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ON ESTIMATING THE EXPECTED REAL RETURN ON THE  
MARKET IN A GENERAL EQUILIBRIUM FRAMEWORK

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by

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On Estimating The Expected Real  
Return On The Market In A General  
Equilibrium Framework

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(Comments Welcome)

Abstract: Two problems in estimating the expected real return on the market are dealt with: (1) the absence of reliable real data, (2) the absence of observations of market (i.e., economy-wide) returns. By combining financial and monetary theory, a general equilibrium model is constructed, both in a single-economy and a multi-economy setting, which indicate the variables to be used to avoid the estimation problems: (1) nominal stock index returns, (2) money supply data, (3) foreign exchange rate data. ...

## I. INTRODUCTION

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It is being noticed continuously that the accurate estimation of a key variable in financial theory, namely the expected real return on the market, is extremely difficult to accomplish, both because of the nominal character of all available data and because a thing such as the "market" is not readily observable. Many financial models assign an important role to the expected real return on the market, the most famous one being the Sharpe-Lintner-Mossin Capital Asset Pricing Model (or CAPM; see Sharpe (1964), Lintner (1965) and Mossin (1966)). Moreover, any investment analysis, whether it relies on those financial models or not, requires a good estimate of the expected real return on the market. For instance, institutional investors such as insurance companies and pension funds must have a clear idea of the aggregate real risk involved in investing in the economy, which is precisely the real market return.

Given the importance of the topic, it is surprising that so little attention has been paid to it. In fact, the only in-depth study of the estimation of the expected return on the market has been Merton's (see Merton (1980)). Merton is, however, more concerned with the huge variance of realized nominal returns on stock market indices. Merton claims that the use of excess realized nominal returns (where the interest rate on a nominally riskless asset is subtracted) properly accounts for any differences between the real return and the nominal return. This is an unhappy statement in an otherwise superb study. Indeed, it is correct only if the real rate of interest is constant over time, which recent evidence

suggests is not always the case. Merton also uses a constraint derived from a well-known theorem which states that the expected real return on any efficient portfolio (in casu, the market) should be at least as large as the return on an asset which is riskless in real terms (see Merton (1982)). However, Merton selects the yield on Treasury Bills for the constraint, but Treasury Bills can hardly be called riskless in real terms.

As mentioned before, the nominal character of all available data prohibits the estimation of real returns. The usual approach is to deflate the nominal returns using any price index. The result has, however, been shown to be not very robust with respect to the price index employed (see Cornell (1984)). Moreover, price index figures are generally available only monthly, and hence, the time series length needed to get accurate estimates of the expected real return may turn out to be longer than any period for which the constancy of the expected real return can safely be assumed. Finally, price indices are devoid of any sensible meaning in an international context, except under very specific circumstances, such as when consumer preferences are homothetic (see Adler and Dumas (1983)). Another way to obtain real returns is to subtract the nominal interest rate from the observed nominal rate of return. This method is justified only for specific eras, as explained before.

The other problem when estimating the expected real return on the market is yet more complex. The purpose is to estimate the expected real return on the total wealth invested in an economy, which is difficult to observe directly. Usually, proxies such as stock market

indices are substituted for total wealth. This practice is bound to fail, however, as was convincingly argued in Roll (1977).

This study is meant to be an exploratory investigation on alternative methods to solve both of these problems, based on an analysis of general equilibrium models. Of course, the second problem is harder to tackle, and, hence, only a very modest step may have been achieved here.

The main idea applied throughout the study is taken from Merton (1980), which is to combine financial theory with any relevant, regularly available information, in order to get a sensible estimator. However, with respect to the expected real market return, not much information is available from which to extract an estimate. Prices of financial securities are the most obvious source, but many of these securities are derivative assets, theoretically linked to underlying variables independent of the latter's expected return. Hence, no information whatsoever on the expected return of the underlying variables is implicit in the prices of these securities. Options on common stock are a case in point (see Black and Scholes (1973)). It seems that only stock market indices contain the necessary information, albeit in a very limited way: indirectly and in nominal terms. Of course, one could use non-market data in the estimation of the expected real return on the market, but it is not clear how to link them to returns, a problem that plainly emerges from the recent controversy about variance bounds on stock price changes (see e.g., LeRoy (1984)).

