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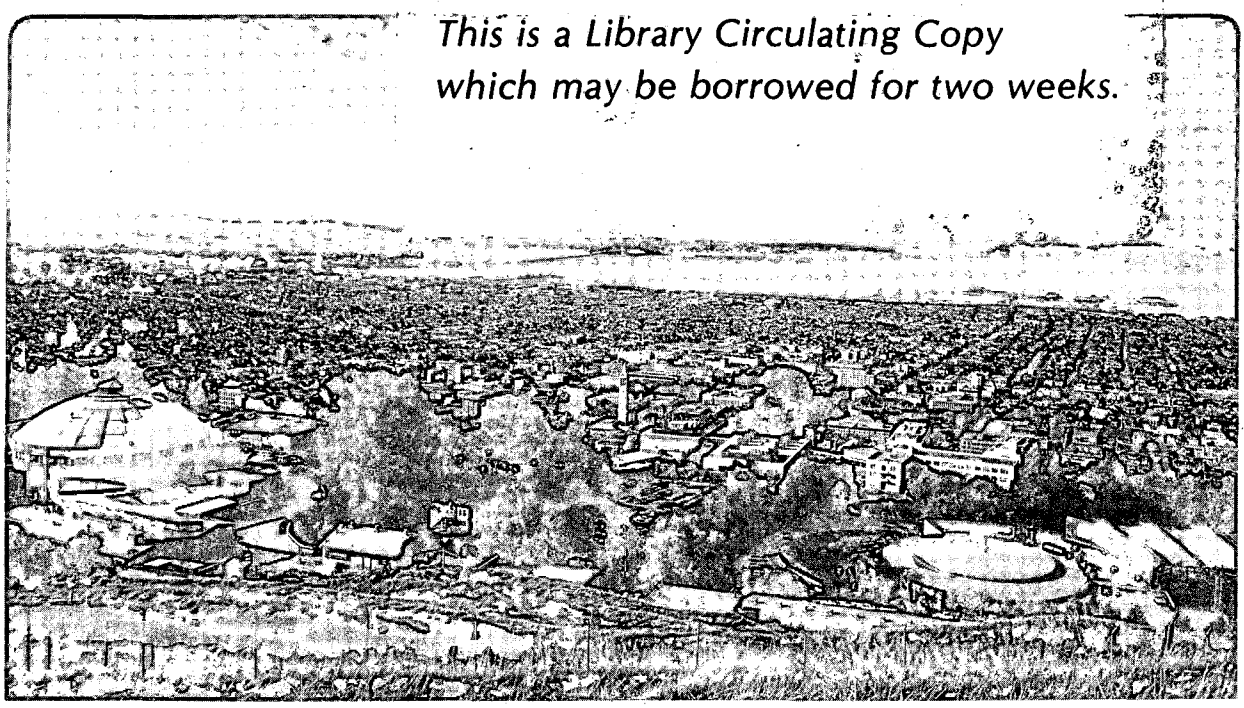

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MINC: AN APPROACH FOR ANALYZING TRANSPORT IN STRONGLY HETEROGENEOUS SYSTEMS

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ABSTRACT. We consider systems in which materials of low-diffusivity occur as islands in pervasive high-diffusivity materials. In these systems, global three dimensional transport occurs in the high-diffusivity materials. Transport in the low-diffusivity materials is local and one-dimensional in nature. MINC (Multiple Interacting Continua) is a method for efficiently simulating transport in such systems. The Integral Finite Difference Method (IFDM) provides a convenient way for implementing MINC. Known information on the shape and the size of the blocks can be judiciously utilized to obtain improved accuracy in estimating transport into the islands. MINC permits handling of continua at several hierarchial levels.

1. INTRODUCTION

In considering the transport of water or chemical species in groundwater systems, heterogeneity is a rule rather than an exception. By the phrase, strongly heterogeneous, we refer to those systems in which materials with strikingly different diffusivities coexist. In particular we focus attention on systems in which materials with very small diffusivities occur as islands within those with very large diffusivities. As examples one may cite the following: transient transport of water in a fractured sandstone in which low-diffusivity rock matrix occurs as islands within the high-diffusivity fractures; water transport in an aquifer with lenses of clay or silt dispersed through the sand; the diffusion of chemical species into the solid matrix surrounded by aqueous phase; and so on.

Under transient conditions, these strongly heterogeneous systems respond to of perturbations in a manner quite different from the response of those systems with homogeneous material properties. In a strongly heterogeneous system, a perturbation is initially conveyed rapidly through the high-diffusivity material over relatively long distances. Following this early-time response, the low-diffusivity material makes its presence felt over later times. Thus, the early-time behavior is dictated by the high-diffusivity material and the late-time behavior influenced by the low-diffusivity material. The over-all response through all time preserves the effects of both the material components.

