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# Seat share distribution of parties: Models and empirical patterns

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## Abstract

We find that strategic sequencing and other factors sort parties roughly into two groups. Low-ranking parties lose part of their inherent support, compared to probabilistic expectations, while high-ranking parties profit from the shift. Our method is to graph the worldwide mean seat shares of parties at various ranks by size against the largest party share (Nagayama triangle format). The resulting empirical pattern looks complex, yet when we adjust a probabilistic model to account for strategic and other factors that may hurt the smaller parties, the fit becomes close. The number of parties that profit from transfers is close to the inverse of the fractional share of the largest party. The model fits best when the transfer is assumed to involve about one-half of inherent minor party support. This is a novel way to estimate the universal average strength of strategic and other factors that work against the smaller parties. The empirical worldwide mean pattern offers us a norm against which seat share distributions in individual countries or single elections can be compared.

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*Keywords:* Seat shares of parties; Strategic sequencing; Nagayama triangle; Logical quantitative models

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## 1. The problem

For a given seat share of the largest party, what is the likeliest distribution of the shares of other parties in a representative assembly? Do seat shares tend to taper off gradually, or do two parties tend to tower above the rest? Could either pattern predominate, depending on how large is the largest share? And what could it tell us about politics?

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For an individual assembly at a given time, almost any distribution can happen, of course. The only formal restriction is that the total of seat shares must amount to 100%. But some distributions are more likely than others. Our thinking about party competition is colored by which size constellations we consider “typical”. Lijphart (1977: 58) uses hypothetical party size distributions with largest share under, at, and over 50% to show the resulting degrees of fractionalization. He does the same to show the resulting disproportionality (Lijphart, 1994: 59) and effective number of parties (Lijphart, 1999: 67). Instead of hypothetical distributions, there could be advantages in using the actual averages for a large number of elections in a large number of countries.

Theoretical considerations are even more important. For a given largest share, we can calculate the most likely distribution of other shares, on the grounds of sheer probability. It would be surprising if the result agreed with the actual average distribution, because it would mean that no strategic considerations or other political factors entered. Because of such factors, we might expect a systematic deviation from the purely probabilistic pattern. We can try to build logical quantitative models to express the average impact of such factors and test whether their predictions agree with the empirical pattern. To the extent there is agreement, we would have made headway in understanding the nature and strength of forces that shape the size relations among parties.

This is what the present study does. For the given largest share, we first determine empirically the mean seat shares of second, third, and lower ranking parties, using a large number of election outcomes in 25 stable democracies, so as to smooth out random variation and path dependent local effects. We estimate the stability of the mean pattern thus obtained. This empirical part is novel and important by itself. But we go further, in a theoretical direction. We establish the probabilistically expected mean patterns. It will be seen that the empirical mean pattern deviates markedly from the probabilistic, favoring the largest parties. We then superimpose strategic and other considerations on the probabilistic model. Compared to sheer probability, some voters can be expected to overlook smaller parties. It will be seen that the resulting adjusted model agrees with the empirical mean pattern when it is assumed that small parties lose about one-half of their potential support to larger parties.<sup>1</sup>

This study operates at a high level of generality, with concomitant loss of detail, to which some may object. It may seem that huge amounts of real-world variation are consigned to nowhere. Actually, they are consigned to a much better place, namely to next-level analysis. Science aims at ferreting out the universal, which by no means implies neglect of detail but does introduce some hierarchy in approach to detail. The planet Neptune was discovered because of minute deviations in the expected movement of other planets, but it could hardly have been found, had all real-world variation been tackled at once. Rather than throwing everything into a grand regression or factorial analysis, as political scientists often do, other sciences

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<sup>1</sup> We use the largest seat share as an intervening variable. This largest share itself is affected by institutional factors such as district magnitude (the number of seats allocated within the district) and the total number of seats in the assembly. The well-known Duverger law and hypothesis (Duverger, 1951, 1954; Riker, 1982) suggest that plurality rule in single-member districts (SMP) tends to correspond to two predominant parties, while proportional representation (PR) in multi-seat districts tends to go with more than two parties. Later extensions of Duverger’s insights (Taagepera and Shugart, 1993) suggest that the largest fractional seat share ( $s_1$ ) tends to decrease gradually with increasing district magnitude ( $M$ ) and also with increasing assembly size ( $S$ ), following the average pattern  $s_1 = (MS)^{-1/8}$ , if the electoral rules are sufficiently simple. If so, then the present study would connect the entire seat share distribution pattern to these institutional factors. Taagepera (2001) made such an attempt, but the empirical pattern deviates markedly from the exponential approximation he used.

mostly have proceeded by first addressing what were guessed to be the main factors. If successful regarding the broad picture, more and more secondary factors were gradually introduced.

We wish success to those who use multi-factor analysis, as well as to those who consider the real world too rich in variation for any broader quantitative analysis. But here we prefer to follow the sequential approach described above, so as to tease out some main features within the enormous variety of seat share distributions. Therefore, we start with the worldwide mean pattern. This approach assumes that there are some basic factors which contribute heavily to this mean pattern, while innumerable other factors are likely to pull a given seat share down or up from this mean, possibly with roughly equal probability. We then try to establish a model of minimal complexity that enables us to reproduce the mean pattern observed empirically.<sup>2</sup>

## 2. The empirical pattern

Our database is Mackie and Rose (1991, 1997), who list party seat shares for over 700 elections in 25 countries. At a given largest party share, we consider the seat shares of the other parties, ranked by size. The scatter is wide for individual elections. The second largest party may have as many seats as the largest (e.g., Netherlands 1901, 1905, 1909, at 25.0%) or as few seats as the third largest (e.g., Germany 1898: 25.7-14.1-14.1-...; Italy 1900: 81.1-6.7-6.5-...). Countries may follow a steady pattern that deviates from the overall average, or the distribution may vary drastically from one election to the next, within the same country.<sup>3</sup> All this variation nonetheless occurs around some worldwide mean, which may have considerable inertia over time and space. We are concerned with establishing this mean pattern – and carrying out some check of its stability.

We calculated the average seat shares of parties at different ranks by size; at largest party share brackets the centers of which range from 20 to 92%. These empirical results are shown in Table 1. For most elections, the *largest share* ( $s_1$ ) ranges from 30 to 50%. Within this range, we used intervals centered on values of  $s_1$  such that  $1/s_1^2 = 4, 5, 6, \dots, 11$ . This choice is based on a simple model (Taagepera, 2001) which proposes that the number of parties ( $p$ ) expected to win at least 1 seat is  $p = 1/s_1^2$ . Table 1 shows that a few more parties actually gain occasional representation, but mostly with an average of less than 1 seat per election. Outside the 30–50% range, we went by largest party share intervals of 5 percentage points, except when the largest share exceeded 85% and the number of cases dropped.<sup>4</sup> In all these intervals, data from at least 4 countries entered, reducing potential country bias.

