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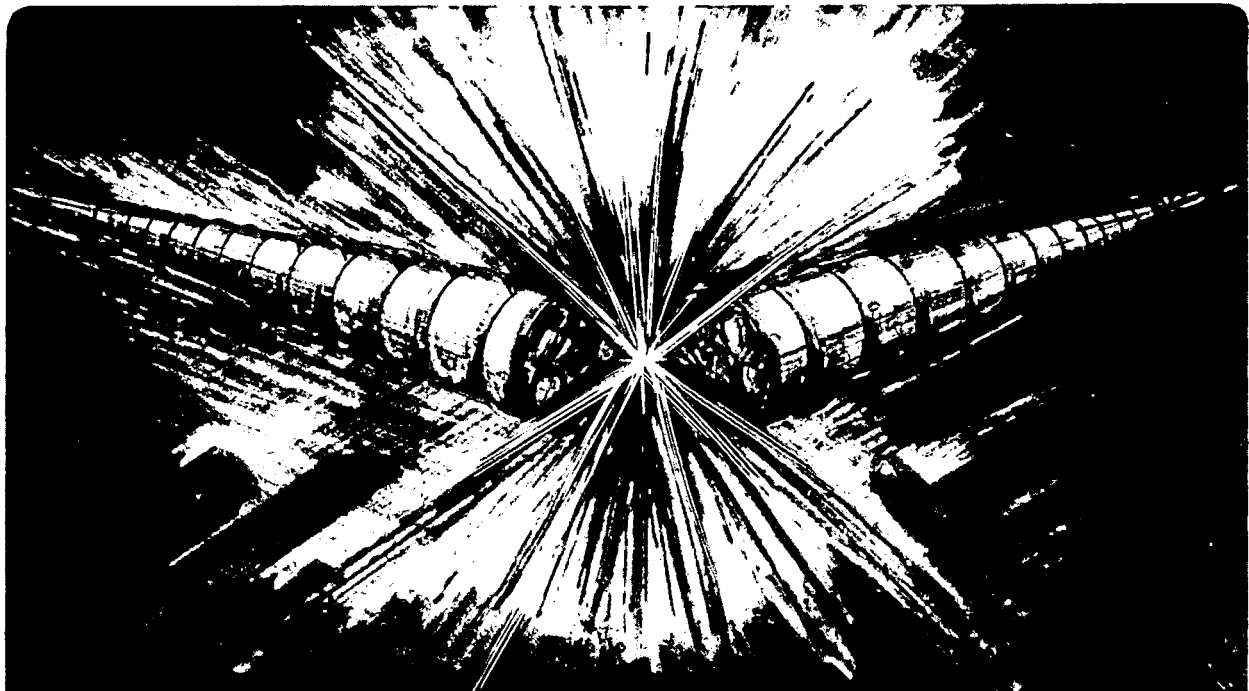
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S. Caspi

July 1994



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Magnetic Field Components in a Sinusoidally Varying Helical Wiggler.*

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Abstract

One may be interested in a pure multipole magnetic field (i.e, proportional to $\sin(n\theta)$ or $\cos(n\theta)$) whose strength varies purely as a Fourier sinusoidal series of the longitudinal coordinate z (say proportional to $\cos \frac{(2m-1)\pi z}{L}$, where L denotes the *half-period* of the wiggler and $m=1,2,3 \dots$). Associated with such a z variation, there necessarily will be present a z component of magnetic field which in the source-free region, in fact, will give rise to both normal and skew transverse fields associated with the functions $A_n(z)$ and $\tilde{A}_n(z)$ as expressed in Reference^{bc}. In this note the field components and expression for the scalar potential both inside and outside a thin pure winding surface are included with additional contributions from a possible high permeable shield. It is also shown that for a pure dipole case of $n=1$ and a pure axial variation of $m=1$ the transverse field can be derived from a simple two dimensional field.

Scalar Potential

We note that in the curl-free divergence-free region near the axis $r=0$ the field components may be expressed as given by $\vec{B} = -\nabla V$ where V is a scalar potential function for which $\nabla^2 V = 0$.

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{\partial^2 V}{\partial z^2} - \frac{n^2 V}{r^2} = 0 \quad (1)$$

The general form for the proposed solution as shown in Reference c can be written in the form that includes both "skew" and "non-skew" terms of all integer harmonic of order n (including $n=0$):

$$V = - \left\{ \sum_{n=0} r^n \sum_{k=0} \frac{(-1)^{k+1} n!}{2^{2k} k! (n+k)!} r^{2k} \left[A_n^{(2k)}(z) \sin n\theta - \tilde{A}_n^{(2k)}(z) \cos n\theta \right] \right\} \quad (2)$$

and the magnetic field components derived accordingly as :

$$\begin{aligned} B_r &= -\frac{\partial V}{\partial r} = \sum_n [g_{rn} r^{n-1} \sin n\theta - \tilde{g}_{rn} r^{n-1} \cos n\theta] \\ B_\theta &= -\frac{n}{r} V = \sum_n [g_{\theta n} r^{n-1} \cos n\theta + \tilde{g}_{\theta n} r^{n-1} \sin n\theta] \\ B_z &= -\frac{\partial V}{\partial z} = \sum_n [g_{zn} r^n \sin n\theta - \tilde{g}_{zn} r^n \cos n\theta] \end{aligned} \quad (3)$$

where

$$\begin{aligned} g_{rn} &\equiv \tilde{g}_{rn} \\ g_{\theta n} &\equiv \tilde{g}_{\theta n} \\ g_{zn} &\equiv \tilde{g}_{zn} \end{aligned} \quad (4)$$

are general functions of r and z that include the appropriate "normal" and "skew" terms $A_n(z)$ and $\tilde{A}_n(z)$ (see Appendix B).

^b 3D Field Harmonics — S.Caspi , M.Helm , and L.J. Laslett , SC-MAG-328 , LBL-30313 , March 1991.

^c An Approach To 3D Magnetic Field Calculation Using Numerical and Differential Algebra Methods — S.Caspi , M.Helm , and L.J. Laslett , SC-MAG-395 , LBL-32624 , July 1992.

Inner Field $r < R$

For the region within the windings (R equals the thin winding radius) of a helical wiggler such functions and even derivatives of order $(2k)$ are expressed as

$$\begin{aligned}
 A_n(z) &= \sum_{m=1} B_{n,m} \cos \left[(2m-1) \frac{\pi z}{L} \right] \\
 \tilde{A}_n(z) &= \sum_{m=1} B_{n,m} \sin \left[(2m-1) \frac{\pi z}{L} \right] \\
 A_n^{(2k)}(z) &= \sum_{m=1} (-1)^k \left[\frac{(2m-1)\pi}{L} \right]^{2k} B_{n,m} \cos \left[(2m-1) \frac{\pi z}{L} \right] \\
 \tilde{A}_n^{(2k)}(z) &= \sum_{m=1} (-1)^k \left[\frac{(2m-1)\pi}{L} \right]^{2k} B_{n,m} \sin \left[(2m-1) \frac{\pi z}{L} \right]
 \end{aligned} \tag{5}$$

and with the substitution of the above expressions into the scalar potential V (Equation 2)

$$V(r, \theta, z) = \sum_{n=1} n! \sum_{m=1} B_{n,m} \left[\frac{2L}{(2m-1)\pi} \right]^n \sum_{k=0} \frac{1}{k!(n+k)!} \left[\frac{(2m-1)\pi r}{2L} \right]^{2k+n} \sin \left[n\theta - \frac{(2m-1)\pi z}{L} \right] \tag{6}$$

and with

$$I_n(\omega_m r) = \sum_{k=0} \frac{1}{k!(n+k)!} \left(\frac{\omega_m r}{2} \right)^{2k+n} \tag{7}$$

where I_n denotes the "modified" Bessel function (of the first kind and order n),

$$\omega_m = \frac{(2m-1)\pi}{L} \quad \text{and} \quad G_{n,m} = n! \left(\frac{2}{\omega_m} \right)^n B_{n,m} \tag{8}$$

we express the scalar potential (Equation 6) as

$$V(r, \theta, z) = \sum_{n=1} \sum_{m=1} G_{n,m} I_n(\omega_m r) \sin(n\theta - \omega_m z) \tag{9}$$

where for a dipole sextupole , decapole etc, $n=1,3,5,\dots$, $m=1,2,3,\dots$, and L = half period.

The transverse field components and z directed field thus become

$$\begin{aligned}
 B_r &= -\frac{\partial V}{\partial r} = -\sum_{n=1} \sum_{m=1} G_{n,m} \omega_m I_n'(\omega_m r) \sin(n\theta - \omega_m z) \\
 B_\theta &= -\frac{1}{r} \frac{\partial V}{\partial \theta} = -\sum_{n=1} \sum_{m=1} n G_{n,m} \frac{1}{r} I_n(\omega_m r) \cos(n\theta - \omega_m z) \\
 B_z &= -\frac{\partial V}{\partial z} = \sum_{n=1} \sum_{m=1} G_{n,m} \omega_m I_n(\omega_m r) \cos(n\theta - \omega_m z)
 \end{aligned} \tag{10}$$

with

$$I_n'(\omega_m r) = I_{n-1}(\omega_m r) - \frac{n}{\omega_m r} I_n(\omega_m r) \tag{11}$$

where the prime denotes differentiation of the Bessel function with respect to its argument.

