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# Probabilistic relational categories are learnable as long as you don't know you're learning probabilistic relational categories

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## Abstract

Kittur, Hummel and Holyoak (2004) showed that people have great difficulty learning relation-based categories with a probabilistic (i.e., family resemblance) structure. We investigated three interventions hypothesized to facilitate learning family-resemblance based relational categories: Naming the relevant relations, providing a hint to look for a family resemblance structure, and changing the description of the task from learning about categories to choosing the “winning” object in each stimulus, which was predicted to encourage subjects to form an invariant higher-order relation. We crossed these variables orthogonally in a factorial design. Of the three, the change in task description had by far the greatest impact on subjects' ability to learn probabilistic relation-based categories. For subjects in the category learning task, naming the relations and the “no single relation” clue both improved performance individually, but in combination, they substantially impaired learning. These results suggest that the best way to learn a probabilistic relation-based category is to discover a higher-order relation that remains invariant over the category's exemplars.

**Key words:** Relational category learning; family resemblance; higher-order relations; relational invariants.

One of the most robust findings in the vast literature on category learning is that people are capable of learning categories with a *family resemblance* structure, in which every member of the category shares some features with every other member, but no single feature is shared by all category members (e.g., Bruner, Goodnow, & Austin, 1956; Kruschke, 1992; Kruschke & Johansen, 1999; Medin & Schaffer, 1978; Nosofsky, 1992; Rosch & Mervis, 1975; Shiffrin & Styvers, 1997; Smith & Medin, 1981). The “prototype” effects that result from such learning (such as our ability to learn a category prototype from the exemplars without ever seeing the prototype itself) are so robust that they led Murphy (2002) to quip that any category learning experiment that fails to demonstrate prototype effects is suspect. It is hardly possible to teach a course in cognitive science without talking about prototype effects: They are among the most ubiquitously observed and widely accepted effects in cognitive psychology.

In reviewing the literature on prototype effects, Kittur, Hummel and Holyoak (2004) noticed that all the studies reporting prototype effects used category structures defined by their members' *features*. For example, if the categories to be learned were fictional animals then they might be defined by features such as the shape of the head, the shape of the tail, etc. Similarly, the vast majority of models of category learning and categorization assume that we represent categories and exemplars as lists of features and assign exemplars to categories by comparing their features (see Kittur et al. for a review). As Kittur et al. observed, this reliance on feature-based categories is a limitation inasmuch as many natural concepts and categories are based, not exclusively on features, but also on *relations*, including both relations between the features of an exemplar (e.g., the seat and back of a chair need to be in a particular spatial relation to serve as a chair) and relations between the exemplar and other objects (e.g., the category *conduit* is defined by a relation between the conduit and the thing it carries; the category *barrier* is defined by the relation between the barrier, the thing to which it blocks access and the thing deprived of that access; even such a basic category as *mother* is defined by a relation between the mother and her child) (see Gick & Holyoak, 1983; Pirolli & Anderson, 1985; Ross, 1987). Relational categories also play a central role in mathematics and science (e.g., denominator, result, carnivore, pressure, magnetic, attraction; Anggoro, Gentner, & Klibanoff, 2005).

The importance of relational categories in human cognition, in combination with their under-representation in one of the largest literatures in cognitive psychology, led Kittur et al. (2004) to pose the following simple question: Can we observe prototype effects with relational, rather than feature-based, categories? That is, if the categories to be learned are defined by the relations between the exemplars' features, rather than the literal features themselves, can human subjects learn categories with a family resemblance structure? And if they do, what will the resulting prototypes be like? Kittur et al. never got an answer to the second question because the answer to the first question turned out to be a resounding No: Using a 2 X 2 design crossing category structure (family resemblance, in which no single

feature or relation always predicted category membership, vs. deterministic, in which one feature or relation remained invariant across all exemplars of a category) with defining property (exemplar features vs. relations between those features), Kittur et al. found that subjects in the relational/family resemblance condition found category learning much more difficult than subjects in the other conditions (an effect that Kittur, Holyoak & Hummel, 2006, used ideal observer analysis to demonstrate could not be attributed to the formal difficulty of the task itself); indeed, the majority of the subjects in the relational/family resemblance condition failed to reach criterion even after 600 trials of learning.

