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# A Chaotic Neural Network Model of Insightful Problem Solving and the Generation Process of Constraints 

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#### Abstract

The solution of an insightful problem needs a drastic change from the "impasse" to the "insight" stage. It is assumed that in this type of a problem, solvers encounter the impasse stage because of special "constraints" like common sense. Abe, Wajima, and Nakagawa (2003) proposed a model of insight problem solving using a chaotic neural network. The model successfully simulates an insight problem. Based on this system, we developed a new model to explain the generation process of new constraints. We hypothesized that once people have solved a problem using insight, the experience of the insight generates new constraints. In order to verify the above hypothesis, we conducted a psychological experiment and executed a computational simulation of the model. In the experiment, participants were instructed to solve the two pictorial puzzles, one was an insight problem and the other was a non-insight problem. The experimental results showed that the solution of the insight problem generated a new constraint, while inhibiting the solution of non-insight problem. We constructed a new model that represents a reinforcement state after solving the insightful problem and several simulations were executed. The result of model's simulation showed a close similarity with the experimental result. The model successfully simulated the process of generation of new constraints.


Keywords: insight; constraint; generation process; neural network; modeling.

## Introduction

Insight is the process by which a problem is suddenly solved after an "impasse," which is the period when the solver is unable to solve the problem (Wallas, 1926). The process of insightful problem solving is the change from the state of "impasse" to the state of "insight," when problem is solved.
In previous studies, it is generally assumed that the impasse stage is due to constraints such as common sense (Knoblich, Ohlsson, Haider \& Rhenius, 1999).
Some constraints can be effective in guiding how people tackle everyday problems, but for problems that require some new insight, such constraints can obstruct the solver from seeing the solution to the problem. It has been suggested that insight is the product of breaking away from such constraints by discovering new effective directions during previous failed attempts at solving the problem.
Abe, Wajima, and Nakagawa (2003) proposed a model for insight problem solving using chaotic neural network.

This model consists of two components, namely, a "constraint component," and an "avoidance component." In order to represent these components, a system of simultaneous differential equations is proposed, with each variable denoting a single node of a neural network. In the model, the "constraint component" is represented by controlling the ease with which the nodes can activate, while the "avoidance component" is represented by the term causing the system to move in the direction in which the value of an evaluation function becomes the largest. This movement corresponds to the avoidance reaction of humans that is a result of failed trials. The model successfully simulates an insight problem.

However, in the previous research, sufficient consideration was not given to the generation process of constraints. Furthermore, the previous research did not refer to the possibility of the insight experience generating a new constraint due to reinforcement, once people had experienced solving a problem using insight.

We report here the development of a new model based on the insightful problem solving model to represent the generation of new constraints by hypothesizing that once people solved a problem using insight, the experience of the insight generates new constraints. In order to verify the above hypothesis, we conducted a psychological experiment and a computational simulation of the present model.
First, we set up an experiment in order to show that new constraints are generated by insight. In this experiment, participants were instructed to solve a non-insight problem after solving an insight problem to examine how the experience of insight subsequently changed the performance.

Second, we constructed a new model, by adding a mechanism to the previous model; a new constraint was generated by reinforcement of insight experience, while computational simulations were executed according to the variation of reinforcement values.

## Hypothesis

We explain our hypothesis of a new constraint generating process by using the "T-puzzle" and the "Arrow-puzzle" solving process.

The T-puzzle is an example of an insight problem (Figure 1, 2). In the T-puzzle, the solver constructs a "T" using pieces of the following four shapes: a triangle, a small trapezoid, a large-trapezoid, and a pentagon with a notch.