In the central range ( $s_1$  ranging from 30 to 50%), where each largest share bracket involves at least 22 elections, the median standard deviation is about 4.4 percentage points for second to fourth largest seat shares. It drops to about 0.5 percentage points for fifth to

<sup>2</sup> Here we are liable to face two charges going in opposite directions, namely that the model is: (1) overly minimal, omitting or lumping together too many important factors, and (2) too complex to be easily followed. We ask for patience from both sides. If we succeed in reproducing the empirical mean pattern, then may be we are on the right track, even if the mathematics may be hard to follow. As a next step, one should tackle the reasons for the variation around the mean pattern, by introducing further factors. We should not be expected to include this next-level analysis within the present study. One can do only so much, within 30 pages.

<sup>3</sup> Still, even a major shift in individual party fortunes need not alter the pattern of parties ranked by size in a major way. When Conservatives in Canada plummeted from 57.8% of the seats in 1988 to 0.7% in 1993, the ranked pattern shifted merely from 57.3-28.1-14.6 to 60.0-18.3-17.6-3.1-0.7-0.3.

<sup>4</sup> The mean  $s_1$  for actual cases in an interval may be slightly off the nominal value. Hence the values of  $s_1$  in Table 1 are not exactly at 20%, 25%, etc. The total number of elections used was 348. In the four brackets centered at 30.3–35.4%, a technical overlap caused 20 of these elections to be double-counted for two neighboring brackets.

Table 1  
Empirical party size distribution in legislative assemblies

Rank	Shares of parties (in percentages)																
1	20.5	25.3	30.3	31.6	32.8	35.4	38.0	40.7	44.7	49.9	55.3	59.9	65.0	70.1	74.5	80.2	91.7
2	17.9	22.5	24.9	25.0	24.7	27.9	29.9	29.3	31.9	37.1	40.6	35.4	29.9	25.4	18.4	12.8	5.5
3	15.0	18.9	17.4	17.5	15.6	18.1	17.0	14.7	13.8	8.0	4.0	3.3	3.4	2.9	4.2	4.6	1.9
4	13.2	12.2	11.8	11.7	9.9	9.4	7.8	9.0	6.4	3.5	0.1	0.9	1.4	0.7	1.8	2.2	0.9
5	11.1	7.1	6.5	6.5	6.6	4.8	3.1	3.8	2.1	1.2		0.4	0.3	0.5	1.1	0.1	
6	8.7	5.2	3.8	3.4	4.1	2.0	1.5	1.5	0.6	0.2		0.1		0.2			
7	5.4	3.7	2.1	2.1	2.4	0.9	0.7	0.4	0.2			0.1		0.2			
8	2.7	1.8	1.4	1.1	1.4	0.7	0.6	0.2	0.1					0.1			
9	2.1	1.2	0.8	0.7	1.0	0.4	0.5	0.1									
10	1.6	0.8	0.5	0.3	0.6	0.2	0.4	0.1									
11	0.8	0.5	0.2	0.1	0.3	0.1	0.3										
12	0.4	0.3	0.1		0.2	0.1	0.2										
13	0.3	0.2	0.1		0.1	0.1	0.1										
14	0.1	0.1	0.1														
15	0.1	0.1															
16	0.1																
SUM	99.9	99.9	100.0	100.0	100.0	100.1	99.9	99.9	99.9	99.9	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Data source: all elections in Mackie and Rose (1991, 1997). Shaded area: parties in excess of  $p = 1/s_1^2$  seat-winning parties.

seventh ranking parties. Stability of the averages reported in Table 1 was checked by dividing the elections in each largest share bracket randomly into 2 equal subgroups. Most subgroup averages for second to fourth largest seat shares differ from overall averages by less than 1 percentage point, the maximum being 2.1 percentage points. For fifth and lower ranking parties, the maximum difference is 1.4 percentage points. This means that the patterns obtained are likely to be preserved within plus or minus 2 percentage points when further data are added.

For graphical representation of these data we apply a format called *Nagayama triangle* by Reed (2001). The shares of the second-running contestants ( $s_2$ ) are plotted against the shares of the top contestants ( $s_1$ ). Their total cannot exceed 100%, nor can the second largest share exceed the largest. These two constraints force the data points to lie within a triangle delimited by  $s_2 = s_1$  and  $s_1 + s_2 = 100\%$ , as shown in Fig. 1. Its left side denotes perfect parity of 2 top contestants, while its right side denotes dominance of the strongest contestant over a single opponent. At the peak, the 2 contestants have equal strength, and there are no other contestants. The left corner area of the triangle corresponds to the presence of multiple contestants.

Nagayama (1997), Reed (2001) and Grofman et al. (2004) have applied this format to the study of candidate voting strengths in individual single-member districts, but it also can be applied to nationwide vote and seat shares (Taagepera, 2004). Moreover, we can go beyond the second largest party, graphing the average party seat shares ( $s_i$ ) at various ranks ( $i$ ) against the largest share ( $s_1$ ). Fig. 1 does so up to the 7th rank. To be complete, it also shows the points that correspond to the largest share itself ( $s_i = s_1$ ). These points are located on left side of the triangle, up to  $s_1 = 50\%$ , and then continue upwards.

The resulting pattern is far from simple. As the largest share increases, the second and third largest shares at first increase and then decrease. The second largest share peaks at 41%, when  $s_1$  is around 55%. The third largest share peaks around 19%, when  $s_1$  is 25%, but it drops below 15% only when the largest share exceeds 40%. Conceivably, the fourth and higher ranked

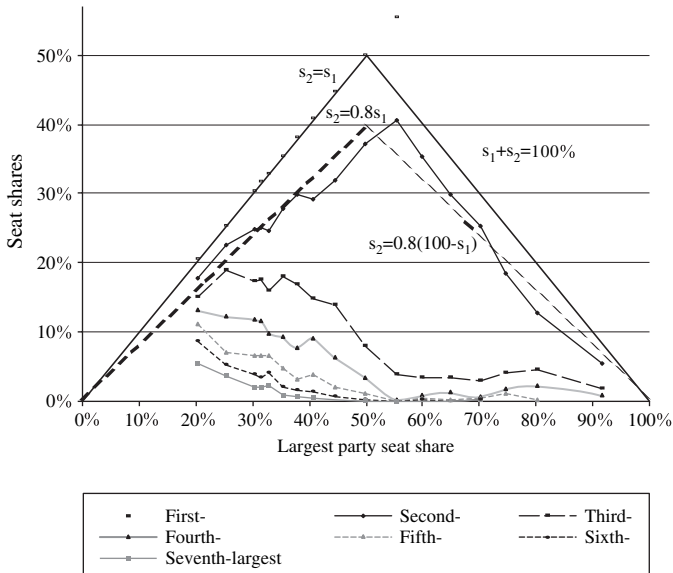


Fig. 1. Average seat shares of parties ranked by size, at a given largest party share. Data from Table 1.

shares might also increase with increasing largest share when  $s_1$  is extremely low, but at the lowest largest party shares observed (around 20%) they decrease.

