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Essays in Housing and Real Estate Economics

A Dissertation submitted in partial satisfaction
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by

Peng Zhao

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In memory of both my grandfathers, and paternal grandmother,

to my father, my mother, and my maternal grandmother,

to my Shiqian & my Aibao,

for your love and support,

I love you all!

ABSTRACT OF THE DISSERTATION

Essays in Housing and Real Estate Economics

by

Peng Zhao

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, June 2020
Dr. Richard J. Arnott, Chairperson

This dissertation consists of three essays in housing and real estate economics which explore housing dynamics, extend housing filtering theory, housing maintenance, construction, and investigate issues in the economic depreciation of housing structures. The first chapter gives an introduction, with special emphasis on the motivations behind this research.

The second chapter develops a spatial, general competitive equilibrium model for metropolitan housing markets. Then the model is parameterized to quantitatively investigate their properties. Using a variety of data sets, a benchmark model is calibrated to the housing market of Riverside, CA, and the equilibrium is then numerically solved using a self-developed algorithm. The chapter investigates the model's equilibrium and compares the housing features in the benchmark model with two alternative steady states. The simulation results replicate the empirical fact that the ratio of maintenance expenditure to the property value is lower for newly constructed housing than for an aged house, and that it is consistent with the one percent maintenance expenditure rule in real estate economics.

Moreover, the numerical results from alternative steady states show that a positive demand shock increases the structural density and decreases the housing space, but it does not increase the rent and property value of a housing unit (although each housing unit's floor area becomes smaller). A positive income shock will increase the rent and the property value of a single housing unit at all levels of housing quality, and also increase the housing space and structural density of new residential construction. This chapter extends and refines housing filtering theory in several aspects. It distinguishes non-depreciable housing space with depreciable housing interior quality, emphasizing the indivisibility of housing services by incorporating a two-dimensional and nonlinear bid-rent function. It provides a quantifiable measure of housing interior quality and demonstrates its scale-independent nature. To the best of my knowledge, it is the first structural model that incorporates two heterogeneous dimensions of housing characteristics in both the demand side and the supply side of the housing market.

The third chapter, motivated by the observation that the land of a property does not depreciate, along with accounting for the capital gains from housing boom and bust cycles, provides a method to separate housing structure depreciation and capital gains on land into two distinct components that make up overall changes in property value. This chapter provides an approach to estimate the housing maintenance technology, and to directly measure the housing structure depreciation while taking account of the maintenance effect and the capital gains on land. The unobservable housing structure value is derived from the difference between the observable property value and the land value associated with the property. The land values are imputed by taking into account the plattage effect (larger parcels have a lower value per unit area of land) and the lot size information of each property. By noticing that general urban accessibility should be controlled when modeling parcel values with this effect, I choose to focus on the New York metropolitan area, where I can separate the urban center from its peripheral regions by utilizing the fact that there are almost no single-family detached houses in Manhattan used for residential occupancy. Using American Housing Survey data, chapter 3 demonstrates that in both the baseline model and the extended model (by incorporating pre- and post-financial recession data), the estimation results are similar and robust.

The fourth chapter explores economic reasons why the average construction cost curve is U-shaped with respect to building height, as noted in previous housing construction literature. A construction cost function is developed to explain such a relationship, particularly with regards to the U-shaped curve. Fixed construction cost is recognized and established as an element in the housing construction function.

The last chapter concludes and summarizes the contributions of this research.

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Chapter 1

Introduction

Housing is an important issue in almost every country's economy. A housing bubble can cause a financial recession, while well-designed housing policy can improve social welfare. My dissertation explores several aspects of housing and real estate economics. The first chapter gives an introduction and my motivations for writing this dissertation.

The second chapter develops a general competitive equilibrium model customized with enriched housing filtering theory for metropolitan housing markets; moreover, it extends and innovates over the housing filtering model in many aspects. The model presented in this second chapter is jointly developed with my advisor Richard Arnott.[1] It expands the previous filtering model, which used a one-dimensional quality index, to have two-dimensional characteristics in both the demand and supply side of the housing market. One dimension is housing space, the other dimension is interior housing quality. When my advisor mentioned his attempt to separate out the treatment of housing floor area from

[1] The second chapter's model was developed in collaboration with Richard Arnott. The rest of the chapter, including the solution algorithm, is solo work by the author. The usual disclaimer applies.

housing quality, it immediately resonated with me. At that time, I was closely observing the real estate market in the Seattle area. The vast majority of the market was second-hand housing, but the interior quality of the homes was different. One reason arises from the construction quality of the housing unit itself and the differing building technology available at the time of construction (e.g., differences in technology level of installed appliances such as a nest learning thermostat vs. mechanical thermostat), but another very important reason I observed is that the previous owner (perhaps due to differences in income, taste, and neighborhood characteristics) maintained the house very differently. The diversity of houses with respect to interior quality and floor area can certainly be created at the time of their construction, but the depreciation of housing quality and endogenous maintenance technology has played an important and under-appreciated role in the diversity of the housing market.

The above is the general motivation in writing the second chapter. Considering the length of this introductory chapter, two examples will be given regarding the specific motivation associated with this chapter. The first example is from the perspective of interdisciplinary connections. Unlike the space of a house, the interior quality of a house is a relatively abstract concept. I very much appreciate the concept of density in physics; it helps me conceptualize the two dimensions in the development and parameterization of this chapter's model (quality is like bubble density in a carbonated drink and housing space is like the volume). The second example is from real-life observations. In the last few years, there has been a large influx of population into Seattle. I had observed a potentially related phenomenon where new houses are built with increasingly small yards and houses are built very close together in Seattle, but the building areas of new houses are roughly similar to pre-existing ones. I then thought of a conversation I had with Ron Loveridge (former mayor of Riverside) where he stressed the importance of well-considered zoning regulations and the ways a lack of zoning regulations could cause land to be ineffectively apportioned. This made me realize localities usually adjust zoning regulations to adapt to population and economic growth, which creates land for construction over time. This observation and connection, together with the realization that the city was increasing in size due to commuting/infrastructure improvements, prompted the introduction of the concept of effective land in Chapter 2, which is to assume the effective land area (total building area) and household population grow at the same rate. This allows for steady-state analysis as well as having practical significance.

The third chapter of the thesis studies the natural depreciation rate of housing structures. The motivations for this come from a few places. One is that when I was a child, the housing price in my hometown - Hangzhou - was still not very high, but it was growing. And my father, utilizing his business acumen, told me that the appreciation in real estate prices would continue, largely due to the appreciation in land value. Later in my high school years, through my observations of the real estate market in Hangzhou, I found that single-family houses had the fastest growth rate in housing prices, followed by townhouses. The high-rise apartment buildings experienced the slowest price gain since every household held a smaller amount of shared-land in an apartment building lot compared to a single-family house. These phenomena have motivated me to separate land value from property value and assume that capital gains on property are primarily driven by land. After living in Pittsburgh, PA for college, I noticed that houses in Oakland (a local neighborhood dominated by college-students) deteriorated very quickly; part of the reason was that they are housing for students, and part of the reason being that most houses in the US are made from wood. These observations from my memory prompted me to provide a method to separate housing structure depreciation and capital gains on land into two distinct components that make up overall changes in property value. This analysis is further developed by distinguishing between the natural rate of structure depreciation (the rate that would occur in the absence of maintenance) and the effects of maintenance on structure value over time. From my later observations in the Seattle real estate market, I found that houses with large lot sizes are not significantly more expensive (relative to the size of the lot) than houses with smaller lot sizes that had similar housing conditions. Discussing this observation with my advisor, I identified it as an example of the plattage effect studied in previous real estate research, which is utilized to model the land value in this chapter.

The fourth chapter discusses the reason for U-shaped average construction cost curves. Since it is not intuitively clear as to why the average cost of construction would initially decrease as the complexity and difficulty in construction increase, further examination of the construction cost's total composition is required. In this chapter, from the perspective of construction cost per unit of land area, a fixed cost is introduced along with a variable construction cost (increasing with building height) to explain the economic reasons behind it. Introducing the fixed cost of construction was inspired by conversations with Bao Huang (a senior construction economist for China Energy Engineering Corporation

(CEEC)) and Xiayi Shen (a Los Angeles based architect and building designer).

The last chapter concludes the dissertation and summarizes the contributions of this research from two perspectives. One is from the perspective of its contributions to the literature and the other is from the perspective of contributions to methodology - algorithmic thinking incorporating economic principles and intuitions.

Chapter 2

Housing Space, Interior Quality, and Housing Dynamics through Filtering

Abstract

This paper develops a spatial, general competitive equilibrium model for metropolitan housing markets and examines its equilibrium in a steady state where household population and total residential construction area grow at the same rate. At a point in time, a housing structure is constructed on internally-owned residential land. The housing structure is described by the structural density (number of stories), the interior quality, and the floor area of each housing unit in the structure. Using a variety of data sets, a benchmark model is calibrated to the housing market of Riverside, CA, and then numerically solved

for its equilibrium. Two other illustrative exercises are undertaken to compare alternative steady states. This paper's model extends housing filtering theory, which emphasizes the heterogeneity of housing by focusing on the change in the quality of housing units after their construction due to their depreciation and endogenous maintenance. Our model innovates over previous filtering models in: i) distinguishing between housing quality and "quantity" (floor area) on both the demand and supply side, ii) developing a quantifiable and scale-independent measure of housing quality, and iii) introducing internally-owned land and the endogenous income derived from land into the economy.

2.1 Introduction

Housing is a commodity that comes with many features including durability, depreciability, multi-dimensional characteristics, and indivisibility. The durability, depreciability, and multi-dimensional characteristics together define the heterogeneity of housing. In a broad sense, there are three classes of economic models of the housing market. One class of housing model treats housing as a general consumption good - a one-dimensional good with linear pricing on divisible and fully durable housing services. This class of housing model remains the workhorse model for housing policy analysis since it is a structural model and convenient to apply in housing market analysis. However, the consequences of assuming a linear housing price are not inconsequential. One consequence is that a linear housing price will induce a uniform housing consumption for a group of people with the same income level. This outcome, however, is not empirically sound since it does not explain why households with similar incomes choose to have different levels of housing services.

Another class of housing models is the hedonic pricing model, which can be traced back to Rosen (1974) who proposed a model of product differentiation in competitive equilibrium (a product is valued by a vector of characteristics that are associated with a set of hedonic prices) and elucidated empirical implications for hedonic price regressions. Later on, the hedonic price regression models become very successful in housing economics. They provide a very rich description of the individual housing unit in terms of not only its structural characteristics but also characteristics associated with its location and neighborhood. They have proved very powerful in various applications of hedonic valuations in the short run, but they belong to reduced-form models. Reduced-form models do not lend themselves to easy or concrete explanations in regards to identifying the underlying causality and transmission mechanisms in a dynamic housing market. For example, if we would like to study the causal relationship between income, population density, housing structural density, and land value, the reduced-form models may not allow us to easily identify the transmission mechanism underlying any economic relations between the variables. Therefore, although the hedonic pricing model is useful in short-run value predictions, its usefulness is limited in housing policy analysis.

Filtering models are the third class of models, which possess many advantages over the previous two. They are an attractive middle ground between workhorse linear-pricing models and hedonic models; they are structural models but admit of the heterogeneity of housing markets since they treat housing as durable but depreciable. Filtering models view the housing market as a stock of durable housing at different discrete quality levels or over a continuum of quality levels, and explicitly address the technological dynamics of housing supply. Housing filtering, in a broad sense, is the idea that newly constructed housing is built for higher-income households, and that as the housing ages, its quality depreciates, leading lower-income households to move in. This phenomenon mirrors the idea of a "hand-me-down." More technically, filtering refers to the phenomena whereby housing units are constructed at high quality and gradually deteriorate in quality at a rate dependent on the level of maintenance expenditure, creating a stock of durable housing at different levels of quality in the market. This description better captures the fact that housing is durable but depreciable and it consists of quality-differentiated submarkets.

Sweeney (1974a, 1974b) formalized this filtering conceptualization in dynamic models and gained wide exposure through urban economics textbooks, such as O'Sullivan (2007). These models introduced housing maintenance. The models viewed housing as having discrete levels of quality and assumed that a housing unit eventually falls into the lowest level of quality and is demolished regardless of how much is spent on maintenance. Braid (1981, 1984) studied housing with a continuous distribution of unidimensional housing quality in static models. Dynamic housing filtering models with a continuous variation in quality were introduced in later filtering models (Arnott, Davidson, and Pines, 1983, 1986; Arnott and Braid, 1997; Arnott, Braid, Davidson, and Pines, 1999). They elucidated the endogenous maintenance theory by incorporating a more flexible maintenance function that permits upgrading as well as downgrading, and they included a construction function for housing being constructed at new quality levels to supply new housing in the economy that would satisfy the increasing demand in housing quantity as the population grows. In addition, they noticed multi-dimensional characteristics of housing services and introduced housing structural density along with housing quality in the supply side of the housing market.

According to all of these theoretical works in filtering, housing filtering is a robust mechanism for long-term modeling of low-income housing. However, despite its conceptual appeal, filtering theory has not been widely applied in housing policy analysis. One reason is that for a long time, economists largely lacked empirical evidence to support whether filtering really occurs in the housing market. The filtering process described by the theoretical literature had been questioned in reality, as it seemed somewhat counterintuitive in some contexts. For example, in tight markets, which experience fast capital gains on real estate investment, the value of existing housing goes up rather than down over time, so it seems unlikely to become affordable by 'filtering'. Rosenthal (2014) resolved this puzzle, showing that housing filtering was in fact robustly supported in most locations within the US by estimating a 'repeat income' model using 1985 – 2011 panel data of the American housing survey (AHS). Rosenthal provided the needed empirical evidence that filtering serves as a primary mechanism for providing affordable housing to lower-income households and through this study found that downward filtering still occurred even in the areas with high housing prices, although the filtering rate was lower. Recently, new studies have yielded further empirical evidence for the applicability of housing filtering. The empirical research by McCarthy and Peach (2015) using 1989 to 2013 panel data of AHS once again yielded empirical results which reinforce confidence in housing filtering theory; they found that higher-price housing units mainly came from new construction, and most of the lower-price housing supply came from the housing units that were previously occupied by higher-income households. Most recently, Mast (2019) used individuals' address history data to identify that even in the short run, new housing construction could, via a 'migration chain' mechanism, eventually improve affordability in low and middle-income neighborhoods.

There have also been important advances in the recent theoretical work on housing that emphasize the indivisibility of housing and resonate with the filtering concept - that housing is heterogeneous and contains quality-differentiated submarkets. In the frameworks applied by several studies (Landvoigt, Piazzesi, and Schneider,2015; Piazzesi and Schneider, 2016; Epple, Quintero, and Sieg, 2020), housing was treated as an indivisible and differentiated good with a continuous distribution of housing quality, with nonlinear pricing on a single dimension quality index so that the prices are determined by the sub-markets differentiated by quality levels. Recent literature on housing has also mentioned the role of housing maintenance. Cocco and Lopes (2019) studied the role of reverse mortgages in homeowners' retirement consumption and also considered the retirees' maintenance expenses in the context of the requirement for reverse mortgages. In their model, retirees' maintenance expenditures are a choice variable in their dynamic optimization problems; it is dependent upon housing value, but does not need to be so high as to counteract depreciation in housing value unless they need to comply with the loan requirement from taking a reverse mortgage.

Our paper differs from their work and innovates on previous housing filtering models in several aspects. First, it distinguishes two important dimensions of housing, interior quality and floor area of a housing unit, whereas the previous filtering literature conflated the two. It emphasizes the relationship between the two as well as the indivisibility of housing services by incorporating a nonlinear bid-rent function of housing floor area and interior quality. It also distinguishes durability and depreciability of housing; the floor area is not depreciable while housing quality depreciates after construction. Second, it provides a quantifiable and scale-independent measure of housing quality. We measure quality as proportional to quality associated construction cost per unit area of floor area in a housing structure of normalized structural density.[1] The depreciation of interior quality and the improvement from maintenance expenditure generates a continuous distribution of housing quality. Third, it introduces endogenous land rent and incorporates internally-owned land into the economy in a way that permits steady-state analysis. It does this by assuming that the effective land (total residential construction area) grows at the same rate as population. This may be defended with the argument that technological improvements in transportation effectively increase the amount of accessible land,[2] and at the same time, it can also be defended via the fact that the zoning regulation adjusts with changes in population density and economic development. As we recently observed in certain US metropolitan real estate markets, as the population increases in these metropolitan areas, we see a trend towards higher density construction (relatively larger portion of construction area with a smaller yard) in certain areas – which increases the effective land area.

This paper is structured as follows. Section 2.2, the heart of the paper, presents the paper's model of the housing market with identical households, demonstrates how equilibrium is obtained, explains the model's economics, and briefly explores some of its prop-

[1] We define it as variable construction cost later in the paper.

[2] Consider the monocentric model. One can describe its spatial structure in terms of a function that gives the amount of land within a given transportation cost distance of the city center. Technological improvements in transportation increase the amount of land within a given transportation cost distance of the city center.

erties. Section 2.3 illustrates the model's solving algorithm. Section 2.4 parameterizes the model and calibrates it to Riverside, CA as a benchmark case. Section 2.5 presents the numerical results of the benchmark case and two other counterfactual steady states and numerically explores some of the model properties in the general equilibrium. Section 2.6 offers concluding remarks.

2.2 Model

Section 2.2.1 describes the model. Section 2.2.2 explains in detail how the model's equilibrium is derived and in the process explains the model's economics.

2.2.1 A thumbnail sketch of the model

The model describes the steady state of a competitive, spatial general equilibrium economy with identical households in which the effective land and the household population grow at the same rate. The economy is endowed with land and the labor services of its resident households. There are two goods, housing units, which are produced using land and labor services, and other goods, which are produced using labor services alone. On the demand side, households derive utility from their housing units and other goods. Developers decide on the housing space (floor area) and construction quality of housing units, and on the structural density (number of stories) of housing structures. Landlords decide on the time path of maintenance of the housing units that they purchase from developers and rent to households. The equilibrium housing unit rent, value functions, and land value are determined such that no agent has an incentive to alter their behavior and all markets clear.

Table 2.1 provides an index of notation for the model.

Table 2.1: Notational glossary

A	fixed costs of construction per unit area of land
BCR	average building coverage ratio
b	shift-scaling parameter of maintenance function
c	shift-scaling parameter of construction function
$c(\mu, q)$	construction cost per unit of floor area
$C(\mu, q)$	construction cost per unit of built-on land area
D	effective household population density
f	floor area of a housing unit
$g(q, m)$	depreciation - maintenance (DM) function
HD	household density
h	housing service
$L(t)$	effective land area (total residential building area) at time t
$m(t)$	maintenance expenditure per unit floor area at time t
n	growth rate of household population and of effective land area
$N(t)$	total household population at time t
$P(f, q)$	property value of a housing unit of floor area f and interior quality q
$q(t)$	interior quality of housing at time t
q_0	construction quality of housing
RAA	residential area ratio
r	interest rate (discount rate)
$R(f, q)$	rent on a housing unit of floor area f and interior quality q
$u(.)$	utility function
V	value of land per unit area
w	household labor income
x	consumption of non-housing goods
y	household total income
β	housing preference parameter
γ	elasticity of quality improvement w.r.t. ft^2 maint.
δ	nature depreciation rate of housing quality
μ	structural density
θ	land rent per household
$\phi(t)$	shadow price of quality at time t
ϕ_0	shadow price of construction quality
ϑ	developer's profit

The demand side

At time t, there are $N(t)$ identical and infinitely-lived households, each of which supplies a fixed amount of labor, w, has tastes described by a utility function with general consumption goods, x, housing floor area, f, and interior quality, q, as its arguments,

$u(x, f, q)$. The utility function is assumed to be strictly concave, and (to avoid having to deal with corner solutions) that x, q, and f are all essential in consumption. The wage rate is normalized to unity, so that each of the households has labor income w. Each household owns an equal share in the economy's land. Thus, where θ denotes land rent per household, a household's budget constraint is $x + R(f, q) = w + \theta$, where $R(f, q)$ is the rent function, giving rent on a housing unit with floor area f and quality q. Each household rents one, and only one, housing unit.

The supply side

1. There is a generic commodity, termed a smoog, produced in a competitive market under constant returns to scale to the single factor, labor. Smoogs are measured in labor units so that the price of a smoog equals unity. Thus, $N(t)w$ smoogs are produced at time t. A smoog may be transformed into either one unit of consumption good, one unit of housing capital, or one unit of housing maintenance.

2. Durable housing structures are produced through construction, by combining land and units of housing capital. The number of stories is denoted by μ, and is referred to as the structural density. As a housing structure ages, its housing units' floor areas remain the same but their quality can be continuously adjusted via expenditure on maintenance.

3. Capital markets are perfect, and, to simplify, it is assumed that the discount rate, r, is constant over time.

4. Housing is the only use of land in the economy. At time t, there is a fixed area of effective land in the economy, $L(t)$. The economy's effective land area (total available construction area) grows at the same exogenous rate as population, so that the economy-

wide population/land ratio remains constant over time. Also, there is no technological progress.

5. Housing interior quality is generally an ordinal concept, and indeed the paper's results are independent of how it is cardinalized.[3] However, to perform a quantitative comparison under different shocks on the equilibrium quality level, a cardinalization is applied.[4] In cardinalizing quality, we use the RSMeans construction cost handbooks, which provide estimates of the construction cost per unit floor area. Specifically, we measure quality as construction cost per unit area in a housing unit (with normalization of structural density) in a particular city and year, as provided in the RSMeans construction cost handbooks. The handbooks also provide construction cost adjustment factors by city (which account to difference in materials costs, factor prices, and weather) and year (which reflect both changes in input prices and technical change in housing construction). Applying these factors leads to a standardized measure of building quality over cities and over time. Letting $c(\mu, q)$ be the (average) construction cost per unit of floor area with housing construction quality q and density μ, it is assumed to be homogeneous of degree one in floor area f, that is $c(\mu, q, f) = fc(\mu, q)$. Let $C(\mu, q)$ be the construction function per unit of land area and $C(\mu, q) = \mu c(\mu, q)$. Then $C(\mu, q, f/\mu)$ is homogeneous of degree one in building area f/μ, that is $C(\mu, q, f/\mu) = fC(\mu, q)/\mu$. Without considering any fixed cost, let $z(q)$ be a mapping from a housing quality to its corresponding construction cost (with controlling the construction density to be 1 and $c(1) = 1$, then we have $C(\mu, q) = \mu c(\mu, q) = z(q)\mu c(\mu)$. Our cardinalization implies that $z(q) = q$.

[3] This holds true as long as the cardinalization is consistent with our definition of housing quality.

[4] It is achieved by adjusting the scaling parameters in both of the depreciation-maintenance function and the housing construction function.

6. In the absence of maintenance, housing depreciates in quality. The technology of maintenance/depreciation is described by $\dot{q}(t) = g(q(t), m(t))$, where $m(t)$ is housing maintenance per unit floor area. $g(q(t), m(t))$ is referred to as the depreciation-maintenance function (DM function). We assume that $g(.)$ is a strictly concave function, with (with subscripts denoting partial derivatives) $g_{q(t)} < 0$, $g_{m(t)} > 0$, and $g_{m(t)m(t)} < 0$ (positive and diminishing marginal returns to maintenance). Furthermore, $g(q(t), 0) \leq 0$.

Economic organization

1. All markets are organized competitively.

2. There are two groups of agents involved in housing supply, profit-maximizing developers and landlords. A developer purchases a unit area of vacant land at its competitively-determined asset price per unit area, V, constructs a housing structure on the vacant land, deciding on the construction quality, q_0, and structural density of the building, as well as on the floor area of the individual housing unit, and then sells the newly-constructed housing unit to landlords according to the competitively-determined property value function, $P(f, q_0)$. A landlord purchases a newly-constructed housing unit, decides on its maintenance program, and collects rent from it. For expositional convenience, we assume that the landlord owns a housing unit over its entire lifetime.

Equilibrium

An equilibrium is a feasible allocation, a housing rent function, $R(f, q)$, a property value function, $P(f, q)$, and an asset price for vacant land, V, land rent per household θ such that: i) No household can improve its utility by purchasing an alternative consumption bundle; ii) No developer can increase profit by constructing at a different structural density, or building housing units of a different floor area and interior quality; iii) No landlord can increase profit by choosing an alternative maintenance program; and iv) All markets clear.

