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LECTURES ON EFFECTIVE INTERACTIONS, EFFECTIVE OPERATORS AND CLOSED SHELL NUCLEI

Larry Zamick

August 1, 1968

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Lawrence Radiation Laboratory
Berkeley, California

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Larry Zamick

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Larry Zamick*

Lawrence Radiation Laboratory
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Berkeley, California

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FIRST SEMINAR - THE IMPORTANCE OF CORE POLARIZATION

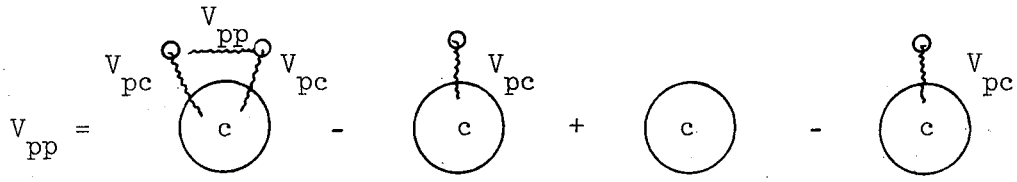
What is the interaction of two nucleons in a nucleus? The simplest thing to consider is 2 nucleons beyond a closed shell and ask for their interaction energy.

Consider for example ^{42}Ca which we first imagine to consist of a closed ^{40}Ca core and two neutrons in the $1f_{7/2}$ shell. Let us use the notation

$p \equiv 1f_{7/2}$ particle

$c \equiv {}^{40}Ca$ core

and let V_{pp} be the interaction of the two $f_{7/2}$ particles? How can we get V_{pp} from experiment? The following diagram is useful



The energy of $^{42}Ca = V_{pp} + 2V_{pc} + V_{cc}$

We see that by introducing ^{41}Ca and ^{40}Ca we can get rid of V_{pc} and V_{cc}

$$V_{pp} = E(^{42}Ca) - E(^{41}Ca) + E(^{40}Ca) - E(^{41}Ca)$$

Using the table of Mattauch et al.,[1] Nuclear Physics, vol. 67 we get

$$V_{pp} = (-361.891 + 350.420 - 342.056 + 350.420)\ \text{MeV}$$

$$= -3.107\ \text{MeV}$$

Suppose we now try to calculate V_{pp}. Fortunately, this was done in a recent paper by T. T. S. Kuo and G. E. Brown[2] (Reaction Matrix Elements in the 0f - 1p shell) and we shall describe what they have done.

They start with the realistic Hamada-Johnston two nucleon interaction with the full complexity of a hard core, a tensor, spin orbit and quadratic spin orbit interaction. But they end up like everybody else with a long column of numbers - the matrix element of the interaction between two particles in given shell model states.

If we take the above picture of ^{42}Ca seriously we would expect the interaction V_{pp} to be given by one of their matrix elements

$$V_{pp} = \langle [f_{7/2} \quad f_{7/2}]^{I=0} V [f_{7/2} \quad f_{7/2}]^{I=0} \rangle$$

But the value they get is -0.869 MeV which is very far away from the empirical value -3.107 MeV.

Does this bad agreement mean that their project of using the realistic interaction in finite nuclei is a failure?

The answer is no. The fault lies not with the interaction but with the wave function. The wave function $[f_{7/2} \quad f_{7/2}]^{I=0}$ is simply too mediocre to give a good estimate of the interaction energy.

The authors proceed to improve the wave function following a procedure which was first developed at Princeton by George Bertsch[3] (this was also done in Japan by Arima and his co-worker) of admitting three particle one hole states to the $f_{7/2}^2$ wave function.

For example you can lift a particle from the $1d_{3/2}$ shell into the $2d_{3/2}$ shell. Also from the $1p_{3/2}$ shell into the $1f_{7/2}$ shell etc. Such

processes involve excitation energies of about $2\hbar\omega$ ($1\hbar\omega$ excitations are forbidden by parity) and this is about 20 MeV for ^{40}Ca

The technique employed to improve the wave function is ordinary Raleigh Schroedinger perturbation theory to first order (and hence to second order in the energy).

The improved wave function is

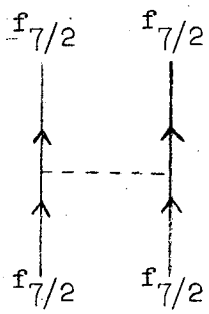
$$\psi = f_{7/2}^{\,2} + \beta\ 3p - 1h$$

$$\beta = \frac{\langle f_{7/2}^{\,2}\ V\ 3p - 1h\rangle}{\Delta E} \qquad \Delta E \approx -2\hbar\omega \approx -20\ \text{MeV}.$$

The correction to the energy is

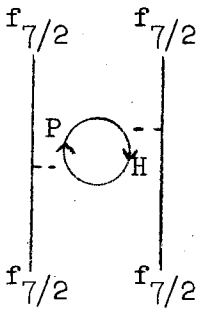
$$\frac{|\langle f_{7/2}^{\,2}\ V\ 3p - 1h\rangle|^2}{\Delta E}\ .$$

Note that this is necessarily negative. But, be careful, this energy is <u>not</u> the pairing energy V_{pp}. Part of it is the pairing energy and part of it is a second order contribution to the energy of a particle with the core V_{pc}. The following graphs will illustrate this division.



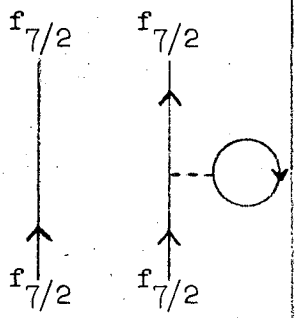
(a)

Bare Pairing Energy = - 0.869 MeV



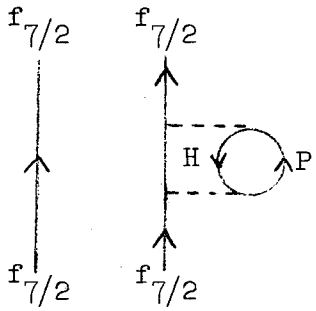
(b)

3p-1b Contribution to Pairing Energy = - 0.938 MeV



(c)

Lowest Order Particle Core Interaction It should be of the Order of - 8 MeV.



(d)

3p-1h Contribution to the Particle -Core Interaction (i.e. to the single particle energy of an $f_{7/2}$ particle)

We see that graphs (b) and (d) together are the total 3p - 1h contribution to the energy of ^{42}Ca. Graph (d) is the charge in energy of ^{42}Ca because the single particle energy of an $f_{7/2}$ nucleon charges due to 3p - 1h admixtures. Kuo and Brown[2] do not list the value of this--they do list the value of graph (b) and we see that it is comparable to and even slightly larger than the 'bare' matrix element.

This illustrates quite dramatically how important core polarization is. The total pairing energy is now - 1.829 MeV compared to the 'BARE' value -0.891MeV. We are still far away from the empirical value -3.107 MeV, however.

So far we have constrained two particles to be in the $f_{7/2}$ shell. The next thing to do is to let them be anywhere in the f - p shell.

$$\psi^{I=0} = C_1 f_{7/2}^2 + C_2 p_{3/2}^2 + C_3 f_{5/2}^2 + C_4 p_{1/2}^2 .$$

But how do we take care of the $3p - 1h$ configurations? Here is where the concept of an EFFECTIVE INTERACTION comes in.

