Title

Heavy quark energy loss in nuclear medium

Authors

Zhang, Benr-Wei
Wang, Enke
Wang, Xin-Nian

Publication Date

2003-09-16

Heavy Quark Energy Loss in Nuclear Medium

Ben-Wei Zhang[a], Enke Wang[a] and Xin-Nian Wang[b,c]
[a] *Institute of Particle Physics, Huazhong Normal University, Wuhan 430079, China*
[b] *Nuclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA*
[c] *Department of Physics, Shandong University, Jinan 250100, China*
(September 5, 2003)

Multiple scattering, modified fragmentation functions and radiative energy loss of a heavy quark propagating in a nuclear medium are investigated in perturbative QCD. Because of the quark mass dependence of the gluon formation time, the medium size dependence of heavy quark energy loss is found to change from a linear to a quadratic form when the initial energy and momentum scale are increased relative to the quark mass. The radiative energy loss is also significantly suppressed relative to a light quark due to the suppression of collinear gluon emission by a heavy quark.

An energetic parton propagating in a dense medium suffers a large amount of energy loss due to multiple scattering and induced gluon bremsstrahlung [1]. In a static medium, the total energy loss of a massless parton (light quark or gluon) is found to have a quadratic dependence on the medium size [2–6] due to non-Abelian Landau-Pomeranchuk-Migdal (LPM) interference effect. In an expanding medium, the total energy loss can be cast into a line integral weighted with local gluon density along the parton propagation path [7–9]. Therefore, the measurement of parton energy loss can be used to study properties of the medium similar to the technique of computed tomography. Recent experimental measurements [10,11] of centrality dependence of high-p_T hadron suppression agree very well [12] with such a parton energy loss mechanism.

Because of the large mass of the heavy quark with a velocity $v \approx 1 - M^2/2E^2$, the formation time of gluon radiation, $\tau_f \sim 1/(\omega_g M^2/2E^2 + \ell_T^2/2\omega_g)$ is reduced relative to a light quark. One should then expect the LPM effect to be significantly reduced for intermediate energy heavy quarks. In addition, the heavy quark mass also suppresses gluon radiation amplitude at angles smaller than the ratio of the quark mass to its energy [13] relative to the gluon radiation off a light quark. Both mass effects will lead to a heavy quark energy loss different from a light quark propagating in a dense medium. This might explain why one has not observed significant heavy quark energy loss from the PHENIX [14] measurement of the single electron spectrum from charm production in $Au + Au$ collisions at $\sqrt{s} = 130$ GeV. In this Letter, we report a study on medium induced energy loss and the modified fragmentation function of a heavy quark. In particular, we will show how the mass effects reduce the total energy loss and how the medium size dependence changes from a linear dependence to a quadratic one when the energy of the heavy quark or the momentum scale is increased. Similar results have been reported in Ref. [15] during the completion of this work.

To separate the complication of heavy quark production and propagation, we consider a simple process of charm quark production via the charge-current interaction in DIS off a large nucleus. The results can be easily extended to heavy quark propagation in other dense media. The differential cross section for the semi-inclusive process $\ell(L_1) + A(p) \longrightarrow \nu_\ell(L_2) + H(\ell_H) + X$ can be expressed as

$$E_{L_2} E_{\ell_H} \frac{d\sigma_{\rm DIS}}{d^3L_2 d^3\ell_H} = \frac{G_{\rm F}^2}{(4\pi)^3 s} L_{\mu\nu}^{cc} E_{\ell_H} \frac{dW^{\mu\nu}}{d^3\ell_H} . \qquad (1)$$

Here L_1 and L_2 are the four momenta of the incoming lepton and the outgoing neutrino, ℓ_H the observed heavy quark meson momentum, $p = [p^+, m_N^2/2p^+, 0_\perp]$ is the momentum per nucleon in the nucleus, and $s = (p+L_1)^2$. $G_{\rm F}$ is the four-fermion coupling constant and $q = L_2 - L_1 = [-Q^2/2q^-, q^-, 0_\perp]$ the momentum transfer via the exchange of a W-boson. The charge-current leptonic tensor is given by $L_{\mu\nu}^{cc} = 1/2{\rm Tr}(\not{L}_1 \gamma_\mu (1-\gamma_5) \not{L}_2 (1-\gamma_5)\gamma_\nu)$. We assume $Q^2 \ll M_W^2$. The semi-inclusive hadronic tensor is defined as,

