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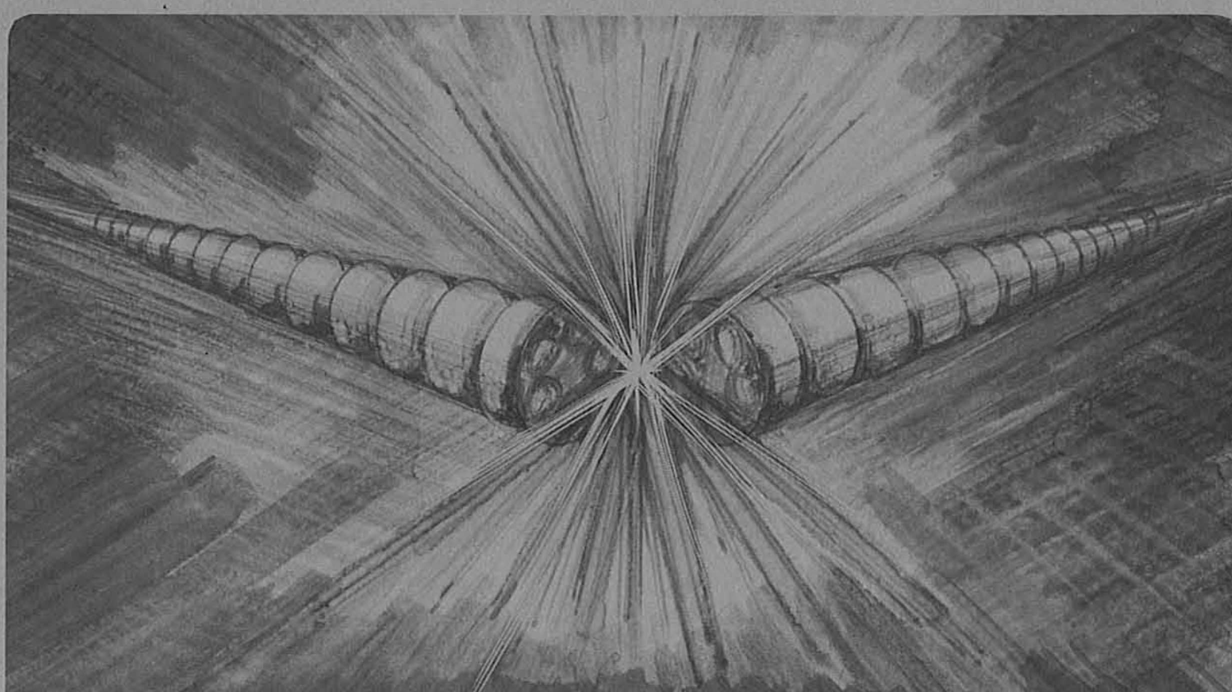

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Abstract

When the gain in a free electron laser is high enough to produce optical guiding, the resonator mode distorts and loses its forward-backward symmetry. We show that the resonator mode in a high gain FEL can be easily constructed using the mode expansion technique taken separately in the interaction and the free-space regions. We propose design strategies to achieve maximal gain and optimal mode quality, and discuss the stability of the optimized mode.

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Introduction

In a high gain free electron laser, optical guiding[1-8] modifies the spatial structures of the optical wave and breaks the forward-backward propagation symmetry within a two-mirror resonator. In this paper, we show that the optimized mode of such resonator can be easily constructed using mode expansion techniques, and we propose design strategies to achieve maximal gain and optimal mode quality. In addition, the stability of the optimized resonator mode is discussed.

In our approach the optical field is expanded in two different set of basis inside and outside of the interaction region. In the forward pass through the gain medium the optical beam profile remains nearly constant due to optical guiding, there the optical field is well characterized by a few low order guided modes. At the exit of the wiggler, the amplified wave is re-expressed in terms of the vacuum modes. These vacuum modes are chosen such that the minimum number is required in the expansion. To make a round trip within the cavity each vacuum mode component is transported through free-space sections and by mirror reflections back to the entrance of the wiggler where the combination of the vacuum modes is taken as input field to the amplifier.

