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Is The Potential For High Investor Leverage A Threat To Social Security Privatization?

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Berkeley**

Is The Potential For High Investor Leverage A Threat To Social Security Privatization?*

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Abstract

How risky and expensive it would be to insure a long-term individual Social Security account invested in stocks against the risk that the portfolio's value would collapse? This paper uses a particular metric to evaluate this risk and cost. This metric is a long-term put option written on such a portfolio. The answer is that for reasonable parameters the Black-Scholes price of such a put option is surprisingly low: just 2-4% of original investment.

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I. INTRODUCTION

The history of the stock market shows that, typically, stocks have been substantially undervalued in the sense that long-term investments in them have yielded very high average returns, on the order of six percent per year in real values[1]. The history of the stock market also shows that the probability of the collapse of a well-diversified leveraged portfolio invested in equities is low. The probability that such a collapse would not be reversed by subsequent mean reversion in stock prices is even lower[2]. Certainly it appears lower than investments in nominal bonds, which bear inflation risk, or trust in government promises–the lavishness of which is a principal source of inflation risk.

Thus diversified long-term investments in stocks largely insure themselves. They carry little risk. How little risk? This paper uses a particular metric to evaluate this risk. It asks how expensive it would be to insure a long-term individual Social Security account invested in stocks against the risk that the portfolio's value would collapse. It asks how expensive standard finance suggests purchasing a long-term put option on such a portfolio should be. The answer is that the Black-Scholes price of such a put option is surprisingly

[1] See Siegel (1995), Siegel (2002) and Smith (1924).

[2] Fama and French (1988) and Poterba and Summers (1988) point out that due to the mean-reverting nature of stock prices, long periods of high stock returns are usually followed by long periods of low stock returns. There is little sign of the same process in bonds and fixed-income instruments. Instead, unpredictable inflation makes real returns on bonds and fixed-income instruments very volatile over the long run, while stocks have proved to provide very good protection against inflation in the long run.

low. It is low even in this world in which the large size of the equity premium reveals that the market price of systematic risk is very high[3].

Therefore, this paper proposes a system in which the government requires that individuals buy insurance on their retirement portfolios at the time as they make their retirement contributions[4]. Such an insurance policy would be incentive-compatible. Those who take extra risks will have to pay higher insurance premiums, thus reducing the key moral hazard problem associated with investing public pensions in individual accounts–that individuals can gamble and anticipate that the government will cover their losses[5].

II. MODEL

According to the proposed system, the government would require that individuals buy insurance on their retirement portfolios at the time as they make their retirement contributions. Thus, at period t individuals invest P_{jt} in some asset (portfolio) j and purchase a European put option for a price Put_{jt} with a strike price S written on this portfolio. At period $t+1$ an option is either exercised or not, depending on the value of P_{jt+1}.

Therefore, by investing $P_{jt} + Put_{jt}$ today, tomorrow investor obtains

$$W_{t+1} = P_{jt+1} + \max[0, S - P_{jt+1}]$$

[3]See Magin (2007) for some preliminary results.

[4]There is of course a complicated and difficult issue of transition to such a system, but due to the lack of space it is not addressed here.

[5]Advantages of individual retirement accounts as opposed to a government-run trust fund are also not addressed here.

It could be easily shown that (with all other parameters fixed) a put option is a monotonically increasing function of the variance of the portfolio the put is written on. Indeed, the higher the variance, the higher the profit that can be realized by exercising the put. But the variance of the portfolio represents total risk of the portfolio. I'm specifically interested in the effect of the systemic component of this risk on the value of the put option. We need to find the derivative of Put_{jt} with respect to β_j.

LEMMA *Suppose that*

1. *CAPM holds for every asset* j, *i.e.,*

$$R_j = R_f + \beta_j R_m + \epsilon_j, \ (1)$$

where $R_f = r_f + 1$ *is risk-free gross rate of return,* R_m *stands for the equity premium for the market portfolio and* ϵ_j *is i.i.d. with* $E\left[\epsilon_j\right] = 0$.