$$E_{\ell_H} \frac{dW_{\mu\nu}}{d^3\ell_H} = \frac{1}{2} \sum_X \langle A | J_\mu^+ | X, H\rangle \langle X, H | J_\nu^{+\dagger} | A\rangle \times 2\pi\delta^4(q + p - p_X - \ell_H) \qquad (2)$$

where $\sum_X$ runs over all possible final states and $J_\mu^+ = \bar{c}\gamma_\mu(1-\gamma_5)s_\theta$ is the hadronic charged current. Here, $s_\theta = s\cos\theta_{\rm C} - d\sin\theta_{\rm C}$ and $\theta_{\rm C}$ is the Cabibbo angle. To the leading-twist in collinear approximation, the semi-inclusive cross section factorizes into the product of quark distribution $f_{s_\theta}^A(x_B + x_M)$, the heavy quark fragmentation function $D_{Q\to H}(z_H)$ ($z_H = \ell_H^-/\ell_Q^-$) and the hard partonic part $H_{\mu\nu}^{(0)}(k, q, M)$ [17]. Here, $x_B = Q^2/2p^+q^-$ is the Bjorken variable and $x_M = M^2/2p^+q^-$.

Similar to the case of light quark propagation in nuclear medium [6], the generalized factorization of multiple scattering processes [16] will be employed. We will only consider double parton scattering. The leading contributions are the twist-four terms that are enhanced by the nuclear medium in a collinear expansion, assuming a small expansion parameter $\alpha_s A^{1/3}/Q^2$. The evaluation of 23 cut diagrams are similar to the case of a light quark [17]. The dominant contribution comes from the central cut diagram, giving the semi-inclusive tensor for heavy quark fragmentation from double quark-gluon scattering,

$$\frac{W^D_{\mu\nu}}{dz_h} = \sum \int dx H^{(0)}_{\mu\nu} \int_{z_h}^1 \frac{dz}{z} D_{Q\to H}(\frac{z_H}{z}) \frac{C_A\alpha_s}{2\pi}\frac{1+z^2}{1-z} \times \int \frac{d\ell_T^2}{[\ell_T^2+(1-z)^2M^2]^4}\ell_T^4 \frac{2\pi\alpha_s}{N_c} T^{A,C}_{qg}(x,x_L,M^2) + (g-\text{frag.}) + (\text{virtual corrections}), \quad (3)$$

where

$$T^{A,C}_{qg}(x,x_L,M^2) = \frac{1}{2}\int \frac{dy^-}{2\pi} dy_1^- dy_2^- \widetilde{H^D_C} \times \langle A|\bar\psi_q(0)\,\gamma^+ F_\sigma{}^+(y_2^-)\,F^{+\sigma}(y_1^-)\,\psi_q(y^-)|A\rangle \times e^{i(x+x_L)p^+y^-}\theta(-y_2^-)\theta(y^- - y_1^-) \quad (4)$$

are twist-four quark-gluon correction functions of the nucleus. Here

$$\widetilde{H^D_C} = c_1(z,\ell_T^2,M^2)(1-e^{-i\widetilde{x}_Lp^+y_2^-})(1-e^{-i\widetilde{x}_Lp^+(y^- - y_1^-)}) + c_2(z,\ell_T^2,M^2)[e^{-i\widetilde{x}_Lp^+y_2^-}(1-e^{-i\widetilde{x}_Lp^+(y^- - y_1^-)}) + e^{-i\widetilde{x}_Lp^+(y^- - y_1^-)}(1-e^{-i\widetilde{x}_Lp^+y_2^-})] + c_3(z,\ell_T^2,M^2)e^{-i\widetilde{x}_Lp^+(y^- - y_1^-)}e^{-i\widetilde{x}_Lzp^+y_2^-} \quad (5)$$

and

$$c_1 = 1 + \frac{(1-z)^2(z^2-6z+1)}{1+z^2}\frac{M^2}{\ell_T^2} + \frac{2z(1-z)^4}{1+z^2}\frac{M^4}{\ell_T^4}, \quad (6)$$