Towards quantifying the transient behavior of this type of systems, Barenblatt et al. (1960) made an insightful advance by considering seepage in a fractured porous rock. The essence of the Barenblatt et al. contribution is that the system can be idealized in

terms of two dynamically interacting continua, one representing the high-diffusivity fractures and the other, the low-diffusivity rock matrix. Although originally conceived as a description of isothermal groundwater flow, the approach of Barenblatt et al is also applicable to other transport problems involving multiphase fluids, heat and chemicals. For convenience, we will designate thec two ontinua by the abbreviations, HDC (high-diffusivity continuum) and LDC (low-diffusivity continuum). On a global scale, perturbations in the potential field are transmitted through the HDC, whereas the interaction between the continua occurs at a local scale. Broadly therefore, transport on a global scale occurs within the HDC in general three dimensions. Within this global framework, the interactions between the HDC and the LDC occur on a local scale and may be conveniently idealized as a one-dimensional process.

The practical utility of this approach in reservoir engineering was soon recognized in the U.S. by Warren and Root (1963), who elegantly analyzed the flow of oil in a naturally fractured reservoir, idealized as containing a set of discrete orthogonal sets of fractures in a porous host rock. Other applications by Odeh (1965), Kazemi (1969) and others followed and soon the new approach acquired the appellation, "double porosity concept."

Recognizing the power of integral numerical methods, particularly the Integral Finite Difference Method (IFDM; Narasimhan and Witherspoon, 1976) in handling complex flow geometries, Pruess and Narasimhan (1985) showed that the double porosity idealization can in fact be be generalized to several interacting continua. Accordingly they coined the term, "Multiple Interacting Continua" (MINC) to characterize the new generalization. The purpose of this paper is to outline the nature and the current status of the MINC methodology.

2. THE CONCEPT

We consider systems in which a high-diffusivity material pervades the flow domain. Recall that diffusivity, by definition, is the quotient of conductivity (e.g., permeability, hydraulic conductivity, thermal conductivity, diffusion coefficient) and storativity (e.g., specific storage, retardation factor, specific heat) of a material of interest. Within this pervasive continuum, a low-diffusivity material occurs as islands of arbitrary shape and size (e.g., polyhedra; spheres). As a result of the striking contrast in the magnitudes of diffusivity of the two materials, one could reasonably assume that the distribution of potential on the surface of any one block of low-diffusivity material at a given instant will be reasonably uniform. Consequently, the surfaces of equal potential within the block at that instant could be expected to be surfaces that are mathematically similar to the outer surface of the block. Therefore, the potential at a given location within the block will be dictated by its proximity to the surface of the block. Within a small region, then, if we consider two blocks of identical shape and size, the potentials at two similar locations within these blocks will be practically identical. Consequently, within this small region of interest, one could collect all points within identical blocks that lie within a certain proximal interval from the surface and assign these points to one continuum. In this manner, one could discretize the low-diffusivity blocks into one or more nested continua. Figure 1 from Pruess and Narasimhan (1985) illustrates a set of nested continua in a two dimensional system representing an idealized square fracture grid, enclosing square blocks of porous rock. In this figure, the HDC is designated as continuum 1

and the remaining LDC continua are designated by numbers 2 through 6. As can be seen from this figure, global transport occurs exclusively through continuum 1 whereas local

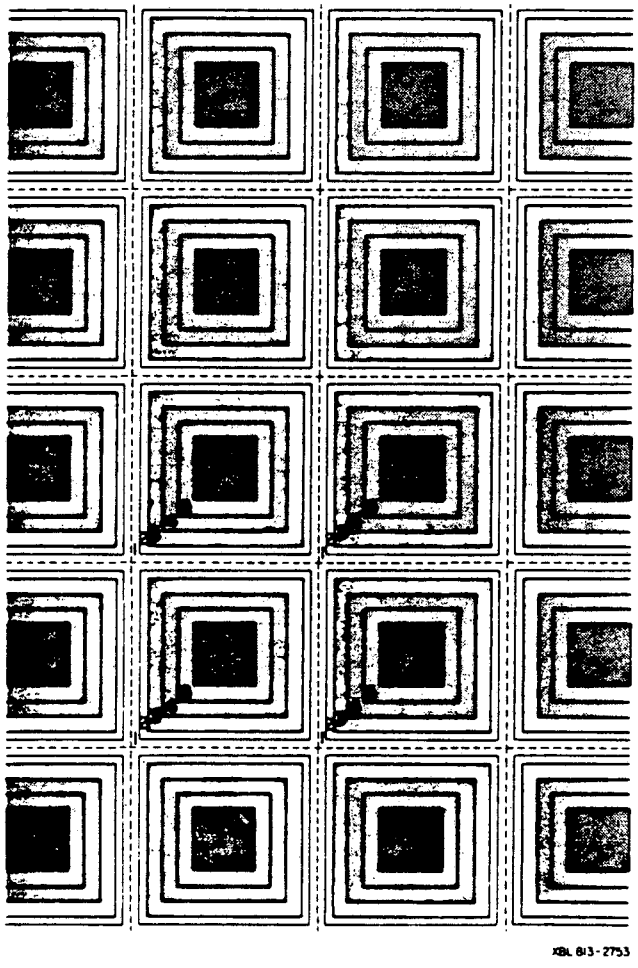

Figure 1: Schematic nested continua in a system in which the high-diffusivity material forms a pervasive contniuum in a square pattern (From Pruess and Narasimhan, 1985)

one-dimensional interactions occur between continua 1 through 6. Note that there is no communication between neighboring blocks because the path of least resistance lies within the first continuum. This interrelationship between global and local flow patterns is schematically shown in Figure 2.

