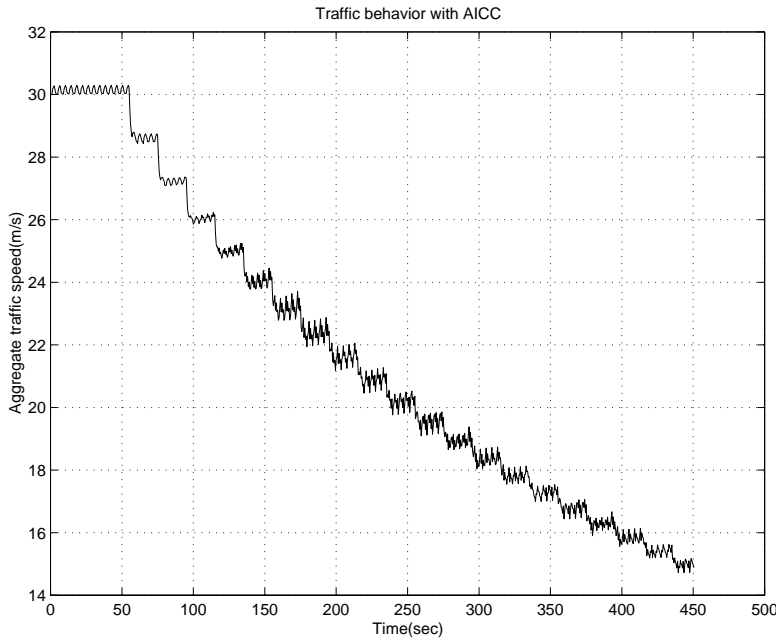


Figure 6: Aggregate traffic speed with AICC with a stream of on-ramp merging vehicles

des parois solides fixes,” published in *Am. Soc. Sc. Druxelles*, Vol. 63, pp. 36-45, 1949.

- [17] Leontief, W. W., “Composite Commodities and the problem of Index numbers,” *Econometrica*, Vol. 4, pp. 39-59, 1936.
- [18] Leutzbach, W., “Introduction to the theory of traffic flow,” Springer Verlag, 1988.
- [19] Liepmann, H. W., Roshko, A., “Elements of gas dynamics,” John Wiley and Sons, 1957.
- [20] Lighthill, M. J., Whitham, G. B., “On Kinematic Waves II. A Theory of Traffic Flow on Long Crowded Roads,” *Proceedings of the Royal Society of London, Series A* 229, pp. 317-345, 1955.
- [21] Lin, C. C., “Theory of hydrodynamic stability,” Cambridge University Press, Cambridge, 1955.
- [22] Looze, D. P., Hout, P. K., Sandell, N. R., Athans, M., “On decentralized Estimation and Control with Application to Freeway Ramp Metering,” *IEEE Transactions on Automatic Control*, AC-23, pp. 268-275, 1978.
- [23] Lyapunov, A. M., Fuller, A. T. (translator), “On the stability of motion,” reprint of doctoral dissertation, published in *International Journal of Control*, March 1992.
- [24] Marsden, J. E., McCracken, M., “Hopf Bifurcation and Its Applications,” *Applied Mathematical Science*, Vol. 19, Springer-Verlag, 1976.
- [25] Orr, W. M., “The stability or instability of the steady motions of a liquid,” *Proceedings of Royal Irish Academy*, Vol. A, no. 27, 1907.

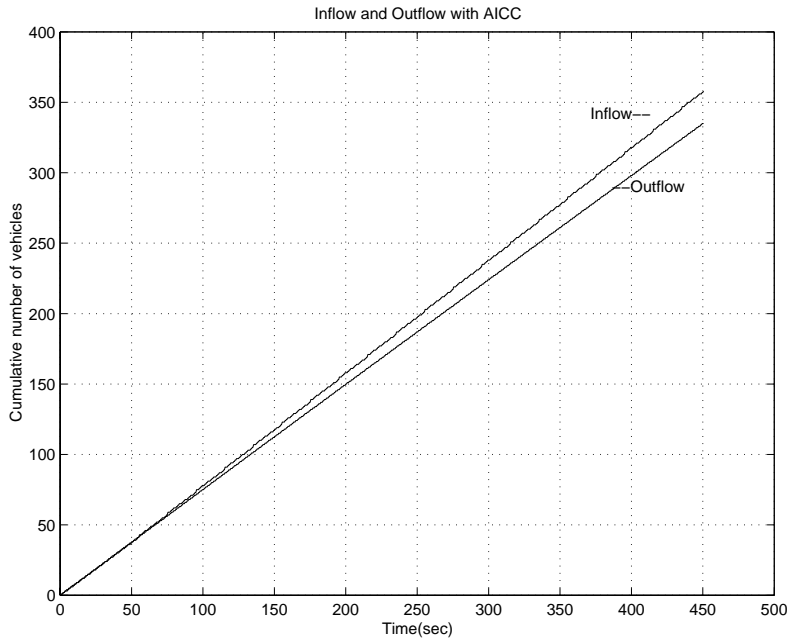


Figure 7: Traffic Inflow and Outflow from the section with AICC with a stream of on-ramp merging vehicles