It is found that maximal gain and optimal transverse mode quality can be achieved simultaneously, and the resonator can be made optimal at high gain when optical guiding occurs and yet stable at low gain when the resonator mode approaches the vacuum mode. In addition, we show that an asymmetric cavity can be used to reduce the power loading on the cavity mirrors without sacrificing either gain or mode quality of the laser.

The Amplifier Region

It has been shown in our previous work[6-8] that 3D evolution of an optical wave through the interaction region in a FEL can be described by an expansion in the guided modes. There are three types of guided modes: exponentially growing and decaying modes and oscillatory modes with constant amplitude. The growing and the decaying modes are bound and discrete. They describe energy transfer between the electrons and the optical wave. The oscillatory modes are continuous and responsible for diffraction. In the high gain regime the decaying and the oscillatory modes are negligible after amplification, and if the discrete modes are nondegenerate[7,8] only a few low order growing modes need to be considered. Therefore a truncated expansion, given by

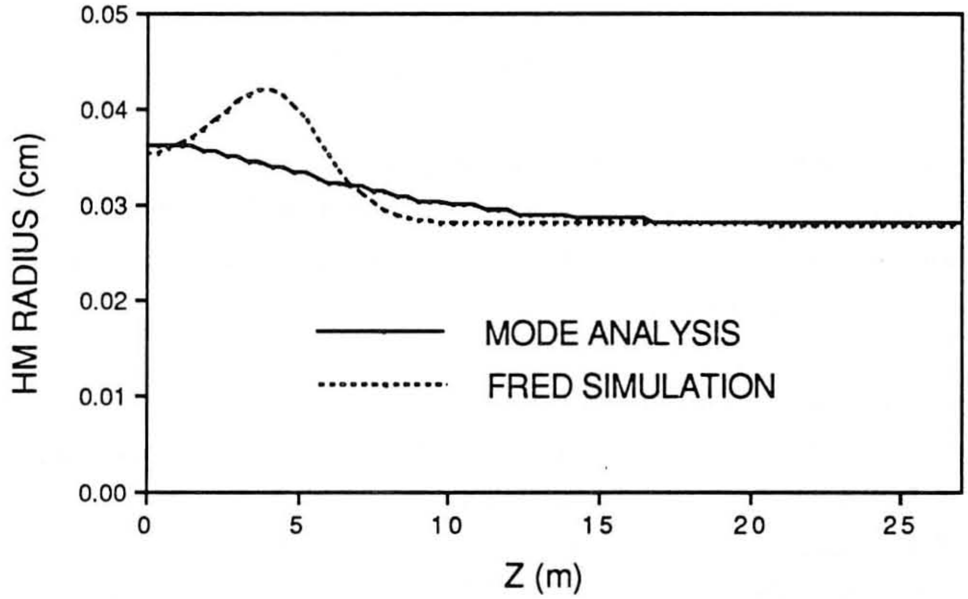
$$E(x,y,z) = \sum_{n=1}^{N} C_n E_n(x,y) \exp(-i\lambda_n z) \ , \tag{1}$$

should approximate result from a full expansion, even though discrepancies are expected near the wiggler entrance. In Eq.(1) E_n and λ_n are the transverse profile and complex propagation constant of a guided mode respectively, and C_n is the input coupling coefficient given in references[7,8].