Outer Field $r > R$

For a configuration in which the magnetic field components are produced by means of currents confined to lie on the surface of a circular cylinder (radius R), it can be of interest to evaluate the character of the magnetic field components that must be present in the external region ($r > R$) and to determine the components (J_z and J_θ , at R) of current density for this configuration. The surface currents will give rise to a discontinuity of the components B_z and B_θ at the interface ($r=R$), but the normal (radial) component will pass continuously through this surface and assume the form

$$B_r = - \sum_{n=1} \sum_{m=1} G_{n,m} \omega_m \frac{I'_n(\omega_m R)}{K'_n(\omega_m R)} K'_n(\omega_m r) \sin(n\theta - \omega_m z) \quad (\text{for } r \geq R) \quad (12)$$

Consistent with B_r written immediately above a scalar potential function V for the external region is given by

$$V = \sum_{n=1} \sum_{m=1} G_{n,m} \frac{I'_n(\omega_m R)}{K'_n(\omega_m R)} K_n(\omega_m r) \sin(n\theta - \omega_m z) \quad (\text{for } r \geq R) \quad (13)$$

where the prime denotes differentiation of the Bessel functions with respect to its argument, and

$$K'_n(\omega_m r) = - \left[K_{n-1}(\omega_m r) + \frac{n}{\omega_m r} K_n(\omega_m r) \right] \quad (14)$$

The remaining field components are found to be

$$\begin{aligned} B_\theta &= - \sum_{n=1} \sum_{m=1} n G_{n,m} \frac{I'_n(\omega_m R)}{K'_n(\omega_m R)} \frac{1}{r} K_n(\omega_m r) \cos(n\theta - \omega_m z) \\ B_z &= \sum_{n=1} \sum_{m=1} G_{n,m} \omega_m \frac{I'_n(\omega_m R)}{K'_n(\omega_m R)} K_n(\omega_m r) \cos(n\theta - \omega_m z) \end{aligned} \quad (15)$$

Surface currents at $r=R$

The discontinuity of the field components at the interface $r=R$ now permit evaluation of the corresponding surface currents on this cylindrical surface. We denote the current system at the interface $r=R$ by $\vec{J} = J_z \hat{e}_z + J_\theta \hat{e}_\theta$ (amp/m), and recall the relation $\frac{1}{\mu_0} \oint \vec{B} \cdot d\vec{l} = I$ (or $\frac{1}{\mu_0} (\Delta B) = J$), where $\mu_0 = 4\pi 10^{-7}$ in MKS-A units. Then

$$\begin{aligned} J_z(\theta, z)|_{r=R} &= \frac{1}{\mu_0} [B_\theta^{ext.} - B_\theta^{int.}] \\ &= \frac{1}{\mu_0} \sum_{n=1} \sum_{m=1} n G_{n,m} \frac{I_n(\omega_m R) K'_n(\omega_m R) - I'_n(\omega_m R) K_n(\omega_m R)}{R K'_n(\omega_m R)} \cos(n\theta - \omega_m z) \end{aligned} \quad (16)$$

and through the use of the Wronskian $I_n K'_n - I'_n K_n = -\frac{1}{\omega_m R}$

$$J_z(\theta, z)|_{r=R} = -\frac{1}{\mu_0} \sum_{n=1} \sum_{m=1} n G_{n,m} \frac{1}{\omega_m R^2} \frac{1}{K'_n(\omega_m R)} \cos(n\theta - \omega_m z) \quad (17)$$

and

$$\begin{aligned}
J_\theta(\theta, z)|_{r=R} &= \frac{1}{\mu_0} [B_z^{int.} - B_z^{ext.}] \\
&= \frac{1}{\mu_0} \sum_{n=1} \sum_{m=1} G_{n,m} \omega_m \frac{I_n(\omega_m R) K_n'(\omega_m R) - I_n'(\omega_m R) K_n(\omega_m R)}{K_n'(\omega_m R)} \cos(n\theta - \omega_m z)
\end{aligned} \tag{18}$$

and again through the use of the Wronskian

$$J_\theta(\theta, z)|_{r=R} = -\frac{1}{\mu_0} \sum_{n=1} \sum_{m=1} G_{n,m} \frac{1}{R} \frac{1}{K_n'(\omega_m R)} \cos(n\theta - \omega_m z) \tag{19}$$

The pair of components satisfy the conservation condition $\nabla \cdot \vec{J} = \frac{\partial J_z}{\partial z} + \frac{1}{R} \frac{\partial J_\theta}{\partial \theta} = 0$ as required.

Contribution of axially-symmetric ferromagnetic shield

We realize that if an axially-symmetric ferromagnetic shield of high permeability is present with a radius $r=a$ ($a > R$), the induced magnetization will contribute supplemental fields ("image fields") that in the region interior to $r=a$ may themselves be derived from a scalar potential ($V_{r < a}^{image}$). The appropriate boundary condition at $r=a$ will be fulfilled if we specify that $V_{r=a}^{image} + V_{r=a}^{direct} = \text{constant}$ or if we conveniently specify that $V_{r=a}^{image} = -V_{r=a}^{direct}$ and specifically

$$V_{r=a}^{image} = - \sum_{n=1} \sum_{m=1} G_{n,m} \frac{I_n'(\omega_m R)}{K_n'(\omega_m R)} K_n(\omega_m a) \sin(n\theta - \omega_m z) \quad (\text{at } r = a) \tag{20}$$

If the iron radius is constant (not a function of z) we can write the scalar potential for $r \leq a$

$$V_{image} = - \sum_{n=1} \sum_{m=1} G_{n,m} \frac{I_n'(\omega_m R) K_n(\omega_m a)}{K_n'(\omega_m R) I_n(\omega_m a)} I_n(\omega_m r) \sin(n\theta - \omega_m z) \quad (\text{at } r \leq a) \tag{21}$$

For the TOTAL magnetic potential function at $r < R < a$, we then have

$$V_{r < R}^{total} = \sum_{n=1} \sum_{m=1} G_{n,m} \left[1 - \frac{I_n'(\omega_m R) K_n(\omega_m a)}{K_n'(\omega_m R) I_n(\omega_m a)} \right] I_n(\omega_m r) \sin(n\theta - \omega_m z) \tag{22}$$

The factor contained within the square brackets is an enhancement factor arising from the inclusion of magnetization developed in the high permeability ferromagnetic shield. For the special 2d case where $L \rightarrow \infty$ or $\omega_m a \ll 1$ this factor becomes approximately

$$\lim_{\omega_m a \rightarrow 0} \left[1 - \frac{I_n'(\omega_m R) K_n(\omega_m a)}{K_n'(\omega_m R) I_n(\omega_m a)} \right] = 1 + \left(\frac{R}{a} \right)^{2n} \tag{23}$$

and the potential

$$V_{r < R}^{total-2D} \approx \sum_{n=1} \sum_{m=1} B_{n,m} \left[1 + \left(\frac{R}{a} \right)^{2n} \right] r^n \sin(n\theta - \omega_m z) \tag{24}$$

as expected for the enhancement of the 2D field. For the above approximation we made use of the following asymptotic relations

$$\begin{aligned}
 s \rightarrow 0 \\
 I_n(s) &\sim \frac{1}{n!} \left(\frac{s}{2}\right)^n \\
 K_n(s) &\sim \frac{(n-1)!}{2} \left(\frac{s}{2}\right)^{-n} \\
 I'_n(s) &\sim \frac{1}{2(n-1)!} \left(\frac{s}{2}\right)^{n-1} \\
 K'_n(s) &\sim -\frac{n!}{4} \left(\frac{s}{2}\right)^{-(n+1)}
 \end{aligned} \tag{25}$$

The square brackets in Equation (22) is plotted in Fig. 4 Appendix A for $n=1$ and $m=1$.

Helical dipole with simple sinusoidal relation

We shall examine a helical dipole with single terms for both series n and m . The choice $n=1$ indicates a pure dipole with no higher harmonics, and $m=1$ indicates a pure $\pi z/L$ variation with no additional frequencies. We express the field components for $r < R$ and $n=m=1$ as

$$\begin{aligned}
 \omega_m = \omega_1 = \frac{\pi}{L} \quad , \quad G_{n,m} = G_{1,1} = \frac{2L}{\pi} B_{1,1} \\
 B_r = -2B_{1,1} I'_1\left(\frac{\pi r}{L}\right) \sin\left(\theta - \frac{\pi z}{L}\right) \\
 B_\theta = -2B_{1,1} \left(\frac{L}{\pi r}\right) I_1\left(\frac{\pi r}{L}\right) \cos\left(\theta - \frac{\pi z}{L}\right) \\
 B_z = 2B_{1,1} I_1\left(\frac{\pi r}{L}\right) \cos\left(\theta - \frac{\pi z}{L}\right)
 \end{aligned} \tag{26}$$

and

$$\vec{J}(\theta, z) = -\frac{2B_{1,1}}{\mu_0} \left(\frac{L}{\pi R}\right) \frac{1}{K'_1\left(\frac{\pi R}{L}\right)} \left[\hat{e}_\theta + \frac{L}{\pi R} \hat{e}_z\right] \cos\left(\theta - \frac{\pi z}{L}\right) \tag{27}$$

We note that a linear relationship exists between the following field components

$$\frac{B_z}{B_\theta} = -\frac{\pi r}{L} \tag{28}$$

and note as well that for $\frac{\pi r}{L} < \frac{\pi}{2}$ or $r < \frac{L}{2}$ the field components can be expressed with less than 1% error as

$$\begin{aligned}
 B_r &= -B_{1,1} \left[1 + \frac{3}{2} \left(\frac{\pi r}{2L}\right)^2 + \frac{5}{12} \left(\frac{\pi r}{2L}\right)^4 + \frac{7}{144} \left(\frac{\pi r}{2L}\right)^6 + \dots \right] \sin\left(\theta - \frac{\pi z}{L}\right) \\
 B_\theta &= -B_{1,1} \left[1 + \frac{1}{2} \left(\frac{\pi r}{2L}\right)^2 + \frac{1}{12} \left(\frac{\pi r}{2L}\right)^4 + \frac{1}{144} \left(\frac{\pi r}{2L}\right)^6 + \dots \right] \cos\left(\theta - \frac{\pi z}{L}\right) \\
 B_z &= B_{1,1} \frac{\pi r}{L} \left[1 + \frac{1}{2} \left(\frac{\pi r}{2L}\right)^2 + \frac{1}{12} \left(\frac{\pi r}{2L}\right)^4 + \frac{1}{144} \left(\frac{\pi r}{2L}\right)^6 + \dots \right] \cos\left(\theta - \frac{\pi z}{L}\right)
 \end{aligned} \tag{29}$$

The representations above will describe a field that formally is both divergence free and curl free — provided that the summations are not truncated. If, however, we wish to truncate these series expressions,

we at best can only do so in such a way that one, but not both, of these conditions is satisfied. Thus, if we wish to preserve the divergence condition $\nabla \cdot \vec{B} = 0$, we should take care that the sum over the k index in the series for B_z should terminate at a value of k that is less by unity than the termination value for this index in the series for the transverse field components B_r & B_θ .

We shall calculate $B_{1,1}$ and compare it with B_{2d} that is produced by a straight long dipole ($L \rightarrow \infty$) carrying the same total current. In the 2D case where a current density (per unit length) of $J(\theta) = J_0 \cos \theta$ and $J_0 = \frac{I_0}{R}$ will produce a dipole field of $B_{2d} = \frac{\mu_0 J_0}{2}$, the dipole field in terms of the total amp-turn is

$$B_{2d} = \frac{\mu_0 I_0}{2R} \quad (30)$$

We shall evaluate the total amp-turn in the helical wiggler by integrating the azimuthal current density along $\theta=0$ using equation (27) (see Fig. 1 below).