- [26] Palmquist, U., "Intelligent Cruise Control and Roadside Information," IEEE Micro 13, 1, pp. 20-8, 1993.
- [27] Papageorgiou, M., "Lecture Notes in Control and Information Sciences," Springer Verlag, Vol. 50, 1983
- [28] Papageorgiou, M., Mayr, R., "Optimal decomposition methods applied to motorway traffic control," International Journal of Control, Vol. 35, pp. 269-280, 1982.
- [29] Papageorgiou, M., Blosseville and Hadj-Salem, H., "Macroscopic modelling of traffic flow on the boulevard peripherique in Paris," Transportation Research, A 24:345-359, 1990.
- [30] Payne, H. J., "Models of Freeway Traffic and Control," Simulation Council Proceeding, vol.1, pp. 51-61, 1971.
- [31] Payne, H. J., Thompson, W. A., Isaksen, L., "Design of a traffic-responsive control system for a Los Angeles Freeway," IEEE Transactions on Systems, Man and Cybernatics, SMC-3, pp. 213-224, 1973.
- [32] Pipes, L.A., "An operational analysis of traffic dynamics", Journal of Applied Physics, vol. 24, pp. 274-281, 1953.
- [33] Reuschel, A., "Fahrzeugbewegungen in der Kolonne", Zeitschrift d. Oesterr. Ing. u. Arch. Vereins, vol. 95, pp. 59-62, 73-77, 1950.
- [34] Richards, P.I., "Shock Waves on the Highway," Operations Research, pp. 42-51, 1956.
- [35] Saxton, L., Schenck, C., "Diversion and Corridor Control Systems in Western Europe", In "World Survey on Current Research and Development on