Figure 1: Pieces comprising the puzzle


Figure 2: T-puzzle


Figure 3: Arrow-puzzle
This puzzle has one constraint: "to fill the notch of pentagon." When participants solve this puzzle, they often fill the notch of pentagon (Suzuki \& Hiraki, 1997). In order to solve the T-puzzle, participants need the insight to escape from this constraint.
By using same four pieces used in the T-puzzle, an "Arrow" is constructed (Figure 3). We refer to this puzzle as the Arrow-puzzle. The Arrow-puzzle has the same constraint as that of the T-puzzle. In order to solve the Arrow-puzzle, participants need to fill the notch. The constraint aids solving the Arrow-puzzle. Thus, the participants do not need the insight acquired while solving the T-puzzle to solve the Arrow-puzzle.
In order to examine the effect of the previous insight on the solution of the next task, we instructed the participants to solve the Arrow-puzzle (non-insight problem) after they had solved the T-puzzle using insight. In this case, we assumed the solving process as follows (Figure 4):

- First, due to the constraint, i.e., "fill the notch of the pentagon," the T-puzzle was not solved.
- Insight was acquired, and participants discovered that the notch need not be filled.
- Reinforcement was acquired by solving the T-puzzle using insight, and a new constraint, i.e., "do not fill," was generated.
- Due to the new constraint, i.e., "do not fill," it became easy to select "not fill the notch of pentagon" when participants solved the Arrow-puzzle after solved the Tpuzzle.
It is considered that this change from filling the notch to not filling the notch is one process that generates constraints. Therefore, we focused on the change of frequency of the filling the notch.


Figure 4: Hypothesis

## Experiment

In this experiment, we examine how the experience of insight subsequently changes the performance. Our hypotheses in this experiment are as follows:

- When participants experienced the insight by escaping from the initial constraint, a new constraint was generated. Under this condition, the participant's performances while solving the next task changed due to the new constraint.
- If participants did not experience insight, the participant's performances did not change.


## Method

Participants. The participants comprised 30 graduate and undergraduate students who had never solved the T-puzzle or the Arrow-puzzle.
Tasks. Training task: Using 3 pieces to make any shapes other than the "T" or the "Arrow" within 3 minutes.

Completion task: Completing the goal shapes (" T " or "Arrow") by referring to the goal image silhouette.
Procedure. Participants were assigned to two groups ("TArrow" or "Arrow-T"). Participants in the T-Arrow group were instructed to first solve the T-puzzle and then the Arrow-puzzle. On the other hand, participants in Arrow-T group were instructed to first solve the Arrow-puzzle and then the T-puzzle.

## Result

In this study, "one trial" is defined as one segment that spans the period from connecting pieces to separating those pieces (Suzuki \& Hiraki, 1997).
The average time to solve the Arrow-puzzle was 433.50s for Arrow-T group and 306.33s for T-Arrow group; there is no significant difference between the two groups. The average trials count to reach the solution was 25.40 for the Arrow-T group and 35.14 for the T-Arrow group; there is no significant difference between the two groups. The average of frequency of filling the notch was 0.55 for the Arrow-T group and 0.28 for the T-Arrow group. The t-test revealed a
significant difference between the Arrow-T group and TArrow group, $(\mathrm{t}(22)=3.97, p=0.0007>0.01)$.
In order to examine the changes in the frequency of filling the notch, all participants' trials for each group was divided into 4 sections in the time series, and the average of the frequency of filling the notch in each section was plotted (Figure 5). In the case of the Arrow-T group, the frequency of filling the notch in the Arrow-puzzle was approximately 0.55 in all sections. In the case of the T-Arrow group, the frequency was approximately 0.2 in $\mathrm{t} 1 \sim \mathrm{t} 3$, and it rapidly increased to 0.61 in 44 .
For the Arrow-puzzle, the frequency change of filling the notch was examined by the analysis of variance, by considering the "Within factor" section and the "Between factor" group. The results indicated significant effects of the section, group, and their interaction. For the effect of group, $\mathrm{F}(1,27)=15.716, p<.001$; for the effect of section, $\mathrm{F}(3,81)$ $=7.894, p<.001$; and for the interaction effect, $\mathrm{F}(3,81)=$ $3.224, p<.05$. This result shows that the change of the frequency of filling the notch was affected by the experience of solving the T-puzzle.
For solving the T-puzzle, the average time to obtain the solution was 972.73 s for T -Arrow group and 849.85 s for Arrow-T group, and thus no significant difference was observed between the groups. The average trials count to solution was 69.47 for the T-Arrow group and 81.0 for the Arrow-T group, and no significant difference was observed between the groups. The average of frequency of filling the notch was 0.50 for the T-Arrow group and 0.45 for the Arrow-T group; there was no significant difference between both the groups.
For the frequency of filling the notch in the T-puzzle, we obtained three common tendencies between both groups: the frequency was approximately 0.5 in t , approximately 0.4 in t 4 , and decreased in $\mathrm{t} 1 \sim \mathrm{t} 4$ (Figure 6).
For the T-puzzle, the frequency change of filling the notch was examined by the analysis of variance by considering the "Within factor" section and the "Between factor" group. The results indicated significant effects of section only. For the effect of section, $\mathrm{F}=(3,81)=0.208, p<.005$. This result shows that the change of the frequency of filling the notch was not affected by experience of solving the Arrow-puzzle.