Data on stock market indices are in nominal terms, whereas an estimate of the return on an economy in real terms is asked for.

Financial theory cannot help us here, since most models are expressed in real terms, and the few exceptions include nominal elements by exogenously specifying a stochastic process for a price index - whatever this index may be (see e.g., Breeden (1979)). However, a link may possibly be established by appreciating the monetary character of prices. Consequently, an incorporation of monetary theory into financial theory is required, as has only recently been recognized by financial economists such as Fama and Farber (1979) and, most notably, Lucas (1982, 1984). Obviously, this requires a restatement of financial models in a general equilibrium framework.

The monetary character of prices will explicitly be considered here in order to find a link between nominal data and the expected real return on an economy. This will be done in a Cox-Ingersoll-Ross type of general equilibrium model (see their 1985 papers). Money will be introduced in the classical quantity theoretical way, contrary to Lucas' cash-in-advance concept (Lucas (1982,1984)) and Fama and Farber's liquidity-demand type of analysis (Fama and Farber (1979)). It will be assumed that no asset exists which is riskless in real terms.

In the next Section, the model will be developed for a single economy. As may have been anticipated, changes in the money supply provide the necessary information to move from nominal data to real returns. Notice that money supply figures are available weekly, enhancing the accuracy of the estimate of the expected real return. In Section III, the model will be developed in a two-economy framework. Here again, changes in the money supply provide the necessary information to

move from nominal to real terms, but there is more. Indeed, the foreign exchange rate, which is the relative value of each economy's currency, will be shown to reflect differences in real returns between the two economies, and, hence, it will supply additional information to estimate real market returns.

With respect to the problem of approximating the return on the market by focusing on stock market indices, it will turn out that a partial solution can be generated by thinking in an international (i.e., multi-economy) framework. The foreign exchange rate does not reflect differences in returns on particular stock indices, but differences in returns of economies as a whole. Consequently, the foreign exchange rate is one of the few prices which directly reflect the real return on the market.

Evidently, this study will at the same time generate an exchange rate determination model. Due to the inclusion of both financial and monetary elements, this model will look very much alike the constructs of the monetary and portfolio approach to the balance of payments in international finance, exemplified by papers of Black (1973), Dornbusch (1976), Frenkel (1976), Kouri (1976), Calvo and Rodriguez (1977), Bilson (1978), Frankel (1979), Stockman (1980), and Mussa (1982). The difference with those models lies in the fact that money will not be considered an asset here, but merely a kind of "grease" for the economy that helps to reduce the losses from trade frictions - as in Fama and Farber (1979).

Section IV constitutes the empirical part of the study.



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Discuss here: statistical properties, and:

(1) estimation in one-economy framework

(2) estimation in two-economy framework

of:

(1) real returns

(2) expected real returns.

Compare with data on economic activity.

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The exploratory nature of this study should be emphasized. Indeed, much of it relies on the links that can be established between monetary and financial theory. Such links have only recently begun to develop, and, consequently, research is just in its initial stage.

## II. A SIMPLE GENERAL EQUILIBRIUM MODEL

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The type of economy to be analysed here is similar to the one introduced in Cox, Ingersoll and Ross (1985a,1985b). Specifically, the following will be assumed.

Assumption 1. The economy possesses one facility that produces a random quantity of a single physical good.

Notice that riskless investment opportunities are hereby ruled out.

Assumption 2. A representative consumer maximizes each period an intertemporal von Neumann-Morgenstern (concave) utility function of consumption subject to a wealth constraint.

This means that the consumer determines each period the fraction of wealth to be allocated to consumption and to investment respectively. Denote the amount of wealth (which, of course, is in terms of the sole physical good available in the economy) at a given moment 0 by  $Y_0$ . The consumer will allocate  $cY_0$  of his wealth to consumption and  $wY_0$  to investment ( $w = 1-c$ ). At the next decision moment (1), the consumer will have acquired the amount  $wY_0(1 + R_1)$  of wealth through the production process, where  $R_1$  denotes the rate of return on investment, a random variable. He will then take a new decision as to consumption and investment. It is clear that the consumer's decision process is thus formulated in real terms. Money illusion is explicitly precluded.