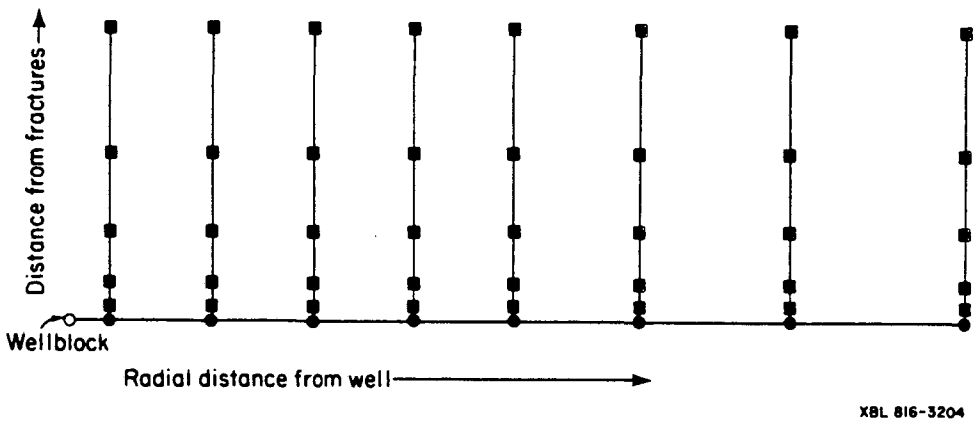

Figure 2 Schematic representation of interrelation between global flow pattern in the HDC and local one dimensional flow in the LDC (From Pruess and Narasimhan, 1985)

We may now develop the governing equations for transient flow in such a MINC system. As was shown by Pruess and Narasimhan (1985), the governing equations can be developed in general for advection as well as diffusion processes involving more than one fluid phase. Nevertheless, recognizing that the purpose of this paper is merely to review and describe the MINC concept, we shall restrict ourselves to the consideration of diffusion-type processes involving a single phase. Extension to advection and multiphase flow may be assumed to be evident in principle.

3. GOVERNING EQUATIONS

We consider a system defined by two state variables, an extensive quantity, M, and an intensive quantity, ψ. The extensive quantity could be mass of water (groundwater system), mass of solute (geochemical system), or heat (thermal system). The corresponding intensive quantity could be potentiometric head, chemical concentration, or temperature. For an elemental volume in such a system, one may combine the equation of state (relating M and ψ under conditions of mutual equilibrium), the equation of motion (expressing the movement of the quantity measured by M in response to spatial variations in ψ) and the axiom of mass conservation to write the following equation for transient transport:

$$\int_{\Gamma} K\vec{\nabla}\psi\cdot\vec{n}d\Gamma = \frac{d}{dt}\int_{V} MdV = Vc\frac{\partial\psi}{\partial t} \qquad (1)$$

in which,

Γ	is the closed surface bounding the elemental volume,
K	is the coefficient of conductivity,
$\vec{n}$	is the unit outer normal to the surface segment $d\Gamma$,
V	is the volume of the element,
c	is the specific capacitance defined as $\frac{1}{V}\frac{\Delta M}{\Delta \psi}$, and,
t	is time.

The surface integral in (1) includes boundary conditions imposed on the external boundary of the system. For completeness one may add source or sink terms to (1).

The expression in (1) is equally applicable to an element in the HDC or in the LDC. Let us follow the convention used in Figure 1, where the HDC is designated as continuum 1 and the LDC is discretized into continua 2 through L, where L is the total number of continua. Then, we may apply (1) to typical elements in the HDC and in the LDC and account for their dynamic interaction through their common interfaces $\Delta\Gamma$ appearing in the surface integral.

We now proceed to develop the appropriate conservation equations for elements the different continua. For the sake of clarity and simplicity we will assume in the following developments that all the blocks (or islands) of the low iffusivity material are of identical shape and size. Let n denote an elemental volume in the HDC over which the variation of potential is sufficiently smooth and gradual so that averages can be reasonably evaluated and defined. We assume that element n will be large enough to contain several blocks of the low-diffusivity material. These blocks will all have identical average properties. The number of blocks in element n is merely equal to the bulk volume of n less the volume of the high-diffusivity material divided by the volume of a single block. Under these conditions, the following conservation statements hold.