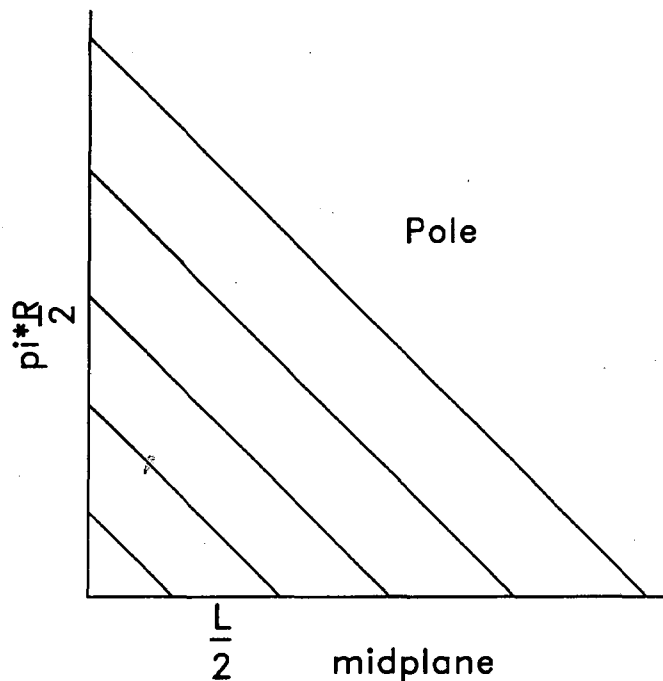
$$I_0 = \int_0^{\frac{\pi}{2}} J_z|_{z=0} R d\theta = \int_0^{\frac{L}{2}} J_\theta|_{\theta=0} dz = -\frac{2B_{1,1}}{\mu_0} \frac{1}{s K_1'(s)} \int_0^{\frac{L}{2}} \cos \frac{\pi z}{L} dz = -\frac{2B_{1,1} R}{\mu_0 s^2 K_1'(s)} \quad (31)$$

By equating the total current in both the 2D dipole and the helical wiggler the ratio of their transverse fields can be reduced to a dimensionless form :

$$\frac{B_{1,1}}{B_{2d}} = s^2 K_1'(s) \quad (32)$$

and note that in the limiting case (using Eq. 25) as $L \rightarrow \infty$

$$\lim_{s \rightarrow 0} \frac{B_{1,1}}{B_{2d}} = 1 \quad (33)$$



as it should be.

The relation between the normalized transverse fields and s (Eq. 32) plotted in Figure 1, reveals a range that surprisingly is grater than 1 where a maximum of 1.0616089 is reached at $s=0.6$. A computational check was made with a cylinder of radius $R=2.0$ cm, surrounded by a current sheet in

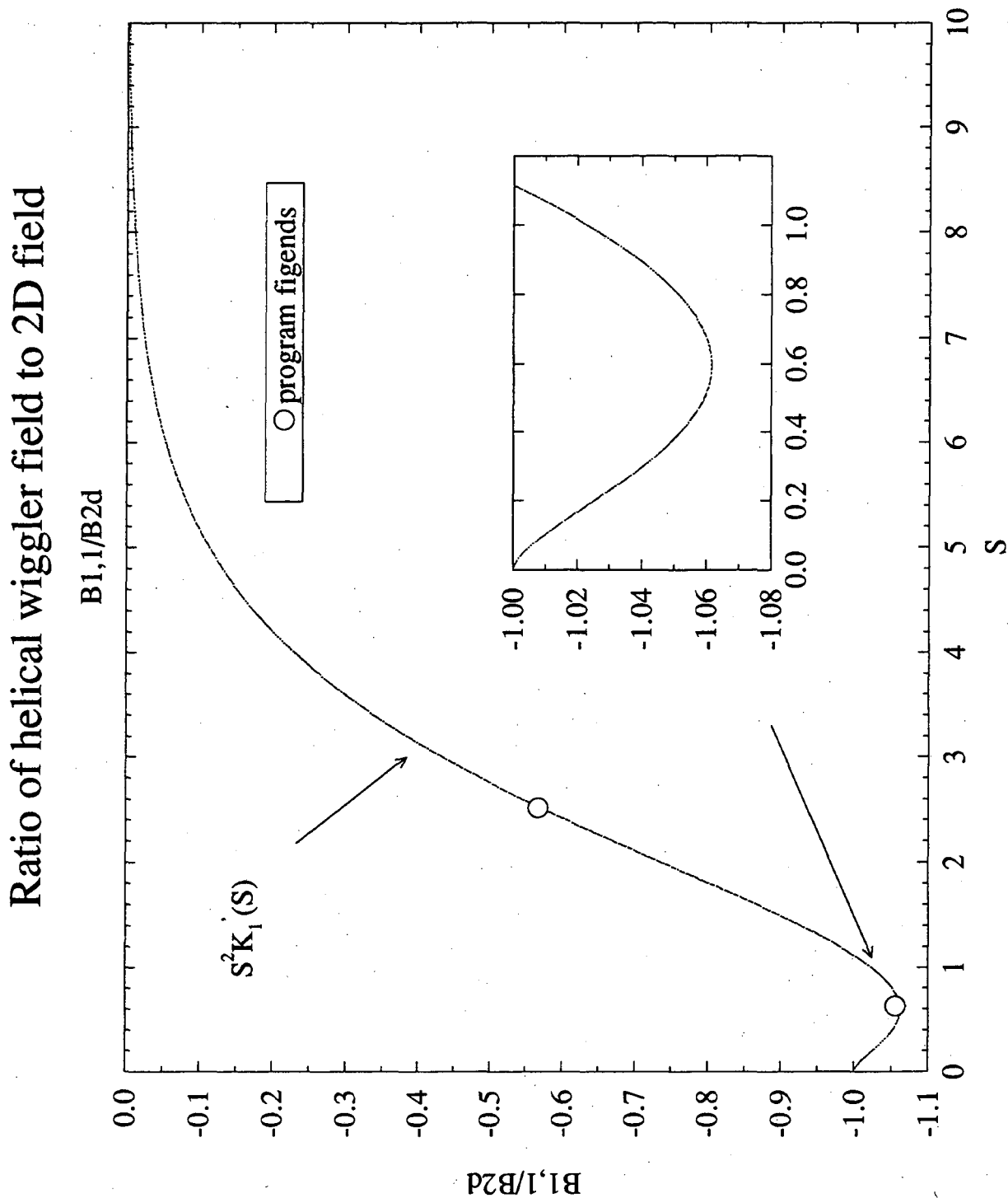


Figure 1 Ratio of wiggler field to 2d dipole field.

a $\cos\theta$ fashion (Figure 2) such that

$$J = \frac{I_0}{R} \cos \theta = 39 \times 10^3 \cos \theta \quad (A/cm) \quad (34)$$

with a dipole field of

$$B_{2d} = \frac{\mu_0 I_0}{2R} = 2.4504 \quad (tesla) \quad (35)$$

(we picked $N=39$ turns, $I=2000$ A and note that $I_0=NI$). A quick check with the 2D program "pkpeak" yields a similar value of $B_{2d}=2.4583$ (tesla). Applying the same current configuration in two examples of a helical wiggler with the same radius R but different periods, such that

$$\begin{aligned} \lambda_1 = 2L = 5 \quad cm, \quad s = \frac{\pi R}{L} = 2.513 \\ \lambda_2 = 2L = 20 \quad cm, \quad s = \frac{\pi R}{L} = 0.6283 \end{aligned} \quad (36)$$

Equation (32) then predicts the following results :

$$\begin{aligned} \frac{B_{1,1}(\lambda_1)}{B_{2d}} = 0.567 \quad or \quad B_{1,1} = 1.3976 \quad (tesla) \\ \frac{B_{1,1}(\lambda_2)}{B_{2d}} = 1.06135 \quad or \quad B_{1,1} = 2.600 \quad (tesla) \end{aligned}$$

With the aid of the 3D program "figends" using a model such as shown in Figure 3, the corresponding values are :

$$\begin{aligned} \frac{B_{1,1}(\lambda_1)}{B_{2d}} = 0.5652 \quad or \quad B_{1,1} = 1.3894 \quad (tesla) \\ \frac{B_{1,1}(\lambda_2)}{B_{2d}} = 1.0518 \quad or \quad B_{1,1} = 2.5858 \quad (tesla) \end{aligned}$$

We comment here that the field components as described by Eq. (26) differs from the corresponding expression written in the Appendix of a paper by J.Blewett et al^d due to possible typographical errors in that paper. We also note that if we express the total current written in equation (31) in a form similar to that expressed in Blewett's paper we arrive at the total current per pole ($= 2I_0$)

$$Current/pole = \frac{5B_{1,1}\lambda_0}{\pi^2\left(\frac{\pi R}{L}K_0 + K_1\right)} \quad (39)$$

where $\lambda_0=2L$ (period). Blewett's expression for the current differs by a factor of $\sqrt{1 + \left(\frac{L}{\pi R}\right)^2}$

$$Current/pole = \frac{5B_{1,1}\lambda_0\sqrt{1 + \left(\frac{L}{\pi R}\right)^2}}{\pi^2\left(\frac{\pi R}{L}K_0 + K_1\right)} \quad (40)$$

For the case of a single pair of current carrying wires wound in a bifilar helix^e this expression is also different from both cases.

$$Current/pole = \frac{5B_{1,1}\lambda_0}{4\pi\left(\frac{\pi R}{L}K_0 + K_1\right)} \quad (41)$$

^d Orbits and fields in the helical wiggler — Journal of Applied Physics, Vol. 48, No. 7, July 1977

^e Static and Dynamic Electricity — W.R.Smythe, p.277.

