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AN OPTIMAL STRATEGY FOR THE PRESENTATION OF PAIRED-ASSOCIATE ITEMS[1]

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This study was concerned with the implementation of certain mathematical results that give optimal ways to present stimulus items in learning experiments. This optimization theory is based on a stimulus-sampling model of a learning experiment. We assumed that this particular model of learning would describe paired-associate learning and then compared the effects of training subjects in several paired-associate experiments by the optimal strategy and by another simple presentation strategy.

Our empirical results did not support the adequacy of the assumed model of learning for representing the learning processes in these experiments. In particular, it appears that suitable models of these learning situations must incorporate some mode of representation of short-term retention.

A PERSISTENT problem in the design of instructional systems is the determination of optimal procedures for presenting instructional stimuli. There are countless ways in which items of instructional material may be arranged, and it is seldom clear which arrangement should be used. These design problems have long been of fundamental concern to educators and learning theorists, and now current research in computer-based instruction has led to a sharpening of interest in these optimal presentation procedures.

There is, of course, a variety of ways to study these design problems, ranging from strongly empirical approaches to formal mathematical modeling. Many different presentation procedures have been tried in a number of learning problems. Most previous methods, however, have been empirically guided. We report here the implementation of formal mathematical methods for modeling and optimizing learning processes in paired-associate learning experiments.

Recent developments in mathematical learning theory (see, for example, Atkinson and Crothers, 1964; Bower, 1961) have resulted in a number of alternative mathematical models of paired-associate learning. Other recent developments in mathematical programming theory (Karush and Dear, 1966) have indicated how to solve for optimal presentation procedures when certain mathematical models of paired-associate learning are assumed. Related work in mathematical programming has also been carried on by Matheson (1964) and Smallwood (1962).

The advantage of the mathematically derived presentation strategy is that the mathematics guarantees the optimum of the stimulus presentation strategy *if* the model of learning is an adequate representation. Our study was carried out to determine whether a mathematical model of a paired-associate experiment represents the learning process adequately, so that the theoretically optimal item presentation procedure will be demonstrably better than a simple, standard presentation procedure.

STIMULUS PRESENTATION STRATEGIES

We shall be considering paired-associate experiments involving m stimulus items, say $I_1, I_2, \cdots, I_m$. An experimenter's stimulus presentation strategy is a complete prescription indicating which of the available items

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is to be presented at every trial in the experiment, given the histories of item presentations and corresponding responses that precede these choice points. A strategy may be illustrated graphically as a tree structure. In Figure 1, a possible complete prescription or strategy for a two-trial experiment involving two stimulus items is illustrated.

To characterize an optimal stimulus presentation strategy, it is necessary to associate a numerical-valued index of worth with each possible strategy. In statistical decision theory, a certain expected loss or risk has been found particularly useful as such an index. The basic losses are expressed for our paired-associate experiments in terms of the state, either conditioned or unconditioned, of each of the stimulus items at the end of the experiment.

To summarize the mathematical results of Karush and Dear (1966), let C_i $(i = 1, 2, \cdots, m)$ represent the state in which the i^{th} item in a stimulus set is conditioned at the end of the experiment, and $\bar{C}_i$ represent the corresponding unconditioned state for the i^{th} item. At the end of the experiment we shall be concerned with the numbers of the m items that remain unconditioned. A numerical-valued loss is associated with the terminal event in which exactly k items of the m are unconditioned. Let b_k be the loss associated with this event, and further impose the reasonable constraints that

$$0 = b_0 < b_1 \leq b_2 \leq \cdots \leq b_m.$$

It will be seen that under the model of learning we assume these terminal events are not observable. However, the model does allow the determination of the probability of each of these events for each path in a strategy tree. Let $\underset{\sim}{x}$ stand for a path of a strategy tree, and define $p(k \mid \underset{\sim}{x})$ as the probability that exactly k items will remain unconditioned at the termination of the experiment, given that a subject has followed path $\underset{\sim}{x}$ in the tree of the experimenter's presentation strategy. An expected loss may then be defined for this path in the strategy tree by averaging the terminal losses over these conditional probabilities of the basic terminal events, that is $\sum_k b_k p(k \mid \underset{\sim}{x})$.