For an element n in Continuum 1,

$$\int_{\Gamma_{11}} K_{HDC}\vec{\nabla}\psi\cdot\vec{n}d\Gamma + \int_{\Gamma_{12}} K_{LDC}\vec{\nabla}\psi\cdot\vec{n}d\Gamma = Vc_{HDC}\frac{\partial\psi}{\partial t} \qquad (2)$$

where,

Γ_{11}	denotes the union of all surface segments bounding element n across which n communicates with another element in continuum 1
K_{HDC}	is the conductivity of continuum 1
Γ_{12}	is the union of all surface segments bounding element n across which element n communicates with continuum 2

K_{LDC} is the conductivity of the low-diffusivity continuum

V is the bulk volume of element n, and

c_{HDC} is the specific capacitance of the high-diffusivity continuum

In regard to K_{HDC} and c_{HDC} we use the convention that they both represent averages over the composite material. That is,

$$K_{HDC} = \frac{K^{*}_{HDC} A_{HDC}}{A_{HDC} + A_{LDC}} \tag{3}$$

where,

K^{*}_{HDC} is the actual intrinsic conductivity of the material making up the HDC,

A_{HDC} and A_{LDC} are the respective areas over which element n communicates with other elements in continuum 1

Similarly,

$$c_{HDC} = \frac{c^{*}_{HDC} c_{HDC}}{c_{HDC} + c_{LDC}} \tag{4}$$

where,

c^{*}_{HDC} is the actual specific capacitance of the material making up the HDC,

c_{HDC} and c_{LDC} are fractional volumes of the two continua.

In view of the foregoing, the following conservation equation holds for the j th continuum in element n,

$$\int_{\Gamma_{j-1,j}} K_{LDC} \vec{\nabla}\psi \cdot \vec{n} d\Gamma + \int_{\Gamma_{j,j+1}} K_{LDC} \vec{\nabla}\psi \cdot \vec{n} = V_j c_{LDC} \frac{\partial \psi}{\partial t}, \quad j \geq 2 \tag{5}$$

where,

$\Gamma_{j-1,j}$ and $\Gamma_{j,j+1}$ denote the total surface area between the respective continua. This total area is equal to the corresponding area per block times the number of blocks in element n

V_j is the bulk volume of continuum j in element n. This bulk volume is equal to the product of the corresponding volume per block and the number of blocks in element n.

Equations 2 and 5 provide a basis for implementing the MINC idealization through numerical models. Central to such an implementation is the task of efficiently evaluating the surface integrals on the left hand side of the two equations. In particular, the second integral on the left hand side of (2) and the two integrals on the left hand side of (5) relate to transport into the low-diffusivity blocks. Because of the MINC idealization, this transfer process is essentially one dimensional in nature and the shape and the size of the one dimensional channel are dependent on the shape and size of the blocks of low-diffusivity material. In practice, these integrals can be evaluated in one of two different ways. The first is to use analytical expressions that are applicable to specific block shapes (e.g., cube, sphere). Such analytic expressions can be obtained for quasi-steady flow or for fully transient in cases where K_{LDC} and c_{LDC} have simple (linear) dependence on ψ. Analytical expressions for the blocks could be combined with numerical expressions for global transport in the implementation of the numerical procedure (e.g., Duguid and Lee, 1977). Or, one could handle the transport process in the blocks through a completely general numerical procedure, using, in the evaluation of the integrals, information on the size and the shape of the blocks. The numerical approach is preferable when K_{LDC} and c_{LDC} depend on ψ in a non-linear fashion.

4. DISCRETIZED EQUATIONS

For numerical solution, the integral expressions in (2) and (5) may be discretized by the finite element method (FEM) or the integral finite difference method (IFDM). The two methods differ primarily in the manner in which they represent and process geometric information. The finite element method specifies coordinates of nodal points and finite elements of prescribed shape, defined by the nodal points occurring at their corners. Geometric quantities such as lengths, surface areas and volumes that are intrinsic to the evaluation of Equations 2 and 5 are generated implicitly through a weighted integration procedure. In the IFDM, on the other hand, the geometric quantities such as lengths, areas and volumes are explicitly provided as input information in the process of evaluating the integrals in (2) and (5).

In the case of a composite system that can be idealized by the MINC approach, we already know the shape and size of the blocks so that the volumetric and area terms needed for the evaluation of the integrals are known as accurately as we may desire. If one wishes to use the finite element method to handle the transport process in the block, one has to indirectly recalculate such information from nodal point coordinates by way of weighted integration. Instead, the IFDM enables one to directly handle the geometric information. For this reason it is quite convenient to describe the MINC method in the context of the IFDM. In the following pages, therefore, we will carry out all our discussions using the IFDM logic. An essential feature of the IFDM procedure is that in discretizing the flow domain, nodal points and surface segments are so designed that the line connecting the nodal points of two adjoining elemental volumes is perpendicular to the surface segment that separates the two volume elements. This enables the direct

approximation of the potential gradient along the normal to the interface.

In view of the foregoing, the conservation Equations (2) and (5) can be expressed as the following discretized equations.