2. *Every risky asset* j *has gross rate of return* R_j *which is lognormally distributed, i.e.,*

$$ln(R_j) \sim N(\mu_j, \sigma_j^2).$$

Then

$$\sigma_j = \sqrt{ln(\frac{{R_f}^2 + \beta_j^2 E[{R_m}^2] + 2\beta_j R_f E[R_m] + E[{\epsilon_j}^2]}{{R_f}^2 + \beta_j^2 E^2[R_m] + 2\beta_j R_f E[R_m]})} \quad (2)$$

PROOF: We know that

$${\sigma'_j}^2 = e^{2\mu_j}[e^{2{\sigma_j}^2} - e^{{\sigma_j}^2}], \quad (3)$$

where

$$\sigma_j{}^2 = VAR[ln(R_j)],$$
$$\sigma_j'^2 = VAR[R_j],$$
$$\mu_j = E[ln(R_j)].$$

Also,

$$ln(E[R_j]) = \mu_j + .5\sigma_j{}^2 \quad (4)$$

and from (4) we obtain

$$e^{2\mu_j} = \frac{E^2[R_j]}{e^{\sigma_j{}^2}}. \quad (5)$$

Substituting (5) into (3) we get

$$e^{\sigma_j{}^2} = \frac{E[R_j{}^2]}{E^2[R_j]} \ . \quad (6)$$

Using (1) we obtain

$$R_j{}^2 = R_f{}^2 + \epsilon_j{}^2 + 2R_f\epsilon_j + \beta_j^2 R_m{}^2 + 2\beta_j R_m R_f + 2\beta R_m \epsilon_j$$

So, taking expectations we obtain

$$E[R_j{}^2] = R_f{}^2 + \beta_j^2 E[R_m{}^2] + 2\beta_j R_f E[R_m] + E[\epsilon_j{}^2] \quad (7)$$

and

$$E^2[R_j] = R_f{}^2 + \beta_j^2 E^2[R_m] + 2\beta_j R_f E[R_m] \quad (8)$$

Hence, substituting (7) and (8) into (6), we have

$$e^{{\sigma_j}^2} = \frac{{R_f}^2 + \beta^2 E[{R_m}^2] + 2\beta R_f E[R_m] + E[{\epsilon_j}^2]}{{R_f}^2 + \beta^2 E^2[R_m] + 2\beta R_f E[R_m]} \quad (9)$$

Set

$$g(\beta) = \frac{{R_f}^2 + \beta_j^2 E[{R_m}^2] + 2\beta_j R_f E[R_m] + E[{\epsilon_j}^2]}{{R_f}^2 + \beta_j^2 E^2[R_m] + 2\beta_j R_f E[R_m]} \quad (10)$$

We can conclude from (9) and (10) that

$$\sigma_j = \sqrt{ln(g(\beta))} = \sqrt{ln(\frac{{R_f}^2 + \beta_j^2 E[{R_m}^2] + 2\beta_j R_f E[R_m] + E[{\epsilon_j}^2]}{{R_f}^2 + \beta_j^2 E^2[R_m] + 2\beta_j R_f E[R_m]})} \quad \blacksquare$$

THEOREM *Consider a two-period economy. At period t, the price of the asset j P_{jt}, the risk-free rate r_f, and the strike price S are observed. $\bar{C}$ is the aggregate consumption. We assume that*

1. $\ln P_{jt+1}$ *and* $\ln \bar{C}$ *are bivariate normally distributed with means* $\left(\hat{\mu}_j,\ \hat{\mu}_c\right)$ *and the variance-covariance matrix:*

$$\begin{pmatrix} \sigma_j^2 & k\sigma_j\sigma_c \\ k\sigma_j\sigma_c & \sigma_c^2 \end{pmatrix},$$

where k is the correlation coefficient between $\ln P_{jt+1}$ and $\ln \bar{C}$.