The question is now whether the analysis should be continued in a discrete-time or a continuous-time framework. The latter has been the usual approach since Merton's 1971 article, but it may lead to untenable conclusions in the present context. Since continuous-time

analysis is based on the assumption that instantaneous rates of return follow a diffusion process, returns over any finite interval are lognormally distributed and their range as such includes values smaller than one. This means in the context of a single-good economy that the yield of the production process, conventionally called the real gross national product, may be negative with a positive probability. In order to avoid this, the subsequent analysis will be carried out in a discrete-time framework<sup>4</sup> with the following stochastic assumption.

Assumption 3.  $R$  is lognormally distributed. Hence  $r (= \ln R)$  is normally distributed with parameters  $\pi$  and  $\sigma^2$ , restricted to be constant over a sufficiently long interval.

(A list of the symbols to be used in this study is given in Table I, where it can be seen that small characters generally denote logarithms).

As should be clear from the Introduction, the estimation of  $\pi$  is the subject matter of this study. The constancy of the parameters is evidently required for any sensible estimation of  $\pi$ . After a sufficiently long interval the parameters are allowed to change randomly, but such changes must be stochastically independent over time, in order to not affect the consumption-investment decision.

As long as no further restriction is put on the utility function apart from it being of the von Neumann-Morgenstern type,  $w$  may fluctuate over time. It will, however, be assumed that utility is such that  $w$  is constant over time, as would be the case with Hyperbolic Absolute Risk Aversion (HARA) utility functions, a fairly general class.

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To be elaborated.  
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So far the real part of the economy has been discussed. But information is available only in nominal terms, hence the nature of prices and their relationship with real variables should be investigated. The price at time 0 ( $P_0$ ) and the price at time 1 ( $P_1$ ) (the subscripts may sometimes be deleted) are defined to be the value in units of the economy's currency of one unit of the physical good at time 0 and 1 respectively. Prices are a monetary phenomenon and the quantity theory of money will be assumed to hold, at least as an approximation.

Assumption 4.

$$P_1 = \frac{M_1}{k(wY_0 R_1)} e^{\epsilon_1}$$

where  $k$  is a constant and  $M_1$  stands for money supply (at time 1), an exogenously given random variable.  $\epsilon_1$  is a normally distributed random variable.

Notice that money is not an asset here, but a kind of "grease" needed in the process of trading the physical good of the economy. It is very difficult to model this transactions motive of money explicitly in a financial model - an attempt is made in Lucas (1982, 1984), so assumption 4 is taken for granted without explicit financial motivation.

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Suggestions are of course welcome !  
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As was pointed out earlier, the basic problem in estimating the expected real return on the market is that  $R$  is not observable directly. But, consider the ratio of the value of total wealth invested in two different periods:

$$S_1 = \frac{P_1 wY_0 (1 + R_1)}{P_0 wY_0}$$

$S$  is not observed either, but is approximated generally by the stock index return  $I$  (the ratio of the aggregate value of the securities traded in a stock market in two different periods).  $S$  includes all assets in the economy, whereas  $I$  only covers stock, hence the following assumption.

Assumption 5.  $I$  is observable each period and  $I = Se^\eta$ , where  $\eta$  is a normally distributed random variable.

Altogether, this model of a simple economy includes two observable variables from which to estimate the realized real return, and, hence, the expected real return: the stock index return ( $I$ ) and the change in the money supply  $(M_1/M_0)^2$ . In the following Proposition the exact relationship between the realized real return ( $R$ ) and the observable variables ( $I$  and  $M_1/M_0$ ) is given. This will be used in the empirical part of the study to be discussed in Section IV.

