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Decision Analysis with Geographically Varying Outcomes: Preference Models and Illustrative Applications

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This paper presents decision analysis methodology for decisions based on data from geographic information systems. The consequences of a decision alternative are modeled as distributions of outcomes across a geographic region. We discuss conditions that may conform with the decision maker's preferences over a specified set of alternatives; then we present specific forms for value or utility functions that are implied by these conditions. Decisions in which there is certainty about the consequences resulting from each alternative are considered first; then probabilistic uncertainty about the consequences is included as an extension. The methodology is applied to two hypothetical urban planning decisions involving water use and temperature reduction in regional urban development, and fire coverage across a city. These examples illustrate the applicability of the approach and the insights that can be gained from using it.

Subject classifications: decision analysis; geographic information systems; multiattribute utility; multiattribute value; additive independence; preferential independence; homogeneity.
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1. Introduction

This paper discusses decision analysis methodology that explicitly accounts for outcomes distributed across a geographic region. As an example, consider a situation where regional planners are considering alternative development plans that could have varying environmental or socioeconomic impacts across a region. To support their policy and planning decisions, planners can consider multiple geographic maps showing current and potential future levels of environmental pollutants, urban development, water availability, air temperature, etc., that could vary across the region in different ways depending on which alternative is implemented.

The outcomes of selecting different alternatives in decisions supported by geographic information can be described in terms of one or more *attributes* (e.g., maximum outdoor air temperature in July) whose *levels* (e.g., 38°C) are known, or can be estimated with some uncertainty over a region. The levels of these attributes may depend on both the alternative that is selected and the geographic location within the region of interest.

As an example, the decision of whether or not to develop a proposed large new landscaped park could affect the maximum July outdoor air temperature in the region near the proposed park. The maximum July temperature at any specific location might differ depending on the direction and distance of that location from the proposed park. We will return to this park example throughout the paper to illustrate analysis concepts that we present.

For this paper, we define a *consequence* as a function that assigns levels to each attribute for an alternative and whose domain is the geographic region of interest. (In the geographic information systems (GIS) context, the assignment of attribute levels could be done in an automated manner by the GIS software.)

Thus, to judge the relative desirability of the alternatives, the decision maker must combine the geographically varying attribute levels for each alternative. The question we address is, when each alternative can be represented by one or more maps showing attribute levels over a geographic region that would result from selecting that alternative, how can a decision maker determine the relative desirability of each alternative?

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This paper discusses preference models for a decision analysis approach to such decisions. We use the term *preference model* to designate (i) conditions on decision maker preferences for the potential consequences from a specified set of alternatives as described by the attributes and (ii) the possible forms of functions that will obey those conditions which can be used to evaluate alternatives. Such functions are called *preference functions*. Decisions in which there is certainty about the consequences resulting from each alternative are considered first, and then decisions with probabilistic uncertainty about the consequences are considered. (The resulting preference functions are called *value functions* for decisions under certainty or *utility functions* for decisions under uncertainty.) We describe procedures to assess value functions of the type presented in this paper and illustrate the use of these preference models with hypothetical applications in two potential problem domains: water use and temperature reduction tradeoffs in regional urban development and fire coverage across a city.

2. Applications of Geographic Information Systems

The use of maps generated by computer-based geographic information systems has become widespread due to increasing computer capabilities and decreasing costs (Obermeyer and Pinto 2007), and such analysis is the basis for a large stream of literature. GIS analyses have been applied, for example, to irrigation and water resource management (Knox and Weatherfield 1999), wildlife habitat selection in Alaska (Pendleton et al. 1998), and deforestation (Kohlin and Parks 2001). Arbia (1993) provides a detailed overview of GIS, including consideration of sampling and modeling errors, but not analysis of uncertainty over consequences or consideration of preferences.

The lack of explicit decision analysis methods is a limitation in most previous GIS work that hinders its use for comparing alternatives or designing new alternatives that would have equivalent or higher value. There has been limited application of preference models to address decisions using GIS data. Instead, most GIS research has focused on statistical approaches for analyzing the data (Bond and Devine 1991) or improved methods to display data to stakeholders (Koller et al. 1995, Slocum et al. 2001). Worrall and Bond (1997) explore some of the reasons GIS tools have yielded fewer benefits than expected in the public sector; one of these reasons is a lack of effective decision support systems. As we will demonstrate, adding a decision analysis component to the analysis of geographic information can increase the power of GIS tools to support policy decision making.

A small subset of the GIS literature addresses decision making. De Silva and Eglese (2000) discuss the development of a decision support system that connects GIS data to a simulation model for evacuations. Malczewski (1999) and Jankowski (1995, 2006) provide more detailed analysis of multicriteria decisions using GIS data. Chan (2005) examines the use of multicriteria decision making in a broad context of spatial applications. However, the decisions considered by these authors do not directly involve preferences over consequences. Keisler and Sundell (1997) use a somewhat different approach for a park planning problem, by modeling multiattribute utility over aggregated attribute levels within a geographic region. These aggregated levels are affected by the decision maker's choice of where the boundary of the region is drawn. In contrast, we consider preference models that directly address attributes whose levels may vary across a geographic region.

3. Preference Models for Spatial Decisions Under Certainty

This section presents four preference models for geographically oriented decision making where potential consequences of alternatives are known with certainty. Two of the preference models are *discrete*, meaning that there are a finite number of subregions and the attribute levels do not vary within any specified subregion (but may be different in different subregions), and two are *nondiscrete*, meaning that the region cannot be divided into subregions within which the attribute levels do not vary. For both discrete and nondiscrete situations, we present both a single-attribute and a multiattribute preference model, so there are a total of four different models.

Keeney and Raiffa (1976) present and demonstrate the use of preference models over a discrete number of attributes. Such models are now widely used to determine the relative desirability of multiple-attribute alternatives, where each alternative takes on a specific level on each of the multiple attributes $Z_1, Z_2, \dots, Z_n$, and z_i designates a specific level of Z_i. (Keeney and Raiffa denote the attributes and levels as X_i and x_i, respectively, but we use Z_i and z_i to avoid confusion with the use of x as a spatial dimension later in this paper.) For example, a decision related to locating a new store branch might have attributes such as the distance from the closest competitor, distance from the warehouse, and population size within a five-mile radius. For decisions with no uncertainty, preferences over the consequences of these alternatives are represented by a *multiattribute value function*, $V(z_1, z_2, \dots, z_n)$. See Debreu (1954, 1964), Fishburn (1970), and Krantz et al. (1971) for expositions of the relevant preference theory. Keeney and Raiffa (1976), Keeney (1992), and Kirkwood (1997) have more elementary presentations. See Keefer et al. (2004) for a review of decision analysis applications.

In the discrete preference models in this paper, the region is partitioned into m discrete subregions, labeled $1, \dots, m$, such that the level of an attribute does not vary within any specified subregion. In the nondiscrete preference models in this paper, the level of an attribute depends on the geographic coordinates x and y that designate a location within the region. In the single-attribute models, the consequences

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are described by functions that assign a single number (the attribute level) to each subregion (for the discrete case) or location (for the nondiscrete case), and in the multiattribute models, the consequences are described by functions that assign a vector of attribute levels to each subregion (for the discrete case) or location (for the nondiscrete case). In both the discrete and nondiscrete cases, the level or levels that are assigned will depend on the alternative that is selected.