2. *Preferences are specified as*

$$U = \frac{1}{1-B} C_t^{1-B} + \rho \frac{1}{1-B} C_{t+1}^{1-B},$$

Then there exists a $\overset{*}{\beta}_j$ large enough such that $\frac{dPut_{jt}}{d\beta_j} > 0$ for all

$$\beta_j > \overset{*}{\beta}_j$$

PROOF: Following Rubinstein (1976) we can conclude that our put here can be priced by Black-Scholes formula. So, differentiating Black-Scholes equation for put with respect to β_j and using above Lemma we obtain

$$\frac{dPut_{jt}}{d\beta_j} = \frac{dPut_{jt}}{d\sigma_j}\frac{d\sigma_j}{d\beta_{\ j}} = \frac{1}{2}\cdot\frac{S\cdot n(Z_S)}{1+rf}ln^{-\frac{1}{2}}(g(\beta_j))g(\beta_j)^{-1}\frac{dg}{d\beta_j},$$

where

$$g(\beta_j) = e^{\sigma_j^2},$$

$n(\cdot)$ is the pdf of a standard normal variable and

$$Z_S = \frac{ln(\frac{P_{jt}}{S}) + ln(1+r_f)}{\sigma_j} - .5\sigma_j.$$

Differentiating now g with respect to β_j we obtain

$$\frac{dg}{d\beta_j} =$$

$$\frac{\beta_j^2 2R_f E[R_m](E[{R_m}^2] - E^2[R_m]) + 2\beta_j(E[{R_m}^2]{R_f}^2 - E^2[R_m]({R_f}^2 + E[{\epsilon_j}^2])) - 2R_f E[R_m]E[{\epsilon_j}^2]}{{R_f}^2 + \beta_j^2 E^2[R_m] + 2\beta_j R_f E[R_m]}^2$$

I claim that there exists a $\overset{*}{\beta_j}$ large enough such that

$$\frac{dg}{d\beta_j} > 0$$

for all

$$\beta_j > \overset{*}{\beta_j}$$

Indeed,

$$\frac{dg}{d\beta_j} > 0$$

if and only if

$$\beta_j^2 2R_f E[R_m](E[{R_m}^2]-E^2[R_m])+2\beta_j(E[{R_m}^2]{R_f}^2-E^2[R_m]({R_f}^2+E[{\epsilon_j}^2]))-2R_f E[R_m]E[{\epsilon_j}^2] > 0$$

Define

$$A = 2R_f E[R_m](E[{R_m}^2] - E^2[R_m])$$

$$B = 2(E[{R_m}^2]{R_f}^2 - E^2[R_m]({R_f}^2 + E[{\epsilon_j}^2])$$

$$C = -2R_f E[R_{mt}]E[{\epsilon_j}^2]$$

$$D = B^2 - 4AC$$

Now, because

$$E[{R_m}^2] - E^2[R_m] = VAR[R_m] > 0$$

we can conclude that

$$A > 0$$

We also know that

$$C < 0$$

Therefore,

$$D > 0$$

Hence,

$$\overset{*}{\beta}_j = \frac{-B + \sqrt{D}}{2A} \quad \blacksquare$$

So, for all betas greater than $\overset{*}{\beta}_j$ put is a monotonically increasing function of systemic risk. Then, as the beta continues to grow, the value of the put slowly increases, asymptotically approaching finite bound $\frac{S}{1+rf}$.

III. CALCULATIONS

According to the proposed system, individuals will be required to invest P_{jt} in some portfolio of assets and also to "insure" by purchasing a put option written on this portfolio, with a strike price S equal to the initial investment P_{jt}[6]. The gross rate of return on this portfolio $\frac{P_{jt+1}}{P_{jt}}$ is lognormally distributed. So, consider investing $P_{jt} = \$1$ for 20 years in a portfolio with $\beta = 1$, with a real expected rate of return of 6.5% and with a standard deviation of .1 annually for 20 years. In addition, investors purchase a put option that guarantees that 20 years from now investors will realize at least a real rate of return of 0% on this portfolio. How likely is it that this put option will be exercised? It is only .69%! At the same time the probability that the initial investment of \$1 is going to be more than doubled is overwhelming.

[6] Why not to set $S = P_{jt}(1 + r_f)^T$? See Bodie (1995) for example. Because this way we will be executing a very expensive strategy. A strategy that guarantees at least an annual risk-free rate with a very low probability and a 6.5% annual return with a very high probability. Because of the No Arbitrage condition this strategy is going to be so expensive that after factoring in all the costs, the resulting rate of return will be equal to r_f.