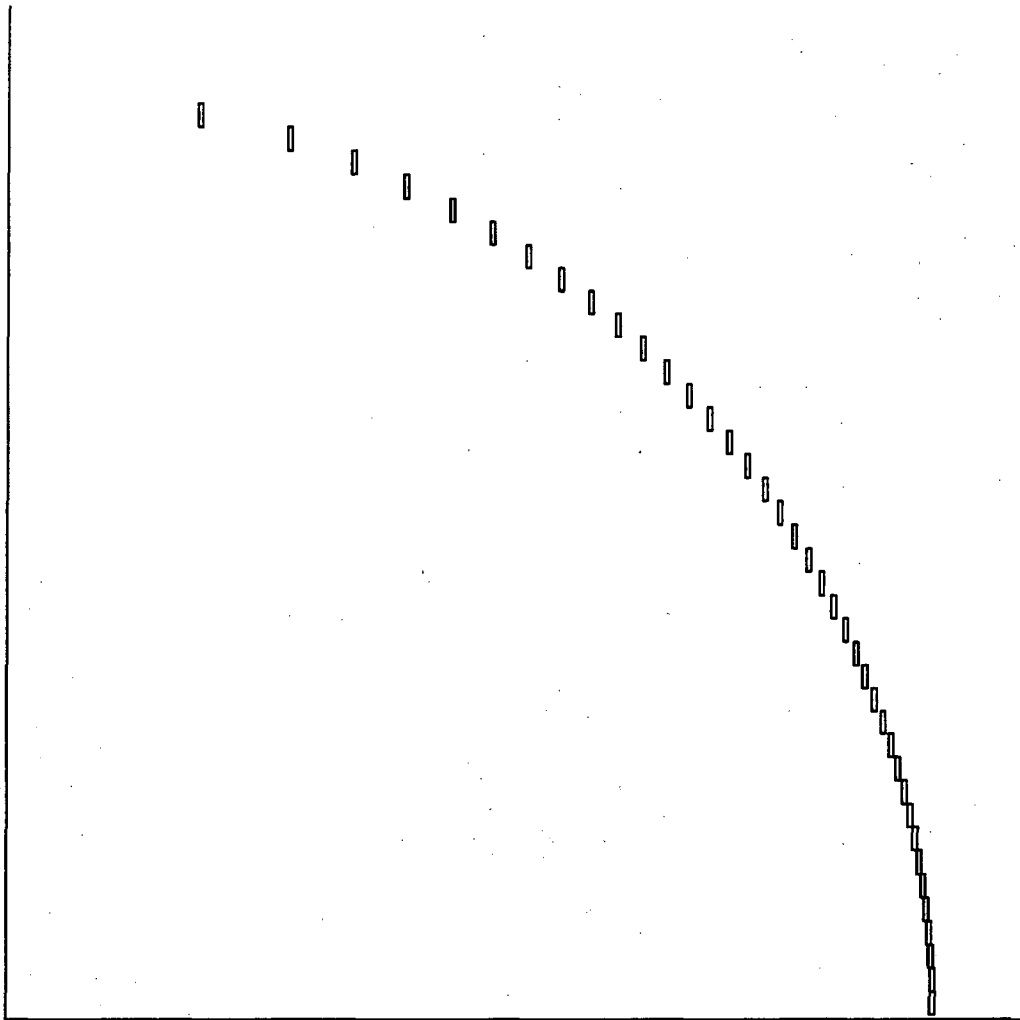


Figure 2 Winding cross section in a $\cos\theta$ configuration.

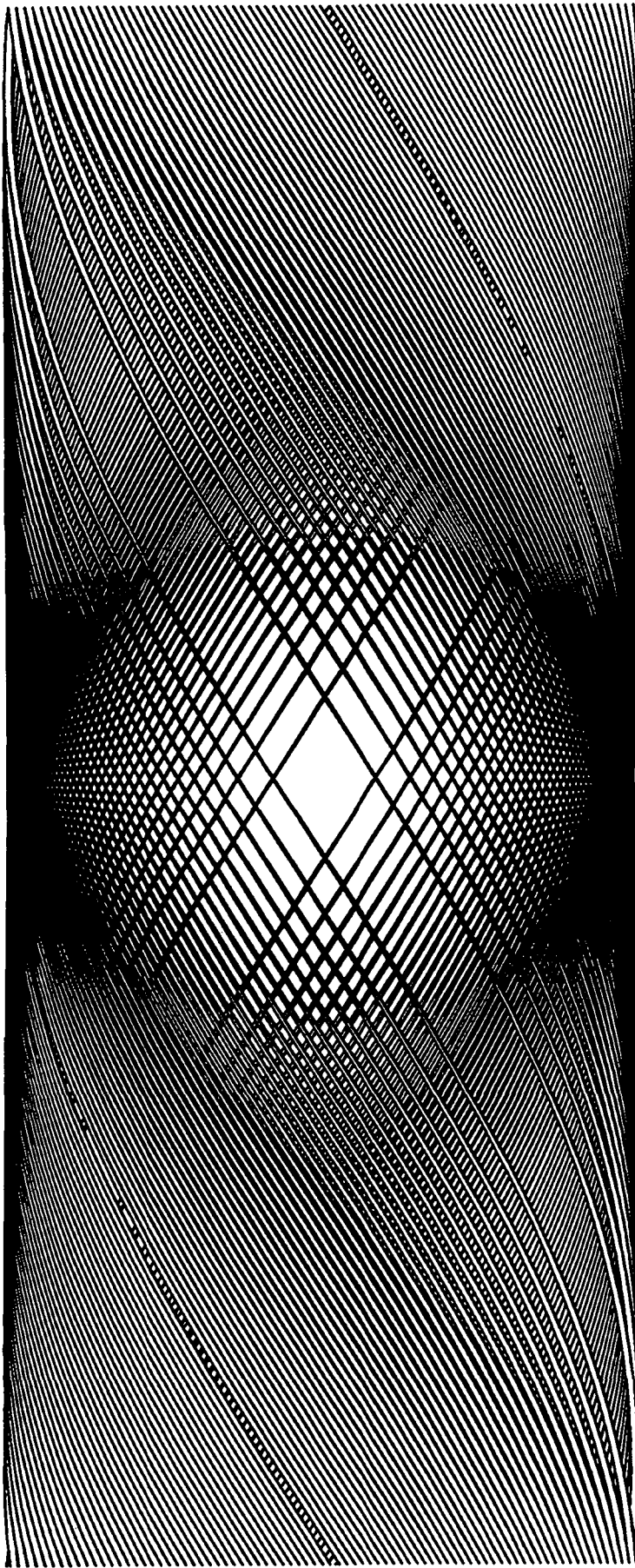


Figure 3 3D windings for half period $L=10$ in a $\cos(\pi z/L)$ configuration.

Appendix A Iron contribution

Equation (22) suggest a field enhancement factor arising from an iron sheet placed at $r=a$. Figure 4 shows such a factor for $n=1$ and $m=1$ as a function of $s = \frac{\pi R}{L}$ with the ratio of a/R used as a parameter.^f

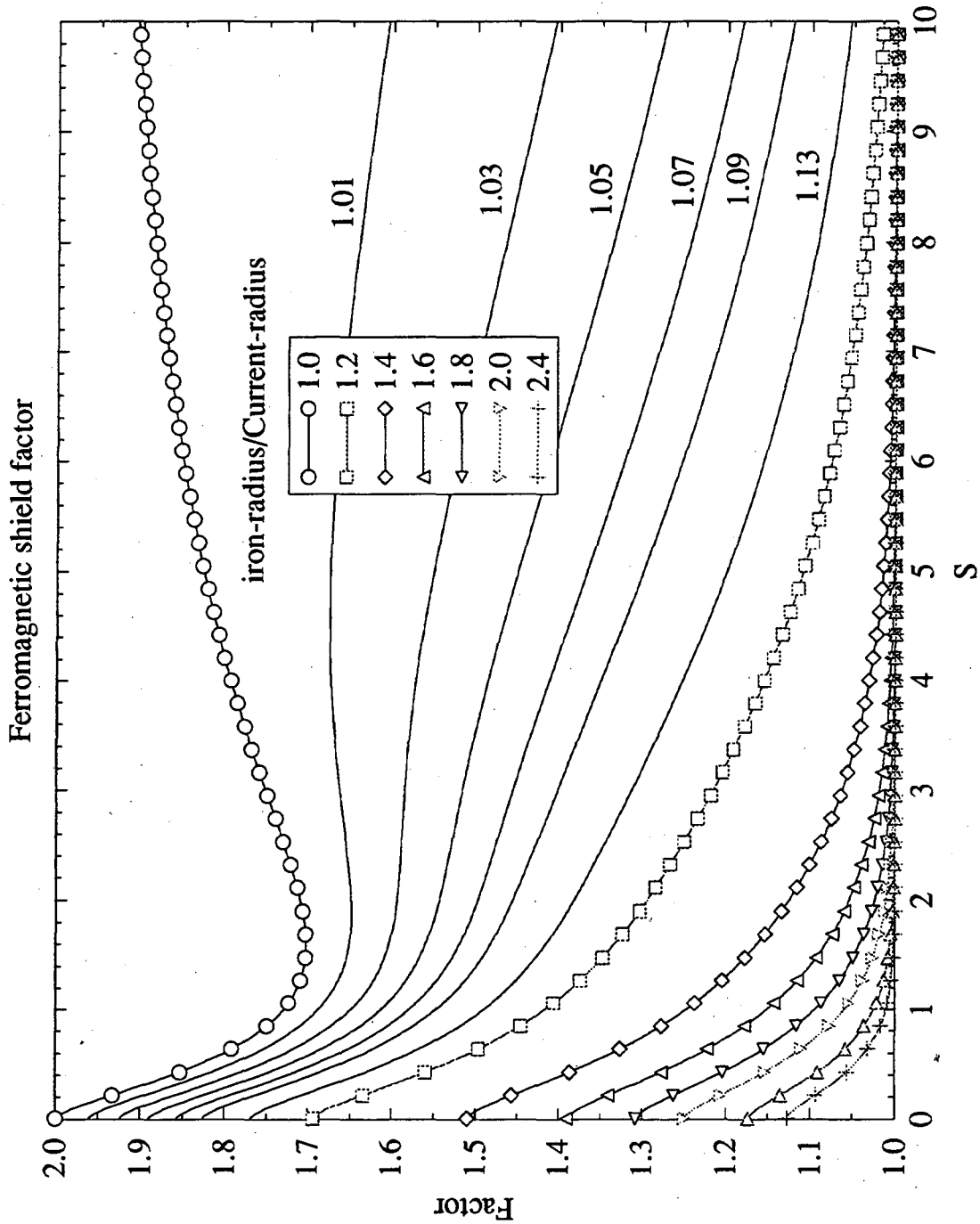


Figure 4 Field compression factor in a helical dipole wiggler.

^f I would like to acknowledge the help I received from Domenico Dell'orco in producing this graph.