The risk of a strategy is then expressed as the expectation over all paths of a strategy tree of these terminal expected losses. For the moment, we shall assume that the probabilities of subjects following the various paths depend on some set or vector of parameters, say $\underset{\sim}{\omega}$. These path probabilities will be designated $p(\underset{\sim}{x} \mid)\underset{\sim}{\omega}$. A stimulus presentation strategy for an N-trial experiment will then be denoted by S_N, and the risk of the

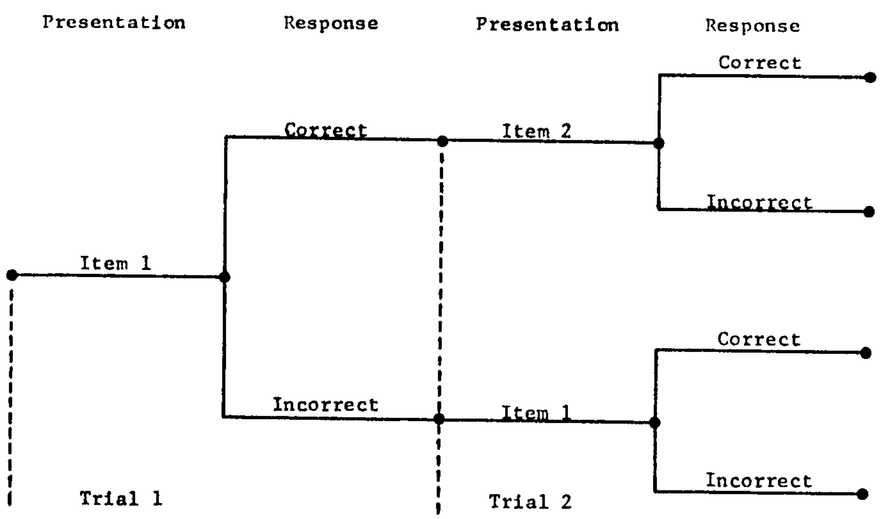

FIG. 1. Tree of a Representative Two-Trial Stimulus Presentation Strategy.

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strategy is defined as the following expected loss:

$$R(\underset{\sim}{\omega}; S_N) = \sum_x \sum_k p(\underset{\sim}{x} \mid \underset{\sim}{\omega}) b_k p(k \mid \underset{\sim}{x}).$$

Thus, to be able to compare strategies in terms of risk as an index of worth, the terms on the right-hand side of this expression must be well defined.

MODEL OF LEARNING

The loss coefficients, b_k, must be supplied by the experimenter to express his teaching objective. The probabilities $p(\underset{\sim}{x} \mid \underset{\sim}{\omega})$ and $p(k \mid \underset{\sim}{x})$ will typically be supplied by assuming that some model of learning will appropriately generate them. In our experiments, we assumed that the learning process for each item in a stimulus set was described by a well-known model of the stimulus-sampling theory of learning, known as the one-element model. We also assumed that the collection of learning processes for a set of items proceeds in a statistically independent fashion.

In the one-element model, a subject is assumed to be in one of two conditioning states with respect to a stimulus item. These are the conditioned state C and the unconditioned state $\tilde{C}$, which we introduced earlier. A subject is said to have learned to respond to an item correctly when he moves from the unconditioned to the conditioned state. When S is in the unconditioned state, it is assumed that he may guess the correct response with a certain probability. These states are assumed to be unobservable. The probabilities of the observable responses of a subject to a stimulus item at a given trial are assumed to depend directly on his state of conditioning at the trial.

In this model, only three parameters are used to generate the probabilities of all conditioning-state and response events that obtain for each stimulus item. These parameters are a guessing probability γ, an initial probability of being in the conditioned state λ^0, and a probability of transition from the unconditioned to the conditioned state θ.

In the model, the conditioning-state process is represented as a first-order Markov chain. Thus, considering the stimulus item I_j, there is a vector of initial or a priori probabilities for its conditioning states:

$$\begin{bmatrix} \lambda_j^0 \\ 1 - \lambda_j^0 \end{bmatrix} = \begin{bmatrix} p(C_j \text{ at the outset of trial } 1) \\ p(\tilde{C}_j \text{ at the outset of trial } 1) \end{bmatrix}.$$

On trials in which I_j is presented and the responses to it are reinforced, the following transition probability matrix is applied:

$$\begin{array}{ll} & \text{state at trial } t+1 \\ & \quad\; C \qquad \tilde{C} \\ \begin{array}{l}\text{state at} \\ \text{trial } t\end{array} & \begin{array}{c} C \\ \tilde{C} \end{array} \begin{bmatrix} 1 & 0 \\ \theta & 1-\theta \end{bmatrix}, \end{array}$$

$$\text{where} \quad 0 < \theta \leqq 1.$$

On trials in which an item is not presented, or presented but the responses are not reinforced, it is assumed that no changes among states take place.

It is important to note that the form of the above transition matrix, along with the assumptions that no changes occur on nonreinforced trials, implies that the conditioned state for an item is an absorbing one. Roughly speaking, this means that once a subject is conditioned to give the correct response to an item, he will not forget that response.