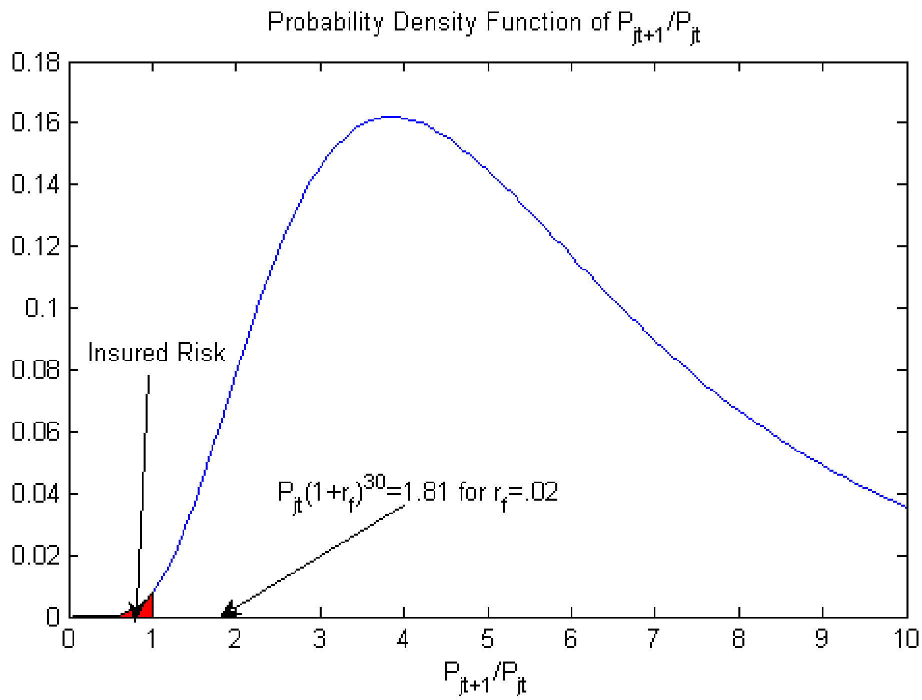

Figure 1:

It is 81.6%. As we move to a 30 years holding period the probability of exercising becomes even smaller. It is only .13%. The probability that the initial investment of \$1 is going to be more than doubled becomes even larger. It is 95.88%. See Figure 1 above. As we shall see below, this low probability of an exercise will make the price of a put option very low indeed.

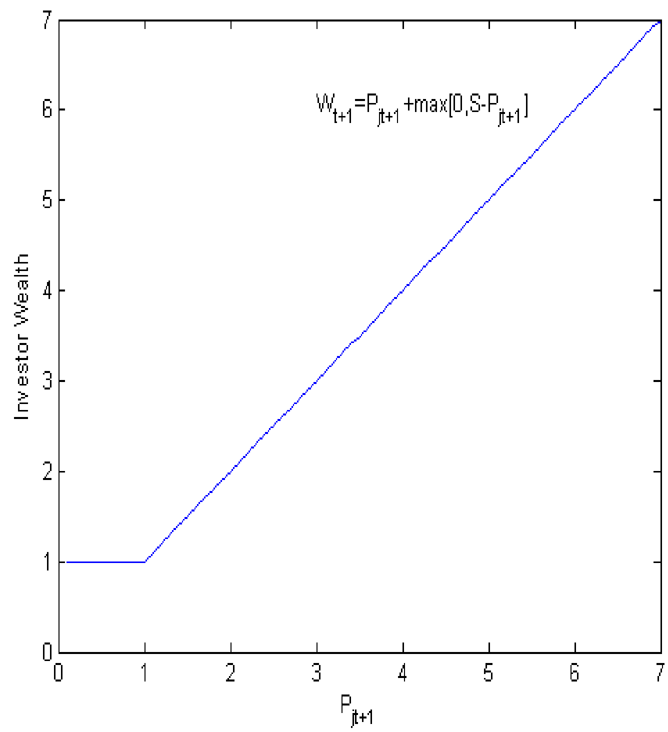

Figure 2:

Figure 2 above shows the investor's end-of-worklife wealth $W_{t+1} = P_{jt+1} + \max[0, S - P_{jt+1}]$ thirty years from now as a function of the investor's end-of-worklife market value $P_{jt+1}$of this asset portfolio. As long as $P_{jt+1} \leqslant S = 1$, the put option will be exercised. Then the investor's wealth $W_{t+1} = S$ will be \$1. If $P_{jt+1} \geqslant 1$, then the investor's wealth will be $W_{t+1} = P_{jt+1}$, represented by a 45 degree line on the figure.