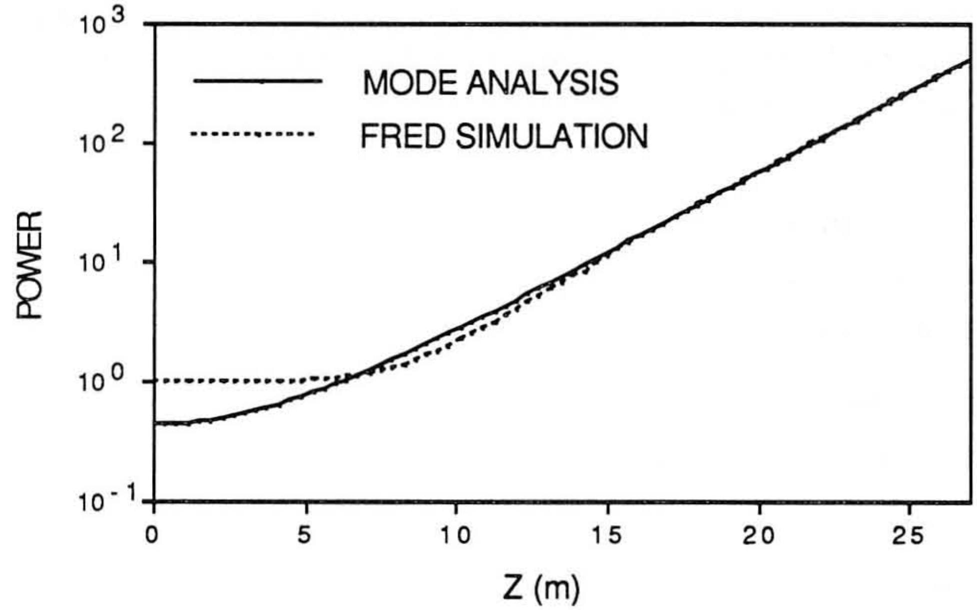
To verify the validity of the truncated expansion optical field in a high gain amplifier is evaluated with Eq.(1) and the results are compared with a 3D simulation performed with the code FRED[9]. The amplifier parameters for this comparison are given in Table 1, and the input field is taken as a Gaussian mode focused at the entrance of the wiggler with a minimum spot size of 600 micron and a Rayleigh range of 4.52 meter. For given input field the power ratio between the fundamental and the next order growing mode is about one order of magnitude at the wiggler entrance, and due to the difference in mode growth rate that ratio increases to three order of magnitude at the wiggler exit. For given parameters five growing and five decaying modes are included in the expansion, even though two growing modes are sufficient at the wiggler exit.

Table 1. Parameters used in calculations.

Electron Lorentz factor γ	2000
Electron beam current I_e (Amps)	270
Electron beam rms radius σ_e (μm)	233
Peak wiggler parameter K	5.74
Wiggler length L (m)	27
Wiggler period λ_w (cm)	11.4
Optical wavelength λ (Å)	2500
Oscillator cavity length Lc (m)	54

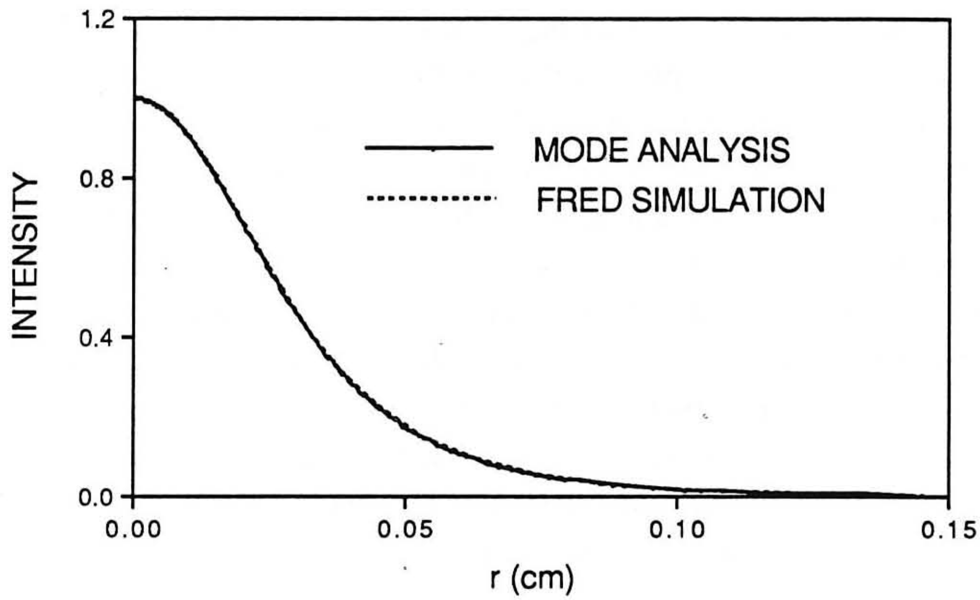


(a)