In our experiments, we considered that the set of responses available to a subject would always be partitioned into two subsets; the correct response and the incorrect response. The probabilities that a subject would make the correct or incorrect response to item I_j at trial t, given his conditioning states at the trial, are assumed to take the following values:

$$p(\text{Correct to } I_j \text{ at trial } t \mid C_j \text{ at trial } t) = 1$$
$$p(\text{Correct to } I_j \text{ at trial } t \mid \tilde{C}_j \text{ at trial } t) = \gamma.$$

The assumption that conditioning proceeds independently for the various items in a set, means that the joint probabilities of the various events which may obtain for the set can be expressed as the product of the probabilities of conditioning for the individual items in the set. For example, the probability that all items in a stimulus set are in the conditioned state at the outset of

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the experiment is given as the product of the a priori probabilities of conditioning for the individual items, that is,

$$p((C_1, C_2, \cdots, C_m) \text{ at trial } 1) = \prod_{j=1}^{m} \lambda_j^0 .$$

OPTIMAL STIMULUS PRESENTATION STRATEGIES

When the loss coefficients and the path and terminal-state probabilities are specified for each presentation strategy in the set of all possible strategies, optimal stimulus presentation strategies are then defined as those in the set that have minimum associated risk. Since there is only a finite number of possible stimulus presentation strategies for experiments involving a finite number of trials, one might assume that with the aid of a large, high-speed computer a variety of enumerative techniques could be used to determine optimal strategies.

It should be apparent, however, that the representation of the complete tree of a given arbitrary strategy in a computer program is infeasible when the number of stimulus items, m, or the number of trials in the experiment, N, is even moderately large. Furthermore, when the risk of each strategy is specified, it will usually be impracticable to solve for an optimal strategy by a sheer enumeration of the risks of the various strategies. There are m^{2^N-1} distinct strategies that would need to be considered by such enumeration procedures.

One would hope that an optimal strategy could be expressed as a simple rule, which would then generate the complete tree of the strategy. Karush and Dear (1966) have shown that, for certain stimulus-sampling models of paired-associate learning, an optimal stimulus presentation strategy may be stated as just such a simple generative rule. The values underlying a simple rule that will generate an optimal strategy are the a priori and a posteriori probabilities that a subject is conditioned on the various items at each trial of the experiment. These probabilities may be determined recursively by the application of Bayes' theorem.

The basic result of Karush and Dear, used in the design of our experiments, is their Theorem I. This theorem may be expressed for paired-associate experiments based on the previously described one-element structure as follows:

Consider an N-trial paired-associate experiment with the one-element stimulus-sampling structure, $N \geq 1$, with arbitrary initial probabilities of being conditioned on the m stimulus items $(\lambda_1^0, \lambda_2^0, \cdots, \lambda_m^0)$, but identical learning rate value θ and identical guessing probability value γ associated with each item. Then an optimal presentation strategy is generated by the rule that at any trial, the stimulus item among the set I_1, $I_2, \cdots, I_m$ that has the smallest current a posteriori probability of being conditioned should be presented.

EXPERIMENTS

Description

We conducted two main experiments, in which special versions of the optimal presentation strategy were compared with another presentation strategy that is frequently employed in paired-associate experiments. This latter procedure involves an initial arrangement of the set of items into an ordered list. The initial ordering is determined by a sampling-without-replacement procedure. Then, throughout the experiment, the items are presented by simply cycling and recycling through the list, whether a subject makes correct or incorrect responses to the items.

The criterion that was used to characterize an optimal strategy deals with the acquisition phase of learning experiments. Consequently, we planned several analyses of the data of our experiments to examine differential effects of the two presentation strategies in terms of characteristics of the acquisition data. However, since the model assumed for these experiments asserts that on nonreinforced trials no changes occur in the conditioning-state probabilities, and hence also no changes in the response probabilities, it was also considered desirable to run a series of posttraining, nonreinforced "test" trials. Thus we were able to compare the effects of the alternative stimulus presentation strategies in terms of both acquisition and posttraining data.

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Subjects and Materials

Eighty-one freshmen and sophomores from the University of California at Los Angeles served as subjects. They were paid for participation in the experiment.

A stimulus set of 32 two-digit integers, selected from a table of numbers containing mean-rated association values (see Battig and Spera, 1962), was partitioned into two subsets of 16 items. Each contained items of approximately equal association value. The set was partitioned into many different pairs of subsets so that an individual item appeared with approximately equal frequency as standard or optimal. The responses were alphabetic letters (that is, a, b, c, d). The correct alternatives for the items in the set were assigned to different response positions for each subject, but the correct options were assigned to each response position (a, b, c, d) with equal frequency.

We attempted to ensure the validity of the assumption of homogeneous learning rates (common value of the parameter θ across items in the stimulus sets) by choosing numbers with "known" association values and then forming reasonably homogeneous sets of items for presentation by the optimal and standard strategy. We attempted to ensure the validity of the common value of the guessing probability γ across items by simply providing the same number of response alternatives for all items in an experiment.