Proposition 1. If Assumptions 1, 2, 4 and 5 hold and  $w$  is constant over time, then

$$\ln \frac{R_1}{1 + R_1} = (m_1 - m_0) - i_1 + \ln \frac{R_0}{1 + R_0} + (\varepsilon_1 - \varepsilon_0) + \eta_1$$

Proof:

$$S_1 = \frac{P_1 w w Y_0 (1 + R_1)}{P_0 w Y_0}$$

(by Assumptions 1 and 2 and the definition of  $S$ )

$$S_1 = \frac{P_1 w (1 + R_1)}{P_0}$$

(because  $w$  is assumed to be constant over time)

But  $P_1 = [M_1 e^{\varepsilon_1}] / [k w Y_0 R_1]$ ,  $P_0 = [M_0 e^{\varepsilon_0}] / [k w Y_{-1} R_0]$ , where

$Y_0 = w(1 + R_0)Y_{-1}$ , by Assumption 4. Hence:

$$\begin{aligned} S_1 &= \frac{M_1 k w w Y_{-1} R_0 e^{\varepsilon_1 - \varepsilon_0}}{M_0 k w w (1 + R_0) Y_{-1} R_1} (1 + R_1) \\ &= \frac{M_1}{M_0} \frac{R_0}{1 + R_0} \frac{1 + R_1}{R_1} e^{\varepsilon_1 - \varepsilon_0} \end{aligned}$$

Also:  $S_1 = I_1 e^{-\eta_1}$ , by Assumption 5, and therefore,

$$I_1 e^{-\eta_1} = \frac{M_1}{M_0} \frac{R_0}{1 + R_0} \frac{1 + R_1}{R_1} e^{\varepsilon_1 - \varepsilon_0}$$

Taking logarithms,

$$i_1 - \eta_1 = (m_1 - m_0) + \ln \frac{R_0}{1 + R_0} - \ln \frac{R_1}{1 + R_1} + (\varepsilon_1 - \varepsilon_0)$$

Rearranging this gives Proposition 1.

It can be inferred from this proof that the (observed) stock index return ( $I$ ) depends on three elements, given Assumptions 1, 2, 4 and 5 and the constancy of  $w$ :

(1) the change in the realized real return of the economy,

(2) the change in the money supply,

(3) errors, due to the following:

(a) the quantity theory of money is assumed to hold only as an approximation (Assumption 4),

(b) the relationship between the nominal return on the market ( $S$ ) and the nominal return on the stock market ( $I$ ) is assumed to be known only up to a multiplicative error (Assumption 5).

The above summarizes the main features of the model.

If in addition the proportion allocated each period to investment, namely  $w$ , is allowed to vary over time, a much richer pattern for observed stock market returns will obtain. The proportion  $w$  may change for instance because of the arrival of new information to the market which indicates a change in the expected real return of the economy, thus eventually affecting the observed return on the stock market.

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Elaborate on such parametric  
changes - give examples.  
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### III. A TWO-ECONOMY GENERAL EQUILIBRIUM MODEL

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The simple general equilibrium model of the previous section can be extended in order to allow for the existence of different economies (or countries), each having its own money. Assumption 1 has to be replaced by the following.

Assumption 1'. There is one physical good; each country possesses one facility which produces a random quantity of this good.

Again, riskless investment opportunities are excluded. For simplicity, the model will be developed here for only two countries (indexed by a superscript  $k = 1, 2$ ).

The representative (world) consumer of Assumption 2 determines each period the fraction of wealth to be allocated to consumption and to investment respectively. As before, denote the amount of wealth (in terms of the sole physical good in the world) at a given moment 0 by  $Y_0$ . The consumer will allocate  $cY_0$  of his wealth to consumption and the remainder  $((1 - c)Y_0)$  to investment. An amount  $w^1Y_0$  will be invested in the first country's production facility, whereas  $w^2Y_0$  will be invested in the other country's facility ( $w^1 + w^2 = 1 - c$ ). At the next decision moment (1), the consumer will have acquired the amount  $Y_1 = w^1Y_0(1 + R_1^1) + w^2Y_0(1 + R_1^2) = Y_0[w^1(1 + R_1^1) + w^2(1 + R_1^2)]$  of wealth through both production processes, where  $R_1^k$  denotes the rate of return on investment ( $k = 1, 2$ ), again a random variable. He will then take a new decision as to consumption and investment.

By restricting the utility functions to the class that generates



linear demand functions,  $c$ ,  $w^1$  and  $w^2$  can be taken to be constant over time, as in the previous section.