## Discussion of experiment

There is a significant difference between the averages of the frequency of filling the notch for the T-Arrow group and the Arrow-T group. The experiment shows that the experience of insight affects the performances while solving the next task.
For the Arrow-puzzle, in first section (t1), there is a considerable difference of the frequencies between the TArrow group and Arrow-T group. This result indicates that the constraint, "to fill the notch" is changed to the constraint, "not filling the notch," by the experience of insight in solving the T-puzzle. Additionally, the frequency in the last section (t4) increased greatly. This increased result meant


Figure 5: Result of experiment for Arrow-puzzle


Figure 6: Result of experiment for T-puzzle
the escaping from the new generated constraint, "not filling the notch" in solving the T-puzzle.
Conversely, with regard to the T-puzzle, there is no significant difference between the T-Arrow group and the Arrow-T group for the solution time, trial count, and the frequency of filling the notch. This result shows that the experience of solving the Arrow-puzzle does not affect the performances while solving the next task.

The result of the experiment clarifies that a new constraint is generated by the experience of insight. However, this result does not show whether there is reinforcement by the insight. In the next section, we construct a new model, which has a mechanism for the generation of a new constraint by reinforcement, and we simulate the generation process of constraints.

## Model

In this study, we construct a new model that includes a component for the generation of new constraints (constraint generating component), based on the previous insightful problem solving model (Abe, Wajima, \& Nakagawa, 2003).

## Previous Model

The model of insightful problem solving has two main components, the "Constraint component" and the "Avoidance component." Along with both the components, the model is represented by a system of simultaneous
differential equations, with each variable denoting each node of the neural network as follows:

$$
\begin{aligned}
& \ddot{x}_{i}+g\left(\dot{x}_{i}, t, x_{i}\right)=\beta\left(\sum_{j} w_{i j} u_{j}+\theta_{i}\right)+\gamma \frac{\partial E v}{\partial u_{i}} \\
& u_{i}=\frac{1}{1-e^{100 x_{i}}} \\
& g\left(\dot{x}_{i}, t, x_{i}\right)=\left(d_{0} \sin (\omega t)+d_{1}\right) \dot{x}_{i}+d_{2} \dot{x}_{i}^{2} \operatorname{sign}\left(\dot{x}_{i}\right) \\
& E v=\sum_{k}^{n} \eta_{k} \log \left\|U_{d k}-U\right\|^{2} \\
& \operatorname{sign}(x)= \begin{cases}0 & : x \leq 0 \\
1 & : x>0\end{cases}
\end{aligned}
$$