Kittur et al. (2004) interpreted their findings in terms of the LISA model of schema induction (Hummel & Holyoak, 2003). Specifically, Kittur et al. reasoned that if a relational category is represented as a schema, as has been proposed by others (e.g., Barsalou, 1993; Gentner, 1983; Holland, Holyoak, Nisbett, & Thagard, 1986; Keil, 1989; Murphy & Medin, 1985; Ross & Spalding, 1994), and if schemas are learned by a process of *intersection discovery*, in which a schema is learned from examples by keeping what the examples have in common and discarding details on which they differ (as proposed by Hummel and Holyoak, 2003; see also Doumas, Hummel & Sandhofer, 2008)<sup>1</sup>, then learning probabilistic relational categories ought to be extremely difficult because the intersection of the examples is the empty set (i.e., there is no single relation shared by all category members).

Although Kittur et al.'s (2004) data are consistent with their conclusions, there are several other reasons why probabilistic relational categories might be hard to learn from examples. In the research reported here, we sought to better understand the difficulty of learning relational categories with a family resemblance structure by investigating circumstances that might make them easier to learn. Specifically, we tested three not-mutually-exclusive hypotheses about what makes family resemblance relational categories difficult to learn. We evaluated these hypotheses using a 2 X 2 X 2 crossed factorial design.

Following Kittur et al. (2004), each of our exemplars was composed of two shapes: a square and a circle (Kittur et al. used an octagon rather than a circle, and they placed their stimuli on a background designed to resemble a "computer chip" whereas we did not, but the stimuli are otherwise isomorphic). In each exemplar, one of the two shapes was *larger* than the other, one was *darker* than the other, one was *in front* of the other, and one was *above* the other. In the prototype of category A (never seen by subjects), the circle was larger, darker, above and in front of the square; in the prototype of category B, the square was larger, darker, above and in front of the circle. In any given exemplar seen by a subject, exactly three of these relations were shared

with the prototype of the exemplar's category and one was shared with the exemplar of the opposite category (e.g., an exemplar of A might have the circle larger, darker and above [A-prototype relations] but behind [a B-prototype relation] the square).

The first hypothesis we explored is that people are simply biased toward learning based on features rather than relations. To test this hypothesis, one factor varied whether the instructions to subjects explicitly stated which (and therefore that) relations were relevant to category membership. To the extent that the results of Kittur et al. (2004) reflect a bias against using relations for categorization, naming the relations should facilitate category learning

The second hypothesis we tested was that, rather than being *unable* to learn relational categories with a family resemblance structure (as Kittur et al., 2004, concluded), people are simply *biased against* assuming that relational categories will have a family resemblance structure. That is, faced with relational categories, perhaps people simply assume that those categories will have some defining (i.e., deterministic; invariant) relation—for example, an *essence* (see Keil, 1989; Medin & Ortony, 1989)—that is shared by all members of the category, and that this assumption caused Kittur et al.'s subjects to adopt a suboptimal learning strategy. To test this hypothesis, the second factor varied whether a clue was given. In the *clue* condition the instructions explicitly informed subjects that no single property would always work as the basis for categorizing the exemplars. In the *no clue* condition, no such clue was provided. To the extent that subjects are biased against assuming a family resemblance category structure given relational categories, providing this clue should help them to adopt a more appropriate learning strategy, especially when the relations were also named.

Our final hypothesis started with Kittur et al.'s (2004) conclusion: If it is difficult to learn relational categories that have a family resemblance structure, then anything that encourages subjects to discover and predicate a property—e.g., a higher-order relation over the first-order relations—that does remain invariant across all members of a category ought to substantially improve relational category learning (since the categories, although probabilistic in the first-order relations, would now be deterministic in the higher-order relation). To test this hypothesis, in the *categorize* condition, subjects were instructed to learn the category of each stimulus, as in Kittur et al. In the *who's winning* condition, we told subjects they would see displays consisting of a circle and a square, and that in each display "either the circle is winning or the square is winning," and that their task was to figure out which one was winning. In all other respects, the who's winning task was identical to the categorize task: In any stimulus that would be categorized as a member of category A, the circle was "winning", and in any stimulus that would be categorized as a B, the square was "winning."