We pretend that the interaction between two $f_{7/2}$ particles in an $I = 0$ state is not the bare value -0.891 MeV but the value obtained with the improved wave function $-0.891-0.938 = -1.829$ MeV. In other words we define an effective matrix element

$$\langle f_{7/2}^{\,2}\ V\ f_{7/2}^{\,2}\rangle_{\text{effective}} = \langle f_{7/2}^{\,2} + \beta\ 3p - 1h|V|f_{7/2}^{\,2} + \beta\ 3p - 1h\rangle_{\text{BARE}} .$$

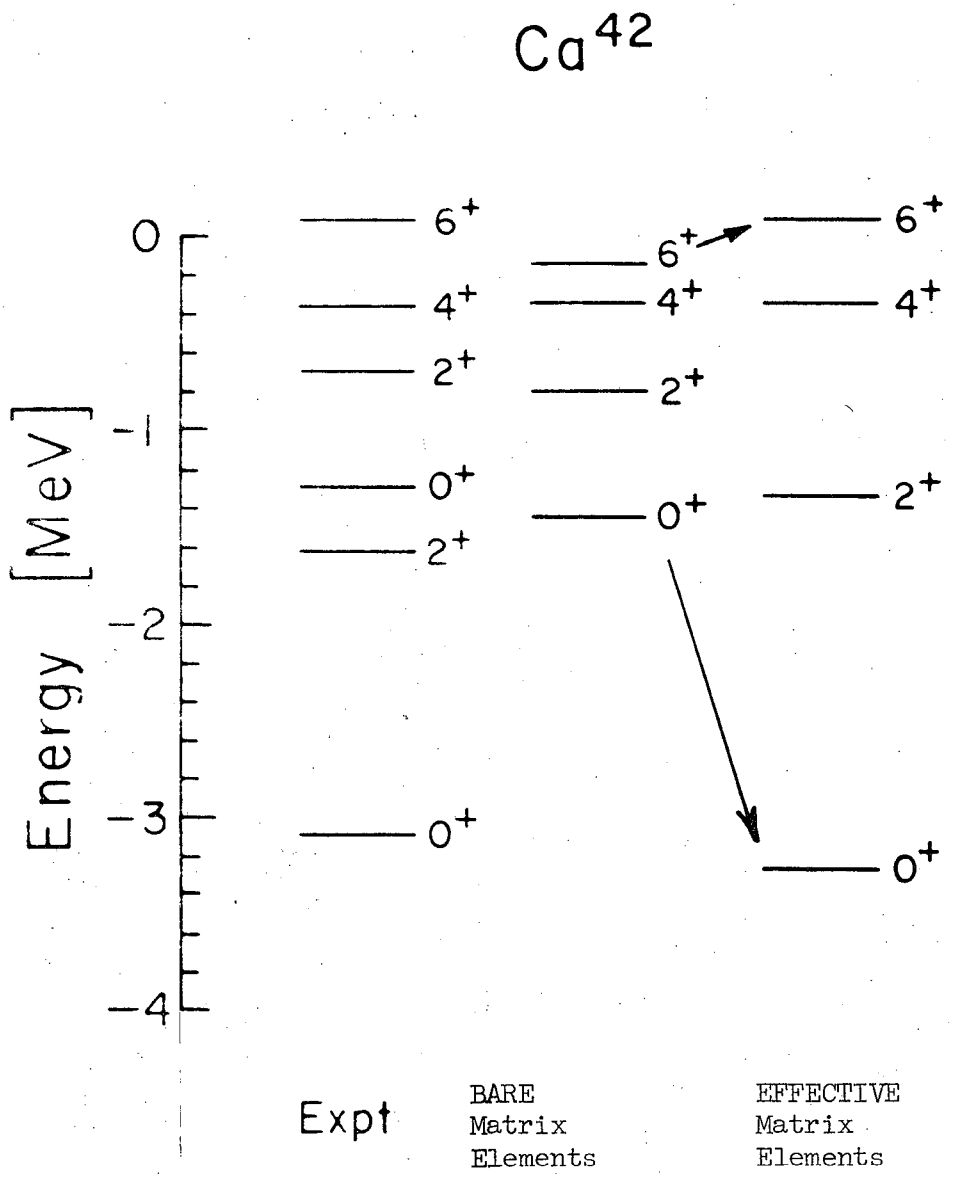
We do the same thing for all $2p$ matrix elements in the $f - p$ shell

$$\langle j_1 j_2\ V\ j_3 j_4\rangle_{\text{effective}} = \langle j_1 j_2 + \beta^{j_1 j_2}\ 3p - 1h|V|j_3 j_4 + \beta^{j_3 j_4}\ 3p - 1h\rangle$$

where $\beta^{j_1 j_2} = \dfrac{\langle j_1 j_2|V|3p\text{-}1h\rangle}{\Delta E}$. (In evaluating the above it is not necessary to include the term in $\beta^{j_1 j_2}\beta^{j_3 j_4}$).

We now carry out the calculation, pretending we have only two particles in the $f - p$ shell but we use the effective matrix elements defined above. Our calculation is now just as easy as a 2 particle calculation with bare matrix elements and we expect that the energy levels thus obtained should be an improvement on those using bare matrix elements. We have to pay a price for the simplicity of the calculation, however, we lose complete knowledge of what the $3p - 1h$ component of our wave function is.

To further dramatize the importance of core-polarization we show the spectrum of ^{42}Ca that Kuo and Brown[2] obtain with the bare and effective matrix elements.


Ca42
Energy [MeV]
0
-1
-2
-3
-4
6+
4+
2+
0+
2+
0+
6+
4+
2+
0+
6+
4+
2+
0+
Expt
BARE
Matrix
Elements
EFFECTIVE
Matrix
Elements

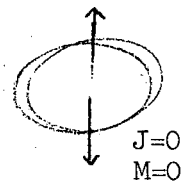
Note, from the above figure, that the 3p - 1h contribution to the interaction can sometimes be repulsive. This is indeed the case for the $J = 6$ state. This point was made by George Bertsch.[3] He noted that the entire 3p - 1h contribution to the energy should be negative--that is the sum of the graph b and d, but that graph b by itself could have either sign.

Points made by the audience:

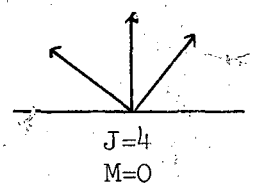
1. Prof. Swiatecki asked for an estimate of the interaction of two particles assuming they move randomly in the nuclear volume. After I was unable to answer, he quickly came up with an estimate.

If the depth of the two nuclear interaction is Vo and the range is a and the radius of the nucleus is R then the answer is Vo $\times$ the ratio of volumes $\frac{4\pi}{3} a3/\frac{4\pi}{3} R^3$. Let $R = \text{Vo } A\frac{1}{3}$. We get $\text{Vo}(\frac{a}{\text{Vo}})^3 \times \frac{1}{A}$. If Vo = 30 MeV and $a \approx$ Vo then for ^{40}Ca we get $\frac{30}{40} = 0.75$ MeV which is quite reasonable. Furthermore this model correctly predicts the $\frac{1}{A}$ dependence of nuclear matrix elements.

2. I attempted to give a physical justification for why the $J = 0$ state in ^{42}Ca lies lower than the $J = 2,4,6$. I said that in a $J = 0$ state the orbits of the two particles overlap most. Assume the orbits are in a plane perpendicular to the angular momentum, then the $J = 0$ state would look like this



with a complete overlap of the orbits.



Poor overlap of the orbits

Bayman criticized the picture on the previous page. The picture suggests that each particle is a definite m state. But both the $J = 0$ $M = 0$ and $J = 4$ $M = 0$ states are mixtures of states of different m ($m = \pm 1/2, \pm 3/2, \pm 5/2, \pm 7/2$). This is true. Redlich later pointed out to me that, nevertheless, the above pictures would be valid in the limit of large j.