$$c_2 = \frac{(1-z)}{2}\left\{1 - \left[\frac{(1-z)(2z^3-5z+8z-1)}{(1+z^2)} + \frac{2C_F}{C_A}(1-z)^2\right]\frac{M^2}{\ell_T^2} - \left[\frac{z(1-z)^4(3z-1)}{(1+z^2)} + \frac{2C_F}{C_A}\frac{(1-z)^7}{(1+z^2)}\right]\frac{M^4}{\ell_T^4}\right\}, \quad (7)$$

$$c_3 = \frac{C_F(1-z)^2}{C_A}\left[1 - \frac{8z(1-z)^2}{1+z^2}\frac{M^2}{\ell_T^2} - \frac{(1-z)^4(z^2-4z+1)}{1+z^2}\frac{M^4}{\ell_T^4}\right], \quad (8)$$

where, $\widetilde{x}_L \equiv x_L + (1-z)x_M/z$ is the additional fractional momentum of the initial quark or gluon in the rescattering that is required for gluon radiation, and $x_L = \ell_T^2/2p^+q^-z(1-z)$. The contribution from gluon fragmentation is similar to that from quark fragmentation with $z \to 1-z$. The virtual correction can be obtained via unitarity constraint. One can recover the results for light quark rescattering [18] by setting $M=0$ in the above equations. Notice that we have embedded the phase factors from the LPM interference in the effective twist-four parton matrix $T^{A,C}_{qg}(x,x_L,M^2)$.

Rewriting the sum of single and double scattering contributions in a factorized form for the semi-inclusive hadronic tensor, one can define a modified effective fragmentation function $\widetilde{D}_{Q\to H}(z_H,\mu^2)$ as

$$\widetilde{D}_{Q\to H}(z_H,\mu^2) \equiv D_{Q\to H}(z_H,\mu^2) + \int_0^{\mu^2}\frac{d\ell_T^2}{\ell_T^2+(1-z)^2M^2} \times \frac{\alpha_s}{2\pi}\int_{z_h}^1\frac{dz}{z}\Delta\gamma_{q\to qg}(z,x,x_L,\ell_T^2,M^2)D_{Q\to H}(\frac{z_H}{z}) + \int_0^{\mu^2}\frac{d\ell_T^2}{\ell_T^2+z^2M^2}\frac{\alpha_s}{2\pi}\int_{z_h}^1\frac{dz}{z} \times\Delta\gamma_{q\to gq}(z,x,x_L,\ell_T^2,M^2)D_{g\to H}(\frac{z_H}{z}), \quad (9)$$

where $D_{Q\to H}(z_H,\mu^2)$ and $D_{g\to H}(z_H,\mu^2)$ are the leading-twist fragmentation functions of the heavy quark and gluon. The modified splitting functions are given as

$$\Delta\gamma_{q\to qg}(z) = \left[\frac{1+z^2}{(1-z)_+}T^{A,C}_{qg}(x,x_L,M^2) + \delta(1-z)\Delta T^{A,C}_{qg}(x,\ell_T^2,M^2)\right] \times\frac{2\pi C_A\alpha_s\ell_T^4}{[\ell_T^2+(1-z)^2M^2]^3N_cf_q^A(x)}, \quad (10)$$

$$\Delta T^{A,C}_{qg}(x,\ell_T^2,M^2) \equiv \int_0^1\frac{dz}{1-z}\left[2T^{A,C}_{qg}(x,x_L,m^2)|_{z=1} - (1+z^2)T^{A,C}_{qg}(x,x_L,M^2)\right], \quad (11)$$

and $\Delta\gamma_{q\to gq}(z) = \Delta\gamma_{q\to qg}(1-z)$. Here we have suppressed other variables in $\Delta\gamma$. Given the twist-four quark-gluon correction functions of the nucleus, $T^{A,C}_{qg}(x,\ell_T^2,M^2)$, one should be able to evaluate the modified heavy-quark fragmentation function.

As seen from the phase factors in the effective twist-four matrix element Eq. (5), the gluon formation time for radiation from a heavy quark is

$$\tau_f \equiv \frac{1}{p^+\widetilde{x}_L} = \frac{2z(1-z)q^-}{\ell_T^2+(1-z)^2M^2}, \quad (12)$$

which is shorter than that for gluon radiation from a light quark. This should have significant consequences for the effective modified quark fragmentation function and the heavy quark energy loss.