3.1. Spatial Decisions with a Single Attribute

3.1.1. Discrete Subregions Case. We first consider the case of one attribute with discrete subregions, and we assume there is a single attribute Z whose domain is a closed interval I, where z designates a specific level of Z. The region is partitioned into m subregions, and the level of Z does not vary within any specified subregion. We designate the level of Z in subregion i by z_i; thus a consequence can be expressed as a vector of levels across all subregions $z = (z_1, \ldots, z_m)$ such that $z_i \in I$ for $i = 1, \ldots, m$.

A relatively simple value function to represent preferences for the possible consequences of selecting different alternatives exists if the preferences meet six conditions. Four of these conditions (*completeness*, *transitivity*, *continuity*, and *dependence on each subregion*) will always be met in practical decision making situations. The other two conditions, *pairwise spatial preferential independence* and *homogeneity*, are more restrictive, but will be reasonable for many decisions. Preferential independence is presented in more detail in Online Appendix A (available as supplemental material at http://dx.doi.org/10.1287/opre.2013.1217). All six conditions are specified explicitly in Online Appendix B.

Described informally, pairwise spatial preferential independence requires that value tradeoffs between the attribute levels in any pair of subregions do not depend on the attribute levels in the other subregions so long as the levels in the other subregions are common; that is, if a decision is between two alternatives that only differ on Z in two subregions, then the preferred alternative will not change if the alternatives are modified by changing the level of Z in the other subregions so long as those levels in the other subregions are the same for the two alternatives. As an example, suppose for the park decision discussed above that two alternatives will result in the same temperature outcomes for all except two specified subregions, and one alternative will give temperatures of 35°C and 45°C in those two subregions, whereas the other alternative will give temperatures of 38°C and 42°C in the two subregions. Suppose the second alternative is preferred to the first. If pairwise spatial preferential independence holds, then this will continue to be true even if the temperatures in the other subregions change, so long as the temperature in each of the other subregions remains the same for the two alternatives. This condition implies that this property will hold for any two subregions. (Situations in which it can fail to hold generally involve decision maker preferences that consider complex interactions between subregions.)

The homogeneity condition requires, stated informally, that relative preferences for different levels of Z within any subregion will be the same for every subregion. More specifically, the homogeneity condition holds if the same *tradeoff midvalue* is found in *every* subregion. The detailed definition of a tradeoff midvalue is in Online Appendix B, and the concept can be illustrated with the park decision for a desert area example: Suppose a decision maker would be willing to give up two degrees, from 43°C to 45°C, in increased maximum temperature in all of the other subregions to decrease the maximum temperature in the specified subregion from 48°C to 40°C, and similarly would be willing to give up two degrees, from 43°C to 45°C, in increased maximum temperature in all the other subregions to decrease the maximum temperature in the specified subregion from 40°C to 36°C. Because the decision maker is willing to incur *the same* losses to improve from 48°C to 40°C as to improve from 40°C to 36°C in the specified subregion, 40°C is a tradeoff midvalue for the interval from 36°C and 48°C in that subregion. The homogeneity condition holds if 40°C is also a tradeoff midvalue between 36°C and 48°C in *every* subregion. (Situations in which this condition can fail to hold generally involve an attribute for which the decision maker believes different levels have qualitatively different implications in different subregions.)

When these two conditions, along with the other four specified in Online Appendix B, hold, then the following theorem specifies the form of the value function. The discussion of why this result holds is in Online Appendix B.

Theorem 1. *For a region with three or more subregions and a preference relation* $\succsim$ *("at least as preferred as") on the set of consequences* $(z_1, \ldots, z_m)$ *such that* $z_i \in I$ *for* $i = 1, \ldots, m$ *for a closed interval* I, *there exists a value function of the form*

$$V(z_1, z_2, \ldots, z_m) = \sum_{i=1}^{m} a_i v(z_i), \tag{1}$$

if and only if the conditions given in Online Appendix B for this theorem hold, where $v(z_i)$ *is a (common) single-attribute continuous value function over* z_i, *and* a_i *is a positive weight on subregion* i.

Thus, when the required conditions hold, it is only necessary to assess one single-attribute value function and a weighting constant for each subregion. This decomposition has intuitive appeal because it separates the preferences over the attribute levels, which are addressed in $v(z_i)$, from the priority or weighting assigned to each subregion, which is encoded in a_i. In some cases subregions will be weighted equally, and then all a_i can be set equal to one so the weights can be eliminated from the equation. For the analogous context of decisions with a stream of outcomes over time, the form of Equation (1) has been applied previously

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with the a_i's being interpreted as time discounting weights (Harvey 1986, 1995). Krantz et al. (1971, pp. 303–305) and Harvey (1986, 1995) show conditions for the existence of a value function of the form of (1), following up on a question raised about this by Fishburn (1970, p. 93).

The conditions required for (1) may seem restrictive for some decision situations with geographically varying consequences. However, the resulting value function given by (1) is more general than most summary metrics typically used in GIS analysis. Those summary metrics are often averages of the attribute levels, which are special cases of the models we present. For example, to specify conditions that imply averages can be used in place of the more general formula in (1) requires including a stronger condition that the tradeoff midvalue of any interval will be the average of the low and high levels in the interval (see Online Appendix B for details). For the park example discussed above, this would mean that the midvalue between any two temperatures will be the average of the two temperatures. For example, the midvalue between 5°C and 10°C will be $(5+10)/2 = 7.5$°C, and similarly the midvalue between 25°C and 30°C will be 27.5°C, and between 40°C and 45°C will be 42.5°C. In this case, as described in Online Appendix B, preferences over the set of consequences can be represented by

$$V(z_1, z_2, \ldots, z_m) = \sum_{i=1}^{m} a_i z_i \tag{2}$$

for some set of weights a_i. If each $a_i = 1/m$, the value function computes the simple (unweighted) average across subregions. A similar condition can be constructed for the other preference models presented below, resulting in analogous linear special cases. Although this may be reasonable in some decision situations, (1) allows for more general preference models.

3.1.2. Nondiscrete Case. In GIS applications with a large number of subregions, a useful modeling approach could be to consider the data to vary in a nondiscrete manner across the region. For example, suppose an application addresses land use policy making for an urban area with 100,000 land parcels. The characteristics of each parcel will vary from the parcel next to it, but not in an extreme manner, except possibly at boundaries between land use categories. In this setting, a nondiscrete model could be easier to analyze than a discrete model with 100,000 subregions. In §5.2, we provide such an example.

A nondiscrete model can be specified as follows: Assume $\succsim$ is a preference relation ("at least as preferred as") over the set of consequences, where the consequences are defined in a nondiscrete manner on locations rather than on discrete subregions. With this assumption, a consequence can be expressed as a function $z(x, y)$ that determines the level z at each location (x, y) in the region of interest. Thus, potential consequences are in the set of $z(x, y)$ such that $z(x, y) \in I$ for a closed interval I for all locations (x, y) within the region. With this formulation, it seems reasonable that an analogous result to Theorem 1 could be developed to provide a specific form for the value function. In the absence of discrete subregions, it is reasonable that the sum in (1) would be replaced with an integral over the region.

