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EXTREME VALUES $A_y = \pm 1$ OF THE ANALYZING POWER FOR SPIN-1 POLARIZATION*

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Abstract:

Extreme values of the component A_y of spin-1 polarization efficiencies are discussed. It is shown that at the energy and angle of such values the other components are also numerically determined. Applying these criteria to experimental data, possible extreme points of this type are indicated in the reactions $^6Li(d,\alpha)^4He$ and $^3He(d,p)^4He$. An experimental value of $A_y \simeq 1.0$ in d-^4He elastic scattering is noted.

Maximum possible values of the analyzing power have been identified at several energies and angles in the elastic scattering of spin-$\frac{1}{2}$ and spin-1 particles from spin-0 nuclei,[1,2] and evidence for at least one extreme value of the tensor component $A_{yy} = 1$ has been found in the $^3He(d,p)^4He$ reaction.[3] The recent analysis by Grüebler et al.[2] of d-^4He elastic scattering data between 3 and 17 MeV has proved the existence of points at which the component A_{yy} of the spin-1 analyzing power reaches its maximum possible value of unity. Also, for points of maximum vector analyzing power, $A_y = \pm 1$, they examined the resulting conditions among elements of the transition matrix M which connects initial and final spin states, $\chi_f = M\chi_i$. At such a point three equations must be satisfied among the M-matrix amplitudes, with the consequence, then, that <u>all</u> the remaining analyzing powers are determined:

$$A_{yy} = +1, \quad A_{xx} = A_{zz} = -\frac{1}{2}, \quad A_{xz} = 0 \qquad (1)$$

In addition to its usefulness in an absolute calibration of deuteron vector polarization, the occurence of such a unique and restrictive point would clearly be of great interest and value, since there the relative values of all the M-matrix elements are determined.

We show in this letter that the necessary, but not sufficient, conditions (1) for the existence of a point $A_y = \pm 1$ are valid for reactions with the general spin structure $1 + a \rightarrow b + c$, where a, b, and c are arbitrary spins. This follows from a property of the spin-1 density matrix, and examination of the 3(2a+1) by (2b+1)(2c+1) M-matrix is not required. We then note regions of energy and angle in specific reactions where the conditions (1) are approached, so these are then regions which can be examined for the possible existence of a point $A_y = \pm 1$. Although such a unique and restrictive point is *a priori* quite unlikely, an experimental value of $A_y \geqslant 0.98$ has been measured in d-^{4}He elastic scattering at $E_d = 26.8$ MeV near $\theta_L = 135°$ (ref. 4). Hence, these criteria are of practical interest.

Methods of specifying a "degree of polarization" for an ensemble of polarized spin-1 particles produced in a nuclear reaction have been given by Lakin,[5] Fano[6] and Minnaert.[7] They are based on the fact that the density matrix of the ensemble is positive semidefinite. Its expansion in terms of an irreducible set of tensor operators τ_{kq} imposes conditions on the tensor moments $t_{kq} = \langle \tau_{kq} \rangle$.[5,7] Identical limitations apply for the polarization efficiencies T_{kq} due to time reversal invariance. They are particularly simple in a transverse coordinate system S' with the z'-axis perpendicular to the reaction plane.

Analagous to the degree of polarization, δ is defined by

$$\delta^2 = \frac{1}{2}\,[(T'_{10})^2 + (T'_{20})^2 + 2|T'_{22}|^2] \leq 1. \qquad (2)$$

More restrictive is the additional requirement

$$(T'_{20} + \sqrt{2})^2 \geq 3(T'_{10})^2 + 6|T'_{22}|^2. \qquad (3)$$

In the space $(T'_{10}, \sqrt{2}|T'_{22}|, T'_{20})$ the maximum value $\delta = 1$ defines a sphere, and Eq. (3) the interior of an inscribed cone (Fig. 1). Points on the surface or the base $(T'_{20} = \frac{1}{2}\sqrt{2})$ of the Lakin cone correspond to restricted combinations of the maximum possible values of the T'_{kq}. In terms of the observables T_{kq} or A_{ij} defined by the Madison convention,[8] the T'_{kq} are:

$$\begin{aligned} T'_{10} &= \sqrt{2}\, i\, T_{11} = \frac{1}{2}\sqrt{6}\, A_y\,, \\ T'_{20} &= -\frac{1}{2}(T_{20} + \sqrt{6}\, T_{22}) = \frac{1}{2}\sqrt{2}\, A_{yy}\,, \\ T'_{22} &= \frac{1}{4}(\sqrt{6}\, T_{20} - 2\, T_{22}) - i\, T_{21} \\ &= \frac{1}{6}\sqrt{3}\,[(A_{zz} - A_{xx}) + i2A_{xz}]. \end{aligned} \qquad (4)$$