Appendix B 3D harmonic coefficients

In order that the series for the potential V_n satisfy the differential equation (Eq. 1) we introduce the functions $A_n(z)$ and express the coefficients g_{rn} , $g_{\theta n}$, g_{zn} as general functions of r and z as shown below :

$$\begin{aligned} g_{rn}(r, z) &= \sum_{k=0} (-1)^{k+1} \frac{n!(n+2k)}{2^{2k} k!(n+k)!} A_n^{(2k)}(z) r^{2k} \\ g_{\theta n}(r, z) &= \sum_{k=0} (-1)^{k+1} \frac{n!n}{2^{2k} k!(n+k)!} A_n^{(2k)}(z) r^{2k} \\ g_{zn}(r, z) &= \sum_{k=0} (-1)^{k+1} \frac{n!}{2^{2k} k!(n+k)!} A_n^{(2k+1)}(z) r^{2k} \end{aligned} \quad (1)$$

Explicitly we can write the above as :

$$\begin{aligned} g_{rn}(r, z) &= -nA_n(z) + \frac{n+2}{4(n+1)} A_n''(z) r^2 - \frac{n+4}{32(n+1)(n+2)} A_n''''(z) r^4 \\ &\quad + \frac{n+6}{384(n+1)(n+2)(n+3)} A_n''''''(z) r^6 - \dots \\ g_{\theta n}(r, z) &= -nA_n(z) + \frac{n}{4(n+1)} A_n''(z) r^2 - \frac{n}{32(n+1)(n+2)} A_n''''(z) r^4 \\ &\quad + \frac{n}{384(n+1)(n+2)(n+3)} A_n''''''(z) r^6 - \dots \\ g_{zn}(r, z) &= -A_n'(z) + \frac{1}{4(n+1)} A_n'''(z) r^2 - \frac{1}{32(n+1)(n+2)} A_n''''(z) r^4 \dots \end{aligned} \quad (2)$$

For the expressions of the skew terms just replace g_{rn} , $g_{\theta n}$, g_{zn} with \tilde{g}_{rn} , $\tilde{g}_{\theta n}$, \tilde{g}_{zn} and $A_n(z)$ with $\tilde{A}_n(z)$

The representation specified above for 3-D magnetic fields, can be written in terms of functions $A_n(z)$ and $\tilde{A}_n(z)$ and their derivatives for the example used in the main part of the paper where $n=1$ and $m=1$, such that :

$$\begin{aligned} A_1^{(2k)} &= (-1)^k \left(\frac{\pi}{L}\right)^{2k} B_{1,1} \cos \frac{\pi z}{L} \\ \tilde{A}_1^{(2k)} &= (-1)^k \left(\frac{\pi}{L}\right)^{2k} B_{1,1} \sin \frac{\pi z}{L} \\ A_1^{(2k-1)} &= (-1)^k \left(\frac{\pi}{L}\right)^{2k-1} B_{1,1} \sin \frac{\pi z}{L} \\ \tilde{A}_1^{(2k-1)} &= (-1)^{k+1} \left(\frac{\pi}{L}\right)^{2k-1} B_{1,1} \cos \frac{\pi z}{L} \end{aligned} \quad (3)$$

In the next series of graphs we include results of such A's (both normal and skew) computed by the program "figends" for one of the example previously noted ($2L=5.0$). We note the sinusoidal periodicity of the A's and their derivatives according to the above relations.

Helical wiggler 2L=5.0 - Normal

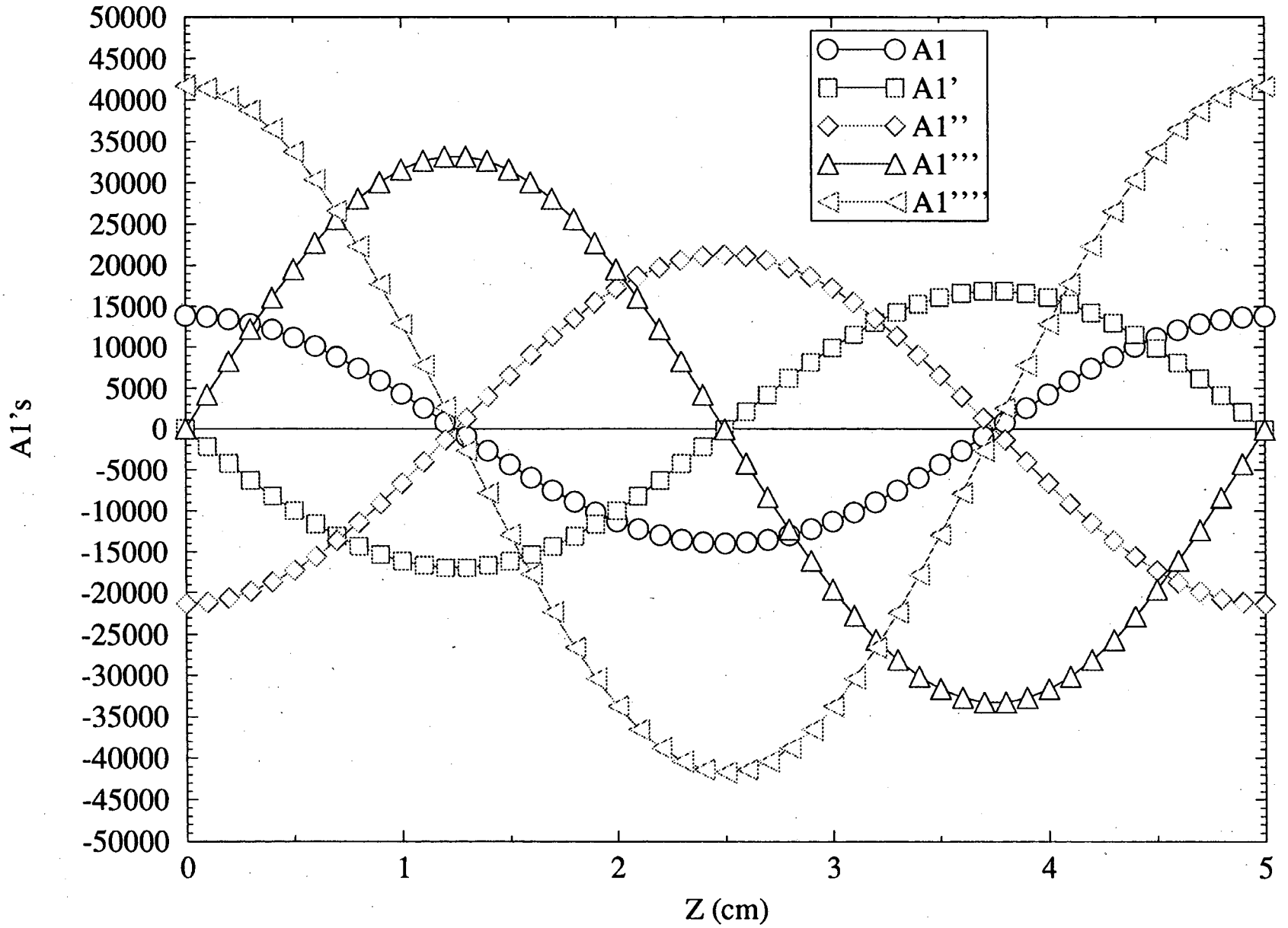


Figure 6 Normal A1 as a function of z over a full period..

Helical wiggler $2L=5.0$ - Normal

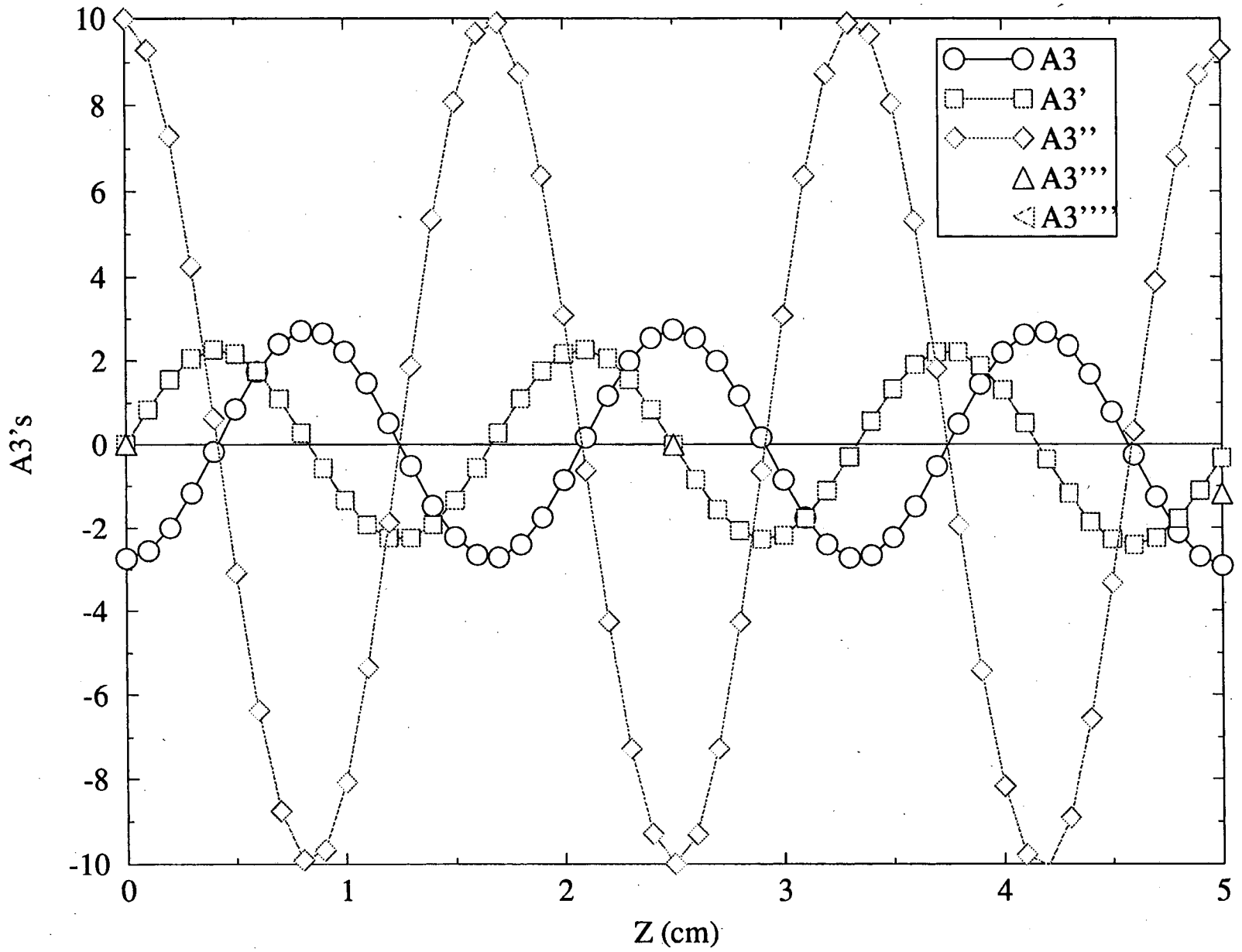


Figure 7 Normal $A3$ as a function of z over a full period..