We provide the following conjecture for this situation.

Conjecture 1. *A preference relation $\succsim$ on the set of consequences such that $z(x, y) \in I$ for a closed interval I for locations (x, y) within a region of interest A can be represented by a value function of the form*

$$V(z) = \int\int_A a(x, y) v[z(x, y)]\, dx\, dy, \tag{3}$$

where a and v are bounded and continuous almost everywhere if and only if the conditions given in Online Appendix C for this conjecture hold, where x and y are coordinates within the region, v is a (common) single-attribute continuous value function over z, and $a(x, y)$ is a weight for location (x, y), which is positive almost everywhere.

This can also be generalized from x and y to any number of indices for the consequences. We thank an editor for pointing this out and noting that practical examples of this include situations where time could be a third index, or the spatial index of interest might be one-dimensional, such as locations along a road or a river. In addition, although we interpret $a(x, y)$ as a weighting function, it could also be viewed more broadly as a generating function for a transform of $v[z(x, y)]$.

Details of how (3) might be developed from preference conditions analogous to the conditions in Theorem 1 are in Online Appendix C, though we do not have a proof of the exact conditions required. Key requirements are nondiscrete analogs of pairwise spatial preferential independence and homogeneity, plus "reasonable" behavior for preferences as the location varies. Stated informally, reasonable behavior means there should not be large abrupt changes in characteristics that impact preferences as x and y vary over small intervals. For example, in the park example, land use should not change back and forth every few feet across the region. (Discontinuities along continuous curves, such as occur at a park boundary, do not cause problems.) It is unlikely that such challenges will arise in practical situations where a nondiscrete preference model might be considered.

3.2. Spatial Decisions with Multiple Attributes

3.2.1. Discrete Subregions Case. Thus far, we have considered only a single attribute defined across a region. Some decisions will address multiple attributes, one or more of which can vary geographically. For example, a park planning decision in an arid region might require consideration of both the maximum July temperature and

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the groundwater level in different subregions. Incorporating multiple attributes is a conceptually straightforward extension provided that the appropriate preference conditions hold. We first consider the situation where there are m subregions, and within each subregion the levels for the attributes do not vary. Let n designate the number of attributes, and let Z_{ij} designate the jth attribute in the ith subregion, where z_{ij} stands for a specific level of that attribute in that subregion, with $z_{ij} \in I^j$ for all i, where I^j is the closed interval domain of the jth attribute. We refer to Z_{ij} as an "attribute–subregion combination." Let $\mathbf{Z}$ designate the set of consequences, where a consequence $\mathbf{z} \in \mathbf{Z}$ specifies z_{ij} for all $m \times n$ attribute–subregion combinations. As above, let $\succsim$ be a preference relation on the set of consequences.

We now present an analogous theorem to Theorem 1 for the case of multiple attributes. The conditions needed for this theorem to hold are similar to those for Theorem 1, except that pairwise preferential independence must hold both across subregions and across the multiple attributes.

Theorem 2. *For a preference relation $\succsim$ on the set $\mathbf{Z}$ of consequences over a region such that $z_{ij} \in I^j$ for all subregions i and attributes j, for closed intervals I^j, there exists a multiattribute value function of the form*

$$V(\mathbf{z}) = \sum_{i=1}^{m} a_i \sum_{j=1}^{n} b_j v_j(z_{ij}), \tag{4}$$

if and only if the conditions given in Online Appendix D for this theorem hold, where $m \geqslant 2$, $n \geqslant 2$, a_i is a positive weight for subregion i, b_j is a positive weight for the jth attribute, and $v_j(z_{ij})$ is a single-attribute continuous value function over the level of the jth attribute in subregion i. (Note that v depends only on the attribute index j.)

This result follows from applying results by Gorman (1968) in combination with homogeneity concepts similar to those studied by Harvey (1986, 1995). A more detailed discussion of this theorem is included in Online Appendix D. We present a two-attribute, 10-subregion example of the use of (4) for an urban development decision in §5.1.

3.2.2. Nondiscrete Case. The following Conjecture 2 extends Theorem 2 to the situation where the (multiattribute) consequences are functions of locations rather than discrete subregions. Thus, this conjecture extends Theorem 2 analogously to the way that Conjecture 1 extends Theorem 1. As in §3.1.2, we conjecture that preferences in such a situation that satisfy a set of conditions can be represented by a value function.

Let $Z^j(x, y)$ designate the jth attribute at location (x, y), and let $z^j(x, y)$ designate the level of $Z^j(x, y)$. Let $\mathbf{Z}$ designate the set of consequences, and let $\mathbf{z}$ designate a specified consequence, where $\mathbf{z}(x, y) = [z^1(x, y), \ldots, z^n(x, y)]$, and $z^j(x, y) \in I^j$ for closed intervals I^j for all j, for all (x, y) within the region of interest. As previously, let $\succsim$ be a preference relation on the set of consequences.

We conjecture the following.

Conjecture 2. *A preference relation $\succsim$ on the set $\mathbf{Z}$ of consequences such that $z^j(x, y) \in I^j$ for all attributes j and locations (x, y) within a region of interest A for closed intervals I^j can be represented by a value function of the form*

$$V(\mathbf{z}) = \int\int_A a(x, y) \sum_{j=1}^{n} b_j v_j[z^j(x, y)]\, dx\, dy, \tag{5}$$

where $a, v_1, \ldots, v_n$ are bounded and continuous almost everywhere if and only if the conditions given in Online Appendix E for this conjecture hold, where x and y are coordinates within the region, v_j is a single-attribute continuous value function for the jth attribute, $a(x, y)$ is a weight for location (x, y), which is positive almost everywhere, and b_j is a positive weight for the jth attribute.

As with Conjecture 1, we have not been able to prove this result, but it should be clear in practical decision situations whether this is a reasonable model. Further details are in Online Appendix E.

4. Value Model Assessment Procedures

This section summarizes approaches for assessing single-attribute value functions, weights, and preference conditions for (1), (3), (4), and (5). Following the usual convention, we assume without loss of generality that (1) single-attribute value functions are scaled so the most preferred level of each attribute that is being considered has a value of one, and the least preferred level that is being considered has a value of zero, and (2) the weights are scaled to sum to one in (1) and (4), or integrate to one in (3) and (5).