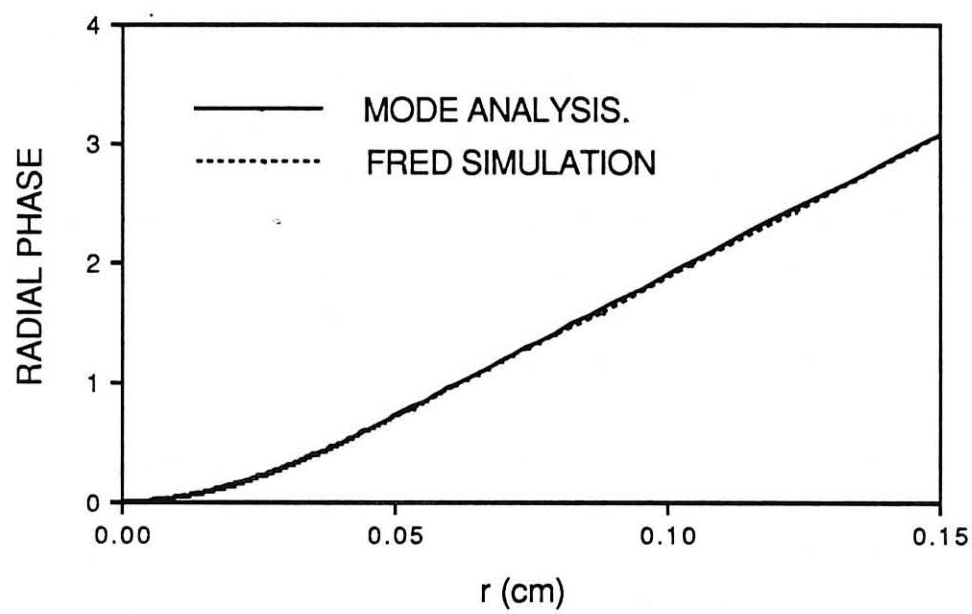


(b)

Figure 1. Half-max radius of intensity (a) and power (b) of the optical field vs. propagation distance along the wiggler.



(a)



(b)

Figure 2. Transverse profile of intensity (a) and phase (b) of the optical field at the exit of the wiggler.

The half-max radius of intensity and the power of the optical wave are plotted in Figure 1 for the 27 meter long wiggler. In the beginning region the two curves are quite different. The simulation shows the expected behavior of near free-space diffraction close to the entrance, whereas the mode expansion fails in this region because the continuous modes are neglected in the expansion (1). However, after a couple of Rayleigh ranges the power in the bound modes grows to a level where diffraction becomes negligible and the two curves converge. Only 43% of input power couples into the bound modes, the remainder goes into the continuous modes. It is noted that the power grows nearly exponentially after about 15 meter from the entrance, indicating the dominance of the fundamental mode from there on. The transverse profiles of the intensity and phase at the wiggler exit is shown in Figure 2. The two curves are almost indistinguishable. The single pass gain is 495 from the mode expansion and 499 from the simulation. These comparisons justify the truncated expansion (1) for high gain, nondegenerate FEL amplifiers.

Resonator Optimization

The major concerns in the resonator design are round-trip gain and transverse mode quality. When optical guiding occurs, the fundamental guided mode grows to dominate the optical field after each pass through the interaction region. In this situation, maximal gain and optimal transverse coherence can be achieved simultaneously by maximizing the input power coupling to the dominant growing mode. It is found[2,7,8] the maximal power coupling is reached if the input mode is a complex conjugate of the fundamental growing mode. This means in particular that the input mode should have a converging phase front, opposite to that of the dominant growing mode.

To construct resonator mode it is convenient to treat the different regions inside the cavity separately. In the forward pass through the interaction region optical field can be expressed by the truncated expansion in the guided modes. Outside of the interaction region and in the whole return pass, propagation occurs in free space where the appropriate modes for the expansion are the well-known Laguerre-Gaussian modes. The transport of these modes through a series of free space drift sections and mirrors outside the gain medium can be followed easily with an ABCD matrix. To make a closed loop in the cavity the mode expansion used to express the field has to be changed at two locations, the entrance and the exit of the wiggler.

There are two free parameters associated with Laguerre-Gaussian modes, the mode waist and the Rayleigh range, or equivalently the spot size and the phase front curvature at any location. These parameters, though may in principal be chosen quite arbitrarily, significantly affect the number of modes required in the expansion. The most convenient and physically meaningful choice is the one which maximize the power coupled from the amplified field at the wiggler exit into the fundamental Gaussian mode. This choice reduces the number of free space modes required in the expansion to a minimum. Following this approach for parameters given in Table 1 we found about 95% of the total power in the fundamental mode and over 99% in 5 low order modes.

