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Authors
Martinelli, E.
Miller, T.
Byrns, R.

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UNIVERSITY OF CALIFORNIA
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ELECTRICAL ENGINEERING REVIEW COURSE

LECTURE IX
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E. Martinelli

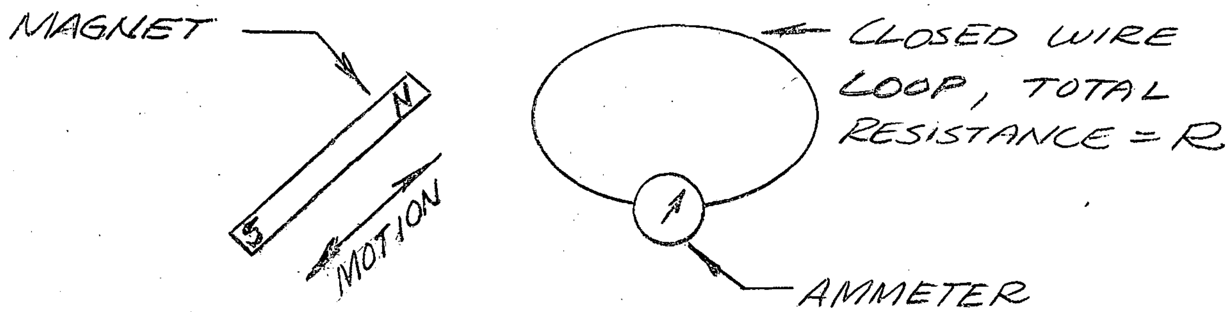
(Notes by: T. Miller, R. Byrns)

FARADAY'S LAW OF INDUCTION AND ENERGY OF A MAGNETIC FIELD

LAW OF INDUCTION:

Faraday's observation: (1831)

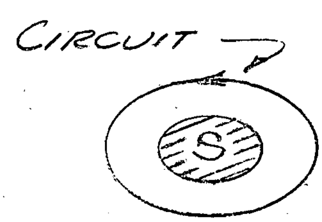
Faraday discovered that relative motion between a magnet and a closed electric circuit would induce an e.m.f. in the circuit and hence current flow.



Faraday's law presents the last part of experimental evidence to be applied to Maxwell's equations and forms the connecting link between electric and magnetic fields.

The law of induction is given by:

$$(1) \quad iR = -\frac{1}{C}\frac{d}{dt}\iint_s \vec{E}\cdot\vec{ds}$$



i = current in wire (e.s.u.)

c = 3×10^{10} cm./sec.

t = seconds

ds = element of surface, S(cm.^{2})

B = magnetic induction or magnetic flux in gauss.

The minus sign indicates that the magnetic field induced by current

flow tends to oppose a change in flux (Lenz's Law)

The surface integral sums the total magnetic flux thru the circuit.

The equation states that the iR drop around any fixed closed loop equals the rate of decrease of total magnetic flux thru the loop.

The equation is valid for slow-varying currents. Generalizing for higher frequencies the wire of resistance R is replaced by an electric field.

Using Ohm's Law:

$\vec{i} = \sigma \vec{E}$ current density = conductivity x electric field

Integrate around the loop:

$$\oint \vec{i}\, A dl = \oint \sigma \vec{E}\, A\, dl$$

A = cross sectional area of wire

$\vec{i}\, A = i$ = total current in wire where $\oint i\, A\, dl$ assumes total current in wire is constant.

Substituting iL for $\oint \vec{i}\, A\, dl$

$$iL = \sigma A \oint \vec{E} \cdot \vec{dl}$$

$$\frac{iL}{\sigma A} = \oint \vec{E} \cdot \vec{dl}$$

But $\frac{iL}{\sigma A} = iR$ where $\frac{L}{\sigma A}$ = total resistance

Then $iR = \oint \vec{E} \cdot \vec{dl}$

Substitute in equation (1) for iR

$$\oint \vec{E} \cdot \vec{dl} = -\frac{1}{C} \frac{d}{dt} \iint_s \vec{B} \cdot \vec{ds}$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{1}{C} \frac{d}{dt} \phi \quad \text{where: } \phi = \text{total flux}$$

The electrostatic field equals the negative rate of change of magnetic flux.