4.1. Assessing Single-Attribute Value Functions

Standard procedures can be used to assess the single-attribute value functions in (1), (3), (4), and (5) (see, for example, Keeney and Raiffa 1976, §3.7.2; Kirkwood 1997, p. 240, Step 3.) Often value functions will increase or decrease monotonically over levels of the attribute, such as value functions for median family incomes or levels of pollution. (If a value function is not monotonic, it can be possible to redefine the attribute as the distance from an "ideal point" level, in which case the redefined attribute will be monotonic.) With monotonic preferences, single-attribute value functions can be assessed using the midvalue splitting approach, using the concept of the tradeoff midvalue described in §3.1.1. For example, suppose a value function is being assessed for profit in subregion i, ranging from \$0 to \$100,000, with higher profits being preferred. The value function is scaled by setting $v(\$0) = 0$ and $v(\$100{,}000) = 1$. Suppose the tradeoff midvalue of [\$0, \$100,000] is determined to be $x = \$40{,}000$, i.e., the profit level x such that the decision maker would accept the same decreases in profit in the other subregions to improve profit in subregion i from \$0 to \$$x$ or from \$$x$ to \$100,000. Then it follows

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directly from (1) that $v(\$40{,}000) = 0.5$. The tradeoff midvalue of [\$0, \$40,000] or [\$40,000, \$100,000] could then be assessed, yielding attribute levels that have single-attribute values of 0.25 and 0.75, respectively, in (1). This procedure could be continued to assess as many specific values as desired. Alternatively, if the preference conditions hold so that a specific functional form is valid for the single-attribute value function, then the parameter(s) for the functional form can be determined to specify the value function. For example, an exponential form for the single-attribute value function is valid if the relative position of the tradeoff midvalue within any specified interval depends only on the length of the interval, and not on its location in the domain of the value function (Kirkwood and Sarin 1980). In this case, only a single parameter is needed to specify the value function, and that can be determined by assessing one tradeoff midvalue. This is illustrated by the examples in §5.

4.2. Assessing Weights

The value tradeoff method (Keeney and Raiffa 1976, §3.7.3; Eisenführ et al. 2010, §6.4.2) can be used to determine the subregion weights a_i in (1). First, have the decision maker consider m distinct hypothetical consequences consisting of the most preferred possible level z^* of the attribute in a subregion i, and the least preferred possible level z^0 of the attribute in all of the other $m-1$ subregions. So, the first hypothetical consequence has the best level in subregion 1, the second has the best level in subregion 2, etc. Then, have the decision maker determine which of these m consequences is most preferred, and let i^* represent the subregion in which the most preferred level of the attribute is achieved. Subregion i^* will have the highest weight, and can be considered the most important subregion to this decision maker. For each other subregion $i \neq i^*$, determine the attribute level z_i' such that a consequence consisting of z_i' in subregion i^* and z^0 in all other subregions is equally preferred to a consequence consisting of z^* in subregion i and z^0 in all other subregions. As stated above, we assume that in (1), $v(z^0) = 0$ and $v(z^*) = 1$. Thus, from the assessed indifference relationship between achieving z^* in subregion i or achieving z_i' in subregion i^*, it follows from direct substitution into (1) that $a_i = a_{i^*} v(z_i')$, $i = 1, \ldots, m$, $i \neq i^*$. Since the weights are assumed to sum to 1, $\sum_{i=1}^m a_i = 1$, and solving the resulting system of m equations yields the set of weights.

The value tradeoff method above can be adapted to determine the weighting function $a(x, y)$ for the nondiscrete case represented by (3), but modifications are needed because there are an uncountably infinite number of weights to be determined. We address this in §5.2 for a specific example.

Analogous procedures can be used to determine a_i, $i = 1, \ldots, m$, and b_j, $j = 1, \ldots, n$, in the discrete multiattribute case represented by (4). The form of (4) allows

Table 1. A pair of indifferent consequences used in determining a set of subregion weights a_1, a_2, a_3 for (4).

Subregion i ($m = 3$)	Attribute j ($n = 4$): Consequence 1: 1	2	3	4	Consequence 2: 1	2	3	4
$1 = i^*$	z_{21}'	—	—	—	—	—	—	—
2	—	—	—	—	z_{21}^*	—	—	—
3	—	—	—	—	—	—	—	—

Notes. Dashes indicate least-preferred attribute levels, and z_{21}^* represents having the most preferred level of attribute 1 in subregion 2. This indifference judgment results in the equation $b_1 a_2 = b_1 a_1 v_1(z_{21}')$.

the two sets of weighting constants to be determined separately, as follows. First determine the subregion weights a_i by considering hypothetical consequences that have the same levels for all attributes except one arbitrary but specified attribute j. Because of the form of (4), it does not matter which attribute j is used for the assessment procedure. Assume for these hypothetical consequences that the other attributes are at their least preferred levels, meaning that their single-attribute values are zero, and then (4) reduces to $v(\mathbf{z}) = b_j \sum_{i=1}^m a_i v_j(z_{ij})$. Since b_j is a constant, this equation has the same form as the single-attribute case represented by (1), and thus the same weight assessment procedure presented above for (1) can be applied to attribute j across the subregions to determine a set of equations $b_j a_i = b_j a_{i^*} v_j(z_{ij}')$, $i = 1, 2, \ldots, m$, $i \neq i^*$, where i^* is the most important subregion, and z_{ij}' is the level of attribute j in subregion i^* that generates the indifference relationship described above for the single-attribute case. The b_j cancels out of the equations, and combining these equations with $\sum_{i=1}^m a_i = 1$ gives a set of m equations that can be solved for the a_i just as was done in the single-attribute case. An example of such an indifference relationship is shown in Table 1, with $m = 3$, $n = 4$, $i^* = 1$, and $j = 1$.

A corresponding procedure can be applied to determine the attribute weights b_j by considering hypothetical consequences that have the same attribute levels in all subregions except one arbitrary but specified subregion i. As before, it does not matter which subregion is used for the assessment procedure, and we can assume the attribute levels for the other subregions to be their least preferred levels, so that Equation (4) reduces to $v(\mathbf{z}) = a_i \sum_{j=1}^n b_j v_j(x_{ij})$. In this case, a_i is constant, and thus the equation again has the same form as (1). Applying the same weight assessment procedure to subregion i across the attributes determines the set of equations $a_i b_j = a_i b_{j^*} v_{j^*}(z_{ij}')$, $j = 1, 2, \ldots, n$; $j \neq j^*$, where j^* is the most important attribute, and z_{ij}' is the level of attribute j^* in subregion i that generates the indifference relationship described above for the single-attribute case. The a_i cancels out of the equations, and combining these equations with $\sum_{j=1}^n b_j = 1$ gives a set

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Table 2. A pair of indifferent consequences used in determining a set of attribute weights b_1, b_2, b_3, b_4 for (4).

	Attribute j ($n=4$)							
	Consequence 1				Consequence 2			
Subregion i ($m=3$)	$1=j^*$	2	3	4	$1=j^*$	2	3	4
1	z'_{12}	—	—	—	—	z^*_{12}	—	—
2	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	—

Notes. Dashes indicate least-preferred attribute levels, and z^*_{12} represents having the most preferred level of attribute 2 in subregion 1. This indifference judgment results in the equation $a_1b_2 = a_1b_1v_1(z'_{12})$.

of n equations that can be solved for the b_j. An example of such an indifference relationship for attribute weights is shown in Table 2, with $m=3$, $n=4$, $j^*=1$, and $i=1$.

It could be useful to conduct this procedure using more than one attribute to check for consistency in the responses and to test whether the decision maker's preferences meet conditions for (4) to be valid. Similarly it could be useful to assess the a_i using more than one subregion.

