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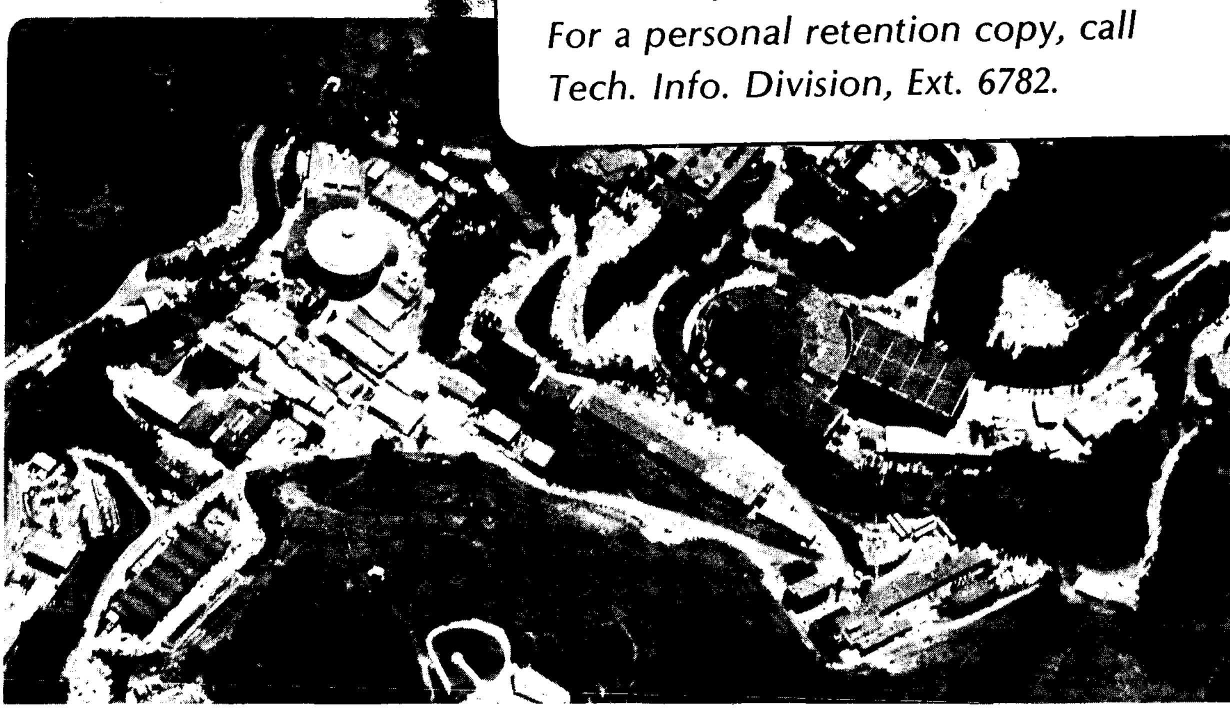

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The Higgs-Higgs Bound State

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Abstract

The binding of two Higgs bosons is calculated using the N/D method. Bound states occur for $m_H > 1.3\,\mathrm{TeV}$. The binding is weak and even when $m_H = 2\,\mathrm{TeV}$ the binding energy is only about 150 GeV. Higgs bosons with masses in the TeV range have widths comparable their masses. Accordingly, so do their bound states.

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I. Introduction

The continuing success of the standard model of electroweak interactions increases the importance of understanding spontaneous symmetry breaking mechanisms. While alternatives are available, the initial proposal of Weinberg [1] and Salam [2] that fundamental scalars are responsible remains a viable approach. Indeed, the search for the Higgs boson, the surviving scalar, is one of the most prominent goals at present accelerators and a major motivation for the construction of new ones.[3]