Element n

$$\sum_{m} U_{nm} (\bar{\psi}_m - \bar{\psi}_n) + U_{nn_2} (\bar{\psi}_{n_2} - \bar{\psi}_n) = V_n c_{HDC} \frac{\Delta \psi_n}{\Delta t} \tag{6}$$

Continuum 2 within element n

$$U_{nn_2} (\bar{\psi}_n - \bar{\psi}_{n_2}) + U_{n_2 n_s} (\bar{\psi}_{n_s} - \bar{\psi}_{n_2}) = V_{n_2} c_{LDC} \frac{\Delta \psi_2}{\Delta t} \tag{7}$$

Continuum j within element n, $j \geq 3$

$$U_{n_{j-1} n_j} (\bar{\psi}_{n_{j-1}} - \bar{\psi}_{n_j}) + U_{n_j n_{j+1}} (\bar{\psi}_{n_{j+1}} - \bar{\psi}_{n_j}) = V_{n_j} c_{LDC} \frac{\Delta \psi_j}{\Delta t} \tag{8}$$

In Equations (6) to (8) U represents *conductance* between adjoining elements and $\bar{\psi}$ represents the time-averaged potential over a time interval of interest. The subscript m denotes an elemental volume in continuum 1 that adjoins and communicates with element n. Subscript n_j, $j \geq 2$ denote the j th continuum within element n.

In the context of finite differences, the conductance between element i and element k is given by,

$$U_{ik} = \frac{K_{ik} \Delta \Gamma_{ik}}{D_{ik}} \tag{9}$$

where,

K_{ik}	is the mean conductivity at the interface
$\Delta \Gamma_{ik}$	is the area of the surface segment
D_{ik}	is the distance between the nodal points on either side of the interface.

Also, the time averaged values of ψ are defined by,

$$\bar{\psi}_i = \psi_i^{\circ} + \lambda \Delta \psi \tag{10}$$

where

ψ_i° is the potential in element i at the beginning of a time interval,

λ is a time-weighting factor, and

$\Delta\psi$ is the change in potential over the interval of time.

A choice of 0, 0.5 and 1.0 respectively for λ leads to the well-known schemes of forward differencing, central differencing and backward differencing.

4.1. Two Special Features

Two special features of the discretized equations are worth discussing. The first of these relates to the amenability to partitioning of the matrix of equations stemming from the discretized equations and the second relates to the possibility of improving the accuracy of the difference approximation.

In the context of the IFDM (Narasimhan et al, 1978), one may define a stable time step or, equivalently, a time constant, for an elemental volume. Such a stable time step is obtained by dividing the capacitance of the element by the total conductance of its bounding surface. The time constant is in fact a generalized diffusivity term which incorporates the material properties of an element as well as its geometric properties. It should be obvious upon a little reflection that the HDC elements will in general have far smaller time constants than the LDC elements which occur imbedded in them. Thus the MINC method may often lead to stiff matrices. One can take advantage of this wide disparity in the time constants by choosing a computational time step that is much larger than the time constants of the HDC elements, but which is smaller than the time constant for the LDC elements. Then, for the particular choice of the time step one could calculate the potential changes in the LDC elements using a simple explicit scheme ($\lambda = 0$) and use an implicit scheme to obtain potential changes only in the HDC elements for which the chosen time step exceeds the stable time step. Essentially, this mixture of explicit calculation in one part of the domain and implicit calculation in another amounts to partitioning a large matrix and obtaining the solution by solving a smaller matrix. This facility could be especially valuable in solving large three dimensional problems. For details of such a mixed explicit-implicit scheme, see Edwards (1972).

Let us now look at the finite difference expression for conductance given in (9). This expression will be strictly accurate if points i and k are located at the ends of a tube of uniform cross sectional area $\Delta\Gamma_{ik}$. Instead, if the flow lines between points i and k were to converge or diverge, then the conductance estimated by (9) will be in error. In situations where the disposition of the flow channels is already known, one could make use of the known geometric information in accurately estimating the flux, rather than using an expression such as (9). Thus, according to Narasimhan (1985),

$$\vec{Q} = \frac{K\Delta\psi}{\int_{x_1}^{x_2} \frac{dy}{A(y)}} \tag{11}$$

where $\vec{Q}$ is the steady state flux through a segment of a stream tube bounded by equal potential surfaces at x_1 and x_2. If we apply this logic to estimate fluxes into the LDC, then,

$$U_{ik} = \frac{K_{LDC}\Delta\psi}{\int_{x_l}^{x_k} \frac{dy}{A(y)}} \tag{12}$$

where x_i and x_k denote the location of nodal points of i th and j th continua along a chosen flow path. The significance of (11) and (12) is that the known geometry of the block enables us to devise a simplified numerical expression to simulate transport in the LDC.

4.2. Proximity Function

In order that the generality of the MINC approximation may be fully exploited, it is desirable to be able to compute the geometric parameters for arbitrary shapes and sizes of the LDC blocks. Towards such an end, Pruess and Karasaki (1982) introduced the notion of a *proximity function*. The proximity function represents the dependence of cumulative LDC volume on position along a path extending from the surface of the block to its core. Ideally, this path will coincide with a macroscopic flow line.

