Lawrence Berkeley National Laboratory

Recent Work

Title

FREE STREAMING AND SPATIAL LANDAU DAMPING

Permalink

<https://escholarship.org/uc/item/0vd2s2dr>

Author

Hirshfield, J.L.

Publication Date

1967-08-01

UCRL-17713

cy. 2

University of California
Ernest O. Lawrence
Radiation Laboratory

FREE STREAMING AND SPATIAL LANDAU DAMPING

J. L. Hirshfield

August 1, 1967

RECEIVED
LAWRENCE
RADIATION LABORATORY
AUG 13 1967
LIBRARY AND
DOCUMENTS SECTION

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545*

UCRL-17713
cy. 2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Submitted to Physics of Fluids (Research Notes)

UCRL-17713
Preprint

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

FREE STREAMING AND SPATIAL LANDAU DAMPING

J. L. Hirshfield

August 1, 1967

FREE STREAMING AND SPATIAL LANDAU DAMPING*

J. L. Hirshfield[†]

Lawrence Radiation Laboratory
University of California
Berkeley, California

August 1, 1967

ABSTRACT

The interpretation of experiments on the spatial Landau damping of an electrostatic disturbance is shown to be complicated by the contribution of the free-streaming, initially perturbed particles. In particular, when measurements give the ratio of the imaginary part to the real part of the wavenumber to be larger than about 0.3, the observation may be understood using a simple model which neglects the mutual interaction between the plasma particles.

FREE STREAMING AND SPATIAL LANDAU DAMPING

A number of experiments have been reported recently which deal with the spatial evolution of electrostatic disturbances in a plasma.¹⁻⁴ Measurements of both dispersion and damping have been made. We wish to discuss the interpretations given to the damping data in an attempt to judge whether or not "spatial Landau damping" has been observed. We use this label to refer only to the spatial decay of a collective mode or modes satisfying an appropriate dispersion relation.⁵ This decay is attributable to a small group of electrons traveling at the phase velocity of the wave.

Our point is to compare the predictions of a simple ballistic model with the experimental results. The ballistic contribution, which is generally taken to decay rapidly enough to be neglected,⁶ can be shown to be added onto the "normal mode" solution.⁷ We thus neglect collective effects altogether. This point of view has also been taken by Ferrar in the interpretation of his experiments on electron waves.⁸ Our detailed considerations are for the electron ballistics, taking the ions to be immobile. Under some circumstances the same behavior would be expected in ion-wave experiments, and it is suggested that the necessary criterion given below for allowing neglect of the ballistic contribution is applicable for disturbances carried either by electrons or by ions.

For simplicity we consider a one-dimensional model. First, the excitation. Both Drummond⁹ and Dobrott and Weitzner¹⁰ have considered the coupling between transparent grids and the plasma. When a (sometimes biased) grid or wire probe, driven from a signal generator at frequency ω , is inserted into the plasma, the space potential ϕ_0 is modified by an

amount $\phi_1 \sin \omega t$ within a distance $\pm L$ of the probe. The distance L is equal to or greater than the Debye length λ_D , depending upon whether $\omega < \omega_p$ or $\omega > \omega_p$. For observations at distances $x \gg L$, we shall see that precise estimates of L are unnecessary, as L drops out of the answer. Let us approximate the true situation by taking $E = E_1 \sin \omega t$ in the interval $-L < x < 0$, $E = -E_1 \sin \omega t$ in the interval $0 < x < L$, and $E = 0$ for $-L > x > L$. A one-dimensional linearized Vlasov equation can then be integrated along an unperturbed trajectory to give the perturbed part of the distribution function $f_1(v > 0, t, x = 0)$ at the source, in terms of the unperturbed distribution function $f_0(v)$:

$$\begin{aligned} f_1(v > 0, t, x \approx 0) &= \frac{eE_1}{m} \frac{\partial f_0}{\partial v} \left[\int_{t-L/v}^t dt' \sin \omega t' - \int_t^{t+L/v} dt' \sin \omega t' \right] \\ &= - \frac{4eE_1}{m\omega} \frac{\partial f_0}{\partial v} \sin \frac{\omega L}{2v} \cos \omega t. \end{aligned} \quad (1)$$

Since the particles are noninteracting, their subsequent streaming is governed simply by $\partial f_1 / \partial t + v \partial f_1 / \partial x = 0$, and

$$f_1(v > 0, t, x > 0) = - \frac{4eE_1}{m\omega} \frac{\partial f_0}{\partial v} \sin \frac{\omega L}{2v} \cos \omega \left(t - \frac{x}{v} \right). \quad (2)$$

The perturbed distribution extends unattenuated in x .