Ideally, an optimal resonator should transport the field at the wiggler exit to it's complex conjugate at the wiggler entrance, assuming the fundamental guided mode is dominant after amplification. To rigorously accomplish this task the resonator mirrors would have to be aberrated due to the aspherical phase front curvature of the fundamental guided mode. This is impractical because, in addition to high manufacturing cost, the profile of the fundamental guided mode itself may not be well-determined due to various operational uncertainties in a real system.

Even though the fundamental guided mode is not exactly Gaussian, it is usually close to Gaussian, and the discrepancies occur mostly in the wings where in most cases the field is distorted or vignetted due to the finite apertures of real optics systems. Therefore the conjugate input coupling condition can be approximated by requiring the input field to be a complex conjugate of the dominant Gaussian mode in the free space expansion at the wiggler exit . The new optimal resonator condition can be easily satisfied with spherical cavity mirrors.

Denote by q_1 and q_2 the complex beam parameters[10] of the dominant free-space Gaussian mode at the exit and the entrance of the wiggler respectively. The conjugate input coupling condition requires:

$$q_2 = - q_1^* \, , \tag{2}$$

note

$$\frac{1}{q_1} = \frac{1}{R} + i\frac{\lambda}{\pi w^2} \, , \tag{3}$$

where R, a positive quantity, and w are the phase front curvature and spot size of the Gaussian mode at the wiggler exit respectively.

The beam parameters q_1 and q_2 are also related by a transport matrix M as follows

$$q_2 = \frac{A + B\,q_1}{C + D\,q_1} \tag{4}$$

where A, B, C and D are matrix elements defined by

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}. \tag{5}$$

The matrix M transports the mode from the wiggler exit through a series of free-space sections and mirror reflections back to the wiggler entrance and can be expressed as a product of the matrices of these elements:

$$M = \begin{bmatrix} 1 & L_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & L_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix}, \tag{6}$$

where L is the wiggler length, L_1 is the separation between the wiggler exit and the downstream mirror, L_2 is the separation between the upstream mirror and the wiggler entrance, L_c is the cavity length, f_1 and f_2 are the focal lengths of the downstream and upstream mirror respectively. Given wiggler length L, four out of five quantities L_1, L_c, L_2, f_1 and f_2 are independent because of a constraint $L_c = L_1 + L + L_2$. In a typical resonator the length parameters L_1, L_2 and L_c are usually fixed, leaving only the two mirror focal lengths as free variables. Substituting the expressions for the matrix elements into Eq.(4), equating the real and imaginary parts with the aid of Eqs.(2), (3), one derives after some rearrangement the following conditions for attainment of optimal condition in the resonator:

$$c_1 f_1^2 + c_2 f_1 + c_3 = 0, \tag{7}$$

$$d_1 f_1 + d_2 f_2 + d_3 = 0, \tag{8}$$

where the coefficients are functions of resonator length parameters, and the mode parameters, w and R. Equation (7) and (8) determine the optimal choice for the focal lengths of two mirrors.

In summary, the design procedure includes the following steps:

(a): Start with an input field to the amplifier, preferably the complex conjugate of the fundamental growing guided mode, calculate the expansion coefficients according to the formula given in references[7,8] for each guided modes in the expansion. The amplified field is then determined by Eq.(1).

(b): Calculate the power coupling from the amplified field at the wiggler exit given in step (a) to a fundamental Gaussian mode with mode parameters w and R, and vary w and R to maximize the power coupling. The mode parameters for the free-space mode expansion are then determined.

(c): With the mode parameters given in step (b), the mirror focal lengths can be solved from Eqs.(7),(8) for given length parameters L_1, L_2 and L_c. This step gives the ABCD matrix through Eq.(6).

(d): Expand the amplified field in free space modes with mode parameters determined in step (b), and propagate the field with the ABCD matrix given in step (c) back to the entrance. This step closes the loop and also prepares the input field to start the next round-trip.

To reach a stable configuration these steps have to be iterated. If the fundamental mode is strongly dominant the profile of the amplified field and hence the transport matrix are quite insensitive to the input field even though the exit power is strongly dependent on the input field. In this case only a few iterations are necessary for even poor initial input field.