The average second largest shares in Fig. 1 tend to approximate 80% of their conceptual maximum (Nagayama triangle), as shown by dashed lines:  $s_2 = 0.8s_1$  when  $s_1 < 50\%$ , and  $s_2 = 0.8(100\% - s_1)$  when  $s_1 > 50\%$ . This is lower than what has been suggested in earlier work.<sup>5</sup>

In general, a party's share tends to be closer to the next smallest share than to the next largest, but the second largest share deviates from this pattern: it is closer to the largest shares than to the third largest. The curves for the 2 largest parties thus bunch together throughout the usual range of the largest share. These 2 main competitors tend to stand out from the crowd even when the largest party share is as low as 25%. It may be that some supporters of smaller parties switch to either of the 2 larger parties as a "lesser evil" or that some electoral rules tend to boost the largest parties. When the largest share is under 25%, the observed pattern suggests that the third largest party may also benefit from the lesser evil considerations.

A curious bump occurs in the curves for third and fourth largest parties. These parties are reduced to almost nothing when  $s_1$  reaches 70% but win a new lease on life, at the expense of the second largest party, when the largest share reaches 80%. United opposition might lose its value when the largest party hegemony becomes overwhelming. Third party supporters who consider the second largest party the lesser evil may consider joining it as long as it has a chance. But when the second largest party drops to a hopelessly low size, the third party supporters may feel that they might as well vote sincerely. Fig. 1 suggests that strategic support for the second largest party runs fades when  $s_2$  drops below 25%.

<sup>5</sup> Taagepera and Shugart (1993) estimated the second-largest party's seat share to about 0.85 of the largest: ( $s_2 = 0.85s_1$ ). This approximation cannot hold beyond a largest party share of 54%, because the total for the two parties would exceed 100%.

Rather than graphing all other seat shares against the largest (as in Fig. 1), one may also graph the *seat shares* at any rank ( $s_i$ ) against their *rank* ( $i$ ). Indeed, this may seem a more natural way of graphing these data. However, we could not build a logical model based on this approach. The empirical results are shown in Appendix A.

This concludes the empirical part. It is now time to consider what kind of patterns we would expect on the basis of sheer probability and how the introduction of strategic and other factors would modify them.

### 3. The probabilistic model

As mentioned earlier, a simple model suggests that the number of parties ( $p$ ) that wins at least one seat should be expected to be around  $p = 1/s_1^2$ . The logic behind this model considers the conceptual extremes. If all parties represented have equal fractional shares, then  $s_1 = 1/p$ . If largest party hegemony is total, then  $s_1$  is close to 1. The geometric mean of these widely diverging extreme possibilities is  $s_1 = 1/p^{0.5}$  or, conversely,  $p = 1/s_1^2$ . Simplistic as the approach might look, the results tend to agree with observation within  $\pm 5\%$ , as an average at various levels of  $p$  (Taagepera, 2001). In Table 1, the number of parties that win at least 0.1% of the seats tends to exceed  $1/s_1^2$  by about 3 parties. These extra parties often win less than 1 seat per average election.

We accept  $p = 1/s_1^2$  as a first approximation. But then one might extend the same reasoning to second ranked parties, and so on. This is a novel step undertaken in this study.

Beyond the largest party, the remaining  $p-1$  parties account for a total fraction  $1-s_1$  of all the seats. The maximum share the second ranked party can possibly have is  $1-s_1$ . This occurs when all other parties are vanishingly small. The minimum possible share of the second largest party occurs when all  $p-1$  parties (except for the largest) have equal shares of  $(1-s_1)/(p-1)$ . The geometric mean of these extremes is  $s_2 = (1-s_1)/(p-1)^{0.5}$ . This is the expected value of  $s_2$ , in the absence of any further information.

For the third ranking party, the same reasoning yields  $s_3 = (1-s_1-s_2)/(p-2)^{0.5}$ . The general formula for the  $i$ -th ranking party is

$$s_i = \left(1 - \sum_{j=1}^{i-1} s_j\right) / (p - i + 1)^{0.5}$$

where the summation ranges from  $j = 1$  to  $i-1$ . Once the largest party's seat share is given, all other shares can be calculated.<sup>6</sup>

The party shares resulting from these theoretical equations are listed in Table 2. Fig. 2 shows the corresponding curves, following the format of previous Fig. 1. The theoretical pattern (Fig. 2) and the empirical mean pattern (Fig. 1) offer broad similarities as well as marked differences in detail. In the probabilistic model, the second largest share occupies middle grounds between the largest and the third largest (except at  $s_1 > 65\%$ ), while actually it is closer to the largest share. In the model, third and fourth largest parties reach their peak sizes at much higher values of  $s_1$  than is actually the case.<sup>7</sup> At  $s_1 = 50\%$ , the predicted values of  $s_3$

<sup>6</sup> For integer values of  $p$ , the last term will be  $s_p = (1 - \sum_{j=1}^{p-1} s_j) / (1)^{0.5} = 1 - \sum_{j=1}^{p-1} s_j$ , and hence the total of all shares is 1, as it should. If  $p = 1/s_1^2$  leads to a non-integer value of  $p$ , then applying the formula to the integer part of  $p$  yields a sum of seat shares slightly smaller than 1. We'll take the remainder to represent a tiny party whose rank exceeds the integer part of  $p$ .

<sup>7</sup> The same is true for the fifth to seventh-largest parties, if we assume that their values at  $s_1 = 20\%$  in Fig. 1 do represent their peak sizes.

Table 2  
 Probabilistic party size distributions with no political adjustment

Rank	Number of parties and the corresponding seat shares (in percentages)															
	50	25	16	11	10	9	8	7	6	5	4	3	2.5	2	1.5	1.2
1	14.1	20.0	25.0	30.2	31.6	33.3	35.4	37.8	40.8	44.7	50.0	57.7	63.3	70.7	81.6	91.3
2	12.3	16.3	19.4	22.1	22.8	23.6	24.4	25.4	26.5	27.6	28.9	29.9	30.0	29.3	18.4	8.7
3	10.6	13.3	14.9	15.9	16.1	16.3	16.4	16.5	16.4	16.0	14.9	12.4	6.8			
4	9.2	10.7	11.3	11.3	11.1	10.9	10.6	10.2	9.4	8.3	6.2					
5	7.9	8.7	8.5	7.8	7.5	7.1	6.6	5.9	4.9	3.4						
6	6.8	6.9	6.3	5.2	4.9	4.4	3.8	3.0	2.0							
7	5.9	5.5	4.6	3.4	3.0	2.5	2.0	1.3								
8	5.1	4.4	3.3	2.1	1.7	1.3	0.8									
9	4.3	3.4	2.4	1.2	0.9	0.5										
10	3.7	2.7	1.6	0.6	0.4											
11	3.2	2.1	1.1	0.3												
12	2.7	1.6	0.7													
13	2.3	1.2	0.4													
14	2.0	0.9	0.3													
15	1.7	0.7	0.1													
Others	8.3	1.6														
Sum	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

and  $s_4$  are about twice the actual. Thus the probabilistic model has some merit but falls short of explaining the details of the actual pattern. We have to consider how the strategic and other factors might modify the pattern.