Remarks

Now that we have described the model, we provide some remarks on why we have constructed the model as we have.

1. We have assumed that there are three groups of agents in the economy: households, developers, and landlords. This assumption is made solely for expositional convenience. The decentralized equilibrium does not depend on the particular institutional arrangement. For example, each household could buy its own land, build its own housing unit, and maintain the housing unit itself.

2. To simplify, we have assumed constant returns to scale in: i) the production of non-housing goods; ii) construction, with respect to lot size or housing unit size; and iii) maintenance, with respect to housing unit size. But we have assumed diminishing returns with respect to maintenance and increasing marginal cost with respect to structural density.

3. Equilibrium in the model is independent of the cardinalization of quality employed. Housing quality is defined to be – quality proportional to construction cost,[5] holding other building characteristics fixed – for three reasons. The first is a technical reason. It results in the economy having all of the convexity/concavity properties needed to readily prove the existence and uniqueness of competitive equilibrium. The second reason for the chosen cardinalization is that the time and city adjustment factors provided in the RSMeans con-

[5]Construction cost here excludes any portion of the cost that is independent of interior quality and structural density, such as land preparation cost.

struction cost handbooks make it easy to apply the same cardinalization to different cities and at different points in time, and hence to apply the model in cross-section (by city), time series, and panel data empirical analysis. The third is that the cardinalization is intuitive, and easy to understand.

4. As noted earlier, we have chosen assumptions so that the economy's equilibrium is in steady state, which considerably simplifies the analysis. The assumption that the supply of effective land grows at the same rate as population can be defended. The reasons include technological progress in transportation.

5. Proving existence and uniqueness of equilibrium is sufficiently routine and intuitive, so we simply sketch the proof here. First, there is a feasible allocation. Second, solve for the equal-utilities social optimum, which maximizes the common utility of households subject to the resource constraints on residential land, smoogs, and to the construction and maintenance technologies. Since the objective function is strictly concave, since the constraint set is weakly convex, and since there is a feasible allocation, there is a unique maximum which corresponds to a Pareto optimal allocation. Third, the Second Theorem of Welfare Economics states that, if appropriate convexity conditions are satisfied, a Pareto optimum allocation can be decentralized as a competitive equilibrium. Since the conditions are satisfied, then the maximum of the social optimization problem can be decentralized as a competitive equilibrium. These three steps together establish existence. Finally, since households have equal utilities in a competitive equilibrium, since there is a unique allocation that maximizes the common utility subject to the constraints, which is Pareto optimal, and since any competitive equilibrium is Pareto optimal by the First Theorem of Welfare

Economics, equilibrium is unique.

2.2.2 Derivation of model equilibrium

The derivation is divided into four modules. The first is the demand side module, which derives the bid-rent function for a housing unit of interior quality q and floor area f, conditional on the utility level u and household income, $w+\theta$. The second is the landlord's profit-maximization problem. The third is the developer's problem. And the fourth is the equilibrium module, which determines the equilibrium housing rent function, the property value function, land value and the allocations.

The housing bid-rent function

The method of derivation of the housing bid-rent function is essentially the same as that used in the monocentric city model (Alonso, 1964). Since all households are identical, and since therefore in equilibrium all households receive the same level of utility, the equilibrium housing rent function coincides with the equilibrium housing bid-rent function. Let $R(f, q; u, w+\theta)$ denote the housing bid-rent function, conditional on u and $w+\theta$. By definition, it is the maximum that a household with income $w+\theta$ can pay in rent for a housing unit with floor area f and quality q, consistent with achieving a utility level of u; thus, where y denotes the sum of wage income w and land rent θ,

$$R(q, f; u, y) = \max\ w+\theta-x \tag{2.1}$$

subject to $u(q, f, x) = u$.

Letting $x(f, q; u)$ denote demand function for non-housing goods x for utility level u, conditional on f and q, we have that $R(f, q; u) = w + \theta - x(f, q; u)$. Concavity of the rent function is implied by concavity of the utility function from which it is derived.

Landlord's problem

To simplify exposition, and without loss of generality, we assume that a landlord purchases a housing unit from a developer immediately after it is constructed and continues to operate it throughout its life. The next subsection shows that, under our assumptions, in equilibrium, all new housing units are the same. Nevertheless, a property value function, $P(f, q)$ is still well defined, being the sum of discounted net-of-maintenance rent associated with the profit-maximizing program, conditional on f and q.

We proceed on the assumption that it is optimal for the landlord to operate her housing unit forever.[6] Where t here denotes the period of time since construction, the landlord's profit-maximizing program solves

$$\max_{m(t)} \int_0^\infty \Big(R\big(q(t); f, u, w + \theta\big) - fm(t)\Big)e^{-rt}dt \tag{2.2}$$

subject to

$$\dot{q(t)} = g(q(t), m(t); f), q(t) \geq 0, \text{ and } m(t) \geq 0$$

for $t \geq 0$, $q(0) = q_0$, and f given.

[6]With additional assumptions that the landlord can costlessly demolish her housing unit (after operating it for a period of her choice) and immediately build another new unit on the site, there may be a possibility that a finite duration demolition trajectory could be the optimal path. However, that requires the 'equal-area condition' to hold (see Arnott, Davidson, and Pines (1983) for a detailed discussion on this condition).

The current-valued Hamiltonian of the problem (2.2) is:

$$\widehat{H}(q(t), m(t), \phi(t); f, u, w+\theta) = R(q(t); f, u, w+\theta) - fm(t) + \phi(t)g(q(t), m(t)),$$

where $\phi(t)$ is the current-valued costate variable on housing quality at time t, which has the economic interpretation as the marginal value (shadow price) of quality of a housing unit at time t. The economic interpretation of the current-valued Hamiltonian is the current net (of maintenance cost and depreciation) economic return from operating the housing unit per unit time, which equals rental income net of maintenance expenditures, less value loss from quality depreciation.

With Pontryagin maximum principle applied, we have

$$-f + \phi(t)g_{m(t)}(q(t), m(t)) = 0, \tag{2.3}$$

which indicates that maintenance should be carried to the point where increasing housing maintenance expenditure by one dollar increases the property value by one dollar.[7] Define $m^*(q(t), \phi(t)/f)$ to be the maximized maintenance expenditure per unit floor area with quality $q(t)$ and marginal value of quality per unit floor area $\phi(t)/f$. Inserting this function into the current-valued Hamiltonian gives the maximized current-valued Hamiltonian:

[7] Increasing maintenance by one unit per unit area increases the housing quality by g_m, which increases the property value by ϕg_m. Spending one more dollar more on maintenance per unit area results in maintenance expenditure on the housing unit increasing by f dollars.

$$H(q(t), m^*(t), \phi(t); f, u, w + \theta) = R(q(t); f, u, w + \theta) -$$

$$fm^*(q(t), \phi(t)/f)) + \phi(t)g(q(t), m^*(q(t), \phi(t)/f))). \quad (2.4)$$

The corresponding state and costate equations are

$$\dot{q(t)} = g(q(t), m^*(q(t), \phi(t)/f)), \quad (2.5)$$

$$\dot{\phi(t)} = r\phi(t) - H_{q(t)}(q(t), \phi(t); f)$$

$$= r\phi(t) - R_{q(t)}(q(t); f, u, w + \theta) - \phi(t)g_{q(t)}(q(t), m^*(q(t), \phi(t)/f)).$$

$$(2.6)$$

The boundary conditions (transversality conditions) that are necessary to characterize an infinite duration optimal path can be written as follows:

$$(i) \quad \phi(0) = \phi_0 \quad (2.7)$$

$$(ii) \quad \lim_{T\to\infty} e^{-rT} H(T) = 0. \quad (2.8)$$

Letting $P(q_0; f, u, w + \theta)$ denote the value of this infinite duration program at construction ($t = 0$), the marginal value of quality at time $t = 0$ is

$$P_{q_0}(q_0; f, u, w + \theta) = \phi(0). \quad (2.9)$$

Also, since the landlord must make the market return on her housing unit at all points in time,

$$H(q(0), \phi(0); f, u, w + \theta) = rP(q_0; f, u, w + \theta). \tag{2.10}$$

Since the optimal control problem is autonomous, the qualitative properties of the landlord's maintenance program can be depicted in a phase plane, which plots the costate variable, $\phi(t)$, on the y-axis, against the state variable, $q(t)$, on the x-axis. The $\dot{\phi} = 0$ and $\dot{q} = 0$ curves are continuous, and it is assumed that the $\dot{q} = 0$ curve intersects the $\dot{\phi} = 0$ curve once. The point of intersection is termed the saddlepoint, and denoted by $S = (q(s), \phi(s))$. The phase-diagram analysis follows that of the Arnott, Davidson, and Pines (1983) (ADP (1983) hereafter). Figure 2.1 plots this phase plane, for which the $\dot{q} = 0$ curve is upward-sloping and the $\dot{\phi} = 0$ curve is downward-sloping.

The marginal cost of quality via construction is ϕ_0. We shall derive later in the section that $\phi_0 = fC_{q_0}(q_0, \mu)/\mu$.[8] We assume that the configuration of the phase plane is such that $\phi(s) > fC_{q_0}(q_0, \mu)/\mu$ – that the marginal value of quality at the saddlepoint exceeds the marginal cost of quality via construction. The arrows indicate the directions of motion of $q(t)$ and $\phi(t)$ in the phase plane, as given by the state and costate equations; the slope of a trajectory in the phase plane is $d\phi/dq = \dot{\phi}/\dot{q}$; trajectories leading to the saddlepoint are called the stable arms of the saddlepoint, and the trajectories leading away from it are termed the unstable arms. ADP (1983) proves that, with this configuration of the phase plane, the optimal infinite duration program starts at W, the point of intersection

[8] Letting $C(q_0, \mu)$ be the per unit land area cost of constructing a housing structure in a building of structural density μ and construction quality q_0, the marginal cost of construction quality of a housing structure (per unit land area) is defined to be $C_{q_0}(q_0, \mu)$. The marginal cost of construction quality of a housing unit is then $fC_{q_0}(q_0, \mu)/\mu$. See remark 5 in Subsection 2.2.1 for details.

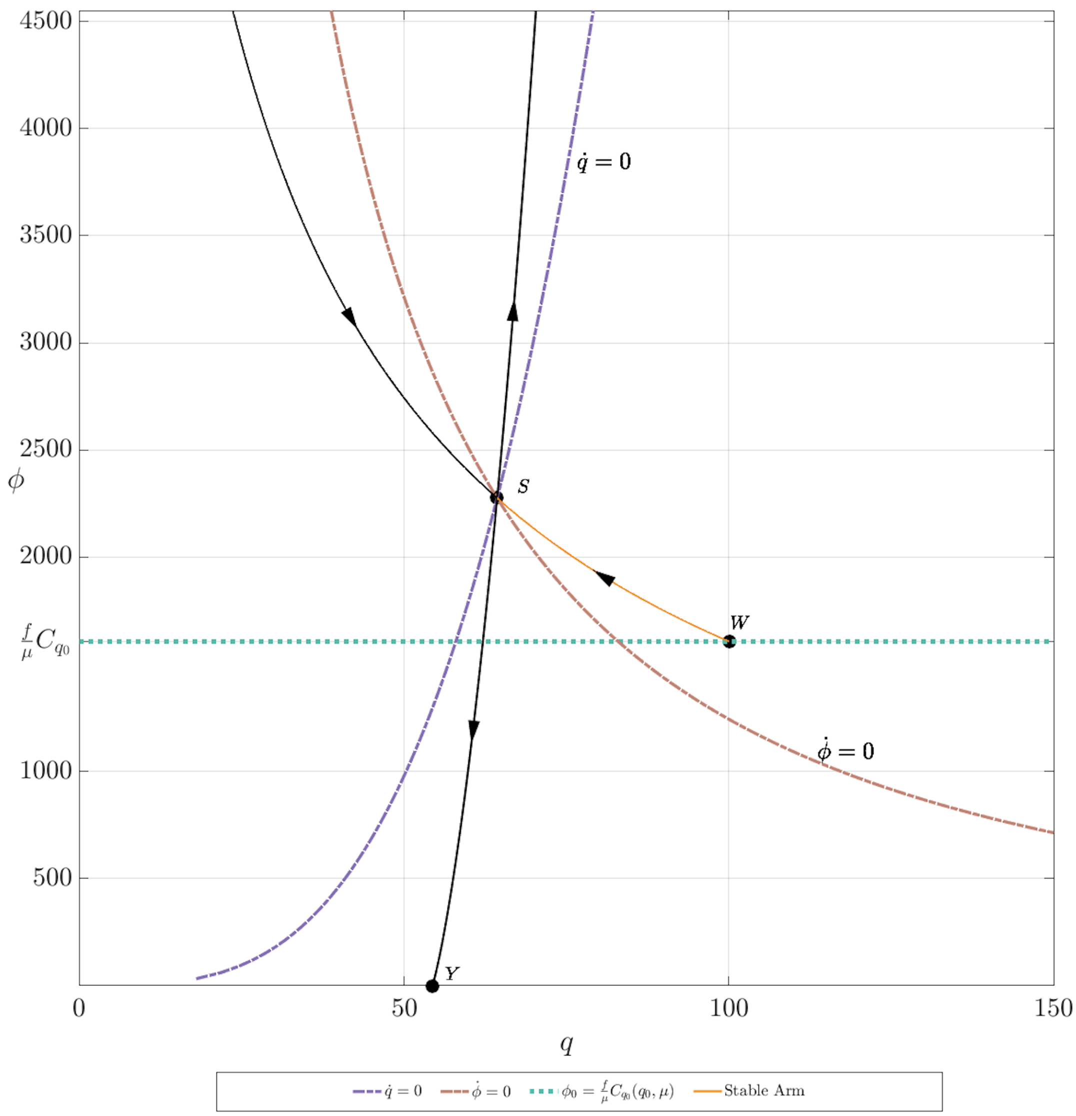

Figure 2.1: Stable arm and housing dynamics through filtering
Note: The data for the figure are those generated from the benchmark model simulation.

of the marginal cost of quality via construction line (labeled $\frac{f}{\mu}C_{q_0}$ in Figure 2.1) and the stable arm, and then travels up the stable arm to the saddlepoint. Thus, the optimal infinite duration program entails initial construction at a quality above saddlepoint quality, and then downgrading along the stable arm from W to the saddlepoint S. With some abuse of notation, we shall let $H(w)$ denote the value of the Hamiltonian at point w in the phase plane. With this notation the value of the optimal infinite duration trajectory is $P(q_0^*; f) = H(W)/r$, where q_0^* denotes construction quality.

With free entry into the competitive housing rental market, the infinite lifespan net revenues a landlord gains from her housing unit are offset by the cost she needs to pay to the developer(the purchase price of a housing unit) at the moment she acquires the newly constructed housing unit. The purchase price (property value) of a newly constructed housing unit is equal to $P(q_0; f, u, w + \theta)$.

Developer's problem

The developer purchases land and chooses the structural density, μ, of the housing structure to construct on the site, the floor area of the housing unit, f, and the construction quality, q_0. The cost of constructing the per unit land area housing structure is $C(q_0, \mu)$. The construction function per unit of land area is assumed to be homogeneous of degree one in building area f/μ.[9] The per unit land area revenue of selling a housing structure is $\frac{\mu}{f}P(q_0, f; u, w + \theta)$. The land value per unit area of land is V. The developer's profit-maximizing problem is

$$\max_{\mu, f, q_0} \vartheta = \frac{\mu}{f} P(q_0, f; u, w + \theta) - C(q_0, \mu) - V, \tag{2.11}$$

where ϑ is the developer's profit.The first-order conditions are:

[9]In Section 2.4, a fixed construction cost is introduced. The construction cost function estimated using the RSMeans data (See Chapter 4) shows a substantial fixed construction cost per unit area of land, which reflects construction costs that are not dependent on the building height, such as site preparation and foundation costs. This complicates the analysis. However, if the fixed cost is low enough that all land remains utilized, then it would have the same effect on the economy's competitive equilibrium as would a reduction in the economy's endowment of smoogs, and hence it would not affect the existence or uniqueness of competitive equilibrium.

$$\mu: \quad P(q_0, f; u, w+\theta) = fC_\mu(q_0, \mu), \tag{2.12}$$

$$f: \quad P_f(q_0, f; u, w+\theta) = P(q_0, f; u, w+\theta)/f, \tag{2.13}$$

$$q_0: \quad P_{q_0}(q_0, f; u, w+\theta) = fC_{q_0}(q_0, \mu)/\mu. \tag{2.14}$$

Eq.(2.12) states that the developer should build up to the point where the value of the marginal housing unit just covers its construction costs. Eq.(2.13) states the floor area should be chosen such that property value per unit floor area is maximized. Eq.(2.14) states that construction quality should be chosen so that marginal increase in the value of a housing unit with respect to construction quality equals the corresponding increase in construction costs.

Thus far, we have not mentioned second-order conditions of the developer's maximization problem. Since it is a sub-maximization problem of the concave programming social optimization problem corresponding to the competitive equilibrium, the second-order conditions are satisfied. Furthermore, we imposed conditions on tastes that guarantee an interior optimum.

The value of land is bid up to the point where developers make zero economic profit when they make the profit-maximizing choices with respect to μ, f, and q_0, and, additionally, when landlords choose the profit-maximizing program. Thus, where *'s denote profit-maximizing values,

$$V = \frac{\mu^*}{f^*}P(q_0^*, f^*; u, w+\theta) - C(q_0^*, \mu^*). \tag{2.15}$$

Using Eq.(2.9), in equilibrium, Eq. (2.14) can be written as

$$\phi_0 - fC_{q_0}(q_0, \mu)/\mu = 0. \tag{2.16}$$

This condition was informally introduced earlier when we discussed the phase diagram for the optimal infinite duration program. Recall that we interpreted ϕ_0 to be the shadow price (or marginal value) of construction quality via depreciation-maintenance technology. Thus, Eq.(2.16) states that in equilibrium, shadow price (or marginal value) of housing quality at construction via depreciation-maintenance technology must be equal to the marginal cost of construction quality via construction technology.

Determination of equilibrium

There are two more conditions for the general equilibrium. First, the demand for housing per unit area of land (which equals the household effective population density, D) equals the supply of housing per unit area of land, μ/f,

$$D = \frac{\mu}{f}. \tag{2.17}$$

This is the equilibrium condition for housing (housing quantity) market clearing. We introduce the growth rate (n) of both population and of effective land area to accommodate steady-state analysis. Note too that the growth rate n, is not present in the equation. It affects only the age distribution of the housing stock. Thus, the profit-maximizing program is unaffected by the growth rate of the population and of effective land area.

The disposition of land rents needs to be considered as well; this is the last condition and it is for the land market clearing. Since the model assumes that land rents are equally distributed among the economy's households, the land rent of each household must equal land rent (per unit area) divided by population density,

$$\theta = \frac{rV}{D}. \tag{2.18}$$

The equations previously derived are:

$$\phi_0 - fC_{q_0}(q_0, \mu)/\mu = 0, \tag{2.19}$$

$$P_f(q_0, f, \phi_0, u, w + \theta) = P(q_0, f, u, w + \theta)/f, \tag{2.20}$$

$$P(q_0, f, \phi_0, u, w + \theta) = fC_\mu(q_0, \mu), \tag{2.21}$$

$$\frac{\mu}{f} = D, \tag{2.22}$$

$$\theta = \frac{rV(q_0, f, \phi_0, u, w + \theta)}{D}. \tag{2.23}$$

Where m_0 denotes the optimal maintenance expenditure at time $t = 0$, the function for property value (P) relating the market value of a housing unit from the landlord's infinite duration program at $t = 0$ is:

$$P(q_0, f, u, \phi_0, w + \theta) = \frac{R(q_0, f, u, w + \theta) - fm_0 + \phi_0 g(q_0, m_0)}{r}, \tag{2.24}$$

where the bid-rent function $R(q_0, f, u, w+\theta)$ and depreciation-maintenance function $g(q_0, \mu)$ will be parametrized in Section 2.4.

From Eq.(3), we can derive the optimal maintenance expenditure per unit floor area at time $t = 0$ by solving:

$$-f + \phi_0 g_{m_0}(q_0, m_0) = 0. \tag{2.25}$$

The function for land value (V), the residual of housing structure value (per ft^2 of built-on land), and the construction cost (per ft^2 of built-on land) is

$$V(q_0, f, \phi_0, u, w + \theta) = \frac{\mu}{f} P(q_0, f, \phi_0, u, w + \theta) - C(q_0, \mu), \tag{2.26}$$

where the construction function $C(q_0, \mu)$ will be parameterized in Section 2.4.

Using Eqs.(2.19) - (2.26), together with the infinite duration optimal maintenance program, and with given values of the exogenous variables and parameters, the equilibrium values of the endogenous variables can be determined.

2.3 Model Algorithm and Numerical Solution

Eqs.(2.19) to (2.26) give 8 equations with 9 unknowns (u, f, q_0, μ, ϕ_0, θ, m_0, V, P). The exogenous variables and parameters include w, D, c, β, b, γ, δ, and r.[10] Solving these equations, along with the landlord's maintenance problem, involves great complexity; the stable arm (characterized by the state and costate equations as well as boundary conditions) does not provide an explicit function form about q_0 and ϕ_0. The derivative of the nonlinear

[10] There may be a fixed cost associated with the housing construction function $C(q_0, \mu)$, such as land preparation and foundation cost. Later on, in the calibration section, we will introduce an exogenous estimated value for the fixed cost (A) in the construction function, so that the land value (V) can be calculated.

bid-rent function $R(q_0, f, u, w + \theta)$ is a component in the costate function. The nonlinear bid-rent function includes two dimensions of housing attributes (interior quality and floor area) along with the utility level (u) and land rent (θ), whose values are all endogenously determined in the general equilibrium. Therefore, the model can not be solved analytically. Due to this circumstance, we have developed algorithms to solve the model numerically instead.

The model is solved numerically via the following method.[11]Without counting technical loops,[12] The heart of the model solving algorithm contains three loops. Let y denote total income of a household, that is $y = w + \theta$. In the inner loop, with the income level y and utility level u given and Eq.(2.5) - Eq.(2.7) from the landlord's problem and Eq.(2.19) - Eq.(2.21) from the developer's problem, we can solve f, q_0, μ, and ϕ_0 by iterating floor area f, such that for a given f, the three first-order conditions are solved yielding $q_0(f; u, y)$, $\mu(f; u, y)$, and $\phi_0(f; u, y)$.[13] To distinguish, we denote $q_0(f; u, y)$ solved from the developer's problem, as $q_0(f; u, y)_s$. In the landlord's optimal maintenance program, we can numerically solve the steady-state housing quality $q(s)$ and the shadow price of housing quality $\phi(s)$ at the saddlepoint from the following three equations in the saddlepoint steady state:

[11]Details of the algorithm pseudocode are provided in Appendix - "Pseudo code of model solving algorithm", including the model solving algorithms, modified Euler's method, and enhanced L'Hospital's rule in determining the slope at the saddlepoint.

[12]Technical loops include loops in the modified Euler's method and loops used in the solving of a system of equations.

[13]$x(f; u, y)$ reads as the value of x generated via iterating f in the current loop, with u and y as constants.

$$-f + \phi(s) g_{m(s)}(q(s), m(s)) = 0, \tag{2.27}$$

$$g(q(s), m(s)) = 0, \tag{2.28}$$

$$r\phi(s) - R_{q(s)}(q(s); f) - \phi(s) g_{q(s)}(q(s), m(s)) = 0. \tag{2.29}$$

We then get $q_s(f; u, y)$ and $\phi_s(f; u, y)$ and use modified Euler's method to travel in reverse down the stable arm and stop at the point when the shadow price of housing quality in the phase diagram equals $\phi_0(f; u, y)$ from the developer's problem. We then calculate the corresponding housing quality and we denote it as $q_0(f; u, y)_d$. By identifying $f(u; y)$ such that $q_0(f; u, y)_d = q_0(f; u, y)_s$, we are able to solve the inner loop and acquire values of $q_0(u; y)$, $f(u; y)$, $\mu(u; y)$, and $\phi_0(u; y)$.

We will then iterate utility level in the middle loop and stop when we meet the equilibrium condition that $\mu(u; y)/f(u; y) = D$, where D is the effective household density, which is exogenously given through calibration (see Section 2.4). Thus, from the middle loop, we acquire the equilibrium utility level $u(y)$ together with $q_0(y)$, $f(y)$, $\mu(y)$ and $\phi_0(y)$, based on income level y. We can then calculate the values for $V(y)$, $R(y)$, $P(y)$, and $\theta(y)$ accordingly based on $q_0(y)$, $f(y)$, $\mu(y)$ and $\phi_0(y)$.