Resonator Stability

There are in general two solutions to the quadratic equation (7). Both of them correspond to resonator configurations satisfying the conjugate input coupling requirement (2), thus give the same round-trip gain and optical mode. At steady state operation before saturation the two configurations should be equivalent. However, under certain circumstances where the gain is not as high optical guiding will be affected and one may expect the mode to approach that of a cold resonator. In fact it is shown in simulations[11] that only the fundamental cold cavity mode remains at saturation. In this situation the two configurations may not be equivalent from the standpoint of stability.

The stability of a cold two-mirror resonator is well described by a so-called stability diagram in which each possible resonator configuration corresponds to a point in a two-dimensional space. A resonator is considered stable if it's parameters fall into certain regions. The stability diagram is most commonly plotted in a coordinate system of "resonator g parameters" defined by $g_1 = 1 - L_c / 2f_1$, and $g_2 = 1 - L_c / 2f_2$.

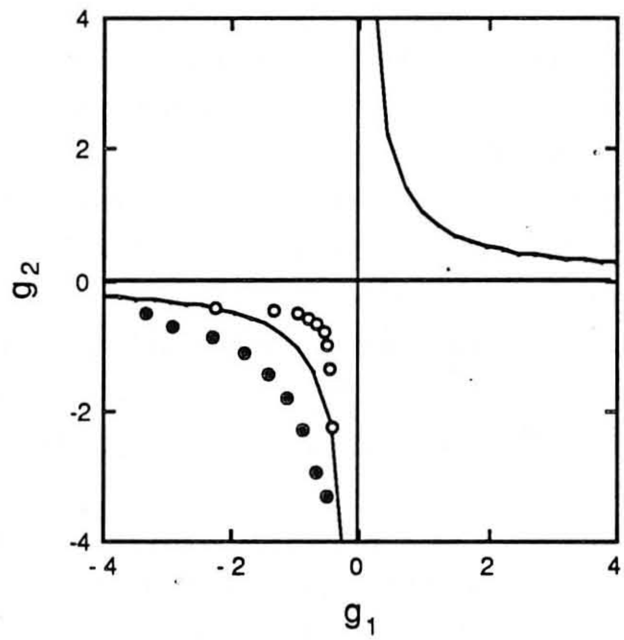


Figure 3. Resonator stability diagram.

Keeping L_c at 54 meter and L at 27 meter while varying L_1 and L_2 under the constraint $L_c = L_1 + L + L_2$, f_1 and f_2 are solved from Eqs.(7),(8) and plotted in a stability diagram in Figure 3. The open circles located in the stable region represent solutions from one branch and filled circles located in the unstable region represent solutions from another branch. Note the two branches are symmetric about the +45° diagonal through the origin in the g plane.