We assume that a receiving probe at position $x \gg L$ connected to a high input-impedance amplifier will measure the local potential $\phi(x)$.

$$\begin{aligned}\varphi(x) &= -\frac{4e^2 E_1}{m\omega\epsilon_0} \int_0^x dx' \int_0^{x'} dx'' \int_0^\infty dv \frac{\partial f_0}{\partial v} \cos \omega \left(t - \frac{x''}{v} \right) \sin \frac{\omega L}{2v} \\ &= \frac{2\omega_p^2 E_1}{n_e \omega^3} \operatorname{Im} \int_0^\infty dv \frac{\partial f_0}{\partial v} v^2 \left[e^{i \frac{\omega}{v} \left(x - \frac{L}{2} \right)} - e^{i \frac{\omega}{v} \left(x + \frac{L}{2} \right)} \right] e^{-i\omega t},\end{aligned}\quad (3)$$

where n_e is the electron concentration. For a Maxwellian velocity distribution, $f_0(v) = n_e (2\pi v_e^2)^{-1/2} \exp(-v^2/2v_e^2)$, Eq. (3) becomes

$$\varphi(x) = \left(\frac{32}{\pi} \right)^{1/2} \frac{\omega_p^2}{\omega^2} \frac{E_1 v_e}{\omega} \operatorname{Im} \left\{ \left[I(z_+) - I(z_-) \right] e^{i\omega t} \right\}, \quad (4)$$

where

$$I(z_\pm) = \int_0^\infty dy y^3 \exp(-y^2 - iz_\pm/y) \quad \text{and} \quad z_\pm = \omega \left(x \pm \frac{L}{2} \right) / 2^{1/2} v_e.$$

The integral $I(z_\pm)$ belongs to a class considered in detail by Laporte,¹¹ and has been evaluated numerically by Kruse and Ramsey.¹² The dominant asymptotic behavior is $\exp -3(iz_\pm/2)^{2/3}$, so that

$$I(z_+) - I(z_-) \approx \frac{\omega L}{\sqrt{2} v_e} I' \left(\frac{\omega x}{\sqrt{2} v_e} \right)$$

provided $x \gg \frac{1}{32} L^3 (\omega/v_e)^2$. In this limit

$$\varphi(x) = \frac{16}{\pi}^{1/2} \frac{\omega_p^2}{\omega^2} \varphi_1 \left[\operatorname{Im} I'(z) e^{i\omega t} \right], \quad (5)$$

with $z = \omega x / 2^{1/2} v_e$. The asymptotic expansion for the function $I'(z)$, obtained from a saddle-point approximation, is¹³

$$I'(z) \sim \frac{1}{2} \left(\frac{\pi}{3} \right)^{1/2} \exp \left[-\frac{3}{2} \left(\frac{z}{2} \right)^{2/3} \right] \left\{ \left[(1 + i\sqrt{3}) \left(\frac{z}{2} \right)^{2/3} + \frac{17}{18} + 0 \left(\frac{z}{2} \right)^{2/3} + \dots \right] \cos \theta \right. \\ \left. - \left[(\sqrt{3} - i) \left(\frac{z}{2} \right)^{2/3} - i \frac{17}{18} + 0 \left(\frac{z}{2} \right)^{2/3} + \dots \right] \sin \theta \right\}, \quad (6)$$

where $\theta = (3\sqrt{3}/2)(z/2)^{2/3}$. Figure 1 shows the in-phase component $|\operatorname{Re} I(z)|$, and the magnitude $|I(z)|$, versus z , as evaluated by numerical integration. The asymptotic expansion [Eq. (6)] agrees with the exact evaluation for $|I(z)|$ to within $\pm 0.1\%$ beyond $z = 10$. Inspection of Fig. 1 shows the decay with z to differ only slightly from exponential, and shows the aperiodicity to be minor beyond the first cycle.

A parameter which has been found useful for comparison with the experiments is $\alpha(x) \equiv - \left[|\phi(x)| d\theta/dx \right]^{-1} d|\phi(x)|/dx$. For a plane wave, $\exp i(\omega t - kx)$, $\alpha(x) = \operatorname{Im} k / \operatorname{Re} k = \text{const.}$ From Eq. (6), we find $\alpha(x) \sim 3^{-1/2} [1 - 2/3(2/z)^{2/3}]$, varying from 0.192 at $z = 2$ to 0.577 as $z \rightarrow \infty$. Our suggested necessary criterion for distinguishing a spatially decaying plasma wave from a signal attenuating on account of free-streaming velocity dispersion is that a measured value of α be less than about 0.3.

