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PHENOMENOLOGICAL ANALYSIS OF μ DECAY

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ABSTRACT

The spectrum, asymmetry, and spirality of the electrons from μ decay are calculated from the most general form of the two-component neutrino theory with lepton conservation. In addition to the nonlocal interactions considered recently by Lee and Yang, terms appearing phenomenologically as derivative couplings may occur. In particular, a spectrum that is, aside from a statistical factor, linear in momentum is possible with $\rho \neq 3/4$. This could be interpreted as evidence that fermions of baryonic mass are responsible for the nonlocality.

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I. INTRODUCTION AND GENERAL FORMULATION

The two-component neutrino theory¹ with lepton conservation allows some rather definite predictions about processes involving neutrinos. At present, only the shape of the μ decay spectrum is in apparent disagreement with experiment. The simple, local theory predicts for the decay

$$\mu^- \rightarrow e^- + \nu + \bar{\nu} \quad (1)$$

an electron spectrum described by a Michel parameter $\rho = 3/4$, while the published experimental values, including radiative corrections, are about 0.68 ± 0.04 .² Lee and Yang have shown recently that the lower ρ value may be explained by a four-fermion interaction taking place over a small space-time region rather than at a single point as is assumed in the usual Fermi theory of β decay.³ In the present paper we will show that the complete analysis of μ decay must include consideration of certain other logically distinct possibilities in addition to those considered by Lee and Yang. It will turn out that these possibilities may be distinguished experimentally.

A ρ value different from $3/4$ can also be interpreted as evidence against the two-component lepton-conserving theory.⁴ The approach followed in this paper as in Ref. 3 is, deferring the question of its confirmation or rejection to the low energy phenomena of β decay, to assume the theory in its simple and restrictive form, and to look to the energetic processes like μ decay or K decay for increasing information about the structure of the

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weak interactions. After all, the apparent universality in strength of these interactions together with the symmetry laws they all seem to violate, suggests some common origin of weak interactions. Such a universal interaction, involving bosons as well as fermions, must give rise to somewhat complex intermediate states, so that from this point of view a non-local interaction would be expected. The last section of this paper touches briefly on some weak decays, other than μ decay, which could yield information about a size structure for weak interactions.

The main point of this paper is that at the present stage these experiments must be analyzed in full generality, without restriction to any preconceived dynamical model. For that reason, without reference to any underlying form of the theory, we proceed to write down directly the most general form of matrix element for μ decay admissible under the proper Lorentz group, assuming the two-component neutrino theory with lepton conservation.

In this theory the neutrino field operators always occur as the projection $1/2 (1-\gamma_5) \psi_\nu$ of the four-component spinor ψ_ν . The vector $\bar{\psi}_\nu \gamma_\mu 1/2(1-\gamma_5) \psi_\nu$ is then the only covariant that can be formed out of two neutrino field operators. The most general form of matrix element for the decay (1) is

$$M = \bar{u}(p) O_\mu u(P) \bar{u}(q) \gamma_\mu 1/2(1-\gamma_5) v(\bar{q}), \quad (2)$$

where $P, p, q, \bar{q}$ refer to the four-momenta of muon, electron, neutrino, and antineutrino respectively, and u and v designate free-particle spinors of positive and negative four-momentum. The most general four-vector that can be constructed from the available four-momenta and Dirac matrices is

$$\begin{aligned} O_\mu = & (g_V - g_A \gamma_5) \gamma_\mu - (i/M) (G_V - G_A \gamma_5) p_\mu \\ & - (i/M) (G_V' - G_A' \gamma_5) (\not{q} - \bar{\not{q}}) \gamma_\mu \\ & - (1/M^2) (G_T - G_{AT} \gamma_5) (\not{q} - \bar{\not{q}}) p_\mu, \end{aligned} \quad (3)$$

where M is the muon mass. Other covariants, such as $(\not{q} + \bar{\not{q}}) \gamma_\mu$, do not occur because they are not linearly independent of those recorded, in virtue of momentum conservation and the Dirac equation.