#### 4. Political adjustments to the probabilistic model

Compared to the probabilistic model (Table 2), the actual mean shares (Table 1) always penalize lower ranked parties in favor of higher ranked parties.<sup>8</sup> The transition point changes as the largest share increases. At  $s_1 = 20\%$ , as many as 6 largest parties have larger shares than predicted by the probabilistic model – they seem to profit from the shift. At  $s_1 = 30\%$  only 4 parties do, and the number drops to 3 at  $s_1 = 35\%$ , and to 2 at  $s_1 = 40\%$ . The inverse of the largest party share ( $1/s_1$ ) is the simplest function of  $s_1$  that comes close to expressing these observations. We will adopt it as an approximation for the number of “large” parties that profits from the shift observed.<sup>9</sup>

What could cause this rather systematic shift away from probabilistic expectations? One may immediately think of Duverger’s mechanical and psychological effects,<sup>10</sup> but the

<sup>8</sup> The only exceptions occur for tiny shares of less than 1%, and at the aforementioned bump at  $s_1$  larger than 70%.

<sup>9</sup> Note that the inverse of the largest party share has been used previously (e.g., by Siaroff, 2003) as a measure of the number of parties ( $N_\infty = 1/s_1$ ) which is lower than the widely used effective number ( $N = 1/\sum s_i^2$ ). Here it acquires a more definite meaning:  $N_\infty = 1/s_1$  is the number of parties that profit from shift to larger parties.

<sup>10</sup> In his ground-breaking study of political parties, Maurice Duverger (1951, 1954) pointed out two effects that tend to reduce minor party support in SMP systems. One has come to be called “Duverger mechanical effect”. Practically all electoral rules offer some advantage to larger parties, so that their seat shares exceed their vote shares, but the impact is the largest in plurality systems. The other effect has come to be called “Duverger psychological effect”. In response to their poor representation in the elected assembly, minor party voters are tempted to shift to some larger party as the “least evil” among those who have some decision making ability. The two effects tend to combine in SMP systems so as to boost two large parties far above the rest (the aforementioned Duverger’s law).



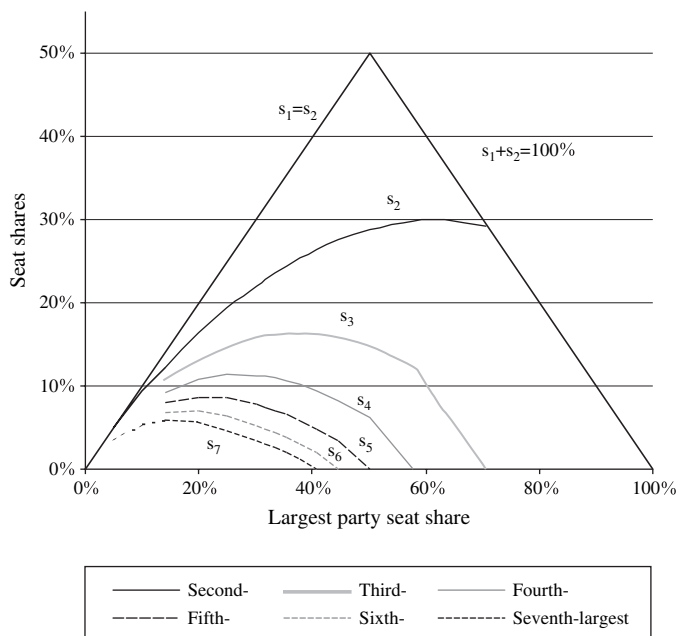


Fig. 2. Probabilistic model for seat shares of parties ranked by size – without political adjustment.

mechanical effect must largely cancel out when we graph the other shares against the largest one, which profits the most from the mechanical effect. Moreover, the Duverger effects are most marked for single-member districts, where the largest share typically exceeds 40%. But here we observe a shift away from the smallest parties for much smaller largest shares, which typically correspond to proportional representation.

One can think of several factors that may disadvantage the smallest parties even under PR. Some countries have legal thresholds of representation that block small parties. They also are disadvantaged by “economics of scale in advertising, raising funds, securing portfolios supplying policy benefits, and so on” (Cox, 1997: 141). Given the limited media coverage of minor parties, some voters could be unaware of the very existence and programs of parties that could appeal to them, if known. The main factor may be strategic concerns broader than Duverger’s psychological effect, of the type Cox (1997: 194–196) has called *strategic sequencing*.<sup>11</sup> Even if a preferred minor party does win seats under PR, some of its voters may still defect, if larger parties offer a better chance to be represented in the government. In the case of single-member districts (which predominate at large values of the largest share) the Duverger psychological effect also enters, plus residuals of the mechanical effect.

How should we proceed with model building, in face of so many possible factors? All logical quantitative models simplify reality. Some can claim that they actually express the main mechanism involved. Some can claim only that reality behaves “as if” the model applied.

<sup>11</sup> We thank a reviewer who helped us get away from a fixation on Duverger’s psychological effect and pointed out that penalization of the smallest parties even under PR rules is a major puzzle raised by our data and probabilistic expectations.

We will use an intermediary approach, combining into a single “political adjustment” parameter the impact of the aforementioned disparate factors that all pull in the same direction.<sup>12</sup>

We’ll assume that the shift always takes away support from the smaller parties. We’ll further assume that it distributes this support roughly equally among the top parties whose size qualifies them as major competitors. We’ll take the number of these “large” parties to be the inverse of the largest share ( $1/s_1$ ), for aforementioned empirical reasons.

The remaining issue is how much the small parties will suffer from the shift. The tiniest ones may well suffer proportionately more, but for the sake of simplicity we’ll assume that all small parties suffer to the same extent. This means that a steady fraction  $m$  of the inherent small party support would shift to one of the large parties.

The previous probabilistic formula is now adjusted accordingly. The calculations become more complex and are given in [Appendix B](#).<sup>13</sup> The open parameter is the fraction ( $m$ ) of support lost by smaller parties. It can in principle run from 0 to 1. In the absence of any other information, we might first try the intermediary value  $m = 0.5$ . This would mean that small parties lose one-half of their inherent seats to larger ones. Surprisingly, this rather extensive transfer agrees with the empirical data.<sup>14</sup>

With  $m$  set at 0.5, the model established in [Appendix B](#) becomes the following. Here  $s_i$  stands for the unadjusted probabilistic value of the  $i$ -th share. Recall that

$$s_i = \left(1 - \sum s_j\right) / (p - i + 1)^{0.5}$$

where  $p = 1/s_1^2$  and the summation ranges from  $j = 1$  to  $i - 1$ . Designate as  $i_0$  the rank index closest to  $1/s_1$ . For small parties (rank index higher than  $i_0$ ), the adjusted seat share ( $s'_i$ , with prime) is

$$s'_i = 0.5s_i \quad [i > i_0]$$

For large parties (rank index lower than  $i_0$ ),

$$s'_i = s_i + 0.5 \left(1 - \sum s_k\right) / (i_0 - 1) \quad [i < i_0]$$

where  $k$  runs from 1 to  $i_0$ .