Thus points with $A_{yy} = 1$ lie on the base of the Lakin cone. Only there, values of $A_y = \pm 1$ can be attained (Fig. 1, points A and B), and then only if $|T'_{22}| = 0$. This results in the conditions (1). As noted, this was shown for d-^{4}He elastic scattering by direct calculation with the appropriate 3 by 3 M-matrix.[2] From the derivation given here, it is clear that Eqs. (1) are valid in general and are independent of the spin space of the process. For all reactions involving polarized spin-1 particles they therefore provide a clear signature of points where $A_y = \pm 1$ is possible.

It should be noted that the agreement of the experimental data with Eqs. (1) are necessary but not sufficient conditions for the existance of an extreme value of the efficiency A_y. Any point along the line AB in Fig. 1

satisfies Eqs. (1). Using a beam with calibrated tensor polarization,[2] it is possible to determine that a point on the line AB is reached. However, the calibration of spin-1 vector polarizations is still so uncertain that it cannot be shown experimentally that point A or B is indeed attained. For proof of that, the fulfillment of the conditions on the appropriate M-matrix would have to be established. For the processes considered in the following, the relevant conditions will be discussed in a subsequent publication.[9] In this paper, Eqs. (1) are applied to experimental data in a search for points where those necessary conditions exist that would make it possible for A_y to reach an extreme value.

<u>$^4He(d,d)^4He$</u>. The first study of extreme values for both A_{yy} and A_y was carried out by Grüebler and collaborators,[2] who determined two points with $A_{yy} = 1$ between 4 and 5 MeV, and one at 11.88 MeV and $\theta_{cm} = 55.1^\circ$. The possible existence of an $A_y = -1$ point at the same energy and angle as the latter $A_{yy} = 1$ point was ruled out by subsequent measurements of A_y in that region.[10] The best candidate for an extreme A_y point lies near 26.8 MeV and $\theta_{cm} = 154^\circ$, where values approaching both $A_y = 1$ and $A_{yy} = 1$ have been measured.[4]

<u>$^6Li(d,\alpha)^4He$</u>. Measurements by Grüebler <u>et al</u>.[11] show values of A_{yy} compatible with unity near 6 MeV and $\theta_{cm} = 30^\circ$ and near 8 MeV and $\theta_{cm} = 90^\circ$ (Fig. 2). Near 8 MeV only A_{yy} can reach unity, since the particle symmetry in the exit channel requires that $A_y(90^\circ) = 0$. Near 6 MeV there are drastic changes in the angular distributions of all efficiencies, which makes a comparison with Eqs. 1 difficult. The component A_y near 30° varies rapidly from negative values at 4 MeV to large positive values at 6 MeV, decreasing again above 7 MeV.[11] It is thus possible that the value $A_y = 1$ is attained. The component A_{xz} changes sign at forward angles between 6 and 7 MeV, while

the values of A_{xx} and A_{zz} are close to $-\frac{1}{2}$. In this region the conditions necessary for $A_y = 1$ may therefore be fulfilled.

<u>$^3He(d,p)^4He$</u>. One point with $A_{yy} = 1$ has been tentatively identified near 9 MeV and $\theta_{cm} = 27°$.[3] Since $A_{zz} \simeq -1$, $A_{xx} \simeq 0$ and $A_{xz} \simeq -1.1$,[11] it is clear that A_y cannot reach the value -1. The situation is different for the possible A_{yy} maximum above 11.5 MeV, where no measurements are available. At 11.5 MeV a value of $A_{yy} \simeq 0.9$ is found near $\theta_{cm} = 120°$ (see Fig. 1 of ref. 3). The vector analyzing power A_y is large and positive both at 11.5 MeV and 15 MeV.[11,12] The values for A_{xx} and A_{xz} below 11.5 MeV are plotted in Fig. 3. Clearly A_{xx} and, due to the condition $A_{xx} + A_{yy} + A_{zz} = 0$, also A_{zz} lie near $-\frac{1}{2}$ at 120°, while $A_{xz} = 0$ obtains at angles moving toward 120° as the energy increases. Thus the necessary conditions for $A_y = 1$ may be met near this angle at some energy between 12 and 15 MeV.