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 Again, to be elaborated.  
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Prices are defined as before, but here each country will have its own set of prices. Trade in a country's output is carried out in that country's currency, such that Assumption 4 continues to hold for each country, i.e.,

$$P_1^k = \frac{M_1^k}{k(w^k Y_0 R_1^k)} e^{\varepsilon_1^k}$$

for  $k = 1, 2$ .

The real return on each country's economy or market  $R^k$  is not observable here either, and it will have to be approximated in order to estimate  $E(\ln R^k)$  ( $= E(r^k) = \pi^k$ ,  $k = 1, 2$ ). Let  $S^k$ , the ratio of the value of total wealth invested in country  $k$  in two different periods, be defined as:

$$S_1^k = \frac{P_1^k w^k Y_1}{P_0^k w^k Y_0} = \frac{P_1^k w^k Y_0 [w^1(1 + R_1^1) + w^2(1 + R_1^2)]}{P_0^k w^k Y_0}$$

(Notice that  $S^k$  is denominated in each country's currency).  $S^k$  will be approximated by country  $k$ 's stock index return  $I^k$ , as in Assumption 5:

$$I^k = S^k e^{\eta^k}$$

It is then straightforward to derive Proposition 2, the analog of Proposition 1 in a two-country setting.

Proposition 2. If Assumptions 1', 2, 4 and 5 hold and  $w^1, w^2$  are constant over time, then

$$\ln \frac{R_1^1}{1+w^1 R_1^1+w^2 R_1^2} = (m_1^1 - m_0^1) - i_1^1 + \ln \frac{R_0^1}{1+w^1 R_0^1+w^2 R_0^2} + (\varepsilon_1^1 - \varepsilon_0^1) + \eta_1^1$$

$$\ln \frac{R_1^2}{1+w^1 R_1^1+w^2 R_1^2} = (m_1^2 - m_0^2) - i_1^2 + \ln \frac{R_0^2}{1+w^1 R_0^1+w^2 R_0^2} + (\varepsilon_1^2 - \varepsilon_0^2) + \eta_1^2$$

where  $w^1$ , and  $w^2$  are given by

$$\frac{w^1}{w^2} = \frac{P_1^k w^1 Y_1}{P_1^k w^2 Y_1}$$

(which is the ratio of invested wealth denominated in terms of a single currency -  $k = 1,2$ ) and

$$w^1 + w^2 = 1$$

Proof: By Assumptions 1' and 2 and the definition of  $S^k$ ,

$$\begin{aligned} S^k &= \frac{P_1^k w^1 Y_1 [w^1(1 + R_1^1) + w^2(1 + R_1^2)]}{P_0^k w^k Y_0} \\ &= \frac{P_1^k [w^1(1 + R_1^1) + w^2(1 + R_1^2)]}{P_0^k} \end{aligned}$$

because  $w^k$  is assumed to be constant over time. But  $P_1^k = [M_1^k e^{\varepsilon_1^k}] / [k w^k Y_0 R_1^k]$ , and  $P_0^k = [M_0^k e^{\varepsilon_0^k}] / [k w^k Y_{-1} R_0^k]$ , where  $Y_0 = Y_{-1} [w^1(1 + R_0^1) + w^2(1 + R_0^2)]$ , by Assumption 4. Hence,

$$\begin{aligned} S_1^k &= \frac{M_1^k}{M_0^k} \frac{k w^k Y_{-1} R_0^k}{k w^k Y_0 R_1^k} e^{\varepsilon_1^k - \varepsilon_0^k} [w^1(1 + R_1^1) + w^2(1 + R_1^2)] \\ &= \frac{M_1^k}{M_0^k} \frac{R_0^k}{[w^1(1 + R_0^1) + w^2(1 + R_0^2)]} \frac{[w^1(1 + R_1^1) + w^2(1 + R_1^2)]}{R_1^k} e^{\varepsilon_1^k - \varepsilon_0^k} \end{aligned}$$

Without loss of generality, we can assume that  $w^1 + w^2 = 1$  (to see

this, substitute  $w^1/(1-c)$  and  $w^2/(1-c)$  for  $w^1$  and  $w^2$  respectively).

