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Authors

Frazer, William R.

Fulco, Jose R.

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 PION-PION SCATTERING RESONANCE ON NUCLEON STRUCTURE*

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Lawrence Radiation Laboratory
 University of California
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The electromagnetic properties of the nucleon have recently been studied by the dispersion-relation method.^{1,2} Although qualitatively successful in accounting for the isotopic vector properties of the nucleon, these treatments proved incapable of explaining simultaneously the value of the magnetic moment and the radii of the charge and moment distributions.³ The purpose of this letter is to show that the inclusion of a strong pion-pion interaction could explain these aspects of nucleon structure.

Let us first consider the general formulation and solution of the dispersion relations. In the notation of reference 1, we write the following representations for the form factors:⁴

$$G_i^V(s) = \frac{1}{\pi} \int_{(2m_\pi)^2}^{\infty} \frac{g_i^V(s') ds'}{s' - s}, \quad (1)$$

$$G_i^S(s) = \frac{1}{\pi} \int_{(3m_\pi)^2}^{\infty} \frac{g_i^S(s') ds'}{s' - s}, \quad i = 1 \text{ or } 2, \quad (2)$$

*

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where $s = (p' - p)^2 = (p_0' - p_0)^2 - (\vec{p}' - \vec{p})^2$, the square of the energy-momentum transfer four-vector. The weight functions $g_1(s)$ are related to a sum over all virtual intermediate states that can be reached from a photon and lead to a nucleon-antinucleon pair. For the isotopic vector functions $g_1^V(s)$, invariance considerations show that the least massive state is the two-pion state. We shall assume, as in previous treatments,^{1,2} that the two-pion contribution dominates in the dispersion integrals. On the other hand, the least massive state contributing to $g_1^S(s)$ is the three-pion state. We have nothing to say here about this contribution and shall limit ourselves to the isotopic vector properties.

In the approximation stated above, $g_1^V(s)$ and $g_2^V(s)$ are proportional to the pion form factor multiplied by the appropriate projection of the amplitude for the process $\langle N\bar{N} | \pi\pi \rangle$ in the state $J = 1$, $I = 1$. Using the Mandelstam representation,⁵ we are able to study the analytic properties of these projections, which we label $J_i(s)$. It can be shown that in the complex s -plane these functions are analytic except for branch cuts on the real axis for $s \leq 4m_\pi^2(1 - (m_\pi^2/4M^2))$ and $s \geq (2m_\pi)^2$. The right-hand branch cut, which was not considered in previous treatments, corresponds to π - π scattering. We shall show that it has an important effect on nucleon structure.

We can then write the dispersion relation

$$J_i(s) = \frac{1}{\pi} \int_{-\infty}^a \frac{\text{Im } J_i(s') ds'}{s' - s - i\epsilon} + \frac{1}{\pi} \int_{(2m_\pi)^2}^{\infty} \frac{\text{Im } J_i(s') ds'}{s' - s - i\epsilon}, \quad (3)$$

where $a \equiv 4m_\pi^2(1 - \frac{m_\pi^2}{4M^2})$. Application of the unitarity condition shows that in the region $(2m_\pi)^2 \leq s \leq (4m_\pi)^2$, the phase of $J_i(s)$ is equal

to the π - π scattering phase shift δ in the $J = 1, I = 1$ state. Considering only the two-pion intermediate state, we shall use this phase relation over the entire range of the right-hand integral in Eq. (3).