As discussed previously [19], one can assume a factorized form of the twist-four parton matrix

$$T^{A,C}_{qg}(x,x_L,M^2) \approx \frac{\widetilde{C}}{x_A}f_q^A(x)\left\{(1-e^{-\widetilde{x}_L^2/x_A^2}) \times[c_1(z,\ell_T^2,M^2)-c_2(z,\ell_T^2,M^2)] + \frac{c_3(z,\ell_T^2,M^2)}{2}\right\}, \quad (13)$$

in the limit $x_L \ll x_T \ll x$. Here $x_A \equiv 1/m_NR_A$ and $x_T \equiv \langle k_T^2\rangle/2p^+q^-z$ is the momentum fraction associated with the initial intrinsic transverse momentum. The coefficient $\widetilde{C} \equiv 2Cx_T f_g^N(x_T)$ should in principle depend on Q^2. With this simplified form of twist-four matrix, one can then calculate the heavy quark energy loss, defined as the fractional energy carried by the radiated gluon,

$$\begin{aligned}\langle \Delta z_g^Q \rangle(x_B, Q^2) &= \frac{\alpha_s}{2\pi}\int_0^{Q^2} d\ell_T^2 \int_0^1 dz \frac{\Delta\gamma_{q\to gq}(z)}{\ell_T^2 + z^2 M^2} z \\ &= \frac{\widetilde{C} C_A \alpha_s^2 x_B}{N_c Q^2 x_A}\int_0^1 dz \frac{1+z^2}{z(1-z)}\int_{\widetilde{x}_M}^{\widetilde{x}_\mu} d\widetilde{x}_L \frac{(\widetilde{x}_L - \widetilde{x}_M)^2}{\widetilde{x}_L^4} \\ &\times \left\{ \frac{1}{2} c_3(z, \ell_T^2, M^2) + \left(1 - e^{-\widetilde{x}_L^2/x_A^2}\right) \right. \\ &\times \left. \left[c_1(z, \ell_T^2, M^2) - c_2(z, \ell_T^2, M^2)\right] \right\}, \qquad (14)\end{aligned}$$

where $\widetilde{x}_M = (1-z)x_M/z$ and $\widetilde{x}_\mu = \mu^2/2p^+q^-z(1-z) + \widetilde{x}_M$. Note that $\widetilde{x}_L/x_A = L_A^-/\tau_f$ with $L_A^- = R_A m_N/p^+$ the nuclear size in the chosen frame. The LPM interference is clearly contained in the second term of the integrand that has a suppression factor $1-e^{-\widetilde{x}_L^2/x_A^2}$. The first term that is proportional to $c_3(z, \ell_T^2, M^2)$ corresponds to a finite contribution in the factorization limit. We have neglected such a term in the study of light quark propagation since it is proportional to R_A, as compared to the R_A^2 dependence from the first term due to LPM effect. We have to keep the first term for heavy quark propagation since the second term will have a similar nuclear dependence when the mass dependence of the gluon formation time is important.

Since $\widetilde{x}_L/x_A \sim x_B M^2/x_A Q^2$, there are two distinct limiting behaviors of the energy loss for different values of x_B/Q^2 relative to x_A/M^2. When $x_B/Q^2 \gg x_A/M^2$ for small quark energy (large x_B) or small Q^2, the formation time of gluon radiation off a heavy quark is always smaller than the nuclear size. In this case, $1-\exp(-\widetilde{x}_L^2/x_A^2) \simeq 1$, so that there is no destructive LPM interference. The integral in Eq. (14) is independent of R_A, and the heavy quark energy loss

$$\langle \Delta z_g^Q \rangle \sim C_A \frac{\widetilde{C}\alpha_s^2}{N_c}\frac{x_B}{x_A Q^2} \qquad (15)$$

is linear in nuclear size R_A. In the opposite limit, $x_B/Q^2 \ll x_A/M^2$, for large quark energy (small x_B) or large Q^2, the quark mass becomes negligible. The gluon formation time could still be much larger than the nuclear size. The LPM suppression factor $1-\exp(-\widetilde{x}_L^2/x_A^2)$ will limit the available phase space for gluon radiation. The integral in Eq. (14) will be proportional to $\int d\widetilde{x}_L[1-\exp(-\widetilde{x}_L^2/x_A^2)]/\widetilde{x}_L^2 \sim 1/x_A$. The heavy quark energy loss

$$\langle \Delta z_g^Q \rangle \sim C_A \frac{\widetilde{C}\alpha_s^2}{N_c}\frac{x_B}{x_A^2 Q^2} \qquad (16)$$

now has a quadratic dependence on the nuclear size similar to the light quark energy loss. Shown in Fig. 1 are the numerical results of the R_A dependence of charm quark energy loss, rescaled by $\widetilde{C}(Q^2)C_A\alpha_s^2(Q^2)/N_C$, for different values of x_B and Q^2. One can clearly see that the R_A dependence is quadratic for large values of Q^2 or small x_B. The dependence becomes almost linear for small Q^2 or large x_B. The charm quark mass is set at $M = 1.5$ GeV in the numerical calculation.