This is the generalized form of Faraday's Law of Induction since the wire has been eliminated. An electric field will exist with or without the wire.

By Stokes' Theorem:

"The surface integral of the curl of a vector function of position in space taken over a surfase S is equal to the line integral of the vector function taken around the periphery of the surface."

$$\iint_s \text{curl}\ \vec{F} \circ \vec{ds} = \oint \vec{F} \circ \vec{dl}$$

$$\iint_s \text{curl}\ \vec{E} \circ \vec{ds} = \oint \vec{E} \circ \vec{dl} = -\frac{1}{C}\frac{d}{dt}\iint_s \vec{B} \cdot \vec{ds}$$

$$\text{curl}\ \vec{E} = -\frac{1}{C}\frac{d\vec{B}}{dt}$$

This is one of Maxwell's equations, Faraday's law in differential form, and relates electric and magnetic fields.

We had:

$$\text{curl}\ \vec{B} = \frac{4\pi}{C}\vec{i}$$

$$\text{curl}\ \vec{E} = -\frac{1}{C}\frac{d\vec{B}}{dt}$$

$$\vec{I} = \vec{i} + \frac{1}{4\pi}\frac{d\vec{D}}{dt}$$

Maxwell's Equations

(2) $\text{div}\ \vec{E} = \frac{4\pi}{r}\rho$

(3) $\text{div}\ \vec{B} = 0$

(4) $\text{curl}\ \vec{B} = \frac{1}{C}\frac{d\vec{D}}{dt} + \frac{4\pi\vec{i}}{C}$

(5) $\text{curl}\ \vec{E} = -\frac{1}{C}\frac{d\vec{B}}{dt}$

Equations (2) and (3) are equations of continuity and relate to the fields created by charges at rest.

Equations (2) states that the divergence of the electric field, or lines of electric force originating from a region in space, equals the charge per unit volume. The electric force lines start and end on charges.

Equation (3), magnetic case analogous to (2) says that the divergence of B, the magnetic induction is equal to zero. The magnetic force lines cannot originate in a source or disappear into a sink; as many lines leave as enter a region.

Equations (4) and (5) deal with fields produced by moving charges.

Equations (4) states that the curl of the magnetic induction equals the rate of change of displacement current plus the current density. Displacement current may be illustrated by considering a capacity which will not pass direct current, but will allow flow of a current varying with time. As the plate of a condenser charges up there develops in the surrounding dielectric what Maxwell called a displacement current.

Equation (5) says that the curl of the electrostatic field equals the negative rate of change of magnetic induction. This equation gives the values of the induced electric fields created by a change of magnetic flux. Thus, as soon as one has a changing magnetic field, an electrical field is present, and vice versa.

<u>Energy Density in Magnetic Field</u>

For the electrostatic field we found:

$$\text{energy density} = u_e = \frac{\vec{E} \cdot \vec{D}}{8\pi} \text{ (gaussian units)}$$

$$u_e = \frac{1}{2} \vec{E} \cdot \vec{D} \text{ (MKS units)}$$

For the electromagnetic field we expect:

$$u_m = \frac{\vec{H} \cdot \vec{D}}{8\pi} \text{ (gaussian units, ergs/cm.}^3\text{)}$$

$$u_m = \frac{1}{2} \vec{H} \cdot \vec{D} \text{ (MKS units, joules/meter}^3\text{)}$$

In general, the above equations for u_m are not true because permeability is not as constant a factor as dielectric. They give energy density provided only soft, unsaturated iron is present, but are invalid for permanent magnets.

In the electrostatic case we found:

$$\text{total energy} = U_e = \frac{1}{2} \int \rho V dv = \frac{1}{2} \sum e_i v i$$

The magnetic form involves fields:

$$U_m = \frac{1}{2} \int \vec{H} \cdot \vec{B} dv$$

For the magnetic field we can't find the scalar potential V, because curl V is not equal to zero.