Helical wiggler 2L=5.0 - Normal

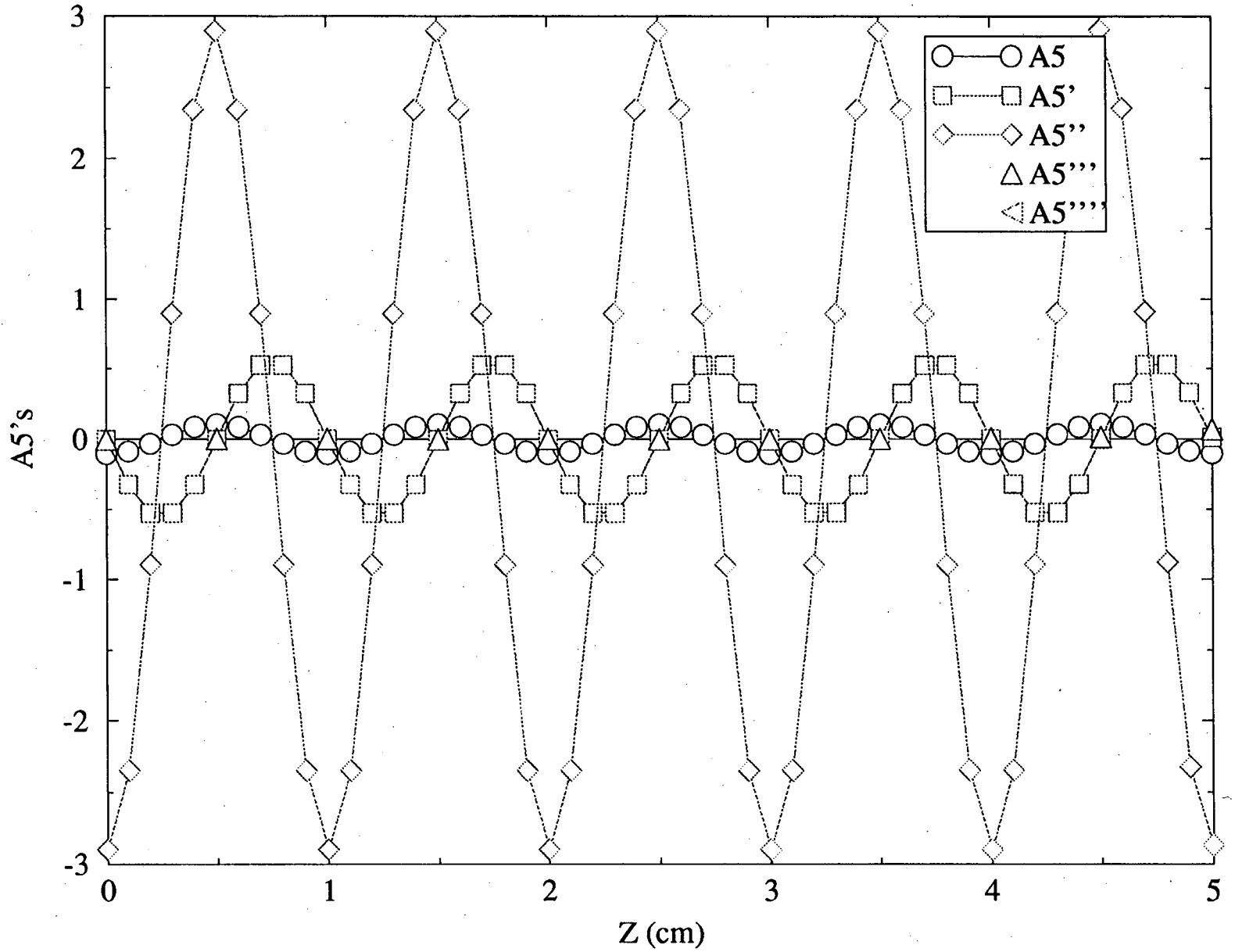


Figure 8 Normal A5 as a function of z over a full period..

Helical wiggler 2L=5.0 - Normal

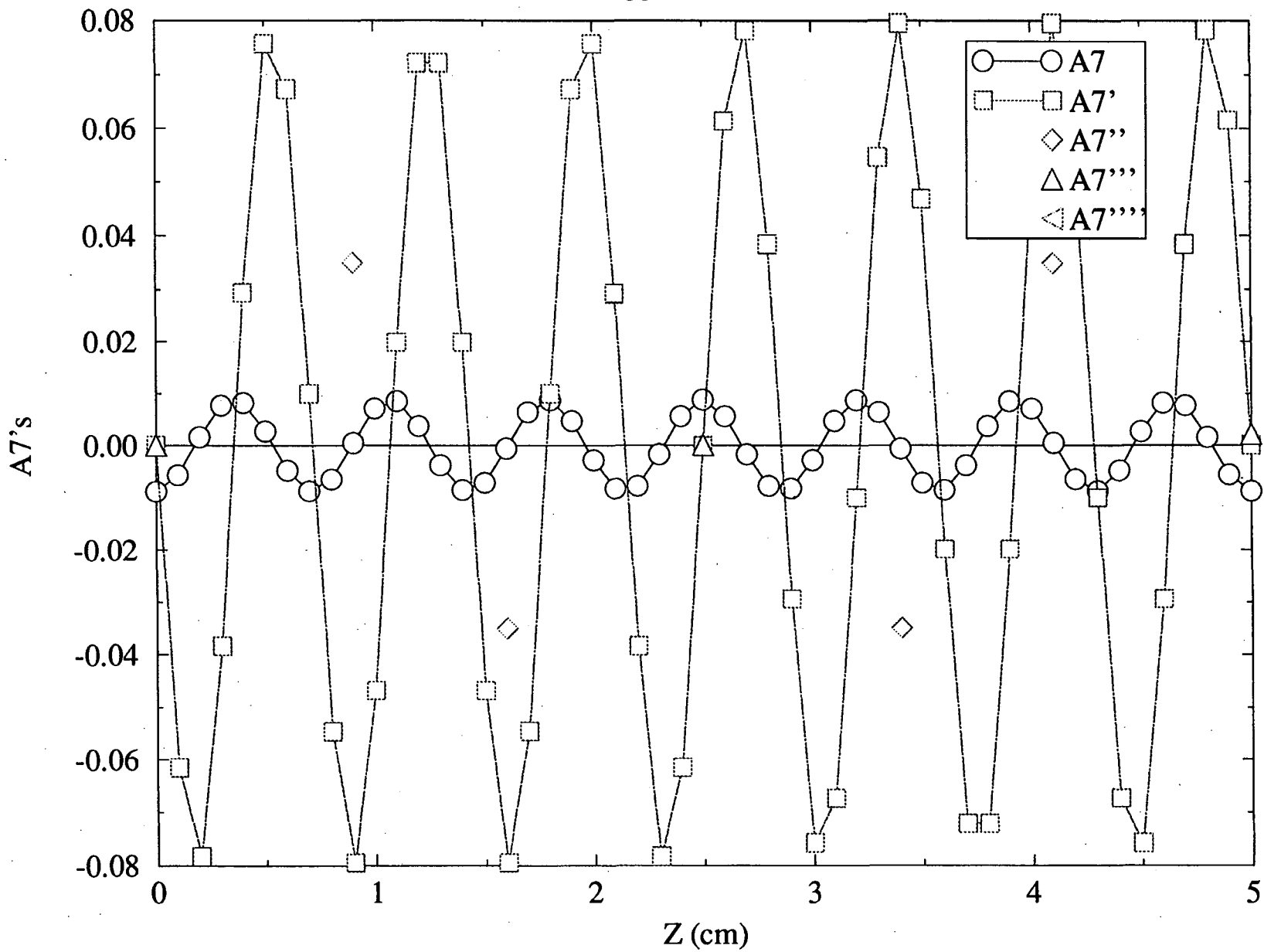


Figure 9 Normal $A7$ as a function of z over a full period.

Helical wiggler 2L=5.0 - Normal

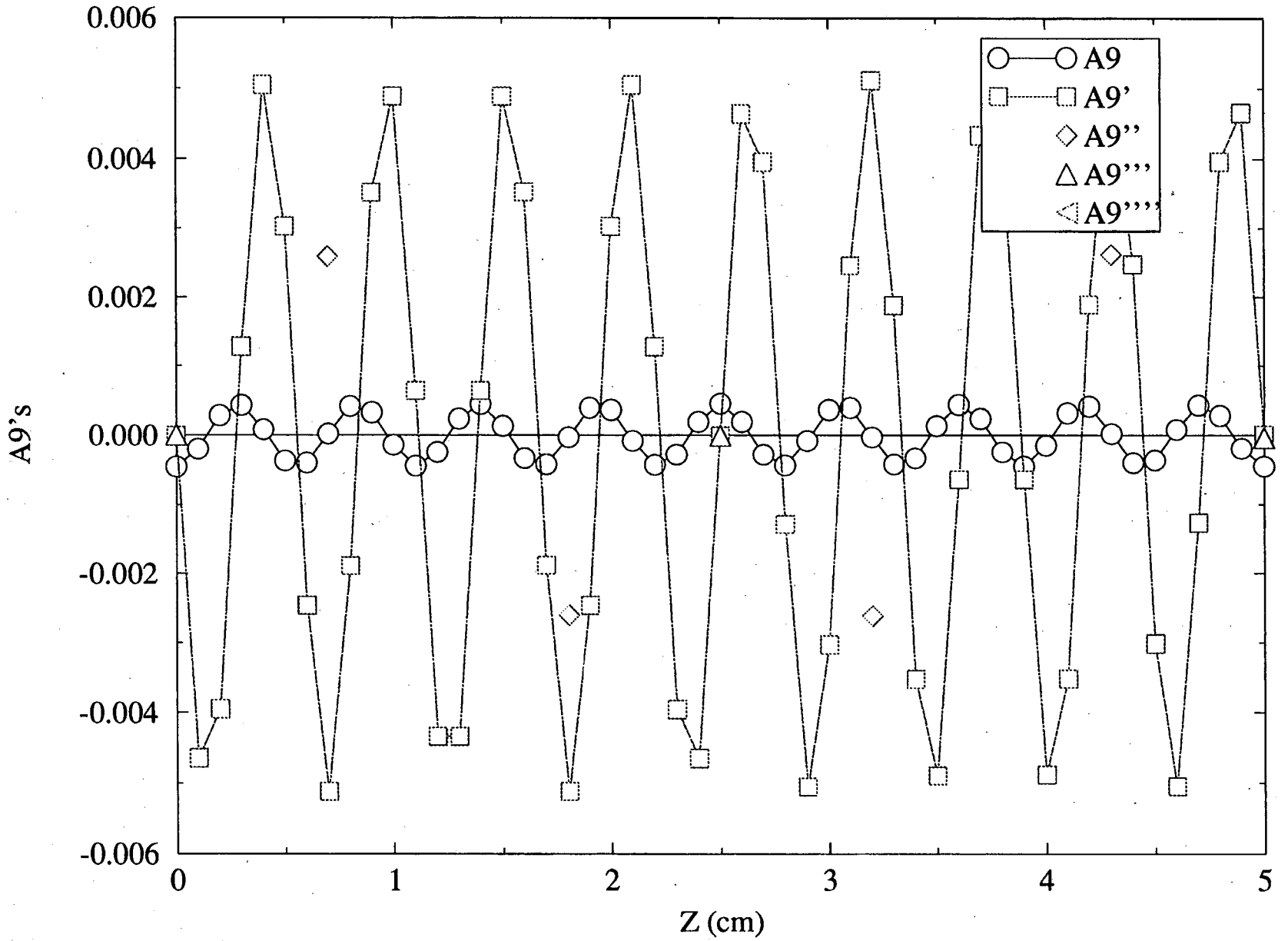


Figure 10 Normal A_9 as a function of z over a full period.