<sup>12</sup> Should we try to include these factors separately? At our present stage of knowledge, they are hard to disentangle. The difficulty in telling the psychological and mechanical effects apart has been highlighted by [Benoit \(2002\)](#). Our first approximation is to use a joint parameter, but we should be on the lookout for resulting discrepancies. Are factors that depend on relative sizes of parties the only socio-political considerations to take into account? Could factors such as duration of democratic rule or per capita GDP also affect seat share distributions? They may well matter if we compare individual countries. But here we are dealing with worldwide averages, over a long time period. Such averages are likely to have considerable inertia over time and space. Rather than throwing everything into a knee-jerk linear OLS analysis, we prefer to apply Occam’s razor and work out in detail the highly nonlinear impact of what we consider basic: the size dependent factor. It will be seen that it explains quite a lot.

<sup>13</sup> We will not try to model the minor resurgence of third parties at  $s_1 > 70\%$ , and the corresponding lowering of the second largest share by about 5 percentage points. The notion of “vanity parties” introduced by [Shugart \(1988\)](#) may be useful for accounting for this resilience of third parties.

<sup>14</sup> More detailed analysis may show that adjustments somewhat less than  $m = 0.5$  also suffice. But [Fig. 3](#) (where  $m = 0.5$ ) is clearly much closer to the empirical pattern than is [Fig. 2](#) (which implies  $m = 0$ ).

Table 3  
Party size distributions with political adjustment

Rank	Number of parties and the corresponding seat shares (in percentages)															
	50	25	16	11	10	9	8	7	6	5	4	3	2.5	2	1.5	1.2
1	16.9	23.9	29.9	38.2	39.0	40.0	41.4	42.9	49.0	51.6	55.3	60.8	64.9	70.7	81.6	91.3
2	15.0	20.2	24.3	30.1	30.2	30.3	30.4	30.5	34.6	34.5	34.2	33.0	31.7	29.3	18.4	8.7
3	13.4	17.2	19.8	15.9	16.1	16.3	16.4	16.5	8.2	8.0	7.5	6.2	3.4			
4	11.9	14.7	11.3	5.6	5.6	5.5	5.3	5.1	4.7	4.1	3.1					
5	10.7	8.7	4.3	3.9	3.7	3.5	3.3	2.9	2.4	1.7						
6	9.6	3.5	3.2	2.6	2.4	2.2	1.9	1.5	1.0							
7	5.9	2.8	2.3	1.7	1.5	1.3	1.0	0.6								
8	2.5	2.2	1.7	1.0	0.9	0.7	0.4									
9	2.2	1.7	1.2	0.6	0.4	0.3										
10	1.9	1.3	0.8	0.3	0.2											
11	1.6	1.0	0.5	0.1												
12	1.4	0.8	0.4													
13	1.1	0.6	0.2													
14	1.0	0.5	0.1													
15	0.8	0.3	0.1													
Others	3.0	0.7														
SUM	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Shaded area: parties that gain seats through political transfers.

One intermediary party (rank index  $i_0$ ) is assumed to lose as much as it gains and thus remains unchanged, as long as the unadjusted largest share remains below 40%:

$$s'_i = s_i \quad [i = i_0, s_1 < 40.0\%]$$

For unadjusted  $s_1$  larger than 40.0%, this intermediary stage is squeezed out. All third parties lose ( $s'_i = 0.5s_i$ ), while for the 2 largest parties the equation becomes

$$s'_i = s_i + (1 - s_1 - s_2)/4 \quad [2 \text{ largest parties, } s_1 > 40.0\%]$$

The results are shown in Table 3. Fig. 3 presents the corresponding theoretical curves, following the format of Fig. 1. Compared to the simple probabilistic model (Fig. 2), the adjusted model in Fig. 3 visibly approaches the empirical pattern in Fig. 1. We'll next consider this degree of agreement in more detail.

### 5. Agreement of the modified model with empirical data

Figs. 4 and 5 show empirical data (from Table 1) superimposed on the model-based curves (Table 3). To avoid crowding, we present separately the empirical averages for second to fourth ranks (Fig. 4) and fifth to seventh ranks (Fig. 5).

Seat shares of the *second largest party* (Fig. 4) follow the model remarkably well as long as the largest party share remains below 50%.<sup>15</sup> When the largest share reaches 50–55%, the

<sup>15</sup> The model developed in Appendix B involves discontinuities around the rank  $1/s_1$ , and here the curves show some artifactual kinks. The actual data points seem to hug even these kinks, as if the discontinuities built into the model had some reality. It is likely to be happenstance, unless confirmed by a more detailed check.

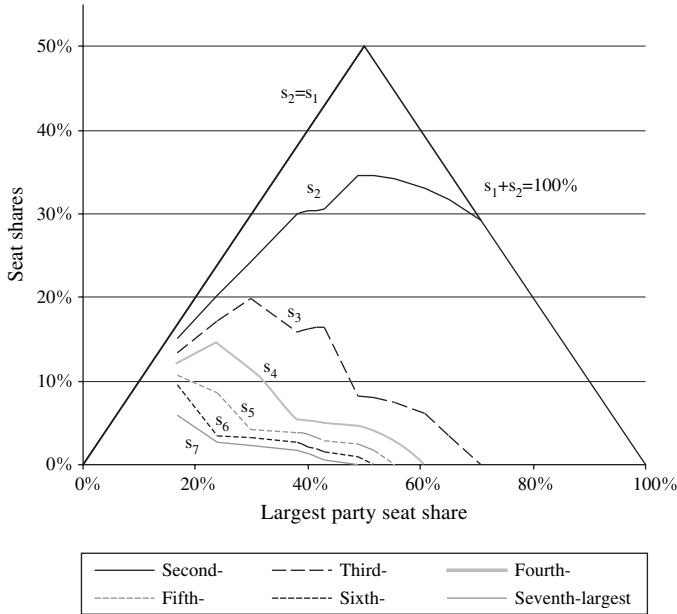


Fig. 3. Seat shares of second- to seventh-largest parties as predicted by the probabilistic model with political adjustment.

empirical average exceeds the model prediction. It may be random fluctuation or impact of residual Duverger mechanical effect. When the largest share grows to beyond 60%, empirical results steadily fall below the model predictions, which do not pretend to explain the aforementioned new rise of third parties in face of a hegemon.