The g 's and G 's in Eq. (3) may be functions of the invariants

$$\begin{aligned} Q_I &= (q + \bar{q})^2 / M^2 = 2(q \cdot \bar{q}) / M^2, \\ Q_{II} &= (p + \bar{q})^2 / M^2 = 2(p \cdot \bar{q}) / M^2, \\ Q_{III} &= (p + q)^2 / M^2 = 2(p \cdot q) / M^2, \end{aligned} \tag{4}$$

formed from the independent momenta in the problem. Only two of these invariants are independent, however, since from momentum conservation we have

$$1 + Q_I + Q_{II} + Q_{III} = 0. \tag{5}$$

The original two-component theory leading to $\rho = 3/4$ is obtained by omitting the last three terms of Eq. (3) and making g_V and g_A constants. The effect of letting g_V and g_A vary with momentum will be considered in Section II, and the consequences of including G terms will be studied in Section III. The three cases in which $g_{V,A}$ are functions of but one of the Q 's correspond to simple nonlocal interactions between fermion pairs, each of which are at the same point. This kind of nonlocality, which was treated by Lee and Yang,³ originates where the nonlocality is propagated by virtual bosons or fermion pairs emitted in local momentum-independent interactions.

The last three terms in Eq. (3) appear phenomenologically as derivative couplings on the electron, neutrino, and electron and neutrino fields respectively. This second kind of nonlocality can occur even where the fundamental point interactions are momentum-independent, if the virtually propagating particles include fermions. This second kind of nonlocality must be considered on an equal footing with the first kind, treated by Lee and Yang.

The two kinds of nonlocality are both expected to be small on theoretical grounds. Experimentally, the μ spectrum is, in fact, described in good approximation by the original local theory without derivative couplings. The effect of the terms in p_μ , q_μ , and $\bar{q}_\mu$, and of the momentum dependence in the coefficient g 's and G 's can for this reason be treated additively. In Section II the derivative terms will be omitted and the dependence of $g_{V,A}$ on the Q considered only in lowest order. In Section III the momentum dependence of the g 's and G 's will be ignored.

In writing the interaction in form (2), a particular ordering $(e\mu)(\nu\bar{\nu})$ has been chosen for the four fermions. Results can be transcribed for a different ordering by using standard formulae of the Dirac algebra.

II. NONLOCALITY OF FIRST KIND

Our general formula (3) is first specialized by setting the G 's equal to zero and expanding

$$g_{V,A}(Q_I, Q_{II}, Q_{III}) = g_{V,A} [1 - \epsilon_{V,A} Q_I - \zeta_{V,A} Q_{II} - \zeta_{V,A}' Q_{III}]. \quad (6)$$

The ϵ, ζ, ζ' are small dimensionless parameters measuring the nonlocal effect. Further specialization reproduces the examples discussed by Lee and Yang.³ When only ϵ_V and ϵ_A are nonvanishing, (Eq. 6) coincides with their case I, in which $(\nu, \bar{\nu})$ and (μ, e) interact at different points and momentum $(q + \bar{q})$ is propagated between the pairs chosen. If only ζ_V and ζ_A are nonzero, then case II is obtained, in which $(e, \bar{\nu})$ and (μ, ν) interact at different points and momentum $(p + \bar{q})$ is thus exchanged. Finally, the case in which only ζ_V' and ζ_A' are finite is identical with their case III in which (e, ν) and $(\mu, \bar{\nu})$ interact at different points and the relevant momentum transfer is $(p + q)$. Because of relation (5), only two of these three cases are linearly independent in a phenomenological sense; Lee and Yang found indeed that the observables for case I were, in a certain well-defined sense, the sum of those for cases II and III.

The distinctive feature of all these cases is that they lead to spectra and asymmetries that are, aside from the statistical factor, quadratic in electron momentum. The deviation of the ρ value from $3/4$ then is fitted by a nonlocal interaction spread over a distance of the order 0.6×10^{-13} cm, corresponding to an intermediate boson of mass 600 times the electron mass.