where $x_{i}, u_{i}$ and $\theta_{i}$ denote the inner state of node $i$, the output of node $i$, and the threshold of node $i$, respectively, $w_{i j}$ means the weight from $j$ to $i, U_{d k}$ represents the problem state of $n^{\text {th }}$ trial, and $U$ represents the current state.
"Constraint component" represents constraints faced by humans while solving problems. "Constraint component" was constructed to control the ease with which nodes are activated. In this model, the larger $\theta_{i}$ is, the easier it is for node $i$ to be activated. $\beta$ represents the effect of the "Constraint component."
"Avoidance component" represents the avoidance for failed trials. Nakagawa's psychological avoidance behavior model has the term that moves the system in the direction in which the value of the $E v$ becomes the largest (Nakagawa, 1978). This move for the term corresponds to the avoidance of failed trials. To construct the "Avoidance component," this model was applied. $\gamma$ represents the effect of the "Avoidance component."

After repeating some failed trials, because of competition between "Constraint component" and "Avoidance component", the model falls into a kind of local minimum where the model can not chose any adequate operator.
In order to escape from this local minimum, we applied the chaotic steepest descent method (CSD) (Tani, 1991). CSD has the chaotic term that unstabilizes the component when it reaches a local minimum. Therefore, the component escapes from a local minimum. The competition of the "Constraint component" and "Avoidance component" is solved by using CSD.

## T-Puzzle and Arrow-Puzzle Solving Model

We constructed the T-puzzle and Arrow-puzzle solving model as an application of the above mentioned "insightful problem solving model." This model selects 2 of the 4 pieces, and decides 6 attributes' states: the direction of the figure to connect, the direction of the figure to be connected, fill or clear of the pentagon notch, types of connections, lengths of connected lines are equal or unequal, and if the
same angle as the goal object exists or not. And the nodes $u_{1} \ldots u_{15}$ represent attributes' states (Figure 7). In order to choose only one length and only one direction, $w_{i j}$ is set as follows,

$\theta_{i}$ is set, as shown in Figure 7, so that 'fill of the pentagon notch' is easily chosen.

Then, the model connects the selected two pieces based on the decided 6 attributes' states. After connecting the pieces three times, the shape of connected pieces is checked to determine if it matches the "T" shape or the "Arrow" shape. The above operations are repeated for the model until the shape of connected pieces matches the " T " or the "Arrow."

## Proposed Constraint Generating Component

In the new model, the constraint generating component increases some $\theta_{i}$ values of attributes that were activated by insight. Consequently it becomes easy to choose those attributes in the following problem solving.
For example, in the model, as the initial state of the notch of pentagon, the "fill" node is easy to activate, while the "clear" node is difficult to activate. However, after solving the T puzzle using insight, the state changes to make the "clear" node easy to activate.

This process is represented by a mechanism that lowers the threshold of the "clear" node $\left(\theta_{8}^{\text {new }}=\theta_{8}-\alpha_{k}\right)$. This process represents the generation of new constraints that makes the Arrow-puzzle difficult to solve, because the solution of the Arrow-puzzle needs the state in which the "clear" node is difficult to activate (Figure 8).

## Simulation

## Simulation 1

First, a simulation that solves the Arrow-puzzle was executed for $\beta=0.1, \gamma=1$ and $\omega=0.07$. In this case, the solution time was $\mathrm{t}=4841.5$, the trial count to solution


Figure 7: T-puzzle and Arrow-puzzle solving model


Figure 8: Constraint generation component
was 33 , and the average of frequency of filling the notch was 0.51 . The change in frequency for all sections was negligible (Figure 9).

## Simulation 2

Several simulations that solved the Arrow-puzzle after solving T-puzzle were executed, changing the $\alpha_{k}$, where $\beta, \gamma$, and $\omega$ were maintained. In the case of $\alpha_{k}=0.1$, solution time was $t=18103.0$, trial count for the solution was 52 , and the average of frequency of filling the notch was 0.33 . The frequency was approximately 0.2 in $t 1 \sim t 3$, and increased rapidly to 0.61 in 4 .