Present opinion is that a light Higgs boson could be best found [4] in $\psi \to H\gamma$ or $\Upsilon \to H\gamma$, or if a $\bar{t}t$ state, Θ, were found, in $\Theta \to H\gamma$. If the Higgs boson has a mass $m_H < 35$ GeV, e^+e^- colliders could find it by observing the decay[5] $Z \to Z^* H \to (\mu^+\mu^- \text{ or } e^+e^-)\, H$. At a very high energy e^+e^- machine, say with $\sqrt{s} \approx 2m_W$, the process $e^+e^- \to ZH$ could be seen for $m_H \leq 60$GeV.[6] It appears that very high energy pp or $\bar{p}p$ colliders ($\sqrt{s} \approx 20-40$TeV) could find a Higgs boson with a mass $m_H \geq 2m_W$ through the signature $H \to W^+W^-$ or $H \to ZZ$.[7] Ordinary Drell-Yan production of W^+W^- and ZZ would be formidable backgrounds since the width of a very heavy Higgs boson is enormous:

$$\Gamma(H \to W^+W^- + H \to ZZ) \approx 50\text{GeV}\left(\frac{m_H}{500\text{GeV}}\right)^3 \tag{1}$$

and thus identifying an excess of events would be extremely difficult if $m_H \geq 500$GeV.[8] (For $2m_b < m_H < 2m_W$, the Higgs boson would decay primarily into $\bar{b}b$ or $\bar{t}t$, which would make it rather difficult to identify.)

The problem of identifying a very heavy Higgs boson, $m_H \geq 500$GeV, is thus a serious one and it is worthwhile to seek indirect tests of its presence. One possibility is to identify low energy parameters whose values might reveal the existence of a very heavy Higgs boson. This has received much theoretical attention, especially from Veltman and co-workers[9], and Appelquist and co-workers[10]. Their perturbative calculations indicate that the effects of very heavy Higgs bosons are small, of order $(g^2/16\pi^2)\log(m_H^2/m_W^2)$.

A large Higgs boson mass requires a strong self-coupling of the Higgs boson and for sufficiently large Higgs boson mass, perturbation theory is no longer reliable. We have investigated a simple non-perturbative effect: the existence of bound states. Our calculations show that two Higgs bosons should bind if $m_H \geq 1.3$ TeV. The binding energy, however, is not very great even if $m_H = 2$ TeV. As a result, the bound state will have a width roughly twice the width of the Higgs boson itself, and thus probably not be any easier to observe than the Higgs boson although the signature of its decay into four W's might be striking.

II. The Higgs Sector

If the gauge interactions are ignored, the conventional Higgs doublet is equivalent to the linear σ-model. We write

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\sigma + i\phi_3) \end{pmatrix},$$

$$\phi^2 = \phi^+\phi^- + \phi^0\overline{\phi^0} = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \sigma^2)$$

$$= \frac{1}{2}(\vec{\phi}\cdot\vec{\phi} + \sigma^2),$$

$$\mathcal{L} = (\partial_\mu\phi)^2 + \mu^2\phi^2 - \lambda(\phi^2)^2$$

$$= \frac{1}{2}\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\lambda}{4}\left(\vec{\phi}\cdot\vec{\phi} + \sigma^2 - \frac{\mu^2}{\lambda}\right)^2 + \frac{\mu^4}{4\lambda} \qquad (2)$$

With $\mu^2 > 0$, this produces spontaneous symmetry breaking and at tree level

$$<\sigma> = \sqrt{\frac{\mu^2}{\lambda}} \qquad (3)$$

When gauge interactions are included, $<\sigma>$ gives rise to the gauge boson masses:

$$m_W^2 = \frac{1}{4}g^2 <\sigma>^2 = \frac{\sqrt{2}g^2}{8G_F} \tag{4}$$

Thus

$$<\sigma> = 247\, GeV \tag{5}$$

In terms of the shifted σ field, $\sigma = <\sigma> + \sigma'$,

$$\mathcal{L}_{int} = -\frac{\lambda}{4}(\vec{\phi}\cdot\vec{\phi} + \sigma'^2 + 2\sigma' <\sigma>)^2 \tag{6}$$

so the σ' has a mass squared $m_H^2 = 2\lambda <\sigma>^2 = 2\mu^2$.