Obviously, the first derivative of the proximity function yields the dependence of cross sectional area of the block as a function of position as one moves from the surface to the core of the block along a flow path. Thus, if P(x) is the proximity function, then, $\frac{dP}{dx}$ is A(x), the function which occurs in Equations 11 and 12.

4.3. Interporosity Flow Parameter

We now briefly examine the transient interaction between the HDC and LDC. As a global perturbation migrates rapidly through the HDC, a block in the LDC will begin to react slowly. As this happens, a perturbation front will migrate into the block. At any instant in time during this front-migration process, the flux into the block can be expressed by,

$$\vec{Q}^* = \frac{K_{LDC}\Delta\psi}{\int_{o}^{x_f} \frac{dy}{A(y)}} \tag{13}$$

where x_f is the distance to the front from the surface of the block and $\Delta\psi$ is the difference in potential between the surface and front. Equation (13), which is based on the concept of geometry imbedded Darcy's Law (Narasimhan, 1985), should, at the

present time be considered an approximate expression. The motivation for writing (13) has been to take into account the fact that a diffusive front travels with a finite velocity. Further work is in progress. It is clear that even if $\Delta\psi$ were to be constant, the flux in (13) would still be a function of time as the front migrates in time because x_f is a function of time. Therefore, conductance is a function of time.

After sufficient time, the front reaches the center of the block. Conceptually, the block may now be visualized as a one dimensional tube of variable cross section, open at one end and closed at the other. If such a tube were subjected to a constant flux condition at the inlet, then the flow system becomes quasi steady as soon as the front reaches the core. Under this condition, the gradients of potentials within the tube will be invariant in time, whereas the potentials themselves will be changing. Under the quasi steady condition, the conductance becomes independent of time.

Using the quasi steady assumption, Warren and Root (1963) introduced an interporosity flow parameter α such that

$$\vec{Q}^* = V\alpha K_{LDC} \left(\psi_n - \bar{\psi} \right) \tag{14}$$

where Q^* is the flux from element n in the HDC to a single block in the LDC, V is the volume of the block and ψ is the "average" potential over the block. In view of (13) and (14), we may express α by the relation,

$$\alpha = \frac{1}{V} \frac{1}{\int_0^{\bar{x}} \frac{dy}{A(y)}} \tag{15}$$

in which $\bar{x}$ is the location at which the "average" potential of the block occurs under quasi steady conditions. For a unit cube, the Warren and Root method yields a value of 60 for α. Using the equivalence given in (15) Narasimhan et al. (1987) have recently been able to independently verify the estimate for α provided by Warren and Root.

5. EXAMPLES

We illustrate the use of the MINC methodology with the help of two examples. The first of these relates to the problem of reactive chemical transport in a groundwater system taking into consideration diffusion in the solid phase. The second concerns the transport of heat in a two-phase geothermal reservoir.

5.1. Solid Diffusion

In considering reactive chemical transport in groundwater systems, diffusion into the solid particles is usually neglected for the reason that for inorganic species, diffusivity in the solid phase at room temperature is known to be $10^{-24}m^2$ per second or less, whereas diffusivity in pure water is known to be of the order of $10^{-10}m^2$ per second. However, in

strongly sorbing systems with very small particles, the very large surface areas available and the strong concentrations on the solid surface may provide enough driving forces to render solid diffusion a non-neglectable mechanism in certain field problems of interest.

To gain insights into the potential importance of solid diffusion, Narasimhan and Liu (1987) investigated reactive chemical transport in a porous medium comprised of spherical particles. The processes considered included advection due to a steady fluid flow field, molecular diffusion and hydrodynamic dispersion in the aqueous phase, adsorption according to a linear isotherm and solid diffusion. In the simulation exercises, the aqueous phase was treated as the HDC. The solid grains are the HDC and these were divided into six interacting continua. The adsorbed material on the surface of the solid was assumed to be uniformly dispersed over a thin shell of the solid with a thickness of the order of a nanometer. The interaction between the HDC and the LDC was handled using this outer skin of sorbed material as an intermediary. The simulations were carried out using computer program TRUMP (Edwards, 1972). Parametric studies were carried out by varying particle size, diffusivity values, and other quantities of interest.

In Figure 3 are presented the break-through curves for a one-meter column with different diffusivities and particle sizes. The results suggest that migration of chemical species into the solid phase could be significant and noticeable on a time scale of 10^3 to 10^4 years. Such a time scale is quite important in geological problems. Such a time scale may also be of potential importance in radioactive waste disposal problems.

5.2. Vapor-Dominated Geothermal Reservoir

The Geysers geothermal field in Lake County, California is a system that produces almost pure steam at a temperature of about 240° C. A hitherto unresolved issue about this reservoir relates to the source water that has been sustaining the steam production. One view proposed by Truesdell and White (1973) postulates a deep, boiling water table. Part of the reason for this suggestion is the assumption that the steam production is indicative of very low matrix saturation in the greywacke (a variety of sandstone) that constitutes the host rock.