Both the inner loop and the middle loop can be optimized by bisection search algorithms. In the inner loop, the utility level (u) is given as a constant. The optimization algorithms are as follows: if $q_0(f; u, y)_d > q_0(f; u, y)_s$, we process the iteration by increasing the floor area $f(u, y)$; if $q_0(f; u, y)_d < q_0(f; u, y)_s$, we decrease floor area f. The economic justification for this judgment is that construction should occur at the quality at which the shadow price of quality via the depreciation and maintenance technology at construction time equals the marginal cost of quality via construction technology. In the middle loop, when housing demand (represented by effective household population density D) is higher than housing supply (number of housing units per unit land area ($\mu(y)/f(y)$), then the household's utility must fall and vice versa. The middle loop identifies equilibrium values of the endogenous variables in housing quantity market clearing.

However, the solving algorithm does not stop here since land is internally owned by households. In the outer loop, we use the land rent θ generated from the middle loop, and add it to the household's income and then enter the updated total income into the inner and middle loops described above. At each iteration, land rent θ will increase, but with diminishing speed; the loop will stop when the value of θ converges. At this time, we have generated the equilibrium set of q_0, f, μ, and ϕ_0, whose values are only based on exogenous parameters. The economic purpose served by this outer loop is to look for the land market clearing that occurs when the equilibrium land value is maximized and simultaneously, the equilibrium household income y is equal to the wage w and equilibrium land rent θ. At period $t+1$, there will be $nN(t)$ units of land parcels that are immediately available for sale, where we denote $N(t)$ as the total number of households at time t and n is the household population growth rate. Competition among developers bids up the land value to the point at which economic profit is zero.

With that, the model is solved. We then can calculate the equilibrium values for V, R, P, and θ from the corresponding equations based on solved values of q_0, f, μ and ϕ_0.

2.4 Data, Model Parameterization, and Parameter Calibration

We parameterize our model to quantitatively investigate its properties. The values of the calibrated parameters in the parameterized model correspond to the Riverside, California housing market. Riverside is a major sub-center within the Los Angeles metropolitan area, located fifty miles from downtown Los Angeles, halfway between downtown Los Angeles and Palm Springs. We choose Riverside because the northeastern part of the city is home to the University of California, Riverside, where we study, teach, and conduct research. We use several data sources to calibrate and estimate the model, including US Census, American Community Survey (ACS), Zillow Research and Zoning (GIS) database from the city of Riverside, etc. The dollar values that we report are real 2018 dollars. The model parameters are shown in Table 2.2. The numerical model is annual. Section 2.4.1 describes the two city features that we have introduced in our model. Section 2.4.2 explains model parameterization. Section 2.4.3 discusses the housing quality in the context of the empirical side by relating it with the theory. Section 2.4.4 explains the target moments for the calibration.

2.4.1 City features

Our model characterizes a city by its two fundamental features: average household income and effective household density (discussed below). A key function of our model is that when we input the above two model features, assuming the model is parameterized and

Table 2.2: Model parameters (all monetary units are in 2018 dollars)

	City features and pref. parameter	
Effective household density[1]	D	
Household density	HD	$4.361 * 10^{-5}$(/ft^2)
Residential area ratio	RAA	0.409
Average building coverage ratio	BCR	*Calibrated*
Median household income	y	\$65,313
Housing preference parameter	β	Calibrated
	Housing maintenance	
Natural depreciation rate of housing quality	δ	0.085
Elasticity of quality improv. w.r.t. ft^2 maint.	γ	0.230
Shift-scaling parameter of maint. function	b	*Normalized*
	Housing construction	
Fixed construction cost per ft^2 land	A	\$93.844 (/ft^2 land)
Shift-scaling parameter of constr. function	c	*Normalized*
	Discount rate	
Interest rate (discount rate)	r	*Calibrated*

Notes: 1. Effective household density = Household density / Residential area ratio /Average building coverage ratio.

parameters are calibrated, equilibrium conditions in the steady state generate the optimized housing space (floor area), housing construction quality, and housing construction density (number of stories) produced by developers for renters (households) with identical income (via landlords). With the value of the parameters given, we can numerically solve our model after obtaining the values of these two features of a city. However, due to data limitations, obtaining the city average ratio of building area divided by lot area[14] was not possible via direct calculation. We calibrate this statistic by identifying the average housing floor area (see Subsection - "Target moments"), then going back to check this calibrated ratio of average lot area over building area in Riverside and identify that it is intuitively reasonable (see the simulation section).[15]

[14]This ratio is one of the three statistics to calculate effective household density; see the discussion on effective household density below.

[15]It is certainly possible to target either average housing quality or average housing density if we do not choose to target average housing floor area. However, from a practical perspective, the accuracy of the estimated moments of other features are very likely to be worse than that of the housing floor area. For instance, in a certain number of cases, it is hard to quantify the housing densities - the total number of floors

Household income

In contrast to reality, the model assumes that all households are identical. Because we want to use a representative income, we have chosen to work with median rather than mean household income. This choice is based on observations of household income distribution in the US. Since the distribution has a large positive skew, the median household income is a better measurement of central tendency compared to the mean, which is less representative due to the extreme incomes of top-end households.[16] We use data from the 2018 American Community Survey (ACS) 5-year estimates for Riverside, CA, which were collected for the 2014-2018 period. The 2018 inflation-adjusted median household income is reported in Table 2.2. Household income reported in ACS is the total income of a household.[17] Household income (y) in our model is also the total income of a household, including income not associated with land (w)[18] as well as income from land rent (θ).

Effective household density

The effective household density of a city is defined as the ratio of the total number of households divided by the total residential building area. This density is the product of household density (total number of households/total land area), and the reciprocal of the residential area ratio (total land area/total residential land area) and the reciprocal of

in the sample, e.g, some houses are built as 'Bi-level' or 'Tri-level' instead of regular one story to three story structures; some have basements, etc.

[16] As documented in Stiglitz(2016), in 2013, the richest 1 percent has 20 percent of the national income and this ratio has doubled from 1980 to 2013.

[17] Total income includes gross wages and salary (before any taxes and deductions are taken out) from all jobs, interest, dividends, net rental income, and so on. The total income does not include the implicit rental income from homeownership. Homeowners pay implicit rent to themselves just as renters pay rent to landlords.

[18] In our model, income not associated with land is the labor income.

the average residential building coverage ratio (total residential land area/ total residential building area).[19] The city of Riverside had 98,745 households in 2018, based on data from Southern California Association of Governments (SCAG),[20] and it had a total land area of 81.23 sq mi ($2.264 * 10^9$ ft^2), based on data from the US Census Bureau.[21] Riverside household density is then estimated to be $4.360 * 10^{-5}$ household(s) per ft^2. Residential area ratio (total residential land area/total land area) is 0.4089 determined from the zoning database (GIS) of the city of Riverside. More details on the procedures used to acquire this statistic are provided in Appendix - "Residential area ratio". The only remaining piece of information needed to calculate the effective household density is the average building coverage ratio. Due to the practical difficulty of obtaining this statistic,[22] we instead determine this parameter through calibration (see details in Subsection - "Target moments").

2.4.2 Model parameterization

Utility function and bid-rent

Households derive utility from their housing (the services derived from their housing units, h) and consumption of other goods (x). We have thoroughly acknowledged that housing is a multi-dimensional, durable, depreciable, and indivisible good. For simplicity, the utility function is assumed to be Cobb-Douglas:

[19] Mathematically, effective household density = (total number of households/total land area) * (total land area/total residential land area) * (total residential area/total residential building area) = total number of households/total residential building area.

[20] Based on an annual publishing book "Profile of the City of Riverside" from SCAG published in May 2018: 2018 total population in Riverside city is 325,860 and 2018 average household size is 3.3. The number of households is then the ratio of the total population divided by the average household size.

[21] Based on 2016 Gazetteer File from US Census Bureau. URL: https://www2.census.gov/geo/docs/maps-data/data/gazetteer/2016$_{Gazetteer}/2016_{g}az_{p}lace_{0}6.txt$

[22] To calculate the average coverage ratio, we would need to know the total building area. Without accessing high-resolution satellite images and performing image analysis on them, the total building area is not easy to obtain.

$$u(h, x) = \beta logh + (1 - \beta)logx. \tag{2.30}$$

The application of Cobb-Douglas utility function for housing and general goods is common in the literature (e.g. in line with Davis and Ortalo-Magne(2011), Michaels, Rauch, and Redding (2012), etc). Two dimensions of the housing service are introduced, which contribute to the durable and depreciable feature of housing: depreciable interior quality, q and non-depreciable housing space (floor area), f. Housing floor area captures the quantity of housing service while interior quality describes the density(quality) of the housing service. We hereby assume the amount of service derived from one's housing unit h is described by the multiplication of interior quality and housing space:[23]

$$h = qf. \tag{2.31}$$

Housing is indivisible. One cannot sell or purchase housing square-by-square. Housing depreciates, and after its construction, it can generate a continuous distribution of housing quality over time through its natural depreciation (depreciation rate with no maintenance expense applied) and improvements from housing maintenance. However, not all the components of housing value depreciate; only housing quality depreciates, not its quantity - e.g. floor area. Based on these factors, instead of simply assuming a linear pricing

[23]This assumption is also inline with Yoshida(2016). Yoshida also introduces this multiplication relationship from the perspective of housing production function and seems to use the term "effectiveness of structure" to refer to "interior quality" - housing structure service = effectiveness of structure $\times$ housing floor area. In a more broad sense, we find a similar relationship in physics: mass m is equal to the volumetric mass density ρ multiplied by volume V, $m = \rho V$. This analogy is not from random thoughts; interior quality refers to how dense or how effective the housing service is from its physical structure. The housing space (floor area) represents its quantity.

schedule for housing, we introduce a bid-rent function in which both housing floor area and housing quality are the independent variables.

The implied housing bid-rent function is then

$$R(f, q; \underline{u}, y) = y - e^{\frac{\underline{u}}{1-\beta}}(qf)^{\frac{-\beta}{1-\beta}}, \tag{2.32}$$

where $\underline{u}$ is the utility level of the household. y is the total household income. Total household income is the sum of wage income (w) and income from land rent (θ). The preference parameter β is set to be 0.27, more details are provided in the Subsection - "Target moments".

Depreciation - Maintenance Function

The depreciation - maintenance (DM) function associated with landlord's problem has the form,

$$g(q, m) = -\delta q + \overbrace{(b_1 b_2)}^{b} m^{\gamma}. \tag{2.33}$$

where δ is the natural rate of depreciation (the rate of depreciation in the absence of maintenance), γ is the elasticity of improvement in housing quality with respect to unit floor area maintenance expenditure, and shift-scaling parameter b is a result of a multiplication of two parameters, shift parameter b_1 and scaling parameter b_2, $b = b_1 b_2$. Shift parameter b_1 captures the technological shift (historical, technological, and/or economic changes in the relevant region) of the maintenance technology while the value of scaling parameter b_2 is determined by the scale of the housing quality (relative to real maintenance expenses) we adopt. The values of δ, γ are 0.085 and 0.230, which are borrowed from the estimation in Chapter 3. More details can be found in Chapter 3 on this subject. Since the shift parameter is a relative concept, we standardize the shift parameter b_1 for Riverside, to be 1. Then $b = b_1$. The value of b is then normalized to be 5.300; technical details of this normalization are provided in the Subsection - "Housing quality". We then have the DM function for the city of Riverside,

$$g(q, m) = -\delta q + bm^{\gamma}. \tag{2.34}$$

Construction function

The housing construction function per unit of built-on land associated with the developer's problem takes the form,

$$C(\mu, q_0) = c_1 A + \overbrace{c_1 c_2}^{c} q_0 \mu^2. \tag{2.35}$$

From RSMeans square foot construction cost data, for a 2,400 - 2,500 ft^2 house made of wood and wood frame, the slope of variable construction cost $c_1 c_2 q_0$ is estimated to be 29.287 and the fixed construction cost to be \$84.392 per ft^2 of built-on land in the year 2018 in terms of the national average. Chapter 4 reports the detailed procedures of the estimation. We use the city construction cost index provided in RSMeans[24] to obtain the construction function for the city of Riverside and we standardize the shift parameter of Riverside to be $c_1 = 1$. Then, $c = c_2$; Fixed construction cost A is equal to \$93.844

[24] According to RSMeans, Cost in City R = (Index for City R / 100) * National Average Cost. City construction cost indexes for the year 2017 are available through the RSMeans database; the city of Riverside's index is 111.2.

and the slope of variable construction cost cq_0 is equal to 32.567. The value of shift-scaling parameter c is then equal to 0.326, which is discussed in detail in Subsection - "Housing quality". We then have the construction function per unit of built-on land area for the city of Riverside to be

$$C(\mu, q_0) = A + cq_0\mu^2. \tag{2.36}$$

2.4.3 Housing quality

Housing quality represents the physical quality of a housing unit - the level of comfort and luxury of the housing unit's interior, which is depreciable. Housing quality does not include quantities such as floor area, nor location factors (e.g. school district) that are part of the land value of the property. When one visits a house, one can get a sense of the housing quality via physically identifiable characteristics. Similarly, housing quality is identifiable in our model. The model can identify housing quality based on real rent, given housing floor area plus the median income level and the effective population density of the city. Rent itself is not housing quality per se, but it gives an indication of quality because it is the price of housing service for a particular floor area and particular level of quality. Housing quality does not need to have an intrinsic cardinalization. Our model does not require us to adopt a particular measurement scale of housing quality, and its prediction is indeed independent of the type of cardinalization.[25] Housing interior quality in our benchmark model is scaled from 0 to 100; that is, we cardinalize quality so that zero quality corresponds to zero variable construction costs and $q = 100$ to construction quality in the

[25] As long as the type of cardinization does not change the way we define housing quality – controlling for the number of stories, variable construction cost per unit area of land is linearly proportional to the housing quality.

benchmark model. We cardinalize quality solely because we can then quantitatively analyze and compare the quantitative changes on housing quality in two counterfactual steady-state exercises - where we alter factors such as median household income and population density. The scaling parameters in the maintenance function and the construction cost function are used to calibrate the housing quality. The technical calibration procedures on the housing quality can be found in Appendix - "Shift-scaling parameters".

2.4.4 Target moments

In the above, from various data sources and our estimations, we have given the values to the baseline model parameters including city features, median household income (y), household density (HD), residential area ratio (RAA), and we have also given the values to the parameters (δ, γ) associated with maintenance function and fixed construction cost per ft^2 land (A) associated with the construction function. We have also elucidated the scale independence property of housing quality associated with our model prediction. By quantifying the housing quality on a 0 - 100 scale, we then acquired the values of scale-shifting parameters (b, c) according to this chosen scale. Before we move to numerically solve the model and examine the prediction of our model, the parameters that are still left to calibrate before we can numerically solve the model are the preference parameter β, discount rate r, and average building coverage ratio (BCR). We perform calibration by adjusting these three parameters for the model to match our target moments: property value to rent ratio, rent to income ratio, and housing floor area. Different sets of values of parameter β, r, and BCR generate different values in the target moments in the equilibrium; however, it is the preference parameter (β) which dominates the average value of the rent to income ratio, while discount rate (r) dominates the average value to rent ratio, and residential building coverage ratio (BCR) (or equivalently, the corresponding effective population density) dominates the value of the construction floor area in the steady-state equilibrium. In Table 2.3, we report the values of the target moments we calculated from the data.

Floor area

We are interested in investigating the steady-state housing market. The steady-state conditions include that income and effective household population density (i.e. when household population growth rate (rate of new construction) is equal to the effective land growth rate) are stable. Different income levels and effective household densities generate different housing stock in the steady state. We, therefore, have decided on calibrating not to the historical housing stock in Riverside, but to what a steady-state housing stock would be if all housing units had been constructed with the same structural density, floor area, and construction quality as contemporary constructed housing. Due to data limitations, we follow an imputation approach to calculate the target moment for housing floor area of recent construction in the city of Riverside. The data are from the American Housing Survey (2017) and Movoto.com, whose data is from multiple listing services or public data sources and provides aggregate monthly city level market statistics data, which includes the median home size on the market. We calculate the average floor area for the city of Riverside's recently constructed homes to be 2500 ft^2. Appendix - "Average floor area" reports the details of our imputation.

Rent to wage ratio

In our model, households have the same income but they do not have to pay the same rent. Households who rent a newly constructed house (after referred to as "new" quality) pay the highest rent while the ones who rent in a house of steady-state quality pay the lowest. The rent to income ratio varies across housing qualities. For simplicity, we take the arithmetic mean of the rent to income ratio at new quality and the rent to income ratio at steady-state quality as the average rent to income ratio. Interestingly, we find that in the steady-state equilibrium, the value of the arithmetic mean of the rent to income ratio at new quality and the rent to income ratio at steady-state quality is quite close to the value of the preference parameter β in the utility function.[26] There is a general rule of thumb in real estate economics that on average, rent accounts for roughly 30 percent of a typical income. We recognize there may exist regional differences in this ratio, therefore we wish to find the average household rent to household income ratio in the city of Riverside, or at least around the local area by taking into consideration both the general flexibility/mobility in renting a home in the local area and the commuting distance between the home and place of work. We use analysis from Davis and Ortalo-Magn'e (2011), which calculates that the median household rent to household wage income ratio in Riverside–San Bernardino–Ontario metro area in 1990 and in 2000 to be 0.28 and 0.27 respectively. The national average of median household rent to wage-income ratio is 0.25 with a standard deviation of 0.02 (across different metropolitan areas and periods) in their sample (1980 - 2000). In the numerical simulations, we assume that the average rent to wage-income ratio

[26]If housing was a simple good, then we could adopt a traditional linear budget constraint. Cobb-Douglas utility function with linear budget constraint would induce the value of housing preference parameter β to be equal to the value of expenditure share on housing.

in Riverside is 0.28. From our model, wage income w can be calculated as the difference between income y and income from land rent θ.

Value to rent ratio

We use the arithmetic mean of housing value to rent ratio at new quality and that at steady-state quality as the average housing value to rent ratio. Zillow research provides monthly median property value to rent ratio data; we have calculated the annual median property value to rent ratio for the city of Riverside in 2018 to be 16.344.[27]

Table 2.3: Target moments

	Target moments (from data)	
Floor area	f	2,500 (ft^2)
Rent to wage ratio	R/w	0.28
Value to rent ratio	P/R	16.34

[27]This average contains January through November 2018. It excludes December 2018 which was not available in the Zillow dataset. Since there were no housing boom and bust shocks happening in December 2018 and since each month's value to rent ratio in 2018 moves around the average, we deem that this average, although missing Dec, is acceptably accurate for 2018. Zillow's property value to rent ratio is first calculated at the individual home level: Zillow's estimated property value is divided by its estimated annualized rent for each individual home, then the median among all value to rent ratios is obtained. Zillow uses its 'Zestimate' technology to estimate the rent and property value for all individual homes in its database. However, we do not exactly know the procedure used by Zillow to estimate the property-to-rent ratio for each property since we do not have any inside knowledge of their 'Zestimate' technology.

2.5 Parameterized Model Results

We solve the model in steady-state equilibrium.[28] Section 2.5.1 introduces a convenient property that our model can utilize to work with different types of housing structures. Section 2.5.2 reports our calibrated parameters. Section 2.5.3 presents the baseline simulation results of the benchmark model. The benchmark model using the calibrated parameters exactly replicates the targeted moments shown in Table 2.3. The values of the replicated targeted moments generated in the baseline simulation are: floor area= 2,500 ft^2, rent to wage ratio= 0.283, and value to rent ratio= 16.340. Sections 2.5.4 and 2.5.5 provide two illustrative comparative steady-state exercises. Section 2.5.4 presents the results under an income shock (a ten percent increase in household income (y)). Section 2.5.5 presents the results under a demand shock (a ten percent increase in population density (D)). Section 2.5.6 presents the results that illustrate the characteristic that quality is scale-independent in our model.

2.5.1 Housing types

Before we discuss further, we point out that our model has a convenient property for working with different types of housing structures: Building coverage ratio (BCR) refers to the lot size of the building structure, regardless of its type, divided by the building area of the structure. The construction density μ refers to the number of building stories, regardless

[28] In this section, since we have the data of the average total household income y, we directly use this statistic and solve the model. That is, we only use the middle loop and inner loop for model solving and compute the land rent income θ from the model simulation. However, if we acquire the data on the labor income w, we then also need the outer loop to solve the model. Both ways are equivalent. From Table 2.7, the yearly labor income $w = y - \theta = \$63,458$. By inputting $w = \$63,458$ in the outer loop, the land rent θ converges to \$1,855, which is exactly the land rent income generated from the middle loop and inner loop for a given average household total income (\$65,313).

of whether the building is a single-family house or an apartment building. The floor area f in the model refers not to the building size, but to a single unit home size, whether for a single-family house, a condo unit, or an apartment unit. Property value P and Rent R, similarly refer to those values for a single unit home, not the building. To avoid confusion, Table 2.4 summarizes their relationship for different housing types.

Table 2.4: Housing types

Housing features	Symbol	Apartment	Condo	House
Building coverage ratio	BCR	Apartment building	Condo structure	House
Construction stories	μ	Apartment building	Condo structure	House
Floor Area	f	Apartment unit	Condo unit	House
Property value	P	Apartment unit	Condo unit	House
Rent	R	Apartment unit	Condo unit	House

2.5.2 Calibrated parameters

We now discuss our calibrated parameters in the benchmark simulation. Table 2.5 reports the values of the calibrated parameters. The average building coverage ratio (BCR) is an important indicator for the accuracy of our model. The calibrated value of BCR is 7.404, which means that for a typical, recently constructed housing structure in Riverside, CA, the ratio of lot size over construction area is 7.404. The housing structure could be a single-family house, a condominium building structure, or an apartment building. That is, roughly speaking, under the targeted moments, the model predicts that for single-story houses, house lot size over house floor area is equal to 7.4. Therefore, based on this model, for two-story houses, house lot size over the house floor area is equal to 3.7, which seems to be reasonable if we look at the average BCR in Riverside's contemporary housing stock.

The calibrated discount rate (r) in 2018 is 0.04439. This value seems reasonable if we consider other factors in the discount rate, such as market risk premium and property tax rate.

The preference parameter (β) is calibrated to 0.270. Interestingly, the calibrated value of β is very close to the arithmetic mean of the rent to wage ratio of the new quality and that of the steady-state quality. It is important to note that while β is the key determinant for the housing expenditure share, interest rate (r) also slightly influences the rent to wage ratio. If we move our calibrated interest rate (r) to its two adjacent percentile points (from 0.044 to 0.04 and 0.05) and keep other parameters the same in our benchmark simulation, as the interest rate rises from 0.04 to 0.05, the average rent to income ratio declines by 3 percent, and the average rent to wage ratio declines by 2 percent. Previous empirical research also lends support for this prediction made by our model. For instance, La Cava (2016) finds a negative correlation between the nominal mortgage interest rate and the housing rent to income (GDP) ratio while controlling for other variables such as property taxes, maintenance, and expected housing capital gains.

Table 2.5: Calibrated parameters

		values
Average building coverage ratio	BCR	7.404
Preference parameter	β	0.270
Interest rate (discount rate)	r	0.044

We normalize housing interior quality on a scale of 0 - 100. Table 2.6 reports the normalized shift-scaling parameters.

Table 2.6: Normalized shift-scaling parameters

		values
Shift-scaling parameter in construction function	c	0.326
Shift-scaling parameter in DM function	b	5.300

2.5.3 Baseline results

Table 2.7 presents the baseline model results. The baseline simulation numerically solves the model with the benchmark parameters. The first row of Table 2.7 is the equilibrium floor area, which our calibration directly targets. Figure 2.1 plots a phase plane of the dynamics of housing quality in the benchmark model. Many other housing features are not targeted by our calibration, but the model predicts them well. For example, simulation results also show that the ratio of maintenance expenditure ranges from 0.6 percent to 1.1 percent of the property value, which is consistent with the 1 percent maintenance expenditure rule in real estate economics. Moreover, the model replicates the empirical fact that for newly constructed housing, the ratio of maintenance expenditure over the property value is lower than that of aged housing. Homeowners usually tend to spend more in maintenance as the housing unit ages. In the meantime, the model predicts that the average number of building story of contemporary housing structures in Riverside is 1.97.