We shall need several diagrams for Higgs - Higgs elastic scattering, which are shown in Fig. 1. The associated amplitudes are[11]

$$-i\mathcal{M}_a = (-i6\lambda <\sigma>)^2 \frac{i}{t - m_H^2} \tag{7a}$$

$$-i\mathcal{M}_b = (-i6\lambda) \times \frac{1}{2} \tag{7b}$$

$$-i\mathcal{M}_c = (-i6\lambda <\sigma>)^2 \frac{i}{s - m_H^2} \times \frac{1}{2} \tag{7c}$$

$$-i\mathcal{M}_d = \frac{(6\lambda)^2}{2}\frac{i\pi^2}{(2\pi)^4}\int_0^1 d\alpha\left[\log\frac{\Lambda^2}{m_H^2 - \alpha(1-\alpha)t} - 1\right] \tag{7d}$$

$$-i\mathcal{M}_e = 3\frac{(2\lambda)^2}{2}\frac{i\pi^2}{(2\pi)^4}\int_0^1 d\alpha\left[\log\frac{\Lambda^2}{-\alpha(1-\alpha)t} - 1\right] \tag{7e}$$

The last two have been cutoff at Λ. If we render them finite by making a subtraction at $t = -m_H^2$ we have

$$-i\mathcal{M}_d = \frac{(6\lambda)^2}{2} \frac{i\pi^2}{(2\pi)^4} \int_0^1 d\alpha \log \frac{1 + \alpha(1-\alpha)}{1 + \alpha(1-\alpha)(-t/m_H^2)} \qquad (7d')$$

$$-i\mathcal{M}_e = 3\frac{(2\lambda)^2}{2} \frac{i\pi^2}{(2\pi)^4} \log\left(-m_H^2/t\right) \qquad (7e')$$

When the subtraction point is chosen to be $t = -m_H^2$, it is understood that our σ^4 coupling constant, λ, is the one which is renormalized at at a mass scale m_H.

III. N/D Calculation

To find the approximate locations of the bound states, we use the N/D method, [12] which we review briefly. The elastic s-wave scattering amplitude,

$$a_0 = \frac{e^{2i\delta_0} - 1}{2ik} \qquad (8)$$

satisfies

$$\mathcal{I}m \frac{1}{a_0} = -k \qquad (9)$$

where k is the center of mass momentum. The partial wave amplitude for $j = 0$ is related to the Lorentz invariant amplitude, $\mathcal{M}$, by

$$a_0 = -\frac{1}{8\pi\sqrt{s}} \cdot \frac{1}{2} \int_{-1}^{1} d\cos\theta \mathcal{M}(k,\theta) \qquad (10)$$

and we use finally rather than a_0

$$A_0 = \frac{\sqrt{s}}{2} a_0 \tag{11}$$

so that

$$Im\frac{1}{A_0} = -\frac{2k}{\sqrt{s}} \equiv -\rho(s) \tag{12}$$

In the N/D method, A_0 is written as $N(s)/D(s)$ where $N(s)$ has only left hand cuts and $D(s)$ has only right hand singularities. Thus on the right hand cut

$$Im\frac{1}{A_0} = -\rho(s) = \frac{ImD(s)}{N(s)} \tag{13}$$

A subtracted dispersion relation may be written for D. If at the subtraction point, s_0, $D(s_0) = 1$

$$D(s) = 1 - \frac{1}{\pi}\int_{s_{th}}^{\infty} ds'\rho(s')N(s')\left[\frac{1}{s'-s} - \frac{1}{s'-s_0}\right], \tag{14}$$

$$A_0(s) = \frac{N(s)}{1 - \frac{1}{\pi}\int_{s_{th}}^{\infty} ds'\rho(s')N(s')\left[\frac{1}{s'-s} - \frac{1}{s'-s_0}\right]}. \tag{15}$$