In Fig. 2, we have collected several published values of α and plotted them versus $(\omega/v_e \operatorname{Re} k)^2$. In addition to results published on electron waves, the result for ion-acoustic waves published by Wong, Motley, and D'Angelo¹⁴ is also given. (Effects of ion drift have been averaged out in presenting this latter datum, and the random ion velocity should be substituted for the random electron velocity in the abscissa on the plot.) The results of Malmberg and Wharton, and the datum of Barrett, Franklin, and Jones satisfy the criterion given above for avoiding a significant contribution due to free-streaming particles. The Malmberg and Wharton

data also exhibit the rather convincing exponentially decreasing dependence upon $(\omega/v_e \operatorname{Re} k)^2$, expected from theory, as well as a great sensitivity to the slope of the tail of the electron distribution function.

The author acknowledges the assistance of Esther Coleman in the numerical work, has benefited from useful conversations with H. Weitzner and W. B. Kunkel, and extends appreciation to C. M. Van Atta for his hospitality.

FOOTNOTES AND REFERENCES

* This work was done under auspices of the U. S. Atomic Energy Commission.

† Permanent address: Department of Engineering and Applied Science, Yale University, New Haven, Connecticut.

¹J. H. Malmberg and C. B. Wharton, Phys. Rev. Letters 17, 175 (1966).

²G. van Hoven, Phys. Rev. Letters 17, 169 (1966).

³H. Derfler and T. C. Simonen, Phys. Rev. Letters 17, 172 (1966).

⁴P. J. Barrett, R. N. Franklin, and H. G. Jones, Dispersion and Damping of Electron Plasma Waves, in Proceedings of the Conference on Physics of Quiescent Plasmas, Frascati, 1967, pp. 464-473.

⁵See, for example, I. B. Bernstein, S. K. Trehan, and M. P. H. Weenink, Nucl. Fusion 4, 61 (1964).

⁶W. E. Drummond, Phys. Fluids 7, 816 (1964).

⁷R. L. Gunshor, Phys. Fluids 9, 1044 (1966). This work lists references to a number of experiments using electron beams.

⁸C. Ferrar, Ph.D. Thesis, University of Connecticut, 1966 (unpublished).

⁹W. E. Drummond, Rev. Sci. Instr. 34, 779 (1963)

¹⁰D. Dobrott and H. Weitzner, Excitation of Longitudinal Waves by a Grid, in Plasma Physics Laboratory Annual Report MATT-Q-24, Princeton University, May 1967, pp. 261-265 (to be submitted for publication).

¹¹O. Laporte, Phys. Rev. 52, 72 (1937).

¹²U. E. Kruse and N. F. Ramsey, J. Math. Phys. 30, 40 (1951).

¹³This form is recognizable as essentially that given by Landau [his Eq. (44)] which can be traced back to the ballistic part of his solution [L. Landau, J. Phys. USSR 10, 25 (1946)]. Variation of the boundary condition at the source does not alter the $\exp -(3/2)(x/2)^{2/3}$ factor or the variation

periodic in θ , but merely the coefficients of the two multiplicative series in the form $\sum_{n=-2}^{\infty} a_n (2/x)^{n/3}$.

¹⁴A. Y. Wong, R. W. Motley, and N. D'Angelo, Phys. Rev. 133, A436 (1964).

FIGURE CAPTIONS

Fig. 1 The integral $I(z) = \int_0^{\infty} dy y^2 \exp(-y^2 - iz/y)$ versus z . Both $|I(z)|$ and $|\operatorname{Re} I(z)|$ are shown.

Fig. 2 Experimental measurement of $\operatorname{Im} k/\operatorname{Re} k$ versus $(\omega/v_e \operatorname{Re} k)^2$, taken from various sources: ∇ , Derfler and Simonen; $\circ$, Malmberg and Wharton; $\square$, Barrett, Franklin and Jones; $\triangle$, Wong, Motley, and D'Angelo. According to the criterion given here, measured values of $\operatorname{Im} k/\operatorname{Re} k$ greater than about 0.3 can represent a decay dominated by the free streaming of noninteracting particles, rather than the spatial Landau damping of an electrostatic plasma wave.

XBL678-3623

Fig. 1.

XBL678-3624

Fig. 2.

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.