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1. Doumas, Hummel, & Sandhofer (2008) predict that intersection discovery is essential, not only to learn entire relational schemas, but also to acquire basic relations, such *above* and *larger-than*.

The “who’s winning” task could encourage subjects to discover an invariant that holds across all members of a “category” (even though subjects in this condition did not know they were learning categories) in at least two ways. First, subjects might semantically “mark” the “winning” role of each relation (e.g., the larger role of *larger*, the darker role of *darker*, etc.) and then notice, at the level of semantic features, that either the circle or square has more “winning features,” effectively changing the “who’s winning” task from a relational judgment to a featural one (namely, “which shape is currently bound to more ‘winning’ features?”; see Hummel & Holyoak, 2003). Alternatively, the “who’s winning” task could activate schemas participants have for situations in which one person or team wins and another loses; such a schema might encourage subjects to predicate a higher-order relation of the form “more winning relations on the circle/square,” which would remain invariant over members of a category. In either case, even though no (nominally relevant) first-order relation would remain invariant over all members of a category, at least one feature or higher-order relation would. If the presence vs. absence of an invariant is key to the learnability of relational categories (as concluded by Kittur et al., 2004), then subjects in the *who’s winning* condition might learn faster than subjects in the *categorize* condition.

## Method

**Participants.** A total of 154 subjects participated in the study for course credit. Each participant was randomly assigned to one of the eight conditions.

**Materials.** Subjects were first given instructions to categorize the stimuli (*categorize* condition) or decide whether the circle or square was winning (*who’s winning* task), which either named the relevant relations (*relations named*) or not (*not named*) and either provided the “no single property will always work” clue (*clue* condition) or not (*no clue*).

After the instructions, all conditions were identical. On each trial following the instructions, an exemplar consisting of a gray circle and a gray square appeared in the middle of the computer screen. The properties of the exemplars were determined by a family resemblance category structure defined over the relevant first-order relations. The prototypes of the categories were defined as [1,1,1,1] for category A and [0,0,0,0] for B, where [1,1,1,1] represents a circle *larger*, *darker*, *on top of*, and *in front of* a square and [0,0,0,0] represents a circle *smaller*, *lighter*, *below* and *behind* a square. Exemplars of each category were made by switching the value of one relation in the prototype (e.g., category A exemplar [1,1,1,0] would have the circle *larger*, *darker*, *on top of* and *behind* the square). Two variants of each logical structure were constructed by varying the metric properties *size* and *darkness*, respecting categorical relations *larger* and *darker*, resulting in eight exemplars per category.

**Design.** The experiment used a 2 (*relations named vs. not named*) X 2 (*clue vs. no clue*) X 2 (*categorize vs. who’s winning* task) between-subjects design.

**Procedure.** Participants were given different instructions based on the condition to which they belonged. After indicating that they fully understood the instructions, participants proceeded to the learning phase. Trials were presented in blocks of 16, with each exemplar presented in a random order once per block. In the *categorize* condition, subjects were instructed to press the A key if the stimulus belonged to category A and the B key if it belonged to B; in the *who’s winning* condition, they were instructed to press the A key if the circle was winning and the B key if the square was winning (i.e., the stimulus-response mapping was identical across tasks, since in all members of A the circle “wins” and in all members of B the square “wins”). Each exemplar remained on the screen until the participant responded. Responses were followed by accuracy feedback (i.e., the correct category label). The experiment consisted of 60 blocks (960 trials) and continued until the participant responded correctly on at least fourteen of sixteen trials (87.5% correct) for two consecutive blocks or until all 60 blocks had transpired, whichever came first. At the end of the experiment, participants answered a debriefing questionnaire that queried them about the strategies they used during the experiment.