Equipment

The experiment was conducted in SDC's Computer-Based Laboratory for Automated School Systems (CLASS). Decimal numbers were presented on a digital display device under control of the computer. Multiple-choice responses were made by button action on the same device. After the responses, the correct answer was indicated by a light opposite the correct letter. Students were given directions in the use of the equipment before the experiment.

Procedures

One group of 44 subjects received training with only two response options available for each item in their presentation sets. The second group of 37 students had the full set of four response options available for each item presented. Since many experiments based on the one-element stimulus sampling model have used only two alternative responses to each item, we chose to do one of the experiments in this study involving four alternative responses as well as one involving two alternatives, to determine if the model's predictive ability is tied, perhaps fortuitously, to the number of response alternatives allowed.

The optimal stimulus presentation strategy used in our experiments was based on certain simple counts of numbers of correct responses to the items in a presentation set. More explicitly, we implemented an optimal strategy by following these rules for each individual subject:

(1) Administer any item in a presentation set to the subject at the first trial.

(2) At the next trial after a subject's incorrect response to an item, present that item again to him.

(3) At the next trial after a subject's correct response to the current presentation, present to him the item to which he has made the smallest number of correct responses following his last incorrect response to the item.

(4) If several items are eligible under rule (3), select from these the item that has had the smallest number of presentations. If several items are still eligible under this condition, select with equal probability from this set.

It may be verified by reference to Karush and Dear's Theorem I that the above rules generate *an* optimal presentation strategy (not *the* optimal presentation strategy, as it is readily shown that many different optimal strategies are derivable from application of the fundamental rule). However, the verification of the optimum of the strategy generated by the four rules above assumes that the probability of being in the conditioned state is zero for each item in a presentation set at the outset of the experiment.

Other rules, in addition to those dictated by this theory, were employed to determine

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our complete stimulus presentation procedures. The one-element model assumes that response probabilities for any item depend explicitly on its current state of conditioning in a subject and not on his previous responses to the item. There is definite evidence (Hellyer, 1962) that this assumption is untenable for paired-associate experiments when an item is allowed to be presented on immediately successive trials. Consequently, to avoid immediately repeated presentations of an item, we forced separations of several trials between presentations of individual items.

The separation was accomplished by first dividing the optimal presentation set of items into two subsets with equal numbers. A similar division of the standard presentation set was also made. Then on trial numbers 1, 5, 9, and so on, presentations were made from one of the standard subsets in accordance with the standard strategy applied to that subset. Items were selected from the first optimal presentation subset in accordance with that strategy for presentation on trial numbers 2, 6, 10, and so forth. In a similar fashion, presentations from the second standard subset were made on trial numbers 3, 7, 11, and so on, and from the second optimal subset on trial numbers 4, 8, 12, and so on. This procedure thus ensured that no item in any of the presentation subsets could be given to a subject and then shown again with less than three other items intervening between the two presentations.

Data from a preliminary experiment were used to estimate θ; as the result of these calculations, we determined that 20 cycles through the subsets of items presented by the standard strategy should be used (or a total of 640 training trials per subject). As soon as a subject completed his training trials he was immediately started on the nonreinforced test trials. In these trials the items in the optimal and standard presentation sets were merged in random order. Thus, one cycle through the test involved the presentation of the complete set of 32 items. Each student completed three test cycles.

RESULTS

Our data-recording and -reduction programs enabled us to examine a number of characteristics of the data from the acquisition trials and from the posttraining test trials. These two types of analyses provide complementary information for evaluating the adequacy of the learning model that was used.

Analyses of data from the acquisition trials

Several analyses of the acquisition-trial data for the two experiments are reported below. The analyses discussed in this section were carried out assuming the validity of the specified stimulus-sampling model of paired-associate learning. By contrast, the analyses of the posttraining data reported in the next section involved no calculations that explicitly use this model.

Numbers of item presentations. Each of these items in the set of 16 presented under the standard strategy was displayed to a student 20 times. Since the number of times any item was presented under the optimal strategy was a random phenomenon depending on the responses of the student to the item, it was of interest to tabulate the frequency distribution of item presentations.

Furthermore, since we made the conventional assumption that the guessing probabilities are equal to the number of response alternatives, for most purposes, the statistics that constitute a sufficient reduction of the sequence of responses to any item are known to be the pair of values: (1) the number of times or trials that the item was presented to a subject, and (2) the trial number of his last error. Because different items were presented at the 640 trials of the acquisition phase, it seemed clearer to refer to these two components of the sufficient reduction of the response sequences respectively as: (1) the number of times that the item was presented, and (2) the number of consecutive correct responses that a subject made after the last error in his response sequence to the given item.