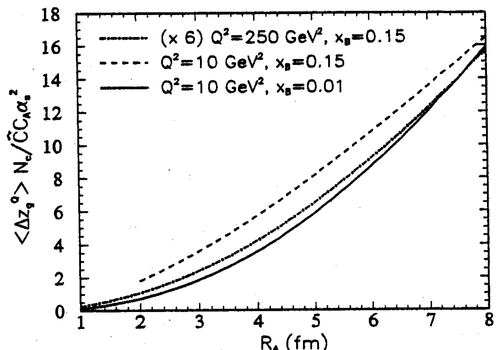

FIG. 1. The nuclear size, R_A, dependence of charm quark energy loss for different values of Q^2 and x_B.

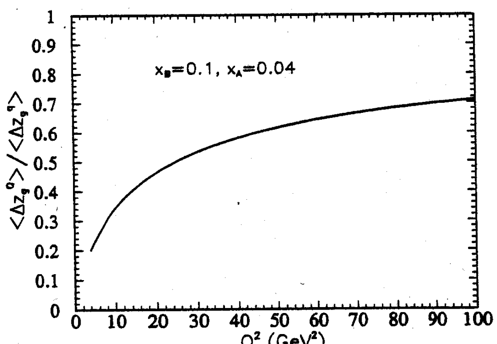

FIG. 2. The Q^2 dependence of the ratio between charm quark and light quark energy loss in a large nucleus.

Another mass effect on the induced gluon radiation is the "dead-cone" phenomenon [13] which suppresses the the small angle gluon radiation. Such a "dead-cone" effect is manifested in Eq. (3) for the induced gluon spectra from a heavy quark which is suppressed by a factor

$$f_{Q/q} = \left[\frac{\ell_T^2}{\ell_T^2 + z^2 M^2}\right]^4 = \left[1 + \frac{\theta_0^2}{\theta^2}\right]^{-4}, \qquad (17)$$

relative to that of a light quark for small angle radiation. Here $\theta_0 = M/q^-$ and $\theta = \ell_T/q^- z$. This will lead to a reduced radiative energy loss of a heavy quark, amid other mass dependence as contained in $c_i(z, \ell_T^2, M^2)$ in Eqs. (6)-(8). Setting $M = 0$ in Eq. (14), we recover the energy loss for light quarks as in our previous study [18]. To illustrate the mass suppression of radiative energy loss imposed by the "dead-cone", we plot the ratio $\langle \Delta z_g^Q \rangle (x_B, Q^2)/\langle \Delta z_g^q \rangle (x_B, Q^2)$ of charm quark and light quark energy loss as functions of Q^2 and x_B in Figs. 2 and 3.

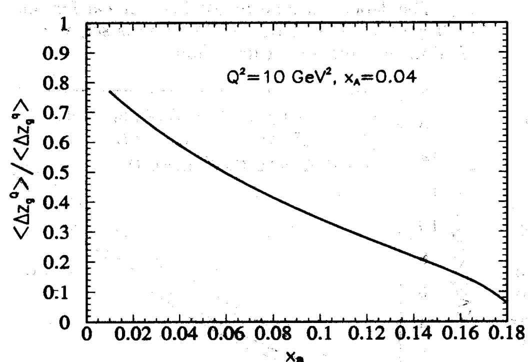

FIG. 3. The x_B dependence of the ratio between charm quark and light quark energy loss in a large nucleus.

Apparently, the heavy quark energy loss induced by gluon radiation is significantly suppressed as compared to a light quark when the momentum scale Q or the quark initial energy q^- is not too large as compared to the quark mass. Only in the limit $M \ll Q$, q^-, is the mass effect negligible. Then the energy loss approaches that of a light quark.