Pruess and Narasimhan (1985) investigated the mechanism of fluid production at The Geysers by considering radial flow to a well in fractured rock of very low matrix permeability. The results suggested that in a fractured porous medium it is possible to have relatively high water saturation in the rock matrix yet producing steam from production wells. When a well is opened in a fractured medium with two-phase (water-steam) conditions in the matrix blocks, initially water is produced from the the fractures. As the pressures fall rapidly in the low-storage fractures, boiling occurs and steam fills the fractures. Simultaneously, a temperature gradient and a pressure gradient is set up from the interior of the blocks to the fractures. If the permeability of the blocks is small (of the order of a few microdarcies), the conductive heat transfer from the blocks to the fracture will be sufficient to provide the latent heat to convert to steam the small quantity of water that moves from the blocks to the fractures. Thus, a steam front will form at the fracture-rock interface and migrate very slowly towards the block interior. It is interesting that if the blocks have high permeability, then the conductive heat transfer cannot provide all the latent heat required and two-phase fluid will discharge from the

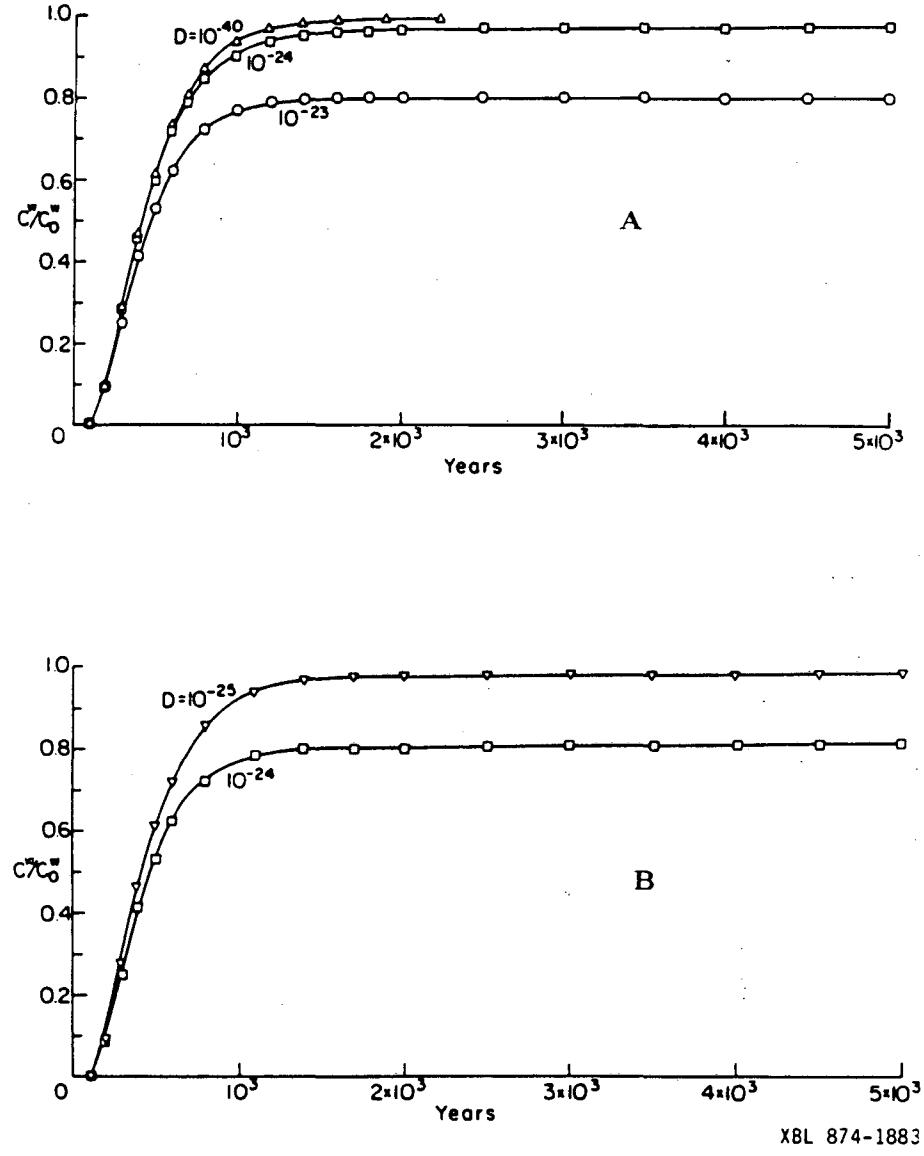

Figure 3 Breakthrough curves for a one-meter column involving advection, diffusion-dispersion, sorption and solid diffusion. D in m^2 per second. Darcy velocity 0.1 m per year. Distribution Coefficient, 2.8 10^{-2} m^3 per kg. (A) particle size 10 microns, (B) particle size 1 micron

matrix blocks into the fracture. An important consequence of this mechanism is the suggestion that the fluid saturation in the blocks may be far larger than residual saturation. If this is indeed so, the longevity of the reservoir may be much greater than hitherto assumed.