Figure 9: Result of simulation 1

## Simulation 3

In the case of $\alpha_{k}=0.02$, the solution time was $\mathrm{t}=$ 12567.6, the trial count for the solution was 41 , and the average of frequency of filling the notch was 0.51 . The frequency change was negligible in all sections (Figure 10).

## Comparison between the experiment result and the simulation result

When the Arrow-puzzle was solved without the experience of the solving the T-puzzle, the experiment results showed that the average trials count was 25.4 and the frequency of filling the notch was 0.55 . The result of the simulation of solving the Arrow-puzzle without the experience to solve the T-puzzle was almost identical to the experiment, the average trials count was 33 and the frequency of filling the notch was 0.51 . Additionally, the frequency of filling the notch in both the experiment and the simulation showed negligible change in all sections. Since the result of


Figure 10: Result of simulation 2 and 3
simulation 1 is similar to that of the experiment of the Arrow-puzzle in the Arrow-T group, our model of solving the Arrow-puzzle represent human performance of solving the Arrow-puzzle.
When the Arrow-puzzle is solved after solving the Tpuzzle, the experiment results showed that average trials count was 81.0 and the frequency of filling the notch was 0.45 . The results of simulation $2\left(\alpha_{k}=0.1\right)$ of the Arrowpuzzle being solved after solving T-puzzle showed that average trials count was 52 and the frequency of filling the notch was 0.33 . The experiment result of the changes of frequency of filling the notch was almost identical the simulation 2's result. These frequencies did not change in section $\mathrm{tl} \sim \mathrm{t} 3$ at 0.2 , and increased rapidly to 0.61 in t 4 . Since the result of simulation 2 is similar to that of the Arrow-puzzle solving experiment in the T-Arrow group, our model of solving the Arrow-puzzle, where $\alpha_{k}=0.1$, represented human performance of solving the Arrowpuzzle after solving the T-puzzle.This result showing that the frequencies in $\mathrm{tl} \sim \mathrm{t} 3$ were lower and increased rapidly in $t 4$ was caused by a new constraint "Not filling the notch" that was generated by insight.
In the case of simulation $3\left(\alpha_{k}=0.02\right)$, average trials count was 41 and the frequency of filling the notch was 0.51 . The change in frequency was negligible at approximately 0.5 in all sections. This result of simulation 3 showed a difference from results of the experiment in the T-Arrow group. Thus, the simulation for $\alpha_{k}=0.02$ cannot represent human performance to solve the Arrow-puzzle after solving the T-puzzle.
In the case of $\alpha_{k}=0.02$, the nodes of filling the notch after operating the constraint generation component was easier to activate than that of not filling the notch. This means a new constraint was not generated.
In the case of $\alpha_{k}=0.1$, it is easier to activate the nodes of not filling the notch after operating the constraint generation component than those of filling the notch. In this state of the nodes, the frequency of not filling the notch increases. This process is identical that observed in the human reinforcement process. These result show that there is a reinforcement process generated by insight.

## Conclusion

In this study, the experiment showed that a new constraint is generated by experience of insight. A new constraint was generated when participants experienced insight, while it was not generated without the experience of insight.
We constructed a new model that has a mechanism where a new constraint is generated by reinforcement when insight is experienced. Simulation results showed that there is a reinforcement process generated by insight. The results also indicated that reinforcement caused by insight generates a new constraint.

People usually experience a strong emotion (happiness, surprise, etc.) when they have solved a problem using insight (Davidson, 1995). It is considered that this strong emotion is very important for reinforcement by insight. We propose the hypothesis that the reinforcement is a result of the reward of the above emotional experiences, and a new constraint is generated by the reinforcement.
In order to verify the above hypothesis, we are planning a new experiment that will examine a new constraint that is not generated if a participant does not experience a strong emotion while acquiring insight.

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