In summary, we have calculated medium modification of fragmentation and energy loss of heavy quarks in DIS in the twist expansion approach. We demonstrated that heavy quark mass not only suppresses small angle gluon radiation due to the "dead-cone" effect but also reduces the gluon formation time. This leads to a reduced radiative energy loss as well as a different medium size dependence (close to linear), as compared to a light quark when the quark energy and the momentum scale Q are of the same order of magnitude as the quark mass. The result approaches that for a light quark when the quark mass is negligible as compared to the quark energy and the momentum scale Q. Similar to the case of light quark propagation, the result can be easily extended to a hot and dense medium, which will have practical consequences for heavy quark production and suppression in heavy ion collisions.

ACKNOWLEDGEMENTS

This work was supported by the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and by NSFC under project Nos. 19928511 and 10135030. E. Wang and B. Zhang thank the Physics Department of Shandong University for its hospitality during the completion of this work.

[1] M. Gyulassy and X.-N. Wang, Nucl. Phys. **B420**, 583 (1994) [arXiv:nucl-th/9306003]; X.-N. Wang, M. Gyulassy and M. Plümer, Phys. Rev. **D** 51, 3436 (1995) [arXiv:hep-ph/9408344].

[2] R. Baier *et al.*, Nucl. Phys. **B483**, 291 (1997) [arXiv:hep-ph/9607355]; Nucl. Phys. **B484**, 265 (1997) [arXiv:hep-ph/9608322];

[3] B. G. Zhakharov, JETP letters **63**, 952 (1996) [arXiv:hep-ph/9607440].

[4] M. Gyulassy, P. Lévai and I. Vitev, Nucl. Phys. **B594**, 371 (2001) [arXiv:nucl-th/0006010]; Phys. Rev. Lett. **85**, 5535 (2000) [arXiv:nucl-th/0005032].

[5] U. Wiedemann, Nucl. Phys. **B588**, 303 (2000) [arXiv:hep-ph/0005129]; Nucl. Phys. A **690**, 731 (2001) [arXiv:hep-ph/0008241].

[6] X. F. Guo and X.-N. Wang, Phys. Rev. Lett. **85**, 3591 (2000) [arXiv:hep-ph/0005044]; X.-N. Wang and X. F. Guo, Nucl. Phys. A **696**, 788 (2001) [arXiv:hep-ph/0102230].

[7] M. Gyulassy, I. Vitev and X. N. Wang, Phys. Rev. Lett. **86**, 2537 (2001) [arXiv:nucl-th/0012092].

[8] E. Wang and X. N. Wang, Phys. Rev. Lett. **89**, 162301 (2002) [arXiv:hep-ph/0202105].

[9] C. A. Salgado and U. A. Wiedemann, Phys. Rev. Lett. **89**, 092303 (2002) [arXiv:hep-ph/0204221].

[10] K. Adcox *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **88**, 022301 (2002) [arXiv:nucl-ex/0109003]; S. S. Adler *et al.* [PHENIX Collaboration], arXiv:nucl-ex/0304022.

[11] C. Adler *et al.*, Phys. Rev. Lett. **89**, 202301 (2002) [arXiv:nucl-ex/0206011]; J. Adams *et al.* [STAR Collaboration], arXiv:nucl-ex/0305015.

[12] X. N. Wang, arXiv:nucl-th/0305010.

[13] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B **519**, 199 (2001) [arXiv:hep-ph/0106202].

[14] K. Adcox *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **88**, 192303 (2002) [arXiv:nucl-ex/0202002].

[15] M. Djordjevic and M. Gyulassy, Phys. Lett. B **560**, 37 (2003) [arXiv:nucl-th/0302069].

[16] M. Luo, J. Qiu and G. Sterman, Phys. Lett. **B279**, 377 (1992); M. Luo, J. Qiu and G. Sterman, Phys. Rev. D**50**, 1951 (1994); M. Luo, J. Qiu and G. Sterman, Phys. Rev. D**49**, 4493 (1994).

[17] B. Zhang, E. Wang and X. N. Wang, in preparation

[18] B. W. Zhang and X. N. Wang, Nucl. Phys. A **720**, 429 (2003) [arXiv:hep-ph/0301195].

[19] J. Osborne and X.-N. Wang, Nucl. Phys. A **710**, 281 (2002) [arXiv:hep-ph/0204046].