## Results

**Trials to criterion.** Since our primary interest is the rate at which participants learn the categories, we report our data first in terms of trials to criterion. These analyses are conservative (i.e., biased against our hypotheses) in the sense that participants who never learned to criterion were treated as though they reached criterion on the last block. Figure 1 shows the mean trials to criterion by condition. A 2 (*relation name vs. no name*) X 2 (*clue vs. no clue*) X 2 (*categorize vs. who’s winning*) between-subjects ANOVA revealed a main effect of task [ $F(1, 145) = 25.826$ ,  $MSE = 2,267,729$ ,  $p < 0.001$ ], reflecting the fact that participants took reliably fewer trials to reach criterion in the *who’s winning* task ( $M = 211$ ,  $SD = 261$ ) than in the *categorize* task ( $M = 453$ ,  $SD = 339$ ). No other main effects were statistically reliable. However, here was a reliable interaction between *relation naming* and *clue*, indicating that the effect of providing the clue was more pronounced for relation *not named* than for *relation named* [ $F(1, 145) = 5.98$ ,  $MSE = 525,066$ ,  $p < 0.05$ ]. Finally, there was a reliable three-way interaction between *relation name*, *clue*, and *task* [ $F(1, 145) = 4.10$ ,  $MSE = 359,946$ ,  $p < 0.05$ ]. As shown in Figure 1, relation naming interacted with the clue differently across the two tasks. With the *who’s winning* task, the effect of the clue was roughly equivalent to the effect of naming the relations, with each reducing trials to criterion. By contrast, for participants given the *categorize* task, naming the relations without providing the clue and providing the clue without naming the relations were both beneficial

relative to doing neither (i.e., there were trends such that trials to criterion were lower in both the *relations named, no clue* and *categorize* and the *relations not named, clue*, and *categorize* conditions than in the *relations not named, no clue* and *categorize* condition, although these trends did not reach statistical reliability in our sample); but both naming the relations and providing the clue together did not facilitate category learning, and in fact the trend went in the opposite direction (i.e., trials to criterion tended toward being greater in the *relations named, clue* and *categorize* condition than in the *relations not named, no clue* and *categorize* condition).

Of particular interest is the fact that the condition that gave rise to the worst performance with the *categorize* task (and overall)—specifically, the *relation named and clue* condition, with only 50% of participants learning to criterion (and a mean of 623 trials to criterion)—gave rise to the best performance with the *who's winning* task (and overall), with 95% of participants learning to criterion (and a mean of 160 trials to criterion). We address the possible reasons for this effect in the Discussion.

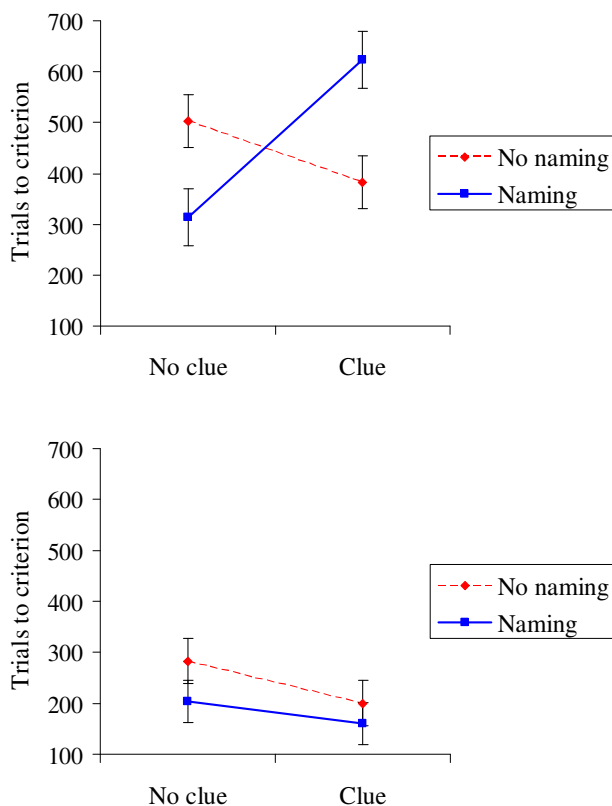


Figure 1. Mean number of trials required by subjects in each task to reach criterion. Top and bottom figure correspond to the *categorize* and the *who's winning* tasks, respectively.