III. NONLOCALITY OF SECOND KIND: DERIVATIVE COUPLING

If the g 's and G 's in Eq. (3) are now treated as constants, the matrix element (2) leads to the following spectrum of electrons of momentum $p = x(M/2)$ emitted into solid angle $d\Omega$ at angle θ with respect to the polarization axis of the parent μ^- :

$$(2\pi)^{-3} (M/2)^5 (1/3) x^2 dx d\Omega (4\pi)^{-1} (\phi_1 + \phi_2), \quad (7)$$

where

$$\begin{aligned} \phi_2 = & |g_2|^2 [(3-2x) - \cos \theta (1-2x)] \\ & + \operatorname{Re}(g_2 G_2^*) [1 - \cos \theta] x + |G_2|^2 [1 + \cos \theta] \frac{1}{4} x^2 \\ & + \operatorname{Re}(g_2 G_2'^*) [3 - \cos \theta] 2(1-x) + |G_2'|^2 [(3-x) - \cos \theta (1+x)] (1-x) \\ & + \operatorname{Re}(g_2 G_2''^*) [-1 + \cos \theta] x(1-x) + |G_2''|^2 [(5-4x) - \cos \theta (3-4x)] (x^2/20) \end{aligned} \quad (8)$$

is proportional to the square of the matrix element for emitting electrons polarized along their direction of propagation. The corresponding quantity for electrons polarized antiparallel to their direction of propagation is given by ϕ_1 , which is obtained from ϕ_2 by the substitutions

$$g_2, G_2 \rightarrow g_1, G_1 \text{ and } G_2', G_2'', \cos \theta \rightarrow -G_1', -G_1'', -\cos \theta.$$

In these formulae, we have

$$\begin{aligned} g_1 &= g_A + g_V, \quad G_1 = G_A + G_V, \quad G_1' = G_A' + G_V', \\ G_1'' &= G_T + G_{AT}; \quad g_2 = g_A - g_V, \text{ etc.} \end{aligned}$$

Cross terms between derivative couplings and also terms proportional to the electron mass have been omitted.

The total transition rate for μ decay is

$$\begin{aligned} \tau^{-1} = & (2\pi)^{-3} (M/2)^5 (1/6) \{ (|g_1|^2 + |g_2|^2) + \frac{1}{2} \operatorname{Re}(g_1 G_1^* + g_2 G_2^*) \\ & + (1/10) (|G_1|^2 + |G_2|^2) + \operatorname{Re}(g_1 G_1'^* - g_2 G_2'^*) + (2/5) (|G_1'|^2 + |G_2'|^2) \\ & + (1/10) \operatorname{Re}(g_1 G_1''^* - g_2 G_2''^*) + (1/30) (|G_1''|^2 + |G_2''|^2) \}. \end{aligned} \quad (9)$$

The spirality of electrons depends on direction and momentum as follows:

$$S = (\phi_2 - \phi_1) / (\phi_2 + \phi_1). \quad (10)$$

After the numerator and denominator are integrated over electron angle θ ,

the resulting expression can be momentum-dependent only if there is some structure of either the first or second kind. However, even if there is structure to the μ decay, the spirality is still momentum-independent if either ρ_1 or ρ_2 vanishes, i. e. if the interactions always produce electrons of only one polarization.

The formulas for μ^+ decay are obtained from those above by interchanging subscripts 1 and 2 on the g 's and G 's. This only changes the sign of the asymmetry and of the angle-integrated spirality.

A. Discussion of spectrum evidence

In $1 + 2$ the various terms of Eq. (3) contribute as follows:

(a) The terms in $g_{1,2}$ alone or crossed with $G_{1,2}$ or $G_{1,2}'$ contribute linearly in x .

(b) The terms in $g_{1,2}$ crossed with $G_{1,2}''$ and in $G_{1,2}$ or $G_{1,2}'$ by themselves contribute quadratically in x .

(c) The terms in $G_{1,2}''$ by themselves contribute cubically in x .