If we approximate $N(s)$ by the "Born amplitude" - the appropriate s-wave projection of the sum of the five amplitudes, Eqs. (7a)-(7c), (7d$'$),(7e$'$), - we obtain an amplitude $A_0(s)$ which satisfies elastic unitarity and which reduces to the "Born amplitude" in the weak coupling limit. Unlike the bootstrap calculation to generate the σ as a bound state of two σ's, we are calculating a loosely bound state of two elementary σ's, so the s-channel σ pole is included in the "Born amplitude". Thus for $N(s)$ we take

$$N(s)=\left(-\frac{1}{16\pi}\right)\frac{1}{4k^2}\int_{-4k^2}^{0}dt\left[\frac{18m_H^2\lambda}{t-m_H^2}+3\lambda+\frac{9m_H^2\lambda}{s-m_H^2}\right.$$

$$\left.+\frac{9\lambda^2}{8\pi^2}\int_0^1 d\alpha\log\frac{1+\alpha(1-\alpha)(-t/m_H^2)}{1+\alpha(1-\alpha)}+\frac{3\lambda^2}{8\pi^2}\log(-t/m_H^2)\right] \tag{16}$$

Letting

$$z=\frac{4k^2}{m_H^2} \tag{17}$$

we have

$$N(s=4m_H^2+zm_H^2)=\frac{9\lambda}{8\pi z}\log(1+z)-\frac{3\lambda}{16\pi}$$

$$-\frac{9\lambda}{16\pi}\frac{1}{3+z}-\frac{9\lambda^2}{128\pi^3}g(z)-\frac{3\lambda^2}{128\pi^3}(\log z-1) \tag{18}$$

where

$$g(z)=\int_0^1 d\alpha\int_0^1 dx\log\frac{1+\alpha(1-\alpha)zx}{1+\alpha(1-\alpha)} \tag{19}$$

Making the subtraction at $s_0=\varsigma m_H^2$ and indicating $z'=(s'-4m_H^2)/m_H^2$,

$$D(s=4m_H^2+zm_H^2)=1-\frac{1}{\pi}\int_0^\infty dz'\sqrt{\frac{z'}{z'+4}}\left[\frac{1}{z'-z}-\frac{1}{z'+4-\varsigma}\right]$$

$$\times\left[\frac{9}{2}\left(\frac{\lambda}{4\pi}\right)\frac{1}{z'}\log(1+z')-\frac{3}{4}\left(\frac{\lambda}{4\pi}\right)-\frac{9}{4}\left(\frac{\lambda}{4\pi}\right)\frac{1}{3+z'}\right.$$

$$\left.-\frac{3}{8\pi}\left(\frac{\lambda}{4\pi}\right)^2(\log z'-1)-\frac{9}{8\pi}\left(\frac{\lambda}{4\pi}\right)^2 g(z')\right] \tag{20}$$

The bound states for the s-wave occur when $D(s)=0$ for $0<s<4m_H^2$.

IV. Binding Energies

For sufficiently large values of $\lambda/4\pi$, D does develop a zero. The corresponding value of the Higgs boson mass is

$$m_H = <\sigma> \sqrt{8\pi\left(\frac{\lambda}{4\pi}\right)} \tag{21}$$

In Fig. 2, the binding energy ,Δ, of the Higgs-Higgs system is displayed as a function of the mass of the Higgs boson. The binding energy is related to the value, z_B, for which D vanishes:

$$\Delta = 2m_H - 2m_H\sqrt{1+\frac{z_B}{4}} \tag{22}$$

We note that the bound state first occurs when $m_H \approx 1.3$ TeV and the binding remains modest even for $m_H \approx 2$ TeV.