3. I attempted to determine "without doing any work" how much $3p - 1h$ was admixed into the $f_{7/2}^2$ $I = 0$ state. Suppose we know the change in energy due to the $3p - 1h$ configuration δE, then we can say

$$\delta E = \frac{|\langle f_{7/2}^2 \; V \; 3p - 1h\rangle|^2}{-2\hbar\omega} \quad .$$

The amount of $3p - 1h$ admixture is given by

$$\beta^2 = \frac{|\langle f_{7/2}^2 \; V \; 3p - 1h\rangle|^2}{2\hbar\omega} = \frac{\delta E}{-2\hbar\omega} \quad \frac{\delta E}{-20} \quad .$$

Unfortunately, Kuo and Brown[2] do not give the entire δE but only the part that contributes to the 'pairing energy' (graph b). Actually the energy of ^{42}Ca will be lowered both because of more pairing and because the single particle energy $\epsilon_{7/2}$ has changed due to $3p - 1h$ contributions.

$$\delta E_{3p\text{-}1h} = \delta E_{\text{PAIRING}} + 2 \; \delta\epsilon_{7/2}$$

where $\delta\epsilon_{7/2}$ is the correction to the energy of an $\epsilon_{7/2}$ particle as given by graph (d).

What is $\delta\epsilon_{7/2}$? But first, what is $\epsilon_{7/2}$ itself? The first number that comes to mind is -8 MeV, the binding energy per particle. Alternately we can say

$$\epsilon_{7/2} = E(Ca^{41}) - E(Ca^{40}) = (-350.420 + 342.056)\text{MeV}$$

$$= 8.354 \text{ MeV} .$$

Since Kuo and Brown[2] don't list $\delta\epsilon_{7/2}$ we will be a little rough and take it from a different calculation--Brown, Evans and Thouless[4]--they say that in ^{41}Ca the f strength is 90%. This presumably means that there is 10% 3p - 1h admixture in the ground state of ^{41}Ca.

Thus $\beta^2_{Ca^{41}} = 10\% = \delta\epsilon_{7/2}/2\hbar w \frac{\delta\epsilon_{7/2}}{20}$. Hence $\delta\epsilon = 2$ MeV.

We now go to ^{42}Ca where $\beta^2 = \frac{\delta E_{Ca^{42}}}{2w} = \frac{\delta E_{\text{PAIRING}} + 2 \times (-2)}{20}$

$$= \frac{-1-4}{20} = 25\% .$$

Note that in perturbation theory two particles polarize the core a little more than twice as much as one particle.

Added Notes Not Given in the Seminar

How much core polarization is present in an arbitrary Calcium Isotope -- ^{40}Ca to ^{48}Ca? To discuss this it is convenient to use a simple formula which relates the binding energy of a given calcium isotope to the interaction of an $f_{7/2}$ neutron with the ^{40}Ca core -- call this C -- and to the interaction of two $f_{7/2}$ nucleons in various spin states I(I=0, 2, 4 and 6) -- we call this V^I.

$$BE(Ca[40+n]) - BE(Ca[40]) = nC + \frac{n(n-1)}{2}\alpha + [n/2]\beta$$

where $[n/2] = n/2$ for n even and $(n-1)/2$ for n odd.

where

$$\alpha = [(2j+2)\bar{V}_2 - V_o]/(2j+1)$$

$$\beta = (2j+2)/(2j+1)\ (V_o - \bar{V}_2)$$

where $j = 7/2$ and $\bar{V}_2$ is the center of gravity of the seniority two states

$$\bar{V}_2 = \sum_{I=2,4,6} (2I+1)\ V^I \ / \ \Sigma\ (2I+1)$$

(Note that the signs of the V^I are defined such that V^o is positive -- hence they are the negatives of the true interaction energies).

The above formula was obtained by Theiberger and Talmi,[5] was applied to the Calcium isotopes by Talmi and Unna[6] whose work we here follow, and to other nuclei by R. C. Barret.[7]

The quantity α has a simple interpretation. The energy required to remove a neutron from an odd n nucleus is $BE(Ca[40+n+1]) - BE(Ca[40+n])$ with n even = $C+n\alpha$. Talmi has made the point that as one adds neutrons to a given shell it should be easier to remove a neutron from the shell. Hence, α should be negative.

Let us list the values that Kuo and Brown[2] get for α.

Let

$V^I(f_{7/2}^2) \equiv$ the bare matrix element

$\delta V^I(f_{7/2}^2)$ ≡ the correction due to 3p-1h mixing in the $f_{7/2}^2$ wave function

V^I_{SPEC} ≡ the energy of the lowest state of given J when all processes (such as 3p-1h mixing and allowing the 2 nucleons to be anywhere in the fp shell) are included.
-- SPEC stands for energy spectrum.

We find

I	$V^I(f_{7/2}^2)$	$\delta V^I(f_{7/2}^2)$	V^J_{SPEC}
0	+0.869	+0.938	+3.28
2	+0.664	+0.121	1.36
4	+0.297	-0.210	0.35
6	+0.120	-0.346	-0.08

The corresponding values of α and β in MeV are:

$$\alpha(f_{7/2}^2) = +0.206 \qquad \beta(f_{7/2}^2) = 0.662$$
$$\delta\alpha(f_{7/2}^2) = -0.358 \qquad \delta\beta(f_{7/2}^2) = 1.296$$
$$\alpha_{SPEC} = -0.042 \qquad \beta_{SPEC} = 3.33$$

The empirical values obtained by Talmi and Unna[6] are $\alpha = -0.23$ and $\beta = 3.33$.

Note that core polarization corrections were necessary in order that α have the correct sign, i.e. in order that the separation energy decreases with increasing n. Note also that the core polarization improves the value of β as well.

Getting back to our original concern -- the core polarization probability in Ca[40+n] -- let us call it λ^2. In perturbation theory this is given by

$$\lambda^2 = \frac{\delta E[n]}{-2\hbar\omega} = \left[n\delta C + \frac{n(n-1)}{2}\delta\alpha + [\tfrac{n}{2}]\delta\beta\right]/2\hbar\omega$$

If we take as before $\delta C = 2$ MeV then we get numerically

$$\lambda^2 \approx \frac{2.0n - 0.36\, n(n-1)/2 + [n/2]\, 1.30}{20}$$

For example for ^{48}Ca this becomes $\approx \frac{16 - 10 + 5}{20} \approx 55\%$.

This is probably an overestimate because, for example, when two $f_{7/2}$ particles simultaneously polarize the core, this should be counted only <u>once</u>, whereas in perturbation theory one counts this twice.

One should say

λ^2 = probability that one and only one nucleon polarizes the core

+ probability that two and only two nucleons polarize the core

+ ...

In the hypothetical case that only δC contributed significantly to λ^2 we would then get, instead of $\lambda^2 = n\delta C/2\hbar\omega$, a new result $\lambda^2 = 1 - (1 - \frac{\delta C}{2\hbar\omega})^n$.

4. Professor Swiatecki asked about the relationship between the two nucleons in ^{42}Ca and the deutron (which has $\approx$-2 MeV binding).

Now the two nucleons in the deuteron are in a $J = 1$ $T = 0$ state whereas the two nucleons on ^{42}Ca are in a $J = 0$ $T = 1$ state. If we go to ^{42}Sc we can have both types of states. An interesting question is why, if for the two nucleon system the $J = 1$ $T = 0$ is lower in energy than the $J = 0$ $T = 1$ state (the former is bound, the latter is unbound), is the reverse true for ^{42}Sc where the $J = 0$ $T = 1$ is the ground state and is about 0.6 MeV. lower in energy than the lowest $J = 1$ $T = 0$ state.