The application of Eqs. (1) have shown that extreme values of the spin-1 analyzing power A_y may occur in the studied reactions. In most processes the rapid changes with energy and angle near the critical point require more closely spaced data in order to establish that A_y and A_{yy} peak at the same point. This determination of location and peak value, as well as the definite identification, require a considerable effort. In view of the importance of these extremes from both the theoretical and experimental point of view, this effort is worthwhile.

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FIGURE CAPTIONS

Fig. 1. Cutaway drawing of the sphere defined by $\delta = 1$ in Eq. (2) and the inscribed cone. The points A and B correspond to the extreme values $A_y = \pm 1$.

Fig. 2. Values of the efficiency A_{yy} in the $^6Li(d,\alpha)^4He$ reaction at 5, 6, 7, 8 and 9 MeV, showing two points with $A_{yy} \approx 1$.

Fig. 3. Components A_{xx} and A_{xz} of the analyzing power in the $^3He(d,p)^4He$ reaction below 11.5 MeV.

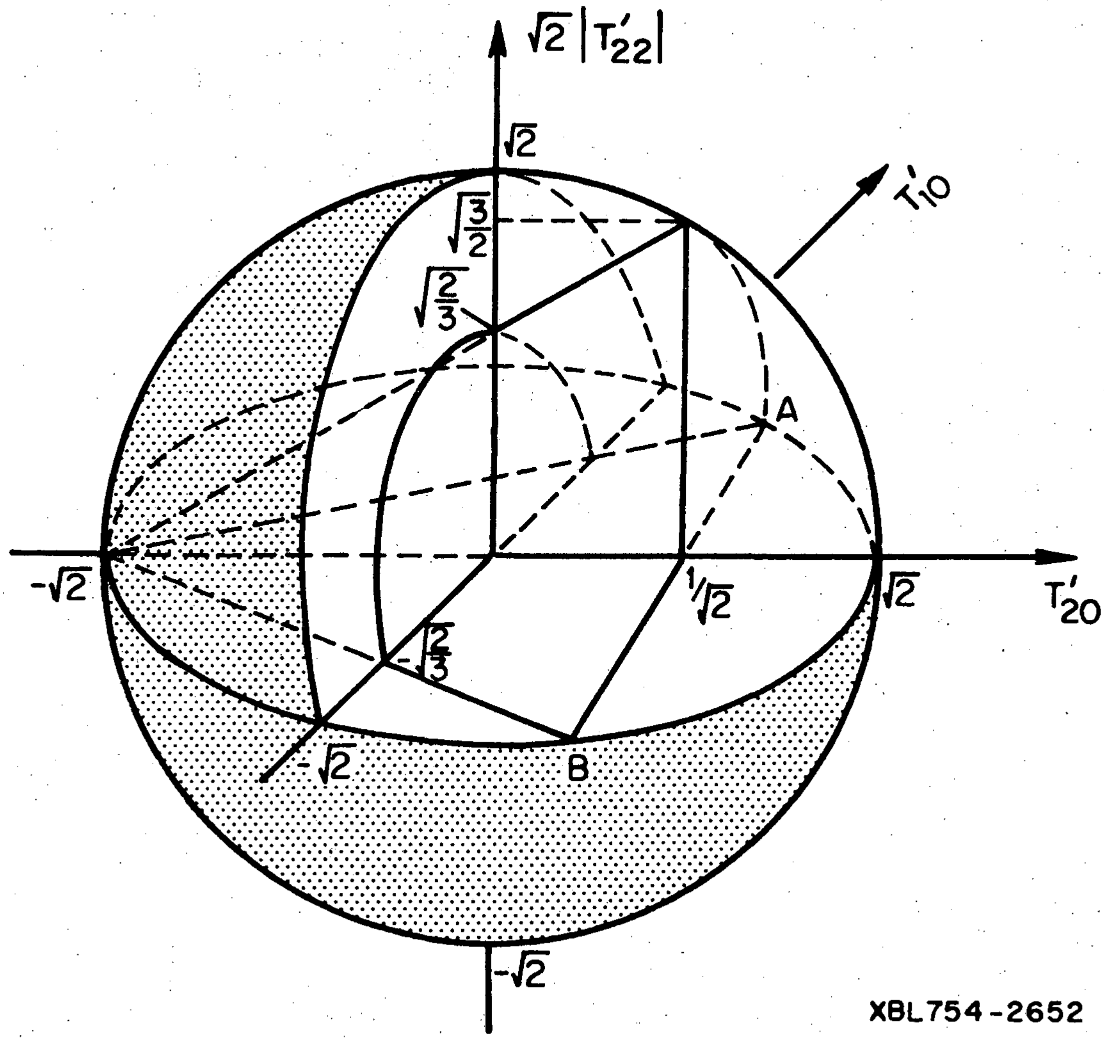

Fig. 1

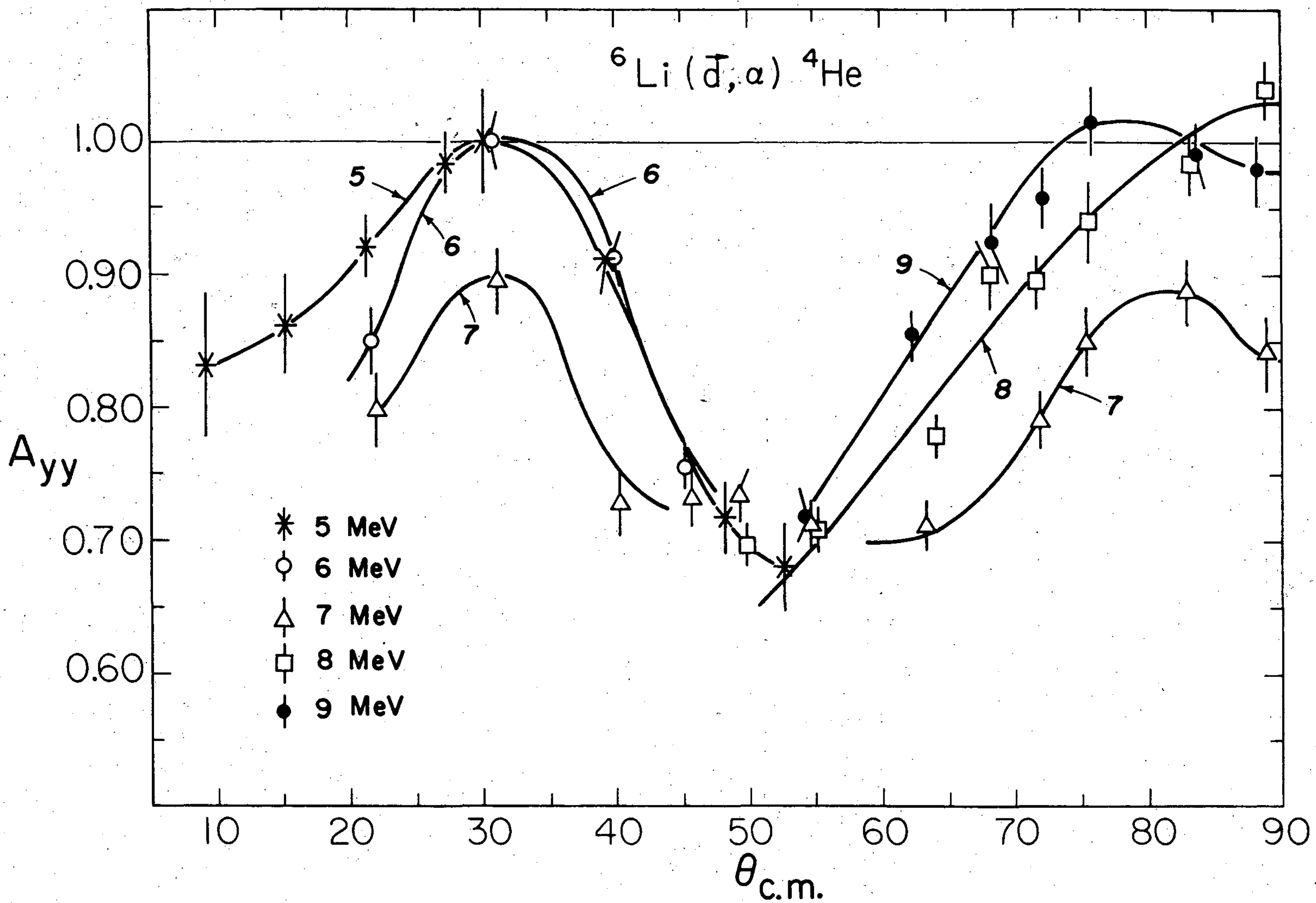

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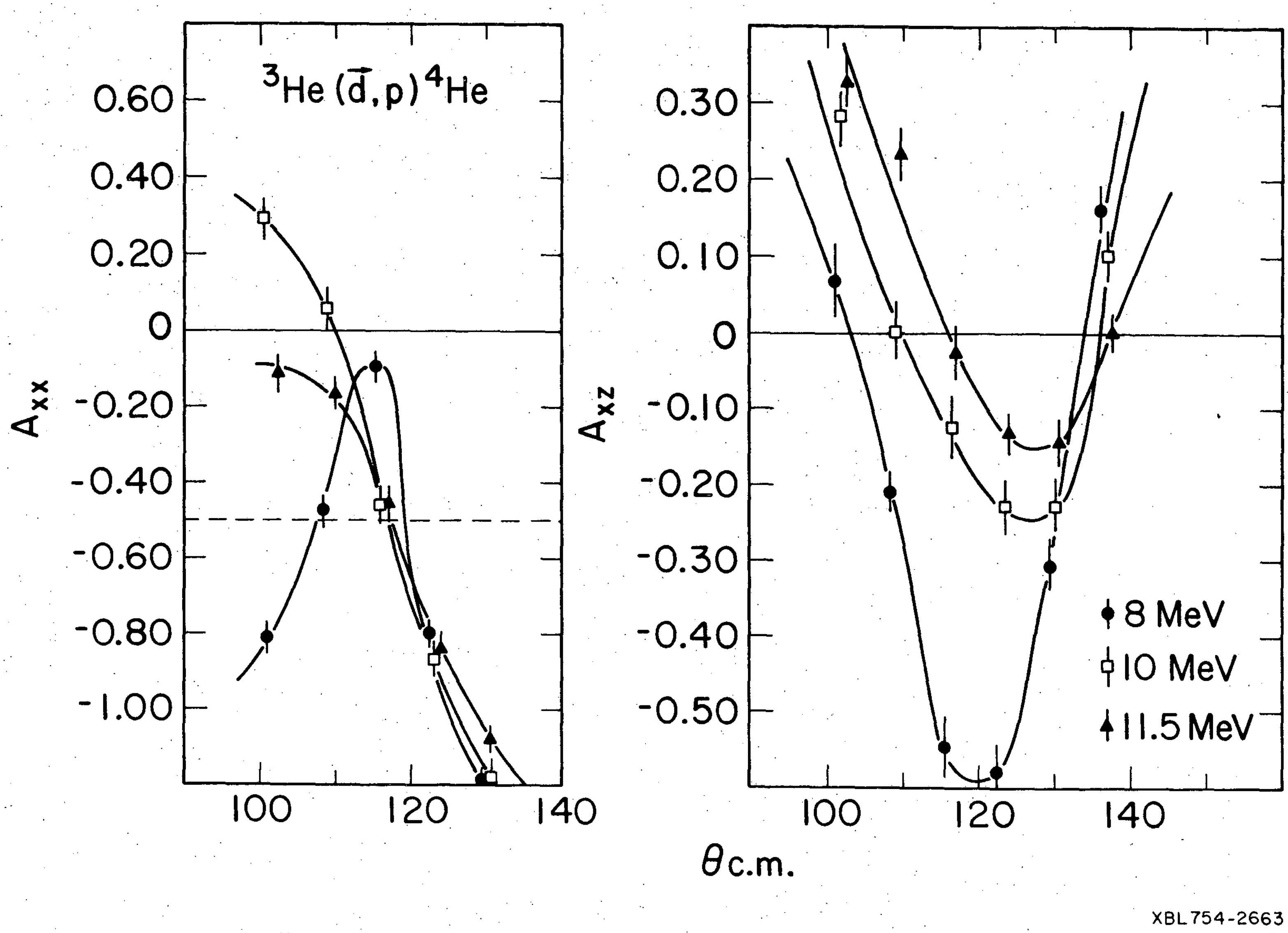
Fig. 2



$^3He(\vec{d},p)^4He$
0.60
0.40
0.20
0
-0.20
-0.40
-0.60
-0.80
-1.00
A_{xx}
100
120
140
0.30
0.20
0.10
0
-0.10
-0.20
-0.30
-0.40
-0.50
A_{xz}
8 MeV
10 MeV
11.5 MeV
100
120
140
θc.m.
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Fig. 3

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