The first question to be answered by the experiments is whether, aside from the statistical factor $x^2 dx d\Omega$, the electron spectrum is linear in x or contains higher momentum dependence. None of the present experiments answer this question to anything like the same accuracy with which the p value is determined. Nevertheless, it seems likely⁶ that the spectrum is overall linear to within about 15%. This indicates that in the phenomenological description the amount of electron or neutrino derivative coupling must be small compared to the amount of local nonderivative coupling, and that the amount of electron-neutrino tensor coupling present is smaller still:

$$g_{V,A} \gg G_{V,A} \quad G_{V,A}' \gg G_{T,AT} \quad (11)$$

This is what one would expect if the apparent derivative coupling was in fact due to heavy fermions in intermediate states. This also justifies our omitting from Eq. (6) any cross terms between derivative couplings, or any momentum dependence in the g 's and G 's where they appear multiplied into derivative couplings.

The momentum dependence of the third and fifth terms of Eq. (8) is quadratic, like that in the nonlocal interaction due to virtual bosons. If the μ meson spectrum, aside from statistical factors, does turn out to be quadratic, it will be difficult to decide between ascribing this to virtual bosons or to virtual fermions producing a moderate amount of effective electron or neutrino derivative coupling.

If the spectrum turns out to be linear, then the interpretation is simplified. In this case the nonlocality cannot be of the type considered by Lee and Yang but must be ascribed to a small amount of electron or neutrino derivative coupling. Under these circumstances, the spectrum is strictly of the Michel form (linear in x):

$$\tau^{-1} x^2 dx d\Omega (4\pi)^{-1} \{ [12(1-x) + 8\rho(\frac{4}{3}x-1)] - \xi \cos \theta [4(1-x) + 8\bar{\delta}(\frac{4}{3}x-1)] \} \quad (12)$$

where the total transition rate is

$$\tau^{-1} = (2\pi)^{-3} (M/2)^5 (1/6) D, \quad (13)$$

and we have

$$\begin{aligned} D &= (|g_1|^2 + |g_2|^2) + \frac{1}{2} \text{Re}(g_1 G_1^* + g_2 G_2^*) + \text{Re}(g_1 G_1'^* - g_2 G_2'^*), \\ \rho &= \frac{3}{4} D^{-1} [(|g_1|^2 + |g_2|^2) + \text{Re}(g_1 G_1^* + g_2 G_2^*)], \\ \xi &= D^{-1} [(|g_1|^2 - |g_2|^2) - \frac{3}{2} \text{Re}(g_1 G_1^* - g_2 G_2^*) - \text{Re}(g_1 G_1'^* + g_2 G_2'^*)], \\ \xi \bar{\delta} &= \frac{3}{4} D^{-1} [(|g_1|^2 - |g_2|^2) - \text{Re}(g_1 G_1^* - g_2 G_2^*)]. \end{aligned} \quad (14)$$

The electron spirality is

$$S = \frac{3\bar{A}(1-x) + 2\bar{\rho}(\frac{4}{3}x-1) - \bar{\xi} \cos \theta [(1-x) + 2\bar{\delta}(\frac{4}{3}x-1)]}{[3(1-x) + 2\rho(\frac{4}{3}x-1)] - \xi \cos \theta [(1-x) + 2\bar{\delta}(\frac{4}{3}x-1)]} \quad (15)$$

where

$$\begin{aligned} \bar{A} &= D^{-1} [(|g_2|^2 - |g_1|^2) + (1/2) \text{Re}(g_2 G_2^* - g_1 G_1^*) - \text{Re}(g_1 G_1'^* + g_2 G_2'^*)], \\ \bar{\rho} &= (3/4) D^{-1} [(|g_2|^2 - |g_1|^2) + \text{Re}(g_2 G_2^* - g_1 G_1^*)], \\ \bar{\xi} &= D^{-1} [-(|g_1|^2 + |g_2|^2) + (3/2)(g_1 G_1^* + g_2 G_2^*) + \text{Re}(g_1 G_1'^* - g_2 G_2'^*)], \text{ and} \\ \bar{\xi} \bar{\delta} &= (3/4) D^{-1} [-(|g_1|^2 + |g_2|^2) + \text{Re}(g_1 G_1^* + g_2 G_2^*)]. \end{aligned} \quad (16)$$