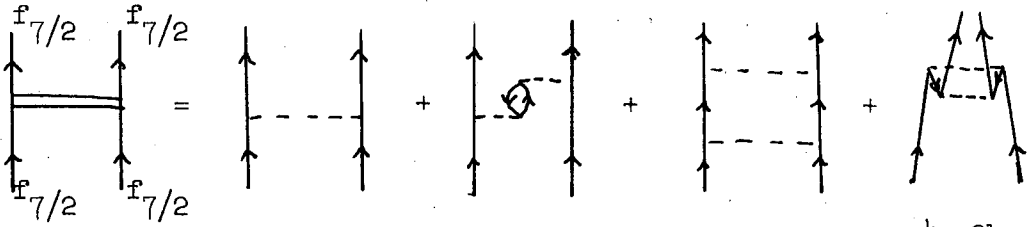
I pointed out that for Light Odd-Odd nuclei with N = Z such as Li^6 and F^{18}, the $J = 1$ state was lowest just like in the deutron but that there was a consistent trend (and some people challenged me on this point) for the $J = 0$ state to come down as we go to heavier nuclei.

A point I should have made but did not is that if we use bare matrix elements in Sc^{42} then indeed, just like in the deutron, the J = 0 state lies above the J = 1 state (but only by 0.6 MeV). The core polarization corrections (3p - 1h) scarcely affect the J = 1 state but push down the J = 0 state below the J = 1 'quasi deutron' state.

Physical insight into why the J = 0 state and not the J = 1 state is pushed down are provided by George Bertsch in his excellent notes 'The Working Man's Shell Model.'

SECOND SEMINAR - THE JUSTIFICATIONS AND LIMITATIONS OF THE EFFECTIVE INTERACTION

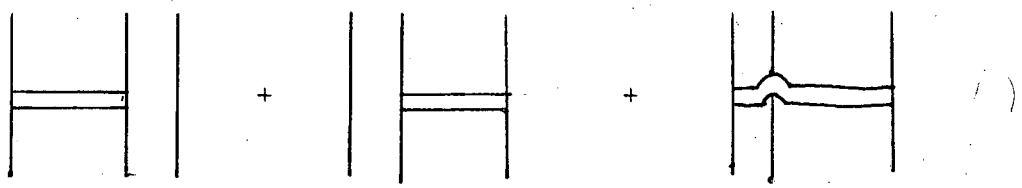
Let us again consider the Calcium isotopes, and assume that the valence nucleons are in the $1f_{7/2}$ shell. But we use an <u>effective</u> interaction. Thus, up to second order in perturbation theory, the following diagrams are included.



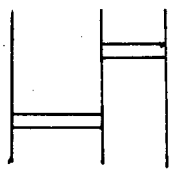
Effective BARE 3p-1h 2p 4p-2h

Suppose we now consider <u>three</u> neutrons in the $1f_{7/2}$ shell, i.e. ^{43}Ca. Can one use the same effective interaction here?

Note that in the $1f_{7/2}$ model of ^{43}Ca there is only <u>one</u> state of each angular momentum J (and only J = 3/2, 5/2, 7/2, 9/2, 11/2 and 15/2 are possible). Hence the energy in terms of an effective interaction is given schematically by

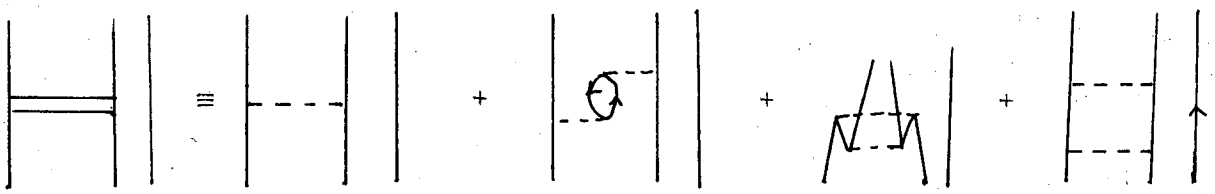


Note that there will <u>not</u> be a term of the form

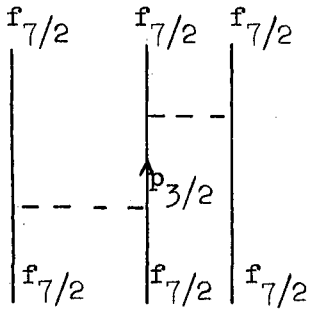


since the intermediate state has to be different from the initial state.

Now the above figure is a shorthand notation. Up to second order we have



Thus the effective interaction takes certain configuration mixing into account correctly. But some configuration mixing is not taken into account -- for example:



In other words, in using an effective interaction one is not correctly taking into account configurations such as $f_{7/2}^{2}\ p_{3/2}$ but one is handling the configuration $f_{7/2}\ p_{3/2}^{2}$ correctly, at least to second order.

It turns out that one gets <u>fantastically</u> good agreement for the binding energies of the calcium isotopes -- see Talmi and Unna[6] -- but that the energy spectra are not nearly as good. For example, in ^{43}Ca using the effective interaction method McCullen, Bayman and Zamick[8] the lowest $J = 3/2^{-}$ state is predicted to lie at 1.4 MeV whereas experimentally it lies at about 0.6 MeV. The reason is that precisely the configuration $f_{7/2}\ p_{3/2}$ is important. This was shown by Engeland and Osnes[9] and by Federman and Talmi.[10] The $3/2^{-}$ state has over 30% of this configuration.

An interesting point is that almost all of the $f_{7/2}^{2}\ p_{3/2}$ configuration in the lowest $3/2^{-}$ state has the two $f_{7/2}$ particles coupled to spin two rather than to spin zero. This means that no $\ell = 1$ strength would show up in the reaction $Ca^{42}(d,p)Ca^{43}$ to the state in question (since in ^{42}Ca the two nucleons couple to $J = 0$). The above example serves as a warning that it is very difficult to guess at wave functions -- most people would have said that the two $f_{7/2}$ neutrons would couple to $I = 0$; nor can one readily construct a wave function by merely looking

at transfer reactions. The above considerations are due mainly to Talmi.

A more severe example of the breakdown of the effective interaction method is the $J = 3/2^-$ state of ^{43}Sc. In the $f_{7/2}$ model the low-lying states consist of one proton and two neutrons in the $f_{7/2}$ shell. The calculation of McCullen, Bayman and Zamick yields a correct result for the ground state --$J = 7/2^-$ --, but the first $J = 3/2^-$ state lies at an excitation energy of more than 3 MeV. 0. But there are at least two $J = 3/2^-$ levels at much lower energies -- one at 400 KeV and the other at 1.1 MeV. A '3 particle' calculation in which the particles are allowed to be anywhere in the 1f-2p shell was carried out by Johnston and Flowers[11] and by Ripka and Zamick[12] and both calculations produced one and only one low-lying $J = 3/2^-$ state. This state has hardly any $f_{7/2}^3$ in it, so it is not surprising that the M.B.Z. calculation <u>failed</u> so badly -- the effective interaction method cannot work miracles. Johnston and Flowers[11] have speculated that the 1.1 MeV state is basically the 3 particle state and that the lowest $3/2^-$ state, by process of elimination, is mostly a 5 particle-two hole state (sometimes called a highly deformed state).

All the Scandium isotopes have a large number of very low-lying states which have nothing whatever to do with the $f_{7/2}^n$ configuration.

THIRD SEMINAR - EFFECTIVE OPERATORS - A CRITICAL EVALUATION OF THE EFFECTIVE CHARGE CONCEPT

There are many electromagnetic transitions which depend one hundred per cent on core polarization for their existence. For example, neutrons do not give off any significant electric radiation and yet the transition rate for the electric decay of the first excited state of O^{17} with $J = 1/2^+$ to the ground state -- $J = 5/2^+$ -- which in the shell model could be pictured as a single neutron in a $2S_{1/2}$ state dropping into a $1d_{5/2}$ state, is almost as strong as if the valence particle were a proton. More generally, in any process in which neutrons and only neutrons are changing their state, the electric transition rate should be negligible; but, in almost every case the transition rate is non-negligible.