So, how expensive would it be to insure a diversified equity portfolio of the kind appropriate for an individual Social Security retirement account against the risk of collapse in its value? I use here a particular metric to price this insurance. This metric is the Black-Scholes price of a long-term put option written on this diversified equity portfolio. The answer is that the price of such a put option is surprisingly low. It is low even in the world we live in, where price of risk is very high.

Fix the risk-free rate at 2%. Consider investing in the S&P 500 portfolio. See Table 1 below. For time horizon of 20 to 40 years the price of the put option is extremely low at only 2.01%-3.66% of the original investment respectively. However, if one invests in a portfolio that is twice as risky as S&P 500 portfolio, then the price of a put option becomes immediately much more expensive at 12-15% of the original investment. Those who take extra risks with their portfolios will have to pay higher "insurance" ex ante.

Table 1

2% Risk-free Rate: the Black-Scholes Price of Insurance Written on and Portfolios as a Percentage of the Original Investment, %

Portfolio's Beta	Time Horizon 20 Years	Time Horizon 25 Years	Time Horizon 30 Years	Time Horizon 35 Years	Time Horizon 40 Years
1	3.66	3.18	2.74	2.35	2.01
2	14.96	14.46	13.78	12.99	12.17

Fix the risk-free rate at 1%. Consider investing in the S&P 500 portfolio (see Table 2 below). Let us calculate the price of a put option written on this portfolio, using the Black-Scholes formula. For a time horizon of 20 to 40 years the price of the put option is only 8.06%-8.58% of the original investment. However, if one invests in a portfolio that is twice as risky as the S&P 500 portfolio, then the price of a put option becomes immediately cost-prohibitive at 23-24% of the original investment. Once again those who take extra risks with their portfolios will have to pay higher "insurance" ex ante, thus eliminating the moral hazard problem.

Table 2

1% Risk-free Rate: the Black-Scholes Price of Insurance Written on and Portfolios as a Percentage of the Original Investment, %

Portfolio's Beta	Time Horizon 20 Years	Time Horizon 25Years	Time Horizon 30 Years	Time Horizon 35 Years	Time Horizon 40 Years
1	8.58	8.59	8.48	8.30	8.06
2	23.06	23.96	24.47	24.69	24.69

V. ARE PRIVATE COMPANIES CAPABLE OF INSURING RETIREMENT PORTFOLIOS?

The calculations performed in this paper above can be interpreted in two ways. First, they can be interpreted as yardsticks of how worried those investing their retirement wealth–specifically, their Social Security wealth–in equities should be of the downside risk that the stock market will permanently collapse. In a well-functioning market, the actuarially-fair (including risk aversion) price of insurance is a convenient yardstick for the seriousness of the potential loss that the insurance would cover. In this situation, the price of insurance is the initial cost of the put option. The fact that the put option is cheap relative to the value of the portfolio reveals that the risk is not of high order. Investors and households have other more important things they need to worry about than this source of risk.

The second way to interpret the calculations is as a trial run for the cost of a implemented system in which the government allows individuals to invest but requires them to buy insurance on their retirement portfolios so that the government is not left holding the bag, forced by simple humanity to compensate after the fact those who took extravagant risks that turned out badly.

Some argue that the solvency of our Social Security system can only be guaranteed by the government acting as a lender of last resort. They use this argument to conclude that the Social Security system should never be privatized. However, insuring retirement portfolios against the permanent collapse of the stock market is the same as insuring against a low-probability catastrophic event. Potential losses associated with this collapse have a risk profile like that associated with events like earthquakes. According to Jaffee and Russell (2006), 1/3 of all residential earthquake policies are written by private insurance companies. Why could not we let private insurance companies insure retirement portfolios? Jaffee and Russell (2006) also point out that the US property casualty industry has a carrying capacity of approximately $400 billion dollars. Hence, insuring retirement portfolios should be well within the reach of the private sector.