Figure 4 from Pruess and Narasimhan (1985) is a plot of enthalpy versus time for the problem investigated using MINC. Note that the highest enthalpy associated with pure steam production occurs in conjunction with the lowest permeability. Note also that a uniform porous medium cannot be used to simulate the pattern of behavior observed in strongly heterogeneous systems.

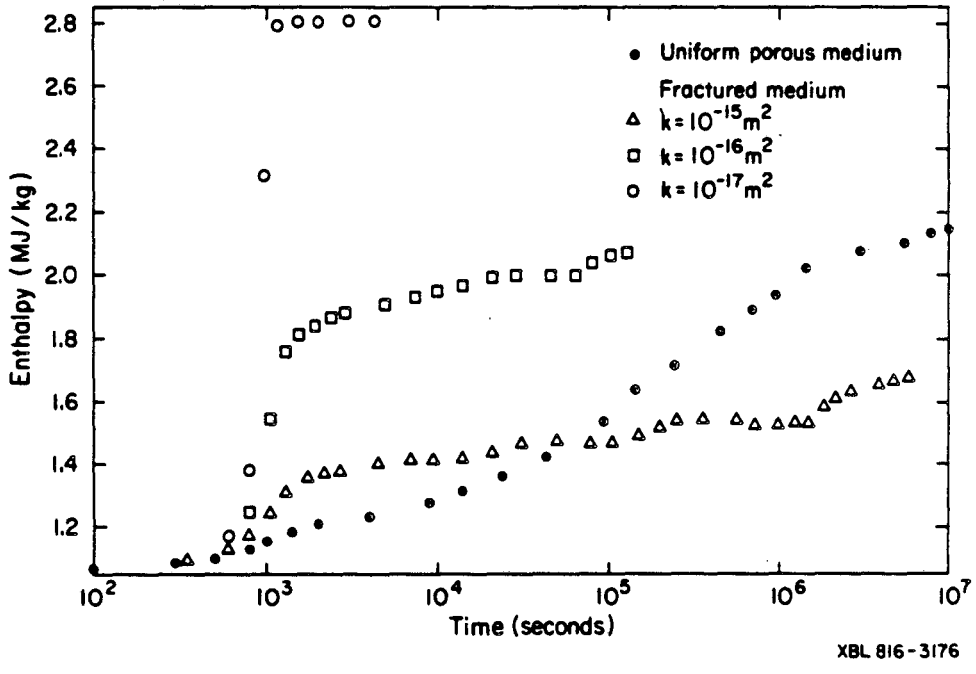

Figure 4 Effect of permeability on enthalpy-time relationship in a fractured porous sandstone reservoir

6. DISCUSSION

Most of the applications of the MINC-method to date have been made for geothermal reservoir engineering studies including fundamental studies of vapor-dominated systems (Pruess and Narasimhan, 1982; Pruess, 1985; Pruess et al., 1987) and heat extraction and reinjection of waste fluids (Bodvarsson et al., 1985; Lam et al., 1985). The method has

also been applied to the study of water flooding of petroleum reservoirs (Wu and Pruess, 1986) and chemical transport in the context of underground waste isolation (Rasmuson and Neretnieks, 1984).

It is appropriate at this juncture to discuss applications of the MINC not hitherto considered. Subject to the availability of data to characterize heterogenieties, the MINC idealization enables the consideration of several hierarchies and branches of continua. As an example, consider chemical transport in a fractured porous medium taking into account advection and diffusion in the aqueous phase and diffusion into the solid phase. In this case, the first level of hierarchy is constituted by the aqueous phase transport process occuring in the fractures (HDC) and in the matrix block (LDC). Transport in the fractures occurs by strong advection and transport in the matrix blocks occurs primarily by molecular diffusion in the stagnant aqueous phase in the interconnected pores and perhaps by very weak advection. Within a matrix block we may define a second hierarchy of continua constituted by the solid particles. Compared to the diffusivity in the aqueous phase, the diffusivity in the solid phase is extremely small. Therefore, one may use the MINC idealization at a second hierarchial level to quantify the interactions between the stagnant aqueous phase and the solid phase on a very large time scale. Hierarchies of contniua may also be formed by fracture sets on different sclaes (Nelson, 1987).

Furthermore, at a given hierarchial level, one may have an ensemble of parallel sets of continua. As an example, consider a fractured porous medium in which all the blocks are not of the same size or shape. In this case, within a single HDC element n, we may define sets of parallel continua, each set being characterized by a particular class of blocks.

Finally, one may also allow for spatial variations in the diffusivities of the HDC on a global scale.

In the context of a flexible algorithm the handling of the various hierarchies and branches is merely a matter of detail. The real limitation is the availability of field data. Observations that have been forthcoming in the last few years, especially in relation to the migration of organic contaminants, suggest that breakthrough curves may exhibit characteristic slope changes at different time scales, suggestive of MINC type of systems.

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