To get a finite transition rate one must somehow bring protons into the picture. In the example of O^{17}, the wave functions $2S_{1/2}$ neutron $\times$ closed O^{16} core and $1d_{5/2}$ neutron $\times$ closed O^{16} core are too mediocre to describe the electric transition rate.

As before, one can improve the wave function by using ordinary Raleigh Schroedinger perturbation theory to first order. This will admix $2p - 1h$ and $3p - 2h$ components into what were originally $1p$ wave functions. Only the $2p - 1h$ component will affect the transition rate in first order and only if the particle excited from the core is a proton will a non-zero result be obtained. The improved initial and final wave functions are:

$$\Psi_i = \Psi_i(1p) + \beta_i \Psi_i(2p - 1h) + \gamma_i \Psi_i(3p - 1h)$$

$$\Psi_f = \Psi_f(1p) + \beta_f \Psi_f(2p - 1h) + \gamma_f \Psi_f(3p - 1h)$$

where $\beta = \langle \Psi(1p)\ V\ \Psi(2p - 1h) \rangle / \Delta E$

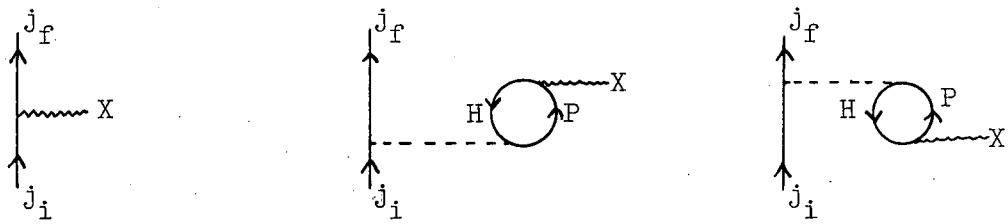
$$\Delta E \approx -2\hbar w \ .$$

In terms of the operator E^L the transition matrix element is

$$\langle \Psi_f(1p)\ E_M^L\ \Psi_i(1p) \rangle$$

$$+ \beta_i \langle \Psi_f(1p)\ E_M^L\ \Psi_i(2p - 1h) \rangle + \beta_F \langle \Psi_f(2p - 1h)\ E_M^L\ \Psi_i(1p) \rangle .$$

For electric transitions in particular $E_M^L = \sum_i r_{(i)}^L \mathcal{Y}_\mu^L(\Omega_i)$. The above three terms can be represented by three Feynman graphs.



In the case of ^{17}O the first term will vanish but the next two terms will give finite contributions.

We now come to EFFECTIVE CHARGE. The idea is that instead of explicitly introducing 2p - 1h components (i.e. core polarization) we pretend that the wave function is still a one particle wave function, but we replace the operator by an effective operator. In the case of the electric multipole operator a popular choice is to replace

$$\sum_{\text{protons}} r^L \mathcal{Y}_M^L$$

by

$$(1 + \epsilon_p) \sum_{\text{protons}} r^L \mathcal{Y}_M^L + \epsilon_N \sum_{\text{neutrons}} r^L \mathcal{Y}_M^L$$

and sometimes one chooses $\epsilon_p = \epsilon_N$. The quantities $1 + \epsilon_p, \epsilon_N$ are the proton and neutron EFFECTIVE CHARGE respectively.

Usually, the parameters ϵ_p and ϵ_N are not calculated but are chosen in some way, and not too much thought is given to the justification of such a procedure.

The perturbation approach that we have just outlined affords a way of calculating the EFFECTIVE CHARGE and in fact this is the way Horie and Arima[13] did the calculation way back in 1954.

The effective charge $\epsilon_{\substack{p \\ N}}$ is clearly the ratio of the first order contribution to the electric multipole matrix element divided by the zeroth order contribution of a one proton state.

$$\epsilon = \frac{\beta_i \langle \Psi_f(1p)\, E_M^L\, \Psi_i(2p - 1h)\rangle + \beta_f \langle \Psi_f(2p - 1h)\, E_M^L\, \Psi_i(1p)\rangle}{\langle \Psi_{f\ \text{proton}}(1p)\, E_M^L\, \Psi_{i\ \text{proton}}(1p)\rangle} .$$

We see immediately that the effective charge should have a more complicated structure than is popularly taken. It apparently depends on the states f and i. $\epsilon = \epsilon(f,i)$. Thus we really have an effective charge matrix.

As an example consider the $2p - 1f$ shell and consider a transition from $n_i\ \ell_i\ j_i \rightarrow n_f\ \ell_f\ j_f$. A priori we should have eight different effective charges depending on the initial state and the final state

$$\epsilon(f_{7/2},\ f_{7/2}) \qquad \epsilon(f_{7/2},\ p_{3/2}) \qquad \epsilon(f_{7/2},\ f_{5/2})$$

$$\epsilon(p_{3/2},\ p_{3/2}) \qquad \epsilon(p_{3/2},\ f_{5/2}) \qquad \epsilon(p_{3/2},\ p_{1/2})$$

$$\epsilon(f_{5/2},\ f_{5/2}) \qquad \epsilon(f_{5/2},\ p_{1/2}).$$

Things look quite complicated now. But Federman and I[14] have shown that things simplify provided one assumes:

1. the two body interaction is central
2. we neglect minor shell energy differences compared with $2\hbar\omega$ energies.

In that case we can show that ϵ depends on n and ℓ but not on j.

$$\epsilon(n_f\ \ell_f\ j_f,\ n_i\ \ell_i\ j_i) = \epsilon(n_f\ \ell_f,\ n_i\ \ell_i).$$

Thus, instead of eight elements in the $2p - 1f$ shell there will only be three

$$\epsilon(1f,1f),\ \epsilon(2p,2p),\ \epsilon(2p,1f).$$

The following things have been learned by calculating the effective charge in perturbation theory:

1. The effective charge is state dependent as indicated above. We find that the bigger the ℓ of the orbit the bigger the effective charge. For example in the $2p - 1f$ shell we found fa ^{41}Ca and ^{41}Sc

$$\epsilon_N(1f,1f) = 0.59 \qquad \epsilon_p(1f,1f) = 0.21$$

$$\epsilon_N(2p,1f) = 0.41 \qquad \epsilon_p(2p,1f) = 0.15$$

$$\epsilon_N(2p,2p) = 0.38 \qquad \epsilon_p(2p,2p) = 0.14\ .$$

We examined our program to see if there was any overwhelming reason why the effective charge increased with ℓ. We did not find anything very clear cut except, possibly that the number of intermediate 2p - 1h states where one of the particles is in an f state is larger than the corresponding number when the particle is in a p state.

2. The neutron effective charge ϵ_N is much bigger than the corresponding proton charge ϵ_p (not to be confused with the <u>total</u> proton charge $1 + \epsilon_p$). This can be seen by the above table. The reason for this is that in order to get a contribution to ϵ, one must excite a <u>proton</u> particle from the core. It is easier for a neutron to excite a proton from the core than a proton because the valence neutron can act in both $T = 1$ and $T = 0$ states with the core protons, whereas the valence proton can act only in $T = 1$.

There is a good deal of empirical evidence to support $\epsilon_N > \epsilon_p$. For example, Halbert, McGrory and Wildenthal,[15] using the same value of ϵ_N and ϵ_p get good fits to nuclei in the 2s - 1d shell containing both neutrons and protons but in the case of O^{18} which contains only neutrons, they are off by at least a factor of two

$$\text{BE } 2\ (2^+ \rightarrow 0^+)_{\text{experiment}} = 6.5 \pm 20\%$$

$$\text{BE } 2\ (2^+ \rightarrow 0^+)_{\text{theory assuming}\ \ \epsilon_N = \epsilon_p = 0.5} = 3.0.$$

By going from $\epsilon_N = 0.5$ to $\epsilon_N = 0.75$ they would get a better fit for this nucleus. They would then have to reduce ϵ_p below 0.5 so that the BE(2) in other nuclei with neutrons and protons would not increase.