2.5.4 Income shock

We are now interested in investigating how the model features react to a 10 percent increase in household income (y). We will do so by holding the population density and the housing supply technologies fixed.

Table 2.8 presents the simulation results of a counterfactual steady state in which household income (y) is 10 percent more than in the benchmark simulation. The fifth column reports the results. The sixth column calculates arc elasticities of the various features (in their equilibrium values) with respect to the 10 percent increase in household income. Figure 2.2 plots a phase plane of the dynamics of housing quality under the income shock. Figure 2.3 takes a closer look at the stable arm and compares the results with that of the benchmark model. In this exercise, we hold the household population density constant, and we assume that the maintenance function and construction function do not change.[29] Note that where the last row of the table reports the elasticity of net quality depreciation rate at steady state, the net depreciation rate at steady state is 0. We put 0 on the elasticity of this row, just to represent there is no change in the statistic under the income shock.

The model reproduced certain aspects of reality remarkably well. These findings can be explained intuitively. A 10 percent increase in real income indicates an increase in household consumption of general goods and housing services (housing floor area and housing quality at all levels). This is caused by the income effect. As expected, the equilibrium housing quality and housing floor area both increase, but quality does not increase as much as the floor area. The intuitive interpretation is that unlike floor area, quality is depreciable and needs to be taken care of by maintenance over the lifetime of the housing unit. Compared to the floor area, quality has a 'cost' over time and is therefore relatively more expensive.

We also see that as income increases, the construction quality increases and the distribution of housing quality shifts up. Let us imagine there is a continuous ranking of

[29] That is, we hold the shift-scaling parameters in the housing maintenance function and construction function the same as in the benchmark model by setting the scale-shifting parameters in the maintenance function and construction function to be 5.300 and 0.326, respectively.

housing quality before and after the income shock.[30] For a household living in a house whose quality level is ranked the lowest (the steady-state quality), the model simulation predicts that the household will have the same expenditure on housing before and after the income shock.[31] The same applies to the households that live in the highest rank of housing quality (the construction quality) pre- and post- income shock. We further conjecture that for any house occupying the same rank pre- and post-income shock, it will require the same share of expenditure on housing regardless of its actual quality level.

In the meantime, the model simulation generates the income elasticity of property rent, property value, and property maintenance expenditure as being equal to 1.[32] This too, can be intuitively explained. First, holding the household's preference in housing consumption and interest rate constant, the income share on housing expenditure remains constant (relative to their rank of housing quality), and so does the share of expenditure on general goods. Second, we observe from the model simulation that with the same rank of housing quality levels, the ratio of maintenance expenditure on property value stays the same after the 10 percent income shock; intuitively, regardless of whether households choose to become homeowners (combining renter's utility maximization problem and landlord's profit maximization problem) or simply be renters, they pay the same expenditure share in general goods consumption. Therefore, the property value also increases by ten percent as income increases by ten percent.

[30]Equivalently, we can also say there is a continuous ranking of housing service since housing service is differentiated only by the housing quality since everyone has the same housing floor area in one steady state.

[31]Assume household has full mobility, it doesn't matter who lives in that rank of house pre- or post-income shock.

[32]Elasticity is calculated as the arc elasticity of each feature from the perspective of general equilibrium.

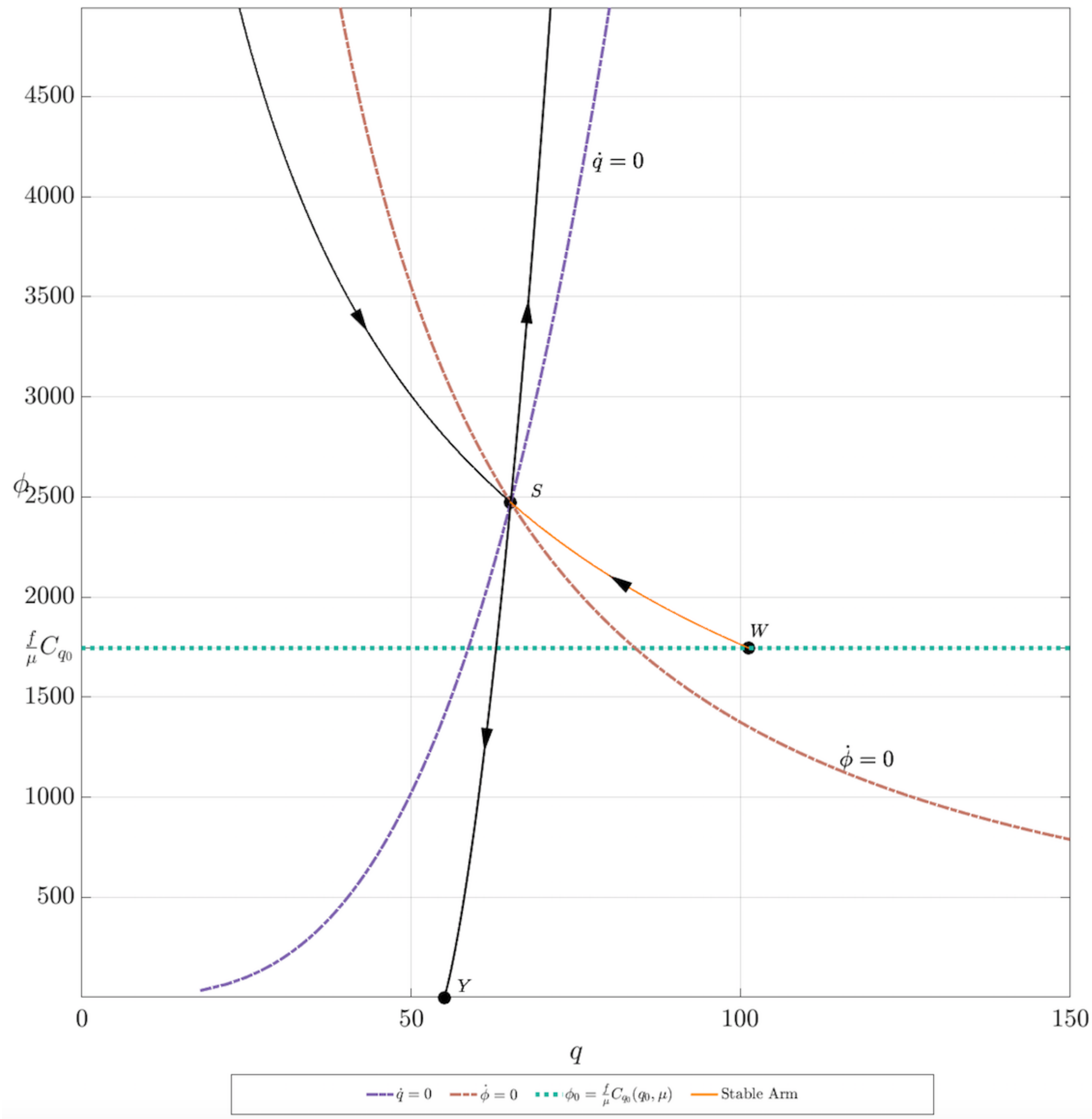

Figure 2.2: Housing dynamics - positive income shock
Note: The data for the figure are those generated from the income shock simulation.

2.5.5 Demand shock

In this exercise, we study how the model features react to a 10 percent increase in household population density (HD) while keeping the residential area ratio in a city, land area, and the average BCR ratio the same as the benchmark case. We will do so by holding

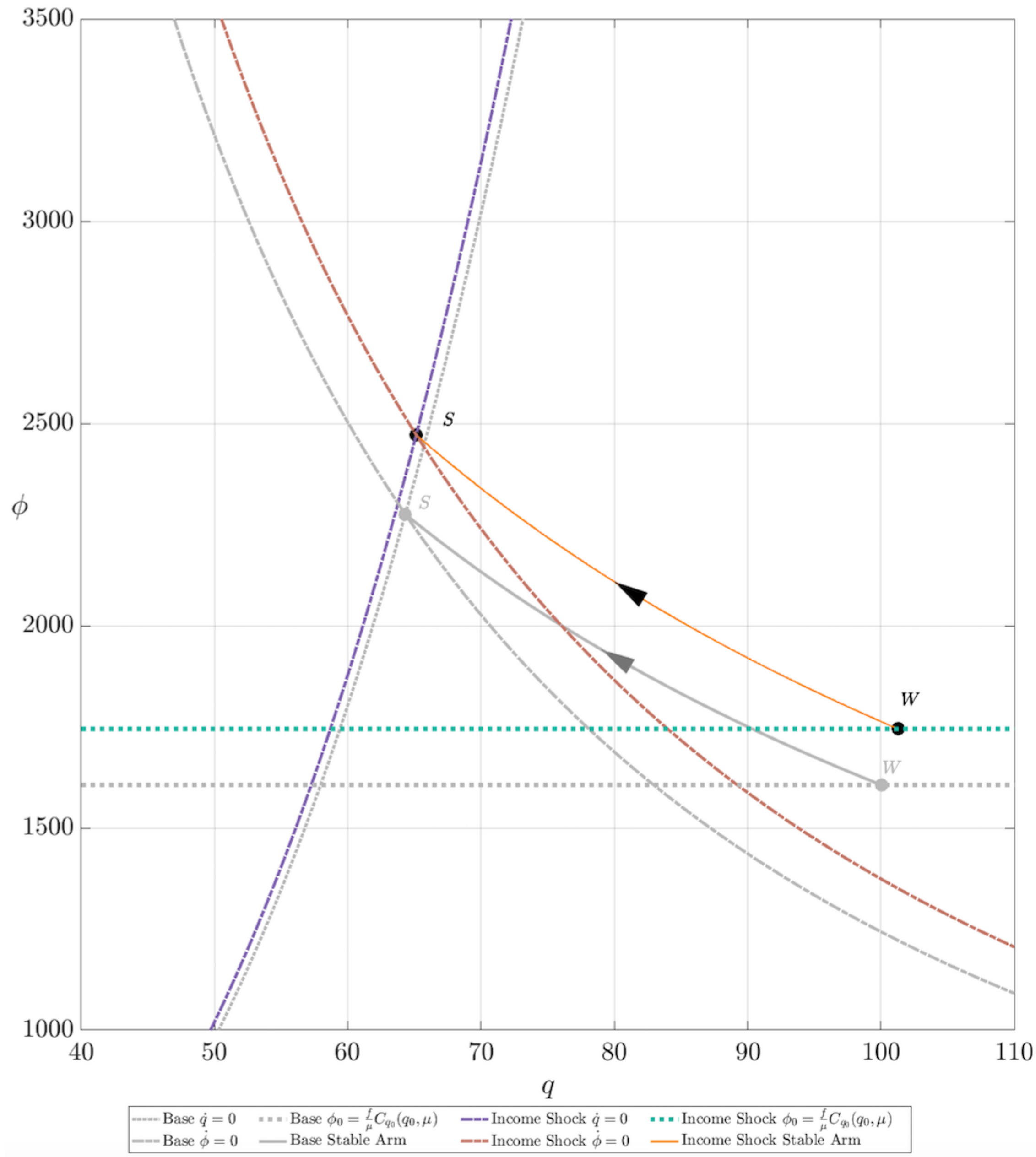

Figure 2.3: Housing dynamics - baseline vs. positive income shock
Note: The data for the figure are those generated from the income shock simulation, as well as the baseline simulation.

the total income level and the housing supply technologies fixed. Table 2.9 presents the simulation results of a counterfactual steady state in which effective household population density (D) is 10 percent more than in the benchmark simulation. Figure 2.4 plots a phase plane of the dynamics of housing quality under the demand shock. Figure 2.5 takes a closer look at the stable arm and compares the results with that of the benchmark model.

The fifth column reports the results. The sixth column calculates arc elasticities of equilibrium features values with respect to the 10 percent increase in population density. In this exercise, we hold the income level of the household (y) constant, and we assume that the maintenance function and construction cost remain the same. As effective household population density increases, space becomes more expensive, causing households to pursue higher housing quality and less housing floor area; this relationship can be explained by the substitution effect between housing quality and housing space. As a result, simulation results show that the equilibrium housing floor area decreases and the range of housing quality (from the construction quality to steady-state quality) shifts up. Simulation results also show as population density increases, housing structural density (housing stories) increases, floor area decreases, and property value per ft^2, and rent per ft^2 increase. These results agree with the empirical facts in populous US cities. Using New York City as an example, although the average household income is comparable to the national average,[33]the housing construction density, housing price (rent per ft^2 and property value per ft^2) are much higher than the national average. Since household preference for housing and interest rates remain the same, and the income of households stay the same, the housing expenditure and general goods expenditure, at the same rank of interior quality, are the same as that of the benchmark case.

[33]According to US census bureau,from 2014 to 2018, the median household income in New York City (in 2018 dollars) is $60,762 per year and the national median household income in the US is $60,293 per year.

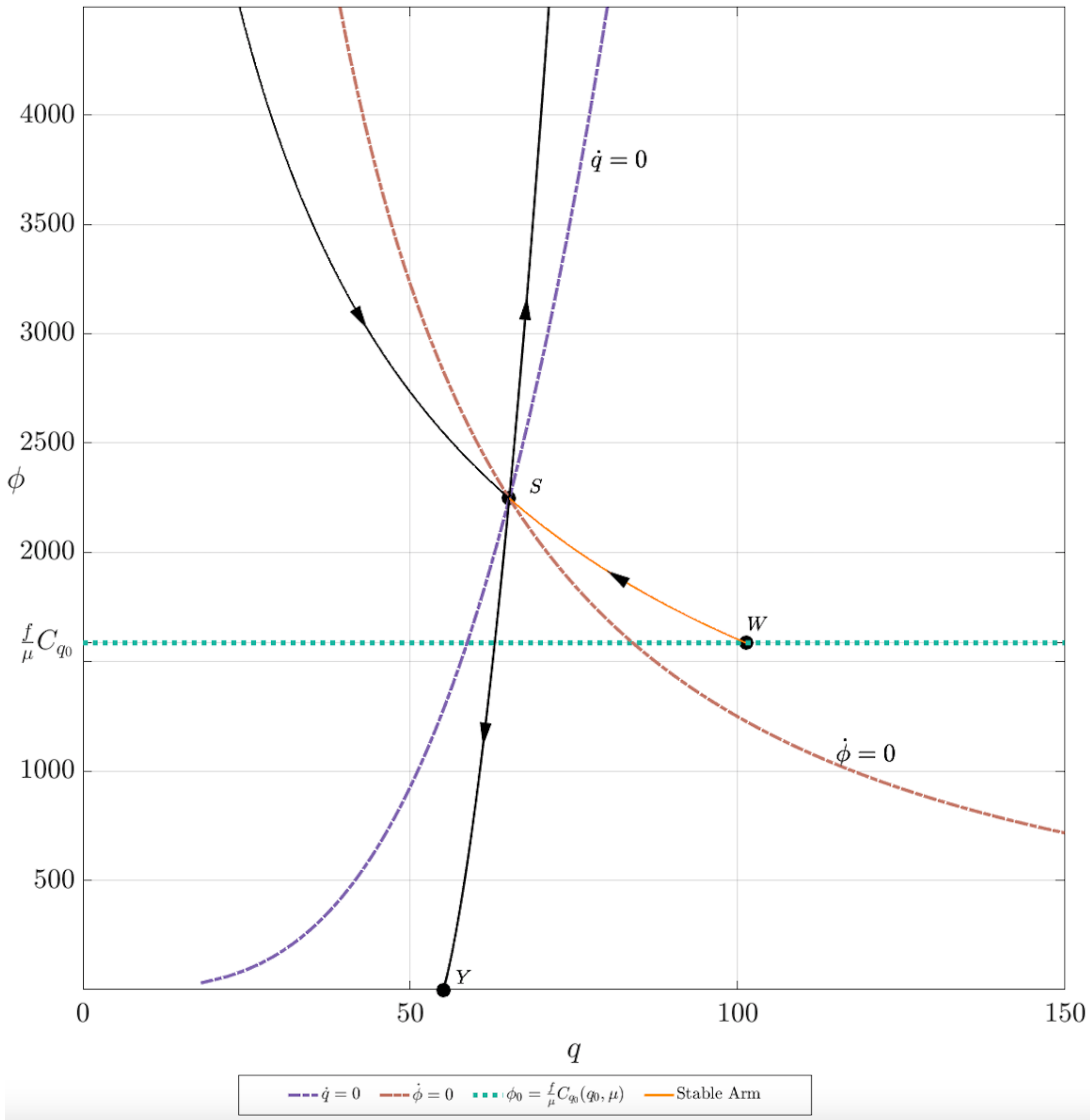

Figure 2.4: Housing dynamics - positive demand shock
Note: The data for the figure are those generated from the demand shock simulation.

2.5.6 Scale-independent nature of housing quality

Table 2.10 presents simulation results under two different scales of housing quality, which illustrate the scale-independent nature of housing quality. Two different scales, 0 - 50 (shrinking the current scale to half) and 0 - 200 (enlarging the current scale by double),

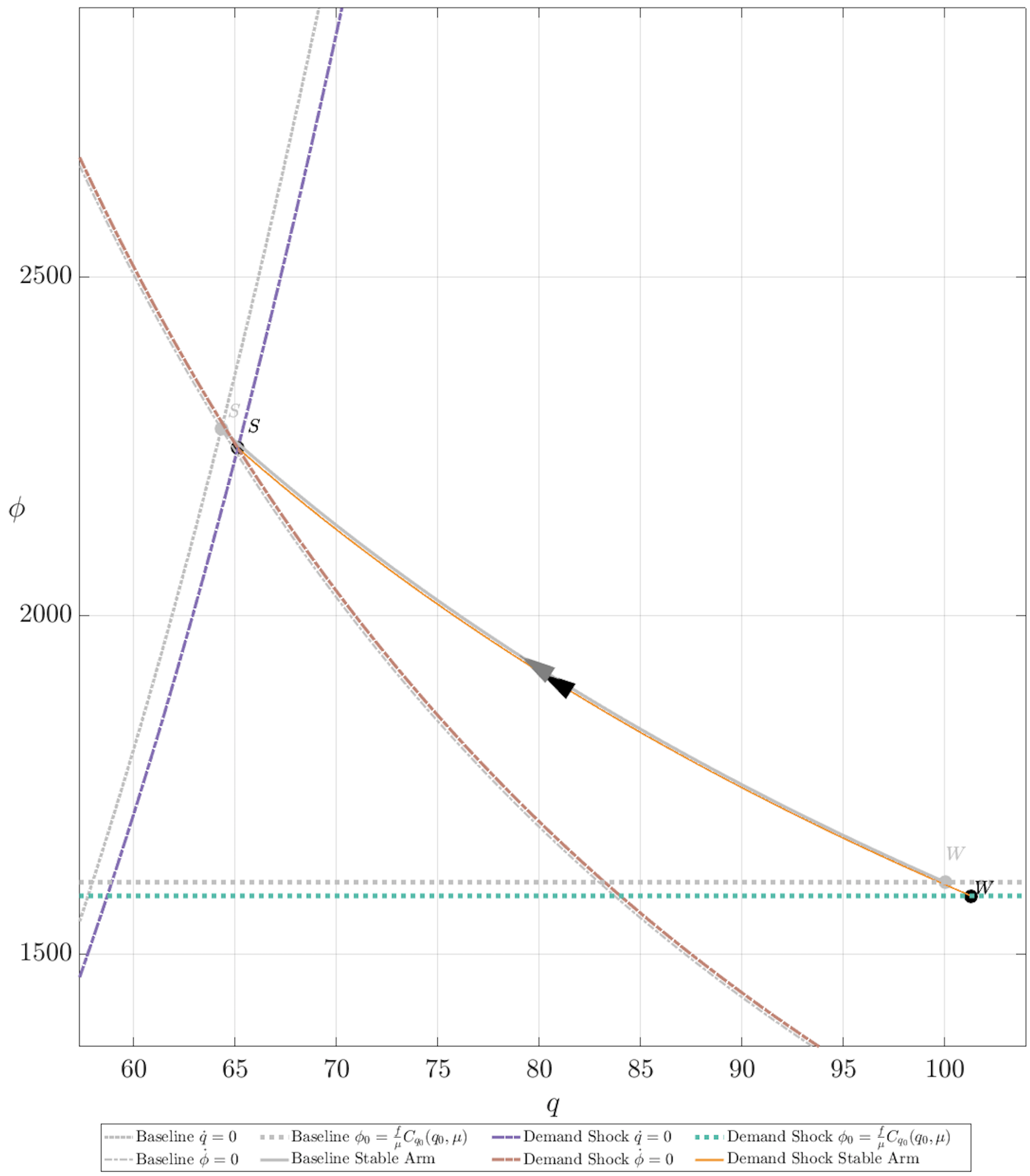

Figure 2.5: Housing dynamics - baseline vs. positive demand shock
Note: The data for the figure are those generated from the demand shock simulation, as well as the baseline simulation.

are adopted. The interior quality is cardinalized by adjusting the shift-scaling parameters in the housing supply functions (See Appendix - "Shift-scaling parameters" for the details). For quality scale, 0 - 50, the shift-scaling parameter associated with the DM function b is

2.6494, and that associated with construction function, c, is 0.6514. For quality scale, 0 - 200, the shift-scaling parameter associated with the DM function, b, is 10.5973, and that associated with construction function, c, is 0.1629. The fourth column reports the baseline results. The fifth column displays the results with a quality scale of 0 to 50. The sixth column reports the results with a quality scale of 0 to 200. The results confirm that the model's equilibrium is independent of the housing scale we choose.[34]

2.6 Conclusion

This paper presents an enriched housing filtering model based on a competitive general equilibrium and numerically examines and compares its equilibrium in steady states. The model elucidates the operation of a competitive market for durable housing and enables policy makers to perform practical policy analysis by providing greater enrichment and refinement of housing filtering theory. It incorporates two heterogeneous dimensions of housing characteristics in both the demand side and the supply side of the housing market; this is done by decomposing housing quality index into two measurable components: housing space (floor area) and interior quality. It also introduces internally-owned land and the endogenous land rent into the model.

By calibrating the model using data from the city of Riverside, CA, the benchmark simulation successfully recovers many key features in Riverside's contemporary housing stock (such as annual rent of a housing unit, housing density, building coverage ratio, maintenance expenditures, maintenance to property value ratio, etc.) and does a good job

[34]Note that the value of the shadow price (marginal value) of interior quality depends on the scale we use, which is exactly explained by the definition of the marginal value of quality.

of generating reasonable values for these features.

However, as shown in the model simulations, although we calibrate the interest rate, preference parameter, average income, and effective population density to match their real world indicators, the model seems to underestimate the housing price. More specifically, the per floor area housing rent, per floor area property value, and the property value of a housing unit are underestimated by the model. Perhaps the financial instruments (e.g., subprime mortgages) and/or frictions in asset markets (e.g., transaction costs) which our model does not consider can come to explain the difference we observe on the housing prices. More importantly, since we focus on the real side of the housing market, one important factor we do not consider as a city feature is income inequality. From a housing filtering perspective, new housing tends to be built for rich households. When rich households get even richer, they demand a higher quality of living and are willing to pay a higher price, even for the same quality of housing services. Poor households then face a higher housing price; so, to survive, they have to lower their standard of living by renting a lower quality house and/or sharing rent with others in a narrower living space. Future research shall extend the model to include income inequality as a third city feature alongside average income and effective population density.