Because the left-hand integral is related to the pion-nucleon scattering amplitude, we shall consider it to be a known function. Equation (3) is then an integral equation, whose general solution has been found by Omnes.⁶ In our case, his solution can be modified into the more tractable form

$$J_1(s) = e^{u(s)} \frac{1}{\pi} \int_{-\infty}^a \frac{ds' \operatorname{Im} J_1(s')}{s' - s - i\epsilon} e^{-u(s')} , \quad (4)$$

where

$$u(s) = \frac{1}{\pi} \int_{(2m_\pi)^2}^{\infty} \frac{\delta(s') ds'}{s' - s - i\epsilon} . \quad (5)$$

It can easily be seen that Eq. (4) reproduces the content of the integral equation (3); namely, $J_1(s)$ has the proper singularities, has the phase of π - π scattering on the right-hand cut, and has the correct imaginary part on the left-hand cut. If the integral defining $u(s)$ fails to converge, one can use the subtracted form

$$u_0(s) = \frac{s}{\pi} \int_{(2m_\pi)^2}^{\infty} \frac{\delta(s') ds'}{s'(s' - s - i\epsilon)} . \quad (6)$$

We now require an expression for the pion form factor, $F_\pi(s)$, which satisfies the dispersion relation:

$$F_\pi(s) = 1 + \frac{s}{\pi} \int_{(2m_\pi)^2}^{\infty} \frac{ds' \operatorname{Im} F_\pi(s')}{s'(s' - s - i\epsilon)} . \quad (7)$$

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Unitarity allows us to conclude that the phase of $F_\pi(s)$ is the π - π scattering phase shift in the $J = 1, I = 1$ state,⁷ and again we shall use this condition over the entire range of integration. In this case Eq. (4) degenerates into the solution

$$F_\pi(s) = e^{u_0(s)}. \quad (8)$$

Combining Eqs. (4) and (8), we find

$$g_1^V(s) = |F_\pi(s)|^2 \frac{1}{\pi} \int_{-\infty}^a \frac{ds' \operatorname{Im} J_1(s')}{(s' - s - i\epsilon)F_\pi(s)}. \quad (9)$$

Equation (9) reveals the important fact that, because of the phase conditions imposed by unitarity, it is the absolute value of the pion electromagnetic form factor which appears in the weight functions $g_1^V(s)$. Thus the well-known condition that $g_1(s)$ be real is satisfied.

Using Eqs. (9) and (1), let us now investigate the effect of π - π scattering on the nucleon structure. It has been shown by Drell³ that in order to obtain agreement with the nucleon magnetic moment and radii, an enhancement of $g_1^V(s)$ by a factor of the order of five is required for $s < M^2$. From Eq. (9) it is apparent that a suitable peak in the pion form factor would produce this enhancement. We shall now show that a π - π resonance would result in such a peak.

An investigation of π - π scattering now in progress by Chew and Mandelstam has shown that the singularities of the partial-wave amplitude in the $J = 1, I = 1$ state are confined to branch cuts along the real axis in the range $s \leq 0, 4m_\pi^2 \leq s$.⁸ In the physical region, the

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effect of the left-hand singularities can be estimated by replacing the branch cut by a pole of appropriate position and residue. This approximation seems reasonable because for nucleon-nucleon scattering it leads to well-known effective-range formulae. Making this approximation, one finds the following solution for the $J = 1$ state, for $\nu > 0$:

$$f_{\pi\pi}(s) = \sqrt{\frac{\nu+1}{\nu^3}} e^{i\delta} \sin \delta = \frac{\Gamma}{\nu_r - \nu [1 - \Gamma \alpha(\nu)] - i \Gamma \sqrt{\frac{\nu^3}{\nu+1}}} \quad (10)$$

where

$$\nu = \frac{s}{4} - m_\pi^2, \quad \text{and}$$

$$\alpha(\nu) = \frac{2}{\pi} \sqrt{\frac{\nu}{\nu+1}} \ln \left(\sqrt{\nu} + \sqrt{\nu+1} \right).$$

A suitably continued form holds for $\nu < 0$. The constants Γ and ν_r are determined by the position and residue of the pole. By examination of the structure of the π - π equations, Chew and Mandelstam have found that the sign of the residue must be positive, corresponding to an attractive force and raising the possibility of a resonance. Further theoretical information about the equivalent pole must await numerical solution of the very complicated π - π equations. We shall now show, however, that if the constants ν_r and Γ are properly chosen, agreement with the nucleon-structure data may be achieved.