The two-way frequency tabulation for the acquisition-trial data of the two-response-alternative experiment is given in Table 1.

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TABLE 1

TWO-RESPONSE-ALTERNATIVE EXPERIMENT

Frequency Counts over Items for Acquisition Trials Summed over Subjects

Numbers of Item Presentations	Number of Consecutive Terminal Correct Responses: 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Row Totals
Optimal Presentation Strategy																						
0																						
1																						
2																						
3																						
4					2																	2
5					2	5																7
6					2	2	3															7
7					2		2	4														8
8		1			4		1	3	1													10
9					4	4	1	4		5												18
10				1	2	2	2	1			4											12
11	2				4	5	6	5			4	2										28
12					3	4	4	4		3	4		2									24
13					5	3	3	10		2	2	4	3	2								34
14				1	4	7	4	2		7	3	1	1	1	1							32
15					4	6	7	7		1	3	4	4	1		3						40
16	1				5	7	5	4	1	4	2	1	4	1	2	1						38
17		1		1	7	5	2	6	1	5	3		3	3		2		3				42
18	1			2	1	2	2	5	1	3	5		3			5			2			32
19		1		2	1	7	6	2	1	4	2	2	3			2						33
20	3	1		4	4	6	4	6	1	2	1		4	2		3		1				42
21	1	1		1	4	3	5	6	1	2	3	2	1	2	1							33
22			1	2	3	5	4		2	4	2		1		1	1						26
23				1	7	2	7	2		4	2	1				1						27
24	1			3	2	4	4	4	1	1	1		1									22
25	1			2	3	4	4		1	1	1		1	1	2			1				22
26	2				5	5	2	1		1	1	2	1									20
27	1				2	2		1				2	2									10
28		3		2	3	1		1	2	2	1		1		1			1				18
29	1			3	2	2	1	2	2	1	1	1	2									18
30 or More	4	2	3	9	16	18	20	1	5	10	5	2	2	1	1							99
Column Total	18	10	4	34	103	111	99	81	20	62	50	24	39	14	9	18	0	6	2	0	0	704
Standard Presentation Strategy																						
20	141	84	57	48	24	35	19	23	17	24	22	29	31	18	30	18	16	17	23	13	15	704

The upper portion of the table lists these frequencies for the items presented, using the optimal strategy. This subtable has been truncated at 30 or more item presentations for convenient display of results. As shown in the table, the smallest number of times that an item was presented was 4, while the largest was 53.

The last row in Table 1 gives frequency tabulations for the items that were presented according to the standard strategy. Since each item in the sets of 16 was presented 20 times when this strategy was employed, there is only a single row in the frequency table for the standard presentation strategy. Note particularly that there were 141 cases of items that terminated with incorrect responses under the standard presentation strategy while there were only 18 such cases among the items presented by the optimal

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TABLE 2
FOUR-RESPONSE-ALTERNATIVE EXPERIMENT
Frequency Counts over Items for Acquisition Trials Summed over Subjects

Numbers of Item Presentations	Numbers of Consecutive Terminal Correct Responses: 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Row Totals
	Optimal Presentation Strategy																					
0																						
1																						
2																						
3				5																		5
4				1	1																	2
5			1	2	3																	6
6					3																	3
7				1	3																	4
8				2	3	1			1													7
9			1	4	3	3	1		1	1												14
10				2	6	4	1		1	7												21
11				2	1	3		1	2			3										12
12			1	5	3		2	1	4	2	1											19
13	1			3	7	2	1	1	1	3	2											21
14			1	6	2	2	1		6		1	2										21
15	2		2	6	7	1	2	2	2	2	1	3		1	2							33
16	3		4	3	3	4	3		3		2	1		2	2							30
17			2	7	6	2		5	5	2	1				1							31
18			2	6	1	2		1	3			1		1	2			3	3			25
19		1	5	6	6	5	1	4	3	2	1	5			1				3			44
20	1		1	7	1	3	1		2	5					2			1				24
21		1		7	3	1	4	4	2	3		1		1	2			2				31
22	1		3	5	3	5	5	1	4	2				1				1				31
23			9	7	2	6	1		5	2		1	1					2				36
24		1	1	5	4	4	1	3	5	1				2								27
25			7	4	3	1		2	2	1		2		1				1				24
26	2	2	3	4	1	2			1	1	1											17
27	1	1	4	1	3	1		2	1			1										15
28	2		4	6	1	2	1	1	2													19
29	1	2	4	4	2	1							1									15
30 or More	2	3	7	17	8	1	4	2	6	2	1	2										55
Column Total	16	12	62	128	89	56	29	30	62	36	11	22	2	9	12	0	0	10	6	0	0	592
	Standard Presentation Strategy																					
20	194	58	35	18	21	38	14	27	16	14	18	15	18	22	18	21	13	9	8	11	4	592

strategy. Since there were 44 students involved in the two-response-alternative experiment and 16 items in the optimal set and the standard set, the two frequency tables of Table 1 each total 704 cases.