The curve for the binding energy as a function of the Higgs mass has been computed for several values of the subtraction point, s_0. The location of a zero in D should, in principle, not depend on the choice of the subtraction point, if the N/D equations are solved consistently. Setting $D(s_0) = 1$ is just a normalization which is compensated by a change in N. In our calculation, however, since the N function is being approximated by the "Born amplitude", without being iterated, we expect that the numerical value of the binding energy will have some dependence on s_0. A reasonable choice is to take $s_0 = -m_H^2$. Then the N/D amplitude coincides with the "Born amplitude" at this point. In Fig. 2, we see that the bindng energy is insensitive to the choice of s_0 as long as s_0 does not approach the location of the zero itself, near $s = 4m_H^2$. This is understandable since we should not force D to be unity near the location of its zero. We take the curve for $s = -m_H^2$ to be a reliable choice.

V. Widths

The small binding of the Higgs bosons in the bound state means that the lifetime of the bound state will be essentially half the lifetime of a constituent Higgs boson. Consequently, the bound state has a width at least half its mass. The difficulty of recognizing such an object is readily demonstrated by recalling the problem of finding resonances in the $\pi - \pi$ s-wave system.

It should be noted that the great width which obscures the bound state is an intrinsic problem. The dominant contribution to the width of a very heavy Higgs boson comes from the decay into longitudinal W's, that is, the Higgs bosons of the σ-model. The $\lambda\phi^4$ coupling which gives the binding also produces the width.

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11. We have ignored the diagrams obtained by crossing $t \rightarrow u$ and compensated in the overall combinatorial factor since we in the end look only at s-wave amplitudes.

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Figure Captions

1. The diagrams contributing to the "Born amplitude" for elastic scattering used in the N/D equations.

2. The binding energy, Δ, of the Higgs-Higgs system as a function of the Higgs boson mass. The different curves represent different choices of the subtraction point, s_0: solid curve ($s_0 = -3m_H^2$), dotted curve ($s_0 = -m_H^2$), dashed curve ($s_0 = m_H^2$), dot-dash curve ($s_0 = 3m_H^2$).