Also,  $S_1^k = I_1^k e^{-\eta_1^k}$ , by Assumption 5, hence,

$$I_1^k e^{\eta_1^k} = \frac{M_1^k}{M_0^k} \frac{R_0^k}{[w^1(1 + R_1^1) + w^2(1 + R_1^2)]} \frac{[w^1(1 + R_1^1) + w^2(1 + R_1^2)]}{R_1^k} e^{\varepsilon_1^k - \varepsilon_0^k}$$

Which, after taking logarithms and rearranging gives the result in the Proposition, for  $k = 1, 2$ . Moreover,  $w^1$  and  $w^2$  are given by

$$\frac{w^1}{w^2} = \frac{P_1^k w^1 Y_1}{P_1^k w^2 Y_1} \quad (\text{trivially})$$

and  $w^1 + w^2 = 1$

Observe that the ratio of total wealth (denominated in one country's currency) invested in each country is perfectly correlated over time (it is equal to  $w^1/w^2$ , a constant). This is a well-known conclusion in intertemporal analysis of investment decisions under linear demand functions.

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 How to observe  $w^1/w^2$  ? Use constancy of  
 $w^1/w^2$  and observations on  $I^1$  and  $I^2$ .  
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Notice also that  $R^1$  and  $R^2$  are determined simultaneously. Hence, as can be inferred from the above proof, the (observed) stock index returns ( $I^1$  and  $I^2$ ) will depend on facts attributable to either country:

- (1) the change in the realized real return of either economy,
- (2) the change in either country's money supply,
- (3) errors due to the approximation in the quantity theory of money and the nominal return on the market, which apply to both

countries.

-----  
Elaborate on the effects of parametric  
changes as in Section II  
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As pointed out in the Introduction, there is yet another observable variable in a multi-economy framework, namely the foreign exchange rate, which will be denoted by  $E$ .

Assumption 6. The foreign exchange rate  $E$  is the purchasing power of each country's money to acquire one unit of the physical good, i.e.,  $P^1 = EP^2$ .

The so-called Law of One Price can be recognized in this Assumption. It also emphasizes prices as a monetary phenomenon. Indeed, Assumption 6 is the cornerstone of the monetary and portfolio approach to the balance of payments. The structure of the multi-economy model introduced here - specifically the fact that trade in a country's output is carried out in that country's currency, and Assumption 6 imply that the foreign exchange rate reflects among other things the differences in realized real returns across countries. The exact relationship is shown in the next Proposition.

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| Proposition 3. If Assumptions 1', 4 and 6 hold and  $w^1$  and  $w^2$  are  
| constant over time, then

$$r_1^1 - r_1^2 = (m_1^1 - m_0^1) - (m_1^2 - m_0^2) + (r_0^1 - r_0^2) - (e_1 - e_0) + (\varepsilon_1^1 - \varepsilon_0^1) - (\varepsilon_1^2 - \varepsilon_0^2)$$

Proof: By Assumptions 1' and 6,  $P_1^1 = E_1 P_1^2$  and  $P_0^1 = E_0 P_0^2$ , and by Assumption 4,

$$E_1 = \left\{ \frac{M_1^1 e^{\varepsilon_1^1}}{k w^1 Y_0 R_1^1} \right\} / \left\{ \frac{M_1^2 e^{\varepsilon_1^2}}{k w^2 Y_0 R_1^2} \right\}$$

$$E_0 = \left\{ \frac{M_0^1 e^{\varepsilon_0^1}}{k w^1 Y_{-1} R_0^1} \right\} / \left\{ \frac{M_0^2 e^{\varepsilon_0^2}}{k w^2 Y_{-1} R_0^2} \right\}$$

Hence,

$$\frac{E_1}{E_0} = \frac{M_1^1 e^{\varepsilon_1^1} R_1^2}{R_1^1 M_1^2 e^{\varepsilon_1^2}} \frac{M_0^2 e^{\varepsilon_0^2} R_0^1}{R_0^2 M_0^1 e^{\varepsilon_0^1}}$$

Taking logarithms and rearranging gives Proposition 3.