Table 2.7: Simulation results (baseline model)

Feature	Symbol	Units	Base Case
Features that are constant over a building's life			
HU[1]floor area	f	ft^2	2,500 (*targeted*)
HS[2] structural density (stories)	μ	1	1.974
HU construction quality	q_0	1	100.0
HS land value per built-on land	V	$\$/\text{ft}^2$	33.00
Land rent income/hsld	θ	$/yr	1,855
Labor income/hsld	w	$/yr	63,458
Features at construction quality			
HU interior quality[3]	q_0	1	100.0
HU shadow price of quality	ϕ_0	$	1,606
HU property value	P_0	$	321,329
HU property value per ft^2	P_0/f	$\$/\text{ft}^2$	128.5
HS property value per built-on land	$\mu P_0/f$	$\$/\text{ft}^2$	253.7
HU annualized rent	R_0	$/yr	21,827
HU value to rent ratio	P_0/R_0	yr	14.72
HU rent to income ratio	R_0/y	1	0.334
HU property maint. exp.	M_0	$/yr	1,820
HU maint. exp. per ft^2	m_0	$\$/\text{ft}^2\ yr$	0.728
HU maint.exp. on property value ratio	M_0/P_0	/yr	0.006
HU maint.exp. to income ratio	M_0/y	1	0.028
HU net quality depreciation rate	$g(q_0, m0)/q_0$	/yr	-0.036
Features at steady-state quality			
HU interior quality	q_s	1	64.32
HU shadow price of quality	ϕ_s	$	2,276
HU property value	P_s	$	253,421
HU property value per ft^2	P_s/f	$\$/\text{ft}^2$	101.4
HS property value per built-on land	$\mu P_s/f$	$\$/\text{ft}^2$	200.1
HU annualized rent	R_s	$/yr	14,111
HU value to rent ratio	P_s/R_s	yr	17.96
HU rent to income ratio	R_s/y	1	0.216
HU property maint. exp.	M_s	$/yr	2,861
HU maint. exp. per ft^2	m_s	$\$/\text{ft}^2$	1.145
HU maint.exp. to property value ratio	M_s/P_s	/yr	0.011
HU maint.exp. to income ratio	M_s/y	1	0.044
HU net quality depreciation rate	$g(q_s, ms)/q_s$	/yr	0.000

Notes: 1. HU:Apartment unit, condo unit or house.
2. HS: Apartment building, condo structure or house.
3. Housing quality at construction is its construction quality.

Table 2.8: Simulation results (income increased by 10 percent)

Feature	Symbol	Units	Base Case	10% $\uparrow$ *in y*	$E_{x:y}$
		Features that are constant over a building's life			
HU floor area	f	ft^2	2,500	2,604	0.415
HS structural density (stories)	μ	1	1.974	2.056	0.415
HU construction quality	q_0	1	100.0	101.3	0.126
HS land value per built-on land	V	$\$/ft^2$	33.00	47.61	4.429
Land rent income/hsld	θ	\$/yr	1,855	2,677	4.429
Labor income/hsld	w	\$/yr	63,458	69,167	0.900
		Features at construction quality			
HU housing quality	q_0	1	100.0	101.3	0.126
HU shadow price of quality	ϕ_0	\$	1,606	1,745	0.863
HU property value	P_0	\$	321,329	353,462	1.000
HU property value per ft^2	P_0/f	$\$/ft^2$	128.5	135.8	0.561
HS property value per built-on land	$\mu P_0/f$	$\$/ft^2$	253.7	279.1	1.000
HU housing annualized rent	R_0	\$/yr	21,827	24,010	1.000
HU value to rent ratio	P_0/R_0	yr	14.72	14.72	0.000
HU rent to income ratio	R_0/y	1	0.334	0.334	0.000
HU property maint. exp.	M_0	\$/yr	1,820	2,002	1.000
HU maint. exp. per ft^2	m_0	$\$/ft^2\ yr$	0.728	0.769	0.562
HU maint.exp. on property value ratio	M_0/P_0	/yr	0.006	0.006	0.000
HU maint.exp. to income ratio	M_0/y	1	0.028	0.028	0.004
HU net quality depreciation rate	$g(q_0, m0)/q_0$	/yr	-0.036	-0.036	0.000
		Features at steady-state quality			
HU housing quality	q_s	1	64.32	65.13	0.126
HU shadow price of quality	ϕ_0	\$	2,276	2,472	0.863
HU property value	P_s	\$	253,421	278,771	1.000
HU property value per ft^2	P_s/f	$\$/ft^2$	101.4	107.1	0.562
HS property value per built-on land	$\mu P_s/f$	$\$/ft^2$	200.1	220.1	1.000
HU housing annualized rent	R_s	\$/yr	14,111	15,522	1.000
HU value to rent ratio	P_s/R_s	yr	17.96	17.96	0.000
HU rent to income ratio	R_s/y	1	0.216	0.216	0.000
HU property maint. exp.	M_s	\$/yr	2,861	3,148	1.000
HU maint. exp. per ft^2	m_s	$\$/ft^2\ yr$	1.145	1.209	0.561
HU maint.exp. to property value ratio	M_s/P_s	/yr	0.011	0.011	0.000
HU maint.exp. to income ratio	M_s/y	1	0.044	0.044	0.000
HU net quality depreciation rate	$g(q_s, ms)/q_s$	/yr	0.000	0.000	0.000

Table 2.9: Simulation results (household population density increased by 10 percent)

Feature	Symbol	Units	Base Case	10% $\uparrow$ *in D*	$E_{x:D}$
		Features that are constant over a building's life			
HU floor area	f	ft^2	2,500	2,367	-0.532
HS structural density (stories)	μ	1	1.974	2.055	0.415
HU construction quality	q_0	1	100.0	101.3	0.127
HS land value per built-on land	V	\$/ft^2	33.00	47.62	4.433
Land rent income/hsld	θ	\$/yr	1,855	2,434	3.121
Labor income/hsld	w	\$/yr	63,458	62,879	-0.091
		Features at construction quality			
HU housing quality	q_0	1	100.0	101.3	0.127
HU shadow price of quality	ϕ_0	\$	1,606	1,586	-0.125
HU property value	P_0	\$	321,329	321,328	0.000
HU property value per ft^2	P_0/f	\$/ft^2	128.5	135.8	0.562
HS property value per built-on land	$\mu P_0/f$	\$/ft^2	253.7	279.1	1.000
HU housing annualized rent	R_0	\$/yr	21,827	21,828	0.000
HU value to rent ratio	P_0/R_0	yr	14.72	14.72	0.000
HU rent to income ratio	R_0/y	1	0.334	0.334	0.000
HU property maint. exp.	M_0	\$/yr	1,820	1,820	-0.001
HU maint. exp. per ft^2	m_0	\$/ft^2 yr	0.728	0.769	0.561
HU maint.exp. on property value ratio	M_0/P_0	/yr	0.006	0.006	0.000
HU maint.exp. to income ratio	M_0/y	1	0.028	0.028	0.000
HU net quality depreciation rate	$g(q_0, m0)/q_0$	/yr	-0.036	-0.036	0.000
		Features at steady-state quality			
HU housing quality	q_s	1	64.32	65.13	0.127
HU shadow price of quality	ϕ_0	\$	2,276	2,247	-0.125
HU property value	P_s	\$	253,421	253,411	0.000
HU property value per ft^2	P_s/f	\$/ft^2	101.4	107.1	0.561
HS property value per built-on land	$\mu P_s/f$	\$/ft^2	200.1	220.1	1.000
HU housing annualized rent	R_s	\$/yr	14,111	14,110	0.000
HU value to rent ratio	P_s/R_s	yr	17.96	17.96	0.000
HU rent to income ratio	R_s/y	1	0.216	0.216	0.000
HU property maint. exp.	M_s	\$/yr	2,861	2,861	0.000
HU maint. exp. per ft^2	m_s	\$/ft^2 yr	1.145	1.209	0.562
HU maint.exp. to property value ratio	M_s/P_s	/yr	0.011	0.011	0.000
HU maint.exp. to income ratio	M_s/y	1	0.044	0.044	0.000
HU net quality depreciation rate	$g(q_s, ms)/q_s$	/yr	0.000	0.000	0.000

Table 2.10: Simulation results (different quality scales)

Feature	Symbol	Units	Quality Scale 100 (baseline)	Quality Scale 50	Quality Scale 200
			Features that are constant over a building's life		
HU floor area	f	ft^2	2,500	2,500	2,500
HS structural density (stories)	μ	1	1.974	1.973	1.973
HU construction quality	q_0	1	100.0	50.00	200.0
HS land value per built-on land	V	\$/ft^2	33.00	32.98	32.99
Land rent income/hsld	θ	\$/yr	1,855	1,854	1,855
Labor income/hsld	w	\$/yr	63,458	63,459	63,458
			Features at construction quality		
HU housing quality	q_0	1	100.0	50.00	200.0
HU shadow price of quality	ϕ_0	\$	1,606	3,213	803.4
HU property value	P_0	\$	321,329	321,330	321,330
HU property value per ft^2	P_0/f	\$/ft^2	128.5	128.5	128.5
HS property value per built-on land	$\mu P_0/f$	\$/ft^2	253.7	253.6	253.7
HU housing annualized rent	R_0	\$/yr	21,827	21,826	21,826
HU value to rent ratio	P_0/R_0	yr	14.72	14.72	14.72
HU rent to income ratio	R_0/y	1	0.334	0.334	0.334
HU property maint. exp.	M_0	\$/yr	1,820	1,820	1,821
HU maint. exp. per ft^2	m_0	\$/ft^2 yr	0.728	0.728	0.728
HU maint.exp. on property value ratio	M_0/P_0	/yr	0.006	0.006	0.006
HU maint.exp. to income ratio	M_0/y	1	0.028	0.028	0.028
HU net quality depreciation rate	$g(q_0, m0)/q_0$	/yr	-0.036	-0.036	-0.036
			Features at steady-state quality		
HU housing quality	q_s	1	64.32	32.15	128.6
HU shadow price of quality	ϕ_s	\$	2,276	4,552	1,138
HU property value	P_s	\$	253,421	253,455	253,474
HU property value per ft^2	P_s/f	\$/ft^2	101.4	101.4	101.4
HS property value per built-on land	$\mu P_s/f$	\$/ft^2	200.1	200.1	200.1
HU housing annualized rent	R_s	\$/yr	14,111	14,112	14,113
HU value to rent ratio	P_s/R_s	yr	17.96	17.96	17.96
HU rent to income ratio	R_s/y	1	0.216	0.216	0.216
HU property maint. exp.	M_s	\$/yr	2,861	2,861	2,861
HU maint. exp. per ft^2	m_s	\$/ft^2 yr	1.145	1.145	1.145
HU maint.exp. to property value ratio	M_s/P_s	/yr	0.011	0.011	0.011
HU maint.exp. to income ratio	M_s/y	1	0.044	0.044	0.044
HU net quality depreciation rate	$g(q_s, ms)/q_s$	/yr	0.000	0.000	0.000

2.7 Appendix

2.7.1 Pseudo code of model solving algorithm

OuterLoop

Let initial $y = w$, where w is given

while θ not converge

$y = w + \theta$

enter *MiddleLoop* and **generate** u, f, μ, q_0, and ϕ_0

compute θ, using (2.22) and (2.23)

MiddleLoop

Input: y from *OuterLoop* **Let** initial $u_0 = u_{min}$ and initial $u_1 = u_{max}$

while $u_0 \leq u_1$

let $u = \frac{u_0+u_1}{2}$, **enter** *innerloop* and **generate** f and μ

if $\frac{\mu}{f} = D$

break the loop and **output** u,f,μ

and **output** q_0, ϕ_0 stored in the *Innerloop*

else if $\frac{\mu}{f} < D$

$u_1 = u$ and **continue** the loop

else

$u_0 = u$ and **continue** the loop

InnerLoop

Input: u from *MiddleLoop*

Let initial $f_0 = f_{min}$ and initial $f_1 = f_{max}$

while $f_0 \leq f_1$

let $f = \frac{f_0+f_1}{2}$

Step 1.1

Input f to (2.19),(2.20) and (2.21)

Solve the systems of equations (2.19),(2.20) and (2.21) with known f

Output μ,ϕ_0, q_0

Step 1.2

Input f to (2.27), (2.28) and (2.29)

Solve (2.27), (2.28) and (2.29) at steady state with known f

Output $\phi(s)$, $q(s)$

Step 2

Input y, u, $q(s)$, $\phi(s)$, ϕ_0, to $\underline{Function : Modified\ Euler's\ Method}$

Output q_{0d}

Step 3

generate $q_{0s} = q_0$

if $q_{0s} = q_{0d}$

break the loop and **output** and **store** f, μ, q_0 and ϕ_0

else if $q_{0s} > q_{0d}$

$f_1 = f$ and **continue** the loop

else

$f_0 = f$ and **continue** the loop

$\underline{Function : Modified\ Euler's\ Method}$

Pass in: y, u, $q(s)$, $\phi(s)$, and ϕ_0 from step 2 of the Inner Loop

input step size h

let $\phi(i) = \phi(s),\ q(i) = q(s)$

Input y, u, $\phi(i)$, $q(i)$, ϕ_s to $\underline{Function : Slope\ of\ Stable\ Arm}$

Output $f(\phi(i), q(i))$

while $\phi_0 \leq \phi(i)$

$q_{0d} = q(i) - h/f(\phi(i), q(i))$

$\phi(i) = \phi(i) - h$

$q(i) = q_{0d}$

pass out: q_{0d}

$\underline{Function : Slope\ of\ Stable\ Arm}$

Pass in: y, u, $\phi(i)$, $q(i)$, $\phi(s)$ from $\underline{Function : Modified\ Euler's\ Method}$

if $\phi(i) = \phi_s$

$$\frac{d\phi(i)}{dq(i)} = \frac{2\delta + r - \sqrt{(2\delta + r)^2 - 4b\frac{\gamma}{1-\gamma}(\frac{\gamma b}{f})^{\frac{\gamma}{1-\gamma}}\phi(s)^{\frac{2\gamma-1}{1-\gamma}}R_{q(s)q(s)}(q(s); f)}}{2b\frac{\gamma}{1-\gamma}(\frac{\gamma b}{f})^{\frac{\gamma}{1-\gamma}}\phi(s)^{\frac{2\gamma-1}{1-\gamma}}}$$

else

$$\frac{d\phi(i)}{dq(i)} = \frac{\dot{\phi}}{\dot{q}} = \frac{(r+\delta)\phi(i) - R_{q(i)}(q(i); f)}{-\delta q(i) + b(\frac{\phi(i)\gamma b}{f})^{\frac{\gamma}{1-\gamma}}}$$

pass out: $\frac{d\phi(i)}{dq(i)}$

Technical Details on Slope of the Stable Arm at Steady-state Quality

At the steady-state quality, from (2.5) to (2.6), we have

$$\dot{q} = -\delta q(s) + b(\frac{\phi(s)\gamma b}{f})^{\frac{\gamma}{1-\gamma}} = 0,$$

$$\dot{\phi} = (r+\delta)\phi(s) - R_{q(s)}(q(s); f) = 0.$$

We then have

$$\phi_{q(s)}(q(s)) = \lim_{q \to q(s)} \frac{(r+\delta)\phi(q) - R_q(q; f)}{-\delta q + b(\frac{\phi(q)\gamma b}{f})^{\frac{\gamma}{1-\gamma}}}.$$

Applying L'Hospital's rule, we have

$$\frac{d\phi(q(s))}{dq(s)} = \phi_{q(s)}(q(s)) = \frac{(r+\delta)\phi_{q(s)}(q(s)) - R_{q(s)q(s)}(q(s); f)}{-\delta + b\frac{\gamma}{1-\gamma}(\frac{\gamma b}{f})^{\frac{\gamma}{1-\gamma}}\phi(s)^{\frac{2\gamma-1}{1-\gamma}}\phi_{q(s)}(q(s))},$$

which can be written as

$$b\frac{\gamma}{1-\gamma}(\frac{\gamma b}{f})^{\frac{\gamma}{1-\gamma}}\phi(s)^{\frac{2\gamma-1}{1-\gamma}}\phi^2_{q(s)}(q(s)) - (2\delta + r)\phi_{q(s)}(q(s)) + R_{q(s)q(s)}(q(s); f) = 0.$$

Solving the above equation for $\phi_{q(s)}(q(s))$, we have

$$\phi_{q(s)}(q(s)) = \frac{2\delta + r \pm \sqrt{(2\delta + r)^2 - 4b\frac{\gamma}{1-\gamma}(\frac{\gamma b}{f})^{\frac{\gamma}{1-\gamma}}\phi(s)^{\frac{2\gamma-1}{1-\gamma}}R_{q(s)q(s)}(q(s); f)}}{2b\frac{\gamma}{1-\gamma}(\frac{\gamma b}{f})^{\frac{\gamma}{1-\gamma}}\phi(s)^{\frac{2\gamma-1}{1-\gamma}}}.$$

Since $R_{q(s)q(s)}(q(s); f) < 0$ and since the slope of the stable arm is negative,

$$\phi_{q(s)}(q(s)) = \frac{2\delta + r - \sqrt{(2\delta+r)^2 - 4b\frac{\gamma}{1-\gamma}(\frac{\gamma b}{f})^{\frac{\gamma}{1-\gamma}}\phi(s)^{\frac{2\gamma-1}{1-\gamma}}R_{q(s)q(s)}(q(s);f)}}{2b\frac{\gamma}{1-\gamma}(\frac{\gamma b}{f})^{\frac{\gamma}{1-\gamma}}\phi(s)^{\frac{2\gamma-1}{1-\gamma}}}.$$

2.7.2 Shift-scaling parameters

Construction function:

$$C(\mu, q_0) = c_1 A + c_1 c_2 q_0 \mu^2. \tag{2.37}$$

DM function:

$$g(q, m) = -\delta q + b_1 b_2 m^{\gamma}. \tag{2.38}$$

In our benchmark, we set $c_1 c_2 q_0$, the slope of variable construction cost per unit area of built-on land with respect to μ, to the estimated value of that slope from RSMeans (the average house in 2017 and 2018, built from wood with wood frame). That is, we assume that the value of variable construction cost per unit of land estimated from RSMeans in 2018 equates to the variable construction cost per unit of land to build a one-story housing unit with construction quality generated by the equilibrium conditions. We therefore match the typical construction quality of Riverside, CA (chosen by RSMeans in 2017 and 2018) to the equilibrium construction quality generated by our parameterized benchmark model. This construction quality is determined by Riverside's housing supply and demand, which are fundamentally driven by factors such as real interest rate, real income level, population size, land supply, etc. The estimated value for the slope is 32.567. We normalize the shift parameter of Riverside where $c_1 = 1$. c_2 is the scaling parameter. Since we put housing quality on a scale of 0 - 100, a housing quality equal to 0 corresponds to a situation where the variable construction cost per unit area of build-on land equals 0. A housing quality equal to 100 corresponds to the construction quality generated from the equilibrium. In that case, c_2 is equal to 0.326. Recall we have another shift-scaling parameter b from the maintenance function, which is the multiplication of b_1 and b_2. Therefore, we must also put the quality in the maintenance function on a scale of 0 - 100. These two scaling parameters (b_2 and c_2) are determined by the value of the quality scale. Therefore, we calibrate b = 5.300 so that construction quality is 100 in the baseline model.[35]

2.7.3 Residential area ratio

We utilize the Zoning GIS database (provided by the planning department of the city of Riverside) to determine the residential area ratio in the city of Riverside. According to the Municipal Code, Riverside zoning contains five categories: Residential Zones, Office Commercial Zones, Mixed-Use Zones, Industrial Zones, and Other Zones (e.g. Public Facilities Zone). Figure 2.6 plots a GIS map containing all land area of Riverside, based on the Zoning GIS database.

The residential zones include agricultural zones (zone codes: RA - 5, RR and RE), residential conservation in environmentally sensitive areas (RC), single-family homes (R - 1) and multi-family dwellings (R - 3). Single-family and multi-family housing are further categorized by their lot density.

The residential area defined in our model is the living area for households; we drop

[35] The above co-determination process is realized by the following method: determine (by iteration) a value of b until we see the value of c equals 0.326, then the scale of quality is set to 0 - 100.

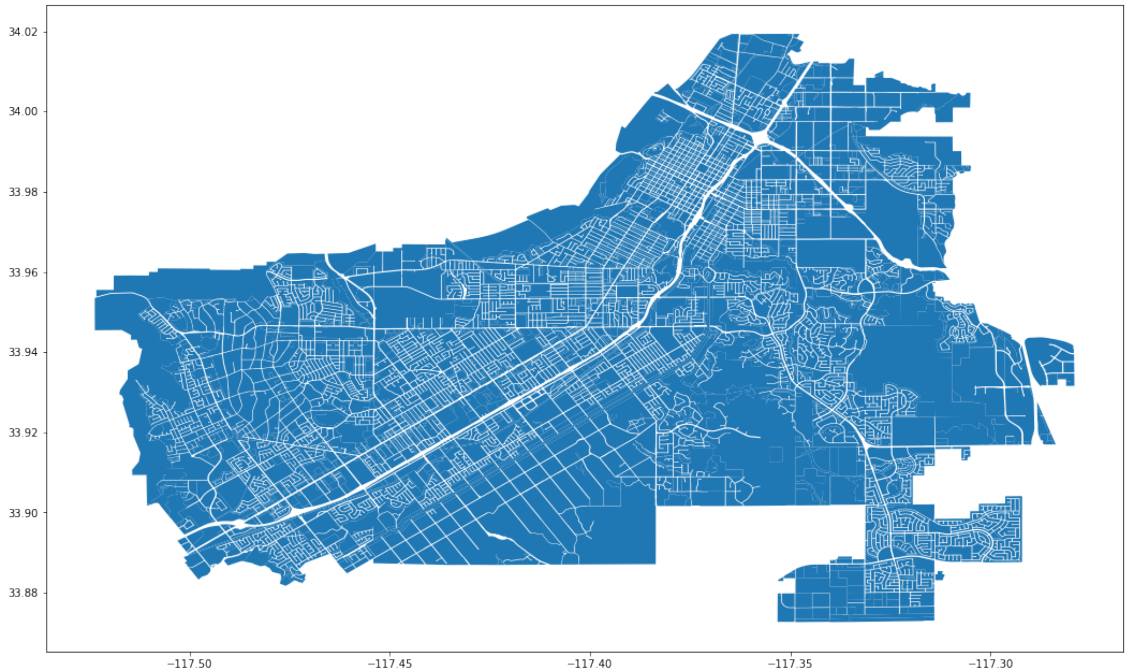

Figure 2.6: Riverside, CA (city zoning GIS data; latitude = Y, longitude = X)

agricultural zones from city-defined residential zones because most of the land area in this category is used for farming and living space for livestock. Figure 2.7 is the map of the residential zones excluding agricultural zones in Riverside.

From Figure 2.7, the residential conservation in environmentally sensitive areas (RC) is located in the mountainous areas of the city of Riverside. They are subject to special environmental regulations and thus the residential density is very low. We thereby exclude this zone in the residential land area we defined in our paper because the very low density results from government regulation in addition to large differences in land quality. Figure 2.8 is the residential land area we defined in this paper, which contains single-family housing (R - 1) and multi-family dwellings (R - 3). The residential land ratio equals the

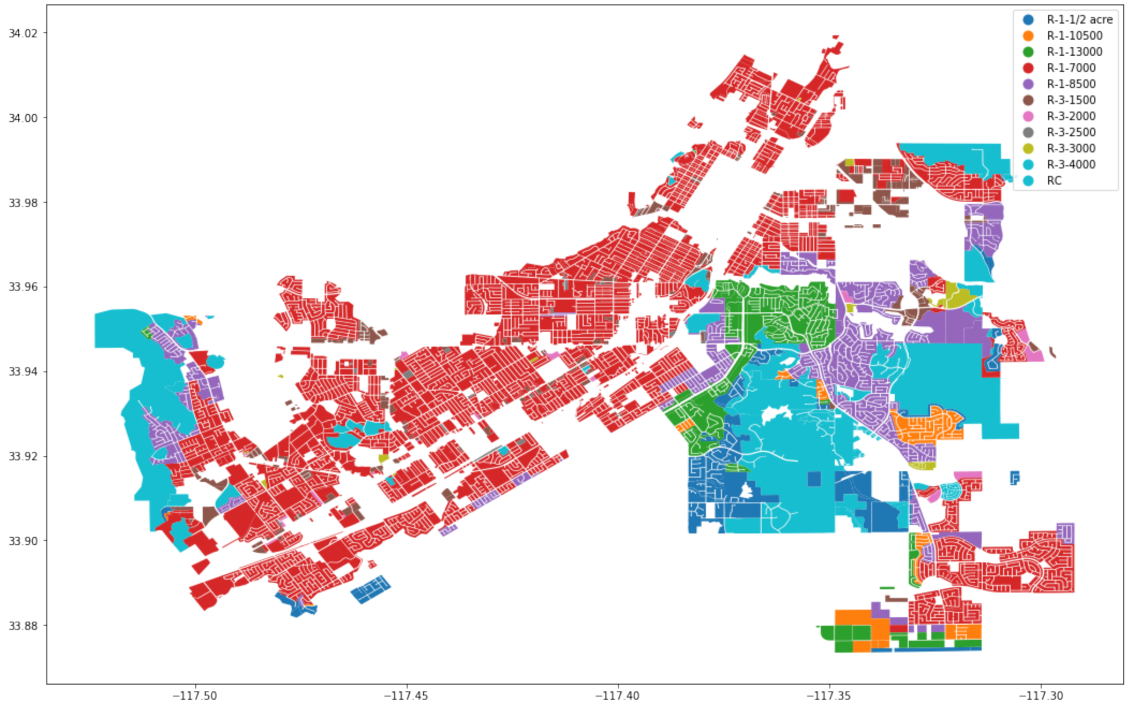

Figure 2.7: Riverside residential zones

total land area of all the single-family and multiple family residential zones (R - 1 and R - 3, as shown in Figure 2.8) divided by total land area of Riverside (as shown in Figure 2.6). We calculate this ratio to be 0.4089.