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A properly normalized solution for $F_\pi(s)$ is

$$F_\pi(s) = \frac{f_{\pi\pi}(s) (s + s_0)}{s_0 f_{\pi\pi}(0)}, \quad (11)$$

where s_0 is the position of the equivalent pole. The justification of this solution is, again, that it has the correct singularities and phase. Equation (11) clearly shows that a resonance in $f_{\pi\pi}$ will be reflected in the form factor F_π . The observed values of the charge and magnetic-moment radii indicate that this resonance should occur at $\nu_r \sim 3.5 m_\pi^2$ (square of the pion momentum in the π - π barycentric system). In Fig. 1 the function $|F_\pi(s)|^2$ is plotted for this value of ν_r and several values of the width Γ . In Fig. 2 the pion form factor is plotted for $s < a$. Since it is less than one over most of this region, its appearance in the denominator of the integral of Eq. (9) will produce an additional enhancement.

In conclusion, our Eq. (9) for the weight functions together with the approximation given in Eq. (10) for the pion-pion scattering amplitude suggests that a π - π resonance of suitable position and width could lead to agreement between dispersion theory and many aspects of nucleon electromagnetic structure. Detailed calculations are in progress.

We are indebted to Professor Geoffrey F. Chew for his advice throughout this work, and for advance communication of some of the results on pion-pion scattering. We also acknowledge the help of James S. Ball and Peter Cziffra in obtaining Eq. (4).