4.3. Testing Preference Conditions

The specific conditions that must be checked depend on which single- or multiple-attribute preference model, in a discrete subregions case or a nondiscrete case, is being applied. As discussed in §3, the key conditions that must be checked are preferential independence and homogeneity. If there are multiple attributes, then preferential independence must be checked for attributes as well as across subregions. (In the online appendices, these conditions are (e) and (f) for Theorem 1 and the corresponding conditions for Theorem 2 and the two conjectures.)

For example, if (1) is to be used for the single-attribute and discrete subregions case, then pairwise spatial preferential independence can be checked by asking the decision maker whether changing the common level of the attribute in the other subregions would cause a preference reversal for alternatives that differ only with respect to a pair of subregions. Homogeneity can be checked by asking the decision maker whether the tradeoff midvalue for a specified interval is the same for different subregions. This was discussed for the park example in §3.1.1. Analogous checks can be made for the single-attribute nondiscrete case.

If the conditions for (1) hold, then the special case of the linear model given by (2) can be used for the single-attribute discrete subregions case if the tradeoff midvalue for any interval for a given attribute is equal to the average of the high and low levels of the interval. In the park example, this would imply, for example, that the tradeoff midvalue of the interval from 36°C to 48°C is $(36+48)/2 = 42$°C.

Similar procedures can be used to check the conditions required for the multiple-attribute models. Kirkwood (1997, pp. 239–240, Step 2) discusses testing for preferential independence in further detail.

5. Examples

This section presents two hypothetical examples that apply the preference models discussed above and shows insights that can be gained from using these models. The first example uses a two-attribute model with 10 discrete subregions, and the second one uses a single-attribute nondiscrete model. The analysis for these applications was conducted using Excel Solver, with some use of Visual Basic for Applications. As discussed in §2, the use of these preference models differs from the approaches in previous applications of GIS data in that decision maker preferences are explicitly specified over spatially varying attributes, rather than assessed at an aggregate level, such as the average attribute level over the region.

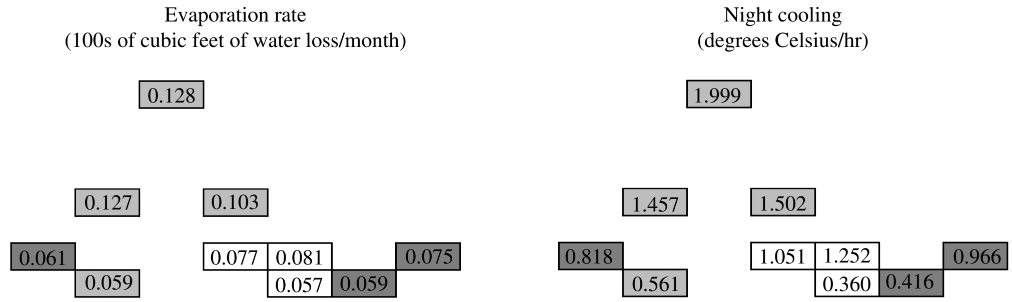
5.1. Water Use and Temperature Reduction in Regional Urban Development

Many decisions involving GIS data address multiple attributes. (For example, Keller et al. 2010 found multiple attributes used by stakeholders in Arizona water resource planning.) The application in this section illustrates the use of Theorem 2 from §3.2.1 in such decisions. The data used in this application come from Gober et al. (2010), who applied a heat flux model to investigate urban heat island effects in Phoenix, Arizona. Urban development has led to increased temperatures in Phoenix, mostly by reducing the amount of night cooling that occurs. As a result, there is motivation to increase the quantity of vegetation, because "green" areas acquire and retain less heat. However, this would also require more water, because green areas lose more to evaporation than developed urban areas. Thus, night cooling and evaporation rate are both important considerations when choosing development strategies.

Using the heat flux model, evaporation rate and night cooling results were estimated by Gober et al. (2010) for 10 different tracts of land with three different land use classifications (industrial, xeric, and mesic) in the greater Phoenix area using each of three potential development strategies for each tract (compact city, oasis city, and desert city). The current levels of evaporation rate and night cooling for the 10 tracts are shown in Figure 1. As shown in the figure, the 10 tracts included in the study are not contiguous. Figure 2 shows the changes that would result from applying each strategy to each tract, as projected by the model. The different shades of the tracts represent the current classifications: the darkest shade represents industrial tracts, the lightest shade xeric (desert vegetation) tracts, and the middle shade mesic (nondesert vegetation) tracts.

Given only the data shown in Figures 1 and 2, it is not clear which development strategy should be implemented

Figure 1. Current evaporation rate and night cooling for each of the 10 tracts.



Note. The darkest shade represents industrial tracts, the lightest shade xeric tracts, and the middle shade mesic tracts.

in each tract, since reductions in evaporation rate (which are desirable) are accompanied by increases in night temperature (which are undesirable), and the magnitudes of these effects vary from tract to tract. Thus, to defensibly choose the optimal development strategy, we should specify a value function to determine an overall value for different combinations of evaporation rate and night cooling across the 10 tracts. If the conditions for Theorem 2 hold, then to determine a value function we need to specify single-attribute value functions for evaporation rate and night cooling, as well as weights b_E and b_N for the two attributes and weights $a_i, i = 1, \dots, 10$, for each of the 10 tracts.

The conditions needed for a single-attribute value function to be exponential were described in §4.1, and these conditions seem reasonable for this example where the changes in evaporation rate and night cooling do not vary drastically among the alternatives. Based on Figures 1 and 2, we can determine that the smallest achievable evaporation rate (most preferred) is 0.047, and the largest (least preferred) is 0.160. Assume for illustrative purposes that the tradeoff midvalue for the specified range of evaporation rate assessed from the decision maker using the procedure in §4.1 is 0.116. We can also determine from the figures that the smallest achievable level of night cooling (least preferred) is 0.031, and the largest (most preferred) is 2.405; assume that 0.470 is the assessed tradeoff midvalue of this range. These assessed midvalues can be substituted into the exponential value function formula to find the single undetermined parameter for each of the two single-dimensional value functions. This yields the following exponential single-attribute value functions for evaporation rate and night cooling, where higher levels of evaporation are less desirable, whereas higher levels of night cooling are more desirable:

$$v_E(z_{iE}) = \frac{1 - e^{-0.905(1-((z_{iE}-0.047)/0.113))}}{1 - e^{-0.905}}, \tag{6a}$$

$$v_N(z_{iN}) = \frac{1 - e^{-3.35((z_{iN}-0.031)/2.374)}}{1 - e^{-3.35}}, \tag{6b}$$

where 0.905 and 3.35 are the exponential constant parameters, z_{iE} represents the evaporation rate in tract i, and z_{iN} represents the amount of night cooling in tract i. The functions are scaled to vary between zero and one over the ranges of possible levels for the two attributes, as graphed in Figure 3.