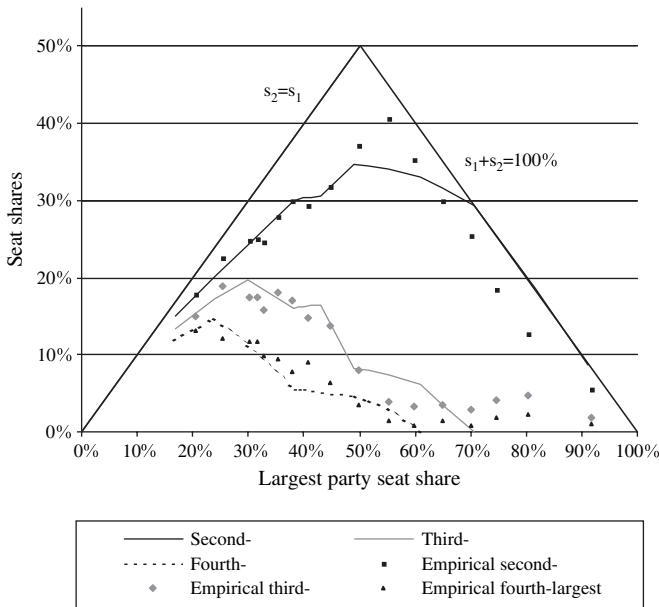


Fig. 4. The model-based curves and actual mean seat shares for the second-, third- and fourth-largest parties.

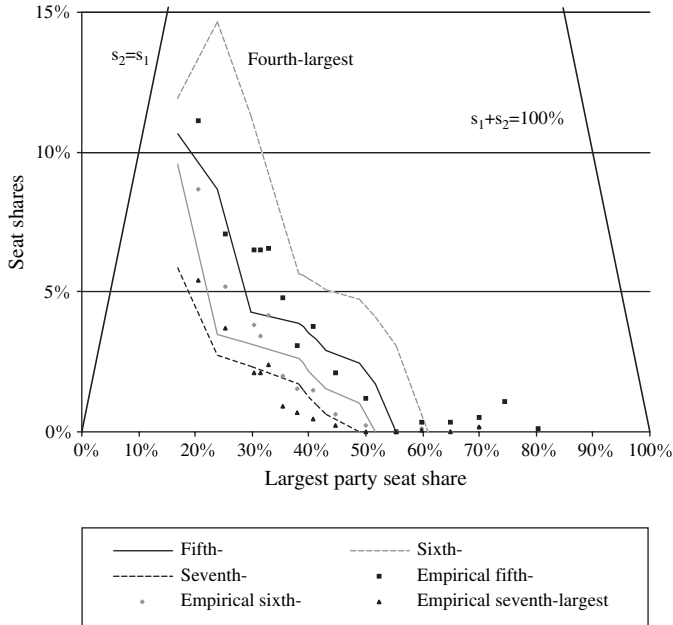


Fig. 5. The model-based curves and actual mean seat shares for fifth-, sixth- and seventh-largest parties.

The *third largest party* at first increases its average seat share with increasing largest party, well in line with the model. The actual peak occurs near the predicted location ( $s_1 = 25\%$ , instead of the predicted 30%) and has the predicted height ( $s_3$  close to 20%). At  $s_1 = 45\text{--}50\%$ , the third party share drops suddenly, as predicted by the model, but then it continues to drop and falls below the predicted level. The new rise at  $s_1 > 60\%$  is beyond the explanatory goal of the model.

The *fourth largest party* peaks at around  $s_1 = 20\%$ , close to the predicted 24%, and almost reaches the expected height of 15%. The sharp drop expected at  $s_1 = 30\text{--}40\%$  is replaced by a more gradual decrease.

The seat shares of fifth to seventh-largest parties (Fig. 5) are low, which enhances relative error. The predicted curves are close to each other. A rough test of the model is whether the data points at a given rank remain contained between the predicted curves for lower and higher ranks.

The empirical points for the *fifth largest party* do indeed remain between the theoretical curves for the fourth and sixth largest, up to  $s_1 = 60\%$ . The highest value observed (at  $s_1 = 20\%$ ) slightly exceeds the prediction. The expected sharp drop at  $s_1 = 25\text{--}30\%$  mutes into a more gradual one, as it did for the fourth largest party.

The *sixth largest party* shares also remain in the expected range, and the same is broadly the case for the *seventh largest*. By now, the shares are so small that nothing more specific can be said. The data themselves become murky, because all too often such minor parties are lumped together as “Others” in Mackie and Rose (1991, 1997).

## 6. Conclusions

What have we achieved? First, we have graphed the seat shares of parties in a novel way. Second, we have explained the main features of the resulting complex pattern in

terms of a model based on probabilistic expectations plus adjustments of a political nature. Third, we find that the model fits best when one-half of the inherent supporters of minor parties switch to larger parties — and this is a novel way to measure the average strength of strategic considerations and other factors that work against the smaller parties.

*On the empirical side*, we have calculated the worldwide averages of seat shares of parties at various ranks, at given largest party share (Table 1). We have graphed them, using the Nagayama triangle format (Fig. 1). These empirical averages offer us a worldwide measuring stick against which seat share distributions in individual countries or single elections can be compared. Since these average party constellations can be considered typical, they could be of use in testing various hypotheses where party sizes enter, such as in coalition formation. They also can be used to test a previously suggested approximate relationship between the effective number of parties and the share of the largest. As shown in Appendix C, this previous approximation is an overestimate.

When the largest share increases beyond 25%, parties are found to divide broadly into 2 groups: the 2 largest and the rest. When the largest party falls short of absolute majority, the second largest share tends to be around 0.8 of the largest share — a somewhat lower ratio than previously suggested. When the largest party has absolute majority, the second largest share tends to be around 0.8 of what is left over by the largest share.

*On the theoretical side*, we have established a model to show how the parties at various ranks are expected to fare as the largest share increases. We first calculated the size distribution of parties that would prevail on the basis of sheer probability. As expected, this pattern is more uniform than what we observe empirically. We then applied in a systematic way the Duvergerian expectation that low-ranking parties lose some of their inherent support, while high-ranking parties profit from the switch.

This model fits the complex empirical pattern surprisingly well when 2 free parameters are given suitable values: the number of parties that profit from transfers, and the strength of the shift. The number of parties that profit from transfers is found to be close to the inverse of the fractional share of the largest party. Most important, the model fits best when we assume that the switch involves about one-half of inherent minor party support. This fraction 0.5 represents an independent quantitative estimate of the average strength of political factors that work against the smaller parties.

Could there be implications for party strategies? It is too early to say because more work remains to be done. In particular, the distribution of vote shares must be investigated, too. Vote shares seem to present a similar picture to seat shares, but a logical quantitative model is harder to establish.<sup>16</sup> In particular, systems with plurality rule in single-member districts, where Duverger mechanical effect enters in a major way, may have to be studied separately.

<sup>16</sup> We started with seats rather than votes because theoretical modeling could be guided by the approximation  $p = 1/s_1^2$ , which depends on having a clear lowest conceivable number of seats (namely 1) with which a party could still have representation in the assembly. The equivalent lowest limit for votes is much harder to define. How would the empirical pattern change, if vote shares were used instead of seat shares? Seat and vote shares are of course similar in PR systems. Under SMP rule, vote shares of the largest and second largest parties tend to be smaller than the corresponding seat shares, but since both parties' shares decrease, this may mean a down movement underneath the Nagayama triangle, still roughly maintaining the 0.8 ratio. Thus the pattern may not change much, but this should be checked empirically.