Helical wiggler 2L=5.0 - Skew

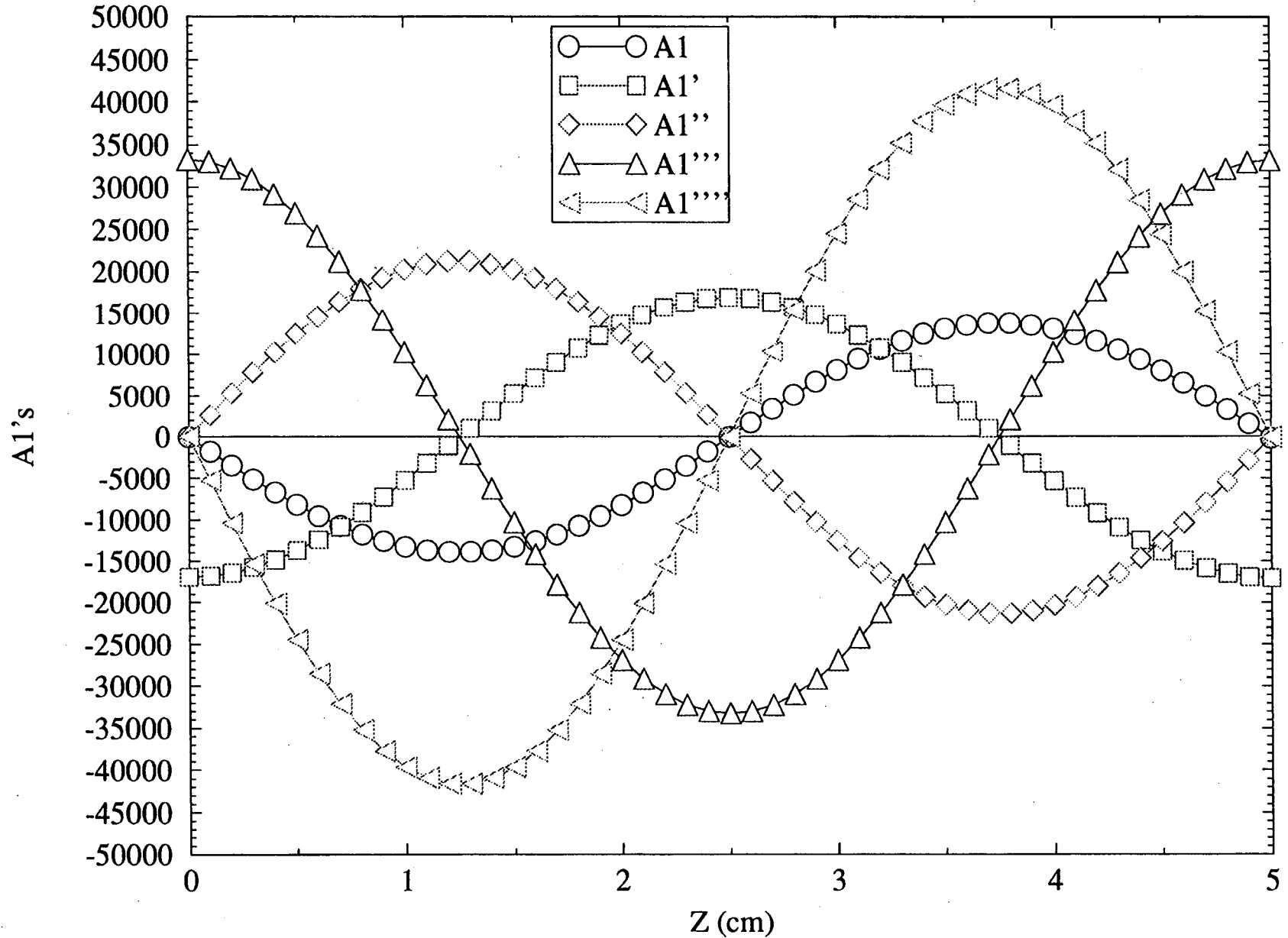


Figure 11 Skew A1 as a function of z over a full period..

Helical wiggler 2L=5.0 - Skew

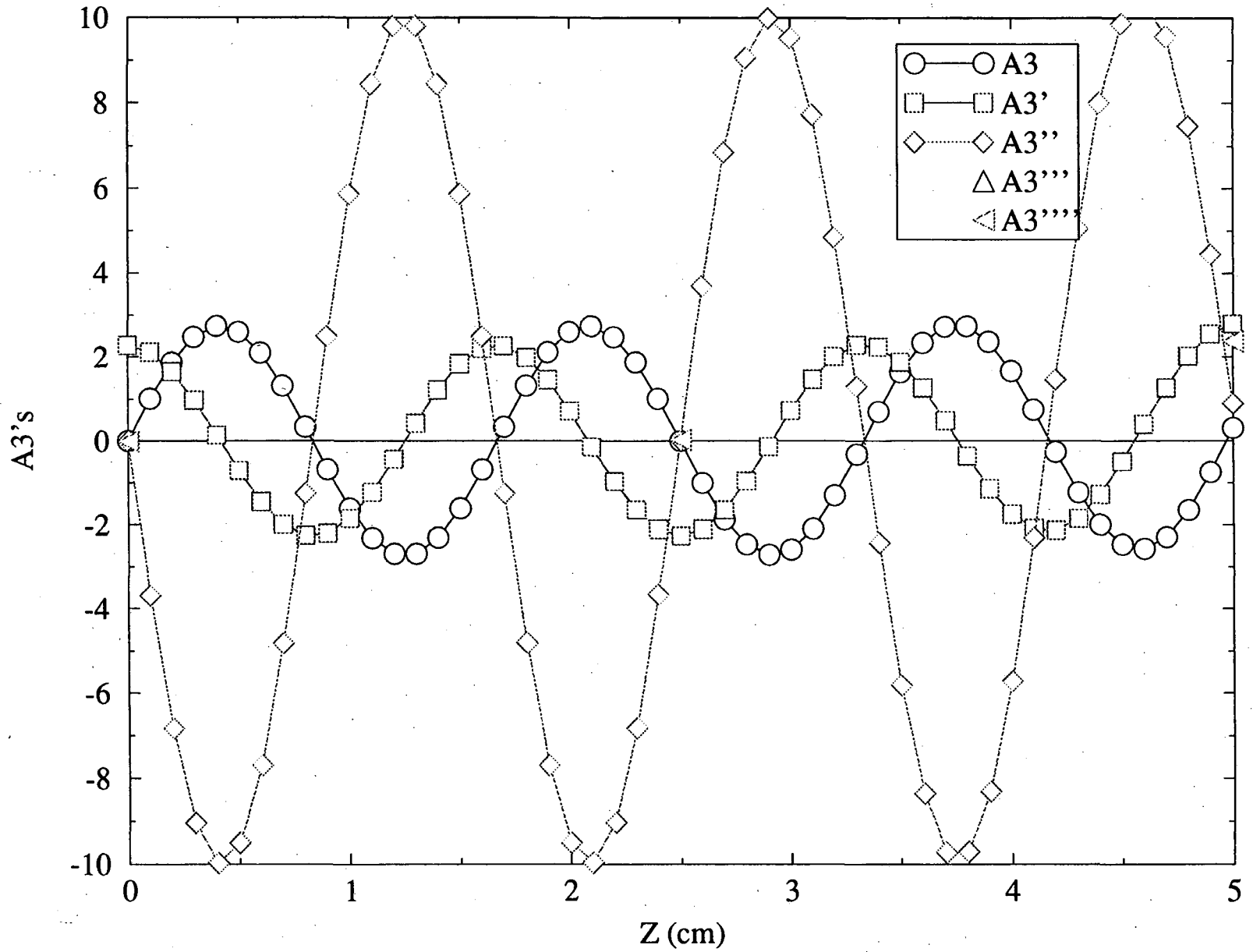


Figure 12 Skew A_3 as a function of z over a full period..