B. Asymmetry evidence

The preliminary evidence gives⁷ $\xi \approx -1$, which suggests $|g_1|^2 \ll |g_2|^2$. If we adopt this as an assumption and write

$$\eta = \text{Re}(G_2/g_2), \quad \eta' = \text{Re}(G_2'/g_2). \quad (17)$$

the above formulae simplify to

$$\begin{aligned} \rho &= (3/4)(1 + (1/2) \eta + \eta'), \\ \xi &= -(1 + 2\eta + 2\eta'), \\ \delta &= (3/4)(1 + (1/2) \eta - \eta'), \\ S &= 1. \end{aligned} \quad (18)$$

The present experimental values $\rho = 0.68 \pm 0.04$,² $\xi = -0.87 \pm 0.12$ ⁷ lead to a fit

$$\eta = 0.0 \pm 0.2, \quad \eta' = -0.08 \pm 0.06, \quad (19)$$

and the prediction

$$\delta = 0.81 \pm 0.07. \quad (20).$$

The entirely tentative value of η' , for example, corresponds to a range of nonlocality $\sim 0.1 \hbar/Mc = 0.2 \times 10^{-13}$ cm, or an intermediate fermion mass of about 2000 electron masses. This is to be contrasted with the corresponding fit to the Yang-Lee kind of nonlocality³ which gave a range about 0.6×10^{-13} cm corresponding to an intermediate boson mass of about 600 electron masses.

IV. CONCLUSIONS AND DISCUSSION

The principal conclusion of this paper is that the detailed shape of the electron spectrum, asymmetry, and spirality may distinguish a different kind of structure in the μ decay than that suggested in Ref. 3. In particular, a ρ value different from $3/4$ can be obtained in the two component lepton conserving theory with a spectrum that remains in the linear Michel form. Assuming the basic interactions to be local nonderivative couplings, such a result suggests the presence in intermediate states of fermions of baryonic mass.

Additional evidence on the interaction between pairs of leptons will be obtained from the corresponding analysis of the processes⁸

$$K \rightarrow \begin{Bmatrix} e \\ \mu \end{Bmatrix} + \pi + \nu. \quad (21)$$

These decays involve even greater momentum transfer than does the μ decay. Since Pais and Treiman, and Werle assumed that the leptons are produced at the same point, the failure of experiment to corroborate their simple predictions would be unambiguous evidence for a nonlocal interaction between the two leptons involved.

Finally the very presence or absence of certain decays in nature gives information on the fermion pairings actually occurring. For example, one might ask whether there is any evidence for a $(\nu \bar{\nu})$ coupling, corresponding to case I of Ref. 3. Such an interaction would allow the decay

$$K^\pm \rightarrow \pi^\pm + \nu + \bar{\nu}, \quad (22)$$

whose pion spectrum is peaked at the maximum kinetic energy, 127 Mev, about midway between the maximum energies of mesons from $K_{\mu 2}$ and $K_{\pi 2}$.

REFERENCES

1. A. Salam, *Nuovo Cimento* 5, 299 (1957);
T. D. Lee and C. N. Yang, *Phys. Rev.* 105, 1671 (1957); and
L. Landau, *Nuclear Physics* 3, 127 (1957).
2. Sargent, Rinehart, Lederman, and Rogers, *Phys. Rev.* 99, 885 (1955);
L. Rosensen, *Phys. Rev.* (to be published); and
K. Crowe, *Bull. Am. Phys. Soc.* 2, 206 (1957).
3. T. D. Lee and C. N. Yang, *Phys. Rev.* (to be published).
4. G. Bouchiat and L. Michel, *Phys. Rev.* 106, 170 (1957); and
T. Kinoshita and A. Sirlin, *Phys. Rev.* (to be published).
5. Our choice of Dirac matrices is that of Lee and Yang, ref. 1.
6. We are grateful to K. Crowe for discussion on the experimental situation.
7. D. Wilkinson (to be published).
8. A. Pais and S. B. Treiman, *Phys. Rev.* 105, 1616 (1957);
J. Werle, *Nuclear Physics* 4, 171 (1957); and
Furuichi, Kodama, Sugahara, and Yonezawa, *Prog. Theoret. Phys.*
(Japan) 16, 64 (1956).