The analogous two-way frequency table of the acquisition data for the four-response-alternative experiment is shown in Table 2. Again, the tabulation of numbers of item presentations is pooled for 30 or more presentations. The number of times that items were presented in this experiment ranged from a minimum of 3 to a maximum of 56.

One notes again that the optimal strategy resulted in very few cases of items whose presentation sequences terminated in incorrect responses (16 instances), while the standard presentation strategy resulted in 194 such cases. Finally, the data of Table 2

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were obtained from the acquisition trials for 37 students; thus the frequencies in each of these tables total 592.

Posterior probabilities of being conditioned. The data in Tables 1 and 2 were used to compute for each student the posterior probability for each stimulus item that he was conditioned to the item at the end of the acquisition trials. Frequency distributions of these posterior probabilities were then tabulated for both the two- and four-response-alternative experiments. These empirical distributions were formed by grouping the probability values into ten equal categories or subintervals of the unit interval.

The basic derivations of the formulas for computing a posteriori probabilities of conditioning to stimulus items under the one-element learning model will be found in Dear (1964). For the present analyses, we used the particular formula that results when the initial probability of being conditioned to an item, λ^0, is assumed to be equal to zero, and the guessing probability parameter γ is also assumed to be known precisely (here we assumed $\gamma = \frac{1}{2}$ or $= \frac{1}{4}$). The learning-rate parameter θ was assumed to be distributed a priori according to the beta distribution with the two parameters η_θ and ξ_θ. Using the result that the Bayes estimate of θ, which we obtained from the data of preliminary experimentation, was approximately equal to .1, we set $\eta_\theta = 1$, and $\xi_\theta = 9$ to give an a priori distribution of θ with mean $\mu = \eta_\theta/(\eta_\theta + \xi_\theta) = .1$.

It may readily be shown that the posterior probability of a student's conditioning to a representative item at the end of M reinforced presentations with his last error occurring at presentation t^0 is:

$$p(C \text{ after presentation } M \mid t^0) = \frac{\sum_{t=t^0+1}^{M} \gamma^{t-T-1} B(\eta_\theta + 1, \xi_\theta + t - 2)}{\left[B(\eta_\theta, \xi_\theta + M - 1) + \sum_{t=t^0+1}^{M} \gamma^{t-T-1} B(\eta_\theta + 1, \xi_\theta + t - 2) \right]},$$

where terms such as $B(\eta_\theta, \xi_\theta + M - 1)$ refer to complete beta functions with integer-valued arguments. (Recall that for such arguments $B(\eta_\theta, \xi_\theta + M - 1) = (\eta_\theta - 1)!(\xi_\theta + M - 2)!/(\eta_\theta + \xi_\theta + M - 2)!$.)

The categorized relative frequency distributions of the posterior probabilities of being conditioned to the items in the optimal and standard sets for the two- and four-response-alternative experiments are given in Table 3. The frequency distribution of these posterior probabilities for the standard presentation strategy has the peculiar property that certain of the categories have zero frequencies. This is due to the fact that each of these items was presented exactly 20 times; hence there are only 21 possible values of the posterior probabilities, and certain of the categories used admit none of these possible values.

Since the two frequency distributions in Table 3 obviously differ with respect to this computational artifact, it would be inappropriate to perform a test of the hypothesis that the two sample distributions have come from the same multinomial population. We are more interested in comparing the relative shift of the category probabilities toward the higher-valued categories under the two presentation strategies. Lacking an adequate formulation of a statistical hypothesis concerning such a shift, we point out the substantial differences between the two strategies in the observed values of their frequencies for the lowest two or three cate-

TABLE 3
RELATIVE FREQUENCY DISTRIBUTIONS OF A POSTERIORI PROBABILITIES

Probability Categories	Two-Response Alternatives		Four-Response Alternatives	
	Optimal Strategy	Standard Strategy	Optimal Strategy	Standard Strategy
$0 \leq p < .1$	.038	.320	.032	.328
$.1 \leq p < .2$	.007	.081	.015	.098
$.2 \leq p < .3$	.017	0	.002	0
$.3 \leq p < .4$	.028	.068	.047	0
$.4 \leq p < .5$	.033	0	.049	.059
$.5 \leq p < .6$	.048	.034	.005	0
$.6 \leq p < .7$	.082	0	.010	0
$.7 \leq p < .8$	.109	.050	.125	.030
$.8 \leq p < .9$	.131	.027	.076	0
$.9 \leq p < 1$	.506	.420	.638	.485

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gories. Although the standard presentation strategy has resulted in a large proportion of posterior probabilities in the highest category, it is quite evident from visual inspection of these distributions that those for the optimal strategy at each subinterval have shifted their mass "to the right" of the corresponding distribution for the standard presentation strategy.