2.7.4 Average floor area

The average floor area was imputed from two data sources, the 2017 American Housing Survey and Movoto.com. Movoto.com acquires real-time data from multiple listing services or public data sources. It provides aggregate monthly city-level market statistics data for the last five years, which includes the median home size on the market.

We utilize the monthly data of the median-sized home on the housing market of Riverside, CA to calculate the mean of a five-year range (2015 June to 2019 May). We

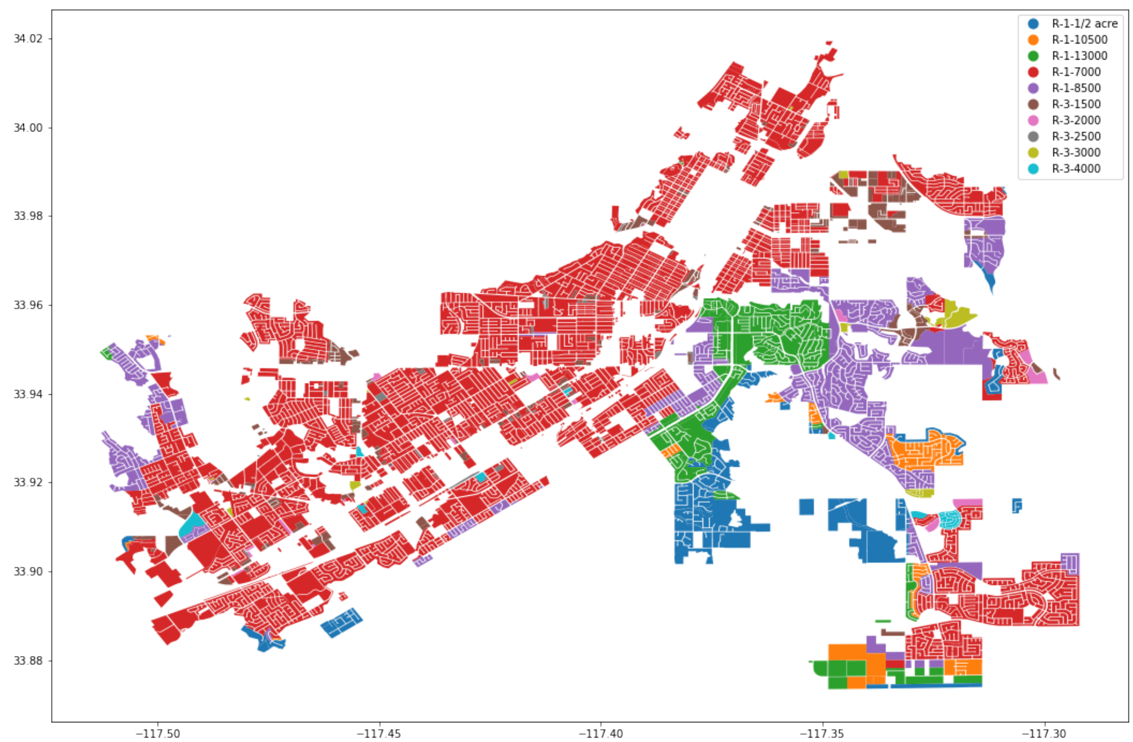

Figure 2.8: Riverside residential land area

determine this statistic to be 1995 ft^2. This value reflects the representative home size in Riverside (including historical and new residential constructions).

We also acquire the average floor size of the recent constructions. From the 2017 American Housing Survey, we determine the average floor area of the Riverside-San Bernardino Metro area to be 1500 ft^2 for all building ages. The average floor area of the residential buildings built from 2010 to 2015 is 1930 ft^2. Then the average floor area built between 2010 and 2015 in Riverside, CA is imputed as $1930/1500 \times 1995 = 2567$ ft^2. Considering there are statistical errors, for convenience, we set this statistic to be 2500 ft^2 for the representative home size of the recently constructed housing in Riverside, CA.

Chapter 3

The Natural Rate of Structure Depreciation: Decoupling Capital Gains and Maintenance Improvement from Property Value Variation

Abstract

This paper, motivated by the observations that the land of a property does not depreciate, along with taking into account the capital gains from housing boom and bust cycles, provides a novel approach to separate housing structure depreciation and capital

gains on land into two distinct components that make up overall changes in property value. This analysis is further developed by distinguishing between the natural rate of structure depreciation (the rate that would occur in the absence of maintenance) and the effects of maintenance on structure value over time. We utilize this approach to estimate the natural depreciation rate of housing structure value and homeowner's maintenance technology of single-family detached housing in the New York metropolitan area using data from the American Housing Survey. This method estimates the natural rate of depreciation of structures to be roughly 9 percent per year; this shows a significant, and often overlooked economic cost is associated with real estate ownership due to value loss from structure depreciation and maintenance expenses.

3.1 Introduction

Scholars and investors in the United States have long debated whether investing in real estate assets is a wise decision compared to investment in the financial market. The expected return on a real estate investment is commonly projected by relying on macroeconomic conditions and government policies (e.g., property tax rates). However, the value loss from structure depreciation and the hidden cost of maintenance improvements are often neglected. These two issues are especially important in the US, where houses are traditionally built with relatively less durable materials like wood and lumber, as compared to brick or concrete. The cost of maintenance builds up over time to have a significant impact on homeowners, particularly in the US. Despite its importance, the natural depreciation rate of housing structures (the rate that would occur in the absence of maintenance) has been overlooked in previous literature. Due to this, we aim to present a method to estimate the exogenous natural rate of structure depreciation by disentangling the various elements of changes in housing property values and explaining their relationships. Not only does this research expand real estate investment and housing depreciation literature, but our model for properly measuring the natural rate of depreciation also contributes to applications such as housing capital valuation and implementation of optimal property tax rates. Moreover, our research has led to an estimated maintenance function that will be useful for developing policies that promote affordable housing.

Our approach is motivated by the following facts. First, not every component of a property depreciates. Only the structure of the property depreciates, but not the land associated with it. A direct estimation of structure depreciation (instead of the property value depreciation) is very necessary since the ratio between land value and the property value varies with different houses, different neighborhoods, and different regions. Second, homeowners routinely spend money on housing maintenance. This slows down the observed rate of depreciation from its natural rate. Third, land is non-reproducible. The inelastic supply of non-reproducible land leads to particularly volatile changes in capital values of housing property due to variations in economic activity and job opportunities associated with the locations of the property that stem from the business cycle.

Consider a house whose market value is observed at two points in time. The measured property value change captures change in the quality of the structure, change in the desirability of the house's location, and macroeconomic conditions. It is, therefore, challenging to achieve an accurate estimation of the depreciation of housing structures.

The housing depreciation estimation is complicated by several facets. The first facet is that even after adjusting for inflation, housing itself (with controlling its quality of the structure) experiences capital gains in the housing boom and bust cycle. Without decoupling capital gains from total change in value of housing property, the housing structure depreciation rate will be understated.

The second facet is the fact that not all the components in a given housing property depreciate. The land of a housing property does not depreciate, while the value of the housing structure do. Since there is no separable market for the housing structure and its lot due to the fact that structure and land are sold as a bundle in real estate transactions, we can not directly observe the change of housing structure values from transaction data. Moreover, the land value of lots differ across different metropolitan areas. The different weights of non-depreciating parts of the properties due to regional market differences could bias the estimation of depreciation rate if we use only the housing property value for depreciation. For a particular housing unit, the higher the lot value of the property is, the lower the depreciation rate will be estimated if we do not subtract out the non-depreciable land value of that property.

The third facet is the effect of maintenance expenditures on the depreciation of housing structure value. Sweeney (1974) and Arnott et al. (1983) provide a theoretical analysis of housing maintenance in the context of housing filtering theory. Housing maintenance expenditures made by the rational landlord or homeowner per year are not a constant number; it is determined by the state of the housing quality. The maintenance expenses will not cover all the depreciation loss unless the housing quality is in a steady state as illustrated in Arnott et al. (1983) and Arnott and Zhao (2019). The estimated depreciation rate without considering the maintenance effect is, in actuality, the net depreciation rate under the effects of maintenance. Much of the existing literature on depreciation rate estimation does not take the homeowner's maintenance expenses into account leading to a blind spot in depreciation analysis this paper purports to correct.

The fourth facet regards age-related depreciation. Literature documents that housing depreciation (net depreciation when considering the maintenance effect) is nonlinear as well as age dependent. Shilling et al. (1991) document that housing capital depreciates fast in its early years. Examining this further, we argue that one important factor that contributes to this nonlinear depreciation rate is the effect of a homeowner's various levels of maintenance expenses at different ages of the housing. Aged houses require higher maintenance expenditures, while homeowners do not usually spend much money on maintenance for newly built housing units. As a result, the net depreciation (after taking into account maintenance expenditures) for housing units in their early years is higher than when they are older. Therefore, the fact that a housing structure depreciates quickly in its early years can be explained by the homeowner spending less on maintenance in the early years of ownership. This observation of housing maintenance and the resulting nonlinear depreciation are also consistent with the simulation results generated by Arnott and Zhao (2019) using our estimated natural rate of structure depreciation and the estimated maintenance technology. Therefore, this aged-related depreciation is partially mitigated in our approach since our method admits a nonlinear (via considering maintenance) depreciation rate. The other factor, in the relatively long run, that contributes to age-related nonlinear depreciation is the obsolescence effect (property potentially loses value due to its outdated housing style).

The last facet we consider is the variation in the natural rate of structure depreciation itself. This arises from the discrepancies associated with the various types of building and construction materials. The structural durability and soundness of various construction materials are different. This fact leads to the natural rate of structure depreciation having some potential variability due to the varying physical durability of different construction materials. With controlling types of housing units to be single-family detached houses, we assume a homogeneous natural depreciation rate for this type of home in the US.[1] The other potential source of variability arises from regional differences in natural conditions such as weather, geographic activities (e.g., earthquake, hurricane). However, by focusing on a particular region, this issue can be accounted for as well.

In the current literature, there are mainly three approaches with respect to housing depreciation. The first approach is a modified repeat sales model approach. For example, Harding et al. (2007) estimate the depreciation rates of housing capital (structure and land) based on the housing asset values provided in the data of the American Housing Survey. One innovation of their approach is that they integrate the housing depreciation and maintenance effect into the repeat sales model; they also estimate the depreciation rate of average housing capital in the US with and without the effect of maintenance. The second approach is based on the hedonic pricing model. Smith (2004) identifies that it is important to remove land values from depreciation estimation and that housing locations play an important role in

[1] It may still be possible that for very high-end luxury and customized houses, the natural rate of depreciation may be slightly different than conventional houses.

the depreciation rates of housing capital. Yoshida (2016) proposes a method based on the hedonic approach to estimate the structure value depreciation based on his estimated structure and land ratio. The third approach is the national account approach, used by Davis and Heathcote (2005). Each approach carries with it its shortcomings as well as benefits.

Previous research variously realizes the need to consider individual elements discussed previously: take the maintenance effect into consideration (Shilling et al., 1991; Harding et al., 2007), separate structure value (Yoshida, 2016), recognize that land will not depreciate, and emphasize the regional heterogeneity of land (Smith, 2004; Yoshida, 2016). However, none accounts for every factor presented in this paper. Among them, it is the hedonic pricing approach that has been employed as a foundation for researchers. However, the works based on the hedonic pricing model did not specify how they can integrate the capital gains on land into their models. By contrast, the modified repeat sales approach incorporates the maintenance effect and housing capital gains, but it does not separate land value from property value and does not account for regional differences in land. The national account approach, make contributions in separating the treatment of housing structures and land, but they did not take into account the effect of homeowner maintenance at different ages of a housing unit.

This paper introduces a novel method to overcome these issues and advances housing depreciation research by providing an approach to estimate the housing maintenance technology and to directly measure the housing structure depreciation while taking account of the housing capital gains, the maintenance effect, the non-depreciability of land, and the regional heterogeneity in the land. In summary, our research contributes to the literature in the following respects. First, we recognize the land value of the lot in a property does not depreciate and housing maintenance slows down the depreciation. By decomposing the property value into land value and structure value, then focusing on the depreciation of housing structure value rather than the housing property value, we mitigate the bias (which before arose from heterogeneity in land price) in estimating the structure depreciation rate. In this way, we can separate the exogenous depreciation of housing structures from the slowing effect of housing maintenance on decreases in structure value. Second, as far as we know, the American Housing Survey (AHS) is the only database that provides maintenance expenditure data. However, AHS does not provide useful information about the land value of the lot, which creates difficulty with directly ascertaining the structure value of the house.[2] The effect of capital gains from the housing boom and bust cycles creates further difficulties in estimating the depreciation rate. We impute land values, taking into account the plattage effect (larger parcels have a lower value per unit area of land) and the lot size information of each property provided by AHS. We then identify the individual contributions from structure depreciation and capital gains derived from changes in land values. Third, we utilize the characteristics of the New York housing market and the plattage effect to account for the heterogeneity of land value. Plattage effect refers to the empirical findings that, with controlling general accessibility, the price of a land parcel rises less than proportionally with its parcel size. Specifically, an important condition to properly apply the plattage effect to impute land value is to control the general urban accessibility. Among

[2]Technically, AHS has a data entry for the land value of a lot. However, possibly due to the difficulty for a homeowner to determine the lot value of his property, it is extremely rare to see house lot value reported in the AHS survey. For comparison, in the 2011 AHS, less than 1 percent of total observations come with the land value while 50 percent of total observations report property value.

the New York metropolitan area, when comparing the ‘urban center’ (Manhattan) with its peripheral regions, the urban accessibility is quite different. We remove the Manhattan housing market (the highest land value in the nation) from our New York metropolitan area dataset, justified by the fact that there are almost no single-family houses on Manhattan used for residential occupancy.[3] Fourth, the estimated exogenous housing depreciation rate and maintenance technology can be applied to the numerical simulations of housing policy using the housing filtering models. In the next section, we describe our model and method. In Section 3.3, we describe our data and data filtering process. In Section 3.4, we present our findings and results. In Section 3.5, we present empirical discussion, robustness checks, and an extended model (with its estimation results).In particular, by introducing time dummies and assuming the maintenance technology remains the same from the year 2005 to the year 2013, we show the robustness of our model as the estimations of the natural depreciation rates and maintenance technology (with combining survey year 2005 and year 2007) are quite consistent with the estimations in Section 3.4. We conclude this paper in Section 3.6.

3.2 Model

The model contains two stages. In the inner stage, the net depreciation of housing structure value is decomposed into two opposing effects: the natural depreciation of the

[3]The New York metropolitan area gives us the largest amount of data from the AHS national survey; this unique characteristic of the New York housing market (there is almost no single family house in Manhattan, the ‘urban center’ of the New York metropolitan area) is the main reason we choose the New York metropolitan area as our example to measure depreciation. Technically, in AHS surveys, it does contain a geographic variable (METRO3) to distinguish the central city and its suburban areas. However, in the AHS codebook, it clearly warns the users to treat this variable with caution for many reasons, such as METRO3 having been altered or assigned with different values to mask the locations of some housing units. The sample unit’s geographic category information varied widely depending on the time when it was added to the survey due to various definitions of century cities /urban areas were adopted at different times.

housing structure itself and the appreciation due to housing maintenance. In the outer stage, the value of depreciable and reproducible housing structure is derived from the difference between the value of housing property and the value of the non-depreciable and non-reproducible land associated with the property, with the land value being modeled using the plattage effect. Table 3.1 provides a notational glossary for our model.

Table 3.1: Notational glossary

Variable	Definition
HV_{it}	real property value of house i [1] at time t
SV_{it}	real structure value of house i at t
sv_{it}	real structure value per unit floor area of house i at t
LV_{it}	real land value of house i at time t
a	age of house i
f_i	floor area of house i
m_{it}	real maintenance expenditure per unit floor area for house i at t
p_t	real normalized price of land
LS_i	lot size of house i
NLS_i	normalized lot size of house i
nls_i	ratio of normalized lot size to floor area of house i
α	shift-scaling parameter in maintenance function
β	elasticity of improvement on sv_{it} with respect to m_{it}
δ	natural rate of structure depreciation
γ	elasticity of land value with respect to land area
$c_{t,t+1}$	rate of real capital gains on land from t to $t+1$
c_t	rate of real capital gains on land averaged across t to $t+2$

[1] Single-family house, detached from any other building

3.2.1 Structure depreciation and maintenance

Housing structure, like capital, depreciates every year. In the meantime, homeowners pay for maintenance expenses (including routine maintenance services and home improvement projects), which cause the housing structure value to 'appreciate.' The model in the inner stage, allows us to identify and separate the changes in housing structure value from these two different effects: natural depreciation of a housing structure and maintenance improvement. Since housing floor space does not depreciate, the depreciation rate of housing structure value is equal to that of housing structure value per unit floor area.

After adjusting for inflation[4], let sv_{it} be house i's real structure value per unit floor area at t, $sv_{i,t+1}$ be house $i's$ real structure value per unit floor area at $t+1$, δ be the annual natural depreciation rate on the structure, m_{it} be the real maintenance expenditure per unit floor area of house i at t, and a be the age of house i. Then, the difference in structure value per unit floor area between two adjacent years is given by:

$$sv_{i,t+1} - sv_{i,t} = g(sv_{it}, m_{it}, \delta, a), \tag{3.1}$$

where $g(sv_{it}, m_{it}, \delta, a)$ is a general function form which reflects the effects from the natural depreciation of the housing itself, homeowner's maintenance expenditure, and obsolescence change associated with the age of the house. The age of house i, a, enters into Eq.(3.1) through the obsolescence.[5]

[4] As being discussed later, under the assumption we make, inflation in quality-controlled replacement cost is equal to inflation at the general level.

[5] Obsolescence refers to the value loss associated with outdated housing styles (old-fashioned architectural design, outmoded amenities, etc.) which is likely caused by changes in consumers' tastes or sudden changes derived from external factors, such as sudden breakthroughs in housing construction technology or changes in housing regulations (e.g., code revisions of earthquake-resistant design of buildings and housing structures).

Here, for convenience, we assume the above general function form is independent of the age of the house. This simplification is valid with the assumption that there is no value loss due to obsolescence over the short-run period we study. While the above discussion is for the age effect on the particular house i from the perspective of our model setup, in the empirical analysis, although it is not related to Eq.(3.1), another effect associated with age can arise from considering multiple houses with different ages at time t. Different houses may be built with different types of construction materials depending on their construction years. Different construction material will lead to variations in the natural rate of depreciation across observations. Our object of study is single-family detached houses in the US; in particular, in the New York metro region. Developers have a long tradition of using wood as the predominant construction material for single-family houses in the US where there is an abundance of wood and timbers. Therefore, we assume a homogeneous natural rate of structure depreciation across the years of construction in our sample. That being said, we can ignore the age of the house provided the above conditions are satisfied and write Eq.(3.1) as:

$$sv_{i,t+1} - sv_{i,t} = \; g(sv_{it}, m_{it}, \delta). \tag{3.2}$$

Eq.(3.2) is similar to the housing quality depreciation function in Arnott et al. (1983) in which housing quality is cardinized as housing (re)construction cost per unit floor area. In this paper, we can also represent the housing structure value per unit of floor area as (re)construction cost per unit floor area. In the real world, with housing being constructed with a multitude of qualities, this induces (re)construction cost (per unit floor area) to be equal to the structure value (per unit floor area) of the house in this wide range. The above arguments, implicitly carry with them the assumption that the quality-controlled and inflation-adjusted replacement costs are stable in the short run. Therefore, housing structure value can be represented as the housing (re)construction cost (in other words, replacement cost), which is also consistent with Davis and Heathcote (2007) and Case (2007).

Strong (additive) separability between housing structure depreciation and housing maintenance associated with the Eq.(3.2) is assumed,[6]

$$g(sv_{it}, m_{it}, \delta) = g_1(sv_{it}, \delta) + g_2(m_{it}), \tag{3.3}$$

where $g_1(sv_{it}, \delta)$ is the function representing the natural depreciation of housing structure and $g_2(m_{it})$ is the maintenance function.

The basis of the above separation is by recognizing that natural depreciation of housing structure and homeowner's maintenance improvements are two separate effects on changes in housing structure value and, more importantly, by assuming the cross partials associated are zero. First, the cross effect of the maintenance expense per unit floor area m_{it} over the marginal effect of the structure value per unit floor area sv_{it} on the depreciation function g_1 is zero, $\frac{\partial}{\partial m_{it}}\left(\frac{\partial g_1}{\partial sv_{it}}\right) = 0$. Intuitively, this assumption makes sense because maintenance expense has no effect on the exogenous process of natural depreciation of housing structure. Secondly, the cross effect of structure value per unit floor area sv_{it} over the

[6]See Su et al. (2015) for a nonparametric test on additive separability. Also, see Matzkin and Altonji (2005) for estimations in nonparametric and nonseparable models. In this chapter, we adopt parametric specifications on models.

marginal effect of the maintenance expense per unit floor area m_{it} on the maintenance function g_2 is zero, $\frac{\partial}{\partial sv_{it}}\left(\frac{\partial g_2}{\partial m_{it}}\right) = 0$. This assumption makes intuitive sense because marginal productivity of maintenance function is independent of the structure value per unit floor area. Finally, the cross effect of natural depreciation rate itself δ over the marginal effect of the maintenance expense per unit floor area m_{it} on the maintenance function g_2 is zero, $\frac{\partial}{\partial \delta}\left(\frac{\partial g_2}{\partial m_{it}}\right) = 0$. The final assumption follows from the intuition that natural depreciation itself has no influence on the marginal productivity of the maintenance function. Then Eq.(3.2) can be written as:

$$sv_{i,t+1} - sv_{i,t} = g_1(sv_{it}, \delta) + g_2(m_{it}). \tag{3.4}$$

In the above equation, $g_1(sv_{it}, \delta)$ characterizes the housing structure depreciation in the absence of maintenance. The natural rate of housing structure depreciation is assumed to, just as the depreciation of capital stock, follow exponential decay (i.e.,depreciation under a constant geometric rate).[7] It follows that, without maintenance, housing structure value decreases by a fixed percent each year rather than by a fixed and equal dollar amount each year. Geometric depreciation of housing structure we assume is in line with Arnott and Braid (1991) and Davis and Heathcote (2005). With the assumption of exponential decay, we have:

[7]Although most of physical depreciation follow exponential decay process - it decreases at a rate proportional to its present amount- with a constant geometric depreciation rate, some external factors (e.g., unusual usage of housing, weather and climate change and etc.) can cause fluctuation in the 'constant' geometric depreciation rate we assumed. In order to avoid complication, we assume the processes of external factors that possibly influence the natural rate of depreciation itself is stochastic so that on average, we have a constant geometric depreciation rate.

$$g_1(sv_{it}, \delta) = -\delta sv_{it}. \tag{3.5}$$

Following Arnott et al. (1983), the housing maintenance function is assumed to be diminishing returns to maintenance expenditure per unit floor area.[8] We also assume, in the short run, there is no technological change in the maintenance function; the function of maintenance technology keeps the same for the time duration we study. Based on the above analysis, a standard function form of diminishing returns to scale, which also carries with it a well-defined, unit-less elasticity, β, is assumed for housing maintenance technology,

$$g_2(m_{it}) = \alpha m_{it}^{\beta}, \tag{3.6}$$

where $g_2(m_{it})$ is the maintenance function and it measures 'appreciation' due to maintenance in the unit of structure value per unit area. m_{it} is maintenance expenditure per unit area at year t; α is the coefficient of maintenance function, whose value is governed by the relative cost of maintenance; β is the elasticity of maintenance expenditure per unit floor area on structure value improvement per unit floor area. Based on the above analysis, we should expect to see that $\beta \in (0, 1)$ and $\alpha > 0$ from data. The validity of our assumptions is supported by the estimation results in Sections 3.4 and 3.5, which both confirm this conjecture.