**Survival function.** We also analyzed how many participants reached criterion by the end of each block. The resulting survival functions for the *who's winning* and *categorize* conditions are shown in Figure 2. (A participant's having "survived" on a given block means they

had not yet reached criterion by that block. Participants who survived to block 60 never reached criterion.) As shown in Figure 2, a higher proportion of participants reached criterion in the *who's winning* condition than in the *categorize* condition, and they did so much faster.

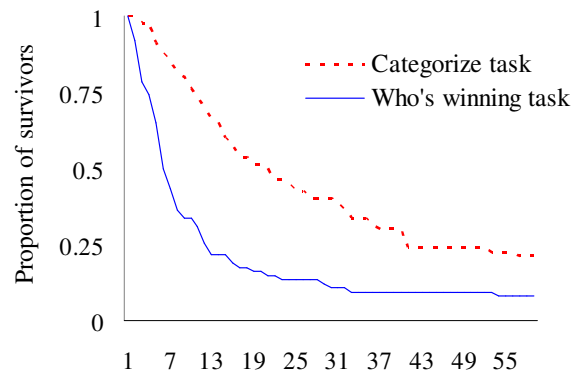


Figure 2. Proportion of survivors per block for each task.

**Response Times.** Since participants in the *categorize*, *relations named*, and *clue condition* required so many more trials to reach criterion than participants in the *who's winning*, *relations named and clue* condition, we also analyzed these conditions in terms of participants' mean response times on individual trials in order to gain insight about the strategies participants in these two conditions may have adopted. Response times in the *relations named, clue* and *categorize* condition ( $M = 1.69$  s) were reliably shorter than those in the *relations named, clue* and *who's winning* condition ( $M = 3.31$  s) [ $t(35) = -4.45$ ,  $p < 0.001$ ]. The reasons for this speed-accuracy tradeoff are addressed in the Discussion.

## Discussion

Kittur et al. (2004) showed that people find relational categories with a probabilistic (family resemblance) structure disproportionately difficult to learn relative to featural categories with a family resemblance structure or relational categories with a deterministic structure. They interpreted this effect in terms of people invoking schema induction (via intersection discovery) when faced with relationally-defined categories, an approach that yields useful relational schemas (Doumas et al., 2008; Hummel & Holyoak, 2003) and succeeds with deterministic category structures, but fails catastrophically with family resemblance category structures.

We sought to better understand this phenomenon by investigating conditions under which people might succeed at learning relational categories with a (nominally) family resemblance structure. Our results showed that recasting category learning as a "who's winning" task substantially improved participants' ability to learn relational categories with a family resemblance structure. Faced with the "who's winning" task, other factors that might sensibly be expected to improve learning—specifically, naming the relevant relations and informing participants that no single relation

will work every time—seemed to improve performance (although not all these trends were statistically reliable in our data). Surprisingly, however, when combined, these factors did not improve the learning of participants charged with the (formally equivalent) task of categorizing the stimuli: Although each factor individually seemed to improve learning of our probabilistic relational categories, when combined they substantially impaired learning.

The reasons for this trend toward an impairment of learning in the *relations named, clue* and *categorize* condition relative to the *relations not named, no clue* and *categorize* condition are not entirely clear, but it is consistent with the pattern that would be expected if participants in the *relations named, clue* and *categorize* condition were attempting to categorize the exemplars based on their features (i.e., the absolute size and darkness of the circle and square) rather than the relations between them. This conclusion is supported by the fact that response times were fastest in the *relations named, clue* and *categorize* condition (1.69 s per trial) and slowest in the *relations named, clue* and *who's winning* condition (3.31s per trial), as though participants in the former condition were responding on the basis of readily perceptible features whereas those in the latter were actively seeking unseen higher-order invariant properties or relations. A post-hoc analysis of participants' end-of-experiment self-reports also supports this conclusion: In their descriptions of the strategies they used during the experiment, participants in the *relations named, clue* and *categorize* condition named stimulus features (e.g., “dark”, “large”, etc.) rather than dimensions (“darkness”, “size”) or relations (“darker”, “larger”) more often than participants in any of the other conditions (19 times vs. a mean of 8.29 times [ $SD = 4.39$ ] across the other conditions).