Helical wiggler 2L=5.0 - Skew

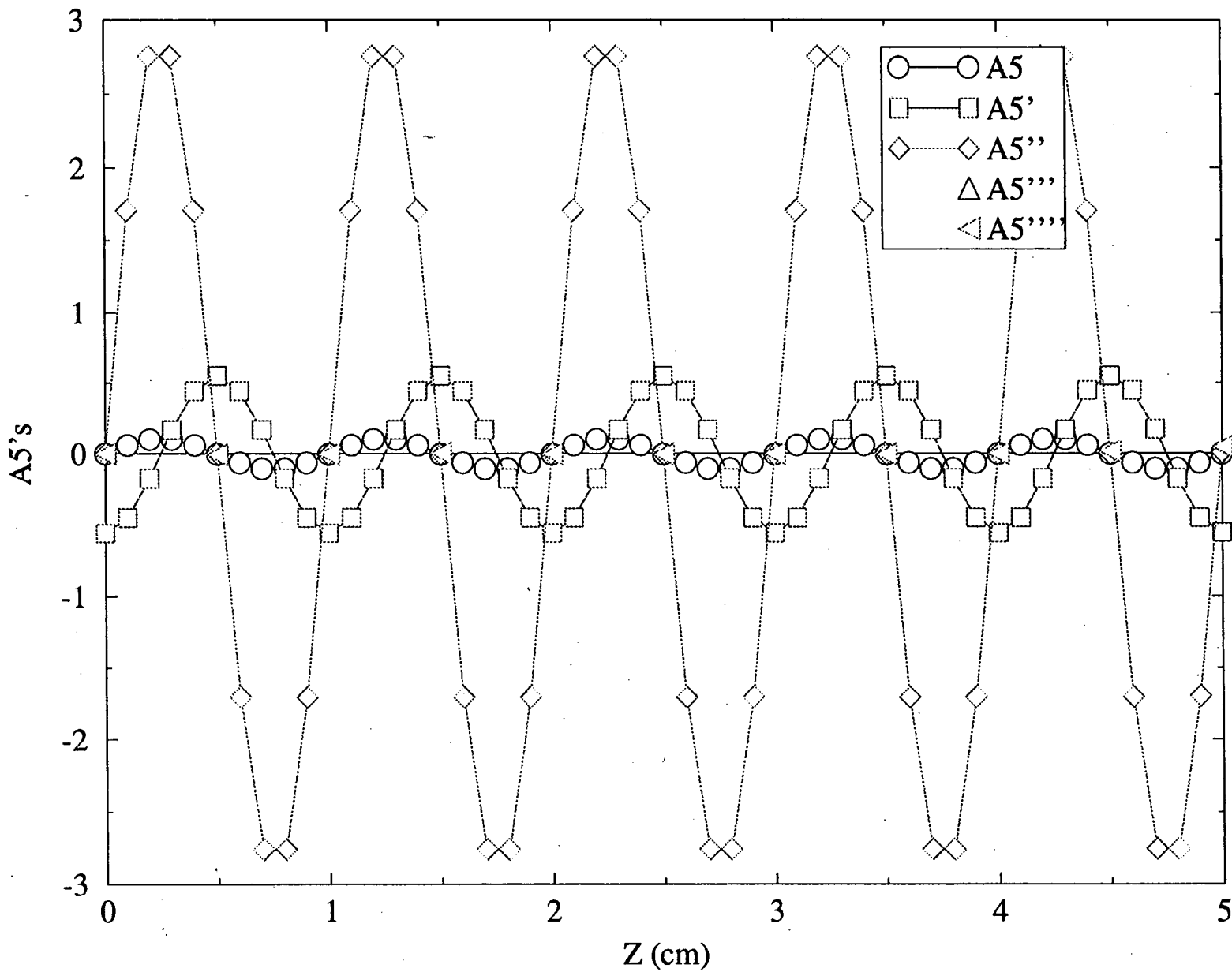


Figure 13 Skew A_5 as a function of z over a full period..

Helical wiggler $2L=5.0$ - Skew

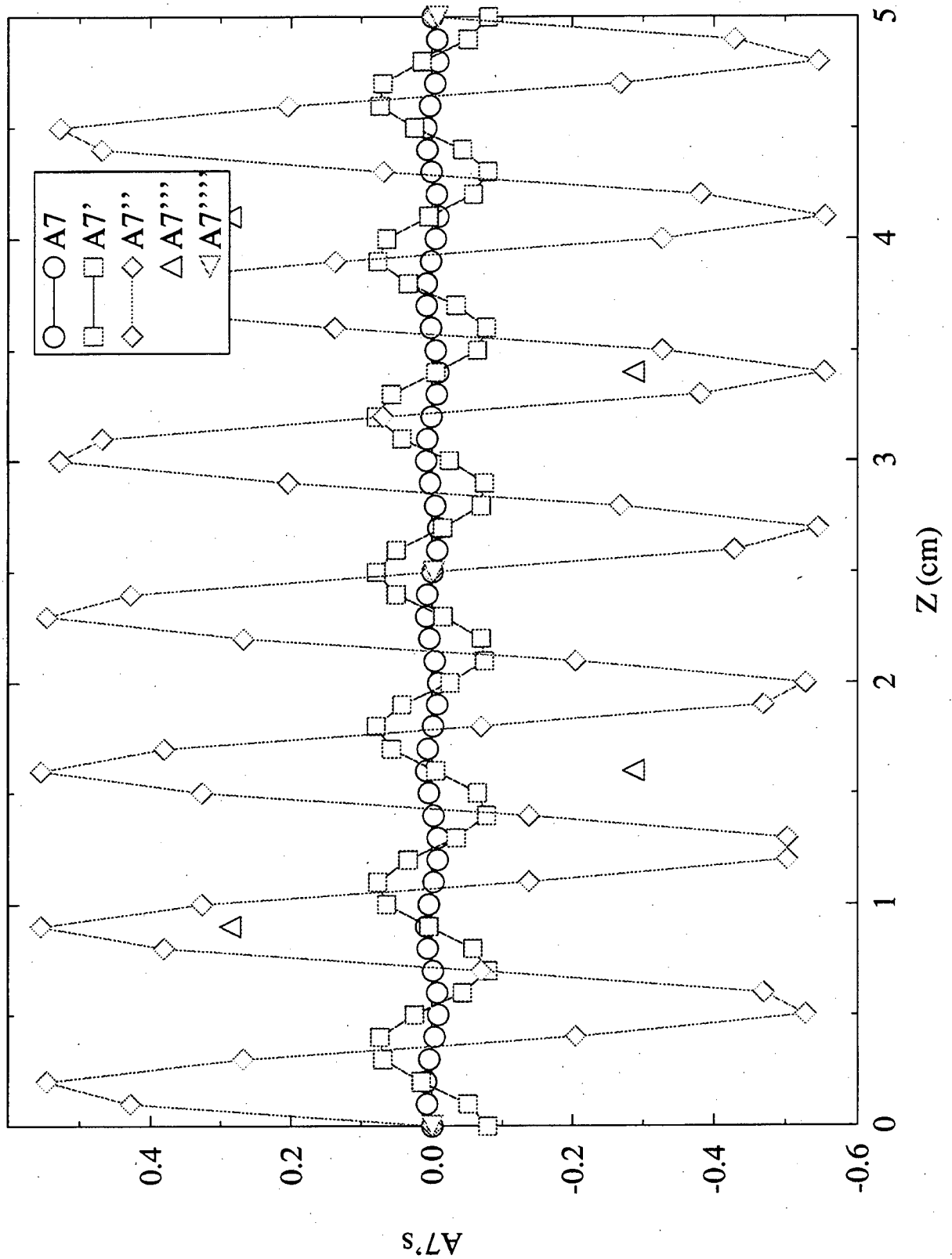


Figure 14 Skew $A7$ as a function of z over a full period..

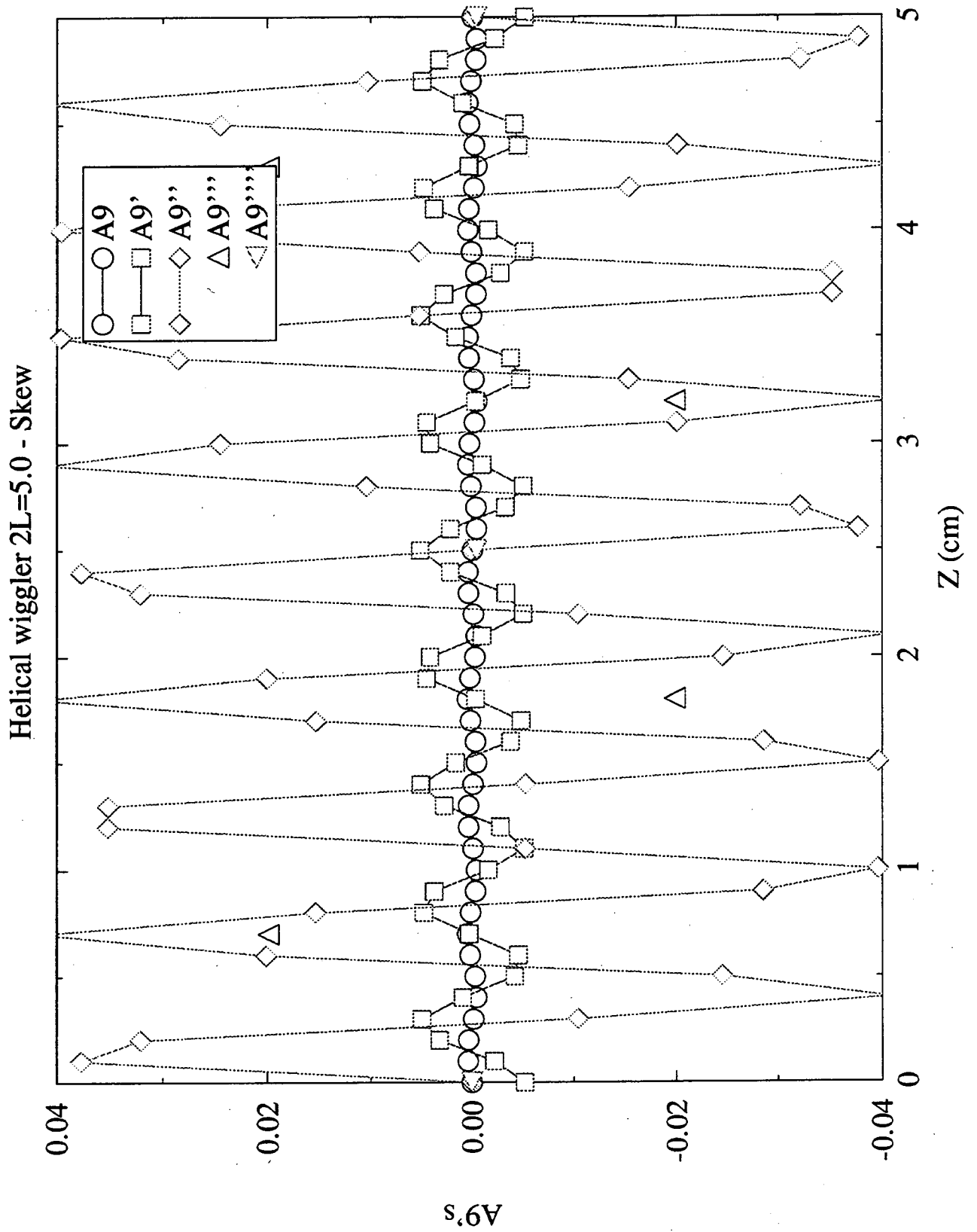


Figure 15 Skew $A9$ as a function of z over a full period..

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