It is also worth noting that the above simplifications, the exclusion of the obsolescence change associated with house i's age, a, in Eq.(3.2) and the assumption of constant

[8] Diminishing returns to maintenance is a crucial assumption, which guarantees that there is unique and interior optimal level of maintenance expenditure per unit floor area for a house at a particular age from the time of construction.

geometric depreciation rate δ in Eq.(3.5), do not shy away from the empirical regularity of the age-related non-linear depreciation of housing structure. The observed depreciation rate in the literature is the net depreciation rate with the effect of housing maintenance. The net depreciation (after taking into account maintenance expenditures) for housing units in their early years is higher than when they are older. Therefore, the observed endogenous depreciation rate under the effect of maintenance is still nonlinear in our model.

By plugging Eq.(3.5) and Eq.(3.6) into Eq.(3.4), the difference in structure value per unit floor area between two adjacent years is given by

$$sv_{i,t+1} - sv_{it} = -\delta sv_{it} + \alpha m_{it}^{\beta}. \tag{3.7}$$

Then,

$$sv_{i,t+1} = (1-\delta)sv_{it} + \alpha m_{it}^{\beta}. \tag{3.8}$$

The above equation concludes the inner stage of our model, which elucidates the change in housing structure value resulting from both the natural depreciation and the appreciation from maintenance.

3.2.2 Land value, capital gains and plattage effect

The outer stage of our model is presented in this subsection. Since the housing unit is sold as a bundle in real estate transactions, the housing structure value is not directly observable, only housing property value is observable. However, we can treat housing structure value as a residual - the difference between property value and the value of the plot of land which the house is built on. The structure value of house i at time t is

$$SV_{it} = HV_{it} - LV_{it}, \tag{3.9}$$

where HV_{it} denotes house i's property value at time t, SV_{it} denotes house i's structure value at time t, and LV_{it} is house i's land value at time t.

The housing structure is durable (with depreciation) and reproducible, whereas the land of the housing unit is non-depreciable and non-reproducible. As discussed earlier, structure value can be represented by housing structure replacement cost. With the assumption that real replacement costs are stable in the short-run, we argue the durable structure is not contributing to the real capital gains.[9] In the short run, all the value change due to capital gains falls on land for the following reasons. First, land is scarce and the supply of land is relatively inelastic. Second, land is non-reproducible and each parcel of land and its location is unique. Third, land embodies the value of the property's geographic location. Various geographic locations provide differing access to a variety of quality of natural and social amenities, such as employment opportunities, shopping malls, schools, medical facilities, etc. In cyclical fluctuations, the social amenities, including the employment opportunity in that location will change, which then causes a change of the land value in that geographic location. Liu et al. (2016) provide empirical evidence that land value and unemployment move in opposite directions over the business cycle. Using

[9]That is to say, we assume, in the short run, the inflation of quality-controlled replacement costs is the same as the inflation in general.

the above reasoning, the land value of house i between time t and $t+1$ is:

$$LV_{i,t+1} = (1 + c_{t,t+1})LV_{it}, \tag{3.10}$$

where $c_{t,t+1}$ is the rate of real capital gains of the land from time t to time $t+1$ for house i.

The land value of a house can be imputed by utilizing the plattage effect, which was first studied by Colwell and Sirmans (1978). The plattage effect refers to the fact that while controlling for general urban accessibility, the value of a land parcel increases less than proportionally with the size of the lot. That is, the price per square foot of a lot decreases as lot size increases. A normalized lot size is then introduced so that the price per unit of normalized land is the same for different sizes of the lot. Let γ denote the elasticity of land value with respect to lot size and p_t denote the price per normalized land at time t. We can represent the normalized lot size of house i , NLS_i as,

$$NLS_i = (LS_i)^{\gamma}. \tag{3.11}$$

Many works in the literature have estimated the elasticity of land value with respect to the lot size under the plattage effect using different methods. Isakson (1997) estimates the value of γ to be 0.6343, 0.6495 and 0.7007 using his three empirical models. Zhang and Arnott (2015) estimate $\gamma = 0.639$ for the value of a developable vacant parcel with respect to lot size. These results are very close to each other which suggests that the elasticity of land value with respect to lot size is very stable across time and space. We take the $\gamma = 0.639$ for land size normalization for the reason that this estimated value is particularly suited for the developable land.[10]

The land value of house i at time t, LV_{it} , is

$$LV_{it} = p_t NLS_i. \tag{3.12}$$

We should also note that urban accessibility is quite different in the center of a metropolitan area compared to its periphery. For example, in the New York metro area, people living in Manhattan enjoy a short walking distance to a high density of quality restaurants, and other social amenities. As a result, the price of the normalized land is very different in Manhattan compared with that in its surrounding areas. By focusing on single-family detached houses in our data in the New York metro area, this issue will be greatly mitigated due to the fact that there are almost no single-family detached houses on Manhattan, limiting this phenomenon's impact on our analysis.[11]

Structure value of house i, SV_{it}, equals structure value of house i per unit floor

[10] Technically, there is a subtle distinction here. The land we study is the developed land – the land on which there is a structure built above. However, since the value of developed land is not directly observable, the datasets employed by the literature to estimate the plattage effect are usually obtained from information on vacant land sales. The value of vacant land is also referred to as the raw value of the site (the land on which there is no structure). An important reason we choose to use the estimation results from Zhang and Arnott (2015) is that they classified land as either undevelopable or developable and reported the estimation results based on this classification. Developable land is quite close to the developed land we study with the exception that developed land has been leveled and connected to utilities (such as water, sewage, gas and etc) whereas developable land has not been serviced and connected to such infrastructure. We assume in our model that the value of the developed land is equal to the value of developable land since the land value we refer to in our model is the location value; this assumption is justified when several factors are considered. First, the value derived from the land grading and utility infrastructure connections are associated with wages and materials; it is not associated with the location of the land parcel, therefore it is not subject to capital gains from the location of the parcel. Second, the land grading and utility infrastructure connections are in essence subsumed within the structure value since both of them depreciate and require maintenance. (e.g., backfilled soil may cause house settling, and connected water and gas pumps may leak.) Thirdly, because we treat the structure value as the (re)construction cost, and the site preparation cost (which includes the cost of land grading) is included in the construction cost according to RSMeans Estimating Handbook.

[11] Technically, there are a few exceptions. For example, there are some historical detached houses in Manhattan, e.g., Alexander Hamilton's House, however, they are no longer belong to residential housing and as a result, they are not in the American housing survey.

area, sv_{it}, multiplied by the total floor area of house i, f_i. Then:

$$SV_{it} = sv_{it} \times f_i. \tag{3.13}$$

Substituting Eq.(3.9), Eq.(3.10), Eq.(3.12) and Eq.(3.13) into Eq.(3.8) yields

$$\frac{HV_{i,t+1}}{f_i} = (1-\delta)\frac{HV_{it}}{f_i} + ((1+c_{t,t+1}) - (1-\delta))p_{it}\frac{NLS_i}{f_i} + \alpha(m_{it})^{\beta}, \tag{3.14}$$

where the notation is as follows:

$\mathrm{HV}_{i,t+1}$	=	real property value of house i at time $t+1$;
HV_{it}	=	real property value of house i at time t;
LV_{it}	=	real land value of house i at time t;
p_t	=	real normalized land price per unit area at time t;
NLS_i	=	normalized lot size of house i;
m_{it}	=	real maintenance cost per unit floor area of house i at time t;
f_i	=	floor size of house i;
δ	=	housing structure (natural) depreciation rate;
$\mathrm{c}_{t,t+1}$	=	rate of real capital gains on land from t to $t+1$;
γ	=	elasticity of land value with respect to lot size;
α	=	shift-scaling parameter of maintenance function;
β	=	elasticity of improvement of housing structure value per floor area with respect to m_{it}.

3.2.3 Empirical model

AHS - national survey is conducted every other year, iterate Eq.(3.14) one period forward while assuming $m_{it} = m_{i,t+1}$:

$$\frac{HV_{i,t+2}}{f_i} = (1-\delta)^2 \frac{HV_{it}}{f_i} + ((1+c_t)^2 - (1-\delta)^2) p_t \frac{NLS_i}{f_i} + (2-\delta)\alpha(m_{it})^\beta, \tag{3.15}$$

where c_t denotes the average rate of real land capital gains between period t to $t+2$; that is, $(1+c_t)^2 = (1+c_{t,t+1})(1+c_{t+1,t+2})$. When the land experiences a capital gain over these two years, then c_t is a positive. When the land experiences negative economic shocks such as resulted from the financial crisis, then the rate of capital gains c_t is negative.

The model is then formulated into a nonlinear regression for estimation:

$$\frac{HV_{i,t+2}}{f_i} = a\frac{HV_{it}}{f_i} + b\frac{NLS_i}{f_i} + k(m_{it})^\beta + u_{i,t+2}, \tag{3.16}$$

where $u_{i,t+2}$ is the error term.

The relationship between the model parameters and the estimated parameters are:

$$\begin{aligned} &a = (1-\delta)^2 \to \delta = 1 - \sqrt{a}\ ; \\ &b = ((1+c_t)^2 - (1-\delta)^2)p_t\ ; \\ &k = (2-\delta)\alpha \to \alpha = \frac{k}{2-\delta}\ ; \\ &\beta = \beta\ . \end{aligned} \tag{3.17}$$

Based on our model and assumptions, we should expect to see the following results from our data. The structure depreciation rate, δ, should be in the range of $0 < \delta < 1$, thus, $0 < a < 1$. Since housing maintenance will slow down the structure depreciation, the shift-scaling parameter, α, should be positive, thus, $k > 0$. Elasticity of maintenance expenditure, should be in the range of $0 < \beta < 1$. The sign of b is an interesting indicator; recall from Eq.(3.15), $b = ((1+c_t)^2 - (1-\delta)^2)p_t$ with $p_t > 0$ and $0 < \delta < 1$. In the situation of a housing boom period, the rate of land capital gain, c_t, is positive, thus, b must be a positive number; in the situation of a housing bust period, land capital gain is negative, and c_t is in the range of $-1 < c_t < 0$. Therefore, the sign of b depends on the magnitude of c_t and δ. In particular, when $-c_t = \delta$, b is 0.

3.3 Data

3.3.1 Data description

The data set we use to test our model and to perform the empirical analysis is the American Housing Survey (AHS). In particular, AHS 2013 national survey and AHS 2011 national survey (in the parameter estimations of the extended model in Section 3.5, we also use AHS 2007 national survey and AHS 2005 national survey together with AHS 2011 and AHS 2013). We focus on the New York metropolitan area (New York-Northern New Jersey-Long Island). AHS national survey is a biennial survey and it also follows every single house by its unique house IDs. AHS contains homeowners' self-reported property value, maintenance expenditures, lot size, and floor area. More importantly, to the best of our knowledge, AHS is the only database that contains the household maintenance data, which includes both project-specific maintenance (for home improvement) and routine maintenance data.[12] All dollar value variables are converted to 2011 dollar values according to the annual average Consumer Price Index published by the Federal Reserve Bank of Minneapolis.

3.3.2 Data cleaning and filtering

Since AHS survey follows every single house by its unique house IDs, the 2011 and 2013 survey year data are merged according to this unique ID. At the same time, we select the New York metro housing units and omit observations which contain missing values for housing value data, lot size, floor size or maintenance data. House prices in the AHS may have been top-coded. As indicated in AHS, high values are usually replaced with maximum values to ensure confidentiality. We then omit the maximum values of house prices of New York Metro in the year 2011 and year 2013. Moreover, there are some data quality issues at the bottom level of the housing price distributions in the AHS. In our sample, roughly 1% of the housing value data is below ten thousands dollars. We then drop the housing units whose values are below ten thousands dollar from our analysis. Then the following necessary data filtering procedures are employed and the final sample includes 377 observations. This restricted and filtered sample will be used in our later analysis. Table 3.2 sums up the

[12] The routine maintenance data (AHS variable CSTMNT) contains the regular maintenance expenditure homeowners spent last year or in a typical year. The home improvement data (AHS variable RAC) contains the aggregate expenditure in all home improvement projects since the last survey. (Technically, the home improvement data is measured either since the last year's survey time or since the move-in date for homes that changed owners. However, we restrict move-in time data to 2011 or earlier.) In our sample, we use maintenance data from the 2013 survey; therefore within our sample, home improvement data covers all projects since the 2011 year survey. Therefore, real maintenance expenditure per unit floor area in Eq.(3.15), m_{it} = (real routine maintenance expense + $1/2 \times$ real home improvement expenditure)/floor area, where real routine maintenance expense is the inflation adjusted 'CSTMNT' and real home improvement expenditure is the inflation adjusted 'RAC'.

statistics for our final sample.

Select only the single-family owner-occupied detached houses

For apartment complexes or condos, which typically have shared community spaces, AHS does not report the individual allocated shared space; thus, we cannot impute the land value for the apartment complex. Renters usually do not spend their own money on the maintenance, so we cannot use their data to estimate the maintenance effect. This procedure also eliminates the sharp effect of the Manhattan housing market in the sample. There are almost no single-family detached houses in Manhattan.

Filter out mobile homes

Mobile homes are, in many aspects of differences compared to regular housing units. In particular, in the American Housing Survey, house values reported by homeowners of mobile homes are different from that of all other regular housing units. The value for mobile homes is the structure value, which does not include the land beneath.

Filter out the data if the floor size or lot size was different across years

In some rare cases, the maintenance costs are used to expand housing space; In order to avoid considering space extension, we filtered out the data if the floor size or lot size was different in the data in the 2011 and 2013 survey years.

Filter out houses whose owners have recently changed

The 2013 AHS survey reports homeowner's maintenance expenses for two years, 2012 and 2013. Homeowner's maintenance includes two types of expenses, the cost of routine maintenance and the cost of home improvement. In some cases, a new owner moved into the house during that two-year period, which could cause a significant discrepancy between the homeowners reported expenses and the actual maintenance cost of that house in the last two years (e.g., the home improvement data is measured either since the last survey year or since the move-in date for homes that changed owners). Because of this potential discrepancy, our final dataset excludes houses where the owners had moved into the house in 2012 or 2013.

3.3.3 Data summary

Table 3.2: Summary statistics for the final sample (all monetary units are in 2011 dollars)

	Std Dev	Mean	25th	50th	75th
House Value					
House Value (2013)	171,863	382,074	289,570	366,789	463,313
House Value (2011)	172,285	406,442	300,000	380,000	490,000
House Info.					
Floor Area (Sq.ft)	832	1,983	1,480	1,900	2,476
Lot Size (Sq.ft)	16,570	13,304	5,500	8,000	14,000
Maintenance					
Home Improvement (2012 and 2013)	17,240	5,425	0	43	4,537
Routine Maintenance (2013)	1,413	1,221	290	772	1,930

3.4 Results

Our model is estimated by nonlinear least square estimation.[13] Table 3.3 shows the estimated primary coefficients from the model given in Eq.(3.16) and the calculated model coefficients based on the primary coefficients.

Table 3.3: Model results

	Pri. Coeff.	Std. Err.	P-values
a	0.839	0.013	0.000
b	-24.376	21.817	0.265
k	30.961	5.836	0.000
β	0.233	0.086	0.007
# of Obs.	377		
R-square	0.968		

	Model Coeff.
δ	0.084
α	16.16
β	0.233

As shown in the above table, the signs of model parameters generated from the data confirmed with our expectations and reasoning discussed in the end of Section 3.2.

[13] Procedures on the algorithm for nonlinear least square regression are described by Davidson and MacKinnon (2004), who also provide the validity of hypothesis testing associated with nonlinear least square regression. Nonlinear regression requires plausible initial values for the estimated parameters, since extreme initial values may cause the algorithm to fail to converge. The initial values used in this paper are : $a = 0$, $b = 0$, $k = 0$, $\beta = 0.1$. It is also important to note that the regression results are stable with different sets of reasonable initial values as long as the values do not lead to the non-convergence of the algorithm.

3.5 Empirical Discussion

3.5.1 Price level irrelevance

Recall our empirical model for nonlinear regression is

$$\frac{HV_{i,t+2}}{f_i} = \overbrace{a}^{(1-\delta)^2} \frac{HV_{it}}{f_i} + \overbrace{b}^{((1+c_t)^2-(1-\delta)^2)p_t} \frac{NLS_i}{f_i} + \overbrace{k}^{(2-\delta)\alpha} (m_{it})^{\beta} + u_{i,t+2}. \tag{3.18}$$

Our previous regression results are based on the 2011 price level. Intuitively and analytically, in our nonlinear model, if we convert the nominal terms (house values and maintenance expenses) to a different base year, we should get the same value for the primary parameter a, the depreciation rate δ, and elasticity β since they are not involved with the price level. The primary parameter b will change accordingly since it contains normalized land price p_t and annual rate of capital gains c_t. Shift-scaling parameter α shall also be expected to change because of the change of base year for real maintenance expense m_{it}. The regression results using the year 2018 as the base year shown in Table 3.4 confirmed this conjecture.

Table 3.4: Price level irrelevance

	Pri. Coeff.	Std. Err.	P-values
a	0.839	0.013	0.000
b	-27.216	24.359	0.265
k	33.690	6.515	0.000
β	0.233	0.086	0.007
# of Obs.	377		
R-square	0.968		

	Model Coeff.
δ	0.084
α	17.58
β	0.233

3.5.2 Estimation of plattage effect from data

We can also estimate the plattage effect from our model. Substituting Eq.(3.11) into Eq.(3.16) yields

$$\frac{HV_{i,t+2}}{f_i} = \overbrace{a}^{(1-\delta)^2} \frac{HV_{it}}{f_i} + \overbrace{b}^{((1+c_t)^2-(1-\delta)^2)p_t} \frac{(LS_i)^\gamma}{f_i} + \overbrace{k}^{(2-\delta)\alpha} (m_{it})^\beta + u_{i,t+2}. \tag{3.19}$$

It is worth noting that if let data determine the elasticity of land value with respect to lot size, γ, instead of setting $\gamma = 0.639$ from literature, a set of similar results including γ generated although the estimation of γ is not of significance.[14]

[14]The estimation results are stable and the algorithm is converged under a range of plausible initial values of γ. This range consists of 0.33 - 1.13.

Table 3.5: Plattage effect

	Pri. Coeff.	Std. Err.	P-values
a	0.838	0.019	0.000
b	-18.022	130.703	0.890
k	30.965	5.848	0.000
γ	0.668	0.685	0.330
β	0.233	0.086	0.007
# of Obs.	377		
R-square	0.968		

	Model Coeff.
δ	0.085
α	16.17
β	0.233

3.5.3 Drop regressor

Significance test on the primary parameter b indicates that it fails to reject the null hypothesis. One conjecture can be used to explain this fact is that the rate of land capital gains, c_t, of the single family detached houses in the New York metropolitan area from the 2011 and 2013 survey years is negative and its absolute value is equal or very close to the housing structure depreciation rate (δ). If it is true, then the model can be written as,

$$\frac{HV_{i,t+2}}{f_i} = \overbrace{a}^{(1-\delta)^2} \frac{HV_{it}}{f_i} + \overbrace{k}^{(2-\delta)\alpha} (m_{it})^{\beta} + u_{i,t+2}. \tag{3.20}$$

If the above conjecture is correct, the regression results should be close to the previous results. Table 3.6 presents the regression results and it indeed shows the conjecture is valid.

The intuition for Eq.(3.20) is that only when the natural rate of structure depreciation is equal to the rate of land capital losses, then the natural rate of structure depreciation equals the natural rate of property depreciation (the depreciation rate that would occur for a particular property in the absence of maintenance).

Table 3.6: Drop regressor

	Pri. Coeff.	Std. Err.	P-values
a	0.829	0.010	0.000
k	27.604	4.987	0.000
β	0.256	0.092	0.006
# of Obs.	377		
R-square	0.968		

	Model Coeff.
δ	0.090
α	14.45
β	0.256

3.5.4 An extended model

In this section, an extended version of this paper's model is provided. The post-financial crisis years illustrate a slow recovery from the economic shock of the 2008 financial crisis. By assuming technology related to housing construction and maintenance remains the same across survey years pre- and post- the financial crisis, time dummies can be introduced to incorporate various years into our model. This extension can also be utilized to examine the robustness of the estimations based on our approach. By incorporating multiple neighboring survey years into our model, it can also increase the sample size and decrease the sampling noise. In the following example, we add the 2005 and 2007 survey years into our original data set. This yields a total of four survey years (2005, 2007, 2011, and 2013) representing years both pre- and post- financial crisis. We expect that housing maintenance technology in the period from 2011 to 2013, will remain almost unchanged from the 2005 to 2007 period. Our regression model specified by Eq. (3.16) requires the the same maintenance technology across our observations. It is worth mentioning that the identical housing construction technology is also required. In our model, housing capital gains are reflected by the capital gains on land. This assumption, no capital gains on housing structure across these years, is primarily based on the reasoning that non-reproducible land is the most sensitive as well as the predominant component contributing to real estate capital gains, however the accuracy of this assumption is predicated on the condition that construction technology (with its quality-controlled real construction cost) stays the same across the years we study. In this way, by inserting two time dummies into Eq.(3.16), we have:

$$\frac{HV_{i,t+2}}{f_i} = a\frac{HV_{it}}{f_i} + D_1 b_1 \frac{NLS_i}{f_i} + D_2 b_2 \frac{NLS_i}{f_i} + k(m_{it})^{\beta} + u_{i,t+2}, \tag{3.21}$$

where $b_1 = ((1 + c_{2005})^2 - (1 - \delta)^2)p_{2005}$ and $b_2 = ((1 + c_{2011})^2 - (1 - \delta)^2)p_{2011}$; D_1 indicates the observation is from the year 2005 while D_2 indicates the observation is from the year 2011. Price level is set to the year of 2011.

We employ the same data cleaning and filtering procedures as described in Section 3.3 on the 2005 and 2007 survey years and then append the dataset to our prepared dataset from the 2011 and 2013 survey years. Table 3.7 reports the regression results.

Table 3.7: Estimation results from the extended model

	Pri. Coef.	Std. Err.	P-values
a	0.827	0.013	0.000
b_1	60.292	22.444	0.007
b_2	-13.356	21.149	0.528
k	30.944	5.442	0.000
β	0.268	0.074	0.000
# of Obs.	599		
R-square	0.953		

	Model Coef.
δ	0.090
α	16.20
β	0.268

The regression results support our previous estimates in Section 3.4. The estimation in Section 3.4 is consistent with the that of the natural rate of housing structure depreciation, elasticity of maintenance expenditure, and the shift-scaling parameter of the maintenance function derived from the regression analysis of this extended model. More importantly, from the historical observations around the financial crisis, from the year 2005 to 2007 (which are the years pre-financial crisis), land has positive capital gains; this means that the primary coefficient b_1 in Eq.(3.21) should be positive according to our model, which is indeed supported by the estimation results of b_1 in Table 3.7. The significance test on b_2 in Eq.(3.21) is consistent with that in Section 3.4. As discussed in subsection 3.5.3, one conjecture to explain this fact is that land capital gains from 2011 to 2013 (which are the years post-financial crisis) in the surrounding region of the New York metropolitan are negative and the rate of capital loss on land is roughly equal to the rate of structure depreciation.

In the extended model we propose, there could be issues of reverse causality. Some houses in the sample of the 2011 and 2013 survey years come from that of the 2005 and 2007 survey years. The value of such a house in 2007 may have had effects on the house value in 2011 and the maintenance expense in 2011. We hereby exclude the 'repeat observations' in the 2005 and 2007 survey years to avoid this possible reverse causality issue; no house in the 2005 and 2007 survey year appears in the 2011 and 2013 survey years in our sample. We report the estimation results in Table 3.8. The regression results are similar and reflect the robustness of our model.