Setting up explicit insurance policies for long-term stock market risk requires that insurance companies engage in dynamic long-term hedging strategies to lay off the risk associated with writing the policies equivalent to the put options. We are going to consider here how a private insurance company might design the continuous time delta-hedge to provide all the necessary funds to cover the insured losses.

Consider portfolio $Z_t = [\Delta, -1]$ consisting of Δ shares of some asset with a price p_t and -1 shares of a put option written on this asset with a strike price S, where

$$\Delta = \frac{\partial Put_{jt}}{\partial P_{jt}} = N(Z_S + \sigma_j) - 1 < 0$$

The resulting portfolio is going to be an instantaneously risk-free one. This hedging strategy is very cheap. Table below illustrates the situation.

Risk-free Rate, r_f	Time to Expiration, T	Put's, Δ
.02	40	-.06
.01	40	-.17
.02	30	-.09
.01	30	-.21

Fix annual standard deviation σat 10%, the initial stock price P_{jt} at 1 and the strike price S also at 1. For a risk-free rate r_f at 2% and a time horizon T at 40 years Δ is only -.06. For a risk-free rate r_f at 1% and a time horizon T at 40 years Δ is smaller -.17. For a risk-free rate r_f at 2% and a time horizon T at 30 years Δ is higher -.09. For a risk-free rate r_f at 1% and a time horizon T at 30 years Δ is only -.21.

The ability to execute this hedging strategy hinges critically on the infrastructural strength of our financial markets. This strength was seriously tested in October of 1987, when a simultaneous sell out of a massive number of shares triggered a meltdown of the DOT system making it impossible to trade. As a result of the market crash of 1987 the Presidential Task Force on Market Mechanisms (1988) was formed. This task force produced a set of recommendations otherwise known as the Brady Commission Report. The thorough implementation of these recommendations has made future infrastructural failures a very low probability event.

Events like the multibillion dollar collapse of Long-term Capital Management and that of Amaranth Hedge Fund in 2006 are very much isolated events. Markets have been not only able to survive them, but thrive despite of them.

A market for these long-term put options does not exist yet, but it could be created once a proposed system is implemented, in which the government requires that individuals buy insurance on their retirement portfolios at the time they make their retirement contributions.

VI. CONCLUSION

Two great empirical regularities in the performance of the aggregate stock market are (a) the high average return earned by investments in equities and (b) the fact of long-run mean reversion in stock prices. Together, these mean that long-run investments in diversified portfolios of equities are both high return and low risk.

Many critics of Social Security privatization fear that retirement portfolios invested in equities are in fact overwhelmingly risky. They fear that such portfolios could not be made safe: that they could not be insured by the private sector because the level of systemic risk that they carry is, in fact, much above what an insurance company will consider unacceptable. They further argue that if the stock market slumps, then those who had invested their Social Security money in risky securities could see their funds go bankrupt—leaving the public with the choice of either watching retirees starve on the street in their old age, or picking up the costs and thus subsidizing those who had made overly risky investments.

Yet, based on two hundred years of American financial history, the probability that a well-diversified leveraged portfolio will lose a substantial part of its value and will not be able to regain it within a short period of time is very low. Hence, insuring well-diversified leveraged retirement portfolio should be cheap. How cheap? We model the situation by assuming that the insurance premium that would be charged to make a diversified portfolio of equities safe has to be equal to the price of a Black-Scholes put option written on this portfolio. For reasonable values of real risk-free interest rates, market risks, time horizons of 20 to 40 years, the price of the put is not more than 2-4% of the value of the portfolio this put is written on.

There are two different ways to interpret calculations performed in this paper. First, they can be interpreted as a measure of how concerned investors are that their retirement portfolios invested in equities will permanently collapse. The low price of the put option reveals that these concerns are not very serious.

The second way to interpret the calculations is as a trial run for a system in which the government would require that individuals buy private insurance on their retirement portfolios at the time they make their retirement contributions.

Private insurance has several advantages. First, the agents will be exposed to risks and will buy insurance against these risks according to their own degree of risk aversion. Second, insurance contracts can be structured to reduce Moral Hazard. By contrast, an explicit insurance guarantee by a government serves, in effect, as an enabler of Moral Hazard.

A combination of a high equity premium and a mean reversion in stock prices create a great potential for making our Social Security System a very safe and a productive vehicle for the retirement funds' accumulation.

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