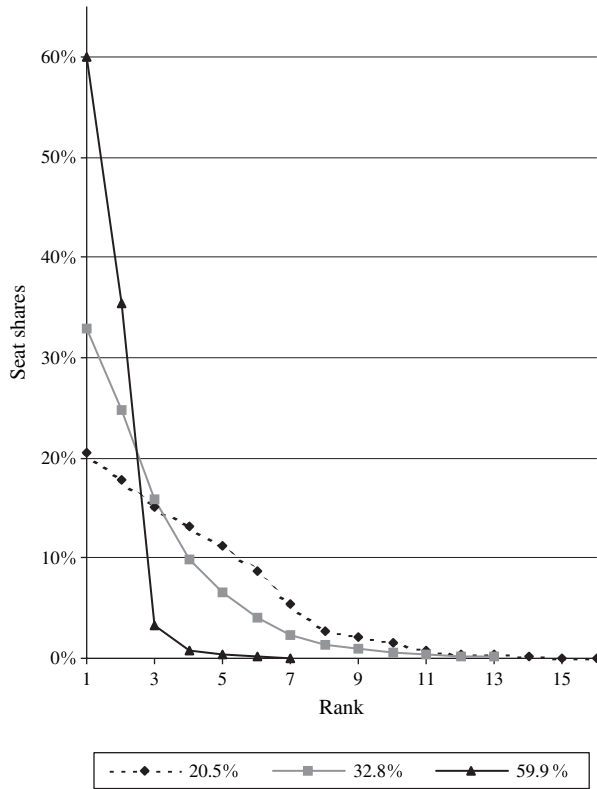


Fig. 6. Average seat shares of parties versus their rank, at various largest party shares.

### Appendix A. Seat shares graphed against rank by size

Rather than graphing all other seat shares against the largest (as in Fig. 1), one can also graph the *seat shares* at any rank ( $s_i$ ) against their *rank* ( $i$ ). Fig. 6 shows the results at 3 different sizes of the largest party: around 20% (about the lowest observed), 32.8% (average), and around 60% (on the high side). The middle curve fits a power function quite well, but in general power functions underestimate the second largest share.<sup>17</sup> No other simple function offers a better fit.<sup>18</sup> Indeed, such an endeavor looks hopeless when one considers the complex patterns in previous Fig. 1.

<sup>17</sup> The general power function format is  $s_i = ai^n$ . Here we subject it to three conditions: for  $i = 1$ , the equation must yield the largest share ( $s_1$ ); for  $i = p + 1$ , it must yield 0 (if  $p$  is the number of parties that have seats); and the total of seat shares must be 1. The result is  $s_i = s_1[(p + 1 - i)/p]^n$ . This equation always satisfies the first two conditions, and  $n$  is adjusted so that sum of seat shares is 1. When  $s_1 = 32.8\%$  and we assume that  $p = 1/s_1^2$ , such adjustment of  $n$  yields  $s_i = .328[10.3 - i]/9.3]^2.68$ . This equation is found to fit the data within  $\pm 1.5\%$ . In general, power functions overestimate the shares around  $i = p/2$  and miss the tails that extend beyond the presumed cutoff at  $i = 1/s_1^2$ . In the extreme case of  $s_1 = 20\%$ , the best power function fit underestimates not only the second largest share but also several next ones.

<sup>18</sup> In particular, the exponential function ( $s_i = s_1e^{-ki(i-1)}$ ) used by Taagepera (2001) tends to underestimate the second and third largest components.

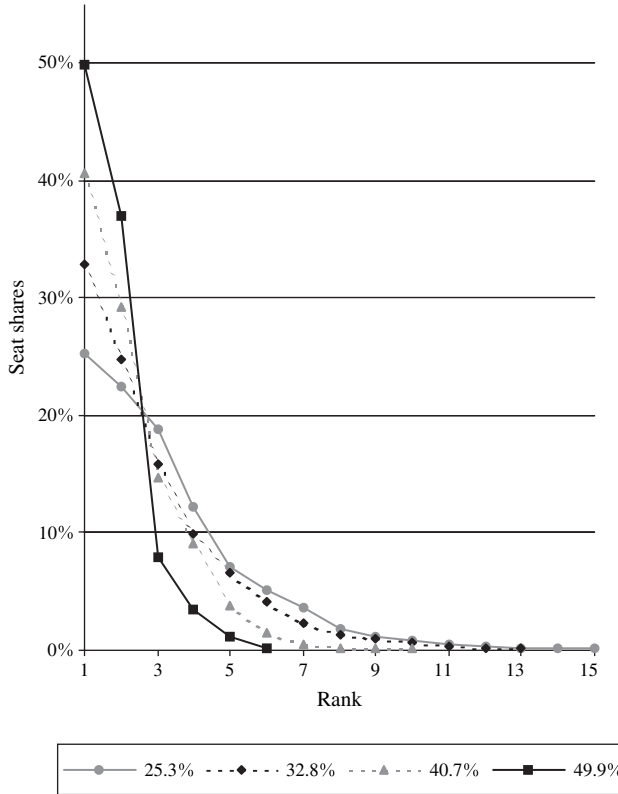


Fig. 7. Average seat shares of parties versus their rank (*i*): crossovers around  $i = 2.6$ .

Fig. 7 shows several analogous curves in the intermediary range of the largest seat share (25–50%). The striking feature here is that all these curves cross at about the location  $i = 2.6$ ,  $s_i = 20 \pm 0.5\%$ . This is not so clearly the case when the largest share is less than 25% or more than 50% (cf. Fig. 6).

**Appendix B. Details of political adjustment to the probabilistic model**

Recall that the unadjusted shares are  $s_i = (1 - \sum s_j) / (p - i + 1)^{0.5}$ , where  $p = 1/s_1^2$  and the summation ranges from  $j = 1$  to  $i - 1$ . Designate as  $i_0$  the rank index closest to  $1/s_1$ . As long as  $i_0 > 2$ , we proceed as follows. The party at rank  $i_0$  is kept unaffected by the adjustment. Smaller parties suffer, and larger parties profit. Because  $i_0$  changes discontinuously when  $s_1$  changes gradually, some discontinuity is built into the model at this point.

Procedure must change when the largest share exceeds 40.0%, so that  $i_0 = 2$ , because otherwise there would be no choice left. Here we stipulate that the second largest party always profits from a shift, even though by the previous rule it no longer would.

As long as the largest party inherently has less than 28.6% of the seats ( $1/s_1 > 3.5$ ), we have  $i_0 > 3$ , and the third largest party profits from the adjustment. When the largest party inherently has between 28.6 and 40.0% ( $3.5 > 1/s_1 > 2.5$ ), we have  $i_0 = 3$ , and the third largest party



profits and suffers to an equal degree. When the largest party inherently has more than  $s_1 = 40.0\%$  of the seats ( $1/s_1 < 2.5$ ), we have  $i_0 < 3$ , and the third largest party begins to suffer from being one of the smaller parties. We stress the word “inherently”, because the adjustment will boost the largest party seat share, too, from the inherent  $s_1$  to a larger adjusted value  $s_1'$ .