These patterns suggest that participants in the *relations named, clue* and *categorize* condition may have simply abandoned the use of the first-order relations (*larger, darker, above* and *in front*) as the basis for categorization and, rather than discovering a useful higher-order relation, simply retreated to a strategy based on the exemplars' features. At the same time, however, it remains unclear why only the participants in this condition would resort to this maladaptive strategy, especially given the absence of evidence that even participants in the *relations not named, clue* and *categorize* condition did *not* resort to this strategy: These latter participants were not even informed that the relations would be important to the task, and yet they show greater evidence of using them than the participants who (in all other respects treated equivalently) were told outright to pay attention to the relations. Perhaps being told what the relevant relations were, in combination with the clue that no single one of them would work every time, had the counterproductive (and counterintuitive) effect of helping these participants know which relations to ignore in their categorizations. Additional experiments will be required to answer this question.

More important for our current purposes is the fact that, as predicted, changing the task from a category learning task to a “who's winning” task substantially improved our participants' ability to discover what separated stimuli requiring an “A” response (i.e., members of category A or, equivalently, stimuli in which the circle was “winning”) from those requiring a “B” response. Importantly, this improvement obtained even though the formal task—i.e., the specific stimulus-response mappings—were identical across the *categorize* and *who's winning* conditions. This improvement is consistent with the idea that the “who's winning” task encouraged participants to search for a relational invariant that remained constant across all members of a category. Our data do not allow us to know whether the invariant participants discovered was some sort of higher-order relation (e.g., “the majority of the first-order relations support such-and-such”) versus a re-representation of the first-order relations in terms of semantic primitives permitting a more feature-based approach to deciding whether the circle or square was “winning”. It is also unclear to what extent the “who's winning?” task improves performance simply by somehow making the first-order relations more meaningful.

These questions are the subject of ongoing research. But regardless of which (if any) of these possibilities turns out to be the case, the findings reported here are consistent with Kittur et al.'s (2004) conclusion that learning a relational categorization is greatly facilitated by the discovery of an abstract invariant of some sort that holds true across all members of a category. As such, our data support the idea that relational category learning may entail some form of intersection discovery.

Our findings also suggest that the traditional format of the laboratory category learning task may, for some reason, inhibit the discovery of the invariants necessary for intersection discovery to succeed. Instead other tasks (such as our “who's winning?” task) may be better suited to this purpose: It appears that probabilistic relational categories may be more learnable if you don't think you're engaged in category learning. Indeed, the fact that participants in our *relations named, clue* and *categorize* condition took the longest of all our participants to reach criterion—and were least likely to reach it—suggests that one of the worst things you can do to a person who is attempting to learn probabilistic relational categories is tell them that they are attempting to learn probabilistic relational categories.

Finally, it is worth noting that our results and those of Kittur et al. (2004, 2006) raise the question of whether natural relational concepts and categories tend to have a deterministic or probabilistic structure. Do ad-hoc categories, such as “things to remove from a burning house”, “ways to escape the mob” and “things to take on a winter camping trip” (Barsalou, 1983) have a relational invariant that holds true across all members of the category? Does our tendency to assume the existence of (invariant) “essences” in biological categories reflect a desire for invariants in relational categories? And do schemas and theories tend to

possess relational invariants? For example, is there a relational core that all members of the category “mother” have in common? Although at first it is tempting to say yes, the differences between birth mothers and adoptive mothers, and between loving mothers and abusive mothers, suggest that the answer might be no. If the answer is no, then do people have difficulty acquiring an all-encompassing schema for the concept “mother”? Or do we simply have multiple “mother” schemas?

The work of Kittur et al. (2004) suggests that schemas, theories and ad-hoc categories must either contain relational invariants or else be difficult to acquire. The findings presented here suggest that they may not be so difficult to acquire, even if they lack invariants among their first-order relations, provided the conditions under which they are learned promote the discovery of an invariant higher-order relation.

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