Estimates of the risks of the strategies. Under the assumptions that were made for our experiments, the explicit expression for the risk of an arbitrary stimulus presentation strategy S_N is

$$R(\lambda_1^0, \lambda_2^0, \cdots, \lambda_m^0, \gamma, \theta: S_N) = \sum_x \sum_k p(\underset{\sim}{x} \mid \lambda_1^0, \lambda_2^0, \cdots, \lambda_m^0, \gamma, \theta) b_k p(k \mid \underset{\sim}{x}).$$

Thus the risk of the strategy involves computing the terminal expected loss $\sum_k b_k p(k \mid \underset{\sim}{x})$ for each path $\underset{\sim}{x}$ of the strategy tree, and then averaging these expected losses over the probability distribution of the paths.

We used to "estimate" the risk of a strategy using the sample analogue method in which the probability of each subject's path is assigned a value equal to the reciprocal of the number of subjects in the sample. These estimates were computed for the teaching objective of seeking conditioning of students on all items in the stimulus sets. For this objective, it is readily shown that the expression for the estimate of the risk of a strategy reduces to

$$\hat{R}(\lambda_1^0, \lambda_2^0, \cdots, \lambda_n^0, \gamma, \theta: S_N) = \frac{1}{s} \sum_i \left(1 - \prod_{j=1}^{n} p(C_{ij} \text{ after presentation } M_{ij} \mid t_{ij}^0)\right),$$

where the probabilities $p(C_{ij}$ after presentation $M_{ij} \mid t_{ij}^0)$ represent the posterior probability that subject i is conditioned to the j^{th} item in his presentation set after M_{ij} presentations of the item to him, resulting in his last incorrect response to this item at the t_{ij}^0 presentation.

TABLE 4
ESTIMATES OF THE RISKS OF THE STRATEGIES

Presentation Strategy	Numbers of Response Alternatives	
	2	4
Optimal	.726	.666
Standard	.947	.939

In Table 4 these estimates of the risks of the optimal and standard presentation strategy are given for the two- and the four-response-alternative experiments. It is readily shown that the risk function based on these loss coefficients can under certain conditions achieve a maximum value of 1, and for other conditions a minimum value of 0. The estimates of the risks of the two strategies given in Table 4 indicate that the optimal strategy is superior to the standard strategy, when the model is assumed to be valid. The standard strategy has resulted in only very slight reduction of risk below the maximum value of 1 in both experiments. The optimal strategy has resulted in a substantial reduction below the maximum risk in both of these conditions.

Analyses of data from the posttraining test trials

The learning model that we assumed asserts that on nonreinforced trials, such as the test trials run after the acquisition phase of the experiments, the probabilities of being conditioned, and consequently also the probabilities of correct responses, are constant throughout these trials. Thus it was of interest to compare the effects of the two strategies in terms of the distributions of certain response or performance scores on the test trials.

The basic test procedure consisted of presenting to a subject each of his two complete sets of 16 items that had been used for training under the optimal and standard presentation strategies. These two sets were intermixed in random arrangements for the test sessions. To increase the precision of our trials, we cycled three times through the combined sets of items so that the complete test session consisted of 96 trials.

We present below the results of testing a number of different null hypotheses concerning the distributions of the posttraining data.

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Since we have in several instances performed significance tests on the same samples of data without the benefit of an adequate simultaneous testing procedure, the significance levels of tests must be regarded as only nominal.

Perfect-performance scores. We chose to run these experiments for a large enough number of training trials to anticipate detectable probabilities that students would have mastered all the items in a presentation set. Consequently, we used as one criterion of performance on the test trials the Bernoulli random variable which considers the events that either a subject responded correctly to all items in a set (optimal or standard) or else he did not. We refer to the values of this random variable as a subject's perfect-performance scores.

For the null hypotheses concerning the equality of the probability that all items in an optimal set will be responded to correctly with the corresponding probability for the associated standard set, the sign test seemed particularly appropriate. This test has the advantage of accounting for statistical dependence that may exist among the pairs of the Bernoulli variables obtained for each subject.

The sample means of the posttraining perfect-performance scores for the two- and four-response-alternative experiments are listed in Table 5. Sign tests were performed to test differences between the scores for the optimal and standard sets of items. They uniformly showed no significant differences, at conventional 1% or 5% levels between the two presentation procedures across each of the three subtests for each of the two experiments.