Table 3.8: Reverse causality

	Pri. Coef.	Std. Err.	P-values
a	0.830	0.015	0.000
b_1	52.002	24.404	0.034
b_2	-17.549	24.480	0.474
k	33.973	6.274	0.000
β	0.237	0.079	0.003
# of Obs.	479		
R-square	0.955		

	Model Coef.
δ	0.089
α	17.78
β	0.237

3.6 Conclusion

In this paper, we study the natural depreciation rate of housing structure value, which has been overlooked in the previous literature. We elucidate a method to estimate this exogenous structure depreciation rate by separating the effect from housing capital gains and housing maintenance improvement from changes in housing property value. Using data from the biennial American Housing Survey for the survey years surrounding the 2008 financial crisis (2005, 2007, 2011 and 2013), the natural rate of structure depreciation is estimated to be roughly 9 percent per year in the New York metropolitan area. It is important to mention that the natural rate of structure depreciation that we study is the exogenous depreciation rate that would occur on the structure value in the absence of maintenance; it is different from the depreciation rate of property value since housing property value also subsumes non-depreciable land. As anticipated, this exogenous depreciation rate of the housing structure itself that we estimate is higher than the rate of property depreciation or the net depreciation rate of the housing structure under the effect of maintenance improvement, which are often estimated in the previous literature studying housing depreciation. This rate of depreciation might vary across regions depending on the construction techniques and building materials employed for various types of housing, reflecting differences in weather, climate, and/or geologic conditions. A more accurate estimation of the rate of structure depreciation which takes into account the principal factors influencing housing value allows for more effective implementation of housing policy and lets real estate investors make more informed investment decisions.

3.7 Appendix

Residual analysis (kernel density)

The proposed model is assumed to capture all the principal economic determinants and explain the economic regularity of housing structure depreciation. Therefore, the error term (or the residual) is assumed to be random noise with a mean of zero. For each individual observation, the value of the dependent variable (property value per sqft) is determined not only by the principal economic factors specified in our model, but is also influenced by stochastic factors not included in the model. The effects of these stochastic factors should tend to cancel out if they are truly random.

In regression models that include a constant intercept, the mean of residuals is inherently shifted to 0; therefore, it is unsurprising to find the mean of the error term to be 0 in such a model. Our model does not carry with it a constant intercept,[15] thus, the mean of residuals close to 0 would indicate a high level of model performance. Examining the base case regression result, we find a mean residual of \$0.787, compared to a mean value of \$251.212 for the dependent variable. Figure 3.1 below displays a kernel density plot of e, the error term in the units of standard deviation of property value per sqft from the base case regression.

[15] According to the theoretical derivation, it should not have a constant intercept.

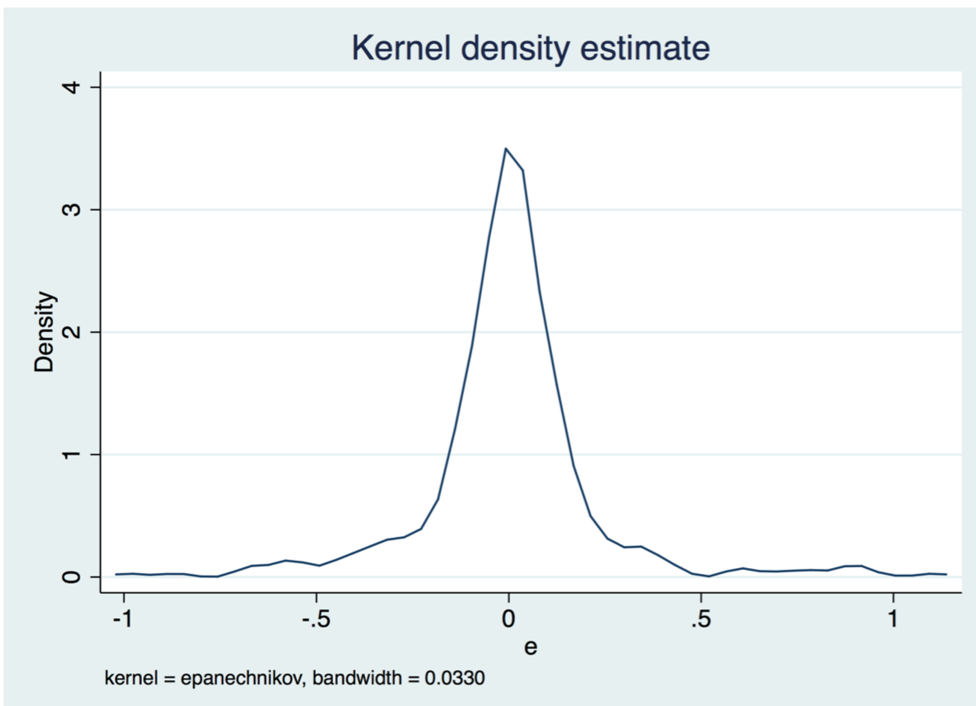

Figure 3.1: Residual analysis (kernel density)

Chapter 4

Housing Construction: A Preliminary Study

Abstract

The relationship between average residential construction costs per unit of floor area and the number of building stories has been noted in the past to take form as a U-shaped curve. Previous literature regarding general building costs have made note of this relationship, but have not provided detailed reasons for the shape of the curve. In order to further study this relationship, a construction cost function, which analyzes the construction costs of single-family detached houses with respect to the number of stories in each building, is proposed. In analyzing total construction costs per unit of built-on land area, this paper focuses on providing an economic explanation for the U-shaped average construction cost curve as well as its justification in single-family housing construction. This

function establishes two components of housing construction costs that will be described in further detail: fixed construction costs, which are independent of building height, and variable construction costs, which do depend on building height. The parameters of which are calibrated and estimated using RSMeans residential housing construction data.

4.1 Introduction

The average cost per square foot of building a home is undoubtedly dependent upon the number of stories constructed. Such a curve exists that relates these two variables. Past studies in housing and real estate construction have identified that this average construction cost curve follows a U-shaped path. In particular, the U-shaped nature of the curve signifies that, initially, the average construction cost per square foot of total floor area is higher for buildings with fewer stories, and lower for buildings with more stories. At a glance, these findings seem unnatural; intuitively speaking, it would seem that for the same type of housing, taller buildings should generally cost more to construct on average than shorter ones. According to Picken and Ilozor (2015), and with regards to the long history of construction economics, most scholars did not recognize the U-shaped housing construction cost curve until the 1970's, and had previously admitted that there was a linear relationship between the height of a building and its construction cost. One exception to this was Nisbet (1961), who, in the background of 1950's to 60's construction technology, identified that two-story buildings had the lowest average construction cost per unit of floor area among low-rise buildings. Furthermore, Picken and Ilozor (2015) also documented that Flanagan and Norman in 1978 questioned this linear model and suggested that a U-shaped average construction cost curve defined the relationship more appropriately. Recent studies by Blackman and Picken (2010) and by Picken and Ilozor (2003), using data from Shanghai and Hong Kong respectively, both confirm the U-shape curves. Needless to say, past studies have acknowledged the existence of the U-shaped average construction cost curve. These studies, while acknowledging the existence of the curve, do not provide detailed economic reasons for the cause of its shape. Since it is not intuitively clear as to why the average cost of construction would initially decrease as the complexity and difficulty in construction increases as building height increases; a further examination of the construction cost's total composition is required.

Previous studies on construction costs have placed particular focus on the average cost per unit of floor area. Since, at the time, most of the data available was reported in these terms, we suspect data availability was the cause of this focus. Using the concept of average cost does not come without its drawbacks. Most importantly, it does not establish the internal relationship between the various components that make up the totality of the construction cost. Additionally, it does not serve to identify the fundamental economic reasons for the trajectory of the U-shaped cost curve. In analyzing total construction costs per unit of land area, this paper introduces two components of construction costs with respect to a building's height (the number of stories): fixed construction costs and variable construction costs. Fixed construction costs refer to the costs, measured in units of land area, that are independent of the number of stories in a building, while variable construction costs capture the costs that depend on a building's height and number of stories. Furthermore, it will shed light on the underlying economic reasons for the trajectory of the U-shaped construction cost curve.

In the next section, we describe our model and method. In Section 4.3, we describe the RSMeans square foot construction cost data, our calibration, and estimation. In our calibration and estimation section, although the data size is relatively small and the information from these small samples is limited, we obtain the best estimate possible of the construction cost function using accessible data from the square foot cost hand book. Such data is gathered from statistical analyses of actual construction project costs. This data shortage stems from the fact that, given our budget, gaining access to large size micro-level construction cost data would have been prohibitively costly. This paper is concluded in Section 4.4.

4.2 A Simple Model

As previously noted, past studies have identified a U-shaped average construction cost curve when relating the cost per square foot of a building to the total number of stories. However, these studies did not provide in depth reasons for the shape of the curve. Intuitively speaking, as the number of stories in a building increases, its height, and therefore, the difficulty and danger involved in construction increases as well. Therefore, it would seem to follow that constructing buildings with more stories would incur a higher average cost, with costs becoming more extreme as the number of stories increases. Consequently, instead of the cost function following a U-shaped path, it should follow as an upward sloping curve, as a positive correlation between the average cost per square foot and the number of stories in the building would exist.

Following this logic, it is not intuitively clear as to why a building with few stories would have a higher cost per square foot than one with more. We hypothesize that this higher average cost stems from the fixed costs involved in construction. These costs are independent of the height of the building and are always present. These fixed costs are made up of tasks such as land preparation and grading, and are partially made up of foundational work, flooring, roofing, etc.

Based on the reasoning above, a construction cost function can be developed that takes into account both the fixed and variable construction costs, with the latter having a direct correlation to the height of the building. Let F denote the fixed construction cost per square foot of land. Let h denote the building height. As the height of the building increases, certain requirements, such as the cost of labor, rental equipment costs, bearing capacity, and cost of vertical transport, all increase. As a result, for simplicity, the marginal cost is assumed to be ch, where c is a constant coefficient.[1] The variable construction cost per square foot of land with respect to height h is then

$$\int_0^h chdh = \frac{1}{2}ch^2 \tag{4.1}$$

When the unit height per story is the same across each building, with the number of stories of a housing structure being denoted as n, we have building height $h = ns$, where s denotes the unit height per story.

[1] The linear marginal construction cost assumption is built on the reasoning that the marginal construction cost (measured from top to bottom) is proportional to the building height. It is certainly possible that another function form exists that also complements the data, e.g. an isoelastic marginal cost function which assumes a constant elasticity coefficient. In a more general sense, we can replace the function form with a more general polynomial regression. We discuss the estimation results from a higher order polynomial regression in Appendix I. Based on the current information, the function form proposed is optimal.

The total cost for an n story housing structure per square foot of built-on land area $C(n)$ is therefore equal to the variable construction cost per square foot of built-on land area and the fixed cost per square foot of built-on land area,

$$C(n) = an^2 + F \tag{4.2}$$

The average construction cost per unit of floor area is therefore $c(n) = an + F/n$, which is a u-shaped curve.

There is the possibility that some costs remain constant throughout the multitude of floors in a building. If that is the case, then the function can be written as, $C(n) = an^2 + bn + F$. Take, for example, the cost of installing a toilet on the second floor of a house as opposed to the first floor. At a glance, the cost of installing the toilets would seem to be constant. However, there are always higher vertical transportation costs associated with building on higher stories. Additionally, there are higher complementary costs to account for, such as longer water pipelines to the second floor, a higher level of seal requirement to avoid leakage, and the possibility that a leak on a higher floor is more serious of an issue than one on a lower floor.

4.3 Data, Calibration and Estimation

We first briefly describe the data we used to calibrate and estimate the simple model in the setting of residual houses we proposed in Section , followed by an analysis of our estimated model.

4.3.1 RSMeans data

RSMeans is a well known North American construction cost database used by professionals for various construction planning and estimation needs. RSMeans data is published every year and is stored in a relational database system; it is available online or in various cost estimation handbooks. We use the RSMeans square foot cost handbook (2017 and 2018) as our data source. In particular, we focus on the residential section of the handbook - the construction costs of single-family houses.[2] In the handbook, the core of the residential construction costs section features the following characteristics: Class of Construction, Type of Residence (Stories), Exterior Wall System and Living Areas.[3]

The relationship between the building costs per square foot and the number of stories is our area of focus. Understandably, the cost per square foot varies depending on the construction quality, as well as other factors such as the material of the exterior wall of the home. Furthermore, we observe an inverse relationship between the total area of a home and its cost per square foot. As the total living area of a building increases, the cost per square foot decreases, controlling for the number of stories, the material of the exterior wall, and category of the construction (as shown by data from RSMeans). Before this issue can be analyzed further, we must introduce the composition of building

[2]We used data from *Square Foot Costs with RSMeans Data 2017* and *Square Foot Costs with RSMeans Data 2018*. In both handbooks, the data was found in the "Base Cost per Square Foot of Living Area" tables located from pages 28-32. In addition, we used the RSMeans Historical Cost Indexes to adjust the 2017 costs to the 2018 level.

[3]The main component of the construction cost entails as following: there are four classes of construction: economy, average, custom, and luxury; Seven types of residence: 1 story, 1.5 story, 2 story, 2.5 story, 3 story, bi-level, and tri-level; four types of material based exterior wall systems and various levels of living areas. Besides the main component, the other additional details regarding residential construction costs (e.g. Basement costs per square foot of living area) are also included in the handbook. For a particular case of a residential unit (in a specific class, type, exterior wall system and living area), there is a corresponding adjustment for those extra additions.

construction costs in RSMeans. They are divided into the following cost sub-categories: site work, foundations, framing, exterior walls, roofing, interiors, specialties, mechanical, electrical, and most importantly, contractor's operational cost and profit.[4]

Operational costs and contractor profits contribute to the nature of the inverse relationship between construction costs per square foot and total square footage. There exists a phenomenon in economics called a volume discount, which refers to the price of a unit becoming lower when a larger quantity is purchased. Take, for example, a builder who is using the same materials to take on 2 construction projects. The first project is to construct a 2000 square foot house, and the second is to construct, one after another, two 1000 square foot houses. While the total floor area between both projects would be the same, it would take significantly longer to build two 1000 square foot houses than to build one 2000 square foot house. The time it takes, we say, is not primarily due to the extension of labor, but rather is due to the waiting period between each construction phase. For instance, when constructing the foundation of a building, concrete should be allowed to cure for a certain number of days. During this waiting period, it is difficult for a single team of contractors to find other comparable jobs, which results in a higher relative opportunity cost. In a perfectly competitive market, this opportunity cost is equal to the accounting profit.

Continuing on, assume that the actual construction cost per unit of built-on land area (from labor and material) is the same. When constructing a large house vs. a small house, the opportunity cost arising from the average waiting time allocated on each square foot is much smaller for the large house. Resultantly, we would observe a much smaller

[4]According to RSMeans, operational cost is referred to as overhead in the cost book.

accounting profit per square foot being made in the large house. In the meantime, other minor factors can cause the above phenomenon to occur (e.g., not as much wall space, window and outside doors per square foot are needed for a larger house). These together explain what we see in the data. Controlling for other factors, the larger the living area, the lower the construction cost per unit area.

In the following subsections, as an illustration, we present the data and method we used to study the relationship between the average construction cost per square foot and the number of stories for one particular case of residence. For this particular case, the type of residence is average, the exterior wall system is wood siding — wood frame, and the floor area is 2000 square feet.[5] We used both the 2017 and 2018 square foot cost books provided by RSMeans. Additionally, an assumption about the data must be made: from 2017 to 2018, the construction quality in the RSMeans data must stay consistent between the same types of residence (average). We use the RSMeans historical construction cost index to perform 'inflation' adjustment - adjusting construction costs measured in 2017 to 2018 levels. In the meantime, we use five types of residences: 1 story, 1.5 story, 2 story, 2.5 story, 3 story.[6] Together, we have 10 observations.

4.3.2 Parameter estimation and model fit

This section reports the estimation results based on the models proposed in Section 4.2. Recall $C(n)$ represents construction cost per square foot of land, which equals the construction cost per square foot multiplies the building stories.

[5]The square footage provided in the data varied based on the number of stories. In some stories, when the floor area provided was not exactly 2000 square feet, we imputed the cost by averaging the price per square foot between the two closest square footage values.

[6]'bi-level' and 'tri-level' are not adopted since they can not be quantified.

$$C(n) = an^2 + F \tag{4.3}$$

Table 4.1: Estimation results

	Coeff.	Std. Err.	P-values
a	31.39	1.558	0.000
F	96.031	8.305	0.000

$$C(n) = an^2 + bn + F \tag{4.4}$$

Table 4.2: Estimation results

	Coeff.	Std. Err.	P-values
a	4.372	4.798	0.392
b	110.42	19.400	0.001
F	-3.246	17.839	0.861

The estimation results in Table 4.2 are inconsistent with the fact that the fixed cost F is positive. Due to this inconsistency, we perform constrained regression, forcing the constant term to be positive. The estimation results are: $a = 5.194$, $b = 106.969$ and the fixed construction cost for the constant term be $5.382 * 10^{-137}$. Since it indicates the fixed costs to be a very small value (nearly zero), this function form is not supported by the information we have.

As shown in the tables above, based on the information we have, the functional form represented in Eq.(4.3) appears to be superior to that of Eq.(4.4). At the same time, we recognize that the power of significance test is limited due to the small sample size. However, the estimation in Eq.(4.3) is of more precision.

We must also consider potential inaccuracy issues resulting from having to define what a half-story is. While there are data reports on 1.5 and 2.5 story residences, it may not necessarily mean that the building costs are, quantitatively, 1 or 2 stories plus half of one of those stories. Potential variation in the layouts of these additional half stories could skew the data, as they are loosely defined. Additionally, the building technology may be intrinsically different from houses that are strictly 1, 2, or 3 stories in height, which could impact the construction cost. Therefore, in the following regression results, we eliminate the observations with 1.5 stories and 2.5 stories. The results are reported in the following two tables.

$$C(n) = an^2 + F \tag{4.5}$$

Table 4.3: Estimation results

	Coeff.	Std. Err.	P-values
a	31.225	1.767	0.000
F	90.148	10.097	0.000

$$C(n) = an^2 + bn + F \tag{4.6}$$

Table 4.4: Estimation results

	Coeff.	Std. Err.	P-values
a	8.371	5.115	0.200
b	93.319	20.674	0.020
F	10.160	18.206	0.616

Similarly, as shown in the above tables, the functional form represented in Eq.(4.5) appears to be superior to that of in Eq.(4.6), achieving higher accuracy.

4.4 Conclusion

This paper uncovers the reasons for the U-shaped relationship between average construction costs and building height. A construction cost function is developed to explain such a relationship, particularly with regards to the U-shaped trajectory of the curve. The method is illustrated using the RSMeans square foot construction cost data gathered from residential constructions. In analyzing these construction costs, this method identifies the fixed costs as well as the variable costs of construction with respect to the number of stories per building. Identifying these two components of the total cost ultimately confirms and explains the U-shaped average construction cost curve per unit of floor area.

4.5 Appendix

Appendix I

A set of polynomial regressions are performed in this section to check, based on the information we have, whether a better function form exists. The results indicate the function form we proposed in Eq.(4.3) is the most suitable with regards to the data we have.

$$C(n) = bn + F \tag{4.7}$$

Table 4.5: Estimation results

	Coeff.	Std. Err.	P-values
b	127.908	2.808	0.000
F	-18.548	5.958	0.014

Since fixed costs cannot be negative, we perform the constrained regression and force the fixed cost term to be positive. The regression coefficient b is 119.665 and the constant term is $e^{-3.04*10^9}$, a number which is infinitely close to 0. Therefore, the average construction cost per unit floor area is $c(n) = b$, inconsistent with the fact that the average construction cost depends on the number of stories in the building.

We have already discussed why we did not choose the function form of the second order polynomial regression $C(n) = an^2 + bn + F$ in Section 4.3. In the next, we will discuss the third order polynomial function form,

$$C(n) = cn^3 + an^2 + bn + F. \tag{4.8}$$

Table 4.6: Estimation results

	Coeff.	Std. Err.	P-values
c	-7.359	9.768	0.480
a	48.528	58.817	0.441
b	28.365	110.739	0.806
F	43.117	64.235	0.527

The average construction cost, based on the third order polynomial regression above, is

$$c(n) = -7.359n^2 + 48.528n + 28.365 + 43.117/n. \tag{4.9}$$

According to the function form above, when the number of stories in a building reaches 8, the average construction cost falls below 0. Therefore, this function form cannot exist. Apart from that, the estimated coefficients are not significant.

Appendix II

Table 4.7 shows estimation results for a 2,400 square foot home, taking into account the volume discount associated with constructing a larger home.[7]As expected, the

[7]Due to data availability, there were no reports of 3 story homes with 2400 square foot, so reports of 3 story homes with 2500 square foot were used instead

estimation of the fixed construction cost and the variable construction cost parameter both fall compared to the values in Table 4.1, which shows results for a 2,000 square foot home.

Table 4.7: Estimation results

	Coeff.	Std. Err.	P-values
a	29.611	1.462	0.000
F	88.518	7.795	0.000

Table 4.8 drops data values with 1.5 and 2.5 stories. This is due to the potential inaccuracy issues with 1.5 and 2.5 story buildings not being explicitly defined in RSmeans.

Table 4.8: Estimation results

	Coeff.	Std. Err.	P-values
a	29.287	1.728	0.000
F	84.392	9.875	0.001

Chapter 5

Conclusions

The contributions of this research can be summarized from two perspectives. The first deals simply with this research's contributions to the literature, while the second regards its melding of economic principles with algorithmic thinking.

This dissertation extends the literature on urban, housing, and real estate economics by: (a) laying out a structural model to account for the multi-dimensional, indivisible, durable, and depreciable nature of housing as a commodity; (b) enriching and refining housing filtering theory with concrete and measurable housing elements so that the model can be applied to practical policy analysis; (c) conceptualizing the scale-independent nature of housing quality and providing a quantifiable measurement on interior quality; and (d) investigating the issues of capital gains in land value, economic depreciation in housing structures, as well as estimating and calibrating for housing maintenance and construction technologies.

Chapter 2 extends and refines housing filtering theory in several ways. It distinguishes non-depreciable housing space with depreciable housing interior quality, thus emphasizing the indivisibility of housing services by incorporating a two-dimensional and nonlinear bid rent function. It provides a quantifiable measure of housing interior quality and demonstrates its scale-independent nature, as well as introducing endogenous land rent and incorporating internally-owned land into the economy in a way that permits steady-state analysis. It does this by assuming that the effective land (total residential construction area) grows at the same rate as population. To the best of my knowledge, it is the first structural model that incorporates two heterogeneous dimensions of housing characteristics in both the demand side and supply side of the housing market. The parametrized benchmark model is calibrated to Riverside CA. The model predicts that annual rent in Riverside ranges from $\$14,000$ to $\$22,000$, the average number of housing stories is 1.98, and the average building coverage ratio is 7.4. It captures the pre-existing intuition that the ratio of maintenance expenditure to the property value is lower for newly constructed housing than for an aged house, and also shows the range of these ratios among different interior qualities is consistent with the one percent maintenance expenditure rule in real estate economics. Moreover, it predicts that a positive demand shock increases the structural density and decreases the housing space, but it does not increase the rent and property value of a housing unit (although each housing unit's floor area becomes smaller). A positive income shock will increase the rent and the property value of a single housing unit at all levels of housing quality, and also increase the housing space and structural density of new residential construction. At all levels of quality, the total maintenance expenditure to income ratio remains constant at different steady states.

Chapter 3 studies the natural depreciation rate of housing structure value. A novel method is proposed to estimate this geometric depreciation rate by separating the effects from housing capital gains and housing maintenance improvement on changes in housing property value. By utilizing the plattage effect to impute the land value, and data from the biennial American Housing Survey for the survey years surrounding the 2008 financial crisis, the natural rate of structure depreciation is estimated to be roughly 9 percent per year in the New York metropolitan area.

Chapter 4 uncovers the economic reasons for the U-shaped relationship between average construction costs and building height. From the perspective of total construction cost per unit of land area, chapter 4 introduces the fixed costs as well as the variable costs of construction with respect to building stories into the model and calibrates the parameters using RSMeans Residential construction data. Identifying these two components of the total cost ultimately explains the U-shaped average construction cost curve per unit of floor area.

In an era of tremendous progress in computing technology, this dissertation shows that economists can utilize the tools from Electrical Engineering and Computer Science (EECS) in their research. As demonstrated in chapter 2, the model is solved by self-developed algorithms. Computer algorithms (e.g., bisection search) are utilized, however, it is not a pure computer algorithm; it is important to note that the model is not solved by inscrutable EECS algorithms without any economic foundation. Rather, the algorithm is designed through the application of economics principles and intuitions. It is economic theory that drives the design of the algorithm which, as demonstrated in chapter 2, is a powerful methodology in model development and solving. Perhaps, a subject like 'Algorithmic Economics' may well be popular in the future.

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