A remarkable fact is that the exchange rate estimates the realized real return differential more efficiently than the stock indices  $I^1$  and  $I^2$ . Indeed, rearranging the result in Proposition 2 gives:

$$r_1^1 - r_1^2 = (m_1^1 - m_0^1) - (m_1^2 - m_0^2) + (r_0^1 - r_0^2) - (i_1^1 - i_1^2) \\ + (\varepsilon_1^1 - \varepsilon_1^2) - (\varepsilon_0^1 - \varepsilon_0^2) + (\eta_1^1 - \eta_1^2)$$

Hence, the variance of the estimated real return differential will be larger when using  $i_1^1$  ( $=\ln I_1^1$ ) and  $i_1^2$  ( $=\ln I_1^2$ ) than when using  $e_1$  ( $=\ln E_1$ ) and  $e_0$  ( $=\ln E_0$ ), if the error terms are approximately uncorrelated. This is because stock indices reflect only partially the total market, whereas the foreign exchange rate clearly completely reflects each economy. Consequently, the use of exchange rate information will help to solve in part the problem underlying Roll's critique (Roll (1977)), as was explained in Section I.

The formal construct introduced here implies at the same time an exchange rate determination model. Two facts emerge from the model:

(1) if the change in the money supply ( $m_1^k - m_0^k$ ) in each country is

assumed to be normally distributed,  $e_1 - e_0$  will also be normally distributed, and, hence,  $E_1 - E_0$  will be lognormally distributed, a quite realistic conclusion;

- (2) the change in the exchange rate is affected by the monetary policy in both countries (i.e., changes in the money supply), errors in the quantity theory of money and differences in the realized real returns across countries.

In summary: a multi-economy general equilibrium model is introduced in this section, from which foreign exchange rate observations appear to be valuable for estimating expected real returns on the market, in addition to data on stock indices and the money supply.

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Study here effects of parametric changes on the exchange rate. Also, generalize the model, e.g. allow for two goods produced in each country; intuitively this should generate a model which fairly well explains recent experience with flexible exchange rates, for instance:

- purchasing power parity need not hold
  - the recent appreciation of the US Dollar and subsequent minor depreciation can be explained in terms of changes in perceived expected real return differences across countries, given rigorous monetary policies.
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## NOTES

(1) see e.g., Samuelson (1969), Fama (1970) and Hakansson (1970).

The basic time interval will be taken to be one week, as this is the frequency with which data on the money supply become available.

(2) Notice that the two nominal returns in the model (I and S) need not be strictly larger than one, contrary to the real return  $(1 + R)$ . Changes in  $w$  or  $M$  may lead to values for I and S strictly smaller than one.

(3) The equation for the exchange rate changes implicit in Proportion 3 is very similar to the one used in some recent studies of the determinants of the exchange rate based on rational expectations models (but where National Income is substituted for the real return on the economy - see Frenkel (1980) and Edwards (1983)).

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 Table 1. Explanation of widely used symbols.  
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$Y_0, Y_1$	Total (real) wealth of an economy in period 0 and 1 respectively
$y_0, y_1$	$\ln Y_0$ and $\ln Y_1$ .
$R_0, R_1$	Total (real) return on an economy in period 0 and 1 respectively (country k when superscripts are present).
$r_0, r_1$	$\ln R_0$ and $\ln R_1$ .
$\pi$	$E(\ln R)$ .
$\sigma^2$	$E\{[\ln R - E(\ln R)]^2\}$ .
$c$	Fraction of wealth the representative consumer allocates to consumption.
$w$	Fraction of wealth the representative consumer allocates to investment in a one-economy model.
$w^k$	Fraction of wealth the representative consumer allocates to investment in country k in a multi-economy model.
$P_0, P_1$	Price of one unit of the physical good (in terms of the currency of country k when superscripts are present), for period 0 and 1 respectively.
$p_0, p_1$	$\ln P_0$ and $\ln P_1$ .
$M_0, M_1$	Money supply (of country k when superscripts are present), for period 0 and 1 respectively.
$m_0, m_1$	$\ln M_0$ and $\ln M_1$ .
$S$	Ratio of the value of total wealth invested in a country (country k when superscripts are present) in two different periods, expressed in that country's currency.
$s$	$\ln S$ .
$I$	Stock market return (of country k when superscripts are present).
$i$	$\ln I$ .
$E$	Foreign exchange rate between two countries.
$e$	$\ln E$ .
$k$	Money velocity constant.

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