TABLE 5
MEANS OF PERFECT-PERFORMANCE SCORES

	1st Subtest (16 Items)	2nd Subtest (16 Items)	3rd Subtest (16 Items)
Two-Response-Alternative Experiments			
Mean Perfect-Performance Scores: Optimal Set	.295	.386	.295
Mean Perfect-Performance Scores: Standard Set	.227	.318	.273
Four-Response-Alternative Experiments			
Mean Perfect-Performance Scores: Optimal Set	.243	.189	.162
Mean Perfect-Performance Scores: Standard Set	.189	.216	.270

Sum-of-correct-responses scores. The objective of maximizing the sum, over a presentation set, of the probabilities of correct responses to items in the set after training has been completed is an objective of general interest. It is applicable to experiments based on a variety of learning models. The random variable that is an appropriate criterion for comparing effects of presentation strategies under this training objective is the sum, over the items in a presentation set, of subjects' correct responses to the items in posttraining test trials.

In Table 6, means of these sum scores are listed for the subtests in the two experiments for optimal sets versus standard items. For the testing of null hypotheses to compare distributions of these sum scores for optimal versus standard sets, the Wilcoxon signed-rank test seemed a particularly suitable and conservative procedure. It turned out again that none of the tests of differences in distributions of scores for optimal sets versus standard sets were significant at conventional 1% or 5% levels. These tests were carried out for the data of each of the three subtests in both experiments and for the data pooled across the three subtests for each experiment.

TABLE 6
MEANS OF SUM SCORES

	1st Subtest (16 Items)	2nd Subtest (16 Items)	3rd Subtest (16 Items)
Two-Response-Alternative Experiment			
Mean Sum Scores: Optimal Set	13.30	13.57	13.25
Mean Sum Scores: Standard Test	13.27	13.07	13.18
Four-Response-Alternative Experiment			
Mean Sum Scores: Optimal Test	11.70	11.43	11.05
Mean Sum Scores: Standard Set	11.16	11.00	11.11

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CONCLUSIONS

The analyses of the two experiments do not support the view that the optimal strategy was a better way to present stimulus items. The several sets of analyses taken together, however, do provide some clues to the major faults in the present mathematical representation of the paired-associate experiment.

The analyses of the acquisition data indicate that the control of experimental conditions (such as choice of stimulus items and alternative responses, general instructions to the subjects, choice of population of subjects) at least reasonably approximated the assumptions of the stimulus-sampling learning model employed. We did not carry out analyses that bear on refined questions about the model—such as the validity of the assumptions of common γ and θ values for all items or the assumption that all subjects are not conditioned to any of the items at the start of the experiment. In view of the stimulus materials used and the alternative responses allowed, it seems unlikely that these assumptions fail to any serious degree.

The analyses of the data of the posttraining trials, taken in conjunction with analyses of the acquisition data, do not support the assumption of this model that no forgetting occurs on nonreinforced trials. Since the analyses of the acquisition data for both experiments support the superiority of the optimal over the standard presentation strategy, the subsequent finding of no significant differences between the two strategies in any of the posttraining performance scores suggests that some sort of forgetting process needs to be incorporated into the model.

An alternative explanation of the finding of no significant differences is that too many training trials had been run. This theory seems reasonable to us for the comparisons based on the sum-of-correct-responses scores. The fact that there were no significant differences between the two strategies in terms of the perfect-performance scores does not seem adequately explained by the possibility that the experiments were continued too long.

Furthermore, since posttraining test trials were run immediately after the acquisition trials, this experimental procedure gave the standard presentation strategy the advantage of more immediate rehearsal of all items in the standard set just prior to entering the test trials. If some sort of forgetting process is operating to an important degree, the fact of more immediate rehearsal of the entire stimulus set under the standard strategy could lead to its favorable comparison with the optimal strategy in the test trials of experiments.

A final remark should be made about the importance of optimal presentation strategies and their relevance to general research toward the improvement of instruction. It may well turn out that even if a satisfactory mathematical model of these sorts of paired-associate experiments is developed and an associated optimal presentation strategy is determined, the risk of such an optimal strategy may not be less than the risks of other simply generated presentation strategies, to any practically important extent. That is, the improvement of teaching procedures by seeking optimal ways of presenting the stimulus materials may have important practical consequences only in situations, for example, where there is a considerable degree of interdependent relations among the stimulus materials. Other experimental variables, such as the modes of reinforcement of responses or the variation of the physical characteristics of the stimulus materials, may turn out to have much greater practical importance in learning situations than the choice of stimulus presentation strategies.

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It is not accidental that all the phenomena of human life are dominated by the matter of daily bread—the oldest link connecting all living things, man included, with the surrounding nature. The food which finds its way into the organism where it undergoes certain changes—dissociates, enters into new combinations, and again dissociates—embodies the vital process, in all its fullness, from such elementary physical properties of the organism as the law of gravitation, inertia, etc., all the way to the highest manifestations of human nature.

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