The weights can be determined using the value tradeoff approach presented in §4.2. To illustrate the procedure, we first consider the 10 subregion (tract) weights a_i, $i = 1, \dots, 10$. Applying the §4.2 procedure, the decision maker considers 10 hypothetical consequences consisting of the most preferred level of night cooling for each single tract in sequence combined with the least preferred level of night cooling in the other nine tracts; that is, one tract will have night cooling level of 2.405, and the other nine tracts will have a night cooling level of 0.031. The set of 10 evaporation rates is identical for each hypothetical consequence. For this example, assume all of these 10 hypothetical consequences are judged to be equally preferred by the decision maker. Then, using Equation (4), it must be true that all of the tract weights are equal, and if the weights for the 10 tracts are scaled to sum to 1, this results in a weight of $a_i = 0.10$ on each tract.

The procedure is analogous to determine the two attribute weights b_E and b_N. Consider two hypothetical consequences that differ only in a single tract i, one with the most preferred evaporation rate (0.047) combined with the least preferred night cooling level (0.031) in tract i, and the other with the least preferred evaporation rate (0.160) combined with the most preferred night cooling level (2.405) in tract i. Assume that the decision maker prefers the second of these hypothetical consequences. From Equation (4), this means that the weight for night cooling b_N must be greater than the weight for evaporation rate b_E.

Now consider a third hypothetical consequence that has the same (worst) level of the evaporation rate as the second (more preferred) hypothetical consequence specified above, but that has less night cooling, and is therefore less preferred. Adjust the level of night cooling until the decision maker is indifferent between the first hypothetical consequence and this third consequence. For example, suppose that indifference is achieved when the night cooling level is set to 0.735. Then, by substitution into Equation (4) and using Equations (6a) and (6b) to obtain $v_E(0.735) = 0.666$

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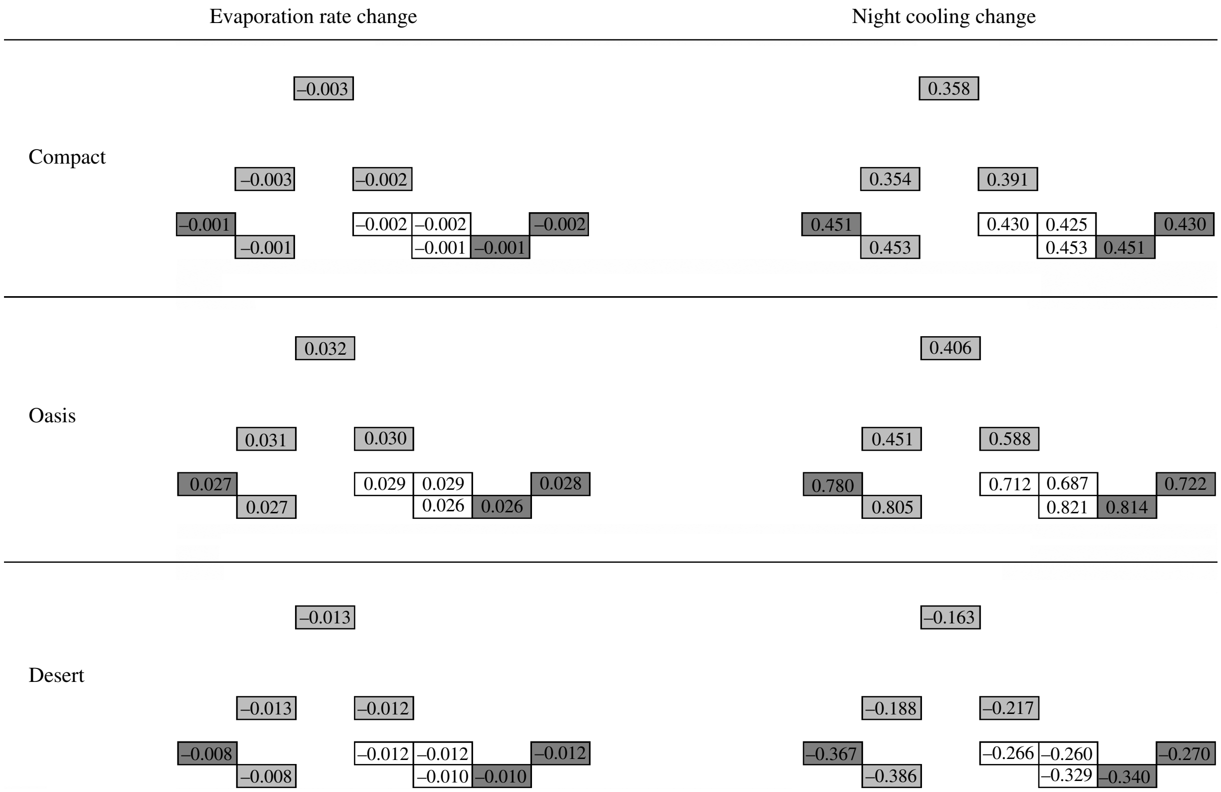
Figure 2. Changes in evaporation rate and night cooling that would result from implementing each of the three strategies in the 10 tracts.



Note. The different shades represent the current classification.

and $v_N(0.031) = 0$, and assuming the weights are scaled to sum to one, it follows that $b_E = 0.4$ and $b_N = 0.6$. Then the overall value function as given by Equation (4) in Theorem 2 is

$$V(z) = \sum_{i=1}^{10} 0.1(0.4v_E(z_{iE}) + 0.6v_N(z_{iN})), \tag{7}$$

and the resulting optimal development plan is shown in Figure 4, assuming there are no constraints on which development strategy can be applied to each tract.

Once this decision was framed using a multiattribute value function, analyzing it became more straightforward. We specified the single-attribute value functions over evaporation rate and night cooling, as well as weights on the two

Figure 3. The exponential value functions over evaporation rate and night cooling, with exponential constants $c = 0.905$ and $c = 3.35$, respectively.

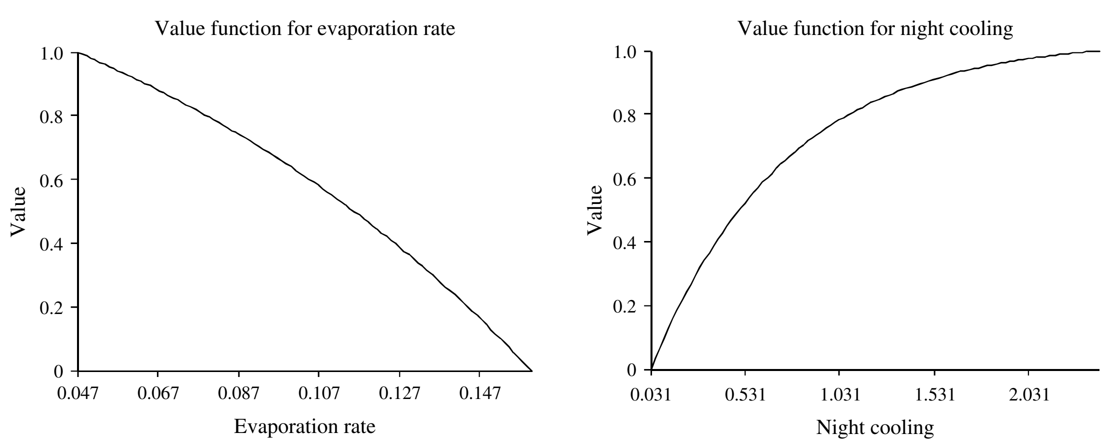


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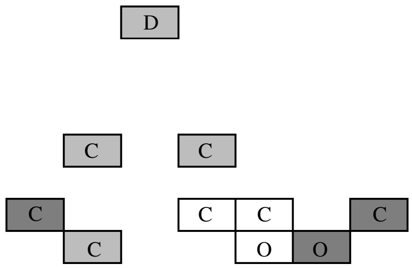
Figure 4. The optimal development plan using (7) and no cross-tract constraints.