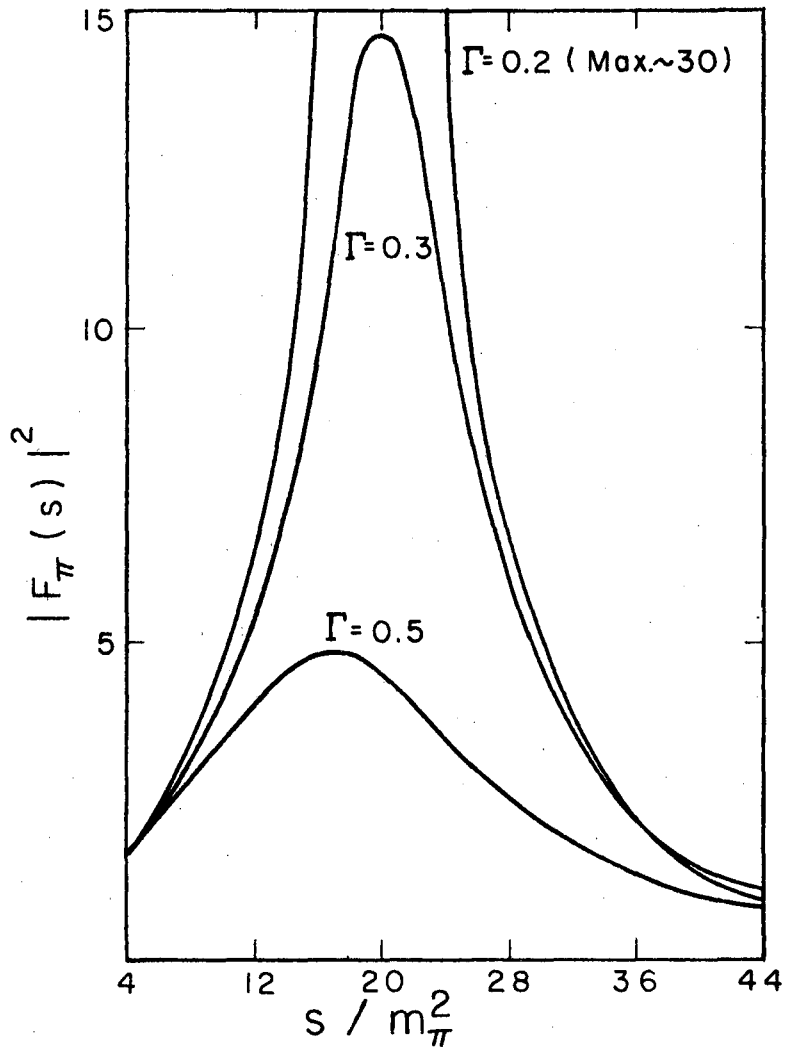
FOOTNOTES

1. Chew, Karplus, Gasiorowicz, and Zachariasen, Phys. Rev. 110, 265 (1958).
2. Federbush, Goldberger, and Treiman, Phys. Rev. 112, 642 (1958).
3. S. D. Drell, Proc. 1958 Annual International Conference on High Energy Physics at CERN (CERN, Geneva, 1958).
4. If the integral in Eq. (1) fails to converge, one can use the usual subtracted form, as discussed in reference 1.
5. S. Mandelstam, Phys. Rev. 112, 1344 (1958).
6. R. Omnes, Nuovo Cimento 8, 316 (1958).
7. See for example Appendix II of Fubini, Nambu, and Wataghin, Phys. Rev. 111, 329 (1958).
8. G. F. Chew, Lawrence Radiation Laboratory, private communication, 1959.

FIGURE LEGENDS

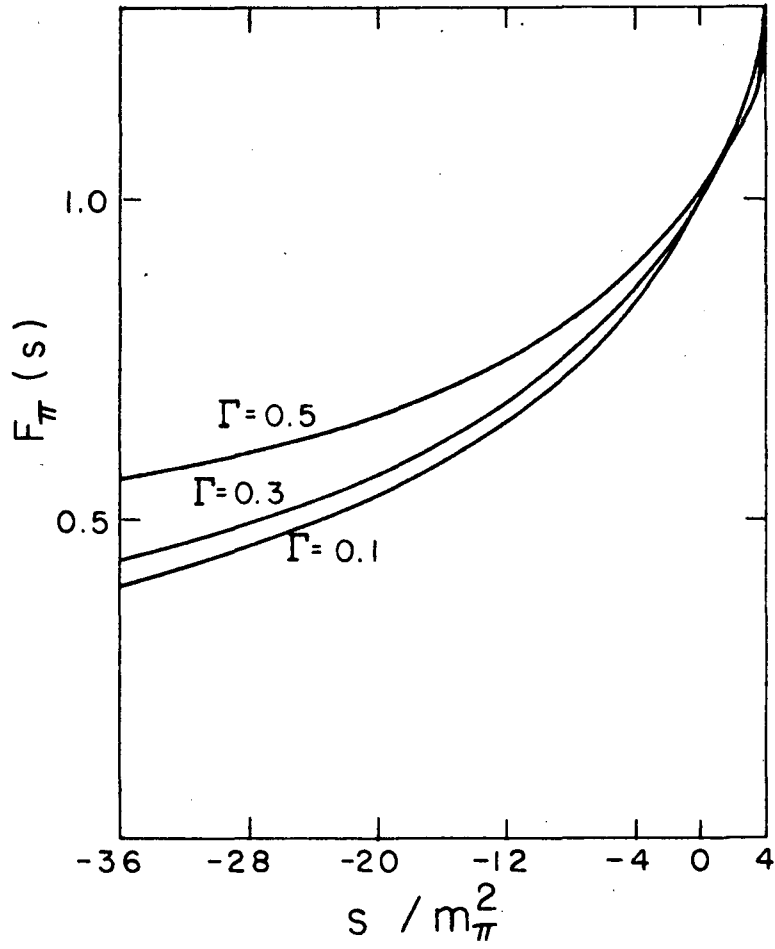
Fig. 1. The square of the magnitude of the pion form factor for $s \geq 4\mu^2$, for three values of the width Γ .

Fig. 2. The pion form factor in its physical region, for three values of the width Γ .



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Fig. 1



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Fig. 2

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