By the rules chosen, as  $1/s_1$  continuously shifts from more than 3.5 to less than 3.5, the third largest party suddenly shifts from recipient to neutral status, and the sudden shift repeats itself at  $1/s_1 = 2.5$ . All other shares also undergo such sudden shifts, and each shift rearranges all shares, including the largest. As a result, the adjusted curves will include artifactual kinks. Avoiding them would require stipulating a smoother transition from donor to recipient status, which of course makes sense but also complicates mathematics. The ungainly simpler model still fits the empirical data to a surprising degree. When  $s_1$  becomes larger than 40.0%, we are forced to make the transition even more abrupt, by stipulating that the second largest party always profits from the shift.<sup>19</sup>

If the model basically holds, we would expect the empirical points to trace the same pattern, but more smoothly. A continuous model would obviously be preferable, but its construction presents difficulties. What we have is an ungainly but efficient approximation.<sup>20</sup>

We have assumed that a steady fraction  $m$  of the supporters of each small party actually vote for one of the large parties. The previous formula  $s_i = (1 - \sum s_j)/(p - i + 1)^{0.5}$  now is adjusted accordingly. For small parties (i.e., those where  $i$  is larger than  $i_0$ ), the adjusted seat share is

$$s'_i = (1 - m)s_i \quad [i > i_0]$$

The combined loss of these small parties is  $m$  times the sum  $\sum s_i$ , where  $i$  runs from  $i_0 + 1$  to  $p$ . But this sum is also 1 minus the sum of all other parties, from  $i = 1$  to  $i = i_0$ . Hence the total loss for small parties can be written as  $m(1 - \sum s_k)$ , where  $k$  runs from 1 to  $i_0$ . This represents also the total gain by the large parties (i.e., those where  $i$  is smaller than  $i_0$ ). Assuming equal distribution of the gain among them, each gets a share  $m(1 - \sum s_k)/(i_0 - 1)$ , where  $k$  runs from 1 to  $i_0$ . This amount is added to their unadjusted share  $s_i = (1 - \sum s_k)/(p - i + 1)^{0.5}$ . Thus the total adjusted share of large parties is

$$s'_i = s_i + m \left(1 - \sum s_k\right) / (i_0 - 1) \quad [i < i_0]$$

where  $k$  runs from 1 to  $i_0$ . For the intermediary party ( $i = i_0$ ), there is no adjustment:

$$s'_i = s_i \quad [i = i_0]$$

For unadjusted  $s_1$  larger than 40.0%, this intermediary stage vanishes, as explained earlier. All third parties lose, in line with  $s'_i = (1 - m)s_i$ , while for the 2 largest parties the equation becomes

<sup>19</sup> Indeed, for unadjusted largest party seat share above 66.7% ( $1/s_1 < 1.5$ ), the general rule would lead to the impossible situation where  $i_0 = 1$ , meaning that the second largest party must lose seats, but the largest party cannot receive them!

<sup>20</sup> Our model starts out with a given largest party seat share ( $s_1$ ). In the course of the adjustment, this largest party seat share itself moves upward – and it does so discontinuously at the aforementioned points where  $1/s_1$  assumes the values 2.5, 3.5, 4.5... The discontinuity is most dramatic at  $s_1 = 40\%$  ( $p^{0.5} = 2.5$ ) where the adjusted largest party share jumps from 41.3 to 48.4%.

Table 4  
Effective number of parties (in percentages)

Largest party share	$N$ on the basis of largest party share alone	$N$ on the basis of average shares
20.5	10.8	7.2
25.3	7.9	5.7
30.3	6.0	4.9
31.6	5.6	4.7
32.8	5.3	4.7
35.4	4.7	4.0
38.0	4.3	3.7
40.7	3.9	3.5
44.7	3.3	3.1
49.9	2.8	2.5
55.3	2.4	2.1
59.9	2.2	2.1
65.0	1.9	2.0
70.1	1.7	1.8
74.5	1.6	1.7
80.2	1.4	1.5
91.7	1.1	1.2

$$s'_i = s_i + m(1 - s_1 - s_2)/2 \quad [2 \text{ largest parties, } s_1 > 40.0\%]$$

Assigning  $m$  the value  $m = 0.5$  yields the equations shown in the main text.<sup>21</sup>

### Appendix C. Effective number of parties and the largest party share

Our empirical results can be used to examine the average relation between the largest party share and the effective number of parties ( $N = 1/\sum s_i^2$ , where  $s_i$  is the share of the  $i$ -th party). The effective number of parties is widely used (e.g., in Lijphart, 1994 and Cox, 1997) and is largely determined by the largest party share ( $s_1$ ). Taagepera and Shugart (1993) have used the approximation  $N = s_1^{-1.5}$ , on probabilistic grounds.<sup>22</sup> It turns out to be an overestimate for most values of  $s_1$ .

Table 4 shows the effective numbers of parties ( $N$ ) as estimated on the basis of the largest party share alone ( $N = s_1^{-1.5}$ ), and the values calculated on the basis of all average shares (from Table 1). The approximation  $N = s_1^{-1.5}$  is seen to fit well only when the largest share is very

<sup>21</sup> Our model starts out with a given largest party seat share ( $s_1$ ) but then adjusts it upward to  $s_1'$ . This adjustment disrupts the presumed relationship  $p = 1/s_1^2$  between the unadjusted largest share and the number of seat-winning parties. The assumption that all minor parties lose a fraction  $m$  of their inherent support implies that the adjustment makes no party vanish completely. Thus  $p$  is maintained, while adjusted  $s_1'$  is larger than the unadjusted  $s_1$ . The result is that  $p = 1/s_1'^2 > 1/s_1'^2$ . This is what Table 1 confirms empirically. Should the inverse square relationship apply to the actual largest share or to the fictional unadjusted one? Its derivation proceeds with the proviso "in the absence of any further information", apart from its extreme limits plus general centralizing tendencies. The unadjusted model proceeds precisely from this mechanical assumption, repeating it for each successive share. The political adjustments do add some "further information" to sheer statistical probabilities, thus affecting the validity of  $p = 1/s_1^2$  to some limited degree.

<sup>22</sup> The underlying reasoning is the following. The effective number  $N$  cannot be less than  $1/s_i$  (equal shares) nor more than  $1/s_i^2$  (completely atomized field, beyond the largest party). The geometric average of these limits is  $1/s_i^{1.5}$ .

large. At lower values, the approximation overestimates the actual  $N$  by 12% at  $s_1 = 33\%$  and by as much as 49% at  $s_1 = 20\%$ . Replacing the estimate  $N = s_1^{-1.5}$  by  $N = 1.06s_1^{-1.25}$  would keep the average error within  $\pm 10\%$  throughout the range.

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