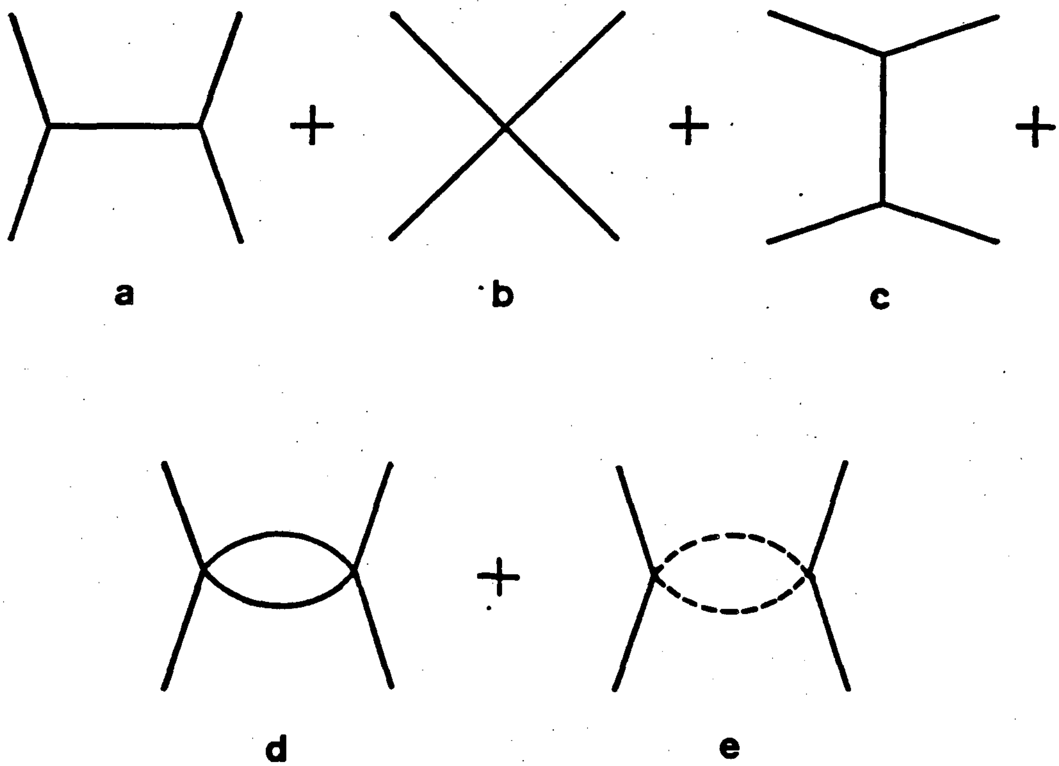

Fig. 1

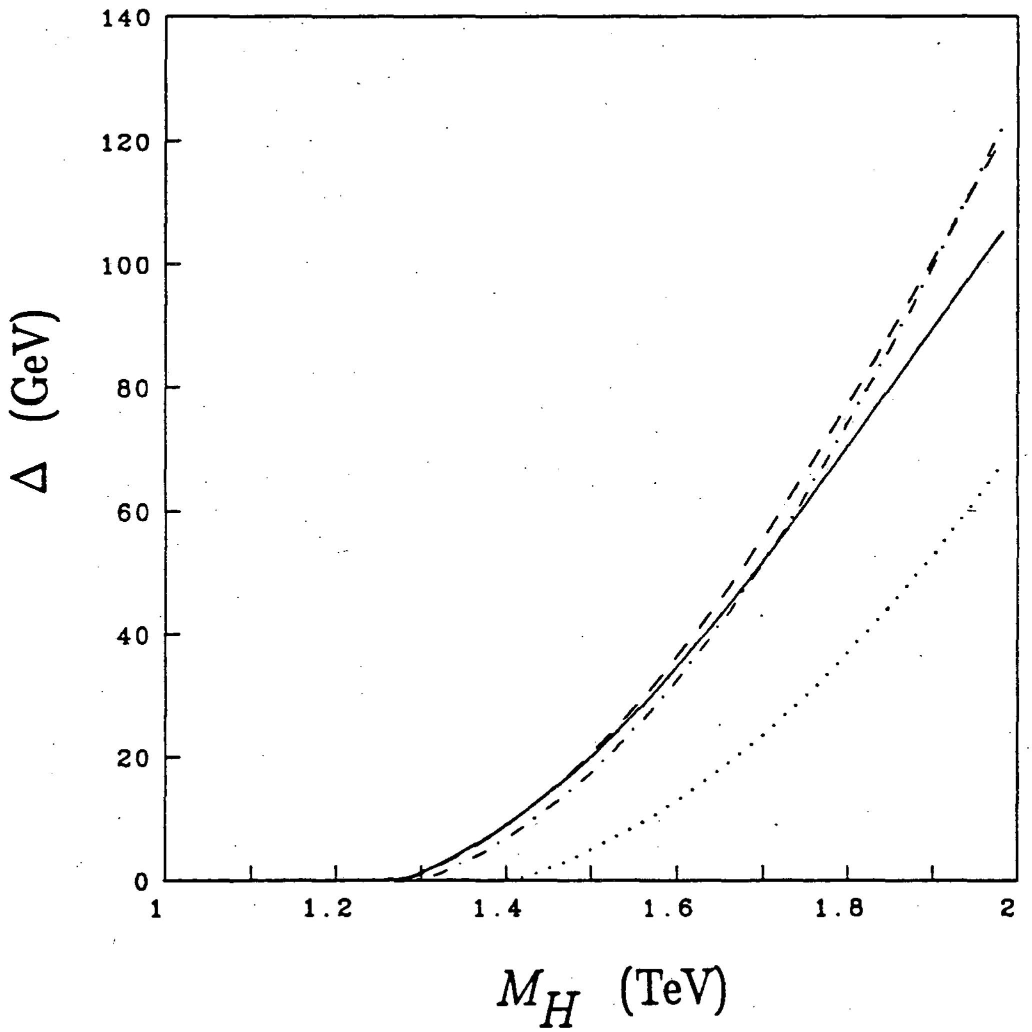

Fig. 2

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