3. We did the calculation both with Ca^{40} as a core and with Ni^{56} (which closes the $f_{7/2}$ shell) as a core. We find a much larger effective

charge with Ni^{56} as a core than with ^{40}Ca. We get for Nickel

$$\epsilon_N(1f,1f) = 1.11 \qquad \epsilon_p(1f,1f) = 0.44$$

$$\epsilon_N(2p,1f) = 1.11 \qquad \epsilon_p(2p,1f) = 0.36$$

$$\epsilon_N(2p,2p) = 0.73 \qquad \epsilon_p(2p,2p) = 0.30 .$$

Part of this comes from the fact that particles in the $f_{7/2}$ shell can be excited into the $2p_{3/2}$ and $1f_{5/2}$ shell--this takes only ~5MeV compared with $2\hbar\omega$ = 20 MeV. Part of it is due to the fact that particles would rather jump out of a filled $f_{7/2}$ shell into the 3p - 2f - 1g shell than jump into an empty $f_{7/2}$ shell from the 1p shell.

4. The effective charge that we calculate is smaller than what is required by experiment. For example we calculate ϵ_N = 1.1f or Ni whereas an empirical analysis requires $1.7 \leftrightarrow 1.9$. In Ca^{41} the calculated value $\epsilon(2p,1f) = 0.41$ is too small by about a factor of <u>three</u> compared with the empirical value for the $BE2(3/2^- \rightarrow 7/2^-)$.

This difficulty seems to be universal--all recent calculations of the effective charge--in different parts of the periodic table by various workers--have yielded results which are too small.

This means that the electromagnetic decays are not yet really understood. G. E. Brown, A. M. Green, W. Gerace and G. Bertsch have advocated that besides 2p - 1h, the 3p - 2h <u>deformed</u> states should be important. They have successfully calculated transition rates by employing such deformed states. As was mentioned before, the 3p - 2h states will not contribute in first order perturbation theory--so maybe it will be necessary to go to second order before one gets a sufficient effective charge.

Magnetic Moments and M1 Transitions

A study of the effect of core polarization on the magnetic moment operator $\vec{\mu} = g_\ell \vec{\ell} + g_s \vec{s}$ (this is used both for M1 transitions and magnetic moments) reveals a behavior which is quite different from the corresponding behavior of the quadrupole operator.

The first difference is that the quadrupole operator can connect between different major shells, whereas this is not true for the magnetic dipole operator (we must be careful what we mean by major shells).

Take O^{17} as an example--in the shell model the ground state is

$$d_M^{5/2} \; O_{core}^{16} .$$

Now, multiply this by the operator $\vec{\mu}$

$$\Psi = \left(g_\ell \, \ell_\alpha + g_s \, s_\alpha \right) d_M^{5/2} \; O^{16} .$$

First the operator acts on the $d^{5/2}$ particle. Since $\vec{\mu}$ depends only on $\vec{\ell}$ and $\vec{s}$ and since ℓ^2 and s^2 are good quantum numbers for $d^{5/2}$ the only thing that can happen is that we get a mixture of $d^{5/2}$ and $d^{3/2}$

$$\vec{\mu}_\alpha \, d_{5/2} = C_1 \, 1d_{5/2} + C_2 \, 1d_{3/2} .$$

The magnetic moment is defined as $\langle \Psi_I^I \, \mu_z \, \Psi_I^I \rangle$. Consider then $\mu_z \, d_{5/2}^{5/2} = C_1 \, d_{5/2}^{5/2} + C_2 \, d_{5/2}^{3/2}$. But $d_{5/2}^{3/2}$ does not exist (M cannot be bigger than j). Hence $\mu_z \, d_{5/2}^{5/2} = C_1 \, d_{5/2}^{5/2}$ and obviously $C_1 = \mu_{Schmidt}$ (Since $d^{5/2}$ is a stretched state $\mu_{schmidt}$ will be the magnetic moment of a free neutron).

$$\mu_z \, d_{5/2}^{5/2} = \mu_{schmidt} \; d_{5/2}^{5/2} .$$

One can show by similar reasoning that $\mu \times O^{16}$ core = 0. $\mu \times O^{16}$ core is a spin 1 state→but to form a spin one state you have to excite a particle from the core. But μ operating on a $1p_{3/2}$ core particle say can only yield $C_1\ 1p_{3/2} + C_2\ 1p_{1/2} \rightarrow$ But this state is still inside the core.

The above results imply that there will be no first order corrections to the magnetic moment operator. This is because in first order the correction will be proportional to

$$\langle (2p - 1h)^{5/2}_{5/2}\ \mu_z\ d^{5/2}_{5/2} \rangle \ .$$

But

$$\mu_z\ d^{5/2}_{5/2} = \mu_{\text{schmidt}}\ d^{5/2}_{5/2} \ .$$

We get

$$\mu_{\text{schmidt}} \langle (2p - 1h)\ d^{5/2}_{5/2} \rangle = 0.$$

We would therefore expect, that for one particle (or one hole) beyond a closed major shell the magnetic moment could be well given by the Schmidt theory.

Here is a table

Nucleus	Ground State	Experimental Moment	Schmidt Moment
N^{15}	$1p_{1/2}^{-1}$	-0.28	-0.26
O^{15}	$1p_{1/2}^{-1}$	0.72	0.64
F^{17}	$1\ d_{5/2}$	4.72	4.79
O^{17}	$1\ d_{5/2}$	-1.89	-1.91
K^{39}	$1\ d_{3/2}^{-1}$	0.39	0.12
Ca^{41}	$1\ f_{7/2}^{-1}$	-1.59	-1.91

The deviations are small although especially in the case of K^{39} and Ca^{41}, they are not insignificant.

Since first order perturbation theory gives no correction we must go to second order.

Let us briefly discuss the formalism of Raleigh Scroedinger perturbation theory in second order. This work was done by Harry Mavromatis as part of his thesis, by G. E. Brown and me.[16] The formalism to be discussed now follows closely the work of Ichimura and Yazaki.[17]

To second order

$$\Psi = N\left[\Phi + \frac{1P}{E - H_o} V\Phi + \frac{1}{E - H_o} P V \frac{1}{E - H_o} P V \Phi\right]$$

where Φ is the shell model wave function e.g. $d^{5/2}_{5/2} \times O^{16}$ closed shell V is the residual interaction and P reminds us not to use the initial state as an intermediate state. The quantity N is the normalization since we have to normalize the wave function up to second order.