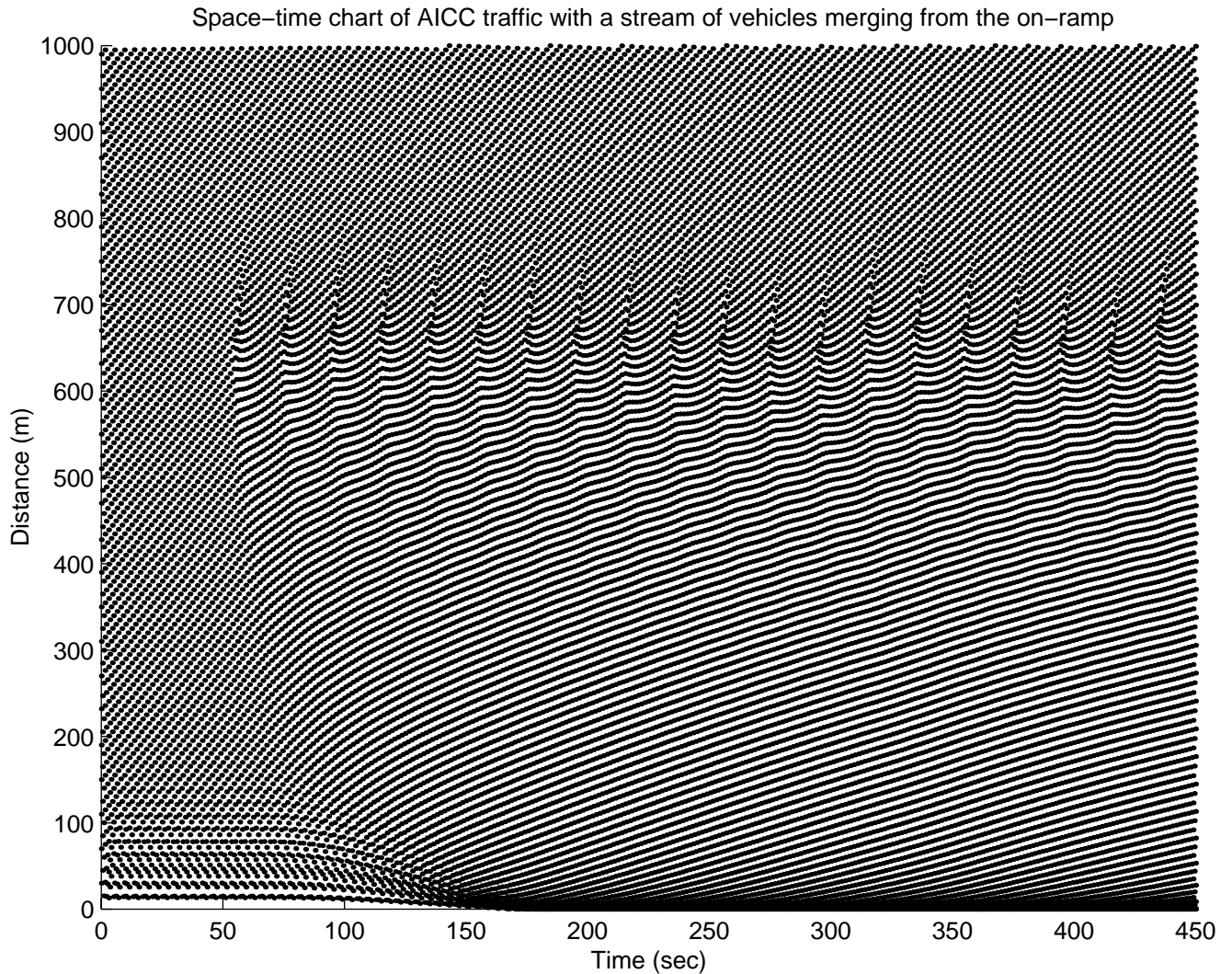


Figure 8: Space-time chart of AICC traffic with a stream of on-ramp merging vehicles

Roads and Road Transport,” International Road Federation, Washington, D.C., pp. 692-723, 1977

- [36] Saridis, G.N., Lee, C.S.G., “On heirarchically intelligent Control and Management of Traffic Systems,” In Engineering Foundations Conference in Computer control of Urban Traffic”, Levine, W. S., Lieberman, E., Fearnside, J.J., Editors, pp. 209-218, 1981.
- [37] Serrin, J., “Mathematical Principles of classical fluid mechanics,” Handbuch der Physik, vol. VIII/1, Springer, 1959.
- [38] Sheikholeslam, S., Desoer, C. A., “Longitudinal control of a platoon of vehicles,” In Proceedings of American Control Conference, pp. 291-296, May 1990.
- [39] Shladover, Steven, “An overview of the Automated Highway Systems Program,” Vehicle System Dynamics Journal, Vol. 24, 1995, pp. 551-595.

- [40] Simon, H. A., Ando, A., "Aggregation of Variables in Dynamic Systems," *Econometrica*, vol. 29, no.2, pp. 111-38, April 1961.
- [41] Straughan, B., "Instability, nonexistence and weighted energy methods in fluid dynamics and related theories," *Research Notes in Mathematics*, Pitman Publishing Inc., 1982.
- [42] Swaroop, D., "String Stability of Interconnected Systems: An application to platooning in Automated Highway Systems," Ph. D Dissertation, Department of Mechanical Engineering, University of California, Berkeley, December, 1994.
- [43] D. Swaroop, R. Huandra, "Variable Spacing Control Strategies for Automated Highway systems," In preparation for submission to the Special Issue on Intelligent Transportation Systems, *Vehicle System Dynamics Journal*, 1998.
- [44] D. Swaroop, J. K. Hedrick, C. C. Chien, P. A. Ioannou, " A Comparison of spacing and headway control strategy for automatically controlled vehicles," *Vehicle System Dynamics*, Vol. 23, no. 8, pp. 597-625, 1994.
- [45] TRB Monograph on Traffic Flow Theory, edited by Nathan Gartner, Carol Messer, and Ajay Rathi, 1998.
- [46] Truesdell, C., "A first course in rational continuum mechanics," Second edition, Vol. 1., Academic Press, 1991.
- [47] Varaiya, P., "Smart Cars on Smart Roads: Problems of Control ," *IEEE Transactions on Automatic Control*, Vol. 38, No.2, pp. 195-207, 1990.
- [48] Vidyasagar, M., "Nonlinear Systems Analysis," Prentice Hall, 1978
- [49] Willsky, A. S., Chow, E.Y., Gershwin, S.B., Greene, C.S., Houpt, P. K., Kurkjian, A. L., "Dynamic Model-Based Techniques for the Detection of Incidents on Freeways," *IEEE Transactions on Automatic Control*, vol. AC-25, no.3, June 1980.