Note. C, Compact; O, oasis; D, desert.

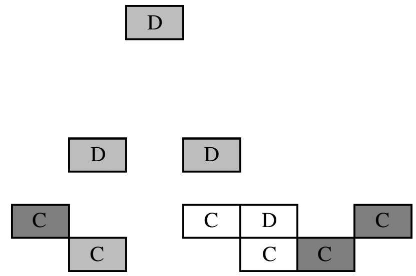
attributes and the 10 tracts. These clarified the geographic value structure, and then it was a straightforward calculation to determine the preferred decision for each tract.

With this formulation, the decision problem is effectively made up of 10 smaller decision problems, one for each tract, which can be solved independently. This is because, using Equation (7), the total value is the sum of the values for each tract, and there is no constraint across tracts. We can extend this model to incorporate cross-tract constraints on the development plans. For example, assume there are constraints on the average decrease in evaporation rate and the average increase in night cooling allowed across the tracts. Restrictions such as these can be included without altering the preference model, and will allow decision makers to consider "what if" questions about the impact of different constraints. For example, Figure 5 shows the optimal plan still using (7), but now requiring a minimum average decrease of 7% in evaporation rate and a minimum average increase of 12% in night cooling across the tracts. In this case, a constrained optimization analysis shows that it is optimal to forgo the oasis development strategy entirely. This is because the oasis strategy leads to increased evaporation rates in tracts where it is imposed, leaving little flexibility in other tracts to satisfy the overall evaporation constraint. This example illustrates the type of analysis that can be done once a value function is determined. This type of analysis is not realistically feasible by simply examining mapped projections of the impacts of various policies, such as those shown in Figure 2.

5.2. Fire Coverage Across a City

The second example is a hypothetical fire coverage problem that illustrates the application of Conjecture 1 from §3.1.2. In this example, the decision is where to locate three fire stations within a city. The example is motivated by Church and Roberts (1983), who argue that traditional coverage models for facility location problems do not sufficiently measure the value obtained from coverage. We consider first a simple model in which each location is equally weighted and the value function is linear over response time. Then we consider a less restrictive model in which

Figure 5. The optimal development plan with constraints on overall levels of evaporation rate and night cooling.



Note. C, Compact; O, oasis; D, desert.

areas can be weighted differently and the value function over response time can be nonlinear.

For the simple model, the preferred solution is to minimize average response time, where average response time is calculated as a continuous function over the city. An optimization model for this is

$$\begin{aligned} \min_K \quad & \int\int_A z(x, y, K)\, dx\, dy, \\ & K = ((K_x^1, K_y^1), (K_x^2, K_y^2), (K_x^3, K_y^3)), \end{aligned} \tag{8}$$

where K is a vector representing the x and y coordinates of the three stations, and z is the average response time for a point (x, y) in the city region A given the locations of the three fire stations; that is, K represents the chosen alternative, and z represents the consequence. For this illustrative example, region A is assumed to be square with dimensions normalized from 0 to 1 in both x and y.

To develop a specific functional form for (8), we assume that for some fraction of incidents α, the fire station assigned to respond is not the closest one. Of those incidents, the same fraction α are not assigned to the next closest station. Finally, of those incidents not assigned to the two closest stations, the same fraction α will end up unassigned to any of the three stations. With these assumptions, the average response time used in Equation (8) becomes

$$z(x, y, K) = \left(\sum_{i=1}^{3} \alpha^{i-1}(1-\alpha) f(d((x, y), K^{(i)})) \right) + \alpha^3 \bar{f}, \tag{9}$$

where $K^{(i)}$ is the location of the ith closest station, $d((x, y), K^{(i)})$ is the distance between (x, y) and $K^{(i)}$, $f(d)$ is the average response time from a station at distance d, and $\bar{f}$ is the average "unassigned" response time (occurring when none of the three stations is properly equipped to respond). For $d(\cdot)$, we use a "metropolitan" distance measure, which is the sum of the x and y distances to account for travel along gridlines in a metropolitan area. The range of $f(d)$ is assumed to be $[0, 1]$. We assume that $f(d)$ is linear in d below an upper bound d'. We can think of d' as a large enough distance between the station and the incident

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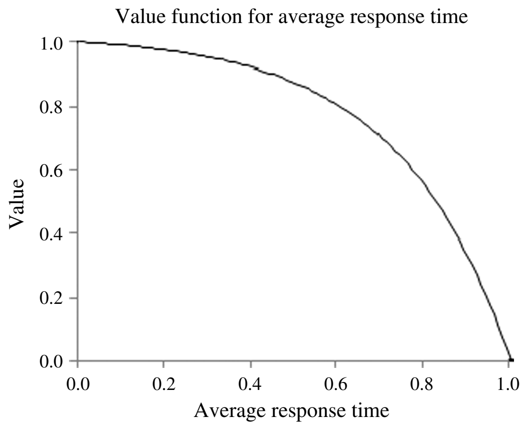
Figure 6. The exponential value function over average response time, with exponential constant $c = 3.86$.



such that there is no benefit to responding, and the station therefore will not respond if $d \geqslant d'$. Provided α is not large, $\alpha^3 \bar{f}$ will be close to zero. Since this term is constant and close to zero, the exact choice of $\bar{f}$ is immaterial, and we assume $\alpha^3 \bar{f}$ can be ignored. In this illustrative example, we set $\alpha = 0.15$. Whereas (9) was developed directly from the definition of average response time, the use of (8) is a nondiscrete special case analogous to (2) where all locations are given equal weight so that $a(x, y) = 1$ for all x and y, and $v(z) = z$, and we will now generalize (8) based on Conjecture 1.

Given the relationship between response time and the size of the fire that the responder will have to fight, it is reasonable to assume there are diminishing returns for decreases in response time from the perspective of a policy maker; that is, high values will be placed on the range of response times that will likely assure the survival of the building(s), and changes in response times slightly above this range will be associated with steeper decreases in value. For example, assume the conditions specified in §4.1 are met for an exponential value function to be valid and that the assessed tradeoff midvalue for the range from zero to one is 0.826, which results in the following single-attribute value function:

$$v(z(x, y, K)) = \frac{1 - e^{-3.86(1 - z(x, y, K))}}{1 - e^{-3.86}}. \tag{10}$$

Equation (10) is shown in Figure 6, normalized so that $v(0) = 1$ and $v(1) = 0$.