To calculate N we note that because of the presence of P the second and third term in Ψ are orthogonal to the first term to second order then

$$N = \left[1 + \left\langle \frac{1}{E - H_o} P V \Phi \middle| \frac{1}{E - H_o} P V \Phi \right\rangle\right]^{-1/2}$$

(terms like $\langle \Phi \, 1 \frac{1}{E - H_o} P V \Phi \rangle$ vanish by orthogonality). The magnetic moment is

$$(\Phi \mu \Phi) = \frac{(\Phi \mu \Phi)}{} N^2 + 2(\Phi \mu \frac{P}{E - H_o} V \Phi)$$

$$- = 0$$

$$+ (\frac{1}{E - H_o} P V \Phi \Big| \mu \frac{1}{E - H_o} P V \Phi)$$

$$+ 2(\frac{1}{E - H_o} P V \frac{1}{E - H_o} P V \Phi \Big| \mu \Phi).$$

The last term is zero because $\mu \Phi = \mu_{\text{Schmidt}} \Phi$ and

$$(\frac{1}{E - H_o} P V \frac{1}{E - H_o} P V \Phi \Big| \Phi) = 0.$$

Note that we need multiply only the first term by N^2 since the other terms are already in second order. Hence,

$$(\Psi\ \mu\ \Psi) = \mu_{\text{Schmidt}}\ N^2 + \langle \Phi V P \frac{1}{E - H_o}\ \mu\ \frac{1}{E - H_o}\ P\ V\ \Phi \rangle .$$

To second order

$$N^2 = 1 - \langle \Phi\ V \frac{P}{E - H_o}\ \frac{1}{E - H_o}\ P\ V\ \Phi \rangle .$$

So we get finally

$$\langle \Psi\ \mu\ \Psi \rangle = \mu_{\text{Schmidt}} + \langle \Psi\ V\ P \frac{1}{E - H_o}\ (\mu - \mu_{\text{Schmidt}}) \frac{1}{E - H_o}\ P\ V\ \Phi \rangle .$$

DIGRESSION: → Mavromatis[18] pointed out the following amusing thing. We can write $\mu = \mu_{\text{isoscalar}} + \mu_{\text{isovector}} = \mu_A + \mu_B \tau_o'$. If V is a central interation then μ_A commutes with V since μ_A involves only $\vec{\ell}$ and $\vec{s}$. But then $\mu_A\ V\ \Phi = V\ \mu_A\ \Phi = V\ \mu_{A\ \text{Schmidt}}\ \Phi$. Hence the isoscalar part of $(\mu - \mu_{\text{Schmidt}})$ vanishes and only the isovector part remains.

But this means that the second order correction will be equal and opposite for a neutron and a proton i. e. for F^{17} and O^{17} or Ca^{41} and Sc^{41}. Hence we expect $\mu(O^{17}) + \mu(F^{17}) = \mu_{\text{Schmidt}}\ (O^{17}) + \mu_{\text{Schmidt}}\ (F^{17})$ → the second order corrections will have cancelled.

The results of the second order calculation are as follows

	DEVIATION EXPERIMENT - SCHMIDT	CALCULATED DEVIATION (Kallio-Kalltveit Interaction)
N^{15}	-0.02	-0.10
O^{15}	+0.08	+0.10
F^{17}	+0.07	+0.16
O^{17}	+0.02	+0.16
K^{39}	+0.27	-0.26
Ca^{41}	+0.32	+0.28

The results are good except for one glaring discrepancy. In K^{39} we get a correction of equal magnitude but opposite sign to what is needed. In other words $\mu_{experiment} = 0.39$ $\mu_{second\ order\ theory} = -0.14$.

FOURTH SEMINAR - TOPICS CONCERNING THE $1f_{7/2}$ SHELL AND ITS NEIGHBORHOOD

1. Is Ni^{56} a good closed shell?

In the shell model picture Ni^{56} closes the $1f_{7/2}$ shell in both neutrons and protons. This, then is the doubly magic nucleus with $Z = N = 28$. How good a picture of Ni^{56} is this?

We can learn a good deal from the beta decay of ^{56}Ni --the $J = 0^+$ $T = 0$ ground state of this nucleus decays 100% to the lowest $J = 1^+$ $T = 1$ state of ^{56}Co --the process is an allowed Gamow-Teller transition. The rate of decay is given by

$$\log ft = 4.4 .$$

The reason that this experiment is so useful is that in the shell model picture both the initial and final state are unique--the wave function of ^{56}Ca is $[f_{7/2_p}^{-1} \; f_{5/2_N}]^{I=1}$ (this is the only one-particle one hole combination that can add up to $J = 1$). We can thus easily calculate log ft

$$\log ft = 3.64 - \log M_{GT}^2 .$$

We find

$$M_{GT}^2 = 9\,6/7 \quad \log ft = 2.5 .$$

The shell model prediction for the transition rate is about ninety five times faster than the experimental one.

Why is M_{GT}^2 so large? Remember that for a free neutron M_{GT} is only <u>three</u>. One reason is that the transition from $J = 0$ to $J = 1$ is three times faster than from one to zero. Secondly, there are many $f_{7/2}$ protons present (about eight) each of which can undergo a beta decay.

A large discrepancy between theory and experiment still exists when a first order perturbation correction to the shell model is performed although

the correction does go in the direction of experiment (log ft is now 2.68).

We conclude that ^{56}Ni is not a good closed shell nucleus

2. Is Ca^{40} a good closed shell?

The nucleus Ca^{40} supposedly closes the $d_{3/2}$ shell. The $lf_{7/2}$ shell should be completely empty.

A way of testing this experimentally is to try to remove particles from the $f_{7/2}$ shell or alternately to try and put particles into the $d_{3/2}$ shell. The extent to which this can be done is the extent to which ^{40}Ca is not a good closed shell.

The relevant pick-up experiment is

$$Ca^{40}\ (p,d)\ Ca^{39}\quad J = 7/2^-$$

$$\text{or}\quad Ca^{40}\ (d, He^3)\ K^{39}\quad J = 7/2^-$$

$$\text{or}\quad Ca^{40}\ (He^3, \alpha)\ K^{39}\quad J = 7/2^-.$$

The stripping experiment is

$$Ca^{40}\ (d,p)\ Ca^{41}\quad J = 3/2^+\ ,\ \text{etc.}$$

In the pick-up experiment the spectroscopic factor to the lowest $7/2^-$ state is about 0.28. What does this mean?

If we assume ^{40}Ca = α closed shell + $\beta\ (2p - 2h)^{T=0}$.

In the proton pick-up experiment $\mathcal{S}$ is the average number of protons in the $f_{7/2}$ shell. In a $(2p - 2h)^{T=0}$ state, the two particles are on the average <u>one</u> neutron and <u>one</u> proton.

Hence the sum of the spectroscopic factors to all $7/2^-$ states

$$\beta^2 \times 1\ \text{(since only one proton)}.$$

Since $\mathcal{S} = 0.25$ for the lowest $7/2^-$ state we conclude

$$\beta^2 \geq 28\% .$$

In the stripping experiment $Ca^{40}(d, p)\ Ca^{41}$ to the lowest $3/2^+$ state different experiments have yielded different values of $\mathcal{S}$. In terms of $\mathcal{S}$ we note that

$$\frac{2J_F + 1}{2J_i + 1} \quad \text{summed over all } 3/2^+ \text{ states} = 4 \Sigma \mathcal{S}$$

should equal the average number of <u>neutron</u> holes in the $d_{3/2}$ shell. Again, in a 2p - 2h state there is on the average <u>one</u> neutron hole. Hence $\beta^2 = 4 \Sigma \mathcal{S}$

$$\beta^2 \geq 4\,\mathcal{S} \quad \text{to the lowest } 3/2^+ \text{ state.}$$

John Schiffer[19] obtained $4\,\mathcal{S} = 0.36$. Hence $\beta^2 \geq 36\%$. But some values ranging from $4\,\mathcal{S} = 0.20$ to $4\mathcal{S} = 0.78$ have been obtained by various experimentalists. Why is this?

A recent analysis by A. Denning, J. G. B. Haigh and G. Brown[20] may have cleared up the situation. They do the reaction at 7, 10, and 12 MeV. Their analysis reveals that the spectroscopic value depends on energy--the three values they get are $4\mathcal{S} = 0.68$, $4\mathcal{S} = 0.21$ and $4\mathcal{S} = 0.19$ respectively. They point out that the reason for this is that at 7 MeV compound nuclear effects are important e.g. whereas the D.W.B.A. calculation fits at small angles, it underestimates the experiment at large angles by a large amount. On a D.W.B.A. theory it is expected that in going from 7 MeV to 10 MeV the cross section at maximum should increase by about 25%. In

actual fact it decreases by a factor of 2.5. At ten MeV and 12 MeV the angular distributions fit so perhaps compound effects are unimportant here.