Given: $\text{div} \vec{B} = 0$

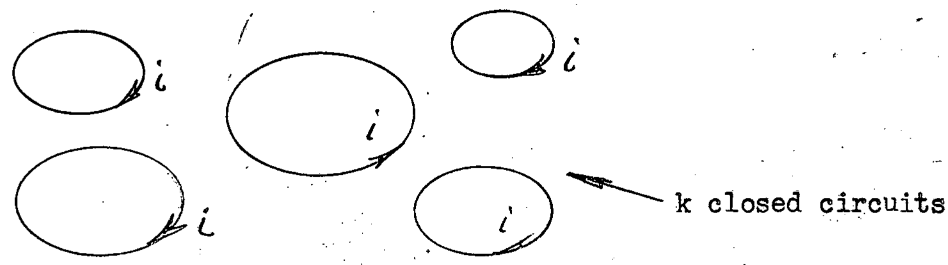
Let: $B = \text{curl} \vec{A}$

Thus vector A is called a vector potential. (As opposed to a scalar potential.)

In general, any vector field can be expressed in terms of a vector potential (solenoidal field) plus a scalar potential (irrotational field)

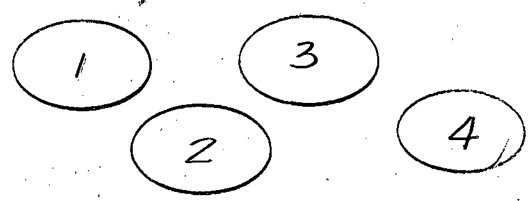
$$U_m = \frac{1}{2}\int_{vol.} (\vec{i} \cdot \vec{A})dv = \frac{1}{2}\sum i_k \oint \vec{A} \cdot \vec{dl}$$

Given a number (k) of closed circuits, i represents the total current in each circuit.



$$U_m = \frac{1}{2}\sum i_k \iint_s \vec{B} \cdot \vec{ds} = \frac{1}{2}\sum i_k \phi_k$$

The above form involves sources.



The flux, ϕ, in loop 1 is given as a function of inductance L by:

$$\phi_1 = L_{1-1}i_1 + L_{1-2}i_2 + L_{1-3}i_3 + \ldots\ldots\ldots\ldots\ldots\ldots$$

ϕ depends on the geometry of the coils and will be a linear function of current.

L_{1-1} is self-inductance.

$L_{H-K} = L_{K-H}$ is mutual inductance

$L_{1-1} = \frac{\phi}{i_1}$

ϕ is total flux in webers
i is current in amperes
L is inductance in henrys

We can now write equations for the magnetic energy in terms of the coefficient of induction (geometrical factors).

$$U_m = \frac{1}{2}\sum\sum L_{k-h} i_h\, i_k$$

$$U_m = \frac{1}{2} Li^2 \quad \text{(for one circuit only)}$$

U in joules
L in henrys
i in amperes

We also have:

$$U_m = \frac{1}{2}\iint_{vol.} \vec{B} \cdot \vec{H}dv$$

U in ergs
H in gauss
v in $cm.^3$

Knowing the values of the fields we can calculate the energy and thus compute L.

In the cyclotron one can assume all the energy is stored in the gap because of iron in the core.

Given: 1500 amps. thru cyclotron magnet
15,000 gauss uniform field
22 inch gap
180 inch diameter

$$\text{Volume of field (cm.}^3) = 22 \frac{\pi}{4} (180)^2 (2.54)^3$$

$$V = 9.16 \times 10^6 \text{cm.}^3.$$

To find L knowing H and i,

$$U_m = \frac{1}{2} \int_{vol.} \vec{B} \cdot \vec{H} dv = \frac{1}{2} L i^2$$

$$L = \int \frac{\vec{B} \cdot \vec{H} dv}{i^2} = \frac{(2.25 \times 10^8)\ (9.16 \times 10^6)}{2.25 \times 10^6\ \ (10^7 \frac{\text{ergs}}{\text{joule}})}$$

$$L = 91.6 \text{ henrys.}$$

This method can also be applied to air solenoids to find L (neglecting end losses). It cannot be used for all geometries and one must know the fields to use, but the approach will give rough approximations.