It is reasonable that a rapid average response time could be more critical in some areas than others, because of differences in, for example, population or economic importance. In such a case, the weight $a(x, y)$ will vary with x and y. Conceptually, the process in §4.2 to assess this weighting function applies, but there are an (uncountably) infinite number of locations for which weights have to be determined, so it is not practical to assess weights for each individual location. Two approaches are practical in this situation: assess weights for a finite number of locations and use a curve fitting method to fit a surface to these points, or assume a specified functional form for $a(x, y)$ that has a small number of unspecified parameters and assess the weights at enough locations to determine those parameters. There is no existing theory to specify preference conditions that determine a functional form for $a(x, y)$ in the way that there is theory to determine when an exponential single-attribute value function is valid. Thus, we will show how such an approach might be used in a practical decision situation, such as the fire station location decision.

For illustrative purposes, assume that development in this city is concentrated along a river extending in a straight line upstream from near the center of the eastern boundary of the city, which is on a bay, to near the southwestern corner, and that the decision maker wants more emphasis placed on protecting areas that are closer to the bay and the river. Using the approach in §4.2, suppose that the decision maker most prefers the hypothetical consequence with the best response time occurring at a location near to the mouth of the river, but slightly inland. Also, assessing indifference relationships for other locations determines that the decision maker wants the weight to decrease with increasing distance from the most preferred location, and also wants the weight for areas along the river to be higher than areas further away from the river for any specified distance from the bay. In addition, the decision maker is unconcerned about the response times at the very edges of the city. The surface shown in Figure 7 has these qualitative characteristics for the weighting function, and these characteristics could be modeled quantitatively using a combination of beta-distribution functional forms for the x and y dimensions. Although there is not a theoretical justification for using this form, it fits the qualitative characteristics of the decision maker's preferences, and we will use it to illustrate how a functional form can be used to specify $a(x, y)$. (In this illustrative analysis, we ignore the impact of the river on response times.)

More specifically, we assume that the value function is specified as the product of an unconditional marginal beta distribution for the x variable and a conditional marginal beta distribution for the y variable, where the conditioning is through a straight-line equation for the mode of the y variable marginal beta distribution as a function of the x variable. Based on the qualitative characteristics presented in the preceding paragraph, the straight line that will be used is the course of the river. Using this specification for the value function, the combined beta function is determined by five parameters: the p and q parameters for the marginal distribution over x, the two parameters that determine the equation for the mode line for the marginal distribution over y as a function of x, and one other parameter that can be used along with the equation of the mode line to determine the p and q parameters of the conditional marginal distribution over y. In addition, an equation must

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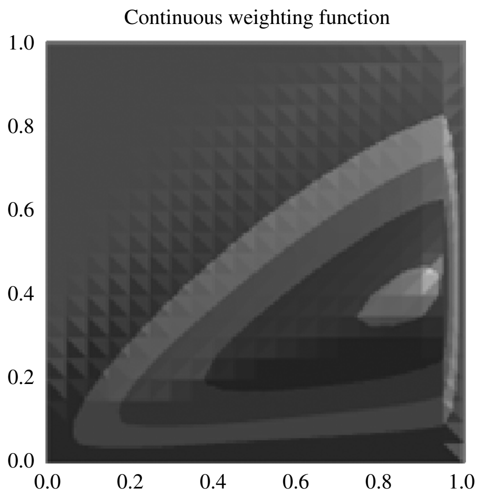
Figure 7. A contour map of the illustrative weighting function expressing the weight assigned to any point in the city.



Note. The maximum weight is assigned to (0.85, 0.4).

be specified to set the scaling for the weighting function (for example, to ensure that it integrates to one over the region of interest), but that does not require an assessment to determine.

Parameters for the combined beta function can be obtained by determining the best fit surface of the combined beta form to tradeoff assessments for a small set of locations where the assessments are made using the procedure in §4.2. To illustrate the possible results of such assessments, the equation for the surface in Figure 7 is specified by

$$a(x,y)=x^{1.1}(1-x)^{0.1}y^{1.5}(1-y)^{(1.425-0.6x)/(0.05+0.4x)}. \tag{11}$$

An alternative procedure to assuming a functional form for $a(x,y)$ would be to do tradeoff assessments for a set of locations and then interpolate a surface through the results without assuming any particular functional form using an interpolation procedure implemented in a mathematical analysis package. Lam (1983) provides a review of many different spatial interpolation procedures.

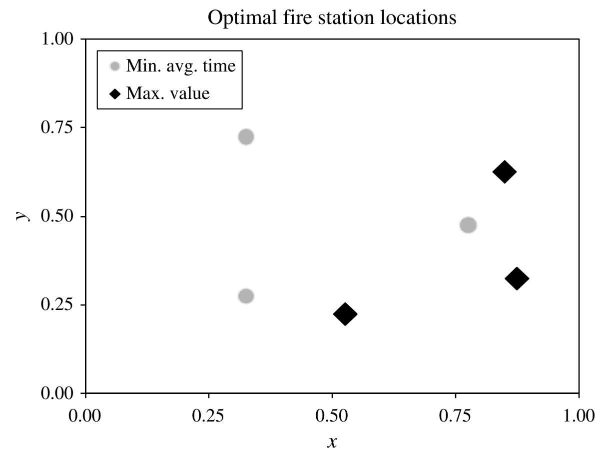
The resulting optimization problem is now

$$\max_K \quad \int\int_A a(x,y)\frac{1-e^{-3.86(1-z(x,y,K))}}{1-e^{-3.86}}\,dx\,dy, \qquad K=((K_x^1,K_y^1),(K_x^2,K_y^2),(K_x^3,K_y^3)), \tag{12}$$

where $a(x,y)$ is given by (11), but normalized to integrate to one over the region of interest. In (12), we are maximizing overall value as expressed by (3) in Conjecture 1.

The optimization problems in (8) and (12) were solved numerically using a grid search with a distance of 0.025 between adjacent points, and a numerical integration that divides the region into 400 cells, computing the average response time in the center of each cell. Changes in the search and integration parameters did not noticeably affect the results. Figure 8 shows the optimal fire station locations for (12) designated with diamonds, along with the locations determined by the unweighted linear value function in (8) designated with circles. Conforming with the preference to protect areas closer to the bay and river, the locations of the fire stations have been "pulled" toward the higher weighted part of the city compared to their locations when only average response time is considered.

Figure 8. Optimal fire station locations when minimizing unweighted average response time, and when maximizing a geographically weighted nonlinear value function.



6. Extensions of the Preference Models to Address Uncertainty

This section, included as Online Appendix F, discusses preference models for decisions where the consequences of alternatives are uncertain, and addresses what conditions are needed for such models.

7. Concluding Comments

This paper presents preference models for decisions based on GIS data. As shown by the illustrative hypothetical applications in this paper, these types of decisions are important in a variety of decision contexts, and with the widespread use of GIS, it is now practical to apply more rigorous decision analysis methods to these decisions. When faced with a decision that has consequences that can vary over a geographic region, formulating specific structures and conditions for the decision stakeholders' preferences will allow an analyst to elicit an appropriate value or utility function using the results in this paper. This can help to provide a more defensible gauge of the desirability of the proposed decision alternatives. We believe the methods in this paper can be applied to a wide range of real-world policy decisions with geographically varying consequences, such as regional development planning, pollution abatement, facility location, and utility service provision.

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Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/opre.2013.1217.

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