We then have $\beta^2 \geq 19\%$.

It should also be mentioned that Rost and also Philpott, Pinkston and Satchler have generated superior form factors to be used in D.W.B.A. programs. Their presumably superior analysis consistently yield larger spectroscopic factors-- $4\mathcal{S} > 0.19$ would be their result.

One further point--Denning, Haigh and Brown[20] get a larger spectroscopic factor in ^{42}Ca (d,p) Ca^{43} $J = 3/2^+$ than they do for ^{40}Ca (d,p) Ca^{41} $J = 3/2^+$ the values are $4\mathcal{S} = 0.35$ and 0.19 respectively. Does this mean that ^{42}Ca is not as good a shell model nucleus as ^{40}Ca? If the above numbers hold up then this may indeed be the case. We have a simple argument of why ^{42}Ca may have more neutron holes in the ground state than does ^{40}Ca.

In perturbation theory the amount of two-hole mixing in ^{40}Ca and ^{42}Ca are respectively

$$\beta_{40} = \langle 0p - 0h | V | 2p - 2h \rangle / \Delta E$$

$$\beta_{42} = \langle 2p | V | 4p - 2h \rangle / \Delta E \ .$$

Now in ^{42}Ca we start with two $f_{7/2}$ neutrons coupled to zero. Let us think of this as a boson. We can think of the above matrix elements as transition matrix elements in which two nucleons from the 2s - 1d shell make a transition into the $f_{7/2}$ shell. Now the presence of a boson (in ^{42}Ca) will cause a <u>stimulated emmission</u> of two neutrons into the $f_{7/2}$ shell. Hence the matrix element

$$\langle 2 \text{ neutrons} | V | 4 \text{ neutrons} - 2 \text{ neutron holes} \rangle$$

should be bigger than

$\langle$no neutrons $|V|$ 2 neutrons - 2 neutron holes$\rangle$.

A detailed calculation shows that this indeed is the case.

3. Holes (Particles) Coupled to Vibrations - ^{39}K .

I am reporting on work done mainly by Phil Goode. It concerns the low lying negative (opposite) parity states of ^{39}K.

We first show the low lying spectra of ^{39}K and just for orientation purposes we put down some proposed configurations. We do not necessarily believe these configurations are correct but they do serve as a starting point

ENERGY	SPIN	PROPOSED CONFIGURATIONS
3.94	$7/2^-$	A BAND OF THE FORM
3.88	$5/2^-$	$[d_{3/2}^{-1}\ \Psi^{3^-}_{VIBRATION}]^{-J}$ J = 3/2,5/2,7/2,9/2
3.60	$9/2^-$	
3.02	$3/2^-$	
2.82	$7/2^-$	$[1d_{3/2}^{-1}\ \Psi^{5^-}_{VIBRATION}]^{J=7/2}$;$[d_{3/2}^{-2}\ I=0\ f_{7/2}]^{J=7/2}$
2.5	$1/2^+$	$2\ s_{1/2}^{-1}$
0	$3/2^+$	$1d_{3/2}^{-1}$.

The first two states of positive parity are the $d_{3/2}$ hole state and the $s_{1/2}$ hole state respectively.

Let us consider two proposals for the $7/2^-$ state. One is the shell model wave function--two $d_{3/2}$ holes couple to $I = 0$ $T = 1$ and then couple

with an $f_{7/2}$ nucleon to $J = 7/2$. The second is the weak coupling model picture--a $d_{3/2}$ hole coupled to a 5^- vibration of ^{40}Ca --one chooses a 5^- rather than a 3^- because this state is excited weakly relative to the second $7/2^-$ state in reactions (p, p'), (d, d') and (α, α'). How do the shell model wave function and the weak coupling model wave function compare?

We can expand the shell model wave functions in terms of weak coupling wave functions and we get

$$[d_{3/2}^{-2}\ I = 0\ f_{7/2}]^{J=7/2} \propto \sum_{I_A} \sqrt{(2I_A+1)}\ \left[d_{3/2}^{-1}\ [d_{3/2}^{-1}\ f_{7/2}]^{I_A}\right]^{7/2}.$$

If we regard $[d_{3/2}^{-1}\ f_{7/2}]^{I_A}$ as a vibration in ^{40}Ca (obviously, this is an oversimplification), we see that the shell model wave function is a strong mixture of all vibrations, with a $\sqrt{(2I_A+1)}$ weighting. It is quite different from the weak coupling wave function. Which of the two pictures is then correct?

Goode performed a calculation in which he coupled a hole in the s - d shell to several low-lying vibrations of ^{40}Ca. The corresponding Shroedinger equation was solved following a procedure of Arnie Sherwood and A. Goswami.

The results were somewhat intermediate between the shell model and the weak coupling model:

1. The $3/2^-$, $5/2^-$ and $9/2^-$ states were almost pure $d_{3/2}$ hole coupled to the lowest 3^- vibration of ^{40}Ca.

2. The two $7/2^-$ states were mixtures of two configurations:

Lowest $7/2^-$	$0.88\, d_{3/2}^{-1}\, \Psi^{5^-}$	$-0.44\, d_{3/2}^{-1}\, \Psi^{3^-}$
Second $7/2^-$	$+0.48\, d_{3/2}^{-1}\, \Psi^{5^-}$	$+0.88\, d_{3/2}^{-1}\, \Psi^{3^-}$.

One may ask--why do not the 2^- and 4^- vibrations mix in (they do mix in strongly in the shell model wave function). The answer is--the shell model wave function-- $d_{3/2}^{-2}$ $I = 0$ $f_{7/2}$--would be an exact wave function of the lowest $7/2^-$ state if all the vibrations in ^{40}Ca $J = 2^-, 3^-, 4^-$ and 5^- were degenerate (and could be described as $d_{3/2}^{-1}\, f_{7/2}$) (The $T = 0$ and $T = 1$ states should be separately degenerate but don't have to be degenerate with each other). But in fact, if one looks at the spectrum of ^{40}Ca one sees that the 3^- and 5^- are considerably lower than the 2^- and 4^-. This is due to the fact that we can have 3^- and 5^- shape oscillations of the nucleus but not 2^- or 4^-.

The electromagnetic transition rates (E3 transitions) that are calculated with our wave functions came out very good. But recently some α, α' reactions by Jerry Peterson[21] and also by Aaron Bernstein[22] have been giving us some trouble. Since the α particle has a large angular momentum--you can see not only the $L = 3$ transfer but also the $L = 5$ transfer--both can occur simultaneously in going from the $3/2^+$ ground state to the $7/2^-$ excited states.

Considerable $L =$ <u>five</u> has been seen in the second $7/2^-$ state but strangely none in the lowest $7/2^-$ state. This is very puzzling because the last four states $J = 3/2^-$, $9/2^-$, $5/2^-$ and $7/2^-$ are all strongly excited and obey the weak coupling formula for $L = 3$ transfer

$$\sigma_{K^{39}}{}^{J}(\text{L transfer}) = \frac{(2J+1)}{(2L+1)\,(2j_H+1)}\ \sigma_{Ca^{40}}(J=0 \text{ to } J=L).$$

Then by process of elimination, the lowest $7/2^-$ state should be mainly $d_{3/2}{}^{-1}\,\Psi^{5^-}$ and this is what we obtain. The absence of a strong $L = 5$ contribution in α, α' to the lowest $7/2^-$ state is indeed giving us some trouble. Goode is working on it but I do not at present know whether the difficulty has been resolved.

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