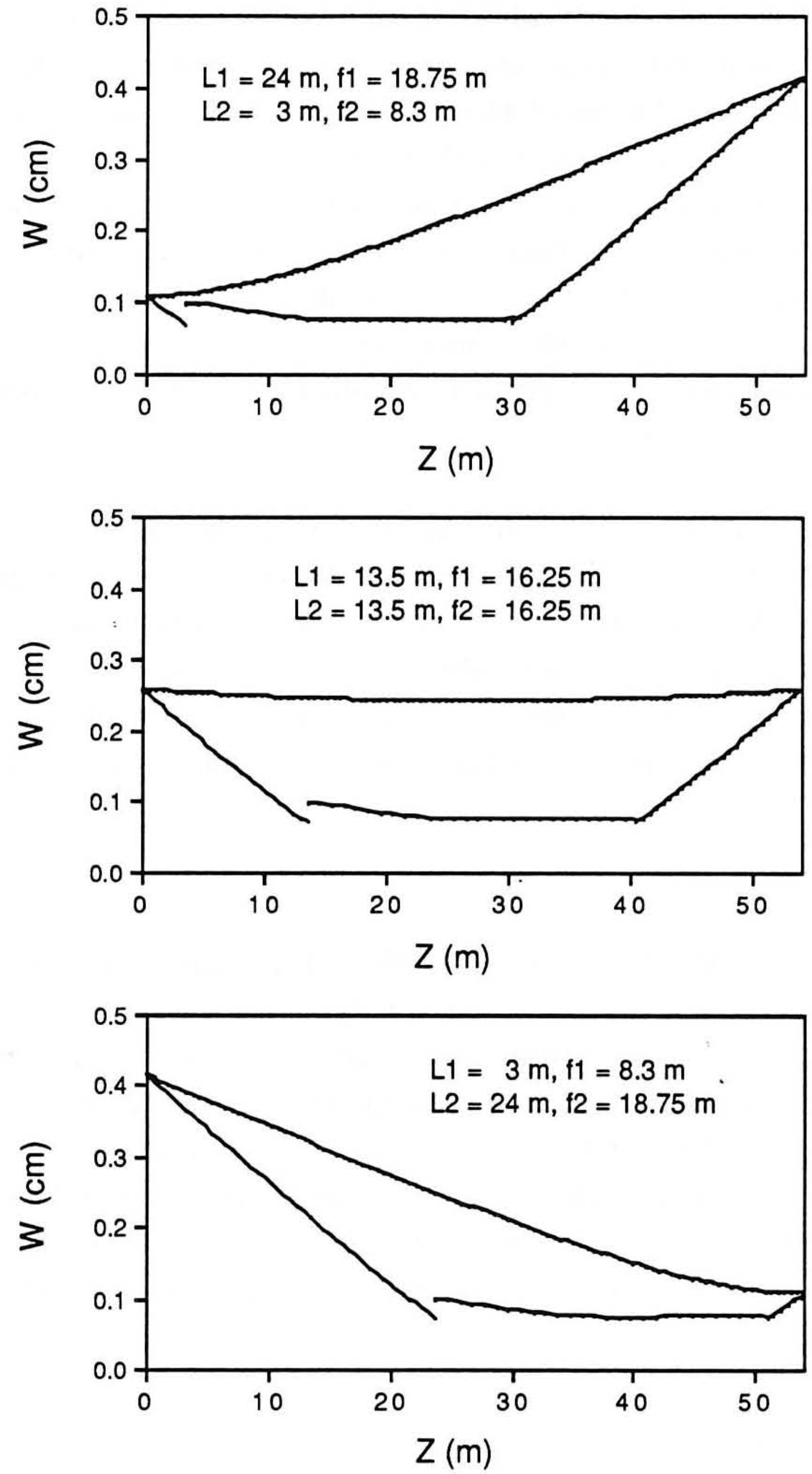


Figure 4. Mode size in round-trip for three stable resonator configurations.

Figure 4 plots the half-max radius of intensity within the 54 meter space between the two end mirrors in both forward and backward passes for three configurations from the stable branch of the solutions. Of the three configurations chosen, two correspond to the outmost asymmetrical points ($g_1 \neq g_2$) and one corresponds to the symmetrical point ($g_1 = g_2$) in the stability diagram. The half-max radius in the three free-space sections are taken as that of the dominant Gaussian mode. Note the features of diffraction in free space as well as the focussing by the two cavity mirrors. In the forward pass through the 27 meter wiggler the half-max radius evolves into a constant value, similar to that shown in Figure 1(a), the discontinuity at the wiggler entrance is due to the neglect of the continuous modes in the truncated expansion (1).

It should be emphasized that all solutions of Eqs.(7),(8), inside or outside the stable region, symmetrical ($L_1 = L_2$, $f_1 = f_2$) or asymmetrical ($L_1 \neq L_2$, $f_1 \neq f_2$), are optimal designs since they offer the same round-trip gain and optical mode. However, it is preferable to choose the one which is also stable at low gain. In addition, the asymmetrical designs allow the downstream mirror to be placed further away from the wiggler exit and are therefore favorable choices for reduction of power loading on the cavity mirrors.

Conclusions

We have presented design strategies for high gain resonator taking optical guiding into account. Our approach is based on the mode analysis and the optimization procedure which maximize the gain in the fundamental guided mode. This approach emphasizes the dominant physical process inside and outside of the interaction regions, and assures optimal mode quality. Our results are valid in the small signal regime. In the saturated regime efficiency, in addition to gain and mode quality, has to be considered in the resonator optimization. Due to strong nonlinearity, studies in this regime so far depend largely on simulations. A simple